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Optimal extraction of water from a groundwater system with two linked aquifers

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With the increase in cost of surface water, groundwater extraction for irrigation has become more competitive. Due to the interdependence of different water resources in a catchment, the management of all water resources should be integrated. An illustrative model is formulated of the dynamic system of two linked aquifers with stochastic recharge, water demand and extraction cost. For each aquifer area, the optimal pumping policy for each state of the overall groundwater system is derived and this policy is applied over a large number of years to derive the expected development over time of extraction and the state of the groundwater system. The implications of the optimal extraction policy for any review of existing allocations and trade arrangements are discussed. Optimal taxes to replace existing allocations are also derived and discussed

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Introduction

Since 1970, groundwater extraction policy in the Murray Darling Basin has moved from an unrestricted policy to a policy of licensing subject to controls. The policy implemented included a range of economic, hydrogeological, engineering and administrative measures requiring cooperation or sometimes coercion (Patterson and Keary 1987). These measures included the proclamation of protected aquifers, licensing with volumetric allocations with some tradability, metering and monitoring of pumping volumes, recharge schemes and an administrative charge to recover costs of metering and monitoring groundwater. Volumetric controls are set to fluctuate inversely with the availability of surface water taking into account the long term average recharge to aquifers. The flexibility introduced in the volumetric controls to deal with fluctuation in surface water supply are intended to reduce the incentive to exceed volumetric allocations and thereby protect the 'integrity of the extraction plan' put forward by the authorities.

Issues

Up to now, the increase in groundwater use has lagged behind the increase in total allocations with the total volume of unused (sleeper) and underused (dozer) allocations increasing overtime. With the cap on the volume of surface water diversions in the Murray Darling Basin and environment flow rules that further restrict the already committed available surface water resources, many sleeper and dozer allocations of groundwater can be expected to be fully used by their holders in the near future. Even though current groundwater use is below the sustainable level, the activation of sleeper and dozer licences expected in the near future could increase the rate of pumping beyond the sustainable level and lower groundwater stocks, particularly in high risk groundwater management areas (GMAs). At that stage, the pumping activities of an individual user impose costs on other users.

An important feature of the current regulatory regime is that the allocations are for annual volumes of groundwater extraction and not the stock of the groundwater resource. Under the existing policy, a licence holder can use groundwater by pumping up to his or her annual allocation regardless of the level of the groundwater stock at that time. Under this system a user knows his or her future allocation levels under some form of contract with the authority. Under this arrangement no user needs to know the state of the system nor whether other users divert from the contracted path. Thus, no user has reason to respond to other users' actions. Each user has some loose form of property right or control over that part of the groundwater resource equal to his or her allocations over the planning horizon. This policy would not be expected to lead to a rebuilding of groundwater stocks from a low to an economically efficient and sustainable level unless the relevant authority intervenes to reduce groundwater allocations to levels below the long term annual average recharge until the sustainable stock level is reached. In order for a groundwater management authority to optimally intervene in this manner, it should have information on the efficient level of pumping for any given stock level taking into account the efficient path of stocks toward the efficient sustainable stock level (ESSL).

In the event of the total annual allocation exceeding the efficient annual extraction level, the authorities could introduce a tax to replace the existing allocations, cut back individual allocations, or a combination of these, so that total annual use will equal the optimal annual extraction for the aquifer. Authorities may also relax current restrictions on inter aquifer trading of groundwater entitlements so that surplus entitlements are allowed to be sold to users in an area where the current cumulative allocation is below the ESSL. Even if selling excess entitlements to users in another aquifer area appears to have averted the potential impact of the activation of sleeper and dozer licences, this action may not have solved the problem completely if the trading aquifers were hydrologically connected. The impact of trade on water users in both aquifers due to hydraulic connectivity between the aquifers needs to be understood. The economic impacts of options for inter aquifer trading need to be evaluated both at the user and catchment levels. Information from hydrogeologists is required about the hydraulic connectivity between the aquifers so that management of the resource can be based on information of all costs and benefits.

A modeling framework, which can be used to investigate the characteristics of a socially optimal extraction policy for a catchment with two linked aquifers is presented in this paper. The characteristics of this socially optimal extraction policy are then compared with that of a policy under which each aquifer manager behaves myopically. An optimal quota policy to guide any review of existing allocations and an optimal tax policy that would yield the same outcomes as optimal annual allocations are also derived.

An illustrative model of two linked aquifers

Consider two aquifers that can be horizontally linked, each with an overlying land area that is irrigated with water pumped from the aquifer. Alternatively, the two aquifers can be vertically linked with the same overlying land area, with some farmers in a part of this area pumping water from the shallow aquifer and the rest pumping from the deep aquifer. The model can also be adapted to a case where an aquifer discharges to a river and the hydraulic head of the river influences discharges that may have economic value downstream.

The aquifers are denoted by subscripts *a* and *b*. For these aquifers, the state transition equations for hydraulic head, the total benefit functions for the use of groundwater for irrigation and the unit cost functions for groundwater pumping in the overlying land area are given in discrete time by equations (1)–(2), (3)–(4) and (5)–(6), respectively.

$$h_{a,t+1} - h_{at} = -\alpha(h_{at} - h_{bt}) - \kappa_a x_{at} + \varepsilon_{a,t+1}, \text{ for } \forall\ t,\ t = 1,2,...,\infty \quad (1)$$

$$h_{b,t+1} - h_{bt} = -\alpha(h_{bt} - h_{at}) - \kappa_b x_{bt} + \varepsilon_{b,t+1}, \text{ for } \forall\ t,\ t = 1,2,...,\infty \quad (2)$$

$$F_{at} = [u_a - (v_a / 2)x_{at}]x_{at}, \text{ for } \forall\ t,\ t = 1,2,...,\infty \quad (3)$$

$$F_{bt} = [u_b - (v_b / 2)x_{bt}]x_{bt}, \text{ for } \forall\ t,\ t = 1,2,...,\infty \quad (4)$$

$$\gamma_{at} = \sigma_a (h_a^{MAX} - h_{at}), \text{ for } \forall\ t,\ t = 1,2,...,\infty \quad (5)$$

$$\gamma_{bt} = \sigma_b (h_b^{MAX} - h_{bt}), \text{ for } \forall\ t,\ t = 1,2,...,\infty \quad (6)$$

Where for aquifers *a* and *b* in year *t*, h_{at} and h_{bt} denote the hydraulic head (metres); h_a^{MAX} and h_b^{MAX}, the maximum hydraulic head (metres); x_{at} and x_{bt}, the volume of groundwater pumped (ML/year); α, the leakage coefficient between the aquifers; κ_a and κ_b, the storage coefficient (m/ML); ε_{at} and ε_{bt}, the stochastic recharge (m/year); σ_a and σ_b, the increase in cost per megalitre pumped per metre drop in the hydraulic head ($/metre.ML), and u_a and u_b the intercepts and v_a and v_b the slopes of the linear inverse demand (marginal benefit) curves for groundwater used in irrigation. For aquifers *a* and *b* in year, *t*, total benefit ($/year) denoted by F_{at} and F_{bt} represent the areas under the respective marginal benefit curves and γ_{at} and γ_{bt} denote pumping cost ($/ML).

The stochastic terms ε_{at} and ε_{bt} are assumed to be stationary and serially independent random variables, each of known distribution and mean. For aquifers, *a* and *b* the terms, $\alpha(h_{at} - h_{bt})$ and $\alpha(h_{bt} - h_{at})$ in equations (1) and (2) respectively represent the effect on the hydraulic head of the water conductivity between the two aquifers. The leakage coefficient, α with values $0 \le \alpha \le 0.5$ captures the transmissivity between the materials of the two aquifers and thus, for aquifers *a* and *b*, the effect on the hydraulic head of water conductivity between the two (*a* and *b*) is a fraction α of the difference between the hydraulic head and that of the adjacent aquifer (hydraulic gradient). When $\alpha = 0$, the two adjacent aquifers are hydrologically separate and when $\alpha = 0.5$, the aquifers are fully connected. The value of α also has some implications for defining property rights as $\alpha = 0$ means no leakage and thus ensures that exclusive property rights can be defined for each of the adjacent aquifers.

A socially optimal extraction policy for the two aquifer system

For an infinite time planning horizon, the problem for the groundwater resource manager for the two aquifer system is to find $x_a(1)$, $x_a(2)$, $x_a(3),\ldots,x_a(\infty)$ and $x_b(1)$, $x_b(2)$, $x_b(3),\ldots,x_b(\infty)$ that maximise the expected present value function

$$V(h_{a0},h_{b0}) = E\left\{\sum_{t=0}^{\infty}\rho^t\begin{bmatrix}[u_a-(v_a/2)x_{at}]x_{at}-\sigma_a\left(h_a^{MAX}-h_{at}\right)x_{at}\\+[u_b-(v_b/2)x_{bt}]x_{bt}-\sigma_b\left(h_b^{MAX}-h_{bt}\right)x_{bt}\end{bmatrix}\right\} \qquad (7)$$

subject to (1)–(2) and initial state values h_{a0} and h_{b0}, where E is the mathematical expectation operator and the discount factor ρ is related to the real discount rate r by $\rho = 1/(1+r)$.

Let's consider the certainty equivalent case of problem (7) after replacing ε_{at} and ε_{bt} with the long term expected values of $\bar{\varepsilon}_a$ and $\bar{\varepsilon}_b$ respectively. This leads to a time evolution of deterministic problems with a current value Hamiltonian function defined for each discrete *t*.

$$\begin{aligned}&\tilde{H}(h_{at},h_{bt},x_{at},x_{bt},\lambda_{at},\lambda_{bt}) = [u_a-(v_a/2)x_{at}]x_{at}-\sigma_a\left(h_a^{MAX}-h_{at}\right)x_{at}\\&+[u_b-(v_b/2)x_{bt}]x_{bt}-\sigma_b\left(h_b^{MAX}-h_{bt}\right)x_{bt}\\&+\rho\lambda_{at+1}\left(-\alpha(h_{at}-h_{bt})-\kappa_a x_{at}+\bar{\varepsilon}_a\right)+\rho\lambda_{bt+1}\left(-\alpha(h_{bt}-h_{at})-\kappa_b x_{bt}+\bar{\varepsilon}_b\right)\end{aligned} \qquad (8)$$

For aquifers *a* and *b* in each year *t*, costate variables for aquifer hydraulic head are denoted by λ_{at} and λ_{bt} respectively. It is assumed that the Hamiltonian is concave in h_{at}, h_{bt}, x_{at} and x_{bt}, the required sufficient conditions are met and an interior solution exists (that is, bounds on x_{at} and x_{bt}, and constraints on h_{at} and h_{bt}, are not binding) and the transversality conditions $lim_{t\to\infty}\,\rho^t\lambda_{at}=0$ and $lim_{t\to\infty}\,\rho^t\lambda_{bt}=0$, are met.

For each year *t*, the maximand on the right hand side of equation (8) represents the total benefits less pumping costs obtained in that year corrected to account for intertemporal effects. The intertemporal effects of groundwater use arise as current pumping decisions affect the state at the end of the period and thus future net benefits. For the two aquifers, future net benefits per unit of hydraulic head are measured, by the costate variables λ_{at} and λ_{bt}. The spatial effects of groundwater are taken into account by summing the net benefits and intertemporal effects over the aquifers. The spatial effects arise as decisions by farms irrigated from one aquifer affect the farms irrigated from the adjacent aquifer because of leakage externalities, which are internalised when the net benefits and intertemporal effects are summed over the aquifers. The economic and policy implications of the necessary conditions for the optimal solution are interpreted as follows.

$$u_a - v_a x_{at} \le \sigma_a \left(h_a^{MAX} - h_{at}\right) + \kappa_a \rho \lambda_{a,t+1}, \text{ (= if } x_{at} > 0\text{), for } \forall\ t \qquad (9)$$

$$u_b - v_b x_{bt} \le \sigma_b \left(h_b^{MAX} - h_{bt}\right) + \kappa_b \rho \lambda_{b,t+1}, \text{ (= if } x_{bt} > 0\text{), for } \forall\ t \qquad (10)$$

For the area irrigated from aquifer *a* or *b* and for each year, *t,* the value of the marginal product of a ML of groundwater cannot exceed the current marginal (and average) pumping cost plus the discounted value of groundwater at the start of the next year. The pumping cost per ML are given by the first terms on the RHS of (9) and (10). The cost of the reduction in the hydraulic head at the end of that year are expressed in the second terms on the RHS of (9) and (10).

$$\lambda_{at} = \rho \lambda_{a,t+1} + \sigma_a x_{at} + \rho\alpha\left(\lambda_{bt+1} - \lambda_{at+1}\right), \text{ for } \forall\ t \qquad (11)$$

$$\lambda_{bt} = \rho \lambda_{b,t+1} + \sigma_b x_{bt} + \rho\alpha\left(\lambda_{at+1} - \lambda_{bt+1}\right), \text{ for } \forall\ t \qquad (12)$$

For the area irrigated by aquifers *a* and *b* at the start of each year *t*, the value of the hydraulic head of groundwater ($/m) equals the discounted value at the start of the next year plus the benefit of lower pumping costs in the year plus the discounted value of the difference in the value of hydraulic head between the aquifers at the start of the next year. If the two aquifers are separate ($\alpha = 0$), the values of the third term on the right hand side of the equations (11) and (12) are zero which means that there is no leakage externality.

Extraction policies if each aquifer is to be managed myopically

For aquifers *a* and *b*, the management rule for a myopic user is obtained by setting the shadow prices of groundwater (λ_{at} and λ_{bt}) to zero in equations (9) and (10) respectively.

$$u_a - v_a x_{at} = \sigma_a \left(h_a^{MAX} - h_{at}\right), \text{ (= if } x_{at} > 0\text{), for } \forall\ t \qquad (13)$$

$$u_b - v_b x_{bt} = \sigma_b \left(h_b^{MAX} - h_{bt}\right), \text{ (= if } x_{bt} > 0\text{), for } \forall\ t \qquad (14)$$

Under a myopic extraction regime, each user does not consider the effect of pumping on hydraulic head of the aquifers and consequently puts zero prices on the hydraulic heads of the two aquifers. As each user only considers pumping cost, pumping rates are initially higher than under regimes where there are positive prices of hydraulic head for the aquifers. In the solution obtained for socially optimal extraction, for aquifers *a* and *b* and year *t*, the aforementioned effects are embodied in the cost given in the second term on the RHS of (9) and (10) respectively. The failure to recognise the aforementioned effects under a myopic extraction regime results in external costs or externalities while in a socially optimal extraction policy, such external costs are internalised in the decision making of users.

Under a myopic management regime, the effect of pumping in the current year on (a) the cost of pumping in the next year and (b) the cost associated with the reduced availability of the stock in the end of next year have been referred to as the pumping cost and stock externalities (Provencher and Burt 1993). In the context of multiple users of an aquifer, Provencher and Burt recognise a third type of external cost, the strategic externality. When there are multiple users of an aquifer, the decision by a user not to pump the marginal unit of the stock in the current year will increase the stock available for pumping in the next year for all the users and thereby stimulate pumping by others. Provencher and Burt developed formal expressions to partition the total external cost in to these three externalities. The stock and the pumping cost externalities arise as the stocks become increasingly limited and a firm's decision to reduce pumping will not be compensated by others and as that decision benefits others, the firms pump water quicker than if such actions are compensated for. The incentives to gain from strategic interactions aggravate the inefficiencies caused by other externalities. Such interactions between water users undermine the efficiency of private property rights both from an economic and hydrological viewpoint and lead to overexploitation of the stocks. Consequently, there will be zero marginal net benefit in every period once the steady state is reached, whereas the marginal benefit is equated to positive social opportunity cost in the case of central control by a benevolent social planner.

Policy interventions

Current groundwater allocations for an area are based on the estimated long term average recharge to the underlying aquifer adjusted for any discharge requirement for dependent ecosystems. There is little evidence that these allocations are based on an extraction policy, which has the characteristics of a socially optimal extraction policy obtained by taking into account both the current states (levels) of and linkages between aquifers. The characteristics of a socially optimal extraction policy given in equations (9)–(12) can be obtained through regulatory measures by imposing optimal quotas or pumping taxes or promoting collusive behaviour between all extractors.

Optimal regulatory measures

Optimal regulatory options require that a benevolent regulatory agency has complete information on the state and dynamics of the system and can determine optimal pumping levels for known pumping costs and benefit functions. Optimal regulatory options require that the regulatory agency has access to each user's private information. However, in a regulatory regime, there is no private incentive for individual user to truthfully reveal private information. In addition to optimal quotas and taxes, other forms of regulation available are the imposition of a critical minimum hydraulic head, restriction of the number of users and the creation of private property rights to the resource.

Optimal quotas

In order to implement a socially optimal pumping policy, in the existing regulatory environment, the current groundwater allocations may be reviewed and new extraction quota may be set at the socially optimal extraction levels. Administration of optimal quotas has some difficulties, as the optimal quotas vary over time and the aquifer hydraulic heads need to be continuously monitored.

Optimal pumping taxes

Replacing existing allocations with an optimal tax can be expected to help achieve the socially optimal extraction policy.

If the existing allocations and associated regulatory measures are removed the users are expected to behave myopically. Equation (13)–(14) suggest that for given initial h_a and h_b levels and values for other parameters a policy that replaces existing allocations with optimal taxes of $\kappa_a \rho \lambda_{at+1}$ and $\kappa_b \rho \lambda_{bt+1}$, for every megalitre of groundwater used in aquifer *a* and *b* areas respectively, can be expected to help achieve the socially optimal pumping policy.

However, administration of such a policy is fraught with difficulties, as the optimal tax rates need to be calculated each year as they vary over time and the aquifer hydraulic head need to be continuously monitored. Imposition of steady state taxes of $\kappa_a \lambda_a^*$ and $\kappa_b \lambda_b^*$, for aquifer *a* and *b* areas respectively, may be an alternative even though it may delay approaching the steady state.

Promoting collusive behavior

Promoting collusive behavior can lead to a socially optimal outcome if exclusive private property rights exist, the cost of negotiation is small, there are only few agents and there is no market power in the output market. As for all the regulatory options, the collusive option assumes complete and symmetric information. Under collusive management, each user needs to know its effect on others and this information is private. Each user is expected to reveal such information if he or she is convinced that it is going to increase profits. Transmission of information is also costly and this needs to be taken into account. If any of the requirements for promoting collusive behavior is absent — for example if there are inadequately defined property rights — then there are incentives to defect from the collusive pumping rates. The incentives to defect arise from the fact that even though each player will be better off pumping at the collusive rate, he or she will be even better off if he or she can pump water at a higher rate while others pump at the collusive rates.

Application of the model

The model was applied to obtain a socially optimal extraction policy and its characteristics so that optimal quotas and taxes could be derived. First, for a socially optimal extraction policy, the optimal value and volume of groundwater pumped and the shadow price are computed for the two-dimensional space of hydraulic head in the two aquifers in the range between minimum and maximum levels. For given initial levels of hydraulic heads of aquifer *a* and *b*, optimal quotas can be fixed at levels determined by the optimal pumping policy. Second, optimal volumes of groundwater pumped with a myopic management regime are derived on the same two-dimensional state space. Third for each aquifer area, optimal taxes are derived on the same two-dimensional state space. Fourth, the model with a myopic management regime is run with pumping taxes imposed at the optimal tax levels and new optimal pumping policy is obtained. Fifth, the expected time paths for the volume of groundwater pumped and the hydraulic head are computed for the socially optimal extraction policy implemented through optimal quotas and optimal taxes and also for the myopic management regime by performing Monte Carlo simulations with 2000 replications per year to account for stochastic recharge over a 300 year period. More details of the solution methods are given in appendix A.

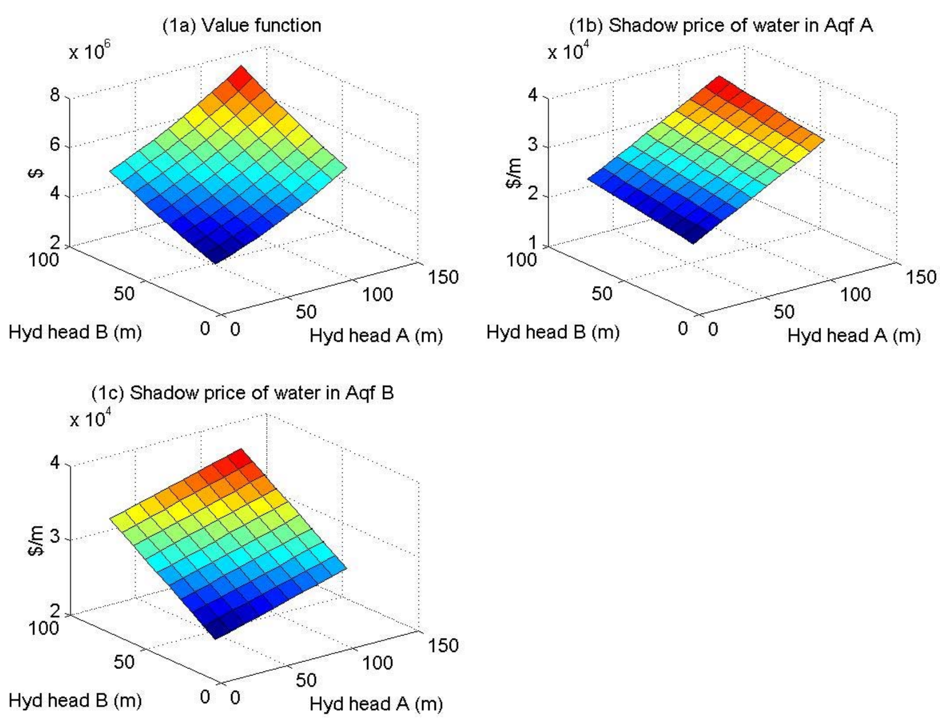
Results and discussion

Characteristics of the socially optimal pumping policy

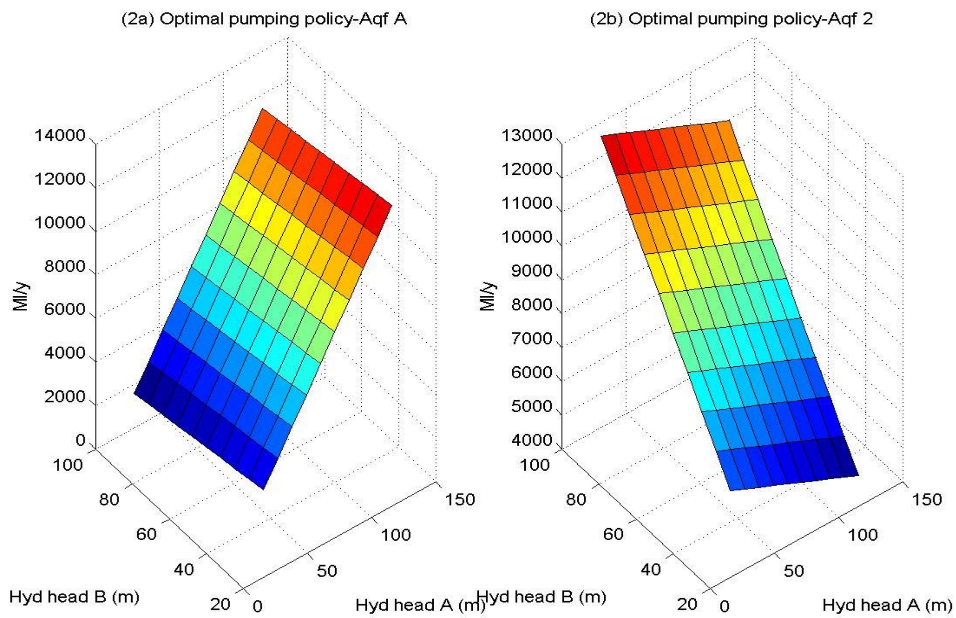
The two-aquifer model given in equation (7) is solved as explained in appendix A with a set of stylised data to illustrate the working of the model. The parameter values used are given in table 1. The value function and shadow price functions presented in figures 1a –1c suggest that the value function is not concave in hydraulic head $[h_a, h_b]$ as one would expect. Knapp and Olson (1995) obtained similar results in their study of optimal pumping policy for the Kern County aquifer in California. The quasi concavity in the value function and the shadow price for each aquifer being an increasing function in hydraulic head of both aquifers $[h_a, h_b]$ can be explained as follows. For aquifers *a* and *b*, every unit reduction in current pumping level increases the hydraulic head by κ_a and κ_b metres respectively, regardless of the level of hydraulic head. However, the higher the hydraulic head the higher the optimal pumping volume and thus the savings in future pumping costs (opportunity cost) due to a unit increase in the hydraulic head. This results in the shadow price, which is the gradient of the value function with respect to hydraulic head, increasing with the hydraulic head (figures 1b and 1c).

Table 1: **The parameter values – two aquifers**

Para-meter	Description	Unit	Aquifer A	Aquifer B
u	Intercept of water demand curve	\$/MLyear	65	65
v	Slope of water demand curve	\$/ML^2	0.003	0.003
κ	Storage coefficient	metre/ML	0.001	0.001
σ	Cost of pumping	\$/metre.ML	0.4	0.4
h^{max}	Maximum hydraulic head	metre	130	100
h^{min}	Initial hydraulic head	metre	15	15
α	Leakage coefficient	1	0.05	0.05
ρ	Discount factor	per year	0.90	0.90
$\bar{\varepsilon}$	Mean annual recharge	metre/year	6	6
σ_ε	Standard deviation recharge	metre/year	0.2	0.2



The optimal pumping policy given in figures 2a and 2b suggests that, for each aquifer and a given hydraulic head of the other linked aquifer, the optimal pumping level increases with the hydraulic head of the aquifer as the marginal cost of withdrawal decreases. The optimal pumping policy for aquifer *a* (*b*) at a given hydraulic head suggests that the optimal pumping level decreases when the hydraulic head of the aquifer *b* (*a*) increases. This is because, for the benevolent social planner, the effect of decreasing pumping cost in aquifer *b* (*a*) is more than offset by the effect of increased movement of water from aquifer *b* (*a*) to aquifer *a* (*b*).



Knowledge of the optimal pumping policy given in figures 2a and 2b may help the manager of the two aquifer system in setting the optimal groundwater allocations for individual aquifers for a given set of hydraulic heads of both aquifers. The optimal pumping policy may be applied as follows.

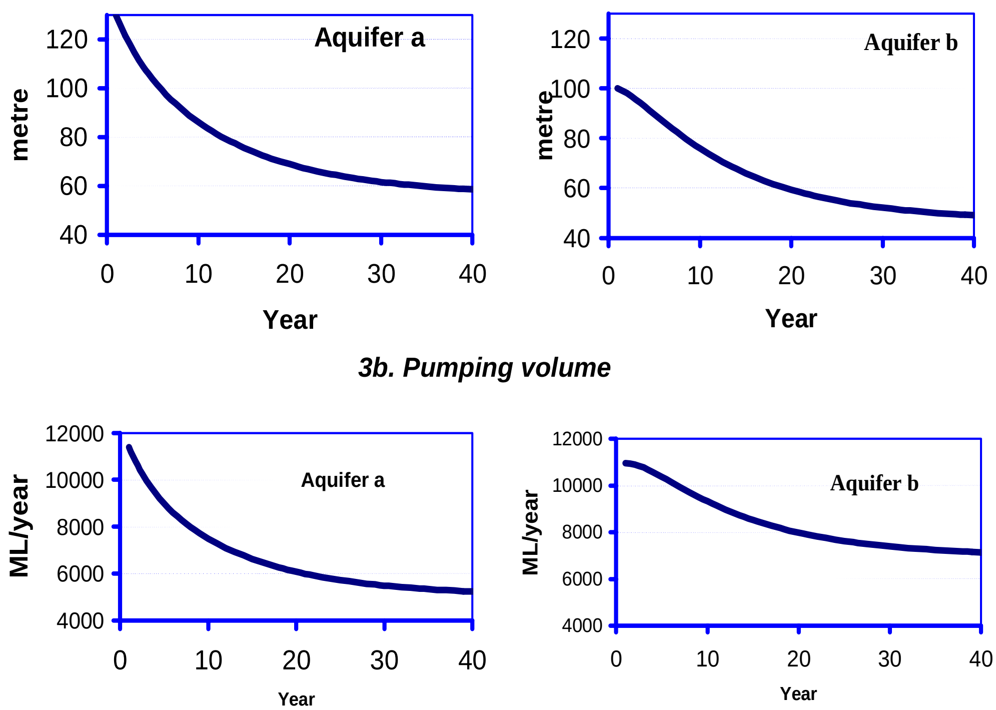
- If the hydraulic heads of the aquifers *a* and *b* in year *t* (h_{at} and h_{bt}) are lower than the corresponding steady state levels (to be discussed shortly), the optimal groundwater allocations in that year (x_{at} and x_{bt}) must be lower than the steady state allocation levels. In this case reduced pumping levels compared to the steady state levels in the early years contribute to building of groundwater stocks overtime.

- If the initial hydraulic heads of aquifers are higher than the corresponding steady state levels, the optimal pumping policy prescribes higher pumping levels compared to that of steady state in the initial years.

In both of the above scenarios, the optimal pumping policy when applied sequentially over a sufficiently long time is expected to lead both the pumping and hydraulic head levels of both aquifers to approach steady state levels as shown in figure 3.

Figure 3 Expected time paths

3a. Hydraulic head



The time paths of the hydraulic heads and pumping levels of the aquifers in the groundwater system when the optimal pumping policy is applied over 300 years with initial hydraulic heads at $\left[h_a^{max}, h_b^{max}\right]$ are obtained and presented for the first 40 years in figures 3a – 3b. The time paths represent the optimal approach to the optimal steady state and suggest that, for each aquifer, optimal pumping level and hydraulic head decrease at a higher rate at the beginning and at gradually declining rate overtime and finally approach the steady state levels. That is, the volume of aquifer which is greater than the steady state is mined over the planning horizon. It should be noted that given stochastic recharges there is no single steady state and the expected paths in figure 3 reach expected or certainty equivalent steady states by around 40th year.

For aquifer *a*, the certainty equivalence policy comprises a steady state pumping level of 5300 ML/year at a hydraulic head of 58 metres. For aquifer *b*, the corresponding estimates are 6600 ML/year and 44 metres respectively. These estimates show that the steady state pumping volume for each aquifer is not equal to the mean recharge volume assumed (6000ML/year). A hydraulic gradient from aquifer *a* to aquifer *b* (aquifer *a*'s initial hydraulic head is higher than that of the aquifer *b*) means groundwater moves from aquifer *a* to *b*. Consequently, the steady state pumping volume of aquifer *a* is less than the mean recharge while the converse is true for the aquifer *b*. It demonstrates that, for each linked aquifer what matters is the mean net recharge which takes into account other influxes to and outfluxes from the aquifer due to horizontal connectivity and the hydraulic gradient between the aquifers in addition to recharge occurring from the outcrop area.

Optimal quota

For a given set of initial hydraulic heads of aquifer *a* and *b*, the optimal quotas for individual aquifer areas can be set at levels determined by the optimal pumping policy given in figures 2a and 2b. In each aquifer area, if the total annual allocation exceeds the annual optimal extraction, the authorities could cut back individual allocations so that total annual use will equal the optimal annual extraction for the aquifer with provision for trading groundwater allocations between users. If for a given aquifer area, the total annual allocation exceeds the optimal annual extraction while the opposite is true for the adjacent area, any restrictions against selling groundwater allocations to the adjacent area can be removed so that trade could result in an increase in the total annual use in the adjacent area up to its optimal annual extraction level provided there is demand for it. In this case, if the combined total annual allocations exceed the combined optimal annual extractions, then individual allocations in the area where the total annual allocation exceeds the optimal annual extraction need to be sufficiently cut back. Administration of optimal quotas has some difficulties, as the optimal quotas need to be calculated for each year as they vary over time and the aquifer hydraulic heads need to be continuously monitored.

Optimal tax

The expected time paths were computed with optimal taxes and are presented in figure 4 along with the expected time paths computed for the socially optimal pumping policy. Optimal tax schedules computed for aquifers *a* and *b* for various combinations of initial hydraulic heads of the two aquifers are given in figure 5 and 6. It is clear that replacing existing allocations in the two areas with linked aquifers with optimal taxes help achieve the socially optimal pumping policy. Figures 5 and 6 show that for each area, the optimal tax level varies with the hydraulic head of both aquifers. The implication of this relationship is that optimal tax levels for two areas with linked aquifers need to be developed simultaneously by taking into account the state of both aquifers and the nature of the hydrological connectivity between them. Just like optimal quotas the administration of optimal taxes also has difficulties, as optimal taxes need to be calculated for each year and this requires continuous monitoring of aquifer hydraulic heads. As an alternative to an optimal tax policy, pumping taxes based on the steady state level of the social opportunity cost of groundwater resources can be introduced. Figure 4 shows that expected paths with steady state taxes did not vary much from that of socially optimal pumping policy; however, they resulted in slightly higher pumping rates in the first five years.

Figure 4 Alternative management regimes for two linked aquifers

4a. Hydraulic head

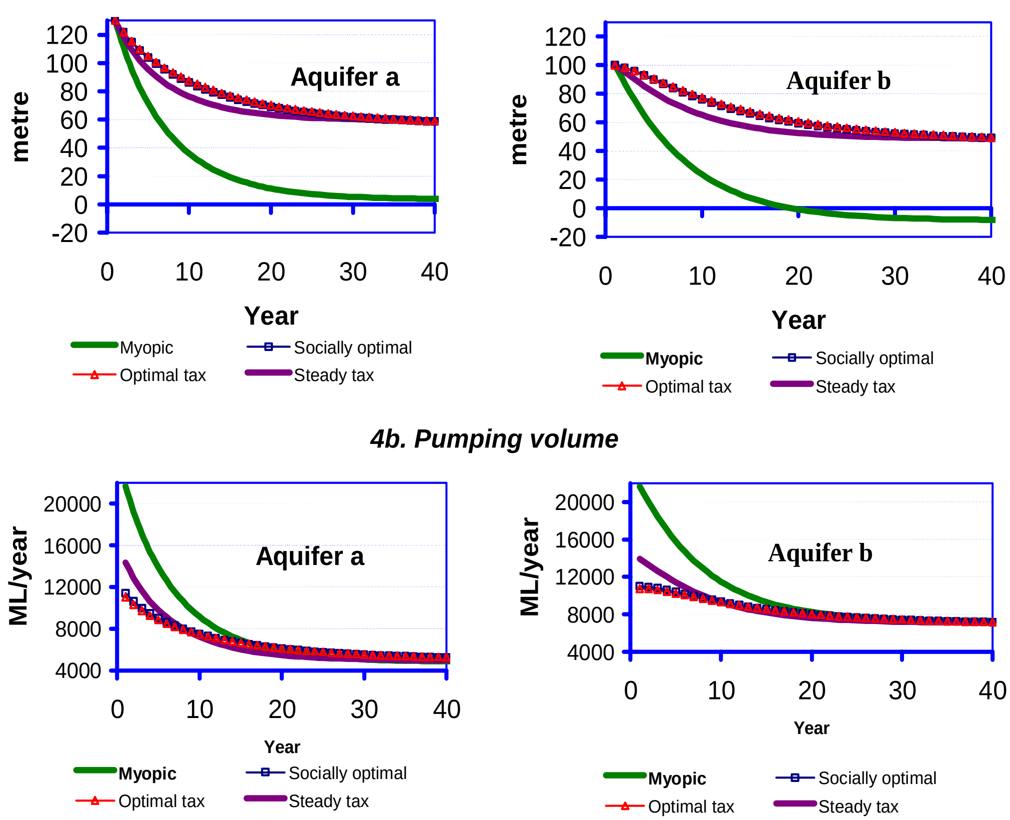


Fig 5 Optimal tax schedule for Aquifer A

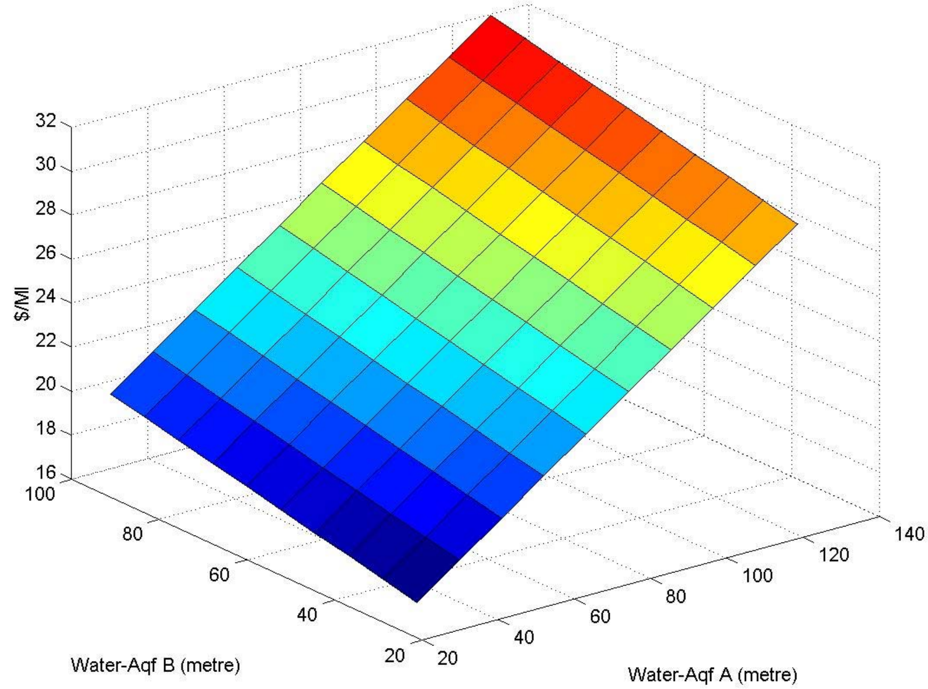
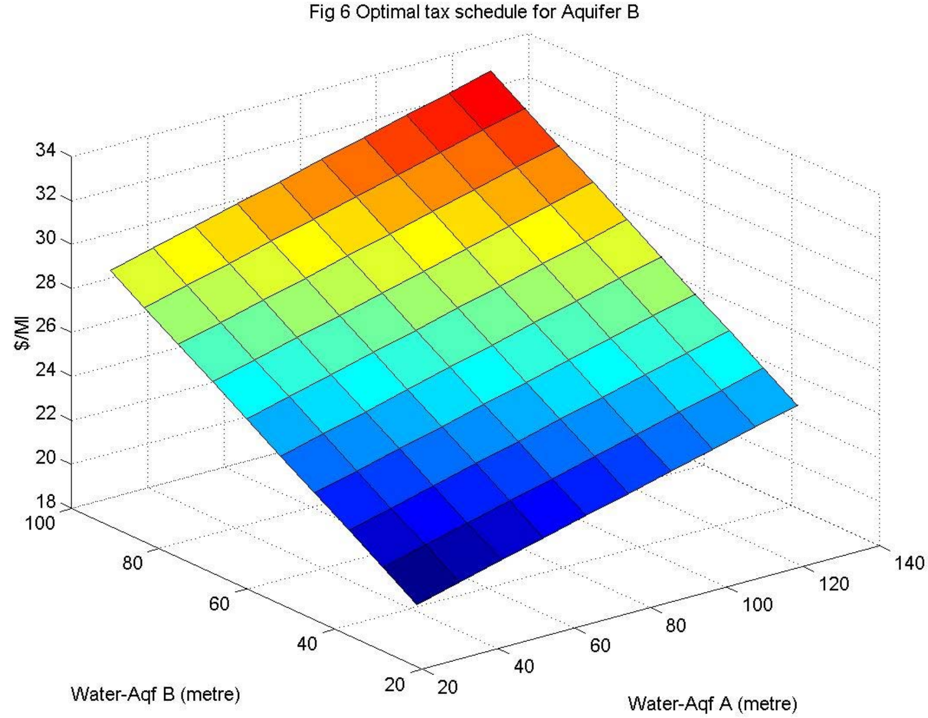


Fig 6 Optimal tax schedule for Aquifer B



Sensitivity of the results to changes in parameter values

The results, particularly the optimal quotas and taxes obtained may be sensitive to changes in the parameter values. The key parameters of interest are the leakage coefficient and the relative size of the aquifer. As long as the initial hydraulic head of the two aquifers remains the same, changes in the leakage coefficient will not affect the optimal quotas and taxes obtained, as there are no net water flows between the aquifers. As the leakage coefficient approaches zero, the connectivity between the two aquifers and thus the interaction between them become increasingly less important. If the leakage coefficient is zero regardless of the hydraulic gradient, the ownership of each aquifer is exclusive and property right is well defined and if it is equal to 0.5 and the hydraulic gradient is insignificant both of the aquifer resources are entirely commonly owned. Unrestricted inter aquifer trade can be allowed in the latter case provided combined total annual allocations equal the combined optimal annual extractions of the two linked aquifers. However, trading arrangements need to be reviewed if there is a significant change in the hydraulic gradient due to some external shocks such as episodic high recharge levels in consecutive years.

Conclusions

In this paper we have demonstrated how a decision support tool can be used to provide a groundwater management authority with information on optimal pumping quotas and taxes that are consistent with a policy of achieving sustainable resource use. The optimal quotas can be used to guide any review of existing groundwater allocations and the arrangements for their trade while optimal taxes can be used to replace existing allocations. The sensitivity of the optimal quotas and taxes to the state of the system and the influxes to and outfluxes from the system highlights the importance of continuous monitoring of the relevant hydrological parameters. A better understanding of the transmissivity of water through aquifer material has important economic implications, particularly in partitioning the resource into components with exclusive property rights. The latter are required in the use of market mechanisms such as tradeable allocations to achieve efficient allocation of groundwater resources. The importance of taking a catchmentwide perspective in addressing groundwater issues was also highlighted. However, given the enormity of scientific resources required in continuous monitoring of dynamic hydrological variables and administrative resources required in implementing optimal quotas and taxes, the costs and benefits of such measures need to be compared with lower cost second best options. More cost effective policy options could be developed. For example, it was demonstrated that instead of an optimal tax regime a steady state tax that can be still applied whatever state of the system to achieve sustainable resource use.

Appendix A: The solution method

Socially optimal pumping policy

The problem is formulated as a discrete time continuous state stochastic dynamic process on a two-dimensional state space. The value function defined on the entire two-dimensional space between the set of a given minimum and the set of given maximum levels of the state variables, $V(\cdot)$, is taken as unknown. The recursive nature in the problem defined in equation (7) is exploited in finding the optimal controls and Bellman's functional recursive equation, $V(\cdot)$, when applied to infinite time horizon is derived and used in the solution method (equation A1).

$$V(h_a, h_b) = \max_{0 \le x_a \le \Omega_b, 0 \le x_b \le \Omega_b} \left\{ \begin{array}{l} [u_a - (v_a / 2) x_a] x_a - \sigma_a (h_a^{MAX} - h_a) x_a \\ + [u_b - (v_b / 2) x_b] x_b - \sigma_b (h_b^{MAX} - h_b) x_b \\ + \rho E_{\varepsilon_a, \varepsilon_b} V \left(\begin{array}{l} h_a - \alpha(h_a - h_b) - \kappa_a x_a + \varepsilon_a \\ , h_b - \alpha(h_b - h_a) - \kappa_b x_b + \varepsilon_b \end{array} \right) \end{array} \right\} \quad \text{(A1)}$$

Where, $V(\cdot)$ is the unknown value function in the Bellman's equation and E is the mathematical operator for expectation. It is assumed that the state space is bounded between a set of minimum, (h_a^{min}, h_b^{min}) and a set of maximum, (h_a^{max}, h_b^{max}), the control space is subject to bounds $\{(x_a, x_b) | 0 \le x_a \le \Omega_a, 0 \le x_b \le \Omega_b\}$, where $\Omega_a = h_a / \kappa_a$ and $\Omega_b = h_b / \kappa_b$ and the net benefit and the state transition equations (the first and the second two terms respectively in the maximand of A1) are twice continuously differentiable functions.

An approximate solution to (A1) is computed using collocation method (Miranda and Feckler 2002). Assume that $\phi_{j_1}^a = [\phi_{1_a}^a(h_a), \phi_{2_a}^a(h_a), \ldots, \phi_{n_a}^a(h_a)]$, and $\phi_{j_2}^b = [\phi_{1_b}^b(h_b), \phi_{2_b}^b(h_b), \ldots, \phi_{n_b}^b(h_b)]$ are the basis functions to approximate the univariate functions along the h_a domain $[h_a^{min}, h_a^{max}]$ and the h_b domain $[h_b^{min}, h_b^{max}]$ respectively with the order of n_a and n_b. For the two dimensional domain, that is $[h_a, h_b]$ on $[h_a^{min}, h_a^{max}] \times [h_b^{min}, h_b^{max}]$, a set of basis functions may be constructed by taking the tensor product of the basis functions from 1 dimensional domain, that is,

$$\phi_{j_1 j_2}(h_a, h_b) = \phi_{j_1}^a(h_a) \otimes \phi_{j_2}^b(h_b) \quad \text{(A2)}$$

where, $j_1 = 1, 2, \ldots, n_a$ and $j_2 = 1, 2, \ldots, n_b$. There are a total $N = n_a n_b$ basis functions.

The collocation method consists of five steps.

First, express the function to be approximated, $V(\cdot)$, as a linear combination of basis functions $\phi_{j_1 j_2}(h_a, h_b)$, that is,

$$V(h_a, h_b) \approx \sum_{j_1=1}^{n_a} \sum_{j_2=1}^{n_b} c_{j_1 j_2} \phi_{j_1 j_2}(h_a, h_b) \qquad \text{(A3)}$$

where, $c_{j_1 j_2}$ are the $j_1 = 1,2,...,n_a$, $j_2 = 1,2,3,...,n_b$ coefficients which are to be estimated

Second, the function $V(\cdot)$given in equation (A3) is approximated on a grid of collocation nodes within a given precision on tolerance[1]. The approximation is equivalent to solving the linear coefficients $c_{j_1 j_2}$, $j_1 = 1,2,...,n_a$ and $j_2 = 1,2,...,n_b$. There are a total of $N = n_a n_b$ coefficients and therefore, $N = n_a n_b$ nodes are required to solve these coefficients. The two dimensional collocation nodes are constructed by taking the Cartesian product of the one-dimensional nodes. Let $\lfloor h_{a_1}, h_{a_2}, ..., h_{a_{n_a}} \rfloor$ and $\lfloor h_{b_1}, h_{b_2}, ..., h_{b_{n_b}} \rfloor$ are the nodes for the one-dimensional state space, then $\lfloor (h_{a_i}, h_{b_j}), i = 1,2,...n_a, j = 1,2,...,n_b \rfloor$ are formed as the nodes of the 2 dimensional state space. Now for each (h_{a_i}, h_{b_j}) node chosen, the Bellman's equation given in (A1) is replaced with a system of N non linear equations in N unknown basis functions. The stochastic expectation is approximated by a finite number of stochastic shocks. The continuous random variables, ε_a and ε_b representing stochastic recharge in the state transition equations are replaced with m_a and m_b discrete approximants, $\varepsilon_{a_1}, \varepsilon_{a_2}, ..., \varepsilon_{a_m}$ and $\varepsilon_{b_1}, \varepsilon_{b_2}, ..., \varepsilon_{b_m}$ with associated probabilities, $w_{a_1}, w_{a_2}, ..., w_{a_m}$ and $w_{b_1}, w_{b_2}, ..., w_{b_m}$ generated using the Gaussian quadrature scheme. Basis functions are defined as Chebychev polynomial functions.

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[1] Initial values of the value function and water pumped from each aquifer derived for all collocation nodes are used to narrow down the search range. These initial values are derived as follows. First, from the state transition equations (1)–(2) and the first order conditions given in equations (9)–(14), the steady state levels of hydraulic head, h_a^* and h_b^*, volumes of water pumped per year, x_a^* and x_b^* and the costate variables, λ_a^* and λ_b^* are analytically solved after dropping the time *t* subscript. Second the parameter values given in table 1 were then used to sequentially derive the certainty equivalent (CE) steady state values of control, costate and state variables by employing analytical expressions obtained in the first step. Third, on each collocation node, a linear approximant of the state transition equations (equations 1 and 2) and a quadratic approximant of the net benefit functions (equation 3 less 5 and equation 4 less 6) are evaluated. First and second order Taylor series expansions around the certainty equivalent steady state values are used in deriving these approximants. Finally the linear quadratic approximant values of the value function and water pumped from each aquifer derived for all the collocation nodes are used as initial values for solving the Bellman's equation using collocation.

$$\sum_{j_1=1}^{n_a}\sum_{j_2=1}^{n_b} c_{j_1 j_2}\phi_{j_1 j_2}\left(h_{a_i}, h_{b_i}\right)=$$

$$\max_{0\le x_{a_i}\le \Omega_a, 0\le x_{b_i}\le \Omega_b}\left\{\begin{array}{l}\left[u_a-(v_a/2)x_{a_i}\right]x_{a_i}-\sigma_a\left(h_a^{MAX}-h_{a_i}\right)x_{a_i}\\ +\left[u_b-(v_b/2)x_{b_i}\right]x_{b_i}-\sigma_b\left(h_b^{MAX}-h_{b_i}\right)x_{b_i}\\ +\rho\sum_{k=1}^{m}\sum_{l=1}^{m} w_{a_k} w_{b_l}\sum_{j_1=1}^{n_a}\sum_{j_2=1}^{n_b} c_{j_1 j_2}\phi_{j_1 j_2}\left(\begin{array}{l}h_{a_i}-\kappa_a x_{a_i}-\alpha\left(h_{a_i}-h_{b_i}\right)+\varepsilon_{a_k}\\ ,h_{b_i}-\kappa_b x_{b_i}-\alpha\left(h_{b_i}-h_{a_i}\right)+\varepsilon_{b_l}\end{array}\right)\end{array}\right\}$$

, for $\forall\ i$ (A4)

Third, the values for $c_{j_1 j_2}$ for all j_1, j_2 are then found by requiring the approximant to satisfy the Bellman's equation at *N* collocation nodes.

Fourth, once the collocation equation has been solved, a diagnostic test is performed to ensure that the computed approximant solves the Bellman's equation at any arbitrary state over the entire two dimensional state space. To do this, a residual function is defined as follows and evaluated at *i'* (=500) equally spaced states over the state space.

$$R\left(h_{a_{i'}}, h_{b_{i'}}\right)=$$

$$\max_{0\le x_{a_i}\le \Omega_a, 0\le x_{b_i}\le \Omega_b}\left\{\begin{array}{l}\left[u_a-(v_a/2)x_{a_i}\right]x_{a_i}-\sigma_a\left(h_a^{MAX}-h_{a_i}\right)x_{a_i}\\ +\left[u_b-(v_b/2)x_{b_i}\right]x_{b_i}-\sigma_b\left(h_b^{MAX}-h_{b_i}\right)x_{b_i}\\ +\rho\sum_{k=1}^{m}\sum_{l=1}^{m} w_{a_k} w_{b_l}\sum_{j_1=1}^{n_a}\sum_{j_2=1}^{n_b} c_{j_1 j_2}\phi_{j_1 j_2}\left(\begin{array}{l}h_{a_i}-\kappa_a x_{a_i}-\alpha\left(h_{a_i}-h_{b_i}\right)+\varepsilon_{a_k}\\ ,h_{b_i}-\kappa_b x_{b_i}-\alpha\left(h_{b_i}-h_{a_i}\right)+\varepsilon_{b_l}\end{array}\right)\end{array}\right\}$$

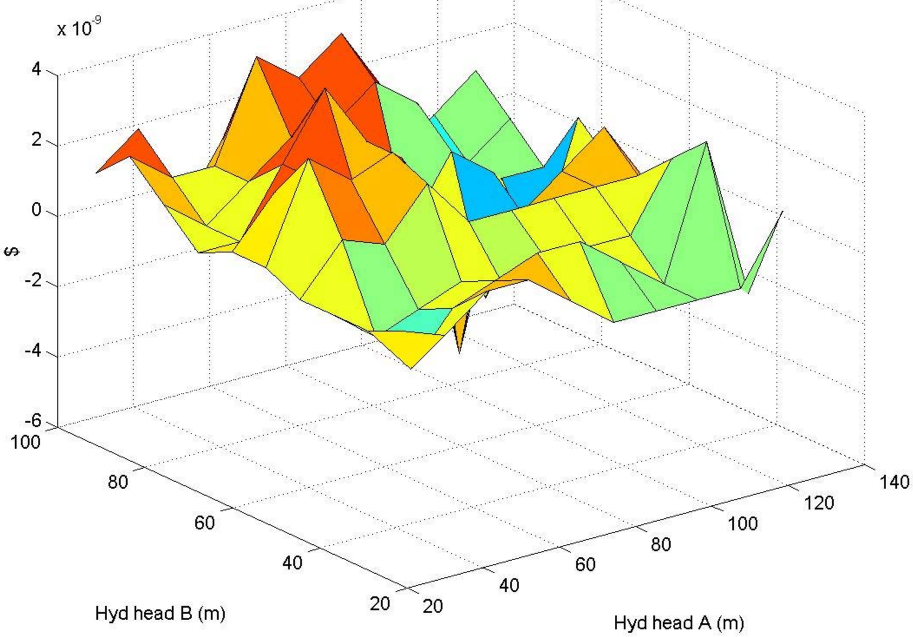
$$-\sum_{j_1=1}^{n_a}\sum_{j_2=1}^{n_b} c_{j_1 j_2}\phi_{j_1 j_2}\left(h_{a_{i'}}, h_{b_{i'}}\right)$$

for $\forall\ i'$ (A5)

Fifth, for each two dimensional node, the optimal value and volume of groundwater pumped and the shadow price are computed using the approximant function.

Before discussing results it is important to make sure that the value function approximated using collocation solves the Bellman's equation at any arbitrary state chosen over the entire two-dimensional state space with a acceptable degree of accuracy. Even though the estimated approximant function was based on only 25 nodes, it is capable of solving the value of the Bellman equation at all the 500 arbitrary points chosen between $\left[h_a^{min}, h^{min}\right]$ and $\left[h_b^{max}, h_b^{max}\right]$ with near zero deviation values (figure A1).

Figure A1 Approximation residual



Myopic pumping policy

Compared to the socially optimal pumping policy, derivation of the myopic pumping policy is straightforward. For each two-dimensional node selected for the previous solution, the myopic pumping policy for aquifer *a* and *b* is derived from equations 13 and 14 respectively are given in equations (A6)–(A7).

$$x_a = \frac{u_a}{v_a} - \frac{\sigma_a}{v_a}\left(h_a^{MAX} - h_a\right) \tag{A6}$$

$$x_b = \frac{u_b}{v_b} - \frac{\sigma_b}{v_b}\left(h_b^{MAX} - h_b\right) \tag{A7}$$

Pumping policy with optimal taxes

Once the optimal tax levels are known, the derivation of the pumping policy with optimal tax is also straightforward. For each two-dimensional node, the pumping policies for aquifer *a* and *b* with optimal taxes are given in equations (A7)–(A8).

$$x_a = \frac{u_a}{v_a} - \frac{\sigma_a}{v_a}\left(h_a^{MAX} - h_a\right) - \frac{\kappa_a \rho}{v_a}\lambda_a \qquad \text{(A7)}$$

$$x_b = \frac{u_b}{v_b} - \frac{\sigma_b}{v_b}\left(h_b^{MAX} - h_b\right) - \frac{\kappa_b \rho}{v_b}\lambda_b \qquad \text{(A8)}$$

Expected time paths

For each solution, the expected paths for groundwater pumping volume and the hydraulic head are computed by performing Monte Carlo simulations with 2000 replications of a 300 year period each.

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