The Optimal Time Path for Carbon Abatement and Carbon Sequestration under Uncertainty: The Case of Stochastic Targeted Stock

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INTRODUCTION

Although there is a consensus in the scientific world hat carbon sequestration should be included in a portfolio of GHG mitigation strategies (WG Ⅲ, IPCC, 2007; Richards and Stokes, 2004) the optimal timing of its implementation is still debated.

An important feature of carbon sequestration that distinguishes it from abatement technologies is its ability to actually reduce atmospheric concentrations of CO_2

This paper explores the optimal time path of carbon sequestration and carbon abatement in stabilizing the level of carbon dioxide in the atmosphere under uncertainty in climate impacts

SETTING

Two periods sequential decision making model

Both controls are treated as investments where current reduction efforts yield future reduction benefits

Uncertainty in climate impacts which effects the desired future stabilization level of the CO_2 stock is assumed in the first period but is resolved prior to the decision on how much to control the stock in the second period.

In the first period the planner knows the mean of the desired stabilization level but is uncertain about the variability around the mean. consider only two possible states of the world at the end of the second period, B_H and B_L

$P_S(u)$ and $P_A(v)$ are well behaved functions.

Marginal cost of abatement (2A) and marginal cost of sequestration (2B) where $P_A(0) > P_S(0)$

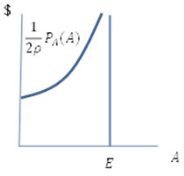


Figure 2A

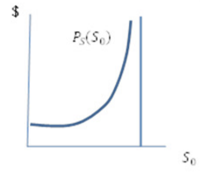


Figure 2B

The key innovation in our paper is that we provide an analytical treatment of the optimal timing of carbon sequestration and abatement under uncertainty. We show that uncertainty can make it optimal to use carbon sequestration either earlier or later and clarify the conditions under which different effects of uncertainty are obtained.

Uncertainty over climate damages is introduced into the model by recognizing that today we cannot be sure of the amount of warming expected at different atmospheric CO_2 concentrations. However, as time progresses, society's understanding of the severity of climate impacts will presumably increase.

MODEL

apply backwards induction starting with the minimization problem for the second period:

$$\min_{S_1} \int_0^{S_1} P_S(u)du$$

subject to:

$$B_1 = X_1 + E - A^* - S_1 - \alpha S_0^* \qquad X_1 \text{ is given} \quad A^* \text{ is given} \quad S_0^* \text{ is given} \quad (1)$$

$$0 \le S_1 \le C_1 \qquad C_1 \text{ is given} \quad (2)$$

Apply Bellman equation for the minimization problem in the first period given the expected optimized value function for the second period:

$$V(A, S_0, 0) = \int_0^A P_A(v)dv + \int_0^{S_0} P_S(u)du + \rho E\left[\int_0^{S_1^*(B)} P_S(u)du\right] \quad (5)$$

Subject to:

$$X_1 = X_0 + E - A - S_0 \qquad X_0 \text{ is given} \quad (6)$$

$$0 \le A \le E \qquad 0 \le S_0 \le C_0 \qquad C_0 \text{ is given} \quad (7)$$

$$C_0^* \ge X_0 + 2(E - A) - B_L \quad (8)$$

Variables list (t=0,1):
$X(t)$ – level of CO_2 in atmosphere
$A(t)$ – rate of abatement
$S(t)$ – rate of sequestration
$C(t)$ – sequestration capacity
$E(t)$ – emissions rate
$0 \le \alpha \le 1$ – future benefits from current sequestration
B_1 – desired stabilization level
$\bar{B}$ – mean of desired stabilization level
σ – variability around the mean of stabilization level
ρ – discount factor
C_0^* - critical capacity of seq required to meet B_L

In the determination of the optimal solution the following two functions are important:

1. Define $f_\gamma(A) = {P_S}^{-1}(\gamma P_A(A))$ as the response function between the rate of abatement deployment and sequestration deployment in the first period where γ is a constant which is depends on the discount factor and the future benefits from current sequestration.

2. Define $R(A,B) = P_S\big(X_0 - 2(E-A) - B - (\alpha+1)f_\gamma(A)\big)$ as the marginal cost of sequestration in the second period which is a function of abatement and the random variable, namely the stabilization level, among other parameters

Proposition 1: under the standing assumption that $\boldsymbol{P_A(0) > P_S(0)}$, the optimal rate of abatement and sequestration in the first period is given by the solution $\boldsymbol{\frac{1}{2}P_A(A^*) = \rho E[R(A^*,B)]}$ and $\boldsymbol{S_0^* = f_\gamma(A^*)}$.

OPTIMAL RATES OF S_0 & A_0

$$S_0^* = P_S^{-1}\left(\frac{(\alpha+1)}{2(1-\rho)}P_A(A)\right) \equiv f_\gamma(A) \ \textit{(11)} \ \textit{and} \ , \frac{1}{2}P_A(A) = \frac{1}{2}\rho\left(P_S\left(\beta(B_H) - 2A - (\alpha+1)P_S^{-1}\left(\frac{(\alpha+1)}{2(1-\rho)}P_A(A)\right)\right) + P_S\left(\beta(B_L) - 2A - (\alpha+1)P_S^{-1}\left(\frac{(\alpha+1)}{2(1-\rho)}P_A(A)\right)\right)\right) \equiv \rho E[R(A,B)] \ \textit{(12)}$$

The above convexity properties of $f_\gamma(A)$ determine qualitatively the respond of the system when uncertainty in the stabilization level exists. Apply simple function forms: Let $P_S(S) = \frac{K}{(C-S)^q}$ where K is a positive constant, $q > 1$ and the rate of sequestration, S, is bounded above the available sequestration capacity, C_0. Similarly, let the marginal cost of abatement be $P_A(A) = \frac{J}{(E-A)^p}$ where J is a positive constant, $P > 1$ and the rate of abatement, A, cannot exceed the rate of emissions, E. In addition, assume $\frac{J}{E^p} > \frac{K}{C^q}$ so that sequestration is cheaper than abatement initially.

The Aggressive Path : Given that p>q then $f_{\gamma(A)}$ is strictly concave (3A) and $R(A,B)$ is strictly convex (3B). Uncertainty in climate impacts calls for higher deployment rates of abatement in the first period relative to no uncertainty in climate impacts.

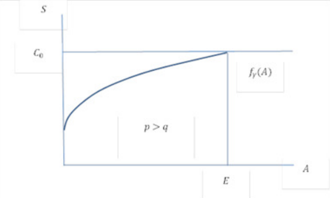


Figure 3A

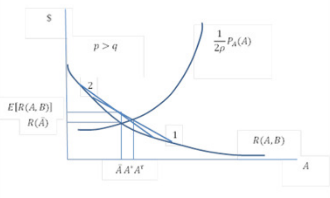


Figure 3B

The Precautionary Path: Given that p<q then $f_\gamma(A)$ is strictly convex (4A) and if C_0 is big enough then $R(A,B)$ is strictly concave (4B). Uncertainty in climate impacts calls for lower deployment rates of abatement and sequestration in the first period relative to no uncertainty in climate impacts.

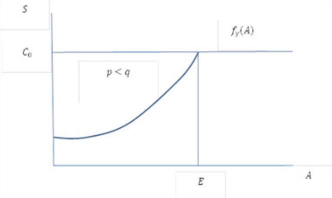


Figure 4A

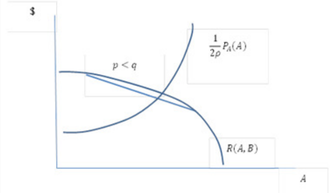


Figure 4B

The Ambiguous Path : Given that $p_1 < q_1 \ \forall A \in (0, A_\varepsilon)$, $p_2 > q_2 \ \forall A \in (A_\varepsilon, E)$ when $A_\epsilon < E$ then $f_\gamma(A)$ is constructed from both convex and concave regions (5A) and if C is big enough then $R(A,B)$ is also constructed from both concave and convex regions (5B). For low (high) rates of abatement uncertainty in climate impacts calls for lower (higher) deployment rates of abatement and, consequently, sequestration in the first period relative to no uncertainty in climate impacts

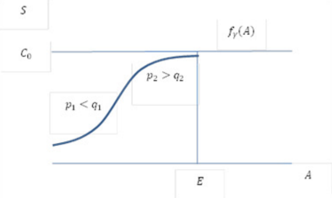


Figure 5A

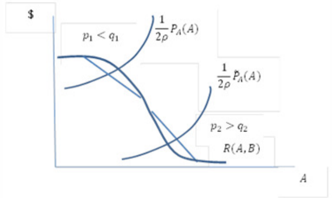


Figure 5B

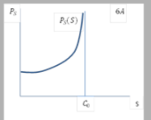
COMPARATIVE STATICS

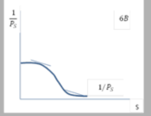
An increase in the level of the discount factor, all else equal, could either result in more or less deployment of abatement (and consequently, either more or less sequestration in the first period) depending on the following condition:

$$\frac{{P_S}'(S_0^*)}{{P_S}^2(S_0^*)} = \frac{1/2[P_S'(S_1^*(B_H) + P_S'(S_1^*(B_L)]}{1/2[{P_S}^2(S_1^*(B_H) + {P_S}^2(S_1^*(B_H)]} \quad (15)$$

$\boldsymbol{P_S(S)}$ and $\boldsymbol{1/P_S(S)}$ are depicted in 6A and 6B, respectively.
Abatement is increasing with a small increase in the discount factor if the rate of change in the first derivative of the reciprocal of the marginal cost of sequestration in the first period is smaller than the expected one in the second period

Comparative statics of (12) with respect to other parameters is suppressed from this presentation.





DISCUSSION

1. two important factors in determining the actual outcome of the system are the magnitude of sequestration capacity and the ratio between the elasticity of excess abatement and the elasticity of excess sequestration capacity
2. The existence of both the precautionary path and the ambiguous path is hinged on the availability of large volume of sequestration capacity. This assumption is consistent with results from recent simulation analysis suggesting a considerable conversion of land to forest depending on predicted carbon-price paths (Sohngen and Mendelsohn, 2003; Richards & Stokes, 2004; Sohngen and Sedjo, 2006)
3. Results for the ambiguous path are not only combining the results for the aggressive path and the precautionary path but also relate the two paths on an abatement deployment rate scale. Here, not only the elasticity of excess sequestration with respect to excess abatement plays a role but also how cheap or expensive abatement is. The combination of both, together with large enough sequestration capacity, will dictate if we are at the precautionary path, the aggressive path or, somewhere in between where uncertainty in climate impacts does not have a real effect on current deployment of abatement and sequestration.
4. The positive dependency between the deployment of sequestration and abatement in the first period is in agreement with previous studies suggesting that sequestration and abatement are compliments in the short run rather than substitutes (Stavins, 1999; Richards & Stokes, 2004). In addition, the cross-temporal dependency in sequestration due to uncertainty in climate impacts reflects either complimentarily or substitutable relationship depending on the ratio between the elasticity of excess abatement and the elasticity of excess sequestration capacity.