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# Voluntary Standards and International Trade: 

# A Heterogeneous Firms Approach 

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## Introduction

Debates over trade policy often center on the concern that increased market integration can lead to a regulatory "race to the bottom", the strategic lowering of domestic regulatory standards in order to increase export competitiveness and attract inward FDI. However, some scholars have argued that increased market integration can have the opposite effect: exerting upward pressure on already low regulatory standards. Vogel (1997) famously argued the demand for automobiles in California led to the diffusion of that state's relatively strict emissions standards to foreign automobile suppliers.

A related body of research has studied the relationship between international trade flows and the recent explosion in the adoption of voluntary industry standards such as ISO 9001 and ISO 14001 (see e.g. Prakash and Potoski, 2006). These authors argue that voluntary certifications actually lead firms to raise production standards in order to stay competitive in international markets. These standards are especially popular in markets for "fair trade" or "sustainable" goods. While some consumers are willing to pay a premium for these types of goods (Loureiro and Lotade, 2005), "fair trade" and "sustainable" are process attributes consumers cannot observe directly in the products they buy. This creates a situation analogous to the "market for lemons" described by Akerlof (1970), whereby firms have an incentive to falsely advertise they employ high labor or environmental standards. If consumers recognize this incentive, they will no longer offer a premium for these attributes and the market for high-standards goods may collapse.

The literature on voluntary standards and international trade has produced a fairly consistent and highly suggestive set of correlations. Besides a few notable exceptions (e.g. Roe and Sheldon, 2009) this empirical work has proceeded without any strong theoretical underpinning. This makes it difficult to interpret parameter estimates and to extrapolate from the empirical results to policy prescriptions. In this paper, I present a model based in Meltiz's (2003) heterogeneous firms and trade (HFT) framework. Employing the HFT framework builds on previous theoretical work by providing for a rich set of firm-level predictions regarding the relationship between voluntary standards and participation in international markets. The HFT framework has also demonstrated an ability to reproduce certain patterns of firm behavior often observed in the data but previously absent from game-theoretic models of trade.

The paper proceeds as follows: Section one introduces the theoretical framework and defines the model equilibrium. Section two demonstrates the derivation of policy-relevant comparative statics. Section three illustrates the model equilibrium and comparative statics using numerical simulation. The derivation of results for sections two and three can be found in the attached mathematical appendix. Section four concludes.

## Theoretical Framework

Adoption of a voluntary certification is best described with a model that can provide a rich set of firm-level predictions. The model presented here is an application of the Melitz (2003) heterogeneous firms and trade (HFT) framework to
the provision of credence goods. The original HFT model only allowed for horizontal differentiation, but subsequent work has modified the original framework to allow for vertical differentiation (e.g. Johnson, 2010). Two important working papers then adapted the vertical differentiation framework to the provision of credence goods, first for the closed economy (Podhorsky, 2010a) and then for an economy with frictionless trade (Podhorsky, 2010b). By assuming zero trade costs, Podhorsky (2010b) eliminated the endogenous exporting decision that distinguished the original HFT model. This assumption also makes it impossible to explore the relationship between trade and voluntary certification. In this section, I present a model of participation in a voluntary standard with fixed export market entry costs and positive transportation costs.

## Consumption

Consumers in each country maximize a utility function characterized by a constant elasticity of substitution ( $\sigma>1$ ) among each of the $\omega \in \Omega$ varieties available in their home market.

Consumers solve:

$$
\begin{align*}
& \max _{x_{i}(\omega)} U=\left(\int_{\omega \in \Omega_{i}}\left(\lambda\left(q_{\omega}\right)^{\frac{1}{\sigma}} x(\omega)\right)^{\frac{\sigma-1}{\sigma}} d \omega\right)^{\frac{\sigma}{\sigma-1}}  \tag{1}\\
& \text { s.t. } \int_{\omega \in \Omega} p(\omega) x(\omega) \leq E
\end{align*}
$$

The quantity variety $\omega$ consumed in country $i$ is $x_{i}(\omega)$. The unit price of variety $\omega$ in country $i$ is $p_{i}(\omega)$. Total expenditures in the country is $E_{i}=w_{i} L_{i}$, where $w_{i}$ is the
wage rate in country $i$, and $L_{i}$ is the total labor supply in $i$. The term $\lambda\left(q_{\omega}\right)$ captures the effect of vertical differentiation on consumer behavior. It acts as a demand shifter, allocating larger budget shares to varieties with higher quality $\left(q_{\omega}\right)$. For simplicity, assume $\lambda\left(q_{\omega}\right)=q_{\omega}^{\gamma}$ and $\gamma \geq 0$.

The consumer maximization problem yields the following demand function:

$$
\begin{equation*}
x_{i}(\omega)=p_{i}(\omega)^{-\sigma} \lambda\left(q_{\omega}\right) \frac{E_{i}}{\tilde{P}_{i}^{1-\sigma}} \tag{2}
\end{equation*}
$$

Where $\tilde{P}$ is the quality-adjusted CES price index:

$$
\begin{equation*}
\tilde{P}_{i} \equiv\left(\int_{\omega \in \Omega_{i}} \lambda\left(q_{\omega}\right) \cdot p_{i}(\omega)^{1-\sigma} d \omega\right)^{\frac{1}{1-\sigma}} \tag{3}
\end{equation*}
$$

Following Podhorsky (2010a), I assume that consumers derive more utility from higher quality varieties, but cannot observe the quality of the variety themselves. Consumers are aware that firms can voluntarily participate in a credible certification that will identify whether they meet the (exogenously determined) minimum quality standard: $q_{\omega} \geq q_{H}$. Consumers therefore perceive the quality of each variety $(\omega)$ as:

$$
q_{\omega}=\left\{\begin{array}{l}
q_{H} \text { if certified } \\
q_{L} \text { otherwise }
\end{array}\right.
$$

We can think of $q_{L}$ as the sum of attributes observable by the consumer. Even in the absence of certification, consumers can perceive $q_{L}$. Since there are no returns to investments in product quality above $q_{H}$ or between $q_{H}$ and $q_{L}$, this specification of consumer preferences turns the firm's choice of optimal quality into a binary decision determined exactly by the firm's optimal certification strategy.

## Production

As in Melitz (2003), firms are monopolistically competitive and heterogeneous in terms of their underlying productivity, here represented by the parameter $\theta$. Assume $\theta$ is distributed Pareto with distribution function $G(\theta)=1-(\theta / \underline{\theta})^{-\varsigma}$, where $\underline{\theta}$ is the lower bound of the support of $G(\theta)$ and $\varsigma>0$ is the scale parameter. Firms must sink an entry $\operatorname{cost} F_{E}$, expressed in labor units, to enter the differentiated products sector. Firms do not know their productivity level before entering the industry. Following entry, each firm will maximize operating profit by choosing an optimal price and quality as a function of their productivity. Firms solve:

$$
\begin{equation*}
\max _{p(\omega), q_{\omega}} \pi_{j}\left(\omega_{i}\right)=p_{j}\left(\omega_{i}\right) x_{j}(\omega)-w_{i} c\left(q_{\omega}\right) x_{i}(\omega) \tag{4}
\end{equation*}
$$

$\pi_{j}\left(\omega_{i}\right)$ refers to the profit earned in country $j$ by the firm producing variety $\omega$ in country $i$. The firm's cost function $c\left(q_{\omega}\right)$ is measured in labor units, paid at wage rate $w$. For simplicity, assume that $c\left(q_{\omega}\right)=1$. When $j=i$, we can solve the profit maximization problem by substituting (2) into (4) and differentiating with respect to $p_{j}\left(\omega_{i}\right)$. This reveals that price is the standard mark-up over marginal cost:

$$
\begin{equation*}
p_{i}\left(\omega_{i}\right)=w_{i}\left(\frac{\sigma}{\sigma-1}\right) \tag{5}
\end{equation*}
$$

When $j \neq i$, firms incur the standard "iceberg" transportation costs when they ship their output to the foreign market. The firm must produce $\tau$ units of output for every unit they sell in the foreign market. The firm therefore solves

$$
\begin{equation*}
\max _{p(\omega), q_{\omega}} \pi_{j}\left(\omega_{i}\right)=p_{j}\left(\omega_{i}\right) x_{j}(\omega)-w_{i} c\left(q_{\omega}\right) \tau x_{j}(\omega) \tag{6}
\end{equation*}
$$

Substituting (2) into (6) and solving for the profit maximizing price yields:

$$
\begin{equation*}
p_{j}\left(\omega_{i}\right)=\tau w_{i}\left(\frac{\sigma}{\sigma-1}\right)=\tau p_{i}\left(\omega_{i}\right) \tag{7}
\end{equation*}
$$

We can use (2) and (7) to calculate the revenue firms from country $i$ earn in each market:

$$
\begin{align*}
& p_{i}\left(\omega_{i}\right) x_{i}\left(q_{\omega_{i}}\right)=p_{i}\left(\omega_{i}\right)^{1-\sigma} \lambda\left(q_{\omega_{i}}\right) \frac{E_{i}}{\tilde{P}_{i}^{1-\sigma}}  \tag{8}\\
& p_{j}\left(\omega_{i}\right) x_{j}\left(q_{\omega_{i}}\right)=p_{j}\left(\omega_{i}\right)^{1-\sigma} \lambda\left(q_{\omega_{i}}\right) \frac{E_{j}}{\tilde{P}_{j}^{1-\sigma}} \tag{9}
\end{align*}
$$

Substituting (7) into (9) and (2) yields:

$$
\begin{equation*}
p_{j}\left(\omega_{i}\right) x_{j}\left(q_{\omega_{i}}\right)=\tau p_{i}\left(\omega_{i}\right) x_{j}\left(q_{\omega_{i}}\right)=\left\{\tau p_{i}\left(\omega_{i}\right)\right\}^{1-\sigma} \lambda\left(q_{\omega_{i}}\right) \frac{E_{j}}{\tilde{P}_{j}^{1-\sigma}} \tag{10}
\end{equation*}
$$

Firm profit in its home market is calculated as:

$$
\pi_{i}\left(\omega_{i}\right)=p_{i}\left(\omega_{i}\right) x_{i}\left(\omega_{i}\right)-w_{i} x_{i}\left(\omega_{i}\right)
$$

Substituting from (5) yields:

$$
\begin{equation*}
\pi_{i}\left(\omega_{i}\right)=p_{i}\left(\omega_{i}\right) x_{i}\left(\omega_{i}\right)\left[1-\frac{\sigma-1}{\sigma}\right]=\frac{p_{i}\left(\omega_{i}\right) x_{i}\left(\omega_{i}\right)}{\sigma} \tag{11}
\end{equation*}
$$

So profits are simply a constant fraction of total revenues. We can perform a similar calculation to find the profit a firm earns in a foreign market:

$$
\pi_{j}\left(\omega_{i}\right)=p_{j}\left(\omega_{i}\right) x_{j}\left(\omega_{i}\right)-\tau w_{i} x_{j}\left(\omega_{i}\right)
$$

Substituting from (6) yields:

$$
\begin{equation*}
\pi_{j}\left(\omega_{i}\right)=\frac{p_{j}\left(\omega_{i}\right) x_{j}\left(\omega_{i}\right)}{\sigma} \tag{12}
\end{equation*}
$$

Equations (11) and (12) show that firm profit depends on its choice of output quality. The specification of consumer preferences adopted here means that firms must choose either high $\left(q_{H}\right)$ or low $\left(q_{L}\right)$ quality. Following Podhorsky
(2010a), I assume that firms who choose to produce high quality goods must pay a fixed cost (denominated in labor units) to have them certified. Firms seeking certification incur the following fixed costs:

$$
\begin{equation*}
\delta(\theta)=\frac{\left(q_{H}-q_{L}\right)}{\theta} \tag{13}
\end{equation*}
$$

Fixed certification costs are increasing in the strictness of the standard $\left(q_{H}-q_{L}\right)$, but decreasing in the firm's productivity. Equations (11) and (12) tell us that profits are higher for high-quality firms at every productivity level, while (13) tells us that the cost of marketing high quality goods falls monotonically with productivity. This implies a cut-off productivity level $\left(\theta^{C}\right)$ beyond which the cost of producing and certifying high-quality goods is small enough to make $q_{H}$ the profitmaximizing level of quality.

Figure 1 illustrates this cut-off condition. Consider a firm deciding whether or not to sell high-quality output in its home market. If the firm sells low-quality output, it will earn a payoff equal to $\pi_{i}\left(q_{L}\right)$. If the firm decides to market highquality output, it will earn a payoff equal to $\pi_{i}\left(q_{H}\right)-w_{i} \delta(\theta)$. Equations (11) and (12) ensure that the payoffs associated with this strategy are non-decreasing and concave in productivity $(\theta)$. Firms with $\theta \in\left[\theta_{\min }, \theta^{C}\right)$ will choose to sell only lowquality products. Firms with $\theta \in\left[\theta^{C}, \infty\right)$ will pay for certification and sell highquality goods.

In keeping with the original HFT framework, firms also face a fixed export cost when they enter a foreign market. We can specify this as:

$$
\begin{equation*}
F_{X}(\theta)=\frac{F_{X}}{\theta} \tag{14}
\end{equation*}
$$

As with (11), we assume that fixed export costs are decreasing in productivity. ${ }^{2}$ If the firm sells output only in the domestic market, it will earn a payoff equal to $\pi_{i}\left(q_{\omega}\right)$. If the firm decides to sell in both the home and foreign markets, it will earn a payoff equal to $\pi_{i}\left(q_{\omega}\right)+\pi_{j}\left(q_{\omega}\right)-w_{i} F_{x}(\theta)$. The result is a cut-off condition similar to the one illustrated for certification. Figure 2 illustrates the profit associated with each strategy. Firms with $\theta \in\left[\theta_{\min }, \theta^{X}\right)$ will choose to serve only the domestic market. Firms with $\theta \in\left[\theta^{X}, \infty\right)$ will sink the fixed export cost and serve both the foreign and domestic markets.

## Model Equilibrium

The model structure outlined above implies firms must choose their export and certification strategies simultaneously. The following matrix illustrates the payoffs to each potential strategy for firm in country $i^{3}$ :

| No Certification | Certification |  |
| :---: | :---: | :---: |
| No Exports | $\pi_{i}\left(q_{L}\right)$ | $\pi_{i}\left(q_{H}\right)-\delta(\theta)$ |
|  | $(\mathrm{LN})$ | $(\mathrm{HN})$ |
| Exports | $\pi_{i}\left(q_{L}\right)+\pi_{j}\left(q_{L}\right)-F_{x}(\theta)$ <br> $(\mathrm{LE})$ | $\pi_{i}\left(q_{H}\right)+\pi_{j}\left(q_{H}\right)-\delta(\theta)-F_{x}(\theta)$ |
|  |  | $(\mathrm{HE})$ |
|  |  |  |

[^1]The highest productivity firms will always choose strategy HE. To see this, note that equations (11) and (12) imply operating profit in any given market is always positive. Equations (8) and (9) imply that operating profit is always increasing in output quality. From the definition of $G(\theta)$, we can see that the support of $G(\theta)$ is such that $\theta \epsilon[\underline{\theta}, \infty)$. As $\theta$ approaches infinity, $F_{x}(\theta)$ and $\delta(\theta)$ go to zero. If we can ignore fixed costs, then firms will always maximize profit by selling high-quality output in as many markets as possible. Similarly, $F_{x}(\theta)$ and $\delta(\theta)$ go to infinity as $\theta$ approaches $\underline{\theta}$, for very small values of $\underline{\theta}$. These firms will maximize profits by minimizing fixed costs, selling only low quality output in the domestic market.

If we place some reasonable restrictions on certain model parameters, it is possible for a subset of firms to adopt the strategy in either the lower-left or upperright hand corners of the above matrix. However, if one of these intermediate strategies is chosen, it will necessarily dominate the other over the relevant range of $\theta$ (see C and D in the appendix). Allowing for both of these intermediate cases means there are two possible definitions of the model equilibrium, depending on which intermediate case dominates the other.

Case 1: LN/LE/HE Equilibrium
Assume that model parameters are set such that firms must choose among strategies LN, LE and HE, as described in the table above. We can proceed with the definition of the model equilibrium using three pieces of information. First, we can use the payoff matrix to define the productivity cut-offs separating each strategy.

Call $\theta^{A}$ the productivity that satisfies:

$$
\pi_{i}\left(q_{L}\right)+\pi_{j}\left(q_{L}\right)-w_{i} F_{x}\left(\theta^{A}\right)=\pi_{i}\left(q_{L}\right)
$$

Or,

$$
\begin{equation*}
\pi_{j}\left(q_{L}\right)=w_{i} F_{x}\left(\theta^{A}\right) \tag{15}
\end{equation*}
$$

This expression defines the firm that is indifferent between selling in the domestic market and sinking $F_{x}(\theta)$ to sell output in both the foreign and domestic markets, given it will only be selling low-quality output.

Call $\theta^{B}$ the productivity that satisfies:

$$
\pi_{i}\left(q_{L}\right)+\pi_{j}\left(q_{L}\right)-w_{i} F_{x}\left(\theta^{B}\right)=\pi_{i}\left(q_{H}\right)+\pi_{j}\left(q_{H}\right)-w_{i} \delta\left(\theta^{B}\right)-w_{i} F_{x}\left(\theta^{B}\right)
$$

Or,

$$
\begin{equation*}
\left[\pi_{i}\left(q_{H}\right)-\pi_{i}\left(q_{L}\right)\right]+\left[\pi_{j}\left(q_{H}\right)-\pi_{j}\left(q_{L}\right)\right]=w_{i} \delta\left(\theta^{B}\right) \tag{16}
\end{equation*}
$$

This expression defines the firm that is indifferent between selling low-quality and sinking $\delta(\theta)$ to sell high-quality goods, given it will sell in both the domestic and foreign markets.

Finally, the model equilibrium is defined by a zero-profit condition, as in Melitz (2003). Firms do not know their productivity draw before they enter the differentiated product sector, but they do know their expected level of operating profit and the expected costs associated with each strategy. Assume further that firms must sink a fixed entry $\operatorname{cost}\left(F_{E}\right)$, denominated in labor units, to enter the industry. Firms will continue to enter until their expected profit, net of their
expected fixed costs, exactly equals the fixed cost of entry. Defining expected operating profits as $E[\pi]$, we can express this condition as:

$$
\begin{equation*}
E_{i}[\pi]-w_{i} E\left[F_{x}(\theta)\right]-w_{i} E[\delta(\theta)]=w_{i} F_{E} \tag{17}
\end{equation*}
$$

Equations (15), (16) and (17) allow us to define $\theta^{A}, \theta^{B}$ and the equilibrium mass of industry entrants $(M)$ in terms of model parameters. As shown in the appendix, making the appropriate series of substitutions yields an expression that defines the export cut-off $\left(\theta^{A}\right)$ only in terms of model parameters (see $C$ in the appendix):

$$
\begin{align*}
& \left(\theta^{A}\right)^{-1} F_{x}\left\{\frac{(2 s+1) \lambda\left(q_{L}\right)-\left[1+\tau^{1-\sigma}\right] s \lambda\left(q_{H}\right)}{\lambda\left(q_{L}\right)(s+1) \tau^{1-\sigma}}\right\}+\left(\theta^{A}\right)^{-(s+1)} F_{x} \\
& \quad+\left(\theta^{A}\right)^{-(s+1)}\left[\frac{\left(\lambda\left(q_{H}\right)-\lambda\left(q_{L}\right)\right)}{\lambda\left(q_{L}\right)}\left[1+\tau^{\sigma-1}\right]\right]^{s+1} \frac{F_{x}^{s+1}}{\left(q_{H}-q_{L}\right)^{s}}=F_{E} \tag{18}
\end{align*}
$$

The model yields no algebraic closed-form solution, but it is still possible to demonstrate the uniqueness and existence of the equilibrium. Call the left-hand side of (15f) $H\left(\theta^{A}\right)$. Assume parameters are fixed such that the first bracketed term in $H\left(\theta^{A}\right)$ is strictly non-negative. It is straightforward to see that $H\left(\theta^{A}\right)$ approaches some positive value as $\theta^{A} \rightarrow \underline{\theta}$, assuming $F_{E}$ is not too high. We can also see that $H\left(\theta^{A}\right)$ monotonically approaches zero as $\theta^{A} \rightarrow \infty$. As long as $F_{E}$ is not too high, then (15f) identifies the unique equilibrium value of $\theta^{A}$ for this model. This equilibrium is illustrated in figure 3. Having identified $\theta^{A}$, we can use (C7) to identify the corresponding equilibrium cut-off for HE:

$$
\begin{equation*}
\theta^{B}=\theta^{A} \frac{\lambda\left(q_{L}\right)}{F_{X}\left[1+\tau^{\sigma-1}\right]} \frac{q_{H}-q_{L}}{\lambda\left(q_{H}\right)-\lambda\left(q_{L}\right)} \tag{C7a}
\end{equation*}
$$

We can also identify an expression for $\theta^{B}$ in terms of only model parameters by making a series of substitutions similar to those we used to derive (18). The appropriate procedure is described briefly in the appendix. The resulting expression is:

$$
\begin{gather*}
\left(\theta^{B}\right)^{-1}\left[q_{H}-q_{L}\right]\left\{\frac{(2 s+1) \lambda\left(q_{L}\right)-\left[1+\tau^{1-\sigma}\right] s \lambda\left(q_{H}\right)}{\left[\lambda\left(q_{H}\right)-\lambda\left(q_{L}\right)\right]\left[1+\tau^{1-\sigma}\right](s+1)}\right\}+\left(\theta^{B}\right)^{-(s+1)}\left[q_{H}-q_{L}\right] \\
+\left(\theta^{B}\right)^{-(s+1)}\left(\frac{\left[q_{H}-q_{L}\right] \lambda\left(q_{L}\right) \tau^{1-\sigma}}{\left[\lambda\left(q_{H}\right)-\lambda\left(q_{L}\right)\right]\left[1+\tau^{1-\sigma}\right]}\right)^{s+1} F_{x}^{-s}=F_{E} \tag{19}
\end{gather*}
$$

Define $H\left(\theta^{B}\right)$ as the left-hand side of (19). Once again, we can see that $H\left(\theta^{B}\right)$ defines a unique equilibrium value of $\theta^{B}$ as long as $F_{E}$ is not too high. Figure 3 illustrates the determination of the equilibrium cut-offs using (18) and (19). Equilibrium cut-offs can be found where $H\left(\theta^{A}\right)=H\left(\theta^{B}\right)=F_{E}$. Equilibrium exists as long as $F_{E}$ is not too large, so that the points of intersection occur at some $\theta^{B}>\theta^{B} \geq \underline{\theta}$.

We can also find the equilibrium mass of entrants to the differentiated products sector using (17) and the equilibrium values of $\theta^{A}$ and $\theta^{B}$ :

$$
\begin{equation*}
M=\frac{L}{\sigma\left\{F_{E}+\left(\frac{s}{s+1}\right)\left(\frac{[q h-q]]}{\theta^{B}}+\frac{F_{X}}{\theta^{A}}\right)\right\}} \tag{20}
\end{equation*}
$$

Figure 4 illustrates the full model equilibrium in productivity and profit space. The payoff associated with each strategy is shown as a concave function. While LN is constant with respect to productivity, LE and HE are both monotonically increasing in productivity. Strategies LE and HE are everywhere steeper in slope than LN, but these payoff functions are shifted downward due to their associated fixed costs.

Strategy HE slopes everywhere more steeply than LE, so this strategy will come to dominate over higher ranges of $\theta$.

Case 2: LN/HN/HE Equilibrium
Assume model parameters are set such that firms must choose among the strategies labeled LN, HN, or HE. As in the previous case, we have three pieces of information to help us define the model equilibrium. We can use the payoff matrix to define the cut-off productivities separating each strategy.

Call $\theta^{C}$ the productivity that satisfies:

$$
\pi_{i}\left(q_{L}\right)=\pi_{i}\left(q_{H}\right)-w_{i} \delta\left(\theta^{C}\right)
$$

or,

$$
\begin{equation*}
w_{i} \delta\left(\theta^{C}\right)=\pi_{i}\left(q_{H}\right)-\pi_{i}\left(q_{L}\right) \tag{21}
\end{equation*}
$$

This expression defines the firm that is indifferent between selling low-quality and high quality goods, given it will only sell in the home market.

Call $\theta^{D}$ the productivity that satisfies:

$$
\pi_{i}\left(q_{H}\right)-w_{i} \delta\left(\theta^{D}\right)=\pi_{i}\left(q_{H}\right)+\pi_{j}\left(q_{H}\right)-w_{i} \delta\left(\theta^{D}\right)-w_{i} F_{x}\left(\theta^{D}\right)
$$

or,

$$
\begin{equation*}
\pi_{j}\left(q_{H}\right)=w_{i} F_{x}\left(\theta^{D}\right) \tag{22}
\end{equation*}
$$

This expression defines the firm that is indifferent between selling only in the home market and selling in both the home and foreign markets, given it will be selling only high-quality goods.

We can use the same zero-profit condition (17) as in the previous case to close the model. Equations (17), (21) and (22) allow us to define $\theta^{C}, \theta^{D}$ and the 13
equilibrium mass of industry entrants $(M)$. As shown in the appendix, making the appropriate series of substitutions yields an expression that defines the export cutoff $\left(\theta^{D}\right)$ only in terms of model parameters (see $D$ in the appendix):

$$
\begin{align*}
& \left(\theta^{D}\right)^{-1} F_{X}\left\{\frac{(2 s+1) \lambda\left(q_{L}\right)-\left(1+\tau^{1-\sigma}\right) s \lambda\left(q_{H}\right)}{\lambda\left(q_{H}\right) \tau^{1-\sigma}(s+1)}\right\}+\left(\theta^{D}\right)^{-(s+1)} F_{X} \\
& \quad+\left(\theta^{D}\right)^{-(s+1)}\left\{\frac{F_{X}}{\tau^{1-\sigma}} \frac{\left[\lambda\left(q_{H}\right)-\lambda\left(q_{L}\right)\right]}{\lambda\left(q_{H}\right)}\right\}^{s+1}\left[q_{H}-q_{L}\right]^{-s}-F_{E}=0 \tag{23}
\end{align*}
$$

Once again, the model yields no algebraic closed form solution, but we can still establish the uniqueness and existence of the equilibrium. Call the left-hand side of (23) $H\left(\theta^{D}\right)$. Once again, assume parameters are fixed such that the first bracketed term in $H\left(\theta^{D}\right)$ is positive ${ }^{4} . H\left(\theta^{D}\right)$ is monotonically decreasing in $\theta^{D}$ and approaches some positive value as $\theta^{D} \rightarrow \underline{\theta} . H\left(\theta^{D}\right)$ also approaches zero as $\theta^{D} \rightarrow \infty$. This implies that a unique equilibrium $\theta^{D}$ exists as long as $F_{E}$ is not too high.

We can use the value of $\theta^{D}$ implied by (23) to solve for the other endogenous variables in the model. From (D5a) we have:

$$
\begin{equation*}
\theta^{C}=\theta^{D} \frac{\lambda\left(q_{H}\right) \tau^{1-\sigma}}{F_{X}} \frac{q_{H}-q_{L}}{\lambda\left(q_{H}\right)-\lambda\left(q_{L}\right)} \tag{D5b}
\end{equation*}
$$

Alternatively, we can make the appropriate series of substitutions to derive a condition that defines $\theta^{C}$ in terms of only model parameters.

[^2]The appendix demonstrates briefly how to derive this condition:

$$
\begin{gather*}
\left(\theta^{C}\right)^{-1}\left(q_{H}-q_{L}\right)\left\{\frac{(2 s+1) \lambda\left(q_{L}\right)-s \lambda\left(q_{H}\right)\left(1+\tau^{1-\sigma}\right)}{\left[\lambda\left(q_{H}\right)-\lambda\left(q_{L}\right)\right](s+1)}\right\}+\left(\theta^{C}\right)^{-(s+1)}\left(q_{H}-q_{L}\right) \\
+\left(\theta^{C}\right)^{-(s+1)}\left\{\frac{\left[q_{H}-q_{L}\right] \tau^{1-\sigma} \lambda\left(q_{H}\right)}{\left[\lambda\left(q_{H}\right)-\lambda\left(q_{L}\right)\right]}\right\}^{s+1} F_{X}^{-s}=F_{E} \tag{24}
\end{gather*}
$$

Defining $H\left(\theta^{C}\right)$ as the left-hand side of (24), we can see this expression defines a unique equilibrium value of $\theta^{C}$ as long as $F_{E}$ is not too high. Figure 5 illustrates the determination of the equilibrium cut-offs using (23) and (24). Equilibrium cut-offs can be found where $H\left(\theta^{C}\right)=H\left(\theta^{D}\right)=F_{E}$. Equilibrium exists as long as $F_{E}$ is not too large, so the points of intersection occur at some $\theta^{D}>\theta^{C} \geq \underline{\theta}$.

We can then (17) along with the equilibrium values of $\theta^{C}$ and $\theta^{D}$ to find the equilibrium mass of entrants (M):

$$
\begin{equation*}
M=\frac{L}{\sigma\left\{F_{E}+\left(\frac{s}{s+1}\right)\left(\frac{[q h-q]]}{\theta^{C}}+\frac{F_{X}}{\theta^{D}}\right)\right\}} \tag{25}
\end{equation*}
$$

Figure 6 illustrates the model equilibrium in productivity and profit space. As before, the profit associated with each strategy is a concave function of productivity. LN is constant, but HN and HE are both monotonically increasing in productivity. Strategies HN and HE are everywhere steeper in slope than LN, but these payoffs are are shifted downward due to their associated fixed costs. Strategy HE slopes everywhere more steeply than HN, so this strategy will come to dominate over higher ranges of $\theta$.

Determining the Prevailing Intermediate Strategy:
We have demonstrated the uniqueness and existence of the model
equilibrium when either intermediate strategy emerges. However, we have not yet described how to determine which intermediate strategy will prevail. Intuitively, the relative magnitudes of the trade and the certification costs will determine how "quickly" firms begun exporting or certifying their output. If certification is expensive relative to the additional profit firms receive from selling high-quality output, firms in the lower ranges of $\theta$ will be more likely to $\operatorname{sink} F_{X}(\theta)$ and enter export markets, instead. Conversely, if exporting is expensive relative to the additional profit from selling output in an additional market, firms in the lower ranges of $\theta$ will be more likely to $\operatorname{sink} \delta(\theta)$ and increasing output quality.

We can make this comparison more concrete by examining (D5) and (C7). Rearranging terms in (C7) yields:

$$
\begin{equation*}
\frac{\theta^{B}}{\theta^{A}}=\frac{\left(q_{H}-q_{L}\right)}{\left[\lambda\left(q_{H}\right)-\lambda\left(q_{L}\right)\right]} \frac{\lambda\left(q_{L}\right)}{F_{X}\left(1+\tau^{\sigma-1}\right)} \tag{26}
\end{equation*}
$$

We know $\theta^{B}>\theta^{A}$, which implies:

$$
\begin{equation*}
\frac{\left(q_{H}-q_{L}\right)}{\left[\lambda\left(q_{H}\right)-\lambda\left(q_{L}\right)\right]}>\frac{F_{X}\left(1+\tau^{\sigma-1}\right)}{\lambda\left(q_{L}\right)} \tag{26a}
\end{equation*}
$$

According to this expression, the cost of certification for a given level of productivity, relative to the additional profit from increasing output quality, must be higher than the cost of entering the export market, relative the benefits of selling low quality output in both markets. This makes certification a less appealing option for firms in lower ranges of productivity, which leads them to adopt the LE strategy over the HN strategy.

We can find a similar expression using (D5):

$$
\begin{equation*}
\frac{\theta^{D}}{\theta^{C}}=\frac{F_{X}{ }^{\sigma-1}}{\lambda\left(q_{H}\right)} \frac{\left[\lambda\left(q_{H}\right)-\lambda\left(q_{L}\right)\right]}{\left(q_{H}-q_{L}\right)} \tag{27}
\end{equation*}
$$

We know $\theta^{D}>\theta^{C}$, which implies:

$$
\begin{equation*}
\frac{\left(q_{H}-q_{L}\right)}{\left[\lambda\left(q_{H}\right)-\lambda\left(q_{L}\right)\right]}<\frac{F_{X} \tau^{\sigma-1}}{\lambda\left(q_{H}\right)} \tag{27a}
\end{equation*}
$$

This expression states roughly the inverse of (20a). In order for a firm to choose the HN strategy over the LE strategy, the cost of certification, relative to its benefits, must be low compared to the cost of exporting, relative to its benefits. The right-hand side of (26a) is strictly greater than the right-hand side of (27a), so these represent two mutually-exclusive statements. Since no parameterization of the model can satisfy both (26a) and (27a), only one of these intermediate strategies can be adopted in equilibrium.

## Comparative Statics:

Although the model yields no closed-form algebraic solution for the cut-off productivities, it is still possible to derive comparative statics for the policy-relevant variables in the model. We assume $q_{H}$ is set by an independent agency, so the parameters that might be of interest to policy-makers in include $F_{E}, F_{X}$ and $\tau$. Part E of the appendix shows how to derive comparative statics for each of these variables using equations (18), (19), (23) and (24).

## Fixed Entry Costs

First, we can show how the equilibrium cut-offs vary with $F_{E}$. Recall that $F_{E}$ is the fixed cost of entering the differentiated products sector. Changing $F_{E}$ is
analogous to raising or lowering the barriers to entry to the industry. As shown in part E of the appendix, deriving the comparative static $\left(d \theta^{i} / d F_{E}\right)$ requires totally differentiating the expression $Q\left(\theta^{i}\right)=H\left(\theta^{i}\right)-F_{E}=0$ with respect to $F_{E}$ and $\theta^{i}$ for $i=A, B, C, D$. The resulting expression is:

$$
\begin{equation*}
\frac{d \theta^{i}}{d F_{E}}=-\left[\frac{\frac{\partial Q\left(\theta^{i}\right)}{\partial F_{E}}}{\frac{\partial Q\left(\theta^{i}\right)}{\partial \theta^{i}}}\right], \quad i=A, B, C, D . \tag{28}
\end{equation*}
$$

The resulting comparative are:

$$
\begin{equation*}
\frac{d \theta^{A}}{d F_{E}}<0, \frac{d \theta^{B}}{d F_{E}}<0, \frac{d \theta^{C}}{d F_{E}}<0, \frac{d \theta^{D}}{d F_{E}}<0 \tag{E7}
\end{equation*}
$$

Figure Raising the barriers to entry to the differentiated products sector will increase rates of participation in both the voluntary standard and export markets. These comparative statics are driven by general equilibrium effects that are not obvious from simply looking at the payoff functions. Examining (20) and (25), we can see the equilibrium number of entrants is decreasing in $F_{E}$ and increasing in $\theta^{i}$ for all $i=A, B, C, D$. This implies the net effect of an increase in $F_{E}$ is to discourage entry, as we would expect. Fewer entrants means a less competitive marketplace, which will raise the profit level of all successful entrants. Firms that were previously just shy of the productivity cut-offs for exporting and certification will now find themselves sufficiently profitable to justify sinking the associated fixed costs.

This implies that the average level of quality produced in the home country
increases with fixed entry costs, but raising the barriers to entry will also decrease the number of firms entering the differentiated products sector. If policy-makers are interested in maximizing the number of domestic certified or export-oriented firms, they would have to balance the increased rates of export participation and certification against the decreased entry to the differentiated products sector.

## Fixed Export Costs

Fixed export costs can be interpreted as institutional or other non-tariff barriers firms must overcome to enter an export market. We can perform a similar analysis to see how the productivity cut-offs change with $F_{X}$, the fixed export cost. As before, we can find this comparative static by totally differentiating $H\left(\theta^{i}\right)$ for all and evaluating:

$$
\begin{equation*}
\frac{d \theta^{i}}{d F_{X}}=-\left[\frac{\frac{\partial H\left(\theta^{i}\right)}{\partial F_{X}}}{\frac{\partial H\left(\theta^{i}\right)}{\partial \theta^{i}}}\right], \quad i=A, B, C, D . \tag{23}
\end{equation*}
$$

The derivation for each cut-off can be found in the appendix. The results are as follows:

$$
\frac{d \theta^{A}}{d F_{X}}>0, \frac{d \theta^{B}}{d F_{X}}<0, \frac{d \theta^{C}}{d F_{X}}<0, \frac{d \theta^{D}}{d F_{X}}>0
$$

Recalling $\theta^{A}$ and $\theta^{D}$ correspond to export cut-offs, the signs of their corresponding comparative statics should not be surprising. Raising $F_{X}$ makes exporting more expensive. Firms that were previously indifferent between exporting and not exporting will choose to serve only the domestic market.

The signs on the comparative statics for $\theta^{C}$ and $\theta^{B}$ are less intuitive. These both represent certification cut-offs. $\theta^{B}$ is the certification cut-off conditional on participating in export markets, while $\theta^{C}$ is the certification cut-off conditional on not participating in export markets. In neither case will a (small) change in $F_{X}$ induce a change in exporting behavior. For $\theta^{C}$, an increase in $F_{X}$ will lower the profits associated with the HE strategy, but it will not lower profits relative to the those associated with the LE strategy. Firms with $\theta$ close to $\theta^{D}$ will not $\operatorname{sink} F_{X}$ regardless of whether it increases or decreases. Changes in $F_{X}$ should therefore have no direct effect on a firm's certification strategy. The relationship between the certification cut-offs and $F_{X}$ must therefore operate through the CES price indices. We know $\frac{d \theta^{A}}{d F_{X}}>0$ and $\frac{d \theta^{D}}{d F_{X}}>0$, so raising $F_{X}$ will reduce the number of foreign firms entering the home market. This will make the home market less competitive overall, and raise profits for domestic firms. Given a higher level of profit at every level of productivity, domestic firms with $\theta$ just below the previous certification cut-off will now be willing to adopt the voluntary certification.

## Transportation Costs

Increasing transportation costs increases the per-unit costs a domestic firm must pay to sell their output in the foreign country. This makes the comparative statics for transportation costs of particular interest because they are a close analogy to tariff barriers in the model.

The comparative statics for the transportation costs can be found by evaluating:

$$
\begin{equation*}
\frac{d \theta^{i}}{d \tau}=-\left[\frac{\frac{\partial H\left(\theta^{i}\right)}{\partial \tau}}{\frac{\partial H\left(\theta^{i}\right)}{\partial \theta^{i}}}\right], \quad i=A, B, C, D \tag{24}
\end{equation*}
$$

The derivation of each comparative static can be found in the appendix. The results indicate that the comparative statics for export cut-offs $\theta^{A}$ and $\theta^{D}$ are unambiguous:

$$
\begin{equation*}
\frac{d \theta^{A}}{d \tau}>0, \frac{d \theta^{D}}{d \tau}>0 \tag{E17}
\end{equation*}
$$

As with $F_{X}$, raising transportation costs unambiguously raises the export cut-offs. The intuition behind this result is simple: raising the costs associated with shipping each unit to the foreign market makes domestic firms less willing to engage in export markets.

The effect of changes in $\tau$ on the certification cut-offs is more ambiguous. As we can see from the appendix, it is possible to impose restrictions on the relative magnitudes of certain model parameters to make the comparative statics for $\tau$ mirror those for $F_{X}$. This result would be reasonable for $\theta^{B}$, where firms near the certification cut-off will not pay $\tau$ regardless of whether it increases or decreases. The primary effect on the certification decision would therefore be through decreased competitiveness in the domestic market as fewer foreign firms enter. An increase in $\tau$ would therefore lead to a decrease in the certification productivity cutoff for import-competing firms: $\frac{d \theta^{C}}{d \tau}<0$.

There is good reason to believe export-competing firms might be more willing to undertake certification if transportation costs fall. We know that increases in $F_{X}$ will lower the certification cut-off for export-competing firms $\left(\theta^{B}\right)$. While export-competing firms considering certification must pay $F_{X}$, the effect of an increase in $F_{X}$ is the same whether they sell high-quality or low-quality goods. This means there is no direct change in the relative profitability of the LE and HE strategies. The same is not true for $\tau$. From equation (6), we can see that changes in $\tau$ affect price-setting behavior in the foreign market. When $\tau$ increases, firms must set a higher nominal price in the foreign market. This will shrink market share and profits, and as we can see from (8a), they will shrink faster for firms producing highquality output. While firms will still indirectly benefit from the general equilibrium effects of decreased market competitiveness, the direct effect will be to discourage investment in the voluntary certification. If the latter effect is sufficiently large, then an increase in $\tau$ will decrease the rate of certification adoption among exportcompeting firms: $\frac{d \theta^{B}}{d \tau}>0$.

## Model Simulation

We can illustrate the model equilibrium and comparative statics using a simple numerical simulation of (18), (19), (23) and (24). Simulations of the baseline equilibrium were performed using the following parameter values:

## Baseline Simulation Parameter Values

| LN/LE/HE Case | LN/HN/HE Case |
| :---: | :---: |
| $q_{L}=10$ | $q_{L}=10$ |
| $q_{H}=12$ | $q_{H}=12$ |
| $\alpha=1.5$ | $\alpha=1.5$ |
| $\sigma=1.8$ | $\sigma=1.2$ |
| $s=1.05$ | $s=1.05$ |
| $\tau=1.1$ | $\tau=1.1$ |
| $F_{X}=2$ | $F_{X}=12$ |
| $F_{E}=2$ | $F_{E}=6$ |

Figure 7 illustrates the determination of the baseline equilibrium productivity cut-offs in the LN/LE/HE case. Figure 8 illustrates the determination of the baseline equilibrium productivity cut-offs in the LN/HN/HE case. In each case, equilibrium is determined by the value of $\theta$ at which the dotted line representing $F_{E}$ crosses the downward sloping $H\left(\theta^{i}\right)$ curves. Examining the figures, we can see that the parameter values specified above yield unique values of $\theta^{A}, \theta^{B}, \theta^{C}$ and $\theta^{D}$ that satisfy $\theta^{D}>\theta^{C}$ and $\theta^{A}>\theta^{B}$.

Figures 9 and 10 illustrate changes in the model equilibrium for changes in $F_{E}$, the fixed entry cost. In Figure $9, F_{E}$ increases from 2 to 2.5 . Since $H\left(\theta^{i}\right)$ is strictly decreasing in $\theta^{i}$ for each $i=A, B$, the increase in $F_{E}$ will decrease both equilibrium productivity cut-offs. In Figure 10, $F_{E}$ increases from 6 to 7. The result is qualitatively similar to the case shown in Figure 9; both equilibrium productivity cut-offs will fall. Note that large changes in $F_{E}$ can destabilize the model entirely. No equilibrium can be found if $F_{E} \geq 4$ in Figure 9 or $F_{E} \geq 9$ in Figure 10.

Figures 11 and 12 illustrate changes in the model equilibrium for changes in
$F_{X}$, the fixed export cost. In Figure 11, $F_{X}$ increases from 2 to $2.5 . H\left(\theta^{A}\right)$ shifts outward as firms find it more expensive to enter export markets at every productivity level. $H\left(\theta^{B}\right)$ shifts inward as firms considering certification benefit from the general equilibrium effects of operating in less competitive markets. The equilibrium value of $\theta^{A}$ increases and the equilibrium value of $\theta^{B}$ decreases. In Figure $12, F_{X}$ increases from 12 to 18 . Once again, the results are qualitatively similar to what we see in Figure 11. The equilibrium value of $\theta^{C}$ decreases while the equilibrium value of $\theta^{D}$ increases.

Figures 13 and 14 illustrate changes in the model equilibrium for changes in $\tau$, the transportation cost. In Figure 13, an increase in $\tau$ from 1.1 to 2.2 shifts $H\left(\theta^{A}\right)$ outward, but causes $H\left(\theta^{B}\right)$ to rotate around a particular value of $\theta^{B}$. This corresponds to the result in shown in equation $E(21)$, which implies the sign of the comparative static with respect to $\tau$ depends on the value of $\theta^{B}$ from which the model is deviating. Given the parameter values described above, an increase in $\tau$ will lead to an increase in $\theta^{B}$. If we were to increase $F_{E}$ to 3 , then an equivalent increase in $\tau$ would decrease the equilibrium value of $\theta^{B}$. In Figure 14, an increase in $\tau$ from 1.1 to 3.0 yields qualitatively similar results to what we see in Figure 12; the equilibrium value of $\theta^{C}$ decreases while the equilibrium value of $\theta^{D}$ increases. We could reverse the sign of the comparative static for $\theta^{C}$ by setting an extremely low value for $F_{E}$, but the result illustrated in Figure 14 is the more intuitive one.

## Conclusion

The model presented here allows us to reexamine the relationship between export participation and voluntary certification through the lens of the HFT framework. Firms are differentiated according to their productivity $(\theta)$, which indexes the ease with which they can sink the fixed costs associated with exporting and certification. Since heterogeneity is constrained to a single dimension, the model permits characterizing two separate equilibria: one where firms make their certification decision conditional on serving only the domestic market and one where firms make their certification decision conditional on participating in export markets.

While the model will not permit both scenarios to emerge under any given parameterization, analyzing the comparative statics under both equilibria can help us understand the forces that drive the relationship between export participation and voluntary certification. For import-competing firms, raising trade barriers can help encourage adoption of voluntary standards by protecting domestic firms from foreign competition. Greater domestic protection raises profits, which will encourage firms to sink the fixed costs associated with certification. The same result will not necessarily hold true for export-competing firms. Raising transportation costs $(\tau)$, analogous to raising tariff barriers, may make it more difficult for firms to recoup the fixed costs associated with adopting the voluntary certification.

The results presented here do not offer a definitive answer to the question of
whether or not freer trade can raise environmental or labor standards in the absence of perfect international legal institutions. In fact, they suggest that the relationship between voluntary certification and trade depends critically on market conditions, including the existing level of exposure to export markets. This might explain the difficulty the empirical literature has had explaining the relationship between trade liberalization and voluntary standards adoption in cross-country studies.

While this model can help us understand the forces at work in the relationship between trade liberalization and voluntary certification, there are several extensions that would help expand the set of model predictions. First, being unable to characterize an equilibrium with both export and import-competing certified firms is an unfortunate consequence of the model's simplifying assumptions. It also makes it more difficult to apply the model to a given country context, where these two cases are likely to coexist. We can avoid this problem by extending firm heterogeneity to two dimensions, or, more simply, specifying asymmetric fixed export costs for high-quality and low-quality firms.

The model would also be improved by relaxing the assumption of strict symmetry between the two countries. This would allow us to see what happens when key model parameters change in only one country at a time. Allowing for trade between a small, developing country and a large, developed country may also change the underlying relationship between liberalization and certification. This would be of particular interest because voluntary standards have been so widely
adopted in the developing world. Allowing for asymmetry between trade partners would help us understand whether or not voluntary standards can make trade liberalization an agent of positive social or environmental change where domestic regulators have failed.

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## Figures

Figure 1
Certification Cut-Off Productivity


Figure 2
Export Cut-Off Productivity


Figure 3
Equilibrium Productivity Cut-Offs in the LN/LE/HE Case


Figure 4
Equilibrium in the LN/LE/HE Case:


Figure 5
Equilibrium Productivity Cut-Offs in the LN/HN/HE Case


Figure 6
Equilibrium in the LN/HN/HE Case:


Figure 7
Model Equilibrium in the LN/LE/HE Case


Figure 8
Model Equilibrium in the LN/HN/HE Case


Figure 9
Comparative Statics for $F_{E}$ in the LN/LE/HE Case


Figure 10
Comparative Statics for $F_{E}$ in the LN/HN/HE Case


Figure 11
Comparative Statics for $F_{X}$ in the LN/LE/HE Case


Figure 12
Comparative Statics for $F_{X}$ in the LN/HN/HE Case


Figure 13
Comparative Statics for $\tau$ in the LN/LE/HE Case


Figure 14
Comparative Statics for $\tau$ in the LN/HN/HE Case


## Mathematical Appendix

A. Eliminating HN from the LE/LN/HE Case

We wish to show that, whenever any subset of firms chooses to export lowquality products, it must be that no firm would choose to sell high-quality products in their home market. If some firms choose the LE strategy, then there must exist some $\theta$ s.t.:

$$
\begin{equation*}
\pi_{i}\left(q_{L}\right)<\pi_{i}\left(q_{L}\right)+\pi_{j}\left(q_{L}\right)-w_{i} F_{X}(\theta) \tag{A1}
\end{equation*}
$$

Or,

$$
\begin{equation*}
w_{i} F_{X}(\theta)<\pi_{j}\left(q_{L}\right) \tag{A1a}
\end{equation*}
$$

This same range of $\theta$ must also satisfy:

$$
\begin{equation*}
\pi_{i}\left(q_{H}\right)+\pi_{j}\left(q_{H}\right)-w_{i} F_{X}(\theta)-w_{i} \delta(\theta)<\pi_{i}\left(q_{L}\right)+\pi_{j}\left(q_{L}\right)-w_{i} F_{X}(\theta) \tag{A2}
\end{equation*}
$$

Or

$$
\begin{equation*}
\left[\pi_{i}\left(q_{H}\right)-\pi_{i}\left(q_{L}\right)\right]+\left[\pi_{j}\left(q_{H}\right)-\pi_{j}\left(q_{L}\right)\right]<w_{i} \delta(\theta) \tag{A2a}
\end{equation*}
$$

Equations (A1a) and (A2a) jointly imply that the HN strategy is strictly dominated. In other words, they imply:

$$
\begin{equation*}
\pi_{i}\left(q_{H}\right)-w_{i} \delta(\theta)<\pi_{i}\left(q_{L}\right)+\pi_{j}\left(q_{L}\right)-w_{i} F_{X}(\theta) \tag{A3}
\end{equation*}
$$

Rearranging terms in (A3):

$$
\begin{equation*}
\left[\pi_{i}\left(q_{H}\right)-\pi_{i}\left(q_{L}\right)\right]-w_{i} \delta(\theta)<\pi_{j}\left(q_{L}\right)-w_{i} F_{X}(\theta) \tag{A3a}
\end{equation*}
$$

Equation (A2a) implies the left-hand side of (A3a) is strictly negative, given our result from (7) and (8) that operating profit is everywhere increasing in quality. Equation (A1a) implies the right-hand side of (A3a) is strictly positive. This ensures
(A3a) holds as long as (A1a) and (A2a) are true. Combined with the qausi-concavity and monotonicity of the payoffs described in the matrix, this ensures that the No Exports/Certification strategy will be strictly dominated over the whole range of $\theta$.
B. Elimination of LE from the LN/HN/HE Case

We wish to show that, whenever any subset of firms chooses to sell highquality products only in the domestic market, it must be that no firm would choose to export low-quality products. If some firms choose the No Export/Certification strategy, then there must exist some $\theta$ s.t.:

$$
\begin{equation*}
\pi_{i}\left(q_{l}\right)<\pi_{i}\left(q_{h}\right)-w_{i} \delta(\theta) \tag{B1}
\end{equation*}
$$

Or,

$$
\begin{equation*}
w_{i} \delta(\theta)<\pi_{i}\left(q_{h}\right)-\pi_{i}\left(q_{l}\right) \tag{B1a}
\end{equation*}
$$

This same range of $\theta$ must also satisfy:

$$
\begin{equation*}
\pi_{i}\left(q_{H}\right)+\pi_{j}\left(q_{H}\right)-w_{i} F_{X}(\theta)-w_{i} \delta(\theta)<\pi_{i}\left(q_{H}\right)-w_{i} \delta(\theta) \tag{B2}
\end{equation*}
$$

Or

$$
\begin{equation*}
\pi_{j}\left(q_{H}\right)<w_{i} F_{X}(\theta) \tag{B2a}
\end{equation*}
$$

Equations (B1a) and (B2a) jointly imply that the Export/No Certification strategy is strictly dominated. In other words, they imply:

$$
\begin{equation*}
\pi_{i}\left(q_{L}\right)+\pi_{j}\left(q_{L}\right)-w_{i} F_{X}(\theta)<\pi_{i}\left(q_{H}\right)-w_{i} \delta(\theta) \tag{B3}
\end{equation*}
$$

Rearranging terms in (B3):

$$
\begin{equation*}
\pi_{j}\left(q_{L}\right)-w_{i} F_{X}(\theta)<\pi_{i}\left(q_{H}\right)-\pi_{i}\left(q_{L}\right)-w_{i} \delta(\theta) \tag{B3a}
\end{equation*}
$$

Equation (B2a) implies the right-hand side of (B3a) is strictly negative.

Equation (B1a) implies the right-hand side of (B3a) is strictly positive. This ensures (B3) holds as long as (B1a) and (B2a) are true. Combined with the strict concavity and monotonicity of the payoffs described in the matrix, this ensures that the Exports/No Certification strategy will be strictly dominated over the whole range of $\theta$.
C. Definition of the Model Equilibrium in the LN/LE/HE Case

We can use (13), (14) and (15) to demonstrate the existence and uniqueness of the model equilibrium in the case where the strategies designated NN, NE, and EC dominate. To begin with, we must establish several preliminary results. Take the definition of the quality-adjusted CES price index:

$$
\begin{equation*}
\tilde{P}_{i}^{1-\sigma}=\int_{\omega \in \Omega_{i}} \lambda\left(q_{\omega}\right) \cdot p_{i}(\omega)^{1-\sigma} d \omega \tag{C1}
\end{equation*}
$$

For the two-country case, we can express the price index as:

$$
\begin{align*}
\tilde{P}_{i}^{1-\sigma}= & M_{i}\left\{\int_{\underline{\theta}_{i}}^{\theta_{i}^{A}} \lambda\left(q_{L}\right) \cdot p_{i}{ }^{1-\sigma} g(\theta) d \theta\right. \\
& \left.+\int_{\theta_{i}^{A}}^{\theta_{i}^{B}} \lambda\left(q_{L}\right) \cdot p_{i}{ }^{1-\sigma} g(\theta) d \theta+\int_{\theta_{i}^{B}}^{\infty} \lambda\left(q_{H}\right) \cdot p_{i}{ }^{1-\sigma} g(\theta) d \theta\right\} \\
& +M_{j}\left\{\int_{\theta_{j}^{A}}^{\theta_{j}^{B}} \lambda\left(q_{L}\right) \cdot\left(\tau p_{j}\right)^{1-\sigma} g(\theta) d \theta+\int_{\theta_{j}^{B}}^{\infty} \lambda\left(q_{H}\right) \cdot\left(\tau p_{j}\right)^{1-\sigma} g(\theta) d \theta\right\} \tag{C2}
\end{align*}
$$

Note the asymmetry between the domestic and foreign contributions to the price index: the index for country $i$ includes all country $i$ firms, but only includes the subset of country $j$ firms that opt into exporting. For simplicity, assume we are dealing with two symmetric countries, in the sense that $L_{i}=L_{j}$.

This implies we can rewrite (C2) as:

$$
\begin{aligned}
\tilde{P}^{1-\sigma}= & M p^{1-\sigma}\left\{\lambda\left(q_{L}\right) \int_{\underline{\theta}}^{\theta^{B}} g(\theta) d \theta+\lambda\left(q_{H}\right) \int_{\theta^{B}}^{\infty} g(\theta) d \theta\right\} \\
& +M(\tau p)^{1-\sigma}\left\{\lambda\left(q_{L}\right) \int_{\theta^{A}}^{\theta^{B}} g(\theta) d \theta+\lambda\left(q_{H}\right) \int_{\theta^{B}}^{\infty} g(\theta) d \theta\right\}
\end{aligned}
$$

Recalling the definition of the distribution function $G(\theta)$, we can rewrite this once again as:

$$
\begin{align*}
\tilde{P}^{1-\sigma} & =M p^{1-\sigma}\left\{\lambda\left(q_{L}\right) G\left(\theta^{B}\right)+\lambda\left(q_{H}\right)\left[1-G\left(\theta^{B}\right)\right]\right\} \\
& +M(\tau p)^{1-\sigma}\left\{\lambda\left(q_{L}\right)\left[G\left(\theta^{B}\right)-G\left(\theta^{A}\right)\right]+\lambda\left(q_{H}\right)\left[1-G\left(\theta^{B}\right)\right]\right\} \tag{C3}
\end{align*}
$$

For convenience, define:

$$
\begin{equation*}
Q_{i}=\lambda\left(q_{L}\right) G\left(\theta_{i}^{B}\right)+\lambda\left(q_{H}\right)\left[1-G\left(\theta_{i}^{B}\right)\right] \tag{C4}
\end{equation*}
$$

This represents the average quality level produced in a given country. Substituting from (C4), we can rewrite (C3) as:

$$
\tilde{P}^{1-\sigma}=M p^{1-\sigma}\left\{Q+\tau^{1-\sigma}\left(Q-\lambda\left(q_{L}\right) G\left(\theta^{A}\right)\right)\right\}
$$

Or,

$$
\begin{equation*}
\tilde{P}^{1-\sigma}=M p^{1-\sigma}\left\{\left(1+\tau^{1-\sigma}\right) Q-\tau^{1-\sigma} \lambda\left(q_{L}\right) G\left(\theta^{A}\right)\right\} \tag{C4}
\end{equation*}
$$

Substituting (2) into (9a) yields:

$$
\begin{equation*}
\pi_{i}\left(\omega_{i}\right)=p_{i}(\omega)^{1-\sigma} \lambda\left(q_{\omega}\right) \frac{E_{i}}{\sigma \tilde{P}_{i}^{1-\sigma}} \tag{C5}
\end{equation*}
$$

This is the profit a firm from country $i$ earns by selling output with quality $q_{\omega}$ in country i. Allowing for symmetry and substituting from (C4) yields:

$$
\begin{equation*}
\pi_{i}\left(\omega_{i}\right)=\lambda\left(q_{\omega}\right) \frac{E}{\sigma M\left\{\left(1+\tau^{1-\sigma}\right) Q-\tau^{1-\sigma} \lambda\left(q_{L}\right) G\left(\theta^{A}\right)\right\}} \tag{C5a}
\end{equation*}
$$

Similarly, we can define the profits a firm in country $i$ earns by selling output with quality $q_{\omega}$ in country $j$. Substituting (C4) into (8a) and allowing for symmetry yields:

$$
\begin{equation*}
\pi_{j}\left(\omega_{i}\right)=\tau^{1-\sigma} \lambda\left(q_{\omega}\right) \frac{E}{\sigma M\left\{\left(1+\tau^{1-\sigma}\right) Q-\tau^{1-\sigma} \lambda\left(q_{L}\right) G\left(\theta^{A}\right)\right\}} \tag{C4b}
\end{equation*}
$$

Substitute this result into (13):

$$
\begin{equation*}
\tau^{1-\sigma} \lambda\left(q_{L}\right) \frac{E}{\sigma M\left\{\left(1+\tau^{1-\sigma}\right) Q-\tau^{1-\sigma} \lambda\left(q_{L}\right) G\left(\theta^{A}\right)\right\}}=w_{i} F_{x}\left(\theta^{A}\right) \tag{C5}
\end{equation*}
$$

Rearranging terms:

$$
\begin{equation*}
\frac{L}{\sigma M\left\{\left(1+\tau^{1-\sigma}\right) Q-\tau^{1-\sigma} \lambda\left(q_{L}\right) G\left(\theta^{A}\right)\right\}}=\frac{F_{\chi}\left(\theta^{A}\right)}{\tau^{1-\sigma} \lambda\left(q_{L}\right)} \tag{C5a}
\end{equation*}
$$

Substituting (C4a) and (C4b) into (14) and rearranging terms yields:

$$
\begin{equation*}
\frac{L \cdot\left[\lambda\left(q_{H}\right)-\lambda\left(q_{L}\right)\right] \cdot\left[1+\tau^{1-\sigma}\right]}{\sigma M\left\{\left(1+\tau^{1-\sigma}\right) Q-\tau^{1-\sigma} \lambda\left(q_{L}\right) G\left(\theta^{A}\right)\right\}}=\delta\left(\theta^{B}\right) \tag{C6}
\end{equation*}
$$

Or,

$$
\begin{equation*}
\frac{L}{\sigma M\left\{\left(1+\tau^{1-\sigma}\right) Q-\tau^{1-\sigma} \lambda\left(q_{L}\right) G\left(\theta^{A}\right)\right\}}=\frac{\delta\left(\theta^{B}\right)}{\left[\lambda\left(q_{H}\right)-\lambda\left(q_{L}\right)\right] \cdot\left[1+\tau^{1-\sigma}\right]} \tag{C6a}
\end{equation*}
$$

Equating (C5a) and (C6a) yields:

$$
\begin{equation*}
\frac{\delta\left(\theta^{B}\right)}{\left[\lambda\left(q_{H}\right)-\lambda\left(q_{L}\right)\right] \cdot\left[1+\tau^{1-\sigma}\right]}=\frac{F_{\chi}\left(\theta^{A}\right)}{\tau^{1-\sigma} \lambda\left(q_{L}\right)} \tag{C7}
\end{equation*}
$$

This expression allows us to define $\theta^{B}$ in terms of $\theta^{A}$ and model parameters, and vice-versa. Defining the equilibrium requires deriving an expression that defines one of our variables of interest only in terms of model parameters. Given (C7), we only need one other expression defining $\theta^{C}$ and $\theta^{X}$ as a function of model parameters.

Finding such an expression requires making use of (15). We can express the expected operating profit term $\left(E_{i}[\pi]\right)$ as:

$$
\begin{gather*}
E_{i}[\pi] \equiv \int_{\underline{\theta}_{i}}^{\theta_{i}^{A}} \pi_{i}\left(q_{L}\right) g(\theta) d \theta+\int_{\theta_{i}^{A}}^{\theta_{i}^{B}}\left[\pi_{i}\left(q_{L}\right)+\pi_{j}\left(q_{L}\right)\right] g(\theta) d \theta \\
+\int_{\theta_{i}^{B}}^{\infty}\left[\pi_{i}\left(q_{H}\right)+\pi_{j}\left(q_{H}\right)\right] g(\theta) d \theta \tag{C8}
\end{gather*}
$$

Substituting from (C4a) and (C4b) and allowing for symmetry allows us to rewrite (C8) as:

$$
\begin{aligned}
E_{i}[\pi]= & \frac{L}{\sigma M\left\{\left(1+\tau^{1-\sigma}\right) Q-\tau^{1-\sigma} \lambda\left(q_{L}\right) G\left(\theta^{X}\right)\right\}}\left\{\lambda\left(q_{L}\right) \int_{\underline{\theta}_{i}}^{\theta_{i}^{A}} g(\theta) d \theta+\right. \\
& {\left[1+\tau^{1-\sigma}\right] \lambda\left(q_{L}\right) \int_{\theta_{i}^{A}}^{\theta_{i}^{B}} g(\theta) d \theta+\left[1+\tau^{1-\sigma}\right] \lambda\left(q_{H}\right) \int_{\theta_{i}^{B}}^{\infty} g(\theta) d \theta }
\end{aligned}
$$

Or,

$$
\begin{aligned}
& E_{i}[\pi]=\frac{E}{\sigma M\left\{\left(1+\tau^{1-\sigma}\right) Q-\tau^{1-\sigma} \lambda\left(q_{L}\right) G\left(\theta^{A}\right)\right\}}\left\{\lambda\left(q_{L}\right) G\left(\theta^{A}\right)+\right. \\
& \quad\left[1+\tau^{1-\sigma}\right] \lambda\left(q_{L}\right)\left[G\left(\theta^{B}\right)-G\left(\theta^{A}\right)\right]+\left[1+\tau^{1-\sigma}\right] \lambda\left(q_{H}\right)\left[1-G\left(\theta^{B}\right)\right]
\end{aligned}
$$

And finally, after substituting from (C3):

$$
\begin{equation*}
E_{i}[\pi]=\frac{E}{\sigma M} \tag{C8a}
\end{equation*}
$$

Substituting this into (15) yields:

$$
\frac{E}{\sigma M}-w_{i} E\left[F_{x}(\theta)\right]-w_{i} E[\delta(\theta)]=w_{i} F_{E}
$$

Or,

$$
\begin{equation*}
\frac{L}{\sigma M}-E\left[F_{x}(\theta)\right]-E[\delta(\theta)]=F_{E} \tag{15a}
\end{equation*}
$$

We can further simplify (15a) by evaluating the expected values of the fixed export and certification costs. Because only a subset of firms will sink $F_{x}(\theta)$ and
$\delta(\theta)$, we must evaluate the remaining terms in (15a) as conditional expectations.
The expected fixed export costs are therefore:

$$
\begin{equation*}
E\left[F_{x}(\theta)\right]=E\left[F_{x}(\theta) \mid \theta \geq \theta^{A}\right]=\int_{\theta^{A}}^{\infty} F_{x}(\theta) \mu(\theta) d \theta \tag{C9}
\end{equation*}
$$

Where $\mu(\theta) \equiv \frac{g(\theta)}{1-G\left(\theta^{X}\right)}$. Substituting this expression and (12) into (C9) yields:

$$
\begin{equation*}
E\left[F_{x}(\theta)\right]=\frac{F_{x}}{1-G\left(\theta^{A}\right)} \int_{\theta^{A}}^{\infty} \theta^{-1} g(\theta) d \theta \tag{C9a}
\end{equation*}
$$

From the definition of $G(\theta), g(\theta)=s \theta^{-(s+1)}$. This implies:

$$
E\left[F_{x}(\theta)\right]=\frac{s F_{\chi}}{\left(\theta^{A}\right)^{-s}} \int_{\theta^{A}}^{\infty} \theta^{-(s+2)} d \theta
$$

And finally,

$$
\begin{equation*}
E\left[F_{x}(\theta)\right]=\frac{s}{s+1} \frac{F_{x}}{\theta^{X}}=\frac{s}{s+1} F_{x}\left(\theta^{A}\right) \tag{C9b}
\end{equation*}
$$

We can derive a similar expression for the expected certification costs:

$$
\begin{equation*}
E[\delta(\theta)]=E\left[\delta(\theta) \mid \theta \geq \theta^{B}\right]=\int_{\theta^{B}}^{\infty} \delta(\theta) \mu(\theta) d \theta \tag{C10}
\end{equation*}
$$

Which implies:

$$
\begin{equation*}
E[\delta(\theta)]=\frac{s}{s+1} \frac{\left[q_{H}-q_{L}\right]}{\theta^{B}}=\frac{s}{s+1} \delta\left(\theta^{B}\right) \tag{C10a}
\end{equation*}
$$

Substituting (C9a) and (C10a) into (15a) yields:

$$
\begin{equation*}
\frac{L}{\sigma M}-\frac{s}{s+1} F_{x}\left(\theta^{A}\right)-\frac{s}{s+1} \delta\left(\theta^{B}\right)=F_{E} \tag{15b}
\end{equation*}
$$

This expression is now in terms of all three of our variables of interest: $M, \theta^{X}$ and $\theta^{C}$. We can proceed by substituting (C5a) into (15b):

$$
\begin{equation*}
\frac{F_{x}\left(\theta^{A}\right)}{\tau^{1-\sigma} \lambda\left(q_{L}\right)}\left\{\left(1+\tau^{1-\sigma}\right) Q-\tau^{1-\sigma} \lambda\left(q_{L}\right) G\left(\theta^{A}\right)\right\}-\frac{s}{s+1} F_{x}\left(\theta^{A}\right)-\frac{s}{s+1} \delta\left(\theta^{B}\right)=F_{E} \tag{15c}
\end{equation*}
$$

Recalling our definition of $Q$ from (C3), we can see that (15c) is an expression in terms of only $\theta^{A}, \theta^{B}$ and model parameters. We can combine (15c) with (C7) to define the equilibrium value of either $\theta^{A}$ or $\theta^{B}$ in terms of only model parameters. Before proceeding to this final expression, we can simplify the bracketed term on the left-hand side of (15c) by substituting from (C3) and the definition of $G(\theta)$.

$$
\begin{gather*}
\left\{\left(1+\tau^{1-\sigma}\right)\left[\lambda\left(q_{L}\right)-G\left(\theta^{B}\right)\left(\lambda\left(q_{H}\right)-\lambda\left(q_{L}\right)\right)\right]-\tau^{1-\sigma} \lambda\left(q_{L}\right) G\left(\theta^{A}\right)\right\}= \\
\left\{\left(1+\tau^{1-\sigma}\right)\left[\lambda\left(q_{L}\right)-\left(1-\left(\theta^{B}\right)^{-s}\right)\left(\lambda\left(q_{H}\right)-\lambda\left(q_{L}\right)\right)\right]-\tau^{1-\sigma} \lambda\left(q_{L}\right)\left(1-\left(\theta^{A}\right)^{-s}\right)\right\}= \\
\left\{\lambda\left(q_{L}\right)+\left(\theta^{B}\right)^{-s}\left(\lambda\left(q_{H}\right)-\lambda\left(q_{L}\right)\right)\left(1+\tau^{1-\sigma}\right)+\tau^{1-\sigma} \lambda\left(q_{L}\right)\left(\theta^{A}\right)^{-s}\right\} \tag{C11}
\end{gather*}
$$

Substituting (C11) into (15c) yields:

$$
\begin{gather*}
\frac{F_{x}\left(\theta^{X}\right)}{\tau^{1-\sigma} \lambda\left(q_{L}\right)}\left\{\lambda\left(q_{L}\right)+\left(\theta^{B}\right)^{-s}\left(\lambda\left(q_{H}\right)-\lambda\left(q_{L}\right)\right)\left(1+\tau^{1-\sigma}\right)+\tau^{1-\sigma} \lambda\left(q_{L}\right)\left(\theta^{A}\right)^{-s}\right\} \\
-\frac{s}{s+1} F_{x}\left(\theta^{A}\right)-\frac{s}{s+1} \delta\left(\theta^{B}\right)=F_{E} \tag{15d}
\end{gather*}
$$

We can rewrite the last two terms from the left-hand side of (15d) as:

$$
\frac{s}{s+1}\left\{F_{x}\left(\theta^{A}\right)+\delta\left(\theta^{B}\right)\right\}
$$

Substituting (C7) into this expression yields:

$$
\begin{aligned}
& \quad \frac{s}{s+1}\left\{F_{x}\left(\theta^{A}\right)+F_{x}\left(\theta^{A}\right) \frac{\left[\lambda\left(q_{H}\right)-\lambda\left(q_{L}\right)\right]}{\lambda\left(q_{L}\right)} \frac{\left[1+\tau^{1-\sigma}\right]}{\tau^{1-\sigma}}\right\}=\frac{s}{s+1}\left\{F_{x}\left(\theta^{A}\right) \frac{\lambda\left(q_{H}\right)}{\lambda\left(q_{L}\right)}\left[1+\tau^{\sigma-1}\right]-\right. \\
& \left.F_{x}\left(\theta^{A}\right) \tau^{\sigma-1}\right\}
\end{aligned}
$$

Replacing this expression in (15d) and collecting terms yields:

$$
\begin{aligned}
F_{x}\left(\theta^{A}\right) \tau^{\sigma-1} & +F_{x}\left(\theta^{A}\right) \frac{\left(\lambda\left(q_{H}\right)-\lambda\left(q_{L}\right)\right)}{\lambda\left(q_{L}\right)} \frac{\left(1+\tau^{1-\sigma}\right)}{\tau^{1-\sigma}}\left(\theta^{B}\right)^{-s}+F_{x}\left(\theta^{A}\right)\left(\theta^{A}\right)^{-s} \\
& -\frac{s}{s+1}\left\{F_{x}\left(\theta^{A}\right) \frac{\lambda\left(q_{H}\right)}{\lambda\left(q_{L}\right)}\left[1+\tau^{\sigma-1}\right]-F_{x}\left(\theta^{A}\right) \tau^{\sigma-1}\right\}=F_{E}
\end{aligned}
$$

Or equivalently,

$$
\begin{gather*}
\frac{F_{x}\left(\theta^{A}\right)}{s+1}\left\{\tau^{\sigma-1}\left[2 s+1-s \frac{\lambda\left(q_{H}\right)}{\lambda\left(q_{L}\right)}\right]-s \frac{\lambda\left(q_{H}\right)}{\lambda\left(q_{L}\right)}\right\}+F_{x}\left(\theta^{A}\right)\left(\theta^{A}\right)^{-s} \\
+F_{x}\left(\theta^{A}\right) \frac{\left(\lambda\left(q_{H}\right)-\lambda\left(q_{L}\right)\right)}{\lambda\left(q_{L}\right)}\left[1+\tau^{\sigma-1}\right]\left(\theta^{B}\right)^{-s}=F_{E} \tag{15e}
\end{gather*}
$$

Substituting from (12) and (C7) again yields:

$$
\begin{align*}
& \left(\theta^{A}\right)^{-1} F_{x}\left\{\frac{(2 s+1) \lambda\left(q_{L}\right)-\left[1+\tau^{1-\sigma}\right] s \lambda\left(q_{H}\right)}{\lambda\left(q_{L}\right)(s+1) \tau^{1-\sigma}}\right\}+\left(\theta^{A}\right)^{-(s+1)} F_{x} \\
& \quad+\left(\theta^{A}\right)^{-(s+1)}\left[\frac{\left(\lambda\left(q_{H}\right)-\lambda\left(q_{L}\right)\right)}{\lambda\left(q_{L}\right)}\left[1+\tau^{\sigma-1}\right]\right]^{s+1} \frac{F_{x}^{s+1}}{\left(q_{H}-q_{L}\right)^{s}}=F_{E} \tag{15f}
\end{align*}
$$

We can derive a similar expression to identify $\theta^{B}$ using only model parameters. We can substitute (C7) into (15b) to yield:

$$
\begin{equation*}
\frac{L}{\sigma M}-\frac{s}{s+1}\left\{\delta\left(\theta^{B}\right)+\delta\left(\theta^{B}\right) \frac{\lambda\left(q_{L}\right) \tau^{1-\sigma}}{\left[\lambda\left(q_{H}\right)-\lambda\left(q_{L}\right)\right]\left[1+\tau^{1-\sigma}\right]}\right\}=F_{E} \tag{15~g}
\end{equation*}
$$

From (C6a), we have:

$$
\frac{L}{\sigma M}=\frac{\delta\left(\theta^{B}\right)\left\{\left(1+\tau^{1-\sigma}\right) Q-\tau^{1-\sigma} \lambda\left(q_{L}\right) G\left(\theta^{A}\right)\right\}}{\left[\lambda\left(q_{H}\right)-\lambda\left(q_{L}\right)\right] \cdot\left[1+\tau^{1-\sigma}\right]}
$$

Substituting from (C7a) into (C11) yields:

$$
\begin{align*}
\left\{\left(1+\tau^{1-\sigma}\right) Q-\tau^{1-\sigma} \lambda\left(q_{L}\right) G\left(\theta^{A}\right)\right\}=\{ & \lambda\left(q_{L}\right)+\left(\theta^{B}\right)^{-s}\left(\lambda\left(q_{H}\right)-\lambda\left(q_{L}\right)\right)\left(1+\tau^{1-\sigma}\right) \\
& \left.+\tau^{1-\sigma} \lambda\left(q_{L}\right)\left(\theta^{B}\right)^{-s}\left[\frac{\left[q_{H}-q_{L}\right] \lambda\left(q_{L}\right) \tau^{1-\sigma}}{\left[\lambda\left(q_{H}\right)-\lambda\left(q_{L}\right)\right]\left[1+\tau^{1-\sigma}\right] F_{x}}\right]^{s}\right\} \tag{C12}
\end{align*}
$$

Substituting (C12) into (C6a) and then into (15g) yields:

$$
\begin{gather*}
\frac{\delta\left(\theta^{B}\right)}{\left[\lambda\left(q_{H}\right)-\lambda\left(q_{L}\right)\right]\left[1+\tau^{1-\sigma}\right]}\left\{\left\{\lambda\left(q_{L}\right)+\left(\theta^{B}\right)^{-s}\left(\lambda\left(q_{H}\right)-\lambda\left(q_{L}\right)\right)\left(1+\tau^{1-\sigma}\right)+\right.\right. \\
\left.\tau^{1-\sigma} \lambda\left(q_{L}\right)\left(\theta^{B}\right)^{-s}\left[\frac{\left[q_{H}-q_{L}\right] \lambda\left(q_{L}\right) \tau^{1-\sigma}}{\left[\lambda\left(q_{H}\right)-\lambda\left(q_{L}\right)\right]\left[1+\tau^{1-\sigma}\right] F_{x}}\right]^{s}\right\}-\frac{s}{s+1} \delta\left(\theta^{B}\right)\left\{1+\frac{\lambda\left(q_{L}\right) \tau^{1-\sigma}}{\left[\lambda\left(q_{H}\right)-\lambda\left(q_{L}\right)\right]\left[1+\tau^{1-\sigma}\right]}\right\}=F_{E} \tag{15h}
\end{gather*}
$$

Or,

$$
\begin{align*}
\left(\theta^{B}\right)^{-1}\left[q_{H}\right. & \left.-q_{L}\right]\left\{\frac{(2 s+1) \lambda\left(q_{L}\right)-\left[1+\tau^{1-\sigma}\right] s \lambda\left(q_{H}\right)}{\left[\lambda\left(q_{H}\right)-\lambda\left(q_{L}\right)\right]\left[1+\tau^{1-\sigma}\right](s+1)}\right\}+\left(\theta^{B}\right)^{-(s+1)}\left[q_{H}-q_{L}\right] \\
& +\left(\theta^{B}\right)^{-(s+1)}\left(\frac{\left[q_{H}-q_{L}\right] \lambda\left(q_{L}\right)}{\left[\lambda\left(q_{H}\right)-\lambda\left(q_{L}\right)\right]}\right)^{s+1}\left(\left[1+\tau^{\sigma-1}\right]\right)^{-(s+1)} F_{x}^{-s}=F_{E} \tag{15i}
\end{align*}
$$

## D. Definition of the Model Equilibrium in the LN/HN/HE Case

We can begin by redefining the price index from (C1) to reflect the new productivity cut-offs:

$$
\begin{align*}
\tilde{P}_{i}^{1-\sigma}= & M_{i}\left\{\int_{\underline{\theta}_{i}}^{\theta_{i}^{C}} \lambda\left(q_{L}\right) \cdot p_{i}{ }^{1-\sigma} g(\theta) d \theta+\int_{\theta_{i}^{C}}^{\theta_{i}^{D}} \lambda\left(q_{H}\right) \cdot p_{i}{ }^{1-\sigma} g(\theta) d \theta\right. \\
& \left.+\int_{\theta_{i}^{D}}^{\infty} \lambda\left(q_{H}\right) \cdot p_{i}{ }^{1-\sigma} g(\theta) d \theta\right\}+M_{j}\left\{\int_{\theta_{j}^{D}}^{\infty} \lambda\left(q_{H}\right) \cdot\left(\tau p_{j}\right)^{1-\sigma} g(\theta) d \theta\right\} \tag{D1}
\end{align*}
$$

Comparing (C2) and (D1), we can see that the domestic component of the price index is more-or-less unchanged. The foreign component reflects the fact that only high-quality varieties are exported in this specification of the model. Recalling the definition of $G(\theta)$ and allowing for symmetry:

$$
\begin{equation*}
\tilde{P}^{1-\sigma}=M\left[p^{1-\sigma}\left\{\lambda\left(q_{L}\right) G\left(\theta^{C}\right)+\lambda\left(q_{H}\right)\left[1-G\left(\theta^{C}\right)\right]\right\}+(\tau p)^{1-\sigma} \lambda\left(q_{H}\right)\left[1-G\left(\theta^{D}\right)\right]\right] \tag{D1a}
\end{equation*}
$$

- We can also redefine our expression for the average level of quality produced in country $i$ :

$$
\begin{equation*}
Q_{i}=\lambda\left(q_{L}\right) G\left(\theta_{i}^{C}\right)+\lambda\left(q_{H}\right)\left[1-G\left(\theta_{i}^{D}\right)\right] \tag{D2}
\end{equation*}
$$

Substituting (D2) into (D1a) yields:

$$
\begin{equation*}
\tilde{P}^{1-\sigma}=M p^{1-\sigma}\left[Q+\tau^{1-\sigma} \lambda\left(q_{H}\right)\left[1-G\left(\theta^{D}\right)\right]\right] \tag{D1b}
\end{equation*}
$$

Substituting (2) and (9a) into (17) yields:

$$
\begin{equation*}
\frac{E p^{1-\sigma}}{\sigma \tilde{P}^{1-\sigma}}\left[\lambda\left(q_{H}\right)-\lambda\left(q_{L}\right)\right]=w \delta\left(\theta^{C}\right) \tag{D3}
\end{equation*}
$$

Substituting from (D1b):

$$
\begin{equation*}
\frac{L}{\sigma M\left[Q+\tau^{1-\sigma} \lambda\left(q_{H}\right)\left[1-G\left(\theta^{D}\right)\right]\right.}=\frac{\delta\left(\theta^{C}\right)}{\left[\lambda\left(q_{H}\right)-\lambda\left(q_{L}\right)\right]} \tag{D3a}
\end{equation*}
$$

We can find an analogous expression for $\theta^{D}$ by substituting (2) and (10a) into (18):

$$
\begin{equation*}
\frac{E \cdot(\tau p)^{1-\sigma}}{\sigma \widetilde{P}^{1-\sigma}} \lambda\left(q_{H}\right)=w F_{X}\left(\theta^{D}\right) \tag{D4}
\end{equation*}
$$

Substituting again from (D1b):

$$
\begin{equation*}
\frac{L}{\sigma M\left[Q+\tau^{1-\sigma} \lambda\left(q_{H}\right)\left[1-G\left(\theta^{D}\right)\right]\right.}=\frac{F_{X}\left(\theta^{D}\right)}{\lambda\left(q_{H}\right) \tau^{1-\sigma}} \tag{D4a}
\end{equation*}
$$

Equating (D3a) and (D4a) allows us to define $\theta^{C}$ in terms of only model parameters and $\theta^{D}$, and vice-versa:

$$
\begin{equation*}
\frac{\delta\left(\theta^{C}\right)}{\left[\lambda\left(q_{H}\right)-\lambda\left(q_{L}\right)\right]}=\frac{F_{X}\left(\theta^{D}\right)}{\lambda\left(q_{H}\right) \tau^{1-\sigma}} \tag{D5}
\end{equation*}
$$

To finish defining the model equilibrium, we need to find at least one more expression in terms of only $\theta^{C}, \theta^{D}$ and model parameters. As before, we can use
(15) to derive such an expression. First, we must redefine our expected profit term $(E[\pi])$ to reflect the new productivity cut-offs:

$$
\begin{equation*}
E_{i}[\pi]=\int_{\underline{\theta}_{i}}^{\theta_{i}^{C}} \pi_{i}\left(q_{L}\right) g(\theta) d \theta+\int_{\theta_{i}^{C}}^{\theta_{i}^{D}} \pi_{i}\left(q_{H}\right) g(\theta) d \theta+\int_{\theta_{i}^{D}}^{\infty}\left[\pi_{i}\left(q_{H}\right)+\pi_{j}\left(q_{H}\right)\right] g(\theta) d \theta \tag{D6}
\end{equation*}
$$

Allowing for symmetry and substituting from (9a) and (10a) yields:

$$
\begin{equation*}
E[\pi]=\frac{E p^{1-\sigma}}{\sigma \tilde{P}^{1-\sigma}}\left\{\lambda\left(q_{L}\right) G\left(\theta^{C}\right)+\lambda\left(q_{H}\right)\left[1-G\left(\theta^{C}\right)\right]+\tau^{1-\sigma} \lambda\left(q_{H}\right)\left[1-G\left(\theta^{D}\right)\right]\right\} \tag{D6a}
\end{equation*}
$$

Substituting from (D1b) and (D2) allows us to simplify (D5a) as in the previous case:

$$
\begin{equation*}
E[\pi]=\frac{L}{\sigma M} \tag{D7}
\end{equation*}
$$

Substituting (D6) into (15) yields the same expression as (15a). The expected fixed cost terms in (15a) can be evaluated largely as before. We need only adjust the expressions to reflect the different productivity cut-offs beyond which firms sink each fixed cost.

$$
\begin{gather*}
E[\delta(\theta)]=\frac{s}{s+1} \frac{\left[q_{H}-q_{L}\right]}{\theta^{C}}=\frac{s}{s+1} \delta\left(\theta^{C}\right)  \tag{D8}\\
E\left[F_{x}(\theta)\right]=\frac{s}{s+1} \frac{F_{x}}{\theta^{D}}=\frac{s}{s+1} F_{x}\left(\theta^{D}\right) \tag{D9}
\end{gather*}
$$

Substituting (D7) and (D8) into (15a) yields:

$$
\begin{equation*}
\frac{L}{\sigma M}-\frac{s}{s+1} F_{x}\left(\theta^{D}\right)-\frac{s}{s+1} \delta\left(\theta^{C}\right)=F_{E} \tag{D10}
\end{equation*}
$$

Substitute (D5) into (D10) to eliminate the $\delta\left(\theta^{C}\right)$ term:

$$
\begin{equation*}
\frac{L}{\sigma M}-\frac{s}{s+1}\left\{F_{x}\left(\theta^{D}\right)-\frac{F_{x}\left(\theta^{D}\right)}{\lambda\left(q_{H}\right) \tau^{1-\sigma}}\left[\lambda\left(q_{H}\right)-\lambda\left(q_{L}\right)\right]\right\}=F_{E} \tag{D10a}
\end{equation*}
$$

We can also eliminate $M$ from (D10) by substituting from (D4a):

$$
\begin{align*}
& \frac{F_{X}\left(\theta^{D}\right)}{\lambda\left(q_{H}\right) \tau^{1-\sigma}}\left[Q+\tau^{1-\sigma} \lambda\left(q_{H}\right)\left[1-G\left(\theta^{D}\right)\right]\right] \\
& \quad-\frac{s}{s+1}\left\{F_{x}\left(\theta^{D}\right)+\frac{F_{x}\left(\theta^{D}\right)}{\lambda\left(q_{H}\right) \tau^{1-\sigma}}\left[\lambda\left(q_{H}\right)-\lambda\left(q_{L}\right)\right]\right\}=F_{E} \tag{D10b}
\end{align*}
$$

As before, we can simplify the first bracketed term by expanding the definition of Q :

$$
\begin{align*}
& Q+\tau^{1-\sigma} \lambda\left(q_{H}\right)\left[1-G\left(\theta^{D}\right)\right] \\
& =\lambda\left(q_{L}\right) G\left(\theta^{C}\right)+\lambda\left(q_{H}\right)\left[1-G\left(\theta^{C}\right)\right]+\tau^{1-\sigma} \lambda\left(q_{H}\right)\left[1-G\left(\theta^{D}\right)\right] \\
& =\lambda\left(q_{H}\right)+G\left(\theta^{C}\right)\left[\lambda\left(q_{H}\right)-\lambda\left(q_{L}\right)\right]+\tau^{1-\sigma} \lambda\left(q_{H}\right)[1- \\
&  \tag{D11}\\
& =
\end{align*}
$$

$\left.G\left(\theta^{D}\right)\right]$

From (D5):

$$
\begin{equation*}
\left(\theta^{C}\right)^{-1}=\left(\theta^{D}\right)^{-1} \frac{F_{X}}{\lambda\left(q_{H}\right) \tau^{1-\sigma}} \frac{\left[\lambda\left(q_{H}\right)-\lambda\left(q_{L}\right)\right]}{q_{H}-q_{L}} \tag{D5a}
\end{equation*}
$$

Which implies:

$$
\begin{equation*}
\left(\theta^{C}\right)^{-s}=\left(\theta^{D}\right)^{-s}\left\{\frac{F_{X}}{\lambda\left(q_{H}\right) \tau^{1-\sigma}} \frac{\left[\lambda\left(q_{H}\right)-\lambda\left(q_{L}\right)\right]}{q_{H}-q_{L}}\right\}^{s} \tag{D5b}
\end{equation*}
$$

Substituting (D5b) into (D11) yields:

$$
\lambda\left(q_{L}\right)+\left(\theta^{D}\right)^{-s}\left\{\frac{F_{X}}{\lambda\left(q_{H}\right) \tau^{1-\sigma}} \frac{\left[\lambda\left(q_{H}\right)-\lambda\left(q_{L}\right)\right]}{q_{H}-q_{L}}\right\}^{s}\left[\lambda\left(q_{H}\right)-\lambda\left(q_{L}\right)\right]+\tau^{1-\sigma} \lambda\left(q_{H}\right)\left(\theta^{D}\right)^{-s}
$$

Replace this in (D10b):

$$
\begin{aligned}
& \frac{F_{X}\left(\theta^{D}\right)}{\lambda\left(q_{H}\right) \tau^{1-\sigma}}\left[\lambda\left(q_{L}\right)+\left(\theta^{D}\right)^{-s}\left\{\frac{F_{X}}{\lambda\left(q_{H}\right) \tau^{1-\sigma}} \frac{\left[\lambda\left(q_{H}\right)-\lambda\left(q_{L}\right)\right]}{q_{H}-q_{L}}\right\}^{s}\left[\lambda\left(q_{H}\right)-\lambda\left(q_{L}\right)\right]+\right. \\
&\left.\tau^{1-\sigma} \lambda\left(q_{H}\right)\left(\theta^{D}\right)^{-s}\right] \\
&-\frac{s}{s+1}\left\{F_{x}\left(\theta^{D}\right)+\frac{F_{x}\left(\theta^{D}\right)}{\lambda\left(q_{H}\right) \tau^{1-\sigma}}\left[\lambda\left(q_{H}\right)-\lambda\left(q_{L}\right)\right]\right\}=F_{E}
\end{aligned}
$$

Or,

$$
\begin{gathered}
\left(\theta^{D}\right)^{-1} \frac{F_{X} \lambda\left(q_{L}\right)}{\lambda\left(q_{H}\right) \tau^{1-\sigma}}+\left(\theta^{D}\right)^{-(s+1)}\left\{\frac{F_{X}}{\tau^{1-\sigma}} \frac{\left[\lambda\left(q_{H}\right)-\lambda\left(q_{L}\right)\right]}{\lambda\left(q_{H}\right)}\right\}^{s+1}\left[q_{H}-q_{L}\right]^{-s}+\left(\theta^{D}\right)^{-(s+1)} F_{X} \\
-\left(\theta^{D}\right)^{-1} F_{x} \frac{s}{s+1}\left\{1+\frac{\left[\lambda\left(q_{H}\right)-\lambda\left(q_{L}\right)\right]}{\lambda\left(q_{H}\right) \tau^{1-\sigma}}\right\}=F_{E}
\end{gathered}
$$

Collecting common terms yields an expression that identifies the unique equilibrium value of $\theta^{D}$ :

$$
\begin{gather*}
\left(\theta^{D}\right)^{-1} F_{X} \frac{(2 s+1) \lambda\left(q_{L}\right)-\left(1+\tau^{1-\sigma}\right) s \lambda\left(q_{H}\right)}{\lambda\left(q_{H}\right) \tau^{1-\sigma}(s+1)}+\left(\theta^{D}\right)^{-(s+1)}\left\{\frac{F_{X}}{\tau^{1-\sigma}} \frac{\left[\lambda\left(q_{H}\right)-\lambda\left(q_{L}\right)\right]}{\lambda\left(q_{H}\right)}\right\}^{s+1}\left[q_{H}-q_{L}\right]^{-s} \\
+\left(\theta^{D}\right)^{-(s+1)} F_{X}=F_{E} \tag{D10c}
\end{gather*}
$$

We can develop a similar expression to identify $\theta^{C}$ in terms of only model parameters. From (D5b), we have:

$$
\begin{equation*}
\left(\theta^{D}\right)^{-s}=\left(\theta^{C}\right)^{-s}\left\{\frac{\lambda\left(q_{H}\right) \tau^{1-\sigma}}{F_{X}} \frac{q_{H}-q_{L}}{\left[\lambda\left(q_{H}\right)-\lambda\left(q_{L}\right)\right]}\right\}^{s} \tag{D5c}
\end{equation*}
$$

Substitute this into (D11):
$\frac{\delta\left(\theta^{C}\right)}{\left[\lambda\left(q_{H}\right)-\lambda\left(q_{L}\right)\right]}\left[\lambda\left(q_{L}\right)+\left(\theta^{C}\right)^{-s}\left[\lambda\left(q_{H}\right)-\lambda\left(q_{L}\right)\right]+\right.$ $\left.\left(\theta^{C}\right)^{-s} \tau^{1-\sigma} \lambda\left(q_{H}\right)\left\{\frac{\lambda\left(q_{H}\right) \tau^{1-\sigma}}{F_{X}} \frac{q_{H}-q_{L}}{\left[\lambda\left(q_{H}\right)-\lambda\left(q_{L}\right)\right]}\right\}^{s}\right]$

$$
\begin{equation*}
-\frac{s}{s+1} \delta\left(\theta^{C}\right)\left\{1+\frac{\lambda\left(q_{H}\right) \tau^{1-\sigma}}{\left[\lambda\left(q_{H}\right)-\lambda\left(q_{L}\right)\right]}\right\}=F_{E} \tag{D10d}
\end{equation*}
$$

Collecting common terms:

$$
\begin{gather*}
\left(\theta^{C}\right)^{-1}\left(q_{H}-q_{L}\right)\left\{\frac{(2 s+1) \lambda\left(q_{L}\right)-s \lambda\left(q_{H}\right)\left(1+\tau^{1-\sigma}\right)}{\left[\lambda\left(q_{H}\right)-\lambda\left(q_{L}\right)\right](s+1)}\right\}+\left(\theta^{C}\right)^{-(s+1)}\left(q_{H}-q_{L}\right) \\
+\left(\theta^{C}\right)^{-(s+1)}\left\{\frac{\left[q_{H}-q_{L}\right] \tau^{1-\sigma} \lambda\left(q_{H}\right)}{\left[\lambda\left(q_{H}\right)-\lambda\left(q_{L}\right)\right]}\right\}^{s+1} F_{X}^{-s}=F_{E} \tag{D10d}
\end{gather*}
$$

## E. Derivation of Comparative Statics

Deriving comparative statics for the policy-relevant parameters in the model requires totally differentiating the expressions that define the equilibrium productivity cut-offs. Using equations (18), (19), (23) and (24), we can find the comparative static for a given parameter $X$, by evaluating:

$$
\frac{\partial Q\left(\theta^{i}\right)}{\partial \theta^{i}} \cdot d \theta^{i}+\frac{\partial Q\left(\theta^{i}\right)}{\partial X} \cdot d X=0
$$

Fixed Entry Costs:
Beginning with the comparative static for fixed entry costs:

$$
\frac{\partial Q\left(\theta^{i}\right)}{\partial \theta^{i}} \cdot d \theta^{i}+\frac{\partial Q\left(\theta^{i}\right)}{\partial F_{E}} \cdot d F_{E}=0
$$

Solving for $d \theta^{i} / d F_{E}$ implies:

$$
\begin{equation*}
\frac{d \theta^{i}}{d F_{E}}=-\left[\frac{\frac{\partial Q\left(\theta^{i}\right)}{\partial F_{E}}}{\frac{\frac{\partial Q\left(\theta^{i}\right)}{\partial \theta^{i}}}{}}\right] \tag{E1}
\end{equation*}
$$

We must evaluate this expression for each $\theta^{i}, i=A, B, C, D$. Beginning with the denominator:

$$
\begin{align*}
\frac{\partial Q\left(\theta^{A}\right)}{\partial \theta^{A}}=- & \left(\theta^{A}\right)^{-2} F_{x}\left\{\frac{(2 s+1) \lambda\left(q_{L}\right)-\left[1+\tau^{1-\sigma}\right] s \lambda\left(q_{H}\right)}{\lambda\left(q_{L}\right)(s+1) \tau^{1-\sigma}}\right\}-(s+1)\left(\theta^{A}\right)^{-(s+2)} F_{x} \\
& -(s+1)\left(\theta^{A}\right)^{-(s+2)}\left[\frac{\left(\lambda\left(q_{H}\right)-\lambda\left(q_{L}\right)\right)}{\lambda\left(q_{L}\right)}\left[1+\tau^{\sigma-1}\right]\right]^{s+1} \frac{F_{x}^{s+1}}{\left(q_{H}-q_{L}\right)^{s}}<0  \tag{E2}\\
\frac{\partial Q\left(\theta^{B}\right)}{\partial \theta^{B}}=- & \left(\theta^{B}\right)^{-2}\left[q_{H}-q_{L}\right]\left\{\frac{(2 s+1) \lambda\left(q_{L}\right)-\left[1+\tau^{1-\sigma}\right] s \lambda\left(q_{H}\right)}{\left[\lambda\left(q_{H}\right)-\lambda\left(q_{L}\right)\right]\left[1+\tau^{1-\sigma}\right](s+1)}\right\}-(s+1)\left(\theta^{B}\right)^{-(s+2)}\left[q_{H}-q_{L}\right] \\
& -(s+1)\left(\theta^{B}\right)^{-(s+2)}\left(\frac{\left[q_{H}-q_{L}\right] \lambda\left(q_{L}\right)}{\left[\lambda\left(q_{H}\right)-\lambda\left(q_{L}\right)\right]}\right)^{s+1}\left(\left[1+\tau^{\sigma-1}\right]\right)^{-(s+1)} F_{x}^{-s}<0 \tag{E3}
\end{align*}
$$

$$
\begin{gather*}
\frac{\partial Q\left(\theta^{C}\right)}{\partial \theta^{C}}=-\left(\theta^{C}\right)^{-2}\left(q_{H}-q_{L}\right)\left\{\frac{(2 s+1) \lambda\left(q_{L}\right)-s \lambda\left(q_{H}\right)\left(1+\tau^{1-\sigma}\right)}{\left[\lambda\left(q_{H}\right)-\lambda\left(q_{L}\right)\right](s+1)}\right\}-(s+1)\left(\theta^{C}\right)^{-(s+2)}\left(q_{H}-q_{L}\right) \\
-(s+1)\left(\theta^{C}\right)^{-(s+2)}\left\{\frac{\left[q_{H}-q_{L}\right] \tau^{1-\sigma} \lambda\left(q_{H}\right)}{\left[\lambda\left(q_{H}\right)-\lambda\left(q_{L}\right)\right]}\right\}^{s+1} F_{X}^{-s}<0  \tag{E4}\\
\frac{\partial Q\left(\theta^{D}\right)}{\partial \theta^{D}}=-\left(\theta^{D}\right)^{-2} F_{X}\left\{\frac{\left.(2 s+1) \lambda\left(q_{L}\right)-\left(1+\tau^{1-\sigma}\right) s \lambda\left(q_{H}\right)\right\}}{\lambda\left(q_{H}\right) \tau^{1-\sigma}(s+1)}\right\}-(s+1)\left(\theta^{D}\right)^{-(s+2)} F_{X} \\
-(s+1)\left(\theta^{D}\right)^{-(s+2)}\left\{\frac{F_{X}}{\tau^{1-\sigma}} \frac{\left[\lambda\left(q_{H}\right)-\lambda\left(q_{L}\right)\right]}{\lambda\left(q_{H}\right)}\right\}^{s+1}\left[q_{H}-q_{L}\right]^{-s}<0 \tag{E5}
\end{gather*}
$$

As we can see from Figures 3 and 5, the equilibrium conditions for each of the productivity cut-offs are everywhere decreasing in $\theta$. The partial differentials are therefore negative. The sign of the comparative statics will therefore depend on the signs of the partial derivatives with respect to the parameter of interest. For the fixed entry cost:

$$
\begin{equation*}
\frac{\partial Q\left(\theta^{A}\right)}{\partial F_{E}}=\frac{\partial Q\left(\theta^{B}\right)}{\partial F_{E}}=\frac{\partial Q\left(\theta^{C}\right)}{\partial F_{E}}=\frac{\partial Q\left(\theta^{D}\right)}{\partial F_{E}}=-1<0 \tag{E6}
\end{equation*}
$$

The bracketed expression in (E1) will therefore be negative for all $i=A, B, C, D$. This implies:

$$
\begin{equation*}
\frac{d \theta^{A}}{d F_{E}}<0, \frac{d \theta^{B}}{d F_{E}}<0, \frac{d \theta^{C}}{d F_{E}}<0, \frac{d \theta^{D}}{d F_{E}}<0 \tag{E7}
\end{equation*}
$$

Fixed Export Costs:
To derive the comparative statics for the fixed export costs $\left(F_{X}\right)$, we need to evaluate:

$$
\frac{\partial Q\left(\theta^{i}\right)}{\partial \theta^{i}} \cdot d \theta^{i}+\frac{\partial Q\left(\theta^{i}\right)}{\partial F_{X}} \cdot d F_{X}=0
$$

Solving for $d \theta^{i} / d F_{X}$ implies:

$$
\begin{equation*}
\frac{d \theta^{i}}{d F_{X}}=-\left[\frac{\frac{\partial Q\left(\theta^{i}\right)}{\partial F_{X}}}{\frac{\frac{\partial Q\left(\theta^{i}\right)}{\partial \theta^{i}}}{}}\right] \tag{E8}
\end{equation*}
$$

Deriving the comparative statics requires evaluating (E8) for each $i=A, B, C, D$. The denominator of the bracketed term in (E8) is identical to (E2)-(E5). We need only evaluate the term in the numerator.

$$
\begin{align*}
& \begin{array}{l}
\frac{\partial Q\left(\theta^{A}\right)}{\partial F_{X}}=\left(\theta^{A}\right)^{-1}\left\{\frac{(2 s+1) \lambda\left(q_{L}\right)-\left[1+\tau^{1-\sigma}\right] s \lambda\left(q_{H}\right)}{\lambda\left(q_{L}\right)(s+1) \tau^{1-\sigma}}\right\}+\left(\theta^{A}\right)^{-(s+1)} \\
\\
\quad+\left(\theta^{A}\right)^{-(s+1)}\left[\frac{\left(\lambda\left(q_{H}\right)-\lambda\left(q_{L}\right)\right)}{\lambda\left(q_{L}\right)}\left[1+\tau^{\sigma-1}\right]\right]^{s+1} \frac{F_{X}^{s}}{\left(q_{H}-q_{L}\right)^{s}}>0
\end{array} \\
& \begin{aligned}
\frac{\partial Q\left(\theta^{B}\right)}{\partial F_{X}}=-s\left(\theta^{B}\right)^{-(s+1)}\left(\frac{\left[q_{H}-q_{L}\right] \lambda\left(q_{L}\right)}{\left[\lambda\left(q_{H}\right)-\lambda\left(q_{L}\right)\right]}\right)^{s+1}\left(\left[1+\tau^{\sigma-1}\right]\right)^{-(s+1)} F_{X}^{-(s+1)}<0 \\
\frac{\partial Q\left(\theta^{C}\right)}{\partial F_{X}}=-s\left(\theta^{C}\right)^{-(s+1)}\left\{\frac{\left[q_{H}-q_{L}\right] \tau^{1-\sigma} \lambda\left(q_{H}\right)}{\left[\lambda\left(q_{H}\right)-\lambda\left(q_{L}\right)\right]}\right\}^{s+1} F_{X}^{-s}<0
\end{aligned}  \tag{E9}\\
& \begin{array}{l}
\frac{\partial Q\left(\theta^{C}\right)}{\partial F_{X}}=\left(\theta^{D}\right)^{-1}\left\{\frac{(2 s+1) \lambda\left(q_{L}\right)-\left(1+\tau^{1-\sigma}\right) s \lambda\left(q_{H}\right)}{\lambda\left(q_{H}\right) \tau^{1-\sigma(s+1)}}\right\}+\left(\theta^{D}\right)^{-(s+1)} \\
\quad+\left(\theta^{D}\right)^{-(s+1)}\left\{\frac{\left[\lambda\left(q_{H}\right)-\lambda\left(q_{L}\right)\right]}{\left.\tau^{1-\sigma \lambda\left(q_{H}\right)}\right\}^{s+1} \frac{F_{X}^{s}}{\left[q_{H}-q_{L}\right]^{s}}>0}\right.
\end{array} . \tag{E10}
\end{align*}
$$

Evaluating $\mathrm{E}(8)$ by combining (E9)-E(12) with $\mathrm{E}(2)-\mathrm{E}(7)$ yields:

$$
\begin{equation*}
\frac{d \theta^{A}}{d F_{X}}>0, \frac{d \theta^{B}}{d F_{X}}<0, \frac{d \theta^{C}}{d F_{X}}<0, \frac{d \theta^{D}}{d F_{X}}>0 \tag{E13}
\end{equation*}
$$

## Transportation Costs

To derive the comparative statics for the transportation costs $(\tau)$, we need to evaluate:

$$
\frac{\partial Q\left(\theta^{i}\right)}{\partial \theta^{i}} \cdot d \theta^{i}+\frac{\partial Q\left(\theta^{i}\right)}{\partial \tau} \cdot d \tau=0
$$

Solving for $d \theta^{i} / d \tau$ implies:

$$
\begin{equation*}
\frac{d \theta^{i}}{d \tau}=-\left[\frac{\frac{\partial Q\left(\theta^{i}\right)}{\partial \tau}}{\frac{\partial Q\left(\theta^{i}\right)}{\partial \theta^{i}}}\right] \tag{E14}
\end{equation*}
$$

Once again, the denominator of the bracketed term in (E14) is identical to $\mathrm{E}(2)$ $E(5)$. We need only evaluate the numerator in the bracketed term:

$$
\begin{align*}
& \frac{\partial Q\left(\theta^{A}\right)}{\partial \tau}=(\sigma-1) \tau^{\sigma-2} \frac{\left(\theta^{A}\right)^{-1} F_{X}}{(s+1) \lambda\left(q_{L}\right)}\left[(2 s+1) \lambda\left(q_{L}\right)-s \lambda\left(q_{H}\right)\right] \\
& \quad+(s+1)\left(1+\tau^{\sigma-1}\right)^{s}(\sigma-1) \tau^{\sigma-2}\left\{\frac{\left(\theta^{A}\right)^{-1}\left[\lambda\left(q_{H}\right)-\lambda\left(q_{L}\right)\right] F_{X}}{\lambda\left(q_{L}\right)}\right\}^{s+1}\left[q_{H}-q_{L}\right]^{-s}>0  \tag{E15}\\
& \frac{\partial Q\left(\theta^{D}\right)}{\partial \tau}=(\sigma-1) \tau^{\sigma-2} \frac{\left(\theta^{D}\right)^{-1} F_{X}}{(s+1) \lambda\left(q_{H}\right)}\left[(2 s+1) \lambda\left(q_{L}\right)-s \lambda\left(q_{H}\right)\right] \\
& \quad+(s+1)(\sigma-1) \tau^{(s+1)(\sigma-1)-1}\left\{\frac{\left(\theta^{D}\right)^{-1}\left[\lambda\left(q_{H}\right)-\lambda\left(q_{L}\right)\right] F_{X}}{\lambda\left(q_{H}\right)}\right\}^{s+1}\left[q_{H}-q_{L}\right]^{-s}>0 \tag{E16}
\end{align*}
$$

Combining $\mathrm{E}(15)$ and $\mathrm{E}(16)$ with $\mathrm{E}(2), \mathrm{E}(5)$ and $\mathrm{E}(14)$ yields:

$$
\begin{equation*}
\frac{d \theta^{A}}{d \tau}>0, \frac{d \theta^{D}}{d \tau}>0 \tag{17}
\end{equation*}
$$

The comparative statics for $\theta^{B}$ and $\theta^{C}$ are more ambiguous. Given (E14) and $\mathrm{E}(2)-$ $\mathrm{E}(5), \frac{d \theta^{i}}{d \tau}>0$ if and only if $\frac{\partial Q\left(\theta^{i}\right)}{\partial \tau}>0$.

Partially differentiating $Q\left(\theta^{i}\right)$ with respect to $\tau$ for $i=B, C$ yields:

$$
\begin{align*}
\frac{\partial Q\left(\theta^{B}\right)}{\partial \tau}= & (\sigma-1)\left[1+\tau^{1-\sigma}\right]^{-2} \tau^{-\sigma}\left\{\frac{\left(\theta^{B}\right)^{-1}\left(q_{H}-q_{L}\right) \lambda\left(q_{L}\right)}{\lambda\left(q_{H}\right)-\lambda\left(q_{L}\right)}\right\}\left(\frac{2 s+1}{s+1}\right) \\
& -(s+1)(\sigma-1)\left[1+\tau^{\sigma-1}\right]^{-(s+2)} \tau^{\sigma-2}\left\{\frac{\left(\theta^{B}\right)^{-1}\left(q_{H}-q_{L}\right) \lambda\left(q_{L}\right)}{\lambda\left(q_{H}\right)-\lambda\left(q_{L}\right)}\right\}^{s+1} F_{X}^{-s}  \tag{E18}\\
\frac{\partial Q\left(\theta^{C}\right)}{\partial \tau}= & (\sigma-1) \tau^{-\sigma}\left\{\frac{\left(\theta^{C}\right)^{-1}\left(q_{H}-q_{L}\right) \lambda\left(q_{H}\right)}{\lambda\left(q_{H}\right)-\lambda\left(q_{L}\right)}\right\}\left(\frac{s}{s+1}\right) \\
& -(s+1)(\sigma-1) \tau^{(1-\sigma)(s+1)-1}\left\{\frac{\left(\theta^{C}\right)^{-1}\left(q_{H}-q_{L}\right) \lambda\left(q_{H}\right)}{\lambda\left(q_{H}\right)-\lambda\left(q_{L}\right)}\right\}^{s+1} F_{X}^{-s} \tag{E19}
\end{align*}
$$

It is not possible to sign (E18) or (E19) without imposing further restrictions on the relative magnitudes of certain model parameters. Given (E11) it would be reasonable to assume $\frac{\partial Q\left(\theta^{C}\right)}{\partial \tau}<0$. Firms with productivity in the vicinity of $\theta^{C}$ will only experience general equilibrium effects given a change in $\tau$. A change in $\tau$ should therefore mirror the effect of a change in $F_{X}$. Rearranging terms in (E19), this implies setting parameters such that:

$$
\begin{equation*}
\frac{s}{(s+1)^{2}}<\left(\theta^{C}\right)^{-s}\left\{\frac{\left(q_{H}-q_{L}\right) \lambda\left(q_{H}\right) \tau^{1-\sigma}}{\lambda\left(q_{H}\right)-\lambda\left(q_{L}\right) F_{X}}\right\}^{s} \tag{E20}
\end{equation*}
$$

Substituting from (D5b), we can rewrite this as:

$$
\frac{s}{(s+1)^{2}}<\left(\theta^{D}\right)^{-s}
$$

We have previously assumed $\underline{\theta}=1$, so both sides of this expression are bound below one. This means that none of our previous assumptions preclude (E20).

There is good reason to suspect that result for $\frac{d \theta^{B}}{d \tau}$ would not mirror the result for $\frac{d \theta^{B}}{d F_{X}}$. While changes in $F_{X}$ do not change the relative profitability of the LE and HE strategies, changes in $\tau$ will. To see this, differentiate (10) with respect to $\tau$
(ignoring general equilibrium effects in $\widetilde{P}$ ):

$$
\frac{\partial \pi_{F}\left(q_{\omega}\right)}{\partial \tau}=(1-\sigma) \tau^{-\sigma} p^{1-\sigma} \lambda\left(q_{\omega}\right) \frac{E}{\tilde{P}^{1-\sigma}}
$$

Given $\sigma>1$ and $\lambda\left(q_{H}\right)>\lambda\left(q_{L}\right)$, profits in the foreign market fall faster for sellers of high-quality goods as $\tau$ increases. Ignoring general equilibrium effects, increases in $\tau$ will change the relative profitability of the LE and HE strategies in a way $F_{X}$ will not, making certification a less attractive option. Rearranging terms $\mathrm{E}(18), \frac{d \theta^{B}}{d \tau}>0$ implies:

$$
\begin{equation*}
\frac{2 s+1}{(s+1)^{2}}>\left(\theta^{B}\right)^{-s}\left\{\frac{\left(q_{H}-q_{L}\right) \lambda\left(q_{L}\right) \tau^{1-\sigma}}{\left[\lambda\left(q_{H}\right)-\lambda\left(q_{L}\right)\right] F_{X}\left[1+\tau^{1-\sigma}\right]}\right\}^{s} \tag{E21}
\end{equation*}
$$

Alternatively, we can substitute from (C7a) and rearrange terms:

$$
\frac{2 s+1}{2 s+1+s^{2}}>\left(\theta^{A}\right)^{-s}
$$

Once again, both sides of the expression are bound below one. None of our previous assumptions violate the condition specified in (E21).

Assuming $\mathrm{E}(21)$ and $\mathrm{E}(20)$ hold, the remaining comparative statics for $\tau$ are:

$$
\begin{equation*}
\frac{d \theta^{B}}{d \tau}<0, \frac{d \theta^{C}}{d \tau}<0 \tag{22}
\end{equation*}
$$


[^0]:    ${ }^{1}$ Corresponding author

[^1]:    ${ }^{2}$ Melitz (2003) assumes that marginal production costs are decreasing in productivity, but this distinction is relatively unimportant. As long as pay-offs are monotonically increasing in productivity and slope at different rates, the assumption I make here makes the model more tractable and produces an identical pattern of firm behavior.
    ${ }^{3}$ For simplicity, I assume firms cannot sell different quality output in different markets.

[^2]:    ${ }^{4}$ Note that this requires an identical assumption about the relative magnitudes of $\mathrm{s}, \tau, \lambda\left(q_{L}\right)$ and $\lambda\left(q_{H}\right)$ as in the first case.

