Sustainability Economics of Groundwater Usage and Management

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1   Introduction

Groundwater economics has been extensively studied in the literature. Pioneering work by Burt and Gardner and Brown identified PV-optimal strategies, while Gisser and Sanchez found that management benefits are not very large. Since then the literature has been expanded in a number of directions including game-theory [Negri, Provencher and Burt], conjunctive surface water use [Olson and Knapp], and spatial models [Noel and Howitt, Brosnivich]. Roumasset developed a model of groundwater lens in a marine environment. Groundwater quality economics include Olson and Conrad, Zeitouni, and Knapp and Baerenklau.

This paper extends the literature to consider groundwater sustainability. Following the capital-resource literature [e.g. Mourmouras, Asheim], sustainability is defined here as efficient use (Pareto-optimality) and intergenerational equity specified as non-declining utility over time. The standard CP and PV-optimal groundwater models cannot be used to assess sustainability since these models report only income streams and the physical variables, while sustainability is measured over consumption. Accordingly, this problem is studied by extending a standard groundwater model to include household utility and saving/dissaving from a financial asset.

Analytic efficiency conditions (Hotellings rule) are first derived, along with an illustration of short-run sustainability interpreted as two successive periods along a sustainable path. Three behavioral regimes are then considered: common property usage (CP), utility maximization defined as the optimal present-value of instantaneous utility (U-opt), and utility maximization subject to a sustainability constraint.

In each instance we analyze the sustainability properties by checking Hotelling's rule and non-declining utility. This is done theoretically for the case of an interior solution and initially high water table. Numerical results are reported under a variety of alternate conditions for sensitivity analysis.

This paper contributes to natural resource economics in several ways. It is the first groundwater paper (to our knowledge) to apply a leading sustainability definition from the capital-resource literature. Some previous groundwater studies have addressed sustainability by lowering the discount rate in a standard PV-opt model. However, this isn't efficient since it would imply a divergence between the rate of return to groundwater management and other investments in the economy, hence violating Hotelling's rule. Other studies enforcing non-declining income with a resource-only model (no other capital stocks) have had to rely on the concept of stepwise-inefficiency [Woodford].

Previous formal sustainability analysis is for capital-resource economies where market equilibrium would naturally lead to efficient allocations. Here we extend this literature to consider a CP resource. While CP usage is clearly inefficient and hence non-sustainable, it could have increasing consumption depending on the parameter values including the household subjective discount rate and market rate of return. And the U-opt regime - while efficient - may have declining instantaneous utility. Our interpretation is therefore that CP is not the only - or even main - cause of non-sustainability, and correcting pumping cost externalities does not alone guarantee sustainability.

There is also relevance to other natural resources. Previous theoretical work has discussed sustainability of competitive equilibrium for a renewable resource [Mourmouras], and provided an axiomatic basis for the sustainability criterion utilized here [Asheim]. However, there is relatively little available work applying the concept to evaluate and calculate sustainable allocations. Finally this work also motivates the need for resource management studies to consider the role of borrowing/saving and household preferences for natural resource management.

2 Model

The model is for an agricultural region overlying a groundwater aquifer. The model components are set out in this section, while specific assumptions for the behavioral regimes are defined later. As depicted in figure 1, the region both imports surface water and extracts groundwater. There are return flows to the aquifer from both surface water imports and deep percolation consequent to irrigation. Aquifer geometry is defined by the following parameters: horizontal area (A), land elevation $(\bar{h})$, and aquifer bottom $(\underline{h})$. All elevations in the model are measured relative to mean sea level (MSL). Agricultural income can also be used for investment in a risk-free financial asset.

Utility over consumption time paths of a representative household is defined by

$$\sum_{t=1}^{T} \alpha^t u(c_t) \tag{1}$$

where $\alpha = 1/(1 + r_h)$ is the discount factor, r_h is the household subjective discount

rate, and c_t is consumption. Instantaneous utility is defined here by the CES function $u(c) = c^{1-\rho}/(1-\rho)$ where $\sigma = 1/\rho$ is the intertemporal elasticity of substitution (IES). The household budget constraint is

$$c_t + \Delta k_t = \pi_t \tag{2}$$

where Δk_t is net savings and π_t is agricultural income. Non-negative consumption $c_t \geq 0$ implies that net savings are constrained by $\Delta k_t \leq \pi_t$.

Annual net benefits from agricultural production are

$$\pi_t = b(q_t) - p_{sw}q_{st} - \gamma_e(\bar{h} - h_t)w_t \tag{3}$$

where $b(q_t) = \int_0^{q_t} p(q)dq$ is benefits from water use, $p(q)$ is the derived demand curve for regional water use, p_{sw} is surface water price, and γ_e is the energy cost of lifting water. Total water use is $q_t = q_{st} + w_t$ where q_{st} and w_t are surface and groundwater quantities respectively. Deep percolation flows to the aquifer are $q_{dt} = \beta_q q_t$ where β_q is the percolation coefficient.

Surface water use is given by $q_{st} = (1 - \beta_s)\bar{q}_s$ where β_s is the surface water infiltration coefficient (canal water loss to the aquifer), and $\bar{q}_s$ is regional surface water availability. Groundwater withdrawals are constrained by $w_t \leq s^y(h_t - \underline{h})A$ where s^y is specific yield, and $\underline{h}$ is the aquifer bottom relative to MSL. The equation of motion for the water table is

$$h_{t+1} = h_t + \frac{\beta_s\bar{q}_s + \beta_q[(1 - \beta_s)\bar{q}_s + w_t] - w_t}{As^y} \tag{4}$$

with the water table height constrained by $\underline{h} \leq h_t \leq \bar{h}$. Equation 4 implies that the

table height increases due to percolation from both irrigation and canal losses, and decreases due to groundwater extractions for irrigation.

Net savings are constrained by $-k_t \leq \Delta k_t$ where k_t represents financial capital. Borrowing is not allowed in this model, so dissaving cannot exceed the available capital stock k_t. The equation of motion for the capital stock is

$$k_{t+1} = (1 + r_m)(k_t + \Delta k_t) \tag{5}$$

with r_m the market interest rate. The constraint on net savings implies a non-negative financial capital stock in all periods $(k_t \geq 0)$.

3 Data

The analysis is for Kern country, California, although some data values are from macro-economic data. Aquifer area is 1.29 million acres, although agricultural production is limited to 0.9 million acres. Data values are given in Table 1.

Empirical estimates for the IES (σ) are available from the macroeconomic literature. Hall (1988) finds elasticities of substitution ranging from $0.03 \leq \sigma \leq 0.48$, while Epstein and Zin (1991) report values in the range $0.18 \leq \sigma \leq 0.87$. Results from more recent studies include those of Favero (2005), in which the author estimates an IES in the range of 0.77 to 0.84. A baseline value of $\sigma = 0.4$ is used here, but with sensitivity analysis. Also assumed is a real rate of return for a risk-free financial asset of $r_m = 0.04$.

A variety of subjective discount rates are considered; however, the baseline value

is $r_h = 0.05 > r_m = 0.04$. The reasoning for this is as follows: Due to transaction costs associated with banks and other financial institutions, there must be a positive gap between the borrowing rate and the saving rate. For borrowing to equal savings in an economy with heterogeneous agents, then, roughly speaking, the subjective discount rate for an average household would need to lie within this gap. Otherwise, assuming away strong non-convexities and income disparities, there would be either positive or negative net saving, and so the market rate would need to adjust for zero net saving in equilibrium. In any case, we will also consider $r_h = r_m$ and $r_h < r_m$ for completeness.

Surface water in Kern County is high quality (low salinity) and comes from three major sources: the California State Water Project, the federal Central Valley Project, and the Kern River. Surface water costs are estimated from data in Vaux (1986) and Kern County Water Agency (1998) with inflation adjustment, and reflect differential costs of alternate sources within the region. Total diversions $\bar{q}_s = 1.97$ acre feet per year reflecting water deliveries in a normal year (Kern County Water Agency, 1998). Pumping costs are \$15.04 per acre ft. per year and are calculated using an energy cost of \$0.148 per acre foot per ft.of lift. Other surface water and aquifer parameter values and data sources are given in Table 1.

The analysis is primarily focused on the life-history of the resource over a finite horizon. Accordingly, initial conditions are generally taken to be a full aquifer $h_1 = \bar{h} - h_z$ where h_z is rootzone depth, and zero net financial assets $k_1 = 0$. The optimization problem is solved using nonlinear programming (NLP) methods over ei-

ther a 60 or 100 year horizon. While these initial conditions are our primary interest, some attention is also given to alternate initial conditions. For example, a formerly unmanaged aquifer might be at a lower initial level than an optimal steady-state, in which case standard PV-optimal management might involve increasing water table levels and consumption, hence sustainability even though this might not be true under similar conditions for h_1 high.

4   Efficiency and sustainability conditions

This section sets out general efficiency and sustainability conditions. Later sections then investigate the extent to which markets and alternate criteria can achieve efficient and equitable solutions.

Efficiency conditions stemming from Pareto-optimality are derived following the concept of short-run efficiency as in Mitra. This involves solving a two-period programming problem along the optimal path with fixed endpoints. To formulate this succinctly, we let $g(w_t) = h_{t+1} - h_t$ represent the second expression on the right-hand side of the aquifer equation of motion (5). An efficient path must be short-run efficient. Consider some period t. Then an efficient path for t and $t+1$ with fixed endpoints must satisfy a programming problem to maximize $u(c_{t+1})$ subject to a specified level of $u(c_t)$, budget constraints and technological conditions.

The programming problem is to maximize $u(c_{t+1})$ subject to

$$u(c_t) = \bar{u}_t$$

$$c_\tau = \pi_\tau - \Delta k_\tau$$

$$h_{\tau+1} = h_\tau + g(w_\tau)$$

$$k_{\tau+1} = (1 + r_m)(k_\tau + \Delta k_\tau) \tag{6}$$

where $\tau \in \{t, t+1\}$, and $\{h_t, k_t\}$ and $\{h_{t+2}, k_{t+2}\}$ are taken as given. This assumes an interior solution, in particular w_t is not bounded by aquifer volume and the water table elevation is such that the no borrowing constraint is not binding.

Since $u(c)$ is univariate and monotone increasing in consumption, one can solve an equivalent problem over consumption without utility. Inspection of the above equations indicates that c_{t+1} can be written solely as a function of w_t. Maximizing this function with respect to w_t results in

$$\frac{\partial \pi(h_{t+1}, w_{t+1})}{\partial w_t} = (1 + r_m)\frac{\partial \pi(h_t, w_t)}{\partial w_t} - \frac{\partial \pi(h_{t+1}, w_{t+1})}{\partial h_t} \tag{7}$$

after re-arranging and making substitutions as appropriate. Since $\partial \pi(h_t, w_t)/\partial w_t = b'(q_t) - \gamma_e(\bar{h} - h_t)$, then letting $b'(q)$ be the agricultural commodity price and with $\gamma_e(\bar{h} - h_t)$ as marginal extraction cost, then this is just Hotelling's rule with a stock cost effect $\partial \pi(h_{t+1}, w_{t+1})/\partial h = -\gamma_e w_t$. An efficient path must satisfy this expression as a necessary condition.

Note that Hotelling's rule is not sufficient as there are an infinite number of paths satisfying this condition starting from an arbitrary value for w_1. For full efficiency,

we would need to select the largest w_1 such that the entire series satisfying Hotelling's rule (7) is consistent with stock feasibility.

Figure 2 illustrates a conceptual (short-run) utility possibilities frontier for $t = 20$ along the baseline PV-optimality results to be reported later. This possibilities frontier is downward-sloping and concave to the origin. Note that the largest consumption level from the simulations considered is that associated with CP extractions and no savings for the given water table height h_{20}. Also drawn on the diagram is a 45 degree line representing equal intergenerational utilities.

Any utility combinations on the curve and above the 45 degree line represent sustainable allocations. This graphically illustrates that there are an infinity of sustainable allocations. The curve also demonstrates sustainable (short-run) discount factors. Letting $-1/(1 + r_e)$ denote the slope of the possibilities frontier at the equal utility point, then any discount factor in the PV-optimality problem with $r_h < r_e$ will generate a sustainable S-R allocation.

5   Common Property

First considered is evolution of the system as an unregulated common property resource (CP). With many relatively small users, the effect of an individual user's current decisions on future water table elevations is borne almost entirely by others. Therefore it is reasonable to assume that under CP, each user maximizes profits in each period without regard for future values of the water table. This is consistent with the Gisser and Sanchez (1980) pumping cost model, and it also implies that

pumping decisions are independent of saving in the CP regime. (This will not be true under efficiency and sustainability.) Therefore the income stream under CP is exogenous to the savings problem, and saving decisions under CP can be optimized once the CP income stream has been computed.

5.1   Aquifer usage

Figure 3(a) displays simulated time-series results for extractions and the water table under CP. Starting from a completely full aquifer (268 feet above MSL), both extractions and the water table inevitably decline until reaching a steady-state after approximately 60 years. Extractions are initially large due to low pumping costs, but as the aquifer declines, pumping costs increase and extractions eventually decrease until reaching a steady-state value.

Figure 3(b) depicts annual net benefits (profits/income) over time. These decline through time as the water table falls and pumping costs increase until the steady-state is reached. Nominally, the common property system would then appear to be unsustainable since income from the aquifer is declining over time, at least until the steady-state. However, as noted earlier, this would not be a correct conclusion since investment possibilities imply that consumption might not be declining, and this is what matters. Note also that net benefits are expected to drop by over 50% during the simulation which is quite substantial.

5.2 Optimal saving

Under CP, the income stream is taken as given for the savings problem. Accordingly, the savings/investment problem facing the household is maximizing the present value of utility (1) subject to the output balance equation (2), the capital equation of motion (5) and the associated bounds. In this problem, the annual income stream π_t is exogenous and computed as above, and the single control variable is the amount saved/dissaved in each year Δk_t.

The Lagrangian for this optimization problem is

$$\sum_{t=1}^{T} \alpha^t [u(c_t) \ - \ \phi_{t+1}[k_{t+1} \ - \ (1 \ + \ r_m)(k_t \ + \ \Delta k_t)] \ + \ \psi_{t+1}(k_t \ + \ \Delta k_t)] \quad (8)$$

where $c_t = \pi_t - \Delta k_t$ is consumption. The Lagrange multipliers ϕ_t are the shadow values for the financial capital stock, and ψ_t are the shadow values on the no-borrowing constraints. The first-order conditions are

$$u'(c_t) = (1 + r_m)\phi_{t+1} + \psi_{t+1} \quad (9)$$

for savings Δk_t, while the co-state equations of motion are

$$\phi_{t+1} = \frac{(1 + r_h)\phi_t}{1 + r_m} - \frac{\psi_{t+1}}{1 + r_m} \quad (10)$$

for ϕ_t.

When the initial aquifer state is full (the primary case of interest here), income will be falling over time. Consequently, households will be interested in saving and not borrowing. This means the borrowing constraint will not be binding and so the shadow value $\psi_t = 0$. The investment optimality conditions then become $u'(c_t) =$

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$(1 + r_m)\phi_{t+1}$ and $\phi_{t+1} = (1 + r_h)/(1 + r_m)\phi_t$. Concave utility then implies that consumption will be falling over time if $r_h > r_m$, constant if $r_h = r_m$, and increasing over time if $r_h < r_m$.

5.3   Numerical results

Figure 3 shows the time-paths for both savings and the capital stock under the baseline assumption that $r_h = 0.045$. In this instance, the household saves during the early years of high income from the aquifer, and then dissaves as income falls over time. Accordingly, the capital stock builds up over time, and then eventually falls back to zero at the end of the 60 year horizon.

Figure 4 shows consumption time-series under different household subjective discount rates, along with the income stream. If the subjective discount rate is sufficiently high relative to the market interest rate (e.g. $r_h = 0.10$), then there will be no savings and consumption just equals the income stream. The baseline subjective discount rate does result in some consumption smoothing. However, when $r_h = r_m$, then, interestingly enough, the household maintains a constant consumption rate over the horizon. When $r_h < r_m$, then consumption is actually increasing over time. Geometrically then, reducing the household subjective discount rate can be viewed as progressively rotating the consumption time path such that early consumption is reduced and later consumption is increased.

Depending on interest rates, the consumption smoothing associated with savings and investment (potential) makes the aquifer less unsustainable (and possibly

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sustainable) in equity terms than what would appear from observing the physical variables and income time path only. Under the baseline parameter value, the aquifer system is still not sustainable since consumption is declining; however, the consumption smoothing does imply less of a decline than that of income. If $r_h \leq r_m$ then the aquifer would be sustainable (equitable) since $c_t \leq c_{t+1}$. Of course, another route to partial sustainability (intergenerational equity) is if the aquifer starts below the common property steady-state (CPSS). This might occur, for example, if the aquifer is initially in CPSS but then energy costs increase so that the new CPSS is higher than the current water table level. This will imply increasing income and consumption as the water table rises to the new CPSS.

5.4   CP usage and sustainability

Summing up, is CP sustainable? CP is not economically efficient due to the pumping cost externality, although an extensive literature has typically found these inefficiencies to be small which is the case here as well. Intergenerational equity is more complicated. If the aquifer starts below the CPSS for some reason, or if $r_h \leq r_m$, then consumption will be constant or increasing over time. However, under the conditions that seem most reasonable for this analysis, CP is not likely to be equitable as defined here, although the inequity may well be less than what nominally appears to be the case from looking at the physical variables and income alone. The next question is then the extent to which this non-sustainability is due to CP usage of the aquifer; that is, will PV-optimality make an unsustainable CP system sustainable?

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6 Utility maximization

U-opt maximizes discounted utility (1) subject to the output and net benefit equations (2-3), the water table and capital equations of motion (4-5), and the associated definitions and bounds. As the full model is optimized, the control variables are now groundwater extractions w_t, and savings/dissavings Δk_t in each year t. This may be interpreted as a competitive equilibrium for generations that live one period and have corrected the pumping cost externality so CE is efficient.

6.1 Optimality conditions

The Lagrangian for this optimization problem is

$$\sum_{t=1}^{T} \alpha^t [u(c_t) - \lambda_{t+1}[h_{t+1} - h_t - \frac{\beta_s \bar{s} + \beta_q q_t - w_t}{As^y}]$$

$$- \phi_{t+1}[k_{t+1} - (1 + r_m)(k_t + \Delta k_t)] + \psi_{t+1}(k_t + \Delta k_t)] \quad (11)$$

where $c_t = b(q_t) - \gamma_e(\bar{h} - h_t)w_t - \Delta k_t$ is consumption and $q_t = (1 - \beta_s)\bar{s} + w_t$. The Lagrange multipliers λ_t and ϕ_t are the shadow values for the resource and financial capital stocks respectively, and ψ_t is the shadow value on the no-borrowing constraint.

First-order conditions are

$$u'(c_t)[b'(q_t) - \gamma_e(\bar{h} - h_t)] = \frac{(1 - \beta_q)}{As^y}\lambda_{t+1} \quad (12)$$

$$u'(c_t) = (1 + r_m)\phi_{t+1} + \psi_{t+1} \quad (13)$$

for withdrawals w_t and savings Δk_t respectively. Co-state equations of motion are

$$\lambda_{t+1} = (1 + r_h)\lambda_t - \gamma_e u'(c_t)w_t \quad (14)$$

$$\phi_{t+1} = \frac{(1+r_h)\phi_t}{1+r_m} - \frac{\psi_{t+1}}{1+r_m} \tag{15}$$

for λ_t and ϕ_t respectively.

6.2 Aquifer management

We begin by considering how the aquifer is managed under U-opt compared to the standard model of maximizing PV of annual net benefits at the market interest rate r_m (this is denoted PV-opt). The latter conditions are standard and reported in the Appendix. Consider first the primary case of interest where the aquifer is initially full. This implies that income will typically be falling over time, consequently households will be interested in saving and not borrowing, and therefore the borrowing constraint will not be binding.

These conditions imply that the shadow value $\psi_t = 0$. The extraction first-order and resource stock co-state equation of motion can therefore be written as

$$b'(q_t) = \gamma_e(\bar{h} - h_t) + \frac{(1-\beta_q)}{As^y} \frac{\lambda_{t+1}}{u'(c_t)} \tag{16}$$

and

$$\lambda_{t+1}/u'(c_t) = (1+r_h)\lambda_t/u'(c_t) - \gamma_e w_t \tag{17}$$

respectively. The investment first-order condition for periods t and $t-1$ and the capital co-state equation of motion (both with $\psi_t = 0$) imply the Euler-type condition $u'(c_t) = (1+r_h)/(1+r_m)u'(c_{t-1})$. Substituting this into the water table co-state equation of motion (17), yields

$$\frac{\lambda_{t+1}}{u'(c_t)} = (1+r_m)\frac{\lambda_t}{u'(c_{t-1})} \tag{18}$$

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as the water table co-state equation under $\psi_t = 0$.

Conditions (16 and 18) are the same as optimizing the PV of income subject to the water table equation of motion (Appendix). In this instance, $\lambda_t / u'(c_t)$ is the water table co-state for the latter PV-optimal problem. Thus, the assumption of perfect capital markets holds under these circumstances, production and consumption are separated, and household preferences do not influence production decisions. However, these conditions do not hold for initially low water tables. In this case, incomes are increasing and the household would like to borrow but cannot, implying that $\psi_t > 0$ and hence the optimality conditions would therefore differ from the standard PV-income conditions. In this instance, then, household subjective parameters can affect aquifer management.

6.3   Utility maximization and sustainability

Efficiency under U-opt is essentially immediate: if the allocation is not Pareto-optimal, then there would be an allocation with utility at least equal to that of the PV-optimal allocation in every period, and strictly greater in one or more time periods. However, this allocation would have a higher PV, contradicting the optimality of the original allocation.

The efficiency condition (7) derived earlier for an interior solution can also be derived for the case when the initial aquifer state is full (the primary case of interest here). This implies that income will be falling over time, consequently households will be interested in saving and not borrowing, and therefore the borrowing constraint

will not be binding. In this circumstance the shadow value $\psi_t = 0$.

The extraction first-order condition for $t + 1$ is then $u'(c_{t+1})\partial \pi_{t+1}/\partial w_t = ((1 - \beta)/(As^y))\lambda_{t+2}$. Substituting for λ_{t+2} from the water table co-state equation (14), and then substituting for λ_{t+1} from the withdrawal first-order condition (12) yields

$$u'(c_{t+1})[\frac{\partial \pi_{t+1}}{\partial w_{t+1}} + \frac{1 - \beta}{As^y}\frac{\partial \pi_{t+1}}{\partial h_{t+1}}] = (1 + r_h)u'(c_t)\frac{\partial \pi_t}{\partial w_t} \tag{19}$$

after re-arranging. The first-order investment condition for period $t + 1$ becomes $u'(c_{t+1}) = (1 + r_m)\phi_{t+2}$. Substituting for ϕ_{t+2} from the capital co-state equation (15) and then substituting for ϕ_{t+1} from the investment first-order condition (13) evaluated at period t yields $(1 + r_h)u'(c_t) = (1 + r_m)u'(c_{t+1})$. Substituting this into (19) and dividing by $u'(c_{t+1})$ leaves the (interior) efficiency condition (7).

Next consider intergenerational equity. The investment optimality conditions under the high initial water table (implying $\psi_t = 0$) are $u'(c_t) = (1 + r_m)\phi_{t+1}$ and $\phi_{t+1} = (1 + r_h)/(1 + r_m)\phi_t$. The financial capital shadow value ϕ_t will be increasing over time if $r_h > r_m$, constant if $r_h = r_m$, and decreasing over time if $r_h < r_m$. Accordingly, concave utility then implies that consumption will be falling over time if $r_h > r_m$, constant if $r_h = r_m$, and increasing over time if $r_h < r_m$.

Intuitively, a low initial water table suggests a binding borrowing constraint if the market interest rate r_m is not too high. With an infinite horizon, the water table under optimal management would rise over time until the OSS. In this case, then, both income and consumption will increase over time, and especially consumption will increase if the market interest rate is sufficiently attractive for saving. This effect will also occur in a finite horizon of sufficient length, except that there are likely to be

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terminal horizon effects so that the water table falls again after some point, although consumption may still be increasing. The overall conclusion is that PV-optimality (CE) may not be sustainable even with the externality corrected; it depends on the relative values of r_h and r_m and the initial conditions.

6.4   Numerical results

Figure 6(a) illustrates the optimal decision rule for extractions as a function of the water table elevation and capital stock. This was estimated by solving the optimization problem over a range of initial water table levels and capital stock values, and then utilizing the optimal first-period extraction rates. Ground water extractions for agriculture are zero for low water tables, but otherwise increasing in hydraulic head. Consistent with the theoretical results, optimal extractions are not affected by the capital stock for high enough values of the water table level. However, they are affected by the capital stock for lower values of the water table.

As in CP, optimal time paths for the water table and extractions [figure 7] also decline over time. However, a comparison of CP to PV-optimality reveals that optimal extractions are less than CP extractions for a given water table level, hence the optimal water table is above that of CP. The difference is substantial: there is a xx foot difference in the terminal period water table levels.

Also in contrast to CP, there is no saving in the baseline case (Figure 7). Presumably this is due to the fact that optimal management already results in some consumption smoothing compared to CP. Optimal management entails a reduction

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in early net benefits to reduce the water table decline rate. This then results in higher annual net benefits in later periods, hence the consumption smoothing effect arising from efficient management. Nevertheless, consumption levels are still declining over time. Thus, although this system is efficient, it does not satisfy the intergenerational equity criteria of non-declining utility and hence is not sustainable.

A sensitivity analysis was also performed for alternate subjective discount rates (Figure 8). The results are qualitatively similar to CP. Some saving is evident with $r_h > r_m$ provided the difference is not too great. When $r_h = r_m$, then consumption is constant through time, while $r_h < r_m$ implies increasing consumption through time as before.

When the initial water table is below the steady-state level (h_{100} with $h_1 =$ high), income and consumption are increasing over time and therefore sustainable. Note that this result is sensitive to the level of r_h and k_1 used; specifically, for high levels of r_h this result will likely not hold.

6.5   Is CP the cause of non-sustainability?

Under the sustainability criterion here, CP is inefficient and therefore not sustainable. Under baseline conditions, the difference is approximately 119.8 ft. in year 60, thus the annual difference can be quite substantial after a sufficient number of years under optimal management. However, basin-wide annualized net benefits over a 60 year horizon are \$76.96 $acre^{-1}yr^{-1}$ and \$73.69 $acre^{-1}yr^{-1}$ under efficiency and common property, respectively. Thus the difference in discounted annual net benefits

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is $3.28 $acre^{-1}yr^{-1}$ or 4.5% of CP discounted income starting from a full aquifer. This is a relatively small amount, something which has been found repeatedly in the groundwater economics literature. [Khoundri, Knapp and Baerenklau].

The other dimension is intergenerational equity. In this case both CP and PV-optimality can result in declining utility over time, hence they are not equitable according to the definition here. Conseqently, while CP is theoretically inefficient, it can't be viewed as the fundamental - or at least only - cause of non-sustainability. This is because even the standard criterion of PV-optimality can lead to non-equitable outcomes. This demonstrates that fixing externalities alone is not sustainability. Externality correction achieves efficiency, but not necessarily equity.

7 Sustainability analysis: Constrained Utility maximization

The previous sections analyze market conditions (with and without externality correction, altruism) to achieve sustainability. This section explores sustainable allocations should the market with externality correction not be sustainable. Note that in general there are an infinity of sustainable allocations, so there is still a social choice problem over the sustainable set.

Sustainable time paths with PV-optimality are generated here with the addition of the sustainability constraint

$$u(c_t) \leq u(c_{t+1}) \tag{20}$$

implying non-declining utility over time (Bromley chapter). This is not the only approach to generating sustainability. In particular, in this model sustainable allocations can simply be generated by appropriate choice of the subjective discount r_h as demonstrated in the previous analysis. However, this approach does not necessarily generalize to more general models with endogenous r_m, hence we pursue the sustainability constraint approach here which is robust across alternate model types.

7.1 Optimality conditions

The Lagrangian for this optimization problem is

$$\sum_{t=1}^{T} \alpha^t [u(c_t) - \lambda_{t+1}[h_{t+1} - h_t - \frac{\beta_s \bar{s} + \beta_q q_t - w_t}{A s^y}]$$

$$-\phi_{t+1}[k_{t+1} - (1 + r_m)k_t - (1 + r_m)\Delta k_t]$$

$$+\psi_{t+1}(k_t + \Delta k_t) + \theta_t [u(c_{t+1}) - u(c_t)]] \tag{21}$$

where $c_t = b(q_t) - \gamma_e(\bar{h} - h_t)w_t - \Delta k_t$ is consumption and $q_t = (1 - \beta_s)\bar{s} + w_t$. The Lagrange multipliers λ_t and ϕ_t are the shadow values for the resource and financial capital stocks respectively, ψ_t is shadow value on the no-borrowing constraint, and θ_t is the sustainability shadow value.

First-order conditions are

$$b'(q_t) - \gamma_e(\bar{h} - h_t) =$$

$$\frac{(1 - \beta_q)}{A s^y} \frac{\lambda_{t+1}}{u'(c_t)} + [\theta_{t+1} - (1 + r_h)\theta_t][b'(q_t) - \gamma_e(\bar{h} - h_t)] \tag{22}$$

for withdrawals w_t and

$$u'(c_t) = \frac{(1 + r_m)\phi_{t+1} + \psi_{t+1}}{1 + (1 + r_h)\theta_t - \theta_{t+1}} \tag{23}$$

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for savings Δk_t. Co-state equations of motion are

$$\lambda_{t+1} = (1 + r_h)\lambda_t - [1 + (1 + r_h)\theta_t - \theta_{t+1}]u'(c_t)\gamma_e w_t \tag{24}$$

$$\phi_{t+1} = \frac{(1 + r_h)}{1 + r_m}\phi_t - \frac{\psi_{t+1}}{1 + r_m} \tag{25}$$

for λ_t and ϕ_t respectively.

7.2 Aquifer management

Aquifer management is first considered for the case of a high initial water table implying $\phi_t = 0$ as before. The withdrawal first-order condition and aquifer co-state equation of motion are

$$b'(q_t) - \gamma_e(\bar{h} - h_t) = \frac{(1 - \beta_q)}{As^y} \frac{\lambda_{t+1}}{u'(c_t)[1 + (1 + r_h)\theta_t - \theta_{t+1}]} \tag{26}$$

and

$$\frac{\lambda_{t+1}}{u'(c_t)[1 + (1 + r_h)\theta_t - \theta_{t+1}]} = \frac{(1 + r_h)\lambda_t}{u'(c_t)[1 + (1 + r_h)\theta_t - \theta_{t+1}]} - \gamma_e w_t \tag{27}$$

in somewhat more convenient forms.

The expression (27) can be converted to an expression for aquifer MUC. The first-order condition for investment (23) is combined with the co-state equation for the capital stock shadow value (25) to yield $u'(c_t) = [(1+r_h)\phi_t]/[1+(1+r_h)\theta_t-\theta_{t+1}]$ while rearranging an analogous expression for $u'(c_{t-1})$ gives $\phi_t = [1 + (1 + r_h)\theta_{t-1} - \theta_t]u'(c_{t-1})/(1 + r_m)$. Substituting the latter into the former yields

$$u'(c_t) = \left(\frac{1 + r_h}{1 + r_m}\right)\left[\frac{1 + (1 + r_h)\theta_{t-1} - \theta_t}{1 + (1 + r_h)\theta_t - \theta_{t+1}}\right]u'(c_{t-1}) \tag{28}$$

as an Euler-type condition.

Utilizing (28) in the water table co-state equation of motion (27) yields

$$\frac{\lambda_{t+1}}{u'(c_t)[1 + (1 + r_h)\theta_t - \theta_{t+1}]} = \frac{(1 + r_m)\lambda_t}{u'(c_{t-1})[1 + (1 + r_h)\theta_{t-1} - \theta_t]} - \gamma_e w_t \quad (29)$$

after re-arranging. It can be checked that the first-order condition (26) and the aquifer co-state equation of motion (29) are therefore equivalent to PV-optimality (Appendix) where $\lambda_{t+1}/[u'(c_t)[1 + (1 + r_h)\theta_t - \theta_{t+1}]]$ is marginal user cost.

Thus when the aquifer starts out full, aquifer management proceeds according to standard PV-optimality, and sustainability is achieved via adjustments in savings/dissavings rates. When the aquifer starts out low, then the no borrowing constraint may be binding and $\phi_t > 0$. This does not necessarily imply the same stock co-state equation of motion as (29) and so the aquifer may be managed differently than PV-optimality depending on the household preference parameters and other parameter values.

7.3   Sustainability

We first consider the efficiency properties of this problem. This is not immediately obvious since utilities have been constrained. [Fn. why P-O proof doesn't work. Proof in 2004 paper?] This is conducted for an initially full aquifer state (the primary case of interest here). In this instance, income will be falling over time, and consequently, households will be interested in saving and not borrowing. This means the borrowing constraint will not be binding and so the shadow value $\psi_t = 0$.

. . . . *Short-run efficiency conditions here* . . . . .

Intergenerational equity. If r_h is sufficiently small, then consumption is rising under the pure PV-optimality problem, hence the sustainability constraints (20) are not binding and $\theta_t = 0$. In the case where r_h is greater than some threshold value, then the sustainability constraints (xx) are binding and $\theta_t > 0$. This implies in turn that $u(c_t) = u(c_{t+1})$ for $1 \le t \le T$, i.e. equalized consumption over time. For efficiency, this would need to be the highest constant level of consumption that could be maintained over the horizon. Intuitively, this would be the annualized value of optimal aquifer management over the horizon, evaluated at $r = r_m$ [Hartwick, Solow, Weitzman].

7.4 Numerical results

Time paths are illustrated in figure 9. Confirming the theoretical results, the physical variables for the sustainability problem are identical to the PV-optimal case. In contrast to PV-optimality with the baseline subjective discount rate, the sustainability solution does have saving in the early years with dissaving in the later years. This maintains a constant level of consumption over the horizon to satisfy the sustainability constraints. As expected, this consumption level does equal the annualized value of optimal aquifer management.

The θ_t shadow values for the sustainability constraints are illustrated in figure 10. These are positive, consistent with the fact that under the assumed parameters (notably $r_h = .045$), utility would be declining over time. These shadow values can be used to specify policy instruments for achieving sustainability, and they can also

be used to identify an equivalent PV-optimal problem. Mathematically, optimizing

$$\sum_{t=1}^{T} \alpha^t [u(c_t) - \lambda_{t+1}[h_{t+1} - h_t - \frac{\beta_s \bar{s} + \beta_q q_t - w_t}{As^y}]$$

$$-\phi_{t+1}[k_{t+1} - (1 + r_m)k_t - (1 + r_m)\Delta k_t]$$

$$+\psi_{t+1}(k_t + \Delta k_t) + \hat{\theta}_t[u(c_{t+1}) - u(c_t)]] \tag{30}$$

over w_t and Δk_t, given the appropriate $\hat{\theta}_t$ values, and subject to the appropriate constraints, will lead to an equivalent solution to the constrained sustainability formulation.

This in turn is equivalent to the unconstrained PV-optimal problem with discount rates such that

$$\prod_{\tau=1}^{t} \frac{1}{1 + r_{st}} = \alpha^t + (1 + r_h)\theta_{t-1} + \theta_t \tag{31}$$

where r_{st} are the equivalent annual discount rates. As the θ are time-varying, it can be anticipated that the calculated discount rates are also time-varying. As it would not be possible to know these in advance of solving the problem, this approach could not be used for actually calculating sustainable allocations. However, for policy work, one application might be to solve the problem above, and then use the estimated r_s for extension to the uncertainty case.

8 Conclusions

Sustainability is a widely used concept but not always explicitly defined. In the authors experience, one possibility is that commentators often conclude lack of sustainability when the resource stock is observed to be declining. However, this is not

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necessarily a correct conclusion as sustainability can't be inferred from the physical variables and/or income alone. Sustainability depends on consumption which in turn depends on how the resource rents are invested and is likely not equal to income. Consumption can be going up even as income from the natural resource is declining. At a minimum, consumption smoothing may imply more sustainability than apparent from the physical variables or income/annual net benefits alone.

Other studies pursue sustainable management of the resource stock alone by either lowering the discount rate in a standard PV-opt analysis or by stepwise-inefficiencies. The difficulty is that these strategies are not efficient because rates of return across asset classes will not be equalized as per Hotelling's rule. Another possibility is that sustainability is an optimal steady-state; the limitation here is that reaching an OSS can take a long-time (groundwater being a good example), and hence the transition path is of considerable practical importance.

This paper pursues sustainable natural resource management by including household preferences and investment opportunities in conjunction with a sector-level resource management model. A formal sustainability criteria (efficiency defined as Pareto optimality and intergenerational equity defined as non-declining utility) is applied to several behavioral regimes. In essence, the conceptual starting point of the paper is that incorporating explicit equity analysis requires moving from the standard sector-level analysis in the direction of dynamic general equilibrium. While this observation is not novel, it does not appear to be widely applied in resource management studies.

CP is not sustainable due to the well-known pumping cost externality. Under baseline conditions here, it is also not equitable due to declining utility over time, although some consumption smoothing is exhibited. Under other conditions, however, consumption can be constant or even increasing even as income decline. Conversely, while U-max corrects the externality, it also exhibits declining utility under baseline conditions. One conclusion is that CP and externalities are not the only - or even fundamental - cause of non-sustainability. While correcting the externality is necessary for efficiency and hence sustainability as defined here, it does not necessarily imply equity and therefore is not a sufficient condition.

As can be seen, allowing for savings can imply a considerably different consumption stream. In particular, the concave utility function in combination with a sufficiently low subjective discount factor implies a significant amount of consumption smoothing over time. This means that the actual non-sustainability of the resource is less than what might be apparent from observing only physical variables or income.

Finally this work also motivates the need for resource management studies to consider the role of borrowing/saving and household preferences for natural resource management. Under standard separation theorems, household decisions are separate from production decisions, and present-value optimization of net benefits at the market interest rate is the appropriate criterion. Most of the natural resource literature utilizes this criterion.

However, this presupposes perfect capital markets where borrowing and saving at a constant rate are possible. In lower income countries, the opportunity for users

to participate in capital markets may be limited, and even in higher-income industrialized countries there are borrowing constraints. Furthermore, market interest rates can be endogenous (e.g. household debt burden, physical capital in economic growth). In this instance, since household preference parameters affect investment and various capital stocks, then they influence market rates of return and hence optimal investment. While not explicitly pursued here, this does point out the need for more research in this area (Just et al reach a similar conclusion).

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Table 1: **Kern County Irrigated Agriculture and Aquifer Parameters**

Symbol	Description	Value [Units]
A	Aquifer area	1.29 [10^6 acres]
$\bar{h}$	Maximum aquifer height	268 [ft. MSL]
$\underline{h}$	Minimum aquifer height	-233 [ft. MSL]
β_q	Irrigation percolation coefficient	0.2
β_s	Surface water percolation coefficient	0.3
$\bar{q}_s$	Maximum surface water flow	1.97 [af yr^{-1}]
s^y	Aquifer-specific yield	0.13 [ft. ft.$^{-1}$]
ω	Natural recharge	0.052 [af yr^{-1}]
γ_e	Energy cost	0.23 [af^{-1}yr^{-1}]
p_{sw}	Price of surface water	37 [\$ af^{-1}]

Figure 1:
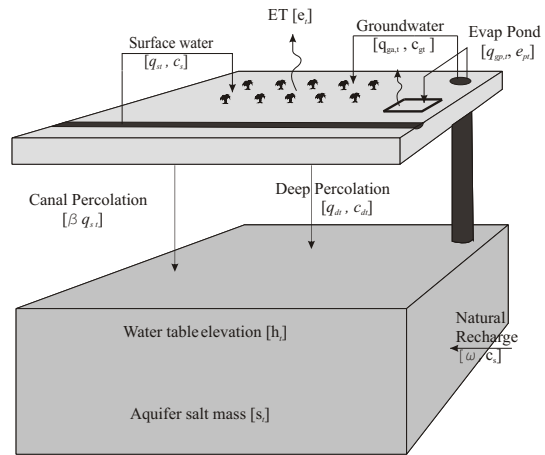
1
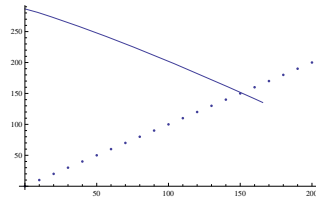
Figure 2: Short-run utility possibilities frontier. [Baseline U-opt with t=20.]

2
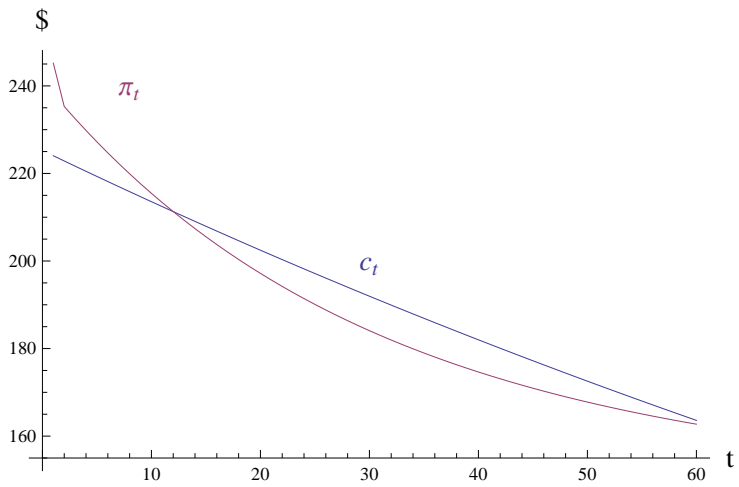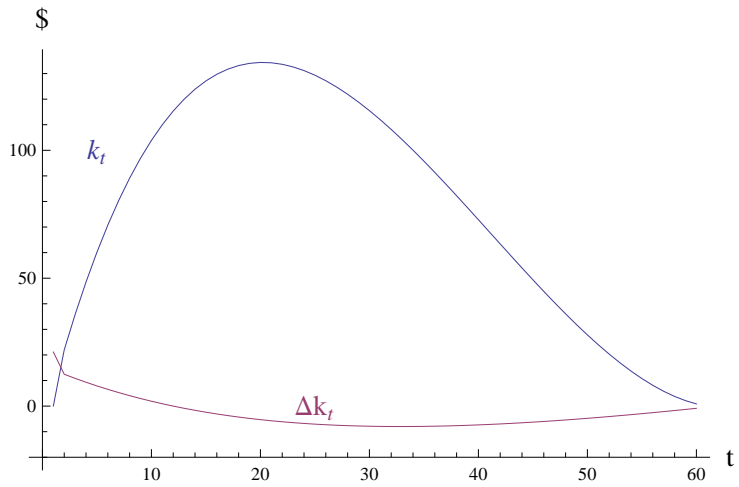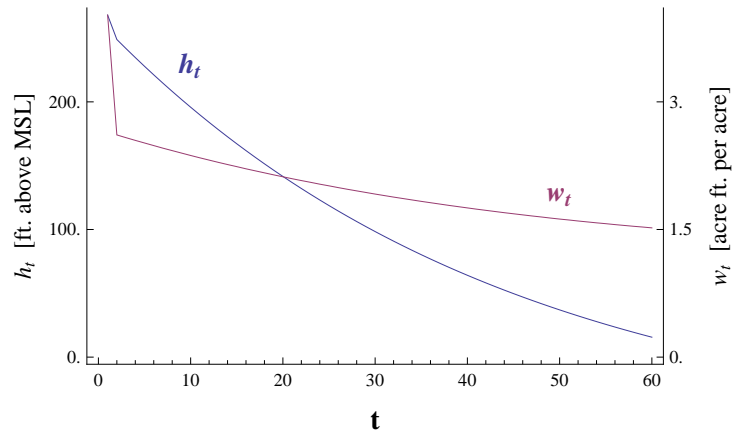
Figure 3: Common Property (i) Aquifer Height and Groundwater Use (ii) Financial Capital and Net Investment (iii) Income and Consumption
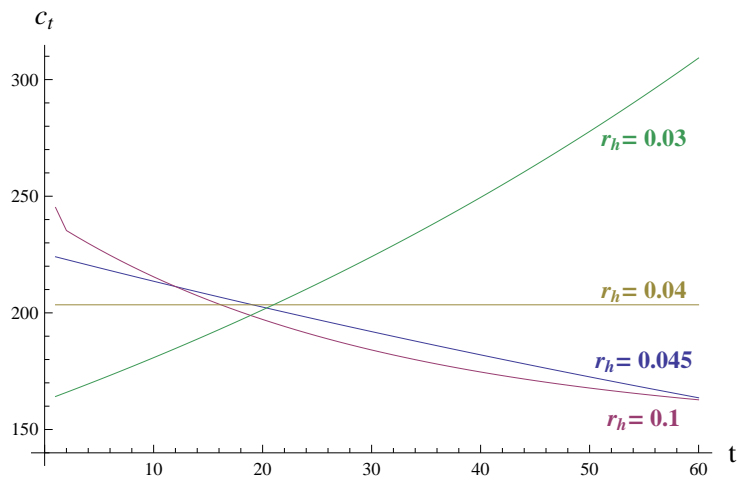
Figure 4: Common Property Consumption as Dependent on Household Discount Rate (r_h)
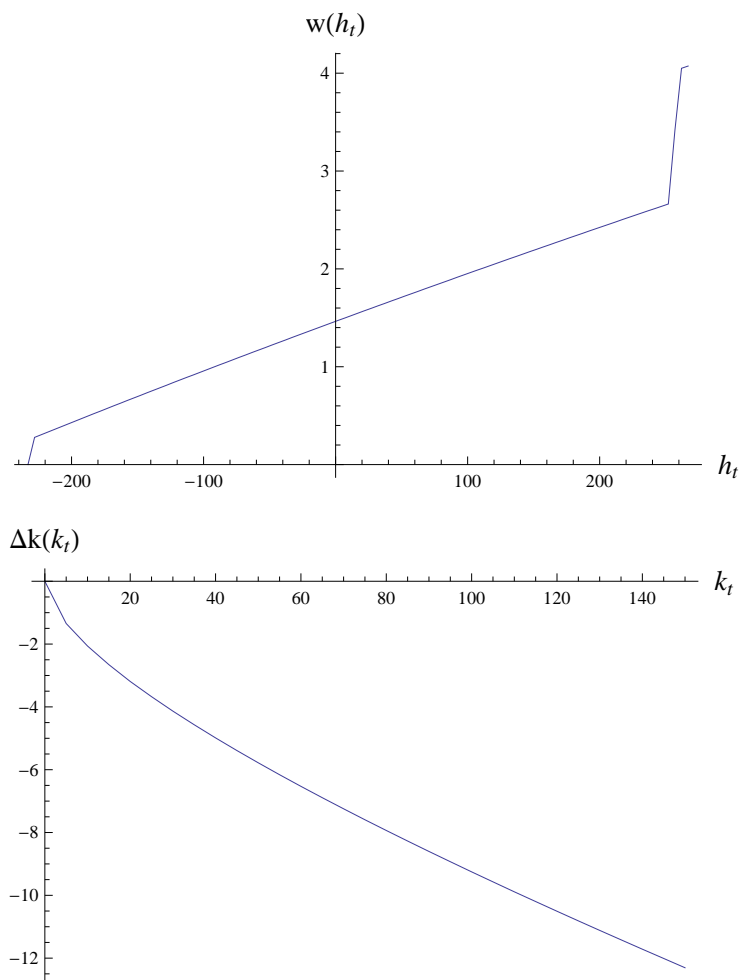
Figure 5: Common Property Decision Rules (i) Groundwater Use (ii) Net Investment
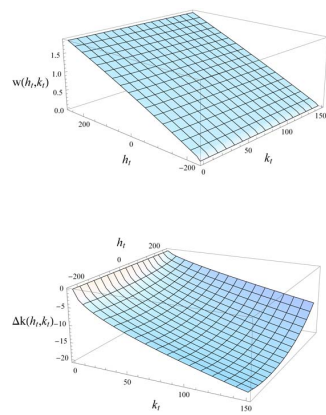
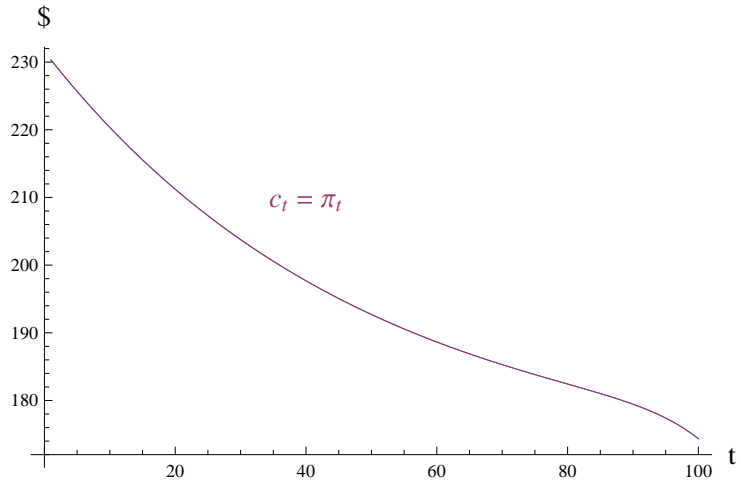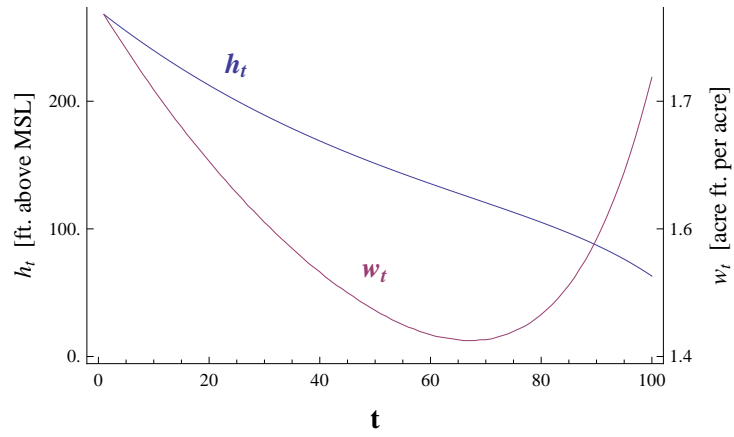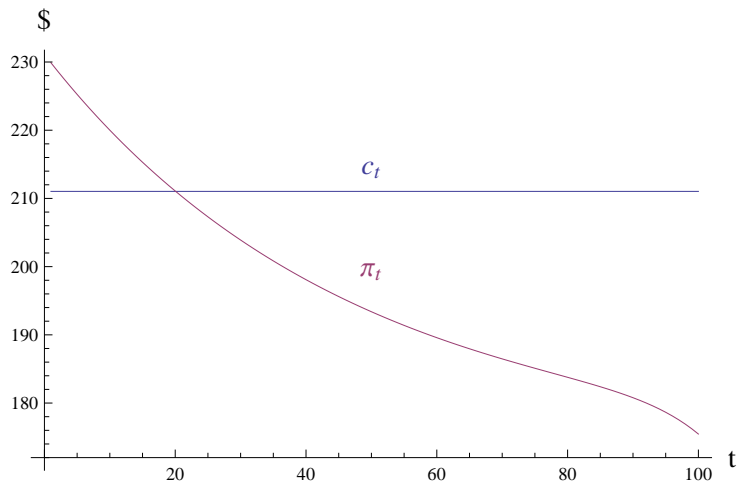Figure 6: PV Optimality Decision Rules (i) Groundwater Use (ii) Net Investment

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Figure 7: PV Optimality (i) Aquifer Height and Groundwater Use (ii) Income and Consumption
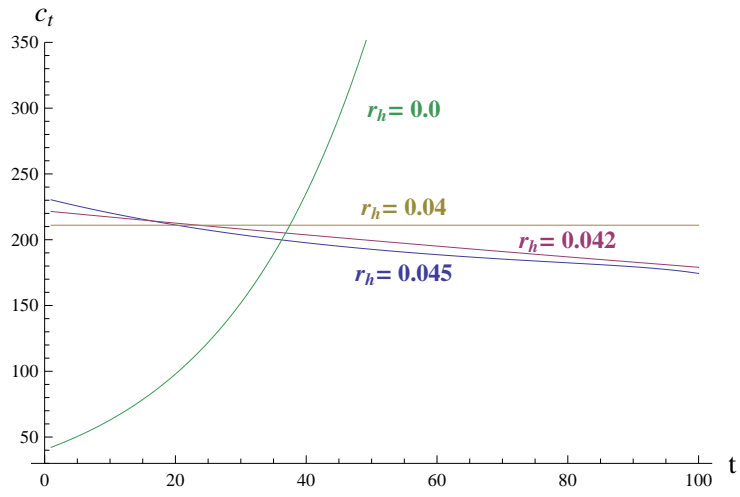
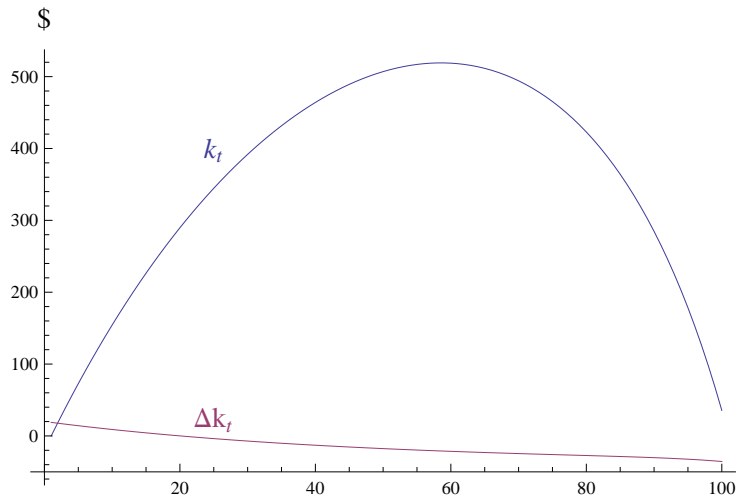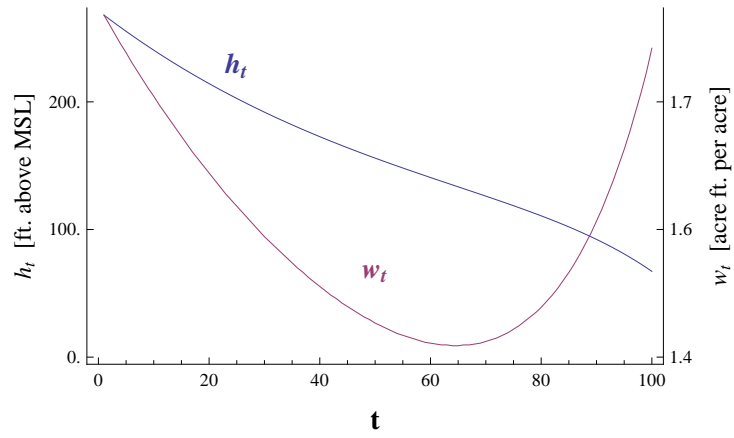Figure 8: PV Optimal Consumption as Dependent on Household Discount Rate (r_h)

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Figure 9: Sustainability (i) Aquifer Height and Groundwater Use (ii) Financial Capital and Net Investment (iii) Income and Consumption
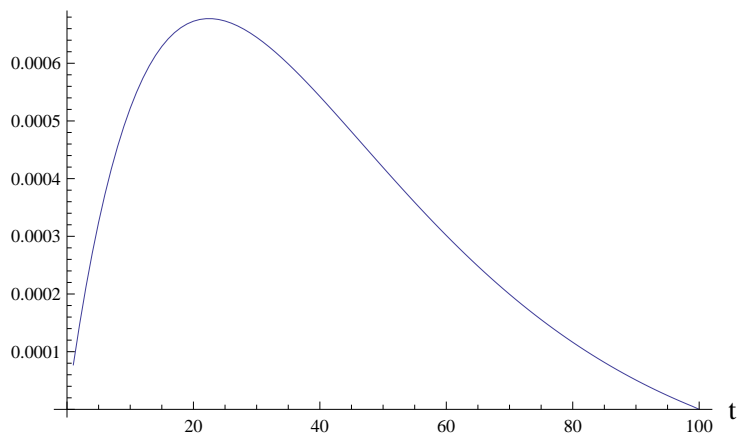
Figure 10: Sustainability: Time series for the sustainability constraint shadow value