Asymmetric Price Transmission: A Copula Approach

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Outline

- Asymmetric price transmission (APT)
- Why copulas?
- An empirical application to the U.S. hog/pork industry
  - Parameter estimation
  - Model selection
  - Model evaluation
- Results and concluding remarks

APT: Why do we care?

- Why do we care about vertical price transmission?
  - Relevant to structure, conduct, and performance issues (i.e., market power)
  - Market behavior often characterized by:
    - Extent of adjustment
    - Permanent versus temporary responses
    - Asymmetric adjustments
- Much of this literature has been directed toward asymmetric adjustments

APT: Empirical literature

- Early analysis used regression and correlation-based tests

$$\Delta p_t^R = \alpha_0 + \alpha_1^+ I^+ \Delta p_t^F + \alpha_2^- I^- \Delta p_t^F + \varepsilon_t$$

$$\Delta p_t^R = \alpha_0 + \sum_{i=1}^{s} \alpha_i{}^+ \Delta p_{t-i}^{F+} + \sum_{i=1}^{q} \alpha_j^- \Delta p_{t-j}^{F-} + \varepsilon_t$$

- More recent attention to time-series properties of the data

$$z_t = p_t^R - \alpha - \beta p_t^W$$

$$\Delta z_t = a + b^+ I_{t-1}^+ z_{t-1} + b^- I_{t-1}^- z_{t-1} + \varepsilon_t$$

APT: Empirical literature

- Current research focusing on regime-switching models (e.g.,threshold, smooth transition error correction models)
- A typical two-parameter and three-regime switching model

$$\Delta z_t = \begin{cases} \alpha_1 + \phi_1 z_{t-1} + \varepsilon_1 & \text{if } z_{t-1} < \theta_1 \\ \alpha_2 + \phi_2 z_{t-1} + \varepsilon_2 & \text{if } \theta_1 < z_{t-1} < \theta_2 \\ \alpha_3 + \phi_3 z_{t-1} + \varepsilon_3 & \text{if } z_{t-1} > \theta_2 \end{cases}$$

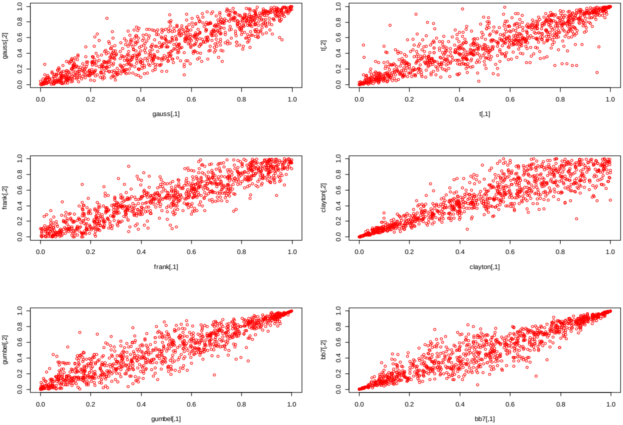
- Smooth transition versions can be obtained by replacing the discrete, regime-changing function by a smooth function
- Goodwin et al. (2011) is the first attempt to introduce copula models into the empirical analysis of price transmissions

Motivations

- A simple linear dependence is assumed, at least, within each regime
  - For a two-variable case, the transmission coefficient is simply a product of the Pearson correlation coefficient and the ratio of standard deviations, i.e. $\hat{\phi} = \hat{\rho}_{\Delta z_t z_{t-1}} \hat{\sigma}_{\Delta z_t} / \hat{\sigma}_{z_{t-1}}$
  - Assume constant variances, and then everything is about $\hat{\rho}$
- Is the linear dependence enough? See a graph for illustration
- General dependence strength versus specific aspects of asymmetry
  - Extreme market conditions
  - Quantile dependence

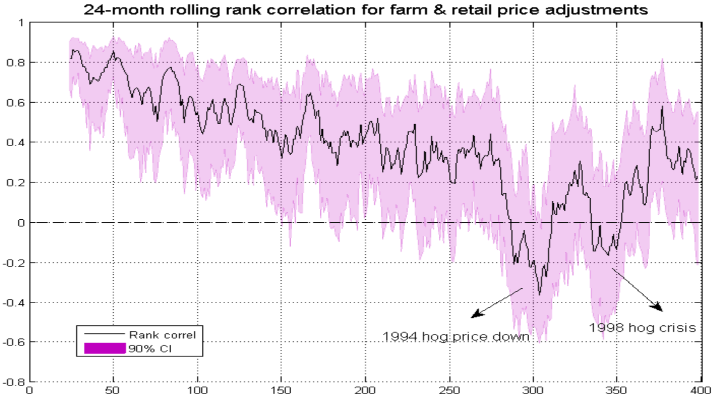
Motivation: Specific aspects of asymmetric adjustments

- Each pair has a linear correlation coefficient of 0.9



Motivation: Adjustments may be time-varying

- Price transmission or adjustment may also have the time-varying feature



Why copulas?

- Motivate the search for more flexible alternative/extra measures of dependency
- The copula approach serves as a promising candidate
  - Copula representations of dependence are free of the linear restriction
  - Copulas enable us to model marginal distributions and the dependence structure separately
  - Copulas allow quantile dependence (taking the tail dependence as an extreme example)
  - Copulas allow dependence varies over time (both parameters change and the copula itself changes)
  - More...

What is a copula model?

- Definition (Copula)
  A d-dimensional copula is a multivariate distribution function C with standard uniform marginal distributions
- Sklar theorem (1959)
  Let H be a joint distribution function with margins $F_1, F_2, \ldots, F_d$. Then there exists a copula $C : [0,1]^d \to [0,1]$ such that $H(x_1, \ldots, x_d) = C(F(x_1), \ldots, F(x_d); \theta)$
- θ is a set of parameters that measures dependence
- Conditional copulas and joint distributions (Patton 2006)

$$H_t(x_1, ..., x_d | W_{t-1}) = C_t(F(x_1), ..., F(x_d) | W_{t-1})$$

where W_{t-1} is the information set.

What is a copula model?

Popular copula families

- Elliptical copulas: Gaussian and t
- Archimedean copulas: Clayton, Gumbel, Frank, Plackett, etc
- Different copulas allow for different dependence structures
- Summary dependence analysis helps narrow down the copula choice
- Model selection is important

Empirical procedure

- 1. Model marginal distribution functions
- 2. Estimate copula parameter(s)
  - Two-stage maximum likelihood method
  - Parametric and semi-parametric models
  - Constant and time-varying copulas (Patton 2006, Creal et al. 2011)
- 3. Model selection
  - Tests of goodness of fit: Kolmogorov-Smirnov and Cramer-von Mises tests (Genest et al. 2009, Remillard 2010)
  - Model evaluation (Rivers and Vuong 2002, Chen and Fan 2006)
- 4. Explore the APT features based on the best fitted copula(s)

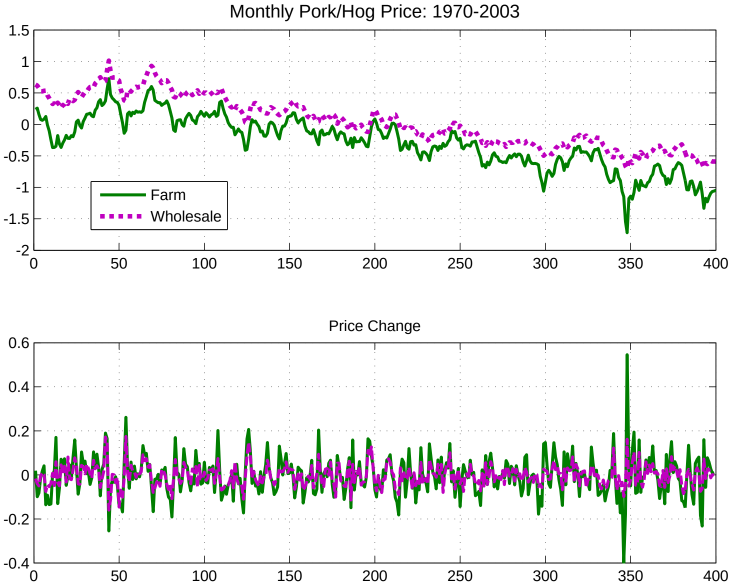
Data

- Data
  - Monthly data on hog (farm) and pork (wholesale and retail) prices covering January 1970 through April 2003
- We are interested in the dependence structure between three pair-wise price changes

$$p_t^F \,\&\, p_t^W,\; p_t^W \,\&\, p_t^R, \text{ and } p_t^F \,\&\, p_t^R$$

where $p_t^i = \log P_t^i - \log P_{t-1}^i$ and P_t^i is the real price of i, $i\ =\ $ farm, wholesale, and retail.

Data

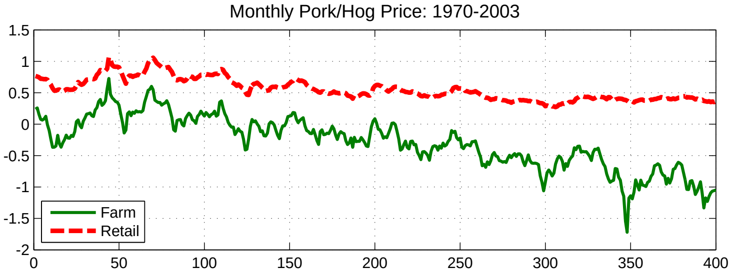


Monthly Pork/Hog Price: 1970-2003
Farm
Wholesale
Price Change

Data


Monthly Pork/Hog Price: 1970-2003
1.5
1
0.5
0
-0.5
-1
0
50
100
150
200
250
300
350
400
Wholesale
Retail
Price Change
0.6
0.4
0.2
0
-0.2
-0.4
0
50
100
150
200
250
300
350
400

Data


Monthly Pork/Hog Price: 1970-2003
1.5
1
0.5
0
-0.5
-1
-1.5
-2
0
50
100
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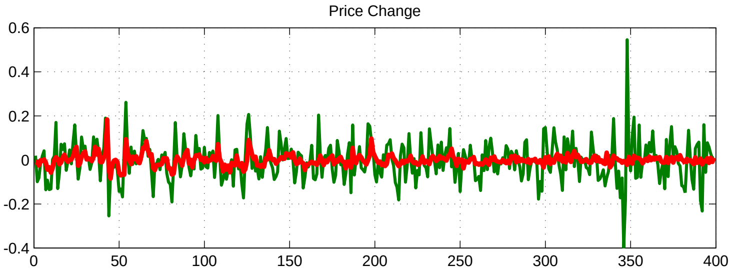
200
250
300
350
400
Farm
Retail


Price Change
0.6
0.4
0.2
0
-0.2
-0.4
0
50
100
150
200
250
300
350
400

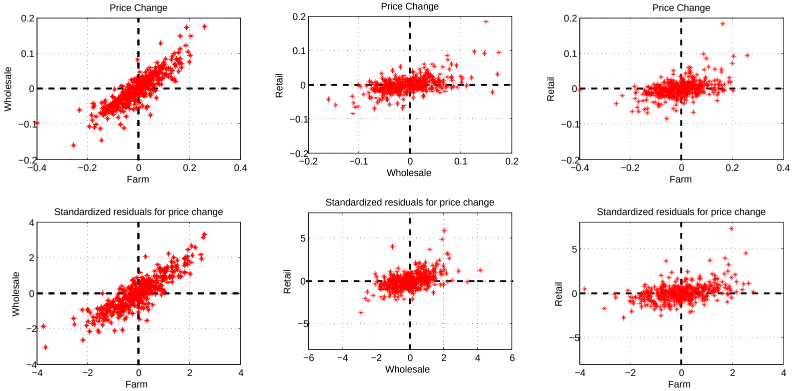
Model marginal distributions

- Price adjustments follow the standard mean-variance structure:

$$\begin{cases} p_t^i = \bar{p}_t^i(W_{t-1}) + \sigma_t^i(W_{t-1})\varepsilon_t^i \\ p_t^j = \bar{p}_t^j(W_{t-1}) + \sigma_t^j(W_{t-1})\varepsilon_t^j \end{cases}$$

- Model the conditional means and variances
  - Estimate the mean and variance: AR-GARCH, models include cross-equation effects when applied
- Model the marginal distributions for standardized residuals
  - nonparametric: empirical DF
  - parametric: regular student t and skew t
- Next step is to estimate the copula parameters. BUT before moving to the copula modeling, we explore the summary dependence characteristics of the data

Explanatory analysis of dependence



Explanatory analysis of dependence

- Table 2. Dependence summary—empirical

	Farm and wholesale	Wholesale and retail	Farm and retail
Pearson	0.87	0.55	0.42
Spearman	0.87 (0.83, 0.89)	0.48 (0.41, 0.54)	0.41 (0.34, 0.48)
Lower tail	0.46 (0.14, 0.83)	0.26 (0.02, 0.65)	0.12 (0.00, 0.54)
Upper tail	0.67 (0.31, 0.95)	0.31 (0.05, 0.71)	0.23 (0.02, 0.66)
Test Low=Upper	0.45	0.51	0.44

90% confidence intervals based on 1000 bootstrap replications are presented in parentheses.

- Summary: All three exhibit positive dependency. May have tail dependence but differences between upper and lower are not significant.

Explanatory analysis of dependence

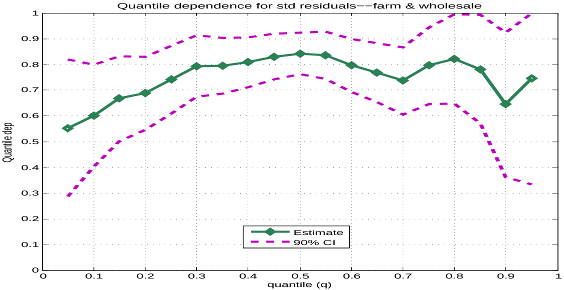
- Quantile dependence

$$\lambda^q = \begin{cases} \Pr[F_{it} \leq q | F_{jt} \leq q], & \text{if } 0 < q \leq 0.5 \\ \Pr[F_{it} \geq q | F_{jt} \geq q], & \text{if } 0.5 < q < 1 \end{cases}$$

- Especially important in APT
  - e.g., under market power of retailer hypothesis, one may expect to observe a larger upper quantile dependence (large positive price adjustments) than a lower one (large negative price adjustments)
  - if the menu cost hypothesis dominate, one might anticipate seeing a relatively larger upper quantile dependence at q=0.7 than at the q=0.6

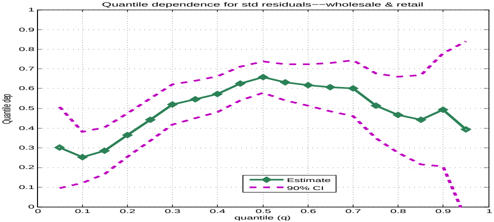
Explanatory analysis of dependence

- Test the asymmetric quantile dependence at $q = 0.1$ and 0.25
- Both reject the null hypothesis of equality in favor of $\hat{\lambda}^{\text{upper}} > \hat{\lambda}^{\text{lower}}$
- Quantile dependence
  - Farm and wholesale

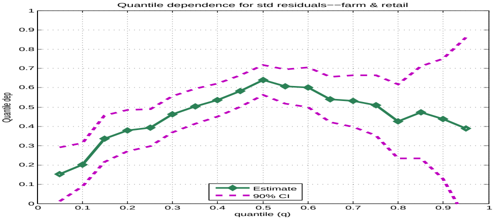


Explanatory analysis of dependence

- Wholesale and retail



- Farm and retail



Estimate constant copulas

- Table 3. Constant copulas

	Parametric			Nonparametric		
Farm & wholesale	**param1**	**param2**	**LL**	**param1**	**param2**	**LL**
Normal	0.874		288.5	0.875		289.2
Clayton	2.321		197.9	2.387		201.5
Frank	9		266.5	9		265.4
Gumbel	3.04		292.1	3.113		298.2
Bootstrapping SE	(0.122)			(0.137)		
Rot Gumbel	2.804		258	2.845		260.3
Student's t	0.877	0.11	292.1	0.88	0.145	294.8
Bootstrapping SE	(0.011)	(0.052)		(0.01)	(0.062)	
Wholesale & retail	**param1**	**param2**	**LL**	**param1**	**param2**	**LL**
Normal	0.506		58.9	0.504		58.4
Bootstrapping SE	(0.06)					
Clayton	0.695		43.9	0.699		42.6
Frank	3.31		51.9	3.329		51.9
Gumbel	1.465		56.7	1.486		59.2
				(0.062)		
Rot Gumbel	1.449		54.2	1.453		53.1
Student's t	0.505	0.107	61.2	0.508	0.13	61.2
Bootstrapping standard error	(0.06)	(0.062)		(0.037)	(0.07)	

Estimate constant copulas

- Table 3. Constant copulas (con't)

	Parametric			Nonparametric		
Farm & retail	**param1**	**param2**	**LL**	**param1**	**param2**	**LL**
Normal	0.433		41.3	0.434		41.4
Clayton	0.502		24.8	0.513		25.7
Frank	2.761		37.2	2.748		36.9
Gumbel	1.367		41.5	1.385		43.1
Bootstrapping SE	(0.061)			(0.034)		
Rot Gumbel	1.333		32.6	1.337		32.8
Student's t	0.436	0.05	41.8	0.441	0.057	41.9
Bootstrapping SE	(0.046)	(0.044)		(0.022)	(0.038)	

Results

- Table 4. Results from tests of TV rank correlation

	0.1	0.5	0.9	Unknown	AR(1)	AR(5)
			Farm & wholesale			
p-value	0.521	0.639	0.748	0.74	0.429	0.473
			Wholesale & retail			
p-value	0.195	0.46	0.951	0.19	0.521	0.109
			Farm & retail			
p-value	0.054	0.188	0.822	0.05	0.575	0.204

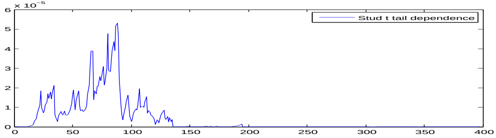
Farm & retail potentially has time-varying dependence

- Estimate time-varying (TV) copulas (Creal et al. 2011) for the farm-retail case for the selected copulas (Normal, Gumbel and t)
- Both have higher log-likelihood and AIC values, compare to their corresponding constant copula models

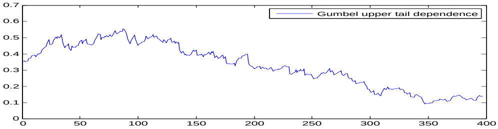
Tail dependence

- For each constant copula, the tail dependence is similar to that obtained from the sample data (however, the t copula usually indicates a larger tail dependence)

- Tail dependence from TV t



- Tail dependence from TV Gumbel



Results

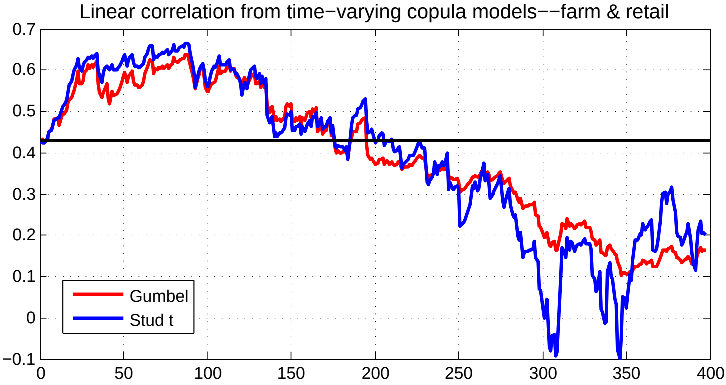
- Table 5. Goodness of fit tests and model selection

	Parametric			Nonparametric		
	KS_R	CVM_R	Rank	KS_R	CVM_R	Rank
Farm & wholesale						
Normal	0.5	0.5	3	0	0	3
Gumbel	1	1	2	0	0	1
Student t	0.5	0	1	0	0	2
Wholesale & retail						
Normal	0.4	0.4	2	0.5	0.75	3
Gumbel	0.4	0.4	3	0.45	0.8	2
Student t	0.95	1	1	0.35	0.55	1
Farm & retail						
Normal	0.95	0.95	3	0.5	0.55	3
Gumbel	0.85	1	2	0.65	0.6	1
Student t	0.9	0.9	1	0	0.25	2
Gumbel-TV	0.35	0.2	NA	0.7	0.2	NA
Student-TV	0.15	0.35	NA	0.3	0.2	NA

Ranks are based on in-sample model comparison (Patton 2012).

Results: Linear correlation coefficients

- For constant copulas, the linear correlation coefficients are calculated from two-dimensional numerical integration. Values are quite similar to those obtained from the sample data.
- For time-varying copulas, the linear correlation coefficients are obtained from simulation.



Summary results

- Farm and wholesale markets are more closely related to each other.
- Retail price adjustment is less dependent on the other two markets.
- Farm-to-retail and retail-to-wholesale price adjustments have relatively constant dependence structures, but farm-to-retail price adjustments exhibit a dynamic, time-varying relationship. Dependency decreases as time goes by (as real prices decrease). This relationship may reflect the market power of retailers.
- Upper quantile dependence is stronger than the lower ones, which indicates that the price is more likely to adjust accordingly when the adjustments of the other price is positive. This is again consistent with the market power hypothesis.

Summary results

- Tail dependence
  - For the farm-to-wholesale and wholesale-to-retail situations: constant tail dependence indicates that markets are linked to each other under extreme market conditions. Shocks in one market would transfer to the other market. The magnitude of dependence varies by the choice of copulas.
  - For the farm-to-retail situation: dependency under extreme market conditions is decreasing dynamically. Under very extreme conditions (i.e., in December 1994 hog prices reach the historical low, and the 1998 hog crisis), lower tail dependence reaches a very low level. This suggests that a retail price does not respond to a dramatic reduction in a farm level price.

Conclusions

- More generally, APT can include many other forms of price co-movements (e.g., long-run and short-run asymmetries, contemporaneous impacts, distributed lag effects, cumulated impacts, and reaction times. See Frey and Manera 2007 for a detailed discussion).
- The copula approach can apply to these APT analyses as well, thus serving as a useful extension and generalization of dependency analysis for modeling APT.
- Growing literature on dynamic copulas could provide increasingly flexible tools for investigating asymmetric price adjustments (e.g., modeling dynamic weights of mixture copulas allows both the asymmetric tail dependence and the asymmetric dependence structure simultaneously).