Commodity Price Adjustment in a Competitive Storage Model with an Application to the US Biofuel Policies

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Abstract

When demand for a commodity shifts permanently outward, the effect of the shift on the level and volatility of prices will differ over the short run and long run depending on: (1) how early the market anticipates the demand shock; (2) the elasticities of supply and demand in the new environment as compared to the old environment; and (3) inventories available when news of the impending demand shock arrives. We analyze the relative importance of these three features on food prices using a modified version of the storage model that allows demand to change over time. Model parameters are found using a grid search that minimizes the difference between simulated and observed data. We then use the model to evaluate the effect of the US ethanol mandate on world food prices. In line with earlier studies, we find that the US ethanol mandate, by shifting the world demand out by more than 5 percent in 2015, can account for 11 to 30 percent of the food price increase between 2005 and 2011 depending on demand and supply elasticities before and after the demand shift. Unlike earlier studies, the analysis shows how the ethanol mandate may have exacerbated the initial price spike or increased volatility during the transition to the new demand environment.

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The recent global surge of ethanol production has drawn a significant proportion of annual grain supply into energy production. In the United States, the Renewable Fuels Standard (RFS) of 2005 and 2007 mandate that fifteen billion gallons corn ethanol be produced by 2015. In order to meet that goal, the US will need about 5.5 billion bushels of corn, which amounts to about forty percent of total US production in 2011. Measured in calories, this equals roughly 7.4 percent of the combined production of the worlds four largest food crops: corn, wheat, rice, and soybeans in 2005 when the policy was first announced. Together with biofuel policies from other countries, the US ethanol mandate has created an outward shift in demand for staple food commodities, raising prices of crops used in biofuel production, as well as major food crops that serve as substitutes for biofuel crops in production or consumption.

Although food commodity prices have soared in recent years, and demand growth from ethanol demand likely has been a contributing factor, there has been considerable debate about the importance of ethanol relative to other factors, like demand growth from rapidly emerging economies (particularly India and China) and weather anomalies. Standard models indicate that, even in stable demand environments, commodity prices will occasionally experience large spikes in both levels and volatility. These spikes occur when random weather or other supply or demand events happen to occur in short succession, thereby drawing down inventories that normally buffer transitory shocks.

In this paper, we investigate how a large, permanent shift in demand, such as that following from rapid growth in ethanol, interacts with a competitive storage market and other kinds of shocks that may occur around the time of the demand shift. Specifically, we use a competitive storage model to examine how prices and inventories adjust in response to both anticipated and unanticipated permanent shifts in demand. This exercise allows us to trace out how prices change in both the short run and long run. It is important to examine these short-run considerations because while most analysis to date suggests biofuel growth can account for only a fraction of price increases experienced in recent years, these analyses do not account for how inventories adjust to the new demand environment. This transition is our main focus.

When world demand for staple grains permanently and unexpectedly shifts out (say, due to ethanol mandates), the higher demand must be satisfied from some combination of higher prices and inventory changes. Adjustment will begin as soon as the market anticipates the demand shift. If the shift happens unexpectedly, and inventories absorb the shock, the initial price change would be modest, although it may bring about greater price volatility during the transition period. If the initial price change is larger, inventories may not be sufficient to buffer ancillary shocks. If the demand shock is anticipated before it occurs, inventory accumulation transfers the price shock backwards to the time when news of the impending demand shock arrived. Thus, key features that ultimately determine the nature of the price transition to a permanent demand shock include: (1) how early the market anticipates the demand shock; (2) the elasticities of supply and demand in the new environment as compared to the old environment; and (3) the level of inventories when news of the impending demand shock arrives. These factors determine whether prices transition gradually to the new equilibrium, initially overshoot the long-run equilibrium, as well as the volatility of prices in response to ancillary shocks (like the weather) during the transition period.

We analyze the relative importance of these three features using a modified version of the storage model that we solve using stochastic dynamic programming. We relax the assumption of *i.i.d* harvest and model land area cultivated as a function of expected future price, while demand remains a function of current price. Due to the temporal delay of supply response, both supply and demand are identifiable. We then solve the infinite-horizon storage model for the year after demand has shifted out completely, assuming model parameters subsequently remain fixed. The solution gives the storage function in the new long run stochastic equilibrium. We then solve recursively for the storage equilibrium one year before the demand shift, accounting for anticipated optimal response after the demand shift, and continue to solve backwards in time for as long the demand shift is anticipated.

We calibrate our model using price and quantity data for corn, soybeans, rice, and wheat between 1964 to 2011 from the US Department of Agriculture website. We convert production and prices of these four grains into total calories and dollars per calorie. These four grains account for roughly eighty percent of the world caloric crop production, and given world prices tend to fluctuate together, they appear to serve as substitutes in production and/or consumption. Following Roberts and Schlenker (2010), we therefore aggregate the four series to obtain a single time series of quantities and prices.

Because the model includes both demand and supply response, we have two more unknown parameters, which makes estimation of all the parameters in the model challenging. Instead of full estimation, we select 2000 sets of parameters, solve the model and generate 5000 years of data for each of them. Based on these simulated data, we compute a GMM-like scores to select the set of parameters with the smallest difference between simulated and observed data. The set of parameters with demand and supply elasticities values of -0.06 and 0.01 respectively are selected as they yield 9 out of the 10 lowest scores from 2000 scores computed using the first four moments of price and the first moment of production. We also consider elasticities estimated in earlier literature.

Using these elasticities, we consider a shift in demand in accordance with recent biofuel policies and examine how prices and inventories adjust assuming demand and supply elasticities stay constant over time. In this case, grain prices spike more in the short run than in the long run, especially when the demand shift is unanticipated. If the market reacts to news of the demand shock after 2007, prices are more volatile during the transition period. If the market anticipated the demand shock in 2006 or 2007, an increase in both inventories and price prior to the demand shift reduces price volatility in the short run. Price and inventories can rise simultaneously because the market rationally expects prices to continue increasing. The size of the initial price spike and the transition dynamics differ markedly depending on inventories when news of the demand shift arrives. Price rises more in the short run and become more volatile the lower are inventories when news of the demand shift arrives.

We then relax the assumption of constant supply and demand elasticities across environments. Even with different supply and demand elasticities, we find transitional price spikes and volatility changes that are similar to those described above. In addition, the more inelastic are supply and demand in the new environment, the greater the level and volatility of prices during the transition. But when demand and supply both become more elastic and the demand shift is anticipated, the initial price spike is smaller and prices approach the new long-run equilibrium more gradually.

Assuming news of the demand shift was anticipated in 2006, we examine how world food prices evolve from 2006 to 2011 using our derived storage demand functions in the transition periods and realized worldwide yield outcomes. Assuming demand and supply become more elastic with the shift, the US ethanol mandate accounts for 31 percent of the food price increase between 2005 and 2006 and 20 percent of the price change from 2005 to 2011. Initial price increases are far larger, however, if elasticities do not change or getting less elastic with the shift. For example, when demand gets less elastic, the initial price increase exceed the level observed in real data in 2006 and accounts for 54 percent of the price change between 2005 and 2011. Yet, in the longer run, the impact on food price subdue and only account for 30 percent of the difference between 2011 and 2005 prices. The result when demand and supply are getting more elastic roughly accords with Helbling et al. (2008) who use a simple partial equilibrium model also found that from 25 to 45 percent of the corn price increase between 2006 and 2007 are due to the US ethanol production. Similarly, Rosegrant (2008) attribute a 30 percent increase in weighted average grain prices from 2000 to 2007 to biofuel demand. Roberts and Schlenker (2010) also find a similar price increase from an econometrically-estimated model, but assume elasticities do not shift with the biofuel mandate. If we use the set of demand and supply elasticities estimated from Roberts and Schlenker (2010) but assume demand and supply become more elastic with the shift, we find that the US ethanol mandate comprise of 22 percent of the price increase between 2005 and 2006 and 11 percent of the price change from 2005 to 2011. These estimates are in line with

Baier et al. (2009) who found that 14 percent of the increase in corn price and 10 percent of the increase in soybeans price can be explained by the US ethanol mandate. Under the assumption that both demand and supply getting less elastic over time, our model indicates slightly relative increase with the US ethanol mandate account for 36 percent of the price difference between 2005 and 2006 and 14 percent of the price increase from 2005 to 2011.

1 The Model

The *rational expectations competitive storage model* has appeared repeatedly in the literature (e.g., Gustafson (1958), Williams and Wright (1991), Deaton and Laroque (1992, 1995, 1996), and Cafiero et al. (2011)). Earlier research considers a market with a demand and a random, *i.i.d.* harvest that is unresponsive to price (ie., the supply curve is vertical). We include supply response in our model by making cultivated land area a function of expected future price. This characterization of supply fits the facts as in Roberts and Schlenker (2010). And because supply response cannot respond to a current price shock, it price variance and autocorrelation structures should differentiate supply response from demand response. We also show how the new storage model can be used to model price transition when demand shifts.

1.1 Competitive Storage Equilibrium

For any time period t, the amount of food commodity on hand (z_t) at the beginning of the period equals unspoiled inventories carried over from the last period (x_{t-1}) plus the new harvest (q_t):

$$\begin{aligned} z_t &= q_t + (1-d)x_{t-1} \\ &= y_t l_{t-1} + (1-d)x_{t-1}, \end{aligned} \tag{1}$$

where d is the decay rate of food stored for one period, y_t is yield and l_{t-1} is the land area cultivated in the previous period. Equation (1) can also be written in terms on consumption and storage this period:

$$z_t = c_t + x_t. \tag{2}$$

Both the inverse food demand and land response functions are linear:

$$p_t = \alpha_d + \beta_d c_t$$

$$l_t = \alpha_s + \beta_s E_t(p_{t+1}). \tag{3}$$

Storage cost is an increasing function of food stored:

$$K(x_t) = k_0 x_t + \frac{1}{2} k_1 x_t^2,$$

with $k_0 > 0$ and $k_1 \geq 0$.[1]

Arbitrage conditions in a competitive market imply:

$$\left\{ \begin{array}{c} p_t + k_0 + k_1 x_t = \frac{1-d}{1+r} E_t(p_{t+1}), \text{ when } x_t > 0; \\ p_t + k_0 + k_1 x_t \geq \frac{1-d}{1+r} E_t(p_{t+1}), \text{ when } x_t = 0 \end{array} \right\}, \tag{4}$$

where r is interest rate. Equation (4) says that, in the presence of storage (i.e. $x > 0$), the discounted expected future price next period cannot exceed the spot price this period by more than the marginal storage cost. Otherwise, storers can arbitrage by storing more food in the current period, reducing current consumption and increasing current price. When current price increases, the gap between current price and discounted expected future price will fall until it equals to marginal storage cost. Similarly, the discounted expected future price can not be smaller than the current price plus storage cost. If the current price plus storage cost is greater than discounted expected future price, storers will store less food, increasing current consumption which will drive down the gap between current price and discounted expected future price until they equal.

A key feature of the model is the non-negativity of inventories. The non-negativity constraint gives rise to the inequality in equation (4), which is strict in the absence of storage and an equality when storage is positive. Zero storage implies $c_t = z_t$, all of the amount on hand in the current period is consumed. In this case price is given by the inverse demand function, $p_t = \alpha_d + \beta_d z_t = F(z_t)$. This will be optimal if anticipated future prices are too low relative to the present to cover the marginal cost of storage. When some of z_t is stored, the amount consumed declines, and current price rises. From equation (4), in the case of positive storage, price must equal the discounted expected future price minus storage cost. Equation (4) therefore can be re-written as:

$$p_t = max \left\{ \beta E_t(p_{t+1}) - k_0 - k_1 x_t, F(z_t) \right\}, \tag{5}$$

[1] In this paper we do not consider *convenience yields*—an apparent ancillary benefit to storers for holding inventories. In earlier literature, convenience yields have been used to explain holding of inventories during times of *contango*, which are times current spot prices exceed futures prices. Positive inventories during contango might alternatively be explained by risk premiums, option values and/or transactions costs associated with delivery of commodities from storage locations to futures market locations. We do not delve into these issues because they are likely of minor importance for the questions investigated in this paper.

where $\frac{1-d}{1+r}$ is replaced by β for simplification.

Equations (4) and (5) indicate the market demand function will not be smooth. When z_t is sufficiently low, all of the amount on hand z_t will be consumed this period and p_t will be high. However, as z_t increases and p_t declines, some of z_t will be stored. At the point where the first unit of commodity is stored, the market demand function has a non-smooth "kink." In other words, there exists a cut-off price below which there will be positive storage and market demand is the sum of consumption demand and storage demand.

Solving for the storage model is complicated due to the non-negativity of storage, the aforementioned kink and because there is feedback between storage and prices. The amount of food stored this period depends on current food availability and on expected prices in future periods. Expected future prices and food availability depend, in turn, on storage in future periods and the last period. Because dependence of current storage on future and past storage, solving for equilibrium requires stochastic dynamic programming. We follow the numerical method initiated by Gustafson (1958) and adapted by Deaton and Laroque (1992, 1995, 1996), and Cafiero et al. (2011). The basic idea is to approximate price p as a function of the amount on hand z using a flexible functional form. We extend the earlier literature by allowing for supply response in storage the model.

Accounting for anticipated supply response, we can rewrite equation (5) as:

$$\begin{aligned} p_t(z_t) &= max\left\{\beta E_t(p_{t+1}(z_{t+1}) - k_0 - k_1 x_t, F(z_t)\right\} \\ &= max\left\{\beta E_t(p_{t+1}(y_{t+1}l_t + (1-d)x_t) - k_0 - k_1 x_t, F(z_t)\right\}. \end{aligned} \tag{6}$$

The state variable z_t is bounded by $[\underline{q_t}, \infty]$ where $\underline{q_t}$ is the lowest harvest possible. The amount stored, x, can also be written as:

$$\begin{aligned} x_t &= z_t - c_t \\ &= z_t - F^{-1}(p_t(z_t)). \end{aligned} \tag{7}$$

Expected future price requires integration over all values of the random yield values y_{t+1}. Integration is difficult given the unknown non-linear form of the price function $p(z_t)$. In order to simplify the model, yield is assumed to have a normal distribution and is discretized into N points so that equation (5) is expressed as:

$$p_t(z_t) = max\left\{\beta \sum_{n=1}^{N} p_{t+1}(y_{t+1}^n l_t + (1-d)x_t) \times Pr(y_{t+1}^n) - k_0 - k_1 x_t, F(z_t)\right\}, \tag{8}$$

where $Pr(y_{t+1}^n)$ is the probability for each of the possible yield from the yield distribution and $\sum_{n=1}^{N} Pr(y_{t+1}^n) = 1$.

Lastly, the land function is re-written as:

$$l_t = \alpha_s + \beta_s(\sum_{n=1}^{N} p_{t+1}(y_{t+1}^n l_t + (1-d)x_t) \times Pr(y_{t+1}^n)). \tag{9}$$

The discretization process is done using Gaussian quadrature.[2] We solve for price as a function of amount on hand (equation (8)) following an iterative process:

1. Select a grid of evaluation points for amount on hand, z. The grid is bounded by $[\underline{z}, \overline{z}]$. While $\underline{z}$ can be set to zero, $\overline{z}$ must to be chosen to equal or exceed the highest feasible amount on hand.

2. Select a flexible functional form for price as a function of amount on hand, $p(z)$. We use cubic spline for approximation.[3]

3. Guess the initial parameters for the price function $p(z_t)$.

4. Use the initial parameters to find the price on the left hand side of equation (8).

5. Calculate the inverse demand curve $F(z_t)$ and storage x_t using equation (7).

6. Given x from the previous step, solve the system of non-linear equations (9) to find land area.

7. Compute price using the right hand side of equation (8).

8. If the price calculated from step 7 equals the one found in step 4, stop the iteration process. If not, we replace the price from step 4 with the price in step 7 and repeat steps 4-7 until the two sides of equation (5) are equal.

The selection of the grid points requires some discussion. Cafiero et al. (2011) selected 1000 equally-spaced grid points to a precise approximation of the "kink" in the storage demand curve, plus any other possible non-linearities. Using this fine-scale grid they obtained a solution that implied a stochastic equilibrium price series with higher levels of autocorrelation. This finding solved a long-standing puzzle identified by Deaton and Laroque (1992, 1996), who used only 20 grid points. Because our model includes supply and thus has more parameters, and we must solve the model thousands of time to find suitable parameters, using a dense grid to finely approximate storage demand is computational expensive.

[2]We use the *CompEcon Toolbox for Matlab* by Miranda and Fackler (2004).

[3]Alternatively one can use linear spline. We found no difference between cubic spline and linear spline for our model.

Since, in practice, storage demand appears to be nearly linear except for the kink at the stockout price, we use fewer grid points but space them unequally, putting most of them near to where we expect the kink to be.

Table 1 reports a comparison of the two approaches. The number of grid points when amount on hand is equally and unequally spaced are 1000 and 50 respectively. The two storage models are identical except for location and number of grid points.[4] There is no meaningful difference between the two sets of results. For the remaining of this paper, we continue to use the approach of unequally spaced grid to solve for our storage models because it is much less computationally expensive.

1.2 Dynamic Equilibrium with Shifting Demand

Theory suggests that when storage is available and not too costly, transitory shocks to supply or demand will be buffered by accumulated inventories if available, tempering price volatility. As a result, inventories and price should tend to move in opposite directions.

One exception to this relationship between price and storage occurs when demand is anticipated to shift in the future. In this case, storers, motivated by inter-temporal arbitrage conditions, accumulate greater inventories with the hope of earning more profit in a future higher-demand environment. This accumulation of inventories withdraws product from the current market, transferring expected future price increases toward the present. In a standard storage model, the spike in current price will temper storers' incentive to transfer goods to the next period. The accumulation of inventories will cease when the total amount on hand (storage plus production) exceeds the new level of amount on hand in the new environment. From that point, because more product is being supplied in the market, the rate of price increase in the transition period will gradually become smaller.

We use backward induction within the storage model to show how price and other variables adjust when an outward shift in demand of food, such as that following from the US Ethanol Mandate, is first anticipated. First, we solve for the storage equilibrium after demand has just shifted out, assuming that the market equilibrium stays constant thereafter. The solution characterizes a terminal condition over the range of feasible prices, storage, and land use that may arise. We then solve backwards in time through the transition years along the demand path. We keep solving backward until we get to the time period when the news

[4] Amount on hand values run from 0 to 10. The quantity in equilibrium without storage is 6. We put 10 points between 0 and 5.5, 20 points between 5.5 and 7, and 20 points for the remaining values. Demand and supply parameters are -0.05 and 0.05 respectively. The decay rate and interest rate are both set at 0.02. Data for the two models are generated for 2000 periods using the same yield draws where yield $\sim \mathcal{N}(10, 1)$. Storage cost is set at 1.6.

of the demand shift was first anticipated by the market, which may occur before any actual shift demand occurs.

A formal description of our solution is as follows. Demand is assumed to shift out completely by period T and news of the demand shift arrives in period $T-m$. Assume that the yield distribution is constant. Denote θ_T and θ'_T as parameters of the demand and inverse demand functions in period T. Storage in period $T-1$ can be written as:

$$\begin{aligned} x_{T-1} &= z_{T-1} - c_{T-1} \\ &= z_{T-1} - F^{-1}(p_{T-1}(z_{T-1}), \theta_{T-1}). \end{aligned} \tag{10}$$

Equation (8) is then modified to account for the fact that $p(z)$ will be different for each period during the transition period $(T-m, T)$. We therefore express the the relationship between $p_T(z_T)$ and $p_{T-1}(z_{T-1})$ over the range of z:

$$p_{T-1}(z_{T-1}) = max\left\{\beta\sum_{n=1}^{N} p_T(y^n l_{T-1} + (1-d)x_{T-1}) \times Pr(y^n) - k_0 - k_1 x_{T-1}, F(z_{T-1}, \theta'_{T-1})\right\}. \tag{11}$$

Equations (10) and (11) show that storage demand in period $T-1$ can be found given $p_T(z_T)$ and l_{T-1}. Recall that land function can be implicitly solved using the equation:

$$l_{T-1} = \alpha_s + \beta_s(\sum_{n=1}^{N} p_{T-1}(y^n l_{T-1} + (1-d)x_{T-1}) \times Pr(y^n)). \tag{12}$$

We then solve for $p_{T-1}(z_{T-1})$ use the following iterative approach:

- Pick l_T as the starting values for land.
- Solve for $p_{T-1}(z_{T-1})$ using equation (11) and l_T.
- Solve for inventory and land using equations (10) and (12).
- If land value replicates itself, stop and use it to solve for $p_{T-1}(z_{T-1})$. Otherwise, use this new land value to solve for price in step 2.

Once $p_{T-1}(z_{T-1})$ is found, we use it to find $p_{T-2}(z_{T-2})$ and solve recursively until we find $p_{T-m}(z_{T-m})$.

Given storage demand functions for each year of the transition, we use simulation to examine how price and other variables evolve with news about the demand shift. We first draw N random observations of yield from the same distribution. For a starting values we use equilibrium values at the points where amount consumed just equal amount on hand (zero storage). Given amount on hand, we can solve for price, consumption, production, land, and inventories in the first period.[5] With inventories from the first period and yield in the second period, we can find all the other variables in the second period. We derive variables of interest until the time $T-m-1$. When the shock occurs at $T-m$, we simply take the amount on hand available at $T-m$, which is known using x_{T-m-1} and y_{T-m}, and storage demand function in $T-m$ to find the associated price. We continue to use the subsequent storage demand functions to generate data for the following time periods until we reach to the last storage demand function in the new environment at time T. Because the effect of the demand shifts depends on inventories and amount on hand, and these are influenced by random production/weather shock, we simulate the demand shift and transition many times to obtain the entire distribution of price changes and other variable changes .

2 Data and Model Calibration

2.1 Data

World production, yield, and inventory data come from the Production, Supply and Distribution dataset (PSD) of the United States Department of Agriculture (USDA). We collect data for the world's four largest principal food commodity crops: corn, rice, soybeans, and wheat from 1964 to 2011.[6] Because the storage model is for a single commodity, we follow the steps of Roberts and Schlenker (2010) to convert variables into caloric energy using the conversion rate from Williamson and Williamson (1942). Given these grain and oilseed prices tend to fluctuate together with similar relative magnitudes, they appear to serve as strong substitutes in production and/or consumption. We therefore aggregate production and inventory data for the four crops into one commodity. Amount on hand is calculated as production this period plus discounted inventory last period.[7] Consumption is calculated by subtracting storage in the current period from amount on hand. World yield is calculated as production-weighted average output per acre harvested of the four commodities.

Figure 1 shows production and yield of the four main crops from 1964 to 2011. Over forty

[5] In implementation, we also express land as a function of amount on hand. Therefore, given amount on hand, we can find price and land at the same time. We also discard the first third of simulated time periods to avoid bias associated with the selection of starting value.

[6] Roberts and Schlenker (2010) estimates that about 75 percent of the caloric content of food production worldwide come from these four main crops.

[7] We use a decay rate of 0.02 for food crop (i.e. one unit of commodity stored this period will yield 0.98 unit of goods next period). The decay rate as estimated by Cafiero et al. (2011) varies from 0.01 for corn to 0.14 for rice using a sparse grid of 20 points. These estimates become much lower when the grid resolution is increased to 1000 points, with the decay rate for corn falling to zero.

years, world production increased more than three-fold, from producing enough calories to feed roughly two billion people in 1964 to have food for more than seven billion people in 2011. The yield trends show the productivity gains explain most but not all of the growth in output. Corn has the largest share of calories while soybeans have the smallest share. In addition, corn is the crop with the greatest variability and rice, which is predominantly irrigated, is the crop with the lowest variability. One explanation for large variability of corn production is the high concentration of production in United States. The U.S. has consistently been the largest corn producer, accounting for 39 percent of world production. Most of its corn is grown on non-irrigated land in the upper Midwest, which makes corn yield and production more sensitive to weather of this region. Other crops' planting areas are less concentrated geographically, so weather anomalies are more likely to average out.

We collect price data from the National Agricultural Statistics Service (NASS) of USDA. Prices are those received by US farmers in the month of December of each year. Like yield data, we convert our price series into calories and calculate the production-weighted averages of the four crops.

Figure 2 plots price versus storage from 1964 to 2011. Price is the US dollar in year 2010 that can be used to buy one year of calories for one person. In general, price of food has been going downward until a recent spike after 2005. Even though the increase in price in 2005 is small compared with other incidents such as the price spike in early 1970s, it is one of the biggest *relative* increases, almost doubling in three years, from forty three dollars in 2005 to eighty dollars in 2008.[8] The second line on Figure 2 shows the ratio of inventory over trend production, derived using a simple linear regression. On average, 20-30 percent of the world annual harvest are carried over from one period to the next, with inventories declining markedly from 1985 until 2005. Due to government accumulation of inventories in many countries, the average level is high relative to competitive storage theory, and stock outs never occur. We do, however, observe a strong negative relationship between inventories and prices. This is consistent with the basic intuition of the competitive storage model where inventories have an important role in smoothing price variability. Storers, motivated by arbitrage profit, store goods in times of abundance and let go in times of shortage. As a result, storage have direct strong impact on the equilibrium of commodity market. For a short period after 2005, prices and inventories go in the same direction, and the subsequent relative price rise was large amid relatively stable inventories. This exception may signal the impending demand shift derived from rapidly growing ethanol production.

[8]The daily data for individual commodities show much larger price spikes, rising three-to-four fold between early 2006 and summer of 2008.

2.2 Storage Model Parameters

There have been several earlier attempts to estimate parameters of the competitive storage model. Deaton and Laroque (1992), under the assumption of i.i.d. supply, use Generalized Method of Moments (GMM) to estimate storage cost, the kink on the storage demand curve, and the discount rate of the expected future price. The authors were unable to back out parameters of the demand curve using GMM. Deaton and Laroque (1996) and Cafiero et al. (2011), still assuming perfectly inelastic supply, use Pseudo maximum likelihood to estimate demand parameters and storage cost. In our research, the added elastic supply give two more unknown parameters of the supply curve. Full estimation of the model parameters is therefore extremely challenging and beyond the scope of this paper. Instead, we apply a grid search over pre-determined set of carefully selected parameters and select the set of parameters that minimized the difference between our simulated data and observed data.[9].

Because our storage model generates data that is stationary over time, and price and quantity data collected both have clearly discernible trends, in order to be able to match observed and simulated data, we detrend the observed data. To do this, we first regress real values of quantity and price data on a linear time trend and obtain predicted values. We then divide each year's data by their predicted values in the same year and multiply that by the predicted value in 2005, which is base year of our analysis. For example, detrended price in year 1990 will be:

$$\widetilde{p_{1990}} = \frac{p_{1990}}{\widehat{p_{1990}}} \times \widehat{p_{2005}},$$

where $\tilde{p}$ is the dentreded price and $\hat{p}$ is the predicted price.

For purposes of calibration, we do not use data after 2005. We do this because this is the last year before the first US ethanol mandate came into effect and to avoid turmoil caused by the subsequent financial crisis and recession.

Figure 3 shows the difference between futures and spot prices (the *basis*) for corn, soybeans, and wheat over time.[10] The bases for the four crops follow different paths until 2006, when all increased several times, signaling that the market was expecting a big increase in the future. Besides the US ethanol mandate, other factors that could have contributed to higher futures prices are emerging markets in Asia, particularly China, and weather anomalies such as the drought in Australia in 2006, 2007, and 2008. The positive basis of food commodities ended with the onset of the financial crisis in 2007 and 2008, which sharply

[9]This approach is essentially a coarse simulated GMM method introduced by McFadden (1989) and further developed by Lee and Ingram (1991) and Duffie and Singleton (1993).

[10]Rice is thinly traded on the futures market and has a shorter history of trading. Rice markets are also heavily influenced by government accumulation of inventories and varying export restrictions, so we omit the commodity from this figure.

reduced anticipated future demand.

Table 2 reports statistics for both the detrended and real values. The detrended data normalized to 2005 show that, on average, the world has enough food production for 7.93 billion persons per year, assuming no waste and that people consume the raw grains.[11] At the baseline average price of 30.35 dollars to feed one person in a year (the 2005 trend value), the world market provided calories to feed 6.2 billion people and 6.18 billion people will consumed the calories provided. The world stores enough food to feed 1.75 billion people on average every year.[12]

We solve our model and simulate data for each of a large set of parameters, summarized in Table 3. Demand elasticities take values from -0.01 to -0.1. Supply elasticities range from 0.01 to 0.1. Parameters of the linear supply and demand curves $\alpha_s, \alpha_d, \beta_s$, and β_d are chosen such that given these parameters, supply and demand intersect at the mean observed consumption of 6.179 and the mean observed price of 30.35 dollars.[13] The interest rate can be either 0.02 or 0.05. The decay rate is set to be at 0.02 since this is the same value we used to construct our amount on hand and consumption data.

The constant marginal storage cost ranges from 0.2 to 2.[14]. We also allow for increasing storage cost when inventory gets too high. Amid the price spike for corn, US farmers doubled their corn inventory from 24.3 million tons to 53.6 million tons between 2004 and 2006, the highest level since the late 1980s. As a result, corn was being stored on the ground for lack of available storage facilities, which results in higher storage cost and commodity loss over time.[15] We specify variable storage cost such that when the amount of inventory exceeds twenty percent of the amount on hand, variable storage cost equals a constant marginal storage cost. When the ratio of inventory over amount on hand is less than twenty percent, marginal storage cost equals zero. This specification of the storage cost function allows for increasing cost of storage when inventory while using just one storage-cost parameter.

[11]Most corn and soybeans are used as animal feed and in processing. Since animals expend most of the energy contained in these commodities, human caloric intake is much smaller. Also, in rich countries people consume more calories than basic caloric requirements.

[12]Compared with the raw data, the detrended data have smaller coefficients of variation for all variables, because trend variance is removed. Because production and consumption increase over time and the base year is 2005, the detrended data have higher mean values. Because price has a declining long-run trend, its detrended mean that is far less that the raw mean.

[13]At the price of 30.35 dollar/one person annual caloric intake, the world market will provide enough food to feed 6.179 billion people.

[14]Cafiero et al. (2011) found that the ratio of storage cost over mean price in equilibrium ranges from 0.002 to around 0.06 depending on the crop. Based on that, and with our mean price in equilibrium selected at 30.35, our range of data is wide enough to capture all combinations of storage cost of different crops.

[15]The New York Times quoted that "In Iowa, the amount of grain being stored on the ground for lack of storage is averaging more than 19 percent, its highest level in at least 25 years."
http://www.nytimes.com/2005/11/09/business/09harvest.html

The sets of parameters in Table 3 yield 2000 combinations of parameters. For each set of parameters, we solve for the storage model assuming yields follow a random normal process, $\sim \mathcal{N}(12.11, 0.368^2)$, which was approximated from the detrended yield data.[16] The lower and upper bound for amount on hand are zero and twenty respectively. We approximate the storage demand function where price is a function of the amount on hand using cubic spline over 50 unequally spaced points of the amount on hand where 20 points are placed between 5.5 and 7, 10 points are in between 0 and 5.5, and 20 points are between 7 and 20. After solving for the storage function, we simulate data for 5000 time periods. The first 100 data points are discarded to avoid bias from selection of starting values.[17]

Figure 4 shows probability density function over 2000 combinations sets of parameters. We compare mean price, standard deviation (SD) of price, mean inventory, mean amount on hand, first order autocorrelation of price (AR(1)), and mean production with their corresponding real world values. We see that while mean price and SD of price closely match with what observed in reality, other simulated moments such as mean inventory, amount on hand, and AR(1) are far lower than their counterparts. Storage theory tells us that, all else equal, low inventory will result in lower total amount on hand. In addition, low level of inventory also means that price are not well-buffered against shocks in the system which results in more price boom and bust otherwise. Because of high frequency of random price fluctuations, price will be less likely to be auto-correlated. The low level of inventory generated by our model can be explained by the fact that in reality, we do not observe any incidence of stock out. Our model, however, allows for the possibility of no storage when expected futures price is smaller than current price plus storage cost. As a result, close to ten percent of our simulated inventory data is zero. In future research, one can try to do a better match between observed and simulated inventory data by allowing for negative storage cost.[18]

From this large set of parameters and solved models, we the one set of parameters that best fits observed data. From 2000 sets of result, we construct a score for each of them using the same objective function used in simulated GMM:

$$Score = \psi(\theta)^T W \psi(\theta), \tag{13}$$

where W is the optimal weight matrix and $\psi(\theta)$ is the difference between simulated and observed data moments given parameters θ. The optimal weighting matrix is calculated as

[16] The continuous normal distribution of yield is approximated with a discrete normal distribution with 10 values of yields, each with its own probability.

[17] We use the equilibrium quantity when there is no storage as our staring values.

[18] This is directly related to the literature of convenience yield where one enjoy the premium of "convenience" of having the goods on hand and do not want to sell it even though the expected futures price is lower than the spot price.

the long run covariance matrix of the observed data which in practice is approximated using the approach developed in Newey and West (1987).[19]The first four moments of price (mean, variance, skewness, and kurtosis) and the first moment of production (mean) are selected to use in $\psi(\theta)$.

Because $\psi(\theta)$ contains the difference between simulated and observed data, it is desirable to get the lowest score possible. Table 4 show the list of parameter sets that yield the lowest score. They all yield a demand elasticity of -0.06. Supply elasticity is either 0.01 or 0.02. Storage cost varies from 0.8 to 2. However, with the difference among these scores are so small, the change in storage cost does not seem to have a big influence on the outcome of the simulated data. We select demand elasticity of -0.06, supply elasticity of 0.01, storage cost of 1.8, and interest rate of 0.02 as our baseline model as they make up the smallest score in our grid search. The intersection of demand and supply curve are selected to be at the mean value of observed price and consumption. Decay rate is set at 0.02.

3 The US Biofuel Mandate

US interest in ethanol fuel is not new but recently re-surfaced in the wake of energy supply insecurity, oil price volatility and growing concern about CO2 emissions. From 1980 to 2001, US ethanol production increased more than 10 times, from 0.175 billion gallon in 1980 to 1.77 billion gallon in 2001.[20]. Between 2001 and 2005, ethanol production doubled to close to 3.9 billion gallon, accounts for 2.7 percent of the total US motor fuel consumption in the same year. Moreover, The US Energy Policy Act of 2005 mandated that 7.5 billion gallons of ethanol be used by 2012. Just two years after that, the Energy Independence and Security Act of 2007 increased the mandate to 36 billion gallon by 2012. Under the 2007 mandate, the all of ethanol produced between 2007 and 2015 are corn ethanol.

The rapid growth of US ethanol production is not without consequences. Because ethanol production exclusively used corn as input until 2009, the US ethanol mandate is considered by many as one of the main driving factors behind the food price crisis of 2008 (Childs and Kiawu (2009), Piesse and Thirtle (2009), Mitchell (2008)). Mitchell (2008) also considers the US demand for corn to produce ethanol is the single greatest cause of price rises. He mentioned that the increased demand for ethanol accounted for 70 percent of the increase in corn prices and 40 percent of the increase in soybean price. Moreover, Collins (2008) found that about 60 percent of the increase in corn price from 2006 to 2008 may have been due to the increase of corn used in ethanol. Recently Roberts and Schlenker (2010) estimated that

[19]We use Barlett window with four lags in approximating the optimal weighting matrix.
[20]http://www.ethanolrfa.org/pages/statistics

the US ethanol mandate results in increases of 20 to 30 percent of world food crop prices under different scenarios.

The US ethanol mandate could have a large direct impact on world food price because of the size of the mandate and relatively inelastic supply and demand of food commodities. As the world biggest corn producer, by producing 15 billion gallon of ethanol in 2015, the US has ethanol manufacturer has used up more than 7 percent of the world combined calories from corn, soybeans, rice, and wheat in 2005 (Figure 5).[21] [22][23] Compared with the level of production when the Energy Policy Act was introduced, by 2015 the US ethanol mandate will be consuming 5 percent more of the world crop calories available in 2005.

4 Results

We measure the impact of the shift in ethanol production on food price by integrating it in our storage model. We first solve for the storage model assuming there was no demand shock in the system. We next allow for the demand curve to gradually shift out accordingly to the US ethanol mandate schedule and generate data for each of the year in the transition period. Note that because we move our demand curve out gradually, we will have a different demand curve for each of the transition year. Our model, therefore, will have an old demand curve for all the periods prior to 2006, a new demand curve for all the periods after 2015, and nine transition demand curves for each of the year from 2006 to 2014.

4.1 Solving the Storage Model for the Transition

We first assume that in the event of the US ethanol mandate, demand and supply elasticities do not change over time. Because supply and demand elasticities do not change over time, for one percent change in quantity, the change in price will be equal to $\frac{1}{e_s - e_d}$ percent with e_d and e_s are demand and supply elasticities respectively. As demand and supply are of linear forms in our model, in order to arrive at the same elasticities of demand and supply over the period from 2006 to 2015, our linear supply and demand curves will have different slopes and intercept for each of the year. The assumption of constant demand and supply elasticities across environments is later relaxed to allow for changes in elasticities. Solution for all of the storage demand curves are presented in Figure 6. We use demand elasticity of -0.06,

[21]We use a conversion rate of 2.7. For each gallon of ethanol produced, 2.7 bushel of corn is needed.

[22]Corn ethanol is capped at 15 billion gallons from 2015 to 2022. All of the ethanol expansion after 2015 comes from increased advanced biofuel production.

[23]Roberts and Schlenker (2010) mentioned that the ethanol expansion account for 5 percent of the total world calories. However, they compared their calories change because of ethanol shift to the level of production in year 2010 while in this study, we look at 2005, before the ethanol mandate was enacted.

supply elasticity of 0.01, decay rate of 0.02, interest rate of 0.02, storage cost of 1.8, and amount on hand ranges from 0 to 20 with 50 unequally spaced points. The two thicker lines are the price functions before policy was known and after the policy has been implemented, respectively. As discussed earlier, the demand curve with storage is not smooth. When current price plus storage cost is higher than the expected futures price, there will be no inventory because one can earn more profit by selling all food crop in the current period. As the spot price declines, inventories slowly increase because more profit can be earn by selling in the future. The dotted line in Figure 6 shows the cut-off price for the demand curve before any demand shift. It says that people will start accumulating inventories when price falls below 31.4 dollars. The nine thinner solid lines are storage demand curves in the transition period from 2006 to 2014. They all lie below the storage demand curve for year 2015 and above the old storage demand curve in year 2005. The higher the line, the closer to 2015 a storage demand function in transition time represents.

Using these storage demand functions, we simulate data for sixty periods from a draw of sixty yield values where yield $\sim \mathcal{N}(12.11, 0.368^2)$. We discard the first twenty points to avoid any bias associated with selecting the starting values, leaving forty points of data. The year 2005 is set to be the at the tenth observation on our series of price data. We draw random yield shocks 10 thousands time to get the mean impact of a demand shock on food price.

4.2 Initial and Long-Run Changes in Prices and Volatility

Figure 8 shows how prices react in the short, medium, and long run when the demand shock shifts out permanently (such as stemming from the US ethanol mandate) with different timing of the demand shock. The first solid line on the left shows what happens if the market knows about the demand shift and immediately reacts to it. When there is no demand shift, our model predicts an average price of 30.05 dollars. With 5.4 percent shift of demand caused by the ethanol mandate, the long run equilibrium price will be higher by 80 percent at 53.6 dollars. As the market reacts immediately to the news of the demand shift, after one year, the mean price climbs to 38.6 dollars, account for 36 percent of the total price increase. The upward trend in price keeps going until it reaches the new long run equilibrium of 53.5 dollars.

As we allow for slower market response, the initial price spike gets larger. If the market internalizes the news of the demand shock one year later in 2007, the price spike in the first year will equals to more than 58 percent of the total food price increase. For all of the timing of the demand shift, the biggest price spike always happens when the demand curve first begins to move out. At the extreme, if the demand shock happens unexpectedly, prices can initially spike up to 286 percent of the total long-run price increase, before the market has time to gradually adjust production and inventories. In other words, if the US wants to have 15 billion gallon of ethanol produced in 2005, the sudden increase in demand will more than double world food price immediately.[24]

Figure 7 shows the evolution of price volatility and inventory dynamics in the short run and long run depending on the timing of the shock. In general, prices become more volatile after demand has completely shifted out, with price standard deviation (SD) increasing by roughly 60 percent (from 8.94 dollars and 14.33 dollars). The change in volatility is somewhat less than the change in the price level, so *relative* variation (the coefficient of variation) actually declines. A key reason for the relative decline in the SD is that we have more inventories in the long run which dampen price volatility in the future.

Prices during the transition period fluctuate more the later the market reacts to news about the demand shock. As the market moves completely to the new higher demand environment, inventories will also rise to accommodate the new demand level. When demand unexpectedly and permanently shift out, the level of inventories will be drawn down in the short run which will lead to a large mean price spike as shown in Figure 8. Prices are generally more volatile during the transition because due to a decline in inventories, which acts as a buffer in time of shortage, price fluctuates a lot more than what it would be in the long run. In fact, the price standard deviation when demand shifts out unexpectedly in the short run is close to double the long run price variations at 26.26 dollars while the level of inventory is cut to one third temporary. Adding and subtracting one standard deviation to the price shock increase the price spike when demand happens unexpectedly to 112.5 dollars and 60 dollars respectively. If the demand shift is known early enough, we can see a build up of inventory in the short run which buffer price volatility. As a result, price standard deviation dips despite an increasing mean price. If the market internalized the demand shock in 2006, the price deviation initially decrease to 5.33 dollars. Adding and subtracting the standard deviation from the mean gives us the prices of 43.85 dollars and 33.2 dollars respectively. Price and inventory can both go up in the short run because the new price level after the first year is still lower than the long run equilibrium of 53.6 dollars. Expecting that current price will continue to rise to reach the new price in the new equilibrium, people will stock up inventories in hope of future arbitrage profit. Inventories only goes down when the total

[24]While being extreme, the case of unexpected demand shift is not unheard of looking at past policies. Facing rapidly increasing food price in 2007 and 2008, India, the third biggest rice exporter, putted a ban on its rice export. Following India, Vietnam, the second biggest rice exporter, and several other countries also banned their rice exporter in early 2008. Together with other factors such as panic buying from big rice importers, rising oil price, a weaker dollar, those unexpected bans by major players in the world market created a net upward shift in demand. As a result, rice trading price more than tripled over a very short period from November 2007 to April 2008.

current amount on hand gets to the level of total amount on hand in the new equilibrium. Our model indicates that the positive relationship between price and inventory happens if the news of the demand shock is internalized after two years (in 2007) at the latest.

4.3 Transitional Prices and Volatility Depend on Initial Inventories

Figure 9 shows price changes during the transition for a range of initial amounts on hand assuming the demand shock is first anticipated in 2006. Price change is the price difference between 2005 and all the subsequent years. Because the first ten points are price data for year 2005, they become zero on the figure. We rank all the price differences by the amount on hand available when the world food demand moved out in 2006. As can be seen, when the level of amount on hand is at its lowest 10 percent, price is 6 dollars higher than when the amount on hand is at its highest 10 percent. In addition, price standard deviation is also higher when amount on hand is at its lowest 10 percent at 11 dollars in compared with 4.7 dollars when amount on hand is at the highest 10 percent. Moreover, while the mean price difference for the lowest 10 percent of amount on hand only stays higher for a year, its price standard deviation remains higher for much longer. So when the level of food available is low, not only does the price spike more in the short run, it also fluctuates more in the short run and medium run. Note that Figure 9 shows the price difference, therefore, when the market gets to its new equilibrium all prices should converge to the same level. However, because of the inverse relationship between price and amount on hand, price in the old environment is typically higher when amount on hand is low and vice versa. Therefore, in the long run, the mean price difference of the lowest 10 percent amount on hand will be lower in compared with other scenarios when market approaches its new long run equilibrium.

Figure 10 is similar to Figure 9 except it shows the price difference when a price shock was not expected. At the lowest 10 percent amount on hand, mean price difference can go up to 101 dollars while the same measurement for the highest 10 percent of amount on hand is only 26 dollars. However, looking at the standard deviation of the price difference, the gap between the lowest and highest 10 percent of amount on hand is only 9 dollars. Because of the big difference for mean and the small difference for standard deviation of prices in the lowest and highest 10 percent of amount on hand, the standard deviation of price for the whole sample is higher than either of the two other cases at 24.3 dollars. The small difference in price standard deviation between the lowest and highest 10 percent cases can be explained by the level of inventories when demand moves out unexpectedly. Because the market has no time to adjust, inventories will immediately being drawn down in the short run to buffer the shock. Our simulation indicates that more than 40 percent of the inventories data are zeros in this case. As a result, prices fluctuate more regardless of the level of amount on hand available.

4.4 Price and Volatility Effects are Larger the More Inelastic Demand and Supply

We next relax our assumption of constant demand and supply elasticity across environments. Supply may become less elastic as remaining arable land declines. Alternatively, supply may become more elastic if there are economies of scale or ethanol-specific factors of production change the scope of production possibilities. Because our baseline supply elasticity is already very small, we skip the case of supply getting less elastic and allow for supply elasticity to increase from 0.01 to 0.02.

In the baseline case, we assume that demand and supply elasticities do not change over the short window of time from 2005 to 2015. However, as price increase, we can expect people to either switch to other cheaper caloric foods or eat less, therefore make food demand more elastic to price.[25] We can also consider cases in which the demand elasticity become less elastic because part of the demand (the demand for ethanol) is perfectly inelastic. We therefore allow demand elasticities to take either of the three values: -0.05, -0.06, and -0.07 at new mean quantity consumed after demand shifts out.

Table 5 shows mean and standard deviation of price across different scenarios of changes in elasticities. We show price in the old environment, price in the new environment, price when demand shift out expectedly in 2006, and price when demand shifts out unexpectedly. Scenarios in Table 5 labelled 1-6. Scenarios 1 to 3 hold the supply elasticity constant and the three demand elasticities. Scenarios 4-6 hold the demand slope constant and consider varying supply elasticities. Scenario 3 is when demand and supply are least elastic and scenario 6 is when demand and supply are most elastic new environment. Across all scenarios price is higher in magnitude and fluctuate more in the new environment. Between old and new environments, price level and volatility change least in scenario 6 and the most in in scenario 3.

When demand shift is anticipated, scenario 3 has the most price change in both absolute and relative terms in the transition period. It also has the highest in absolute value the first year after demand shifts out unexpectedly. Scenario 6 has the lowest price increase in both absolute and relative term when a price shock is expected as well as the lowest price increase

[25]For example, during the centrally planned economy era in Vietnam, when rice price gets higher and people can barely afford it, it was fairly common to see rice being reserved for the children while parents eat sweet potatoes because it was cheaper than rice.

in absolute value when a demand shock happens unexpectedly in transition times. Scenario 1 and 5 are interesting because they both have a total absolute elasticity value of 0.07. While the long run price equilibrium are really close, price in scenario 5 varies more than price in scenario 1. The difference can also be seen when looking at the price change in the short run when demand shifts out both expectedly and unexpectedly. Either case, price in scenario 5 increase more with more volatility in compared with the price in scenario 1. The difference between these two scenarios can be explained by show supply and demand react to price in times of shock in the system. While demand response to price in the current period, supply only response to expected futures price. The amount of food supplied this period depends on the area planted from last period and current weather shock. Supply therefore can not adjust like demand does in the current period. As a result, in scenario 5 with less elastic demand and more elastic supply, price varies more than price in scenario 1.

4.5 Simulated Impact of the US Biofuel Mandate on Food Prices 2006-2011

Assuming the market first reacted to the US biofuel policy in 2006, we simulate a series of price change from 2006 to 2011 showing how the price could have evolved in response to the news of the coming demand shift. We do this in several steps. First we pick the average level of amount on hand in the old environment (before 2006) as our staring value and generate the simulated price data in 2005 using the old storage demand function.[26] Next we generate price data for each year from 2006 until 2011 using the corresponding storage demand functions in the transition period and real yield values. Lastly, we scale this simulated price data series so that the price in 2005 equals what we observe in real world data.[27]

Table 6 show different world food price evolutions after 2005 under the US ethanol mandate for different scenarios of future demand and supply elasticities values. As can be seen, under scenarios 1, 3, and 5 price in 2006 is higher than what we see in reality. These are scenarios where demand elasticities either stays the same at -0.06 or gets lower to -0.05 when supply elasticity stays the same at 0.01 . One possible explanation is that demand actually gets more elastic over time. Recall that the total ethanol demand shift accounts for

[26]Note that because price is a function of amount on hand, the mean value of prices from our simulation will be different from the price as a function of the mean simulated amount on hand.

[27]One alternative approach is to pick the real price in 2005 as the starting value and find the amount on hand using the inverse form of the storage demand function. From there, continue to solve for price data between 2006 and 2011 using real yield data and the storage demand in the transition periods. However, the observed price in 2005 was at \$43.6 which is higher than the cut-off price in our model of \$31.4 in the old environment (Figure 6). As a result, there will be no storage data (which is needed for price data) generated in the first year. This is not desirable since in 2005, the real level of inventory is not zero.

a little over 5 percent of the total world food crop demand, in reality, it is highly possible that this inelastic part of the food demand curve get over-compensated by the total food demand curve getting more elastic as prices increase. If we take out scenarios 1, 3, and 5, the remaining scenarios 2, 4, and 6 yields 84, 83, and 69 percent of the total increase in food price between 2005 and 2006. Our most conservative estimate is with scenario 6 when demand and supply both get more elastic over time. Under scenario 6, 31 percent of the price increase between 2005 and 2007 are caused by the US ethanol mandate. At a longer horizon, the US ethanol mandate accounts for 20 percent of the price increase between 2005 and 2011.

4.6 Robustness Checks

Because supply and demand elasticities have big direct impact on the price transition, we regenerate the price transitions using demand elasticity of -0.05 and supply elasticity of 0.1 as found in Roberts and Schlenker (2010). We also allow for possible future demand and supply elasticities changes. Demand elasticities can be either -0.04, -0.05, or -0.06. Similarly, supply elasticities can take any of the three values 0.09, 0.1, and 0.11.

Table 7 summarizes results across scenarios using the new set of demand and supply parameters. In general, we can see similar patterns of price transition as discussed earlier using our estimated parameters. However, because demand and supply are more responsive to price now, the initial price spike are smaller than before for both expected and unexpected demand shocks. In relative term, the initial increases in price also account for smaller percent of total price increases when demand shift is anticipated in 2006.

When demand shifts out unexpectedly, transitional prices are also less volatile as compared to using our estimates. For example, when both demand and supply become more elastic, using estimates from Roberts and Schlenker (2010) yields price standard deviation of 16.82 dollars compare with 20.59 dollars earlier. Price volatility when the demand shift is anticipated are higher compared to results from our estimates. This phenomenon can be explained by lower inventory accumulation resulting from lower expected price increase in the future. As less inventories are stored, prices become more volatile in the short run than before.

Lastly, the differences between price equilibriums also become smaller. Prices difference between new and old equilibriums when demand and supply both get more elastic reduce from 18 dollars to 10 dollars. Using our estimates, the coefficient of variation before and after demand shifts assuming constant elasticities across environments are 29.46 and 26.7 respectively. With new elasticities, the coefficient of variations declines from 26.07 to 24.39

with the demand shift.

Table 8 simulates how world food price would have been under the impact of the US ethanol mandate using the new set of demand and supply elasticities. Similar to simulated results from our estimates, the biggest price spike comes in 2006, especially when both demand and supply become less elastic. Yet, because of larger elasticities, the price increases are lower than before. Even when both demand and supply get less elastic over time which lead to the biggest increase in price, the US ethanol mandate only accounts for 14 percent of the total increase in price between 2005 and 2011. When both demand and supply are more elastic in the new environment, the price spike generated by the model between 2005 and 2011 composed 11 percent of the observed price change.

5 Conclusion

In this paper, we extend the rational competitive storage model by explicitly allow for land to be a function of expected future price. In addition, we also show how the storage model can be used to show how price evolve in the short, medium, and long run when a demand shock happens both expectedly and unexpectedly in the model. We are able to show how price change over time by having a unique storage demand function for each year in the transition period. The storage functions in transition time are solved using backward induction. First, the storage model is solved when demand has completely shifts out, assuming that the model stays stationary after that. Second, given the storage function in the new equilibrium, we solve backward in time to get the storage function one year prior knowing all the terminal conditions in the next period. We keep solving backward until we reach the storage function in the old equilibrium.

The storage model is used to see how the world food price is affected by the US ethanol mandate which shifts the world crop demand out by more than 5 percent from 2005 to 2015. In general, we found that price gets to a higher level with higher volatility in the new environment. Moreover, price spike as soon as the news of the demand shock is internalized by the market. The later the market knows about the demand shock, the higher the price increase in both magnitude and volatility in the short and medium run. In addition, the level of amount on hand readily available when the shock happens also play a crucial role on price change and price volatility. Lastly, we see that prices increase more with more volatility when demand and supply become less elastic in the new environment.

Assuming that market internalized news of the demand shift in 2006, we simulate price data from 2006 to 2011 using observed yield data. We found that under the assumption of demand and supply getting more elastic over time, the US ethanol mandate accounts for one fifth of the food price increase between 2005 and 2011. These results are roughly accords with findings from Helbling et al. (2008) and Rosegrant (2008). When demand get less elastic and supply response does not change, the price paths simulated from our model accounts for 30 percent of the price increase between 2005 and 2011. Using an alternative set of elasticities estimates from Roberts and Schlenker (2010), we found that from 11 to 14 percent of the food prices increase between 2005 and 2011 are caused by the US ethanol mandate depending on the nature of future demand and supply elasticities. The second set of results are in line with Baier et al. (2009) who attribute 14 percent of the rise in corn prices and nearly 10 percent of the rise in soybean prices to the increase in US biofuel production.

The remaining puzzle of this paper comes from our estimates of supply and demand elasticities. Even though demand and supply elasticities values of -0.06 and 0.01 had the lowest score among our grid search values, they could not replicate the level of inventories, amount on hand, and first level auto-correlation of price as observed in real data. Besides, data statistics from Table 2 suggests that production varies more than consumption with the coefficient of variation values of 0.035 and 0.021 respectively. Likewise, Roberts and Schlenker (2010) using a similar set of data from FAO looking at world crop supply and demand elasticities also found supply to be more response to price than demand. The inelasticity of supply and demand generated from our model cause the initial price spike in 2006 to exceed observed prices if demand and supply are not getting more elastic over time. Future research can try to fill in the gap between simulated and observed data by allowing for negative storage cost which would induce more storage in the model. In addition, one can also try to estimate the storage model parameters by pushing further the simulated GMM approach using alternative sets of moments. Another possibility is to estimate the model parameters using the Pseudo Maximum Likelihood Estimation as previous done by Cafiero et al. (2011), Deaton and Laroque (1995), and Deaton and Laroque (1996) but also account for supply response.

References

S. Baier, M. Clements, C. Griffiths, and J. Ihrig. Biofuels impact on crop and food prices: using an interactive spreadsheet. *FRB International Finance Discussion Paper No. 967*, 2009.

Carlo Cafiero, Eugenio S.A. Bobenrieth H., Juan R.A. Bobenrieth H., and Brian D. Wright. The empirical relevance of the competitive storage model. *Journal of Econometrics*, 162 (1):44–54, May 2011.

N.W. Childs and J. Kiawu. *Factors behind the rise in global rice prices in 2008*. US Dept. of Agriculture, Economic Research Service, 2009.

K.J. Collins. *The Role of Biofuels and Other Factors in Increasing Farm and Food Prices: A Review of Recent Developments with a Focus on Feed Grain Markets and Market Prospects*. K. Collins, 2008.

A. Deaton and G. Laroque. On the behaviour of commodity prices. *The Review of Economic Studies*, 59(1):1 –23, January 1992.

A. Deaton and G. Laroque. Estimating a nonlinear rational expectations commodity price model with unobservable state variables. *Journal of Applied Econometrics*, 10(S1):S9–S40, 1995.

A. Deaton and G. Laroque. Competitive storage and commodity price dynamics. *The Journal of Political Economy*, 104(5):896–923, 1996.

D. Duffie and K.J. Singleton. Simulated moments estimation of markov models of asset prices. *Econometrica*, 61(4):929–952, 1993.

Robert L. Gustafson. *Carryover levels for grains: a method for determining amounts that are optimal under specified conditions*. U.S. Dept. of Agriculture, 1958.

D.L. Helbling, V. Mercer-Blackman, C.T.N. Dao, N. Erbil, and E. Oakes. Is inflation back? commodity prices and inflation. *World Economic Outlook*, 2008.

B.S. Lee and B.F. Ingram. Simulation estimation of time-series models. *Journal of Econometrics*, 47(2):197–205, 1991.

D. McFadden. A method of simulated moments for estimation of discrete response models without numerical integration. *Econometrica: Journal of the Econometric Society*, pages 995–1026, 1989.

M.J. Miranda and P.L. Fackler. Applied computational economics and finance. *MIT Press Books*, 1, 2004.

D. Mitchell. A note on rising food prices. *World Bank Policy Research Working Paper No. 4682*, 2008.

W.K. Newey and K.D. West. A simple, positive semi-definite, heteroskedasticity and autocorrelation consistent covariance matrix. *Econometrica*, 55(3):703–708, 1987.

J. Piesse and C. Thirtle. Three bubbles and a panic: an explanatory review of recent food commodity price events. *Food Policy*, 34(2):119129, 2009.

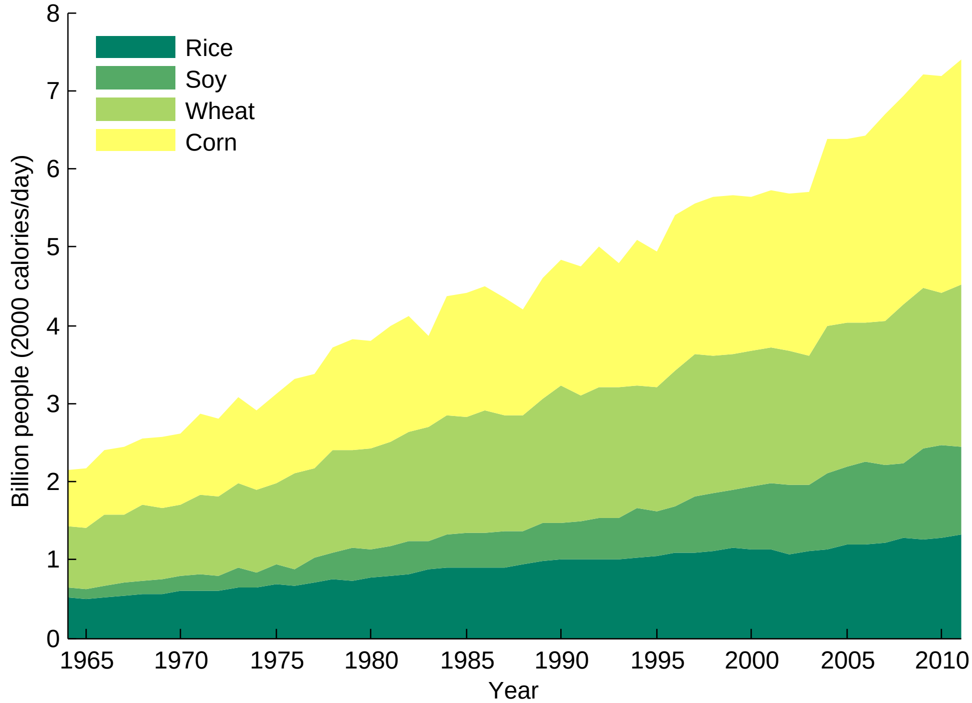
Michael J. Roberts and Wolfram Schlenker. Identifying supply and demand elasticities of agricultural commodities: Implications for the US ethanol mandate. *National Bureau of Economic Research Working Paper Series*, No. 15921, April 2010.

M.W. Rosegrant. Biofuels and grain prices: impacts and policy responses. *Testimony for the US senate committee on homeland security and governmental affairs*, 7, 2008.

Jeffrey C. Williams and Brian Wright. *Storage and commodity markets*. Cambridge University Press, 1991.

L. Williamson and P. Williamson. What we eat. *Journal of Farm Economics*, 24(3):698–703, 1942.

Figure 1: World Production and Yield in Calories of Corn, Soybeans, Rice, and Wheat



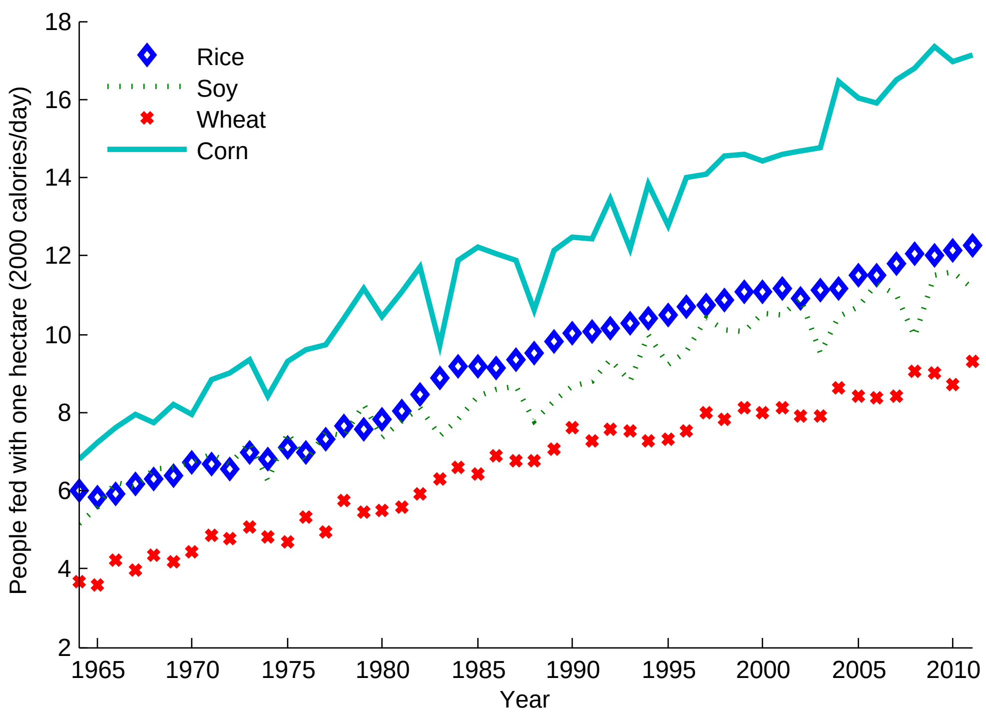


Figure 2: Food Price and Inventory 1964-2011

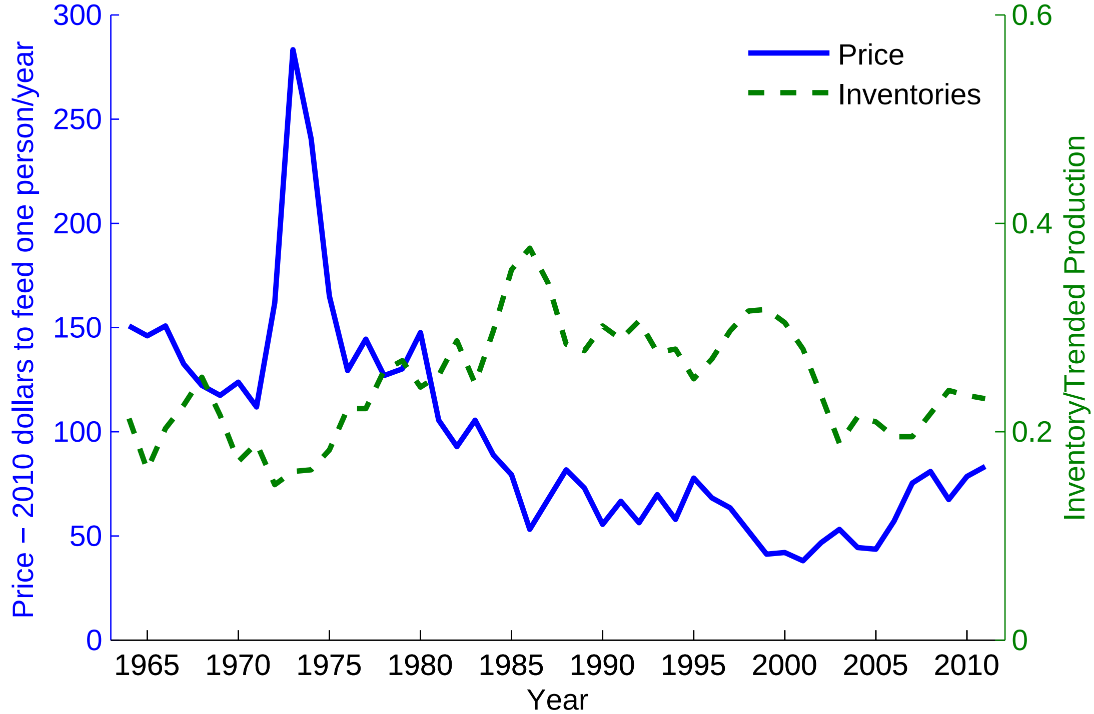
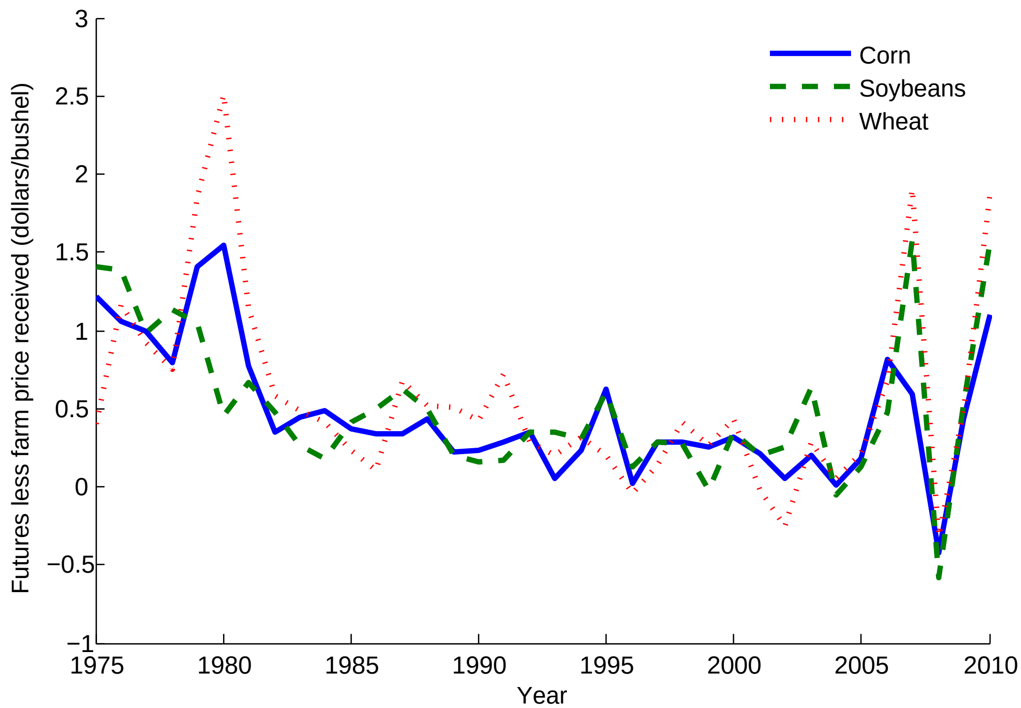
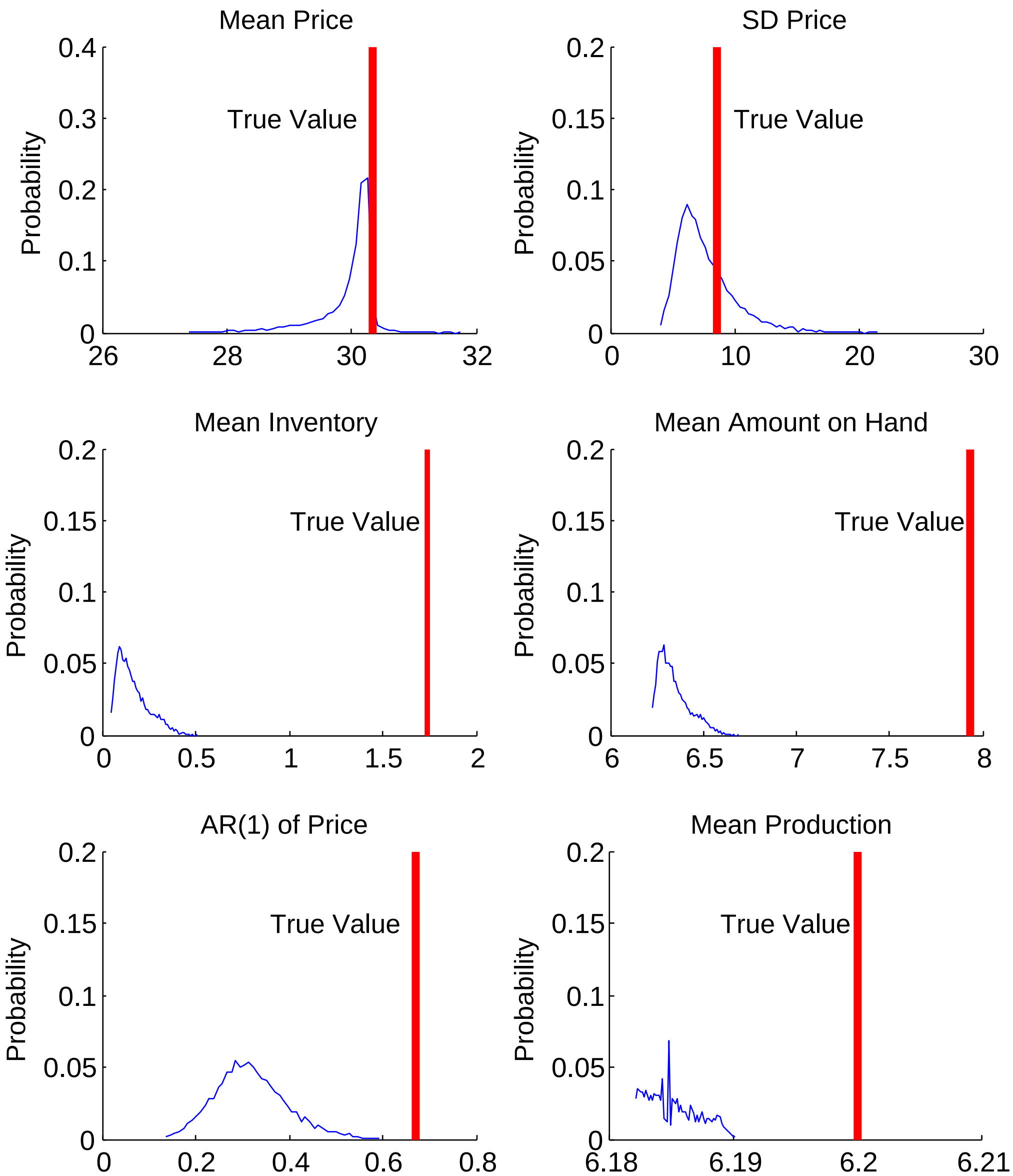


Figure 3: Differences between Futures and Spot Price



Notes: Data come from the Season-Average Price Forecasts dataset of USDA. Spot price is price received by US farmers in December. Futures is December settlement price for the nearby futures contract.

Figure 4: Data Distribution over 2000 Sets of Model Parameters



Notes: The thin line show probability density function over 2000 sets of parameters. For each set of parameters, 5000 points of data was simulated. The first 100 data points are discarded to avoid bias resulting from the selection of starting values. The thick vertical line is what we observe from the real word data.

Figure 5: US Corn Ethanol Mandate in Term of Percent of World Crop Calories in 2005

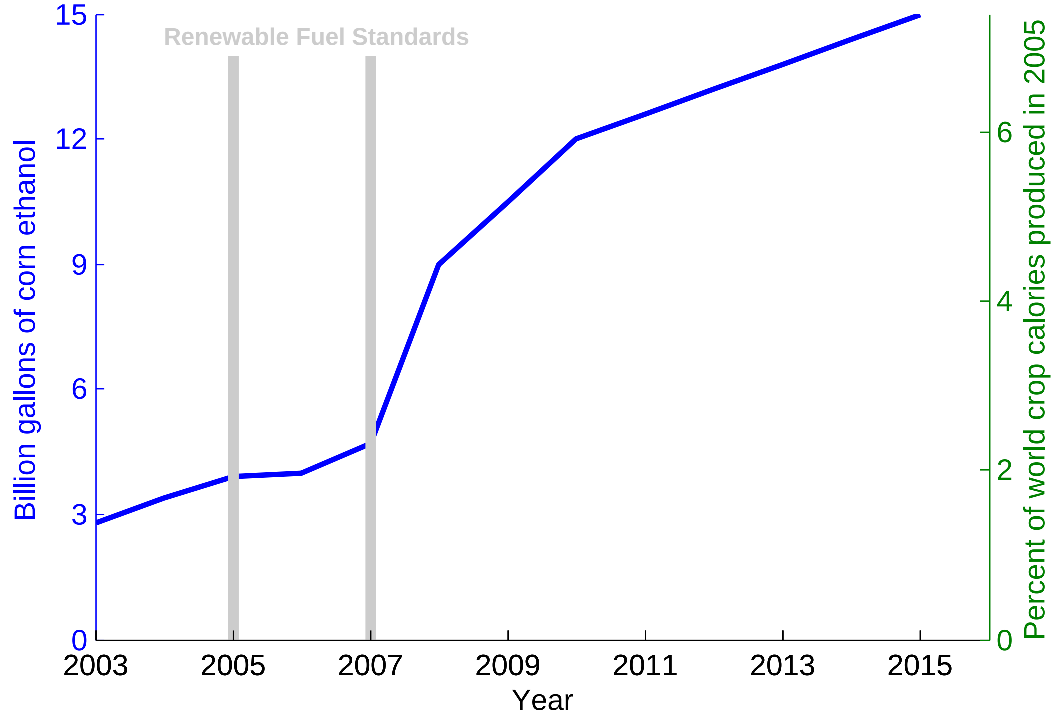
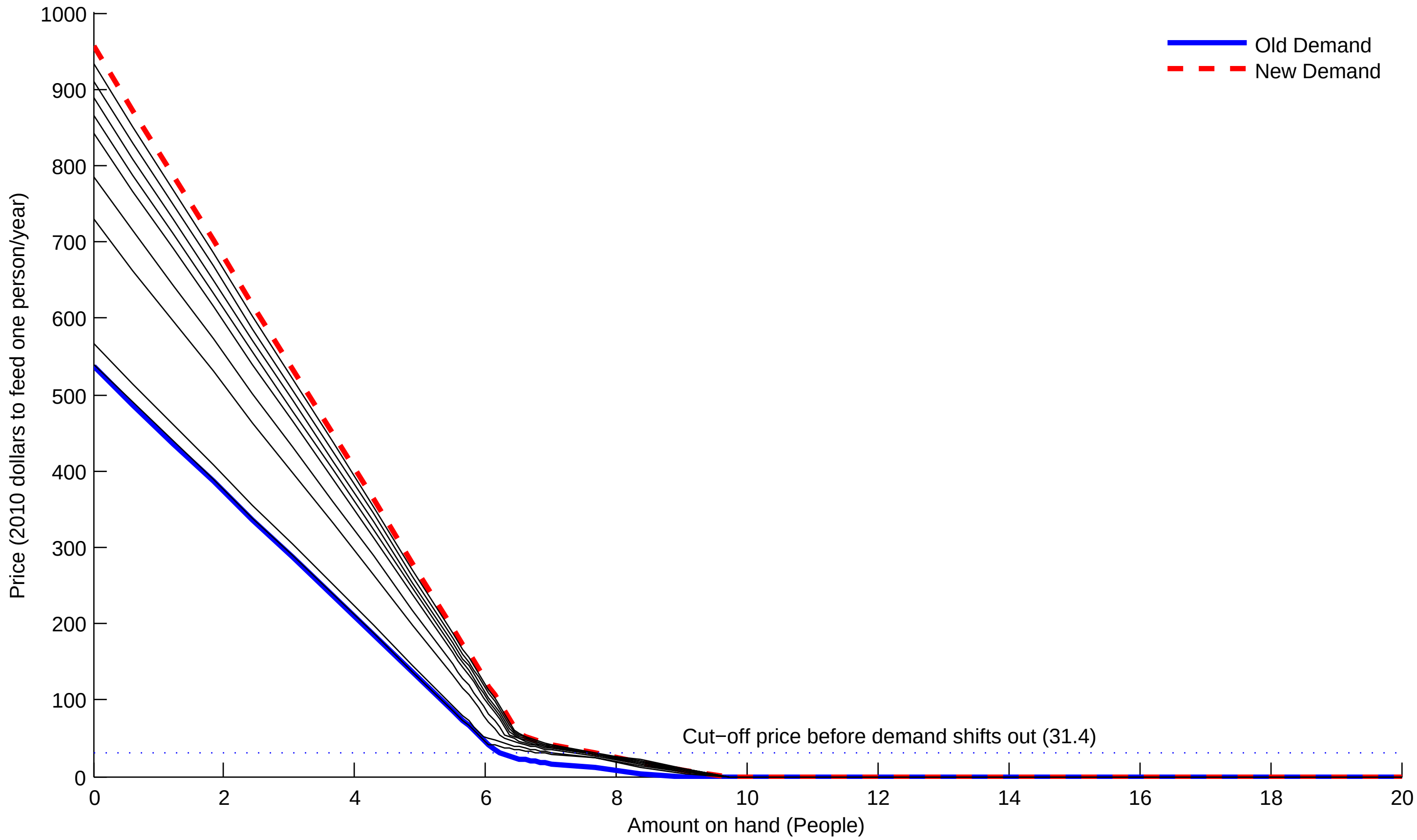
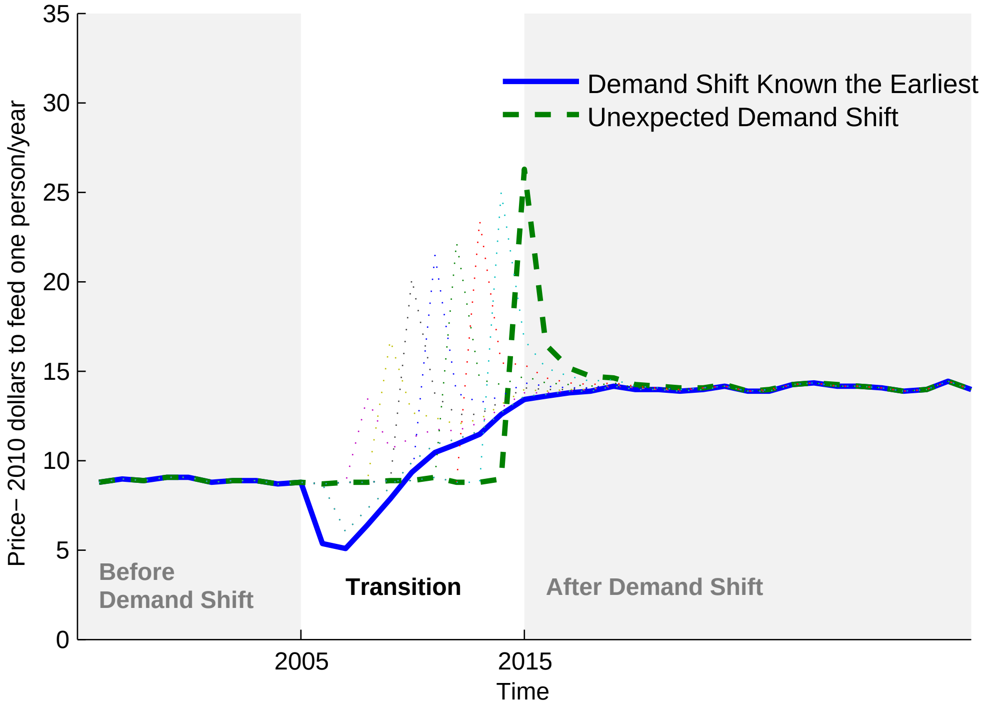


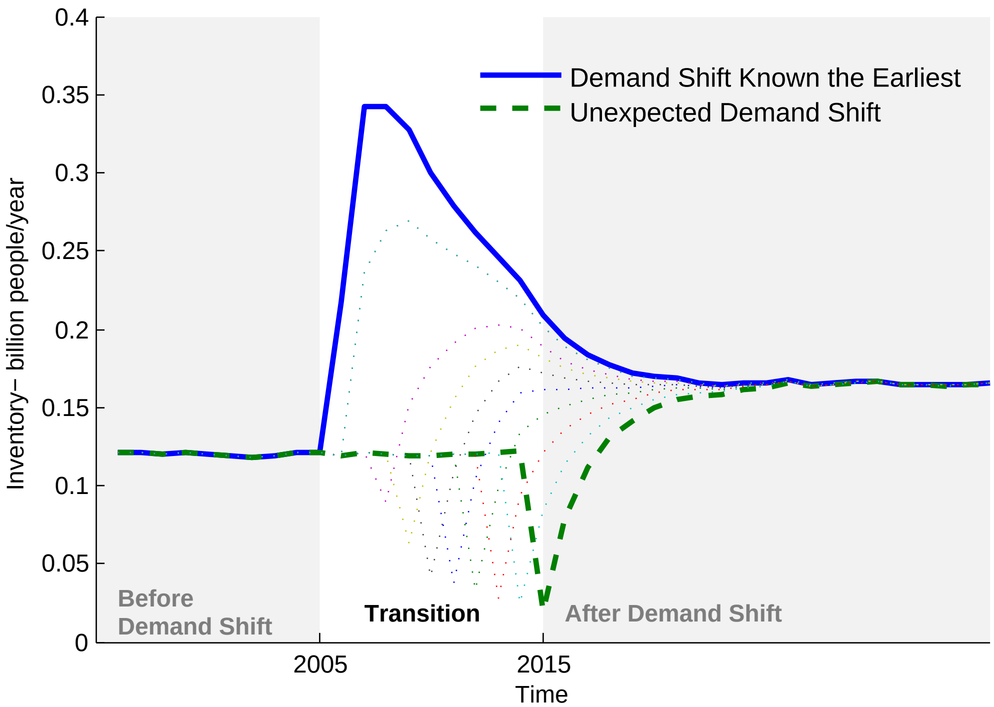
Figure 6: Solve for the Storage Model When Demand Shift is Anticipated



Notes: Dashed line is the storage function after demand shifts out. Thick solid line is the storage function before demand shift. Thinner solid lines are storage demand functions in transition time. The higher the thin line, the closer to the actual demand shift (year 2015). Cut-off price is the price at which for any price higher than that, storage become zero. Before demand shift, the storage model was solved using demand elasticity of -0.06 and elasticity of 0.01. Price is approximated over 50 unequally spaced points of amount on hand ranging from 0 to 20. Decay rate is 0.02, interest rate is 0.02, and storage cost is 1.8. The linear demand and supply curve intersect at p=30.35 and q=6.179.

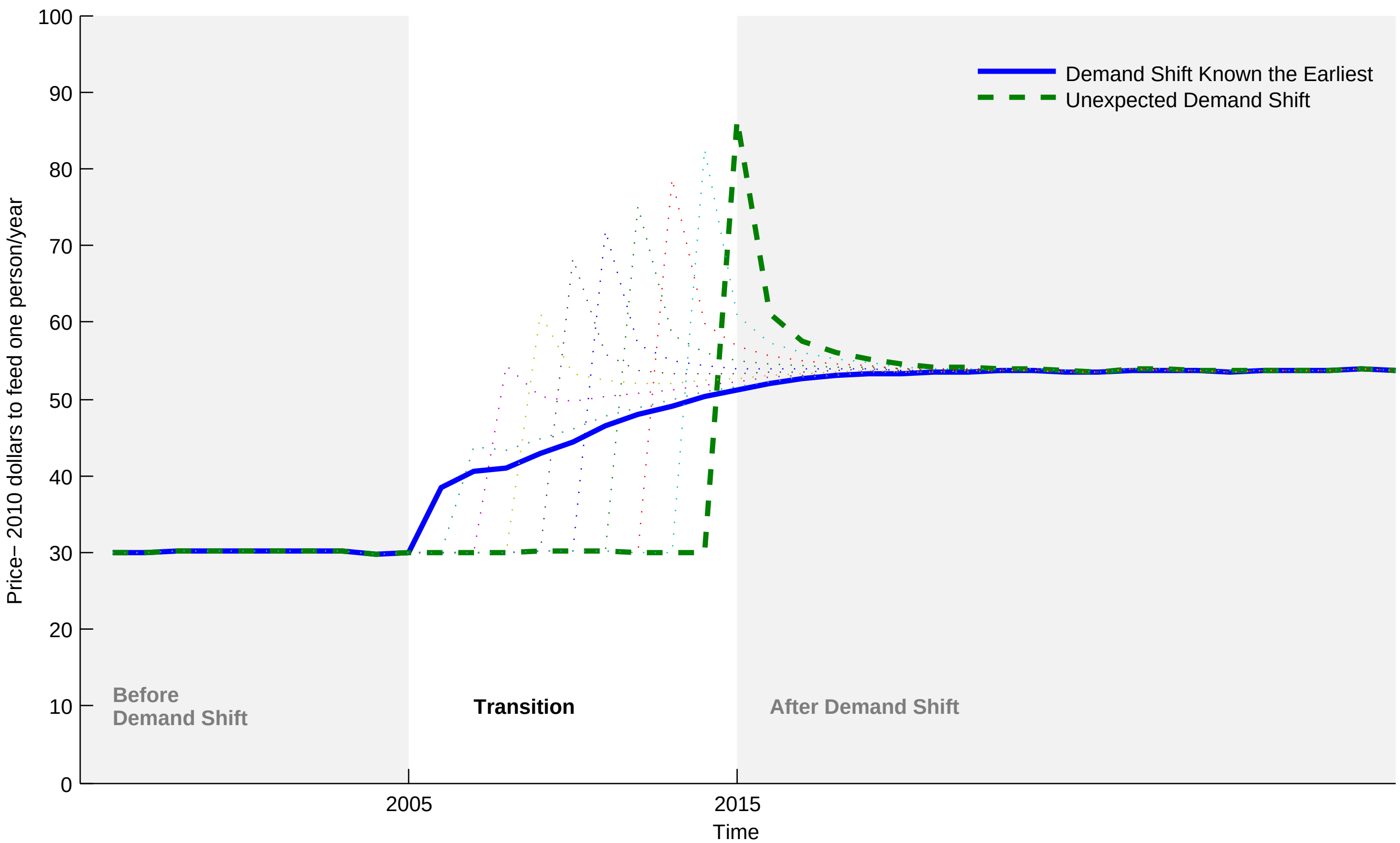
Figure 7: Price SD and Mean Inventory by Timing of the Demand Shock (10,000 Simulation)





Notes: Parameters of the storage models stay constant across environments. Top panel shows price standard deviation (SD) over 10,000 simulations and how price SD change associate with the timing of the demand shock. Bottom panel shows mean inventory over time.

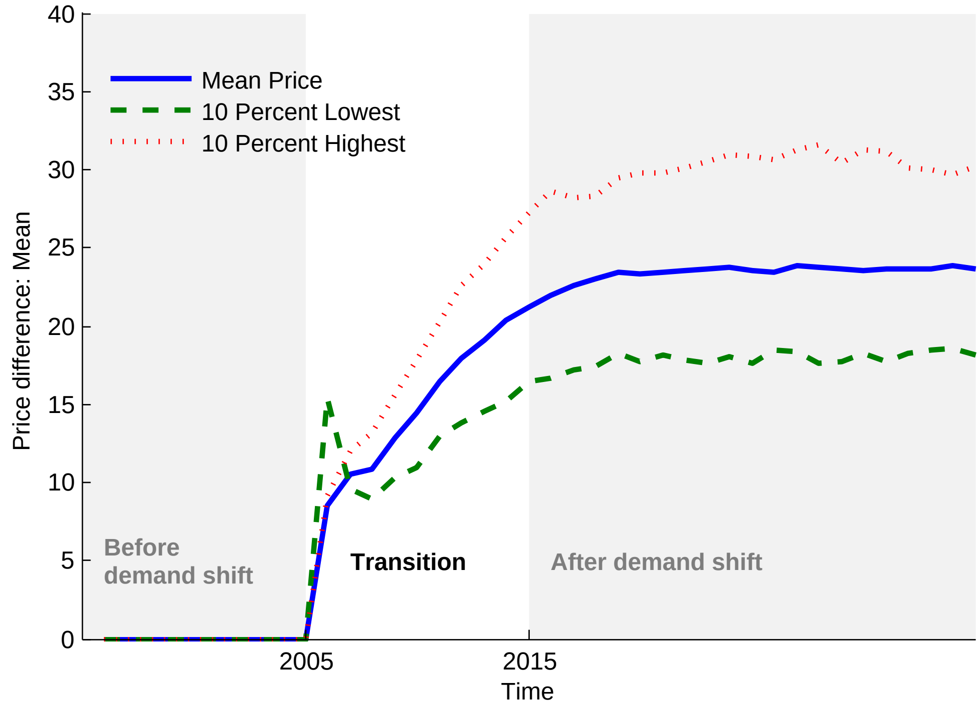
Figure 8: Mean Price Transition by Timing of the Demand Shock (10,000 Simulations)

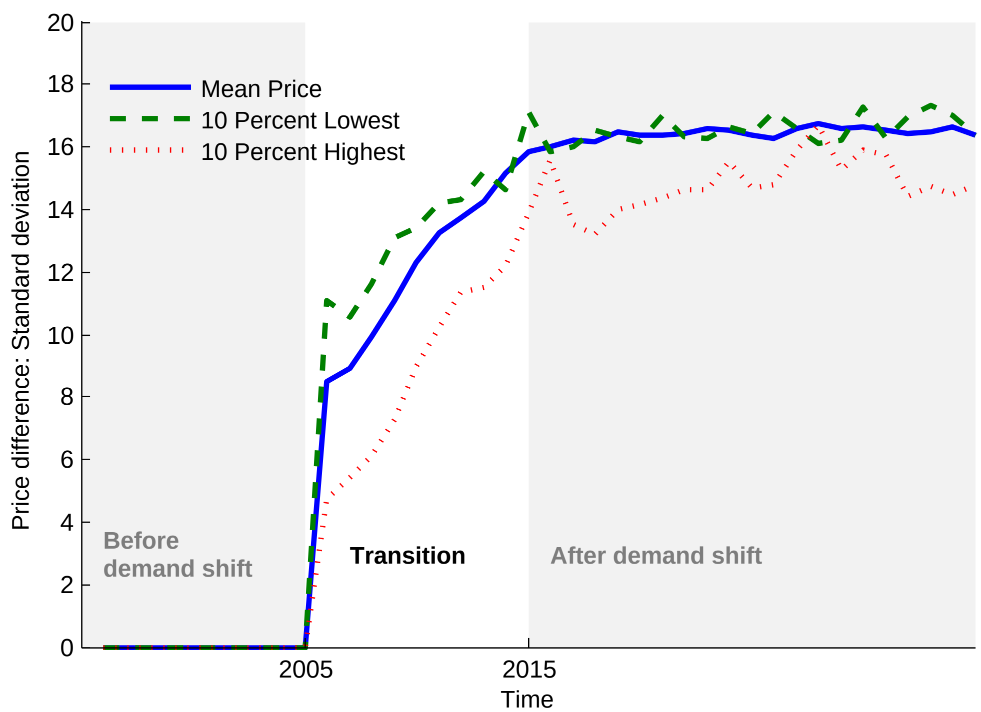


Notes: Parameters of the storage models stay constant across environments.

Figure 9: Price Difference by Amount on Hand

(Demand Shock Anticipated)

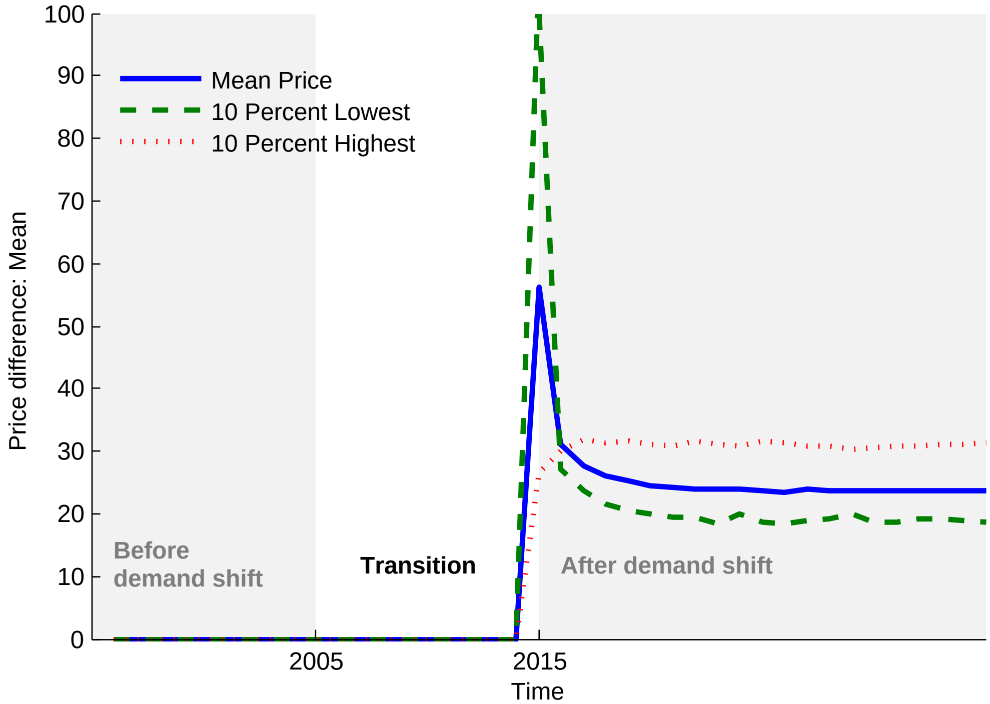




Notes: Parameters of the storage models stay constant across environments. Figure show price change after the demand shock by level of amount on hand available when the shock happens expectedly (in 2006). Mean price is the average change over 10,000 simulations. Ten percent lowest is the average over 1000 observations with the lowest amount on hand. Likewise, ten percent highest is the average over 1000 observations with the highest amount on hand.

Figure 10: Price Difference by Amount on Hand

(Demand Shock Unexpected)



Notes: Parameters of the storage models stay constant across environments. Figure show price change after the demand shock by level of amount on hand available when the shock happens unexpectedly. Mean price is the average change over 10,000 simulations. Ten percent lowest is the average over 1000 observations with the lowest amount on hand. Likewise, ten percent highest is the average over 1000 observations with the highest amount on hand.

Table 1: Solve for the Storage Model with Two Different Grids of Amount on Hand

Variables	**1000 points equally spaced**	**50 points unequally spaced**	**T-test**
Cut off price	39.980	40.023	
Price	27.701	27.693	0.014
	(18.272)	(18.336)	
Amount on hand	6.834	6.829	0.212
	(0.728)	(0.726)	
Production	6.037	6.037	-0.002
	(0.553)	(0.553)	
Inventory	0.811	0.806	0.274
	(0.580)	(0.578)	
Land	0.606	0.606	-0.020
	(0.010)	(0.010)	

Notes: Variables values reported are mean and standard deviation in parenthesis. Two storage models are identical except for number of amount on hand grid points and how they are spaced apart. Demand and supply parameters are -0.05 and 0.05 respectively. Decay rate and interest rate are both set at 0.02. Amount on hand range from 0 to 10. The quantity and price in equilibrium with no storage are 6 and 30 respectively. For 50 points unequally spaced, 10 points are put between 0 and 5.5, 30 points are put between 5.5 and 7, and 20 points are put between 7 and 10. Data for the two models are generated for 2000 periods using the same yield draw where yield $\sim \mathcal{N}(10, 1)$. Storage cost is set at 1.6.

Table 2: Data Statistics

Variables	**Detrended Values**	**Real Values**
Price - 2010 dollars to feed one person per year	30.35	99.82
	(8.58)	(53.54)
Yield- People fed per hectare per year	12.11	8.79
	(0.37)	(2.00)
Amount on hand- Billion people	7.93	5.22
	(0.34)	(1.65)
Production- Billion people	6.20	4.17
	(0.22)	(1.22)
Consumption- Billion people	6.18	4.13
	(0.13)	(1.22)
Inventory- Billion people	1.75	1.09
	(0.33)	(0.46)

Notes: Mean and standard deviations are reported. Data for corn, soybeans, rice, and wheat from 1964 to 2005 are from US Department of Agriculture. Data are converted into calories and then to people assuming a daily 2000 caloric intake. Yield and price data are constructed by production-weighted average of the four crops. Real data is what we observed. Detrended data are data with a linear trend removed and bring back to the level in 2005

Table 3: Grids of Parameters

Parameters	**Values**
Demand Elasticity	-0.01, -0.02, -0.03, -0.04, -0.05, -0.06, -0.07, -0.08, -0.09, -0.1
Supply Elasticity	0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.07, 0.08, 0.09, 0.1
Price in Equilibrium (2010 dollars)	30.35
Quantity in Equilibrium (Billion people)	6.179
Storage Cost	0.2, 0.4, 0.6, 0.8, 1, 1.2, 1.4, 1.6, 1.8, 2
Interest Rate	0.02, 0.05
Inventory Decay Rate	0.02
Yield (Mean and SD)	12.11, 0.368

Notes: At the global scale, units for quantity is in billion. For example, at the price of 30.35 dollar/a year of calories for one person, the world market will provide enough food crop to feed 6.179 billion people in a year. Crop yields enough harvest to feed 12.11 people from one hectare, on average.

Table 4: The Ten Lowest Scores from Grid Search

Demand Elasticity	**Supply Elasticity**	**Storage Cost**	**Interest Rate**	**Score**
-0.06	0.01	1.8	0.02	0.0174
-0.06	0.01	2	0.02	0.0193
-0.06	0.01	1.6	0.02	0.0194
-0.06	0.01	1	0.05	0.0203
-0.06	0.01	1.2	0.05	0.0206
-0.06	0.01	0.8	0.05	0.0236
-0.06	0.01	1.4	0.05	0.0246
-0.06	0.01	1.4	0.02	0.0251
-0.06	0.02	2	0.02	0.03
-0.06	0.01	0.6	0.05	0.0304

Notes: The table shows the sets of parameters that will minimize the difference between simulated and observed data. Parameters for equilibrium price and quantity and decay rate are omitted because they stay constant over 2000 sets of parameters.

Table 5: Simulated Price Data over Different Scenarios (10,000 Simulations)

Scenarios	**Old Equilibrium (1)**	**New Equilibrium (2)**	**Shock Anticipated (3)**	**Shock Unexpected (4)**	**(3-1)/ (2-1)**	**(4-1)/ (2-1)**
1	30.05	53.67	38.52	86.24	0.36	2.38
	(8.94)	(14.33)	(5.33)	(26.26)		
2	30.05	50.74	36.55	76.98	0.31	2.27
	(8.94)	(12.47)	(4.82)	(21.52)		
3	30.05	57.60	41.33	99.75	0.41	2.53
	(8.94)	(16.96)	(6.03)	(33.42)		
4	30.05	50.71	36.99	81.45	0.34	2.49
	(8.94)	(13.10)	(5.70)	(24.88)		
5	30.05	53.64	39.12	92.87	0.38	2.66
	(8.94)	(15.17)	(6.51)	(31.23)		
6	30.05	48.44	35.46	73.47	0.29	2.36
	(8.94)	(11.57)	(5.14)	(20.59)		

Notes: Mean and standard deviation in parenthesis are reported. Before demand shifts out, demand and supply elasticities are: $e_d = -0.06,\ e_s = 0.01$. Scenarios are demand and supply elasticities in the new environments ranging from 1 to 6 with demand and supply elasticities values equal $e_d = -0.06,\ e_s = 0.01$; $e_d = -0.07,\ e_s = 0.01$; $e_d = -0.05,\ e_s = 0.01$; $e_d = -0.06,\ e_s = 0.02$; $e_d = -0.05,\ e_s = 0.02$; and $e_d = -0.07,\ e_s = 0.02$ respectively.

Old equilibrium is the price before the demand shock. New equilibrium is the price after demand completely shifts out. Shock anticipated is the price shock when the news of the demand shift is realized immediately (in 2006). Shock unexpected is the price shock when demand shifts out unexpectedly (i.e. The US Ethanol mandate requires 15 billion gallons of ethanol to be produced in 2005 instead of 2015).

T-statistics (not reported) between (1) and each of (2), (3), (4) range from 64 to 202.

Table 6: Simulated Price Transition 2005-2011

Scenarios	**Year** 2005	2006	2007	2008	2009	2010	2011
Observed price	43.61	59.47	84.04	95.31	84.88	105.83	120.54
1	43.61	59.95	61.49	60.63	58.80	61.11	63.60
		(1.03)	(0.44)	(0.33)	(0.37)	(0.28)	(0.26)
2	43.61	57.02	58.70	58.15	56.64	59.03	61.53
		(0.85)	(0.37)	(0.28)	(0.32)	(0.25)	(0.23)
3	43.61	64.09	65.46	64.20	61.92	64.09	66.44
		(1.29)	(0.54)	(0.40)	(0.44)	(0.33)	(0.30)
4	43.61	56.76	58.30	58.07	56.55	58.91	61.31
		(0.83)	(0.36)	(0.28)	(0.31)	(0.25)	(0.23)
5	43.61	59.69	61.09	60.61	58.77	61.03	63.39
		(1.01)	(0.43)	(0.33)	(0.37)	(0.28)	(0.26)
6	43.61	54.59	56.28	56.19	54.98	57.33	59.69
		(0.69)	(0.31)	(0.24)	(0.28)	(0.22)	(0.21)

Notes: Number in parenthesis are the ratios between simulated price change and observed price change. Before demand shifts out, demand and supply elasticities are: $e_d = -0.06,\ e_s = 0.01$. Scenarios are demand and supply elasticities in the new environments ranging from 1 to 6 with demand and supply elasticities values equal $e_d = -0.06,\ e_s = 0.01$; $e_d = -0.07,\ e_s = 0.01$; $e_d = -0.05,\ e_s = 0.01$; $e_d = -0.06,\ e_s = 0.02$; $e_d = -0.05,\ e_s = 0.02$; and $e_d = -0.07,\ e_s = 0.02$ respectively.

Table 7: Simulated Price Data over Different Scenarios Using Demand and Supply Parameters Estimated From the Literature

Scenarios	**Old Equilibrium (1)**	**New Equilibrium (2)**	**Shock Anticipated (3)**	**Shock Unexpected (4)**	**(3-1)/ (2-1)**	**(4-1)/ (2-1)**
1	29.77	40.76	32.49	68.65	0.25	3.54
	(7.76)	(9.94)	(6.46)	(20.64)		
2	29.77	40.05	32.26	67.48	0.24	3.67
	(7.76)	(9.68)	(6.54)	(20.32)		
3	29.77	41.58	32.77	69.98	0.25	3.41
	(7.76)	(10.24)	(6.38)	(21.00)		
4	29.77	40.17	31.77	62.94	0.19	3.19
	(7.76)	(9.16)	(5.55)	(17.05)		
5	29.77	39.54	31.58	61.98	0.18	3.30
	(7.76)	(8.95)	(5.62)	(16.82)		
6	29.77	40.88	32.00	64.03	0.20	3.08
	(7.76)	(9.40)	(5.48)	(17.32)		
7	29.77	41.39	33.51	77.04	0.32	4.07
	(7.76)	(10.86)	(7.86)	(26.08)		
8	29.77	40.57	33.23	75.57	0.32	4.24
	(7.76)	(10.54)	(7.95)	(25.63)		
9	29.77	42.33	33.84	78.74	0.32	3.90
	(7.76)	(11.24)	(7.75)	(26.61)		

Notes: Demand and supply parameters from Roberts and Schlenker (2010). Mean and standard deviation in parenthesis are reported. Before demand shifts out, demand and supply elasticities are: $e_d = -0.05,\ e_s = 0.10$. Scenarios are demand and supply elasticities in the new environments ranging from 1 to 9 with demand and supply elasticities values equal $e_d = -0.05,\ e_s = 0.10$; $e_d = -0.05,\ e_s = 0.11$; $e_d = -0.05,\ e_s = 0.09$; $e_d = -0.06,\ e_s = 0.10$; $e_d = -0.06,\ e_s = 0.11$; ; $e_d = -0.06,\ e_s = 0.09$; $e_d = -0.04,\ e_s = 0.10$; $e_d = -0.04,\ e_s = 0.11$, and $e_d = -0.04,\ e_s = 0.09$ respectively.

Old equilibrium is the price before the demand shock. New equilibrium is the price after demand completely shifts out. Shock anticipated is the price shock when the news of the demand shift is realized immediately (in 2006). Shock unexpected is the price shock when demand shifts out unexpectedly (i.e. The US Ethanol mandate requires 15 billion gallons of ethanol to be produced in 2005 instead of 2015).

T-statistics (not reported) between (1) and each of (2), (3), (4) range from 21 to 180.

Table 8: Simulated Price Transition 2005-2011 Using Estimates from the Literature

Scenarios	**Year**						
	2005	2006	2007	2008	2009	2010	2011
Observed price	43.61	59.47	84.04	95.31	84.88	105.83	120.54
1	43.61	48.04	49.23	49.99	49.47	51.50	53.28
		(0.28)	(0.14)	(0.12)	(0.14)	(0.13)	(0.13)
2	43.61	47.63	48.73	49.49	49.02	51.03	52.73
		(0.25)	(0.13)	(0.11)	(0.13)	(0.12)	(0.12)
3	43.61	48.53	49.82	50.57	50.00	52.05	53.88
		(0.31)	(0.15)	(0.13)	(0.15)	(0.14)	(0.13)
4	43.61	47.54	48.84	49.61	49.12	51.22	52.97
		(0.25)	(0.13)	(0.12)	(0.13)	(0.12)	(0.12)
5	43.61	47.18	48.39	49.19	48.74	50.80	52.45
		(0.22)	(0.12)	(0.11)	(0.12)	(0.12)	(0.11)
6	43.61	47.96	49.35	50.10	49.57	51.70	53.54
		(0.27)	(0.14)	(0.13)	(0.14)	(0.13)	(0.13)
7	43.61	48.71	49.75	50.52	49.95	51.88	53.62
		(0.32)	(0.15)	(0.13)	(0.15)	(0.13)	(0.13)
8	43.61	48.21	49.16	49.92	49.40	51.32	53.02
		(0.29)	(0.14)	(0.12)	(0.14)	(0.12)	(0.12)
9	43.61	49.31	50.45	51.23	50.59	52.55	54.32
		(0.36)	(0.17)	(0.15)	(0.17)	(0.14)	(0.14)

Notes: Demand and supply parameters from Roberts and Schlenker (2010). Number in parenthesis are the ratios between simulated price change and observed price change. Before demand shifts out, demand and supply elasticities are: $e_d = -0.05,\ e_s = 0.10$. Scenarios are demand and supply elasticities in the new environments ranging from 1 to 9 with demand and supply elasticities values equal $e_d = -0.05,\ e_s = 0.10$; $e_d = -0.05,\ e_s = 0.11$; $e_d = -0.05,\ e_s = 0.09$; $e_d = -0.06,\ e_s = 0.10$; $e_d = -0.06,\ e_s = 0.11$; ; $e_d = -0.06,\ e_s = 0.09$; $e_d = -0.04,\ e_s = 0.10$; $e_d = -0.04,\ e_s = 0.11$, and $e_d = -0.04,\ e_s = 0.09$ respectively.