MEASUREMENT AND DECOMPOSITION OF FLEXIBILITY OF MULTI-OUTPUT FIRMS

Swetlana Renner, Thomas Glauben, Heinrich Hockmann

Selected Paper prepared for presentation at the International Association of Agricultural Economists (IAAE) Triennial Conference, Foz do Iguaçu, Brazil, 18-24 August, 2012.

Abstract:

Flexibility can be considered as a crucial factor of competitive advantage, especially under conditions of dynamically changing environments. Based on the classical microeconomic definition of flexibility, as introduced by Stigler, and some recent concepts developed in the production economics, this article proposes a primal flexibility measure for multi-product firms. When decomposed, this measure offers useful insights into possible sources of flexibility, especially by investigating the role of both scale and scope economies. This approach provides the theoretical basis to investigate the magnitude and sources of flexibility in the Polish agricultural sector during the transition period.

Key Words: cost function, duality, input distance function, flexibility, Poland, scale economies, scope economies

JEL codes: D24, Q12, L25

1 INTRODUCTION

Firms in all sectors of the economy are facing changing economic, legal, and political conditions. From this point of view, the competitiveness of a firm and even its long-term viability depend not only on its efficiency and productivity, but also on its ability to adequately respond to changes in prices or policies with adjustments of its production programs. A flexible or adaptable production technology is required to meet this challenge.

In agricultural economic literature, there is a large body of research that, for one thing, assess efficiency and productivity, and for another, investigate the impact of economies of scale and scope (e.g. Hallam 1991; Brümmer, Glauben and Thijssen 2002; Paul, Nehring and Banker 2004; Chavas 2008). However, only little attention has yet been paid to the role of flexibility in the agricultural sector (Pasour and Bullock, 1975). Moreover, in their recent overview on production economics research, Chavas, Chambers and Pope (2010, p. 370) point to the "need to refine our understanding of the role of risk/uncertainty in agriculture" in analyzing the ability of farmers to adjust their management strategies in response to changing conditions. This article therefore aims to contribute to filling this research gap by elaborating a new concept to measure flexibility in the agricultural sector in three ways: (i) linking well known concepts of scale and scope economies with flexibility, (ii) extending established methodological approaches to the calculation of flexibility and (iii) applying these new concepts to the agricultural sector.

Researchers have been interested in firms' flexibility since the topic was introduced in literature by Stigler (1939). He defined flexibility as those attributes of cost curves that determine how responsive output decisions are to demand fluctuations. Since in this context flexibility is discussed in terms of relative convexity of the average cost curve, a flatter average curve thus indicates greater flexibility. Though more intuitive and descriptive in nature, Stigler's definition of flexibility strongly influenced subsequent theoretical and empirical analyses in the microeconomic literature. However, flexibility literature is not consistent with regard to the choice of measurement of flexibility. Marschak and Nelson (1962) suggested relating flexibility to the second derivative of the *average* cost function,[1] whereas some other authors (cf. Tisdell 1968; Mills 1984; Zeller and Robison 1992;

[1] In addition, Marschak and Nelson (1962) defined flexibility more broadly as the extent to which an initial action allows for the best subsequent action which may be taken as reaction to the future environmental state. This implies to define an initial action such that a sufficiently high number of alternative actions are offered to the decision maker in the subsequent period. However, this approach neglects the question how costly such actions are.

Zimmermann 1995) used the second derivative of the *total* cost function. On the other hand, Mills and Schumann (1985), and Das, Chappell and Shughart (1993) proposed the supply elasticity, while Fuss and McFadden (1978) suggested the use of the elasticity of substitution between inputs as a measure for (input) flexibility.

However, as already criticized by Mills (1986) and Hiebert (1989), all these measures are a priori limited in reliability and significance because they relied on the restrictive assumptions of squared cost functions which are known to be less flexible representations of production technology and which depend only on the output. It would therefore be more appropriate to use more general functional forms and to consider input prices as well as is standard in microeconomic theory assuming cost minimizing behavior. But there is another drawback. All these measures were formulated under the assumption of single-product firm, which in fact is far from realistic for most sectors, in particular agriculture. For a multi-product company flexibility does not only mean the ability to adapt total production to changes but also the capacity to adjust or even to transform the structure of production (i.e. the product mix) according to changing market conditions (Carlsson 1989; Ungern-Steinberg 1990). Taking into account the just mentioned aspects, a more general concept has recently been developed and applied to multi-product firms by Cremieux et al. (2005). Being closely related to Stigler's definition, their measure enables to calculate multi-product flexibility of the firm using any flexible cost function depending on both outputs and factor prices.

Most empirical studies on flexibility focus on the industrial sector. In contrast to the theoretical literature, almost all of them approximate flexibility with the variance of the aggregated production (cf. Fiegenbaum and Karnani 1991; Das, Chappell and Shughart 1993; Nor et al. 2007); a procedure that is based on the main hypothesis of Mills and Schumann (1985), that companies with more flexible technologies react to fluctuations in demand with a stronger variability of output than less flexible companies. Altogether, these studies - applying econometric models to examine the interrelation between flexibility and the size of a company - confirm the existence of a negative correlation between flexibility and firm size (cf. Mills and Schumann 1985; Fiegenbaum and Karnani 1991; Das, Chappell and Shughart 1993; Zimmermann 1995; Nor et al. 2007); indeed, a finding which coincides with the hypotheses proposed by Mills (1984). Surprisingly little attention has so far been given to the flexibility in agricultural economics. To our knowledge, there is only one study by Weiss (2001), that empirically investigates the impact of some economic and socio-demographic factors on the variability of the aggregated output (as an approximation for tactical flexibility) and product-mix-changes (using for operational flexibility) of agricultural companies in Austria. [2]

In summary, this article provides theoretical, methodological and empirical contributions to production economics and to the discussion of flexibility. First, applying Stigler's flexibility concept (Stigler 1939) in combination with the dual multi-product measure of Cremieux et al. (2005), it provides a decomposition of the derived multi-product flexibility measure into three additive components so that it is possible to separately analyze the impact of the three factors (scope and scale economies, and resource scarcity) on flexibility. Following a procedure similar to that of Chavas and Kim (2010), we distinguish between three different cost responses that are associated with the three different aspects of production, namely the level of production (scale effect), the structure of production or diversification (scope effect or complementarity effect), and the shape of the marginal cost function determined by resource scarcity (convexity effect). Second, we develop a primal flexibility measure from the dual relationships between the variable cost and input distance function. In doing so, we make use

[2] There are two theoretical studies in agricultural economic literature that address flexibility issues: Zeller and Robison (1992), and Pasour and Bullok (1975). These papers, however, are limited in relevance to our analysis as they remain theoretical and do not contain any methodological issues or empirical application.

of the primal measures for economies of scale and scope as proposed in the economic literature (Färe, Grosskopf and Lovell 1986; Hajargasht, Coelli and Prasada Rao 2008). The advantage of the primal flexibility measure is that flexibility can be derived even if it is impossible to estimate cost functions econometrically due to poor data availability or other specification problems. All of the derived measures are formulated for both the short and the long run in order to capture the impact of fixed costs on flexibility. Third, the article provides an empirical contribution by applying the proposed flexibility concept to the case of Polish agriculture. The sector is characterized by a large number of small family farms that despite their relatively low productivity (Latruffe et al. 2005), did not disappear either during the transition period or after Poland's accession to the EU. One possible explanation provided in our study is that small farms use more flexible technologies as a source of their competitive strength in an uncertain environment.

The article is organized as follows: it begins with presenting an appropriate flexibility measure based on the average cost function and formulated for the single-output and multi-product case. The section also discusses the economic interpretation of the flexibility measure and its components. In the ensuing section we develop a primal flexibility measure for both short and long run, which both are calculated from the elasticities of the short run input distance function. After the discussion of econometric issues in the following section, we introduce the data set used for our study and present the estimated results. The final section summarizes the analysis and offers some concluding remarks.

2 DEFINITION, MEASUREMENT AND DECOMPOSITION OF FLEXIBILITY

We consider flexibility as an attribute of production technology to accommodate output variations at lower costs. In this respect we adopt Stigler's (1939) definition of flexibility according to which flexibility varies inversely with the curvature of the average cost curve. Thus, the steeper the slope of the average cost curve, the less flexible the production technology of the firm can respond. Since the curvature of a curve is measured by the second derivative, flexibility for the *single-output* case can be formally expressed as follows:

(1) $$Flex = \frac{\partial^2 (C/y)}{\partial y^2} = \frac{1}{y^3}\left[C_{yy} \cdot y^2 - 2C_y \cdot y + 2C \right],$$

where C is a cost function, satisfying the usual homogeneity, monotonicity and curvature properties (see Chambers 1997). Further, C_{yy} and C_y are second and first order derivatives of the cost function with respect to the output y.[3]

Cremieux et al. (2005) extended this measure for the *multi-output* case using the concept of ray average cost (Baumol, Panzar and Willig 1988)[1], so that, in analogy to the single-output case, multi-output flexibility is given by:

(2) $$Flex = \mathbf{y}'\mathbf{C}_{\mathbf{yy}}\mathbf{y} + 2C \cdot \left(1 - \mathbf{1}_\mathbf{J}'\mathbf{E}_\mathbf{y}\right),$$

where $\mathbf{C}_{\mathbf{yy}}$ denotes the *(JxJ)*-Hessian matrix of second order derivatives of the cost function with respect to output y_j *(j=1, ..., J),* $\mathbf{1}_J$ *is the (Jx1)*-vector of ones and $\mathbf{E}_\mathbf{y}$ is the *(JxJ)*-vector of partial cost elasticities with respect to output y_j : $\varepsilon_{y_j}^C = \frac{\partial C}{\partial y_j} \cdot \frac{y_j}{C}$.

[3] Cost elasticity is defined as the percentage change in costs caused by a 1% increase of output: $\varepsilon_y^C = \frac{\partial C}{\partial y} \cdot \frac{y}{C}$.

Smaller values of the second derivative of the average cost function with respect to output correspond to flatter average cost curves. Thus, lower values of *Flex* imply more flexible technologies.

For a more detailed analysis of the sources of flexibility the Hessian matrix is separated into two matrices: $\mathbf{C_{yy}} = \mathbf{C}_{\mathbf{yy}}^{Diag} + \mathbf{C}_{\mathbf{yy}}^{-Diag}$ with $\mathbf{C}_{\mathbf{yy}}^{Diag}$ the diagonal matrix containing only the diagonal entries of $\mathbf{C_{yy}}$ and, correspondingly, matrix $\mathbf{C}_{\mathbf{yy}}^{-Diag}$ containing off-diagonal elements of $\mathbf{C_{yy}}$ and zero values on the main diagonal.

(3) $\quad Flex = \mathbf{y'C}_{\mathbf{yy}}^{-Diag}\mathbf{y} + \mathbf{y'C}_{\mathbf{yy}}^{Diag}\mathbf{y} + 2C\cdot\left(1-\mathbf{1_J}'\mathbf{E_y}\right).$

Thus, the multi-product flexibility measure in (3) is the sum of three terms corresponding with scope (or complementarity), convexity, and scale effect, in respective order.

Scope effect

The first component of (3) $\mathbf{y'C}_{\mathbf{yy}}^{-Diag}\mathbf{y}$ measures the impact of cost savings resulting from economies of scope. The term “economies of scope” refers to cost reductions due to diversification of production (Panzar and Willig 1977). Positive economies of scope may arise from sharing or joint utilization of input resources by diversified production technologies. According to Baumol, Panzar and Willig (1988), economies of scope depend on the signs of the second order derivatives of the multi-product cost function with respect to outputs. Negative values indicate weak cost complementarities among two outputs, which, at the same time, is a sufficient condition for the existence of economies of scope (Deller, Chicoine and Walzer 1988). Indeed, complementarity in outputs implies a reduction of the marginal cost of a particular output y_j when the production of another output $y_{p\neq j}$ is increased.[4] Thus, the scope effect indicates how strong the utilization of economies of scope contributes to a firm’s ability to meet demand fluctuations by adjusting output levels.

Convexity effect

The second term of (3) $\mathbf{y'C}_{\mathbf{yy}}^{Diag}\mathbf{y}$ can be interpreted as convexity effect. As it consists of the diagonal elements of the Hessian matrix, which can take on both negative and positive values, this effect reflects the shape of the marginal cost function. Thus, its sign depends on whether the cost function is concave or convex in y. If the technology is homothetic, the cost function is quasi-convex in outputs (Färe and Lehmijoki 1987). In this case, marginal costs are increasing, reflecting increasing resource scarcity (Chavas 2008), which, in turn, affects the value of flexibility negatively. But, on the contrary, if the convexity effect for the *j*-th output of **y** is negative (e.g. decreasing marginal costs), flexibility will increase. Therefore, the lower the growth rate of costs, the more flexible the production technology responds.

Scale effect

Finally, the third component of (3) $2C\cdot\left(1-\mathbf{1_J}'\mathbf{E_y}\right)$ reflects the concept of economies of scale associated with the relationship between (variable) cost and output changes, hereafter referred to as scale effect. Scale elasticity measures the proportional change (increase or decrease) in cost resulting from a proportional increase in the level of output. According to Brown, Caves and Christensen (1979), and Christensen and Greene (1976), scale elasticity may be expressed as denoted in the bracketed term, i.e. as one minus the sum of cost elasticities with respect to

[4] A few examples may help explain what is meant by this: Cost complementarities arise e.g., when land, labor and management resources across farm activities and enterprises are allocated within a given period, so that these inputs can be jointly utilized more efficiently. They also occur when production technologies rely on intermediate products, as e.g. the use of manure as organic fertilizer in crop production or the utilization of crops and their residues in animal feeding.

outputs, thus yielding positive values for economies of scale and negative values for diseconomies of scale. In other words, the scale effect measures cost responsibility due to economies of scale achieved by the intensification of production (increase in the output level of individual products). Thus, firms with higher economies of scale are less flexible. Such firms have a greater economic incentive to increase output in order to reduce scale inefficiency, which increases flexibility at the same time. However, firms which incur diseconomies of scale are faced with a tradeoff between flexibility and scale efficiency, i.e. increasing output leads to more flexibility, though at the expense of becoming even less scale efficient.

From this it follows that the flexibility measure can be decomposed into a scale effect as well scope and convexity effects. Thus, it offers a way to detect possible sources of flexibility for the multi-product firm. In particular, decision makers need to be aware that increasing flexibility through the scale effect may enhance scale efficiency if farms operate in the area of economies of scale but could lead to scale efficiency losses if they possess diseconomies of scale. Moreover, as flexibility is in addition affected by resource scarcity and complementarity between outputs, which might lead to divergent if not opposite adjustments, the overall effect cannot be determined theoretically. Thus, to avoid such unintended responses, it makes sense to empirically quantify impact and magnitude of the mentioned effects.

Short and long run flexibility measures

Furthermore, to investigate the impact of fixed costs on flexibility, it is necessary to determine measures that capture short and long run effects. In the short run, some of the inputs (e.g. land, buildings, machinery, equipment, etc.) are considered to be quasi-fixed factors as they cannot easily be varied during the investigation period. Consequently, short-term production adjustments can only be made by changing the input of variable factors given the amount of quasi-fixed factors. In the long run a decision maker is able to vary or reallocate all production factors and, thus, determine optimum factor input by minimizing the *total costs* of production. Long run flexibility therefore benefits from additional durable sources of cost savings with regard to both variable and overall costs of production, and thus is expected to be greater than short run flexibility; an assumption that follows immediately from the Le Chatelier Principle (Samuelson 1947), according to which the responsiveness of the decision variable decreases with the number of constraints imposed to the system. Additionally, since the long run average cost function is an envelope of the short run cost functions it is less steeply sloped than the short run functions, which, in turn, induces higher flexibility in the long run.

Formally, the short run flexibility measure is based on the short run variable cost function (VC), derived from the cost minimization problem for given quasi-fixed factors:

$$(4) \qquad VC(\mathbf{w},\mathbf{y},\mathbf{k}) = \min_{\mathbf{x}>0} \left\{ \mathbf{w}'\mathbf{x} \middle| \mathbf{x} \in V(\mathbf{y},\mathbf{k}) \right\}$$

with $\mathbf{x}$, the *(I x 1)* vector of variable inputs, $\mathbf{y}$, the *(J x 1)* vector of outputs, $\mathbf{w}$, the *(I x 1)* vector of input prices, $\mathbf{k}$, the *(M x 1)* vector of quasi-fixed factors and $V(\mathbf{y},\mathbf{k})$ denotes the input requirement set. Applying VC instead of C in formula (3) immediately provides the flexibility indicator for the short run and its decomposition.

Long run flexibility is defined similarly. However, the estimation of the long run total cost function requires information on prices of quasi-fixed factors. If such data are not available, this flexibility measure can alternatively be computed based on the estimated results of the short run variable cost function. The long run total cost function TC can be defined as:

$$(5) \qquad TC(\mathbf{w},\mathbf{r},\mathbf{y}) = \min_{\mathbf{k}>0} \left\{ VC(\mathbf{w},\mathbf{y},\mathbf{k}) + \mathbf{r}'\mathbf{k} \right\}$$

with **r** - *(M x 1)* vector of the quasi-fixed factor prices.

Simple derivation leads to the Hessian matrix of the long run cost function as expressed below (see appendix A):

$$(6) \qquad \mathbf{TC_{yy}} = \mathbf{VC_{yy}} - \mathbf{VC_{yk}} \left(\mathbf{VC_{kk}}\right)^{-1} \mathbf{VC_{ky}} .$$

From equation (6) we can calculate the second order derivatives of the long run total cost function using second order own and cross-partial derivatives of the short run variable cost function with respect to **y** and **k**.[5] Off-diagonal elements of the resulting matrix $\mathbf{TC_{yy}}$ in (6) provide information on the long run scope effect, while diagonal elements represent the long run convexity effect.

To proceed further, the scale effect needs to be developed from a long-run measure for economies of scale ($Scale^l$). For this purpose, following Caves, Christensen and Swanson (1981), and Mosheim and Lovell (2009) and performing appropriate operations on the relationships between total and variable cost functions derived from (5) we can determine the long-run measure for economies of scale ($Scale^l$):

$$(7) \qquad Scale^l = 1 - \mathbf{1_J}'\mathbf{E_y}\left(1 - \mathbf{1_I}'\mathbf{E_k}\right)^{-1}..$$

And finally, from (6) and (7) follows the *long run flexibility measure* $Flex^l$ using elasticities and derivatives of the short run variable cost function:

$$(8) \qquad Flex^l = \mathbf{y}'\left(\mathbf{VC_{yy}} - \mathbf{VC_{yk}}\left(\mathbf{VC_{kk}}\right)^{-1}\mathbf{VC_{ky}}\right)\mathbf{y} + 2\left(VC - \mathbf{VC_y}'\mathbf{y} - \mathbf{VC_k}'\mathbf{k}\right).$$

3 PRIMAL MEASURE OF FLEXIBILITY BASED ON THE INPUT DISTANCE FUNCTION

For an empirical analysis of flexibility the measures in (2) and (8) may be directly derived from the elasticities of the econometrically estimated short run multi-output variable cost function. However, the estimation of the cost function may be problematic for some reasons. A first problem might arise from missing data as the input prices necessary for estimating cost functions are not always available. Second, due to the unpriced nature of many inputs in family farm agriculture (e.g. family work, family owned land), it may be difficult to estimate the overall cost. Finally, even if there is a way to approximate input data to some degree, the estimated parameters of the cost function are likely to be inconsistent with the theoretical assumptions (e.g. when quasi-concavity in input prices is not fulfilled) due to inappropriate calculation of price data.

In order to circumvent these drawbacks, the dual approach can be applied as an alternative representation of the production technology. The estimation of input distance functions offers an appropriate way because these functions are equivalent specifications to the multi-product cost functions; however the estimation of the input distance function only requires data on input and output quantities but no price data.

Applying duality theory we can define the short run input distance function, which is dual to the short run variable cost function of equation (4), as follows:

$$(9) \qquad D(\mathbf{x},\mathbf{y},\mathbf{k}) = \inf_{\mathbf{x}>0}\left\{\mathbf{w}'\mathbf{x} : VC(\mathbf{y},\mathbf{w},\mathbf{k}) \geq 1\right\}.$$

[5] Hereafter we use the following notation: $VC_\bullet$ is the vector of first-order partial derivatives and $VC_{\bullet\bullet}$ is the matrix of second-order or cross-term derivatives of the short run variable cost function *VC(w,y,k)* with respect to the corresponding vectors **k** and **y**.

We assume that input distance function in (9) satisfies the standard properties: (i) decreasing in each output level, (ii) increasing in each input level, (iii) homogeneous of degree one and (iv) concave in all inputs (see Färe and Primont 1995).

Finally, we obtain the primal flexibility measure by applying the functional relationships between the cost and input distance function. In doing so we make use of primal measures of economies of scale and scope proposed in the economic literature. Following Färe, Grosskopf and Lovell (1986), the multi-output measure of economies of scale (*Scale*) computed from the input distance function and expressed in terms of cost elasticities can be written as:[6]

$$(10)\quad Scale = 1 - \mathbf{1_J}'\mathbf{E_y} = 1 + \mathbf{y}'\mathbf{D_y} \cdot \mathbf{D}^{-1}.$$

According to Hajargasht, Coelli amd Prasada (2008) we derive the matrix of the second order derivatives of the short run variable cost function with respect to the output vector in terms of the derivatives of the input distance function as follows:[7]

$$(11)\quad \mathbf{VC_{yy}} = VC \cdot \left[\mathbf{D_y D_y'} - \mathbf{D_{yy}} + \mathbf{D_{yx}} \left(\mathbf{D_{xx}} + \mathbf{D_x D_x'} \right)^{-1} \mathbf{D_{xy}} \right].$$

It is important to note here that the interpretation of the matrix $\mathbf{VC_{yy}}$ is analogous to the interpretation of $\mathbf{C_{yy}}$; hence the elements on the main diagonal convey the convexity effect, those of the off-diagonal the scope effect. .

To proceed further, we derive the relationship between cost and distance function[8] by using the solution of the first order condition of the multi-product cost minimization problem with production constraints expressed by the input distance function and the optimal value of the Lagrangian multiplier $\lambda^*(\mathbf{y},\mathbf{w})$ which is equal to the cost function $C(\mathbf{y},\mathbf{w})$. The relationship can be expressed by:

$$(12)\quad VC = \left(\mathbf{1_I}' \cdot \mathbf{D_x} \right)^{-1}$$

with 1_I the (Ix1)-vector of ones and D_{x_i} (Ix1)-vector of the first derivatives of the input distance function with respect to particular input i.[9]

After replacing the corresponding parts of formula (2) with (10), (11) and (12), the primal measure of short run flexibility of a multi-product firm ($Flex^s$) based on the parameters of the input distance function reads:

$$(13)\quad \begin{aligned} Flex^s = {} & \left(\mathbf{1_I}' \cdot \mathbf{D_x} \right)^{-1} \cdot \mathbf{y}' \left[\mathbf{D_y D}_y' - \mathbf{D_{yy}} + \mathbf{D_{yx}} \left(\mathbf{D_{xx}} + \mathbf{D_x D}_x' \right)^{-1} \mathbf{D_{xy}} \right] \mathbf{y} + \\ & + 2 \left(\mathbf{1_I}' \cdot \mathbf{D_x} \right)^{-1} \left(1 + \mathbf{y}'\mathbf{D_y} \cdot \mathbf{D}^{-1} \right). \end{aligned}$$

Finally, to arrive at the *long run flexibility analysis* two more steps a necessary. First, we make use of the dual relationships between the shadow price of the quasi-fixed factor and the short run input distance function:[10]

---

[6] The equality of the primal and dual measures of economies of scale holds for convex input sets; see Färe, Grosskopf and Lovell (1986).

[7] Hereafter we use the following notation: $D_\bullet$ is the vector of first derivatives and $D_{\bullet\bullet}$ is the matrix of second-order derivatives of the input distance function *D(x,y)* with respect to the corresponding vectors **x** and **y**.

[8] For derivative properties and dual relationships between cost and distance functions see Färe and Primont (1995), p.51ff.

[9] Assuming, that input prices are normalizes such that $\sum_{i=1}^{I} w_i = 1$.

(14) $\sum_m \varepsilon_{k_m}^{VC} = \mathbf{k}'\mathbf{VC_k}\left(VC\right)^{-1} = -\mathbf{k}'\mathbf{D_k}D^{-1}$.

In order to obtain the Hessian matrix of the long run total cost function from the derivatives of the short run distance function, we have to replace $\mathbf{VC_{yy}}$, $\mathbf{VC_{yk}}$, $\mathbf{VC_{kk}}$ and $\mathbf{VC_{ky}}$ in formula (6) with their corresponding dual vectors and matrices (see Appendix B). In so doing, we finally obtain the primal long run flexibility measure $Flex^l$ and its components similar to the short run measure in (13):

(15) $Flex^l = \mathbf{y'TC_{yy}y} + 2\left(\mathbf{1_I}'\cdot\mathbf{D_x}\right)^{-1}\left(1+\mathbf{y'D_y}D^{-1}+\mathbf{k'D_k}D^{-1}\right)$

with

$$\mathbf{TC_{yy}} = \left(\mathbf{1_I}'\cdot\mathbf{D_x}\right)^{-1}\Big[\mathbf{D_yD_y'} - \mathbf{D_{yy}} + \mathbf{D_{yx}}\left[\mathbf{D_{xx}}+\mathbf{D_xD_x'}\right]^{-1}\mathbf{D_{xy}} - \left(\mathbf{D_yD_k'} - \mathbf{D_{yk}} + \mathbf{D_{yx}}\left[\mathbf{D_{xx}}+\mathbf{D_xD_x'}\right]^{-1}\mathbf{D_{xk}}\right)$$
$$\left(\mathbf{D_kD_k'} - \mathbf{D_{kk}} + \mathbf{D_{kx}}\left[\mathbf{D_{xx}}+\mathbf{D_xD_x'}\right]^{-1}\mathbf{D_{xk}}\right)^{-1}\left(\mathbf{D_kD_y'} - \mathbf{D_{ky}} + \mathbf{D_{kx}}\left[\mathbf{D_{xx}}+\mathbf{D_xD_x'}\right]^{-1}\mathbf{D_{xy}}\right)\Big].$$

4 ECONOMETRIC SPECIFICATION

For the econometric estimation of the short run multi-product input distance function we specify a translog functional form that allows considering different technologies by adding dummy variables in order to capture, for the one, specialized production processes, and for the other, technological change. Thus, a parametric specification of the short-run multi-product input distance function with respect to the individual firm *f* can be expressed as:

$$\ln D_f = \beta_0 + \beta_d D\_Sp + \beta_\tau t + \beta_{\tau\tau}t^2 + \sum_{j=1}^{J}\beta_j \ln y_{jf} + \sum_{i=1}^{I}\beta_i \ln x_{if} + \sum_{m=1}^{M}\beta_m \ln k_{mf} +$$
$$+\sum_{j=1}^{J}\beta_{dj}\ln y_{jf}\cdot D\_Sp + \sum_{i=1}^{I}\beta_{di}\ln x_{if}\cdot D\_Sp + \sum_{m=1}^{M}\beta_{dm}\ln k_{mf}\cdot D\_Sp +$$
$$(15)\qquad +\sum_{j=1}^{J}\beta_{j\tau}\ln y_{jf}\cdot t + \sum_{i=1}^{I}\beta_{i\tau}\ln x_{if}\cdot t + \sum_{m=1}^{M}\beta_{m\tau}\ln k_{mf}\cdot t +$$
$$+0.5\sum_{j=1}^{J}\sum_{p=1}^{P}\beta_{jp}\ln y_{jf}\ln y_{pf} + 0.5\sum_{i=1}^{I}\sum_{s=1}^{S}\beta_{is}\ln x_{if}\ln x_{sf} + 0.5\sum_{m=1}^{M}\sum_{n=1}^{N}\beta_{mn}\ln k_{mf}\ln k_{nf} +$$
$$+\sum_{i=1}^{I}\sum_{j=1}^{J}\beta_{ij}\ln x_{if}\ln y_{jf} + \sum_{m=1}^{M}\sum_{j=1}^{J}\beta_{mj}\ln k_{mf}\ln y_{jf} + \sum_{i=1}^{I}\sum_{m=1}^{M}\beta_{im}\ln x_{if}\ln k_{mf}$$

where y_{jf} represents the quantity of the *j*th output (*j*=1,2, …, *J*), x_{if} the quantity of the *i*th variable input (*i*=1,2, …, *I*), k_{mf} the quantity of the *m*th quasi-fixed factor (*m*=1,2, …, *M*), and $f=1,...,F$ denotes the number of the considered agricultural firm. The above mentioned dummy variables are denoted *t* and *D_Sp* accounting for technological change (time) and specialization in production processes (crop production, grazing livestock or granivores), respectively.[11]

[10] This relationship is based on the envelope theorem applied to the cost minimization problem $\mathbf{VC_k} = -VC(\mathbf{w},\mathbf{y},\mathbf{k})\cdot\mathbf{D_k}(\mathbf{x}(\mathbf{w},\mathbf{y},\mathbf{k}),\mathbf{y},\mathbf{k})$, where both sides were multiplied by $\mathbf{k}'(VC)^{-1}$ which leads to $\mathbf{k}'\cdot\mathbf{VC_k}\cdot(VC)^{-1} = -\mathbf{k}'\cdot\mathbf{D_k}\cdot D^{-1}$.

[11] Firms are considered as specialized when they produce more than 50% of the overall production in one of the three output groups - crop production (*D_sp=1*), grazing livestock (*D_sp=2*) or granivores (*D_sp=3*); otherwise they are considered as non specialized or diversified (*D_sp=0*).

Following the standard practice in the stochastic frontier literature we set $0 = \ln D_{ft} + v_{ft} - u_{ft}$, where the random disturbance term v_{ft} is assumed to be normally distributed with a zero mean and u_{ft} as one-sided random variable according to a half normal distribution. The term u_{ft} is usually interpreted as technical inefficiency (because technical inefficiency measures the distance to the frontier). The restrictions required for symmetry are: $\beta_{jp} = \beta_{pj}, \beta_{is} = \beta_{si}$ and $\beta_{mn} = \beta_{nm}$. Input homogeneity is imposed by normalizing the distance function by the variable x_0 that is chosen from the set of variable inputs x_i such that the regression model takes the form: $\ln x_{oft} = \ln D^*_{ft} + v_{ft} - u_{ft}$ with $D^*(y;x_i/x_0) = D(x,y)/x_0$ the short run distance function with variable inputs normalized by x_0.[12] Parameters of the stochastic input distance function were estimated running the maximum likelihood procedure in Limdep 9.0.

5 DATA AND EMPIRICAL RESULTS

In the empirical application we used a data set comprising records of 580 Polish agricultural farms over the period of eight years from 1994 to 2001, thus totaling 4,640 observations. The data set was provided by the Polish Institute of Agricultural and Food Economics - National Research Institute (IERiGZ-PIB). Variables contain both farm-specific accountancy information and socio-demographic characteristics. All monetary variables given in current values were deflated by the corresponding price indices provided by the Statistical Office in Poland (GUS var. issues, a, b) with base year 1994.

Our empirical analysis proceeds in two steps. First, we estimate the short run input distance and calculate production elasticities, which are used to obtain farm-specific short- and long-run flexibility indicators. Second, we identify possible determinants of flexibility and its components using regression analysis.

The estimation of the input distance function considered the three variable sets of outputs y_j, variable inputs x_i and quasi-fixed factors k_m, each including a subset of three variables (so that J=3, I=3, and M=3). In detail, the set of output comprised total output values of crops and crop products (y_1), output from grazing livestock including milk production, cattle, sheep and goats (y_2), and granivores (y_3) including pigs, poultry and other animal production.[13] These figures include sales, home consumption and stock changes. From the set of variable inputs we specified the implicit quantity index for specific inputs of crop (x_1) and animal production (x_2) respectively, and "other variable costs" (x_3). The specific input variables (x_1, x_2) were obtained by deflating the components of their respective variable costs in crop or animal production. The variable "other variable costs" was deflated by the national price index for fuel, oils and technical lubricants since these categories of expenditure account for 80% of the total costs of "other variable inputs". This variable was chosen as normalizing variable. The vector of quasi-fixed factors ($\mathbf{k}$) contained the variables labor (k_1), land (k_2), and capital (k_3). In detail, labor was specified as agricultural working units for both family and hired labor. Land input was approximated by the sum of arable land and grassland in use excluding unused land in order to obtain an indicator of land used in production as accurate as possible.

---

[12] Some authors (e.g. Krumbhakar and Lovel, 2003) argue that normalized distance functions could create endogeneity problems, since one of the arbitrary chosen input variables is considered as exogenous while all other inputs are assumed to be endogenous. In spite of that we argue with Brümmer, Glauben and Thijssen (2002) that "the problem is not likely to be more severe than in any production function type of study."

[13] The classification of outputs into crops, grazing livestock and granivores was according to the standard grouping used by the Farm Accountancy Data Network (FADN) within the European Commission.

Capital input was approximated by the sum of expenditure on capital services and depreciation of building, machinery and equipment, deflated by the price index of agricultural investment. All necessary information on definitions and descriptive statistics of the data used in our econometric model are summarized in table 1.

Table 1. Descriptive Statistics by farm type

Variable	Definition	Total*	Mixed farms	Specialist crops	Specialist grazing livestock	Specialist granivores
Outputs:						
Crops	Total output crops & crop production in zł	13772.6 *(16705.2)*	9881.7 *(8075.9)*	22226.4 *(25070.2)*	8911.1 *(7505.7)*	13818.5 *(15434.4)*
Grazing livestock	Total output grazing livestock (milk products, cattle, sheep etc.) in zł	8694.1 *(12741.0)*	8265.1 *(6984.3)*	4551.3 *(6563.0)*	20660.1 *(23105.9)*	2913.5 *(4600.5)*
Granivores	Total output granivores (pigs, poultry and other granivores) in zł	9269.3 *(17405.9)*	7921.4 *(7694.4)*	5594.1 *(9908.0)*	2686.8 *(4252.3)*	31867.2 *(37499.0)*
Quasi-fixed factors:						
Land	Total arable land and grassland in use in ha	14.8 *(14.7)*	12.7 *(9.3)*	17.4 *(20.7)*	15.6 *(12.3)*	14.7 *(14.7)*
Labour	Total labour input in annual work unit	3863.8 *(1821.6)*	4024.5 *(1604.2)*	3610.9 *(2007.5)*	4044.0 *(1763.4)*	3663.5 *(2033.6)*
Capital	Depreciation of farm assets plus expenditure on services in zł	4110.3 *(2958.1)*	3504.8 *(2093.9)*	4633.9 *(3387.4)*	4219.2 *(3082.7)*	4768.9 *(3696.7)*
Variable inputs:						
Input crop	Specific costs of crop production in zł	3087.6 *(5160.5)*	2088.9 *(2055.7)*	5002.7 *(8363.2)*	2177.7 *(2331.6)*	3220.8 *(4099.2)*
Input animal	Specific costs of animal production in zł	9191.7 *(12570.3)*	8315.1 *(6122.5)*	5831.6 *(6499.0)*	9530.1 *(15585.1)*	19610.1 *(24097.4)*
Other var. inputs	Other variable costs in zł	2726.5 *(2475.1)*	2292.6 *(1887.3)*	3127.9 *(2938.4)*	2669.1 *(2309.2)*	3321.9 *(2937.7)*

*Standard deviations are given in parenthesis

For convenience, we normalized all variables by their geometric means such that the coefficients of the first-order effects can be interpreted as the corresponding elasticities at the point of approximation. The results of the maximum likelihood estimation of the stochastic short run input distance function model are listed in table 2.

The monotonicity requirements for inputs and outputs were fulfilled for about 85% of the observations. The distance function is quasi-concave in variable inputs as the (I-1) eigenvalues of the negative semidefinite Hessian Matrix turned out negative; a result that was valid for 97% of the observations. In order to avoid misinterpretation, in two subsequent steps we excluded values, for one thing, those that proved improper, i.e. inconsistent with the theoretical properties of the distance function, and for another, all outliers for flexibility indicators, thus reducing the sample from 4,640 to 4,056 and finally to a total of 3,910 observations.

To proceed further the estimated parameters of the *short run* input distance function in (15) were used to determine the indicators for scale and scope economies as well as flexibility values for each firm according to the suggested primal measures in (10) – (14) and (B.4) – (B.9). Figures for descriptive statistics of the flexibility components are represented by different types of farms in table 3 and table 4 which take the short-run and long-run measures, respectively. Interpreting the results it is important to recall that lower values for the flexibility indicator ($Flex^s$, $Flex^l$) correspond to a flatter average cost curve with slower increase (decrease) of cost per unit of output, which, in turn, indicates a more flexible production technology. In contrast, farms with high levels of the flexibility indicator operate on less flexible production technologies as indicated by the steeper slope of the average cost curve.

Table 2. Estimated parameters of the stochastic short run input distance function
First order effects

Variable	Coeff	Time	Specialization-dummies for		
			Crops	Grazing	Granivores
Time	0.026***	-0.005***			
Crops	-0.439***	0.001	0.000***	0.000***	0.000***
Grazing	-0.104***	-0.001	0.000***	0.000***	0.000***
Granivores	-0.200***	0.001	0.000***	0.000***	0.000***
Input crops	0.250***	-0.004*	-0.023	-0.019	0.062**
Input animals	0.586***	-0.001	-0.116***	0.017	-0.069***
Labor	-0.123***	-0.001	-0.086***	0.044*	-0.045*
Land	-0.050***	0.006	0.134***	0.021	-0.011
Capital	-0.113***	-0.009**	-0.009	-0.041	-0.038
Constant	0.553***		0.167***	-0.113***	-0.134***

Second order effects

Variable	Crops	Grazing	Granivores	Input Crops	Input Animal	Labor	Land	Capital
Crops	-0.068***	-0.003***	-0.005**	0.020	0.011***	0.089***	-0.051**	0.116***
Grazing		-0.006***	0.007***	0.004***	-0.003***	-0.002**	0.000	0.000
Granivores			-0.009***	0.011***	-0.006***	0.003	0.007***	0.000
Input crops				0.094***	-0.021***	-0.131***	-0.012	0.025
Input animals					0.026***	-0.001	-0.003	0.011***
Labor						-0.107***	-0.003	0.015
Land							0.119***	-0.097***
Capital								-0.112***

Number of observations: 4635 Lambda: 0.81088
Log likelihood value: 1321.841 Sigma: 0.21057
Note: ***, **, * denote variables significant at 1%, 5%, and 10% level respectively

In some detail and with respect to farm type, the results in table 3 reveal that *mixed farms* prove to be more flexible than other types of farming, which is indicated by the lowest average value of the overall flexibility index in the short run across all types, amounting to 0.142 as compared to total average of 0.360. Being more diversified, these farms can better exploit economies of scope as evident from the lowest value across all scope effects (0.215). Moreover, mixed farms perform at production levels closer to constant economies of scale than farms of other categories; a fact that contributes to flexibility positively. In this context it is important to point out that farms of all categories were operating under increasing returns to scale in the short run, thus yielding positive values of scale elasticity in 99.9% of observations. This result is consistent with Latruffe at al. (2005), who reported increasing returns to scale for the majority of Polish farms by applying data envelopment analysis. Indeed, it is not surprising given the small sized structure of Polish agricultural sector.
Looking at farms specializing in *livestock production* (grazing livestock, granivores), the situation is different. The corresponding figures (average overall flexibility are 0.661 and 0.608, respectively) indicate them less flexible in the short run. In particular, this concerns specialists in granivores which benefit less than other farms from scope and scale economies. Indeed, these farms attain the highest positive values for the scope effect, and thus are

significantly affected by diseconomies of scope. Farms specializing in production of pigs, poultry and other animals show economies of scope only in 3 of 322 cases. In all other cases there were no complementarities with grazing livestock or crop production.

Table 3. Short run flexibility (average values)

Type of farming	Overall flexibility	Scope effect	Convexity effect	Scale effect
Mixed farms	**0.142**	**0.215**	-0.779	**0.706**
n = 1853	(0.922)	(0.510)	(0.652)	(0.690)
Specialist crops	0.467	0.357	**-0.674**	0.784
n = 1011	(0.824)	(0.363)	(0.547)	(0.928)
Specialist grazing livestock	*0.661*	*0.386*	*-0.657*	*0.931*
n = 724	(1.781)	(0.919)	(0.746)	(1.052)
Specialist granivores	0.608	*0.504*	**-1.123**	*1.227*
n = 322	(0.823)	(0.457)	(0.865)	(1.162)
Total	0.360	0.307	-0.758	0.810
n = 3910	(1.126)	(0.583)	(0.677)	(0.887)

Note: Numbers in parenthesis are standard deviations.

Although this high share of diseconomies of scope may point out a characteristic for livestock farms, other farm types are also affected by negative cost complementarities: In 92% of all observations marginal costs of the considered outputs rise when the production of another output is increased. However, it should be noticed that this result is likely to be overestimated due to limitations resulting from aggregation level of outputs. We can only observe the cost complementarities between crop and grazing livestock, crop and granivores, and grazing livestock and granivores. Certainly, using more disaggregated information, we would probably observe more farms operating on economies of scope resulting from complementarities between different crops that are produced with joint inputs. However, more disaggregated variables would increase the number of parameters of the production function, hence making the estimation of a stochastic frontier model impossible.

Beside these short run indicators, we calculated the corresponding indicators in the *long run* assuming quasi-fixed factors to be utilized at their optimal long run equilibrium levels. The results - average values and standard deviations of the long run indicators - are reported in table 4. Altogether, farms are more flexible in the long run and that holds for all components of flexibility. Indeed, this result comes as no surprise recalling Samuelson's Le Chatelier Principle (Samuelson, 1947), according to which a firm responds to market changes the stronger, the fewer inputs are held fixed. In the long run, farms were more likely to exploit cost complementarities between outputs rather than in the short run. The explanation is easy to find because common production factors such as capital, land and labor are considered to be variable in the long run. In more than one third of the cases the scope effect yielded negative values, which is to be interpreted as a positive effect of diversification. To put this in perspective, it is necessary to compare this proportion to that of only 7% of observations indicating economies of scope in the short run. As in the case of short run flexibility, it is *mixed farms* that benefit more from scope economies than specialized farms, and therefore show the lowest average scope effect among all farm types amounting to 0.063. More than 50% of these farms possess positive economies of scope. As regards overall flexibility, across all types of farms, specialists in granivores yielded the lowest negative value -0.976 resulting mainly from a scale effect of -0.560 which again is the lowest across all farm types. Almost all of these farms operate with diseconomies of scale, which, in turn, as aforementioned, positively affects flexibility. Firms specializing in grazing livestock, including milk production, are less flexible compared to farms of the other categories in the long run, which is mainly explained by significant diseconomies of scope. The lack of flexibility by crop farms can be explained through scale and convexity effects. Having less convex cost functions, they fail to benefit either from scale economies or from a decline in the growth rate of marginal cost, which makes them quite inflexible in the long run.

Table 4. Long run flexibility (average values)

Type of farming	Overall flexibility	Scope effect	Convexity effect	Scale effect
Mixed farms	-0.852	**0.063**	-0.803	-0.123
n = 1853	(2.855)	(1.568)	(1.382)	(0.168)
Specialist crops	-0.272.	0.325	*-0.600*	*0.003*
n = 1011	(1.706)	(0.929)	(0.965)	(0.154)
Specialist grazing livestock	*-0.169*	*0.592*	-0.637	-0.124
n = 724	(2.856)	(1.979)	(1.403)	(0.198)
Specialist granivores	**-0.976**	0.564	**-0.980**	**-0.560**
n = 322	(4.195)	(2.361)	(1.952)	(0.471)
Total	-0.586	0.270	-0.743	-0.122
n = 3910	(2.770)	(1.618)	(1.355)	(0.255)

Note: Numbers in parenthesis are standard deviations.

In the second step of the empirical analysis, the regression models are estimated in order to identity possible determinants of flexibility. It is done by relating the calculated flexibility indicators and their corresponding components to the set of economic and socio-demographic variables. Economic variables include farm size, efficiency levels, share of hired labor used for agricultural production, share of non-agricultural income, capital intensity, access to bank credits and degree of commercialization measured by the share of sales in gross agricultural production. These variables are based on accounting data and vary over time. Since the Polish agriculture is dominated by family farms, we also include a set of socio-demographic variables in order to control for the role of particular family and individual characteristics on farms' adjustment ability. These variables include family size, agricultural and general education, age and gender of the farm head. Furthermore, we include dummy variables to capture for differences in flexibility by farms specializing in production processes (mixed farms, grazing livestock or granivores)[14]. Variables from the second group vary across the farms but not over the time.

Taking into account the data's panel structure, which contains both the time-variant and time-invariant variables, we used the two-step procedure, proposed by Hsiao (2005). In the first stage we estimate the panel fixed-effects model including only the first group of time-variant (economic) variables on the right hand side. These regressions provide the vector of mean effects of all neglected variables, including the effect of time-invariant variables. In the second stage we regress the vector of the fixed effects on variables included in the second group to obtain estimates for the socio-demographic and other time-invariant variables (specialization dummies). The estimation results are reported in Tables 5 and 6.

[14] We drop one category- namely, crop farms - to avoid multicollinearity.

Before starting with an interpretation of results, the following should be mentioned: Larger values of estimated flexibility imply steeper average cost curve, and, thus, less flexible production technology. Therefore, we have to turn over the sign of the estimated parameters to define the direction of the relationship between flexibility and explanatory variables, i.e. positive sigh would mean that investigated variable influences flexibility negatively. Describing our results we concentrate only on the interpretation of some selected statistically significant coefficients.

Regression results for the sort-run flexibility and its components are presented in table 5. The parameter estimates *(*coefficients*)* for *farm size* has a positive sign in all flexibility models except of convexity effect, providing evidence for the negative relationship between the farm size and the overall flexibility on the short run. This finding is in line with previous research on flexibility mentioned in the introduction and corroborates the hypothesis that smaller farms can better adjust their production programs to changing conditions by using more flexible production technologies. More specific, higher flexibility by small farms is caused by cost savings resulting from economies of scale and scope. As mentioned above, the majority of farms operate under diseconomies of scope under increasing returns to scale in the short run. This would thereby mean that lower diseconomies of scope and lower positive economies of scale by small farms were the source of higher flexibility by Polish farms during investigated period.

Table 5. Determinants of the short-run flexibility: Estimates of the two-step fixed-effects regression

	Variables	Overall flexibility	Scope effect	Convexity effect	Scale effect
Economic	Farm Size	+ 0.009***	+ 0.002***	- 0.002***	+ 0.009***
	Efficiency	- 1.240*	- 1.479***	+ 2.172***	- 1.933***
	Hired Labour	- 0.034	+ 0.206	+ 0.233	- 0.473***
	Nonagr. Income	+ 0.178	- 0.208***	+ 0.059	+ 0.327***
	Capital Intensity	+ 0.678	+ 0.027	+ 0.668***	- 0.018
	Access to credits	+ 0.038	+ 0.011	+ 0.053***	- 0.026***
	Commercialisation	+ 0.087	- 0.008	+ 0.111	- 0.017
Socio-demographic	Dummy Mixed Farms	- 0.113*	- 0.069*	- 0.012	- 0.032
	Dummy Specialist Grazing	+ 0.220***	+ 0.051	+ 0.063	+ 0.105
	Dummy Specialist Granivores	- 0.091	+ 0.031	- 0.234***	+ 0.113
	Family Size	+ 0.018	+ 0.021**	- 0.031**	+ 0.029*
	Agr. Education	+ 0.042*	+ 0.027**	- 0.081***	+ 0.097***
	Gen. Education	+ 0.012	+ 0.002	+ 0.006	+ 0.004
	Age	+ 0.001	- 0.001	+ 0.005***	- 0.004
	Gender	+ 0.025	+ 0.061	- 0.281***	+ 0.245***

Further we investigate the relationship between flexibility and technical (input*) efficiency*, which is calculated for each observation using input distance function. The estimated coefficient for the scope effect is negative (- 1.479), providing that higher technical efficiency leads to lower diseconomies of scope, and thus increases the farms flexibility. Negative coefficient for the scale effect (-1.933) means in the short run, that more technical efficient farms tend to be more scale efficient (because the lower value of scale elasticity, or the closer it is to zero, the closer farms are to their optimal size), which in turn, affects flexibility

positively. In spite of the negative convexity effect, positive scope and scale effects are prevailing. All together, technical efficiency contributes to higher overall flexibility.
Estimated parameters of other economic variables indicate that these factors influence flexibility components in opposite directions. In most cases, these effects compensate each other, so that the effect on the overall flexibility is neglected.
Coefficients, estimated for *specialization* dummy variables, support the results presented above in the table 3. Negative coefficient for mixed farms (-0.133) reveals that farms of this category benefit from more flexible production technology, which is mainly caused by gains achieved from scope economies. At the same time, positive significant coefficient for specialists in granivores (+0.220) indicates them less flexible in the short run. Regarding the role of socio-demographic factors, our findings reveal a significant influence of agricultural education, family size, age and gender on certain components of flexibility. However, they have no impact on the overall flexibility. Thus, we do not interpret these results in current study.
Interestingly, the regression results for *long rung flexibility* measures, presented in table 6, reveal that the negative relationship between farm size and flexibility holds only in the short rut. The impact of the variable capturing for farm size on particular components of flexibility goes in opposite directions. Although scope effect is affected by farm size negatively, it is compensated by the positive impact on both the scope and the scale effects. As a result, the overall flexibility is not affected by the farm size, indicated by not statistically significant estimated coefficient. This result emphasizes the need for differentiating between the short and long run view in investigating flexibility. According to Mills and Schumann (1985), and Nor et al. (2007) small firms rely more on variable inputs, whereas larger firms focus more on capital and other quasi-fixed factors, which reduces their adaptive capability, especially in the short run. In the long run, when all factors are free to be adjusted to their optimal levels, all firms become more flexible irrespective of their size.

Table 6. Determinants of the long-run flexibility: Estimates of the two-step fixed effects regression

	Variables	Overall flexibility	Scope effect	Convexity effect	Scale effect
Economic	Farm Size	+ 0.001	+ 0.004***	- 0.002*	- 0.002***
	Efficiency	+ 3.150	- 0.909	+ 4.530***	- 0.472***
	Hired Labour	+ 1.930**	+ 1.211**	+ 0.765*	- 0.046
	Nonagr. Income	- 0.388	- 0.262	+ 0.032	- 0.158***
	Capital Intensity	+ 3.061**	+ 1.787**	+ 1.342**	- 0.068
	Access to credits	+ 0.045	+ 0.007	+ 0.031	+ 0.007
	Commercialisation	- 0.060	- 0.059	+ 0.129	- 0.131***
Socio-demographic	Dummy Mixed Farms	- 0.205	- 0.075	- 0.027	- 0.102***
	Dummy Specialist Grazing	+ 0.242	+ 0.250**	+ 0.088	- 0.095***
	Dummy Specialist Granivores	- 0.397*	+ 0.064	- 0.129	- 0.332***
	Family Size	- 0.046	- 0.006	- 0.045**	+ 0.004
	Agr. Education	- 0.071	+ 0.030	- 0.090***	- 0.012**
	Gen. Education	+ 0.055	+ 0.031	+ 0.016	+ 0.008
	Age	- 0.011**	- 0.010***	- 0.003	+ 0.001*
	Gender	- 0.541***	- 0.160	- 0.360***	- 0.021

6 CONCLUSIONS

Flexible technology enables farms to cope with demand fluctuations and helps them to survive under rapidly changing market conditions. If output adjustments to these changes lead to a significant increase in average cost, farmers need to be able to identify sources of inflexibility. This article presents an extended methodological approach to measuring flexibility for multi-product firms based on a primal as well as dual formulation of the problem. Thus, the measure can be obtained by estimating both cost and input distance functions, each formulated for the short and long run. Employing the primal index, flexibility can be measured when the econometric specification of the cost function fails due to data availability problems. The proposed decomposition of the multi-output flexibility measure not only provides valuable insights into the role of scale and scope economies as well as convexity properties of the production technology but also proves its applicability to assessing the ability to adapt to changing demand. Since these adaptive capabilities are reflected in the flexibility measures, and vice versa, decision makers should be aware of how great flexibility might be affected by such factors as high positive scale economies, resource scarcity and/or absence of cost complementarities between outputs.

In the empirical application we analyzed flexibility, both its sources and the magnitude for different types of farming using data on Polish farms during the transition period. Our results indicate that mixed farms, being more diversified, are more flexible in the short run due to gains from economies of scope, whereas farms specializing in granivores prove more flexible in the long run due to scale and convexity effects. As hypothesized in flexibility literature, the empirical results provide strong evidence that small Polish farms used a more flexible production technology in the short run. In the long term however, no differences could be found in the adjustment ability between small and large farms.

REFERENCES

Baumol, William J.; Panzar, John C. and Willig, Robert D. (1988): Contestable markets and the theory of industry structure. San Diego: Harcourt Brace Jovanovich.

Brown, R. S.; Caves, D. W.and Christensen, L. R. (1979): Modelling the structure of cost and production for multiproduct firms. In: Southern economic journal 46: 256–273.

Brümmer, B.; Glauben, T. and Thijssen, G. (2002): Decomposition of productivity growth using distance functions: The case of dairy farms in three European countries. In: American Journal of Agricultural Economics 84: 628–644.

Caves, Douglas W.; Christensen, Laurits R. and Swanson, Joseph A. (1981): Productivity Growth, Scale Economies, and Capacity Utilization in U.S. Railroads, 1955-74. In: The American Economic Review 71: 994–1002.

Carlsson, Bo (1989): Flexibility and the theory of the firm. In: International journal of industrial organization 7: 179–203.

Chambers, Robert G. (1997): Applied production analysis. Adual approach. Reprint. Cambridge: Cambridge Univ. Press.

Chavas, J. P. (2008): On the economics of agricultural production. In: Australian Journal of Agricultural and Resource Economics 4: 365–380.

Chavas, J. P.; Chambers, R. G. and Pope, R. D. (2010): Production economics and farm management: A century of contributions. In: American Journal of Agricultural Economics 92: 356–375.

Chavas, Jean-Paul and Kim, Kwansoo (2010): Economies of diversification: A generalization and decomposition of economies of scope. In: International Journal of Production Economics 126: 229–235.

Christensen, L. R. and Greene, W. H. (1976): Economies of scale in US electric power generation. In: The journal of political economy 84: 655–676.

Cremieux, P. Y.; Ouellette, P.; Rimbaud, F. and Vigeant: (2005): Hospital Cost Flexibility in the Presence of Many Outputs: A Public-Private Comparison. In: Health Care Management Science 8: 111–120.

Das, Bhaskar J.; Chappell, William F. and Shughart, William F. (1993): Demand fluctuations and firm heterogeneity. In: The journal of industrial economics 41: 51–60.

Deller, S. C.; Chicoine, D. L. and Walzer, N. (1988): Economies of size and scope in rural low-volume roads. In: The Review of Economics and Statistics 70: 459–465.

Färe, Rolf; Grosskopf, Shawna; Lovell and C. A. Knox (1986): Scale economies and duality. In: Journal of economics 46: 175–182.

Färe, Rolf and Lehmijoki, Ulla (1987): On Quasi-Convexity of the Cost Function. In: Scandinavian Journal of Economics 89: 115-118.

Färe, Rolf and Primont, Daniel A. (1995): Multi-output production and duality. Theory and applications. Boston: Kluwer Acad. Publ.

Fiegenbaum, Avi and Karnani, Aneel (1991): Output Flexibility-A Competitive Advantage for Small Firms. In: Strategic Management Journal 12: 101–114.

Fuss, M.; McFadden, D. (1978): Flexibility versus efficiency in ex ante plant design. In: Production Economics: A Dual Approach to Theory and Applications, 311–364.

GUS (var. issues a): Statistical Yearbook of the Republic of Poland. Central Statistical Office of Poland, Warszawa.

GUS (var. issues b): Statistical Yearbook of the Polish Agriculture. Central Statistical Office of Poland, Warszawa.

Hajargasht, Gholamreza; Coelli, Tim and Prasada Rao, D. S. (2008): A dual measure of economies of scope. In: Economics letters 100: 185–188.

Hallam, Arne (1991): Economies of size and scale in agriculture. An interpretive review of empirical measurement. In: Review of agricultural economics 13: 155–172.

Hiebert, L.Dean (1989): Cost Flexibility and Price Dispersion. In: The journal of industrial economics 38: 103–109.

Kumbhakar, S. C. and Lovell, C. A.K. (2003): Stochastic frontier analysis: Cambridge Univ Pr.

Latruffe, Laure; Balcombe, Kelvin; Davidova, Sophia and Zawalinska, Katarzyna (2005): Technical and scale efficiency of crop and livestock farms in Poland. Does spezialization matter? In: Agricultural economics 32: 281–296.

Marschak, Thomas and Nelson, Richard (1962): Flexibility, uncertainty, and economic theory. In: Metroeconomica 14 : 42–58.

Mills, David E. (1984): Demand fluctuations and endogenous flexibility. In: The journal of industrial economics 33: 55–71.

Mills, David E. (1986): Flexibility and firm diversity with demand fluctuations. In: International journal of industrial organization 4: 202–215.

Mills, David E. and Schumann, Laurence (1985): Industry structure with fluctuating demand. In: The American economic review . - Nashville, Tenn. : American Economic Assoc. 75: 758–767.

Mosheim, Roberto; Lovell, C. and Knox, A (2009): Scale Economies and Inefficiency of U.S. Dairy Farms. In: American Journal of Agricultural Economics 91: 777–794.

Nor, Norashidah Mohamed; Nor, Nor Ghani Md.; Abdullah, Ahmad Zainuddin and Jalil, Suhaila Abd. (2007): Flexibility and small firms' survival: further evidence from Malaysian manufacturing 00014: 931–935.

Panzar, John C. and Willig, Robert D. (1977): Economies of scale in multi-output production. In: The quarterly journal of economics 91 3: 481–493.

Pasour jr., E. C. and Bullock, J. Bruce (1975): Implications of uncertainty for the measurement of efficiency. In: American Journal of Agricultural Economics 57 2: 335–339.

Paul, C. J. M.; Nehring, R. and Banker, D. (2004): Productivity, economies, and efficiency in US agriculture: a look at contracts. In: American Journal of Agricultural Economics 86: 1308–1314.

Samuelson, P. A. (1947): Foundations of economic analysis. Cambridge, Mass.

Stigler, George J. (1939): Production and distribution in the short run. In: The journal of political economy .47:305-327.

Tisdell, Clement Allan (1968): The theory of price uncertainty, production, and profit: Univ. Press.

Ungern-Sternberg, Thomas Roman von (1990): The flexibility to switch between different products. In: Economica 57: 355-369.

Weiss, Christoph R. (2001): On flexibility. In: Journal of economic behavior & organization 46: 347–356.

Zeller, Manfred and Robison, Lindon J. (1992): Flexibility and risk in the firm. In: European review of agricultural economics 19: 473–484.

Zimmermann, Klaus F. (1995): Flexibility in the face of demand fluctuations. Employment, capacity utilization, and industry structure. In: International journal of industrial organization 13: 179–193.

APPENDIX A: DUAL LONG-RUN FLEXIBILITY MEASURE

In the long run equilibrium, a firm minimizes its long run total cost by choosing the optimal value for the quasi-fixed factor ***k*** where market prices of the quasi-fixed factors equal their shadow prices, which are defined as the derivative of the short run variable cost functions with respect to the quasi-fixed factor **k**, so that $r_m = -\partial VC/\partial k_m$. Thus, in terms of elasticities the long run total cost function can be rewritten as:

(A.1) $TC = VC\left(1 - \sum_m \varepsilon_{\mathbf{k}_m}^{VC}\right)$ with $\varepsilon_{k_m}^{VC} = \frac{\partial VC}{\partial k_m}\frac{k_m}{VC} = \frac{\partial \ln VC}{\partial \ln k_m}$,

where VC is now evaluated at the optimal quasi-fixed factor input vector $\mathbf{k}(\mathbf{w},\mathbf{r},\mathbf{y})$, which minimizes long run total cost such that $VC = VC(\mathbf{w};\mathbf{y};\mathbf{k}(\mathbf{w},\mathbf{r},\mathbf{y}))$.

Based on this relationship we can derive both a flexibility measure and its decomposed components - convexity, scope and scale effects - for the long run. Deriving (A1) with respect to *y* yields the following relationship:

(A.2) $\mathbf{TC_y} = \mathbf{VC_y} + \mathbf{K'_y} \cdot \mathbf{VC_k} + \mathbf{K'_y} \cdot \mathbf{r}$

where $\mathbf{K_y}$ is the *(M x J)* matrix of partial derivatives of the fixed factors with respect to different outputs y_j. In the long run equilibrium it holds: $\mathbf{r} = -\mathbf{VC_k}$, thus the expression in (A.2) leads to:

(A.3) $\mathbf{TC_y} = \mathbf{VC_y}$.

Differentiating $\mathbf{r} = -\mathbf{VC_k}$ with respect to $\mathbf{y}$ leads to: $0 = -\mathbf{VC_{ky}} - \mathbf{VC_{kk}}\mathbf{K_y}$, from which we can now obtain $\mathbf{K_y} = -\left(\mathbf{VC_{kk}}\right)^{-1}\mathbf{VC_{ky}}$. Differentiating both sides of (A3) with respect to $\mathbf{y}$ yields: $\mathbf{TC_{yy}} = \mathbf{VC_{yy}} + \mathbf{VC_{yk}}\mathbf{K_y}$. After replacing $\mathbf{K_y}$, the expression in (6) follows immediately: $\mathbf{TC_{yy}} = \mathbf{VC_{yy}} - \mathbf{VC_{yk}}\left(\mathbf{VC_{kk}}\right)^{-1}\mathbf{VC_{ky}}$.

APPENDIX B: PRIMAL LONG-RUN FLEXIBILITY MEASURE

Dual relationships between the variable cost and input distance function are based on both the first order conditions and the envelope theorem. Both procedures are applied to the following cost minimization problem:

(B.1) $VC(\mathbf{w},\mathbf{y},\mathbf{k}) = \min_{\mathbf{x}>0}\left\{\mathbf{w'x} : D(\mathbf{x},\mathbf{y},\mathbf{k}) \geq 1\right\}$.

Applying the envelope theorem leads to:

(B.2) $\mathbf{VC_y} = -VC(\mathbf{w},\mathbf{y},\mathbf{k}) \cdot \mathbf{D_y}(\mathbf{x}(\mathbf{w},\mathbf{y},\mathbf{k}),\mathbf{y},\mathbf{k})$,

(B.3) $\mathbf{VC_k} = -VC(\mathbf{w},\mathbf{y},\mathbf{k}) \cdot \mathbf{D_k}(\mathbf{x}(\mathbf{w},\mathbf{y},\mathbf{k}),\mathbf{y},\mathbf{k})$.

After differentiating both equations (B.2) and (B.3) with respect to **y** and **k**, using the first order condition $\mathbf{w} = VC(\mathbf{w},\mathbf{y},\mathbf{k}) \cdot \mathbf{D_x}(\mathbf{x},\mathbf{y},\mathbf{k})$ and some rearranging we obtain:

(B.4) $\mathbf{VC_{yy}} = VC\left(\mathbf{D_y}\mathbf{D_y}' - \mathbf{D_{yy}} + \mathbf{D_{yx}}\left[\mathbf{D_{xx}} + \mathbf{D_x}\mathbf{D_x}'\right]^{-1}\mathbf{D_{xy}}\right)$,

(B.5) $\mathbf{VC_{yk}} = VC\left(\mathbf{D_y}\mathbf{D_k}' - \mathbf{D_{yk}} + \mathbf{D_{yx}}\left[\mathbf{D_{xx}} + \mathbf{D_x}\mathbf{D_x}'\right]^{-1}\mathbf{D_{xk}}\right)$,

(B.6) $\mathbf{VC_{ky}} = VC\left(\mathbf{D_k}\mathbf{D_y}' - \mathbf{D_{ky}} + \mathbf{D_{kx}}\left[\mathbf{D_{xx}} + \mathbf{D_x}\mathbf{D_x}'\right]^{-1}\mathbf{D_{xy}}\right)$,

(B.7) $\mathbf{VC_{kk}} = VC\left(\mathbf{D_k}\mathbf{D_k}' - \mathbf{D_{kk}} + \mathbf{D_{kx}}\left[\mathbf{D_{xx}} + \mathbf{D_x}\mathbf{D_x}'\right]^{-1}\mathbf{D_{xk}}\right)$.

After substituting the relationships (B.4) - (B.7) into formula (6) we can derive the dual long-run measures for scope and convexity effects

(B.8)

$$\mathbf{TC_{yy}} = \left(\mathbf{1_I}' \cdot \mathbf{D_x}\right)^{-1}\Big[\mathbf{D_y}\mathbf{D_y}' - \mathbf{D_{yy}} + \mathbf{D_{yx}}\left[\mathbf{D_{xx}} + \mathbf{D_x}\mathbf{D_x}'\right]^{-1}\mathbf{D_{xy}} - \left(\mathbf{D_y}\mathbf{D_k}' - \mathbf{D_{yk}} + \mathbf{D_{yx}}\left[\mathbf{D_{xx}} + \mathbf{D_x}\mathbf{D_x}'\right]^{-1}\mathbf{D_{xk}}\right)$$
$$\left(\mathbf{D_k}\mathbf{D_k}' - \mathbf{D_{kk}} + \mathbf{D_{kx}}\left[\mathbf{D_{xx}} + \mathbf{D_x}\mathbf{D_x}'\right]^{-1}\mathbf{D_{xk}}\right)^{-1}\left(\mathbf{D_k}\mathbf{D_y}' - \mathbf{D_{ky}} + \mathbf{D_{kx}}\left[\mathbf{D_{xx}} + \mathbf{D_x}\mathbf{D_x}'\right]^{-1}\mathbf{D_{xy}}\right)\Big].$$