# Joint Estimation of Risk Preferences and Technology: Further Discussion

Feng Wu
Research Associate
Gulf Coast Research and Education Center
University of Florida

Zhengfei Guan
Assistant Professor
Gulf Coast Research and Education Center &
Food and Resource Economics Department
University of Florida

Selected Paper prepared for presentation at the Agricultural & Applied Economics Association's 2012 AAEA Annual Meeting, Seattle, Washington, August 12-14, 2012.

In a recent paper in this journal, Lence (2009) investigated the hypothesis whether it is possible to recover producers' risk preferences parameters from typical production data. To answer this question, the author designed a Monte Carlo experiment with risk calibrated to match historical farm data. In the experiment, optimal input data are generated from an expected direct utility maximization model. Employing such data and a flexible utility function, the author applied a GMM estimation method based on unconditional moment restrictions and jointly estimated risk preferences and production technology. Although the experiment is designed to "favor the likelihood of obtaining the good estimates of the risk-aversion structure", the results showed that risk preference parameters are estimated without reasonable precision. Therefore, the author challenged the decades long effort that pursues recovery of risk preferences in the literature and concluded that the structure of risk preferences is unlikely to be recovered from actual production data.

In this comment we discuss the reasons why the author was not able to recover the parameters in his simulation -- the unconditional moment restrictions may not globally, or can only weakly, identify the true parameter. This potential identification problem has often been found in macroeconomic models (see a survey in Stock, Wright, and Yogo (2002)). In particular, weak identification arises frequently in nonlinear models, which can break down conventional GMM procedures and make standard GMM point estimates unreliable. We tested for weak identification in Lence nonlinear GMM model and the results indicate presence of weak identification. So it is not that there is no sufficient information that causes the problem as the author concluded; it is a result of the identification problems in the setup of the estimation procedure in the paper. This study further recommends alternative procedures robust to the identification problems in non-linear models.

**GMM and Global Identification**

Economic models are often characterized by conditional moment restrictions in the underlying economic variables. The input decision making model gives rise to three conditional moment restrictions, which consists of production technology and two first-order conditions for input optimization (equations (10) and (11) in his paper). Assume

(1) $\phi(X_n, \theta) = [\varepsilon_{y,n}(\alpha)\ \ \varepsilon_{A,n}(\alpha_{A,}\gamma)\ \ \varepsilon_{B,n}(\alpha_{B,}\gamma)]'$,

where $X_n$ is a 7-dimensional vector $[y_n, W_{0,n}, p_n, r_{A,n}, r_{B,n}, x^*_{A,n}, x^*_{B,n}]'$, and $\theta$ is a 5-dimensional parameter vector $[\alpha_0, \alpha_A, \alpha_B, \gamma_0, \gamma_1]'$. Variables in $X_n$ represent output, initial wealth, price of output, prices of inputs *A* and *B*, optimal inputs *A* and *B*, respectively. $\alpha$ and $\gamma$ are parameters of production technology and utility function, respectively; the latter are the risk preferences parameters to recover. Lence's model is defined by the following conditional moment restrictions

(2) $E[\phi(X_n, \theta_0)|Z_n] = 0$,

where $\theta_0$ is the true parameter vector and $Z_n$ is the information set known at the time of decision making. The conditional moments in (2) are postulated to uniquely identify the parameter $\theta_0$ in his simulation experiment. Lence used the typical approach – a set of unconditional moment restrictions implied from the conditional ones – when estimating the parameters of interest:

(3) $E[\phi(X_n, \theta_0) \otimes \imath_n] = 0$,

where $\imath_n$ is the selected instruments set, a subvector of $Z_n$; then he used Hansen's (1982) generalized method of moments (GMM) estimator to estimate $\theta_0$ by minimizing quadratic form with respect to the unknown parameters:

(4) $\hat{\theta} = argmin_{\theta}[g(X_n, \imath_n, \theta)' V_N g(X_n, \imath_n, \theta)]$,

where $g(X_n, \imath_n, \theta) = 1/N \sum_{n=1}^{N} \phi(X_n, \theta) \otimes \imath_n$ and $V_N$ is a positive definite weighting matrix.This estimation procedure is often performed because of its computational attractiveness. However, the prerequisite for the validity of this unconditional GMM-based approach is the potential global identification assumption – the parameters identified in the conditional moment restrictions can be globally identified by the implied unconditional moment restrictions, i.e.,

(5) $E[\phi(X_n, \theta) \otimes \imath_n] = 0 \Rightarrow \theta = \theta_0$.

That is, the identification hinges on uniqueness of the solution to the unconditional moment conditions. But the assumption is often not satisfied. Dominguez and Lobata (2004) argued that the unconditional moment restrictions do not guarantee global identification of the parameters of interest. For example, in one extreme case the identifying set $\Theta_{\mathrm{I}} = \{\theta \in \Theta: E[\phi(X_n, \theta) \otimes \imath_n] = 0\}$ by the unconditional moment restrictions may not be a singleton $\Theta_{\mathrm{I}} = \{\theta_0\}$. As a result, the GMM objective function may have several global minima. In this case, the GMM method cannot identify the true parameters and thus leads to inconsistent estimates. Another less extreme case is that there exists a large set of $\Theta_{\mathrm{I}}$, under which $E[\phi(X_n, \theta) \otimes \imath_n]$ is fairly small while nonzero, so that parameters are only weakly identified. At all events, the conversion to unconditional moments from conditional moments may introduce an identification issue. The failure or near-failure of global identification in unconditional restrictions is quite common in nonlinear models regardless of whether the instruments $\imath_n$ are optimally chosen or not. Dominguez and Lobta (2004) provide some examples where (5) is not satisfied, one of which is as follows:

"Assume that the random variable $Y$ satisfies the simple nonlinear model$E(Y|X) = \theta_0^2 X + \theta_0 X^2$. Suppose that $\theta_0 = 5/4$ and that $V(Y|X)$ (the conditional variance of $Y$) is constant. Assume that the research properly specifies the model and, instead of an arbitrary instrument, she chooses the optimal instrument, given by $W_0 = 2\theta_0 X + X^2 \cdots$. … in practice the researcher just knows the form of the optimal instrument, given by $W = 2\theta X + X^2$. In this case the parameter $\theta_0$ is not identified again, since the equation model $E[(Y - \theta^2 X - \theta X^2)W] = 0$ is also satisfied for $\theta$ = -5/4 and for $\theta$ = -3 when $X$ follows an $N(1, 1)$ random variable."

Besides this problem, the unconditional GMM based approach results in efficiency loss because it does not utilize all information from the conditional moment restrictions (2) to estimate $\theta$. A general loss of information from (3) is due to it being an implication of (2); not equivalent (Dominguez and Lobta, 2004). The conditional version of the first-order condition of the optimization problem contains distributional information about future production and price and thus may be used to improve the precision of the estimates. In fact, some prior studies, e.g., Saha, Shumway, and Talpaz (1994), adopted an estimation procedure that is directly based on the conditional moment restrictions defining the parameters of interest. Such estimators preclude identification issues as well as loss of information arising from using unconditional moments. In Kumbhakar (2001, 2002a, and 2002b) effort was also made to derive closed-form conditional moment restrictions with functional form approximations when more flexible-utility functions are allowed for.

**GMM with Weak Identification**

The global identification problem in unconditional restrictions so often appears in the form of weak identification. In nonlinear GMM, empirical economists often confront that $E[\phi(Y_n,\theta)\otimes \imath_n]$ is very nearly 0 for a large set of $\theta$. This in turn implies that the GMM population objective function has large regions of plateaus that are close to its minimum value. Thus the objective function has only limited ability to identify among a large set of parameter values. In such circumstances $\theta$ can be thought of as being weakly identified.

In Lence's study, the GMM estimation yields extremely large point estimates which makes no sense from an economic standpoint. For example, the upper bound of the 95% CIs for risk aversion preference ($\gamma_1$) in most scenarios is very far from the calibrated value of the parameter and fairly distant even from high degrees of risk aversion found in some empirical applications.[1] Additionally, the estimates systematically overestimate the risk preference parameter in terms of the medians. The 95% CIs in most scenarios fail to contain the true risk preference parameter. In fact, these erratic results of conventional GMM procedures in nonlinear models have been well documented in the economics literature. For example, Hansen, Heaton, and Yaron (1996) examined GMM estimators of various consumption-based capital asset-pricing models (CCAPM) using a Monte Carlo design calibrated to match U.S. data. They found that the time nonseparability preference models would result in a large number of very large estimates of CRRA, just like Lence did, and the risk preference parameters are not estimated with any reasonable precision. They used two-step, iterative, or continuous-updating estimator, but none saves the CCAPM. Weak identification is considered a frequent cause of the breakdown of conventional GMM procedures. GMM is easily contaminated by weak identification and in turn leads to unreliable point estimates. Also, other problems of GMM under failure or near-failure of identification condition are well documented. For instance, the sampling distribution of GMM estimators is not normal under these circumstances, that is, conventional Gaussian asymptotic theory would provide a very poor approximation to the actual sampling distribution of estimators; and hypothesis tests of parameter values and tests of overidentifying restrictions can exhibit substantial distortions so that any inference based on it is unreliable (Stock, Wright, and Yogo, 2002).

**Detection of Weak Identification**

Whether the imprecise parameter estimates from unconditional moments are resulted from the identification failure can be tested. Although various tools are now available for detecting and handling weak identification in linear IV models, the development of a reliable statistic to detect weak identification in nonlinear GMM still remains an open challenge. Stock and Wright (2000) and Stock, Wright, and Yogo (2002) pointed to several symptoms of weak identification that can be readily detected in empirical work. If those symptoms are present, a diagnosis of weak identification is appropriate. We will examine whether the GMM estimators exhibit those symptoms in Lence unconditional moment restrictions. We replicate Lence Monte Carlo experiment focusing on the case which combines CRRA ($\gamma_0 = 0$, and $\gamma_1 = 3$) with high uncertainty regarding output and price shocks. In the replication, the only difference is the probability distribution of output shocks. Since historical corn yields for Iowa farms are not available to the reader, we simply assume that output risk is following a lognormal distribution with the same mean and standard deviation as calibrated by Lence.

Stock and Wright (2000) developed asymptotic distribution theory for GMM estimators when some or all of $\theta$ are weakly identified. In the proof of theorem 1, they remarked that if the weakly identified parameters $\alpha$ are known, the well identified parameter vector, i.e., $\hat{\beta}$, a subvector of $\theta$, would be $\sqrt{T}$-consistent. But the weakly identified parameters either could not be consistently estimated or converge at a very slow rate even if $\beta_0$ were known. This would further impart a nonzero bias to the estimates of $\beta$. Before checking the symptom, we examine a priori which risk preference parameter in Lence model is weakly identified.[2] As shown in Lence estimation results, $\hat{\gamma}_0$ is more widely dispersed than $\hat{\gamma}_1$ in terms of the 95% CIs implying $\gamma_0$ more difficult to pin down. Since $W_{0,n}$ is of an even larger order than $\gamma_0$, the effect of an even large change in $\gamma_0$ on $\left(\gamma_0 + p_n y_n - r_{A,n} x_{A,n}^* - r_{B,n} x_{B,n}^* + W_{0,n}\right) / \left(\gamma_0 + W_{0,n}\right)$ would be so small that the optimization input restrictions are flat over a wide range of $\gamma_0$. Therefore, $\gamma_0$ enters the model weakly and can be considered weakly identified. The test is to perform estimation of

Lence restricted utility specifications by fixing either $\gamma_0$ or $\gamma_1$ at its true value or other reasonable values and estimating the other. The first test is to fix $\gamma_0$ at its true value 0 and estimate $\gamma_1$. We then set $\gamma_0 = -12$ to see how much bias is imparted to $\hat{\gamma}_1$ from the inconsistency of $\gamma_0$ . Finally, we fix $\gamma_1$ at its true value 3 and check whether $\gamma_0$ can be consistently estimated. Tables 1, 2, and 3 report results from these three CRRA restricted utility specifications, respectively. Besides for sample sizes of 100, 500 and 1000, we also report the median and the 95% CIs of the estimates for sample size of 10000 to see whether one parameter converges to its true value when the other is known.

Different from the estimation results in Lence's restricted utility estimation specifications (see table 6 and 7, Lence (2009)), our parameter estimates $\gamma_1$ converge to its true value across sample sizes if $\gamma_0$ is fixed at its true value 0, as shown in table 1. This finding is in agreement with the conclusion in Theorem 1 of Stock and Wright (2000). Furthermore, $\gamma_1$ estimates have distributions that are much more concentrated around the true parameter value than Lence's estimates do. Meanwhile, we did not observe that the medians systematically overestimate the true value of $\gamma_1$. Although the 95% CIs for small samples (100-observation samples) are wider and a little skewed to the right, the median of $\hat{\gamma}_1$ is fairly close to the true value. The precision of parameter estimates, as measured by the width of the 95% CIs, increases with sample size. This suggests that the variability in the price and output shocks as specified in the experiment is adequate to identify the level of relative risk aversion.

In sum, with prior knowledge of the parameter $\gamma_0$, parameter $\gamma_1$can be consistently estimated. However, $\gamma_0$ cannot be estimated precisely if $\gamma_1$ is fixed at its true value as reported in table 2. The medians for $\hat{\gamma}_0$ have larger bias and the dispersion is substantially wider than that of $\gamma_1$. Even in 10,000-observation samples, the dispersion of $\hat{\gamma}_0$ is considerably large as measured by the 2.5% and 97.5% quantiles. However, $\gamma_0$ does seem to converge albeit at a slow rate.[3] Finally, the bias of $\hat{\gamma}_0$ would negatively impact $\hat{\gamma}_1$. Table 3 reports $\gamma_1$ estimates when we set $\gamma_0$ at -12, instead of its true value 0. Although the 95% CIs of $\hat{\gamma}_1$ cover its true value in the small sample cases, $\hat{\gamma}_1$ is systematically underestimated.

The second symptom for weak identification is that two-step estimates are sensitive to instrument choice when simultaneously estimating the weak and well identified parameters. We again perform GMM estimation with different sets of instruments for the flexible-utility specification. Tables 4 report statistics summarizing the properties of the estimates when instrument sets $[1, W_{0,n}, p_{0,n}, r_{A,n}, r_{B,n}, x^*_{A,n}, x^*_{B,n}]$. Similar estimate statistics can be reached when $[1, W_{0,n}, p_{0,n}, r_{A,n}, r_{B,n}]$ are used, respectively. The optimal inputs are excluded from the second instrument set. The 95% CIs do cover the true value, but are widely dispersed. In our simulation, we observed that, when the sample size equals 100, some estimations produced some odd results with very large values. Further examination shows that the gradient of moment conditions in those cases has no full rank and the weighting matrix is close to singular and the algorithm actually did not converge, which might well have occurred in Lence's simulation. Our reported statistics exclude the estimates in those non-convergence cases. Further comparing estimates one-by-one (pairwise), we find a number of interesting phenomena. First, in many cases, the estimate of $\gamma_0$ is substantially different in terms of its sign and magnitude when different instruments are used for an identical sample set. Second, convergence not achieved with one instrument set could be achieved with the other set, and vice versa. Finally, inferences based on the $J$ statistic often differ.

The third symptom is that point estimates of risk preference parameters and inference based on GMM estimators are substantially affected by the procedure for constructing the weighting matrix $V$. Besides 2-step GMM estimator, we use continuous-updating estimator in which the weighting matrix is changed for each hypothetical parameter value (Hansen, Heaton, and Yaron, 1996). These two estimators are asymptotically equivalent under the conventional GMM theory. Continuous-updating GMM estimation results are summarized in table 5 while 2-step GMM results are in table 4. The median bias of $\gamma_0$ for the continuous-updating estimator is much greater, whereas the medians of $\gamma_1$ for the 2-step and continuous-updating estimators are similar. The distribution for the continuous-updating estimator is also more dispersed, as evidenced by the larger spread. This is consistent with findings in other Monte Carlo studies that the continuous-updating estimates have much fatter tails and tend towards arbitrarily large values (Stock and Wright, 2000). We also observed that inference from the over-identification tests under two equivalent GMM estimators are often in conflict.

The symptoms exhibited in the simulation suggest that weak identification is present, in which case, estimation results should be interpreted with caution, and more robust approach may be needed. This is particularly important in empirical estimation when the sample size is small.

**An Alternative Approach**

In addition to the issues discussed above, conventional asymptotic normality will provide a poor approximation to the sampling distribution of GMM estimators and the conventional Wald test will not in general be valid. Researchers have made much progress proposing new approaches to address these issues. The proposed approaches all give up point estimation and directly construct tests of hypotheses concerning the parameters of interest where the asymptotic distribution of the test statistic is not affected by the identification issue (Wright, 2010). Among them, Stock and Wright (2000) developed test statistics for constructing confidence sets immune to weak identification. They referred to these confidence sets as S-sets and found that the S-sets lead to substantially different conclusion than the conventional GMM analysis. The following introduces the way to construct S-sets.

As we know, the GMM continuous-updating estimator $\hat{\theta}$ minimizes the objective function $S_N$ over θ ∈ Θ:

(6) $S_N(\theta) = g(Y_n, \imath_n, \theta)' V_N(\theta) g(Y_n, \imath_n, \theta)$

where $V_N(\theta)$ is a weighting matrix, continuously evaluated at the parameter values used for the moments. Stock and Wright (2000) showed that at the true values of the parameters, the objective function has a standard asymptotic $\chi^2$distribution if an efficient weighting matrix is used:

(7) $S_N(\theta_0) \overset{d}{\to} \chi_k^2,$

where $k$ is the number of moment restrictions. This theorem provides a straightforward method for constructing asymptotically valid hypothesis tests and confidence set. For example, to perform an asymptotically valid test of the hypothesis $\theta = \theta_0$, reject if $S_N(\theta_0)$ exceeds the appropriate $\chi_k^2$ critical value. Joint confidence sets for whether well identified or weakly-identified parameters are constructed by directly comparing $S_N(\theta)$, evaluated over the entire parameter space, with the chi-squared critical value. For example, an asymptotic 100(1-$r$)% confidence set is such as $\{\theta_0 : S_N(\theta_0) \le \chi_{k,r}^2\}$, where $\chi_{k,r}^2$ is the $100 * r$% critical value of the $\chi_k^2$ distribution. Also, they derived another theorem (see Theorem 3 in Stock and Wright (2000)) to construct CIs for the weakly identified parameters. Assume that weakly-identified parameter subvector $\alpha$ is $g$-dimensional. Then

(8) $S_N\big(\alpha_0, \hat{\beta}(\alpha_0)\big) \overset{d}{\to} \chi_{k-g}^2$

The theorem holds under the condition that $\beta$ is well identified. Likewise, a confidence set for $\alpha$ can be constructed by searching the acceptance regions based on the concentrated objective function $S_N(\alpha_0, \hat{\beta}(\alpha_0))$.

Disagreement between inferences based on S-sets and conventional GMM in the case of weak identification has been documented in the literature (e.g., Stock and Wright, 2000; Yogo, 2004). It is important that the weak identification be tested in empirical studies, especially in small sample.

**Concluding Remarks**

Through a Monte Carlo Study of an input optimization model, Lence found significant biases risk preference parameter estimates in a flexible utility specification. He consequently concluded that "typical production data are unlikely to allow identification of the structure of risk aversion". Our simulation results showed that recovery of risk preference structure is possible. We found that weak identification in nonlinear models can make it difficult for the conventional GMM procedure to pin down the true value of some parameters, in our case the parameter $\gamma_0$. Additionally, unlike Lence, we found the relative risk aversion parameter $\gamma_1$ can be well identified in one-parameter utility function (e.g., when $\gamma_0$ equals zero).

The inadequacy of conventional asymptotics in the case of weak identification requires alternative, robust approaches.[4] The S-sets proposed by Stock and Wright (2000) are constructed. This approach yields robust inference regardless of whether parameters are well or weakly identified. Although point estimates are often preferred by researchers to be reported, it is helpful to have a diagnostic to examine whether the identification is sufficiently strong in nonlinear models when using GMM estimation.

[1] For example, Kocherlakota (1990) estimated the CRRA to be as high as 13.7, which is much greater than the one (i.e., 3) set by Lence.

[2] All technology parameters in the (log)linear model are well identified as evidenced by the consistent estimates in the simulation. Note that the bias in alpha0 resulted from the logarithmic transformation of production function and can be corrected by adding the expectation of log normally distributed error.

[3] The rate of convergence in weak identification is the square-root T as assumed by Stock and Wright (2000). If the convergence rate is slower, Antoine and Renault (2009) proposed a framework which ensures that GMM estimators of all parameters are consistent.

[4] Stock, Wright, and Yogo (2002) gave a brief survey of the literature on detecting weak identification and on procedures that are robust to weak identification.

**Reference:**


Antoine, B., and E. Renault. 2009. "Efficient GMM with Nearly-Weak Instruments." *Econometrics Journal* 12: 135-71.

Dom ńguez, M. A., and I. N. Lobato. 2004. "Consistent Estimation of Models Defined by Conditional Moment Restrictions." *Econometrica* 72:1601-15.

Hansen, L. P., J. Heaton, and A. Yaron. 1996. "Finite Sample Properties of Some Alternative GMM Estimators." *Journal of Business & Economic Statistics* 14: 262 – 80.

Kocherlakota, N. R. 1990. "On Tests of Representative Consumer Asset Pricing Models." *Journal of Monetary Economics* 26: 285-304.

Kumbhakar, S.C. 2001. "Risk Preferences under Price Uncertainties and Production Risk." *Communications in Statistics—Theory and Methods* 30: 1715–35.

——. 2002a. "Risk Preference and Productivity Measurement under Output Price Uncertainty." *Empirical Economics* 27:461–72.

——. 2002b. "Specification and Estimation of Production Risk, Risk Preferences and Technical Efficiency." *American Journal of Agricultural Economics* 84:386–90.

Lence, S. H. 2009. "Joint Estimation of Risk Preferences and Technology: Flexible Utility or Futility?" *American Journal of Agricultural Economics* 91:581-98.

Stock, J. H., and J. H. Wright. 2000. "GMM with Weak Identification." *Econometrica* 68: 1055–96.

Stock, J. H., J. H. Wright, and M. Yogo. 2002. “A Survey of Weak Instruments and Weak Identification in Generalized Method of Moments.” *Journal of Business and Economic Statistics* 20: 518-29.

Saha, A., C. R. Shumway, and H. Talpaz. 1994. “Joint Estimation of Risk Preference Structure and Technology Using Expo-Power Utility.” *American Journal of Agricultural Economics* 76:173–84.

Wright, J. H. 2010. “Testing the Adequacy of Conventional Asymptotics in GMM.” *Econometrics Journal* 13: 205-217.

Yogo, M. 2004. “Estimating the Elasticity of Intertemporal Substitution When Instruments are Weak.” *Review of Economics and Statistics* 86: 797-810

**Table 1. GMM Parameter Estimates for Restricted-Utility Estimation Specification ($\gamma_0 = 0$)**

| | | Parameter Estimates | | | |
|---|---|---|---|---|---|
| Risk | Sample | Utility | Technology | | |
| Structure | Size | $\hat{\gamma}_1$ | $\hat{\alpha}_0$ | $\hat{\alpha}_A$ | $\hat{\alpha}_B$ |
| CRRA | 100 | 4.529 | 2.862 | 0.203 | 0.609 |
| | | (0.52,11.60) | (2.59,3.12) | (0.19,0.22) | (0.56,0.66) |
| CRRA | 500 | 3.393 | 2.862 | 0.201 | 0.602 |
| | | (1.86,5.26) | (2.75,2.98) | (0.19,0.21) | (0.58,0.62) |
| CRRA | 1,000 | 3.217 | 2.863 | 0.200 | 0.601 |
| | | (2.07,4.48) | (2.78,2.95) | (0.20,0.21) | (0.59,0.62) |
| CRRA | 10,000 | 2.998 | 2.866 | 0.200 | 0.600 |
| | | (2.63,3.39) | (2.84,2.89) | (0.20,0.20) | (0.60,0.60) |

Note: for each parameter, the table reports the median and the 2.5% and 97.5% quantiles (within parentheses) for sample sizes of 100, 500, 1,000, and 10,000, respectively. CRRA risk structures correspond to [$\gamma_0$, $\gamma_1$] equal to [0, 3].

**Table 2. GMM Parameter Estimates for Restricted-Utility Estimation Specification ($\gamma_1 = 3$)**

| Risk Structure | Sample Size | Parameter Estimates: Utility | Technology | | |
|---|---|---|---|---|---|
| | | $\hat{\gamma}_0$ | $\hat{\alpha}_0$ | $\hat{\alpha}_A$ | $\hat{\alpha}_B$ |
| CRRA | 100 | -7.319 | 2.885 | 0.200 | 0.600 |
| | | (-19.60,98.13) | (2.62,3.18) | (0.19,0.21) | (0.56,0.64) |
| CRRA | 500 | -2.180 | 2.868 | 0.200 | 0.600 |
| | | (-11.46,24.51) | (2.76,2.99) | (0.19,0.21) | (0.58,0.62) |
| CRRA | 1,000 | -1.150 | 2.867 | 0.200 | 0.600 |
| | | (-8.90,17.50) | (2.79,2.95) | (0.20,0.20) | (0.59,0.61) |
| CRRA | 10,000 | -0.002 | 2.866 | 0.200 | 0.600 |
| | | (-3.80,4.65) | (2.84,2.89) | (0.20,0.20) | (0.60,0.60) |

Note: for each parameter, the table reports the median and the 2.5% and 97.5% quantiles (within parentheses) for sample sizes of 100, 500, 1,000, and 10,000, respectively. CRRA risk structures correspond to [$\gamma_0$, $\gamma_1$] equal to [0, 3].

**Table 3. GMM Parameter Estimates for Restricted-Utility Estimation Specification ($\boldsymbol{\gamma_0 = -12}$)**

| Risk Structure | Sample Size | Parameter Estimates: Utility $\hat{\gamma}_1$ | Technology $\hat{\alpha}_0$ | Technology $\hat{\alpha}_A$ | Technology $\hat{\alpha}_B$ |
|---|---|---|---|---|---|
| CRRA | 100 | 2.932 | 2.870 | 0.198 | 0.605 |
| | | (0.14,7.37) | (2.60,3.17) | (0.19,0.22) | (0.56,0.66) |
| CRRA | 500 | 2.213 | 2.874 | 0.200 | 0.600 |
| | | (1.02,3.44) | (2.76,3.00) | (0.19,0.21) | (0.58,0.62) |
| CRRA | 1,000 | 2.067 | 2.875 | 0.200 | 0.599 |
| | | (1.22,2.90) | (2.80,2.96) | (0.20,0.20) | (0.58,0.61) |
| CRRA | 10,000 | 1.878 | 2.879 | 0.200 | 0.600 |
| | | (1.59,2.14) | (2.86,2.90) | (0.20,0.20) | (0.59,0.60) |

Note: for each parameter, the table reports the median and the 2.5% and 97.5% quantiles (within parentheses) for sample sizes of 100, 500, 1,000, and 10,000, respectively. CRRA risk structures correspond to [$\gamma_0$, $\gamma_1$] equal to [0, 3].

**Table 4. GMM Parameter Estimates for Flexible-Utility Estimation Specification with Instruments [1, $W_{0,n}, p_{0,n}, r_{A,n}, r_{B,n}, x^*_{A,n}, x^*_{B,n}$]**

| | | Parameter Estimates | | | | |
|---|---|---|---|---|---|---|
| Risk | Sample | Utility | | Technology | | |
| Structure | Size | $\hat{\gamma}_0$ | $\hat{\gamma}_1$ | $\hat{\alpha}_0$ | $\hat{\alpha}_A$ | $\hat{\alpha}_B$ |
| CRRA | 100 | 0.935 | 6.182 | 2.851 | 0.204 | 0.612 |
| | | (-17.40,396.73) | (0.19,60.32) | (2.58,3.15) | (0.19,0.22) | (0.56,0.66) |
| CRRA | 500 | 0.351 | 3.957 | 2.857 | 0.201 | 0.603 |
| | | (-15.36,231.10) | (1.19,22.41) | (2.74,2.98) | (0.19,0.21) | (0.58,0.63) |
| CRRA | 1,000 | 0.073 | 3.520 | 2.861 | 0.200 | 0.602 |
| | | (-13.66,119.4) | (1.51,12.58) | (2.78,2.95) | (0.20,0.20) | (0.59,0.62) |
| CRRA | 10,000 | 0.001 | 3.011 | 2.867 | 0.200 | 0.600 |
| | | (-6.62,11.08) | (2.37,4.19) | (2,84,2.89) | (0.20,0.20) | (0.60,0.60) |

Note: for each parameter, the table reports the median and the 2.5% and 97.5% quantiles (within parentheses) for sample sizes of 100, 500, and 1,000, respectively. CRRA risk structures correspond to [$\gamma_0$, $\gamma_1$] equal to [0, 3].

**Table 5. GMM Parameter Estimates for Flexible-Utility Estimation Specification with The Continuous-Updating Estimator**

| Risk Structure | Sample Size | Parameter Estimates | | | | |
|---|---|---|---|---|---|---|
| | | Utility | | Technology | | |
| | | $\hat{\gamma}_0$ | $\hat{\gamma}_1$ | $\hat{\alpha}_0$ | $\hat{\alpha}_A$ | $\hat{\alpha}_B$ |
| CRRA | 100 | 2.241 | 6.198 | 2.838 | 0.204 | 0.613 |
| | | (-17.70,707.15) | (0.30,102.10) | (2.53,3.15) | (0.19,0.23) | (0.56,0.67) |
| CRRA | 500 | 4.668 | 3.874 | 2.856 | 0.201 | 0.603 |
| | | (-16.07,316.40) | (1.11,24.22) | (2.74,2.99) | (0.19,0.21) | (0.58,0.62) |
| CRRA | 1,000 | 2.927 | 3.385 | 2.863 | 0.200 | 0.601 |
| | | (-14.75,132.77) | (1.36,13.56) | (2.78,2.95) | (0.20,0.21) | (0.59,0.62) |

Note: for each parameter, the table reports the median and the 2.5% and 97.5% quantiles (within parentheses) for sample sizes of 100, 500, and 1,000, respectively. CRRA risk structures correspond to $[\gamma_0, \gamma_1]$ equal to [0, 3]. The set of instruments contains $[1, W_{0,n}, p_{0,n}, r_{A,n}, r_{B,n}, x^*_{A,n}, x^*_{B,n}]$.