A Semiparametric Varying-Coefficient Model of Monotone Auction Bidding Processes

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Introduction

We propose an econometric model of the price processes of online auctions. Since auction bids monotonically increase within individual auctions but can differ considerably across auctions, we construct a monotone series estimator with a common relative price growth curve and auction-specific slopes. Furthermore because the impacts of auction-specific attributes may evolve along the course of auctions, we employ a varying coefficient approach to accommodate these possibly time-varying effects.

We present an algorithm to solve the proposed partially linear panel model with a nonparametric monotone component. We apply the proposed model to eBay auctions of a Palm PDA. The estimates capture closely the overall pattern of online auction price processes and are robust against alternative specifications.

The Model

Suppose we observe bid histories of J similar auctions. Assuming some common structure among them, we can estimate the common structure in question by pooling bid histories of these auctions. Our data now consist of an unbalanced panel of bids. To account for unobserved persistent auction-specific effects, we assume that each auction has an individual effect that is constant over time. Let $P_j(t)$ be the price process of auction j. Assuming for now that all auctions share a common price growth process, we can express the price process of auction j as the following,

$$P_j(t) = a + \beta_j m(t) + \boldsymbol{x'_j \gamma}(t) + \eta_j + \varepsilon_{j,t},$$

where $\boldsymbol{x_j}$ is a vector of attributes for auction j, η_j is an auction-specific individual effect, and $\varepsilon_{j;t}$ is an error with mean zero and finite variance, which is iid across auctions and over time. The model assumes a common relative growth curve with a auction-specific slope, and time varying effects of auction-specific attributes.

Gathering the specifications of various components discussed above, we write our model as follows:

$$\begin{aligned} p_{i,j} &= \alpha + \sum_{j=1}^{J} \beta_j D_j m(t_{i,j}) + x'_j \gamma(t_{i,j}) + \eta_j + \varepsilon_{i,j} \\ &= \alpha + \sum_{j=1}^{J} \beta_j D_j \int_0^{t_{i,j}} \exp\left(\sum_{k=1}^{K} c_k \phi_k(u)\right) du + \sum_{s=1}^{S} \sum_{k=0}^{K_s} x_{j,s} d_{s,k} \psi_{s,k}(t_{i,j}) + \eta_j + \varepsilon_{i,j}, \end{aligned}$$

where $x_s = \{x_{j,s}\}_{s=1}^{S}, j = 1, \ldots, J.$

Data

our empirical investigation of the price processes of online auctions, we use a data set of eBay auctions of Palm M515 Personal Digital Assistant (PDA). At the time of data collection, these PDAs were popular items on eBay and had a market value of $249, according to the popular online store Amazon.com.

All auctions examined in this study are hard-closed with a duration of seven days. To reduce external sources of variation, we include only auctions of completely new items with no added-on features, having at least 10 bids, and where the seller did not set a secret reserve price. Every auction in our data resulted in a sale. All auctions were listed on eBay between March 14th and May 25th of 2003 and are transacted in US dollars. The number of auctions is 125 and the total number of bids is 3,351.

For each auction, we observe the sequence of bids, their arrival times and auction-specific characteristics including seller's rating and opening bids.

We only include auctions of brand-new Palm M515 with no add-on features, the final prices exhibit a substantial dispersion, ranging from $177 to $250. The distribution of the final prices is centered around $230 and approximately symmetric.

Estimation Results

We use power series to parameterize the relative price growth curve $m(\cdot)$ and time varying coefficients in model (7). For simplicity, we use a power series of degree two to model the the varying coefficients of opening bids and seller's ratings. As for the price growth curve $m(\cdot)$, we use the method of generalized cross validation (GCV) to determine the penalty coefficient λ and the specification of $m(\cdot)$ simultaneously in a data driven manner. Our preferred model suggested by the GCV specifies the the shape polynomial for $m(\cdot)$. The auction-specific individual effects and slopes are modeled as random effects and fixed effects respectively. Since the large sample properties of the proposed estimator can be rather complicated, we use bootstrap methods for inferences. The bootstrap procedure is repeated for 300 times.

The estimated results are reported below, where $x_{1;j}$ is the logarithm of seller's rating and $x_{2;j}$ is the opening bid, j = 1, 2, . . . , J. Bootstrap standard errors are reported in parenthesis below their corresponding coefficients.

$$\hat{P}_j(t) = \underset{(9.52)}{-25.71} + \sum_{j=1}^{J} \hat{\beta}_j D_j \hat{m}(t) + (\underset{(1.39)}{4.63} - \underset{(3.16)}{10.99}t + \underset{(0.26)}{1.38}\, t^2) x_{1,j} + (\underset{(0.13)}{1.10} - \underset{(0.15)}{0.31}\, t + \underset{(0.01)}{0.03}\, t^2) x_{2,j}, \qquad (9)$$

where

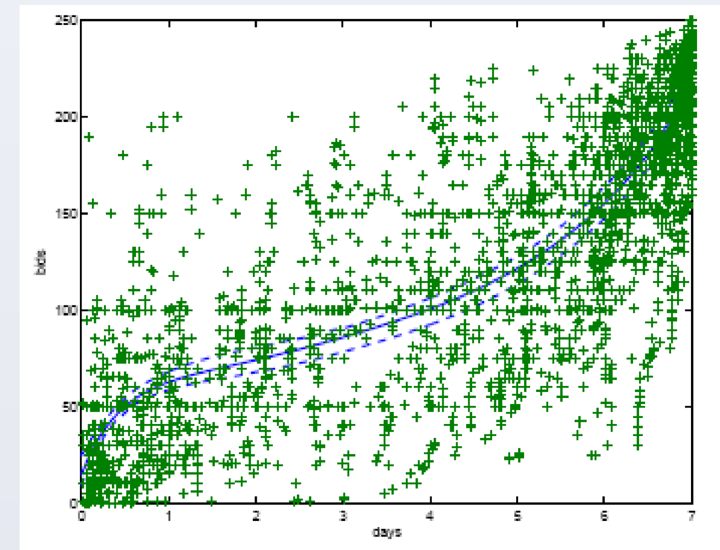
$$\hat{m}(t) = \int_0^t \exp(-\underset{(0.175)}{1.655}\, u + \underset{(0.159)}{0.784}\, u^2 - \underset{(0.039)}{0.193}\, u^3 + \underset{(0.003)}{0.016}\, u^4) du.$$

Results

Figure 1 plots the estimated price process (represented by a solid line) evaluated at sample averages of auction attributes and the average slope for $m(\cdot)$ across all auctions. In other words, the plotted price curve is constructed as,

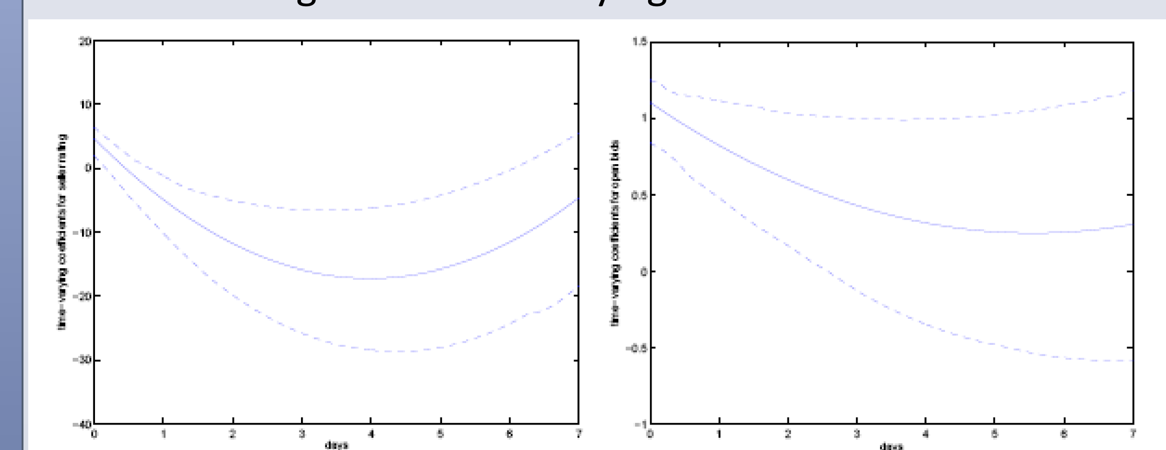
$$\hat{P}(t) = -25.71 + \bar{\hat{\beta}} \hat{m}(t) + (4.63 - 10.99t + 1.38t^2)\bar{x}_1 + (1.10 - 0.31t + 0.03t^2)\bar{x}_2,$$

Figure 1: Estimated price process



We next examine the estimated time-varying coefficients for the two auction-specific attributes. The results are plotted in Figure 2, in which the estimated curves are represented by solid lines and bootstrap 95% confidence bands by dash lines. Both coefficients are estimated with high precision and show distinct time-varying patterns. Regarding seller's rating, the initial effect is positive. For most of the middle duration of the auctions, its effects are shown to be negative.

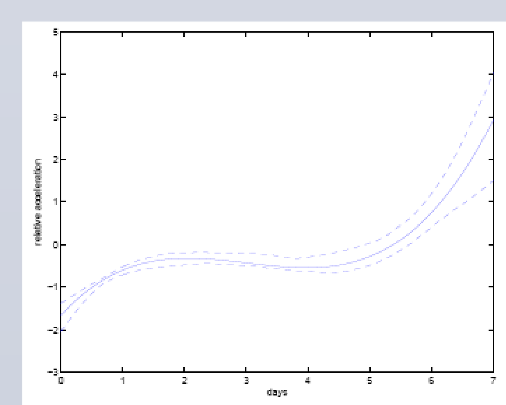
Figure 2: Time-varying coefficients



(a) Time-varying coefficients for seller's rating (b) Time-varying coefficients for opening bids

Next recall that the relative acceleration of price process, $m''(t)/m'(t)$, is captured by $w'(t)$, where $w(\cdot)$ is given in (4). Figure 3 plots the estimated $w'(\cdot)$. The estimated curve is represented by a solid line, with bootstrap 95% confidence bands in dash lines. It is seen that this relative acceleration curve is estimated with high precision.

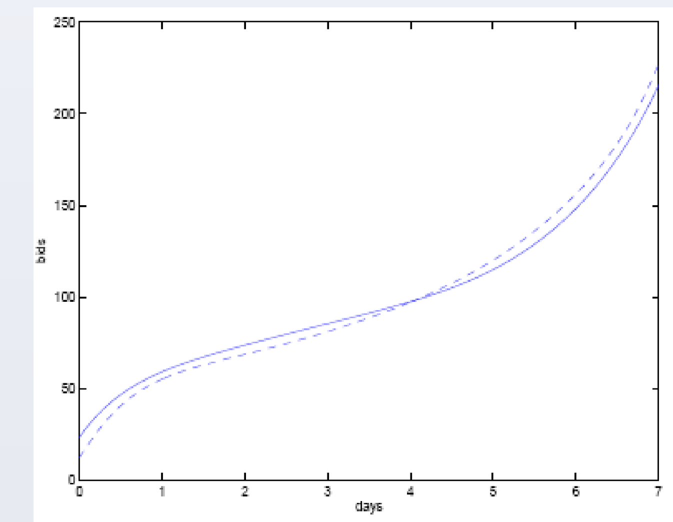
Figure 3: Estimated relative acceleration of price process



SPECIFICATION AND ROBUSTNESS CHECKS

The estimated price processes based on $p(t)$ (solid) and $q(t)$ (dash) are reported in Figure 5 below. It is seen that our nonparametric models adapt to the transformation of the dependent variable and therefore the estimated price process is barely affected by the proposed monotone transformation of the dependent variable.

Figure 4: Estimated price process (solid: price; dash: log price)



Conclusions

In this study we note some important and economically interesting features of the price dynamics of online auctions. Based on our examination of online auction bidding process and guided by economic theories, we present a panel model of online auction price process that has the following features: (a) a common monotone price growth curve with a multiplicative auction-specific slope; (b) an additive auction-specific effect; (c) time varying effects for auction-specific attributes. A monotone nonparametric estimator is used for the common price growth process. The auction-specific slopes and intercepts are modeled using panel data methods. A varying coefficient representation is used to model the time varying effects of auction attributes. Lastly, a penalized nonlinear least square estimator is proposed to estimate the coefficients and select the smoothing coefficients simultaneously. We applied the proposed model to eBay auction data of a Palm PDA. The results are shown to capture the overall pattern of online auction data closely.

The proposed model and estimator can be useful for researchers interested in modeling online auction price processes and auction predictions. Future work that examines the price process and bid arrival process jointly will be of interest.