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# Research Benefits in a Multimarket Setting: A Review

Julian M. Alston \*

The literature on measuring the size and distribution of returns to research has paid increasing attention of late to questions that require a multimarket framework. These questions include the distribution of benefits among stages of a multistage process or among factors of production (i.e. the vertical incidence) and the distribution across different markets either for the same product in different places or for different products (the horizontal incidence). The latter may be regarded as including quality change which has attracted recent attention. In much of the literature, a linear elasticity modeling approach has been used to obtain measures of the consequences of research-induced supply shifts for prices and quantities which, in turn, are used to evaluate the size and distribution of welfare effects. This review summarizes the state of the art of that work. It is a selective treatment, beginning with a simple basic model of research benefits and proceeding to consider increasingly complicated problems of vertically and horizontally related markets. The recurring theme is the theoretical and empirical questions surrounding (a) how to represent technical changes, and model their effects on equilibrium displacements, in a theoretically consistent manner, and (b) how to translate those equilibrium displacement effects into welfare measures.

# 1. Introduction

There is a significant interest among agricultural economists and in agricultural research funding agencies in obtaining evidence on the size and distribution of the returns to agricultural research and development (R&D). This interest is reflected in a large number of studies of the ex ante or ex post returns to agricultural R&D. In some studies the primary interest is in the total benefits from research-induced technical change while the distribution of those benefits is of secondary interest or no interest at all. In many contexts, however, the distributional consequences of technical change are of significant — if not paramount — interest. Alternative investments in R&D may differ in important ways in terms of the ultimate incidence of benefits and costs among different producing and consuming groups, income classes, regions, internationally and domestically. In some settings

these differences are important to policy-makers or other groups involved in making R&D investment decisions.

A simple economic surplus model of research benefits has been used extensively to show the size of total benefits and the distribution of those benefits between "producers" and "consumers" (for a discussion of much of the relevant literature, see Norton and Davis 1981). This article extends that analysis to consider the size and distribution of benefits in the context of multiple factors and multiple product markets. To a great extent this is simply an effort to disaggregate more finely the measures of benefits that are obtained from the basic model (such as to allocate the "producer surplus" among individual productive factors as quasi-rents). At the same time, however, the extension of the analysis into multiple market settings also may lead to a different perspective on the implications of technical change for the total economy as well as from the viewpoints of particular interest groups. While the focus of this article is on benefits from research, the models used are drawn from the more general literature on modeling market displacements and welfare economics. By

\*Department of Agricultural Economics, University of California, Davis CA, 95616, U.S.A. This review is a revision of my ACIAR/ISNAR project paper no. 18, "Research Benefits in a Multimarket Setting." The work for that paper was partially supported from funding made available to the International Service for National Agricultural Research (ISNAR), by Deutsche Gesellschaft für Technische Zusammenarbeit (GTZ) under contract no. 87.7860.7-01.100/60034878. I thank ISNAR for this support.

A number of people provided helpful comments on drafts or helped in other ways. Some of them are responsible for substantive parts of some of the content of the paper. Without implicating them any further or more specifically than this I thank Jim Chalfant, John Constantine, Geoff. Edwards, John Freebairn, Garth Holloway, Brian Hurd, Cathy Kling, Alex McCalla, John Mullen, George Norton, Nancy Ottum, Phil Pardey, Dick Perrin, Wally Thurman, Dave Vidaver and Mike Wohlgenant.

Review co-ordinated by Frank Jarrett.

the same token, many of the results could be interpreted more generally as applying to market equilibrium displacements caused by government policies or other events.

The discussion proceeds as follows. First, the standard model of research benefits is presented in summary form with a discussion of the total benefits and their distribution between "producer surplus" and "consumer surplus". That model is then extended into multi-factor settings, multi-markets for a single product, and (with less success) multi-product settings.

The first set of extensions considers multiple factors. The measures of benefits in the standard model are disaggregated between final consumers and two factors of production and the cases of fixed and variable proportions between those two factors are compared. This set of results may also be interpreted as representing two stages of a multistage production process. Next, those results are extended to the case of three factors of production (or three stages of production) and then from there to an arbitrary number of factors. The second set of extensions to the standard model considers multiple markets for a single product. This permits an examination of the international (or inter-regional) incidence of benefits and costs from researchinduced technical change, the incidence between adopters and non-adopters, and implications of international "leakage" of technology. The third set of extensions considers multiple products. We begin with the case where the products are independent in production but related in consumption and go from there to consider jointness in production and quality change.

Some important simplifying assumptions are retained throughout. First, supply and demand curves are treated as being linear and shifting in parallel as a consequence of R&D-induced technical changes. Second, a static (single period) model is used and the important dynamic issues are put aside. Third, competitive market clearing is imposed. Fourth, Harberger's (1971) "three postulates" are invoked so that the standard surplus measures may be used as measures of welfare change<sup>1</sup>. These assumptions have been shown to have major implications in the standard model of research benefits, and no

doubt these concerns carry over — perhaps more importantly in some cases — into the multi-market analysis.

#### 2. The Basic Model

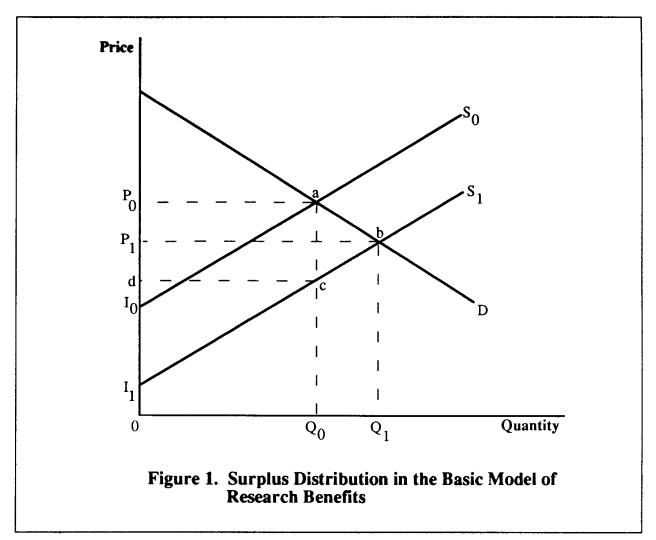
## 2.1 Surplus Distribution in the Basic Model

The basic model of research benefits in a closed economy is shown in Figure 1. In this model D represents demand for a homogeneous product, and  $S_0$  and  $S_1$  represent, respectively, the supply of the product before and after a research-induced technical change. The initial equilibrium price and quantity are  $P_0$  and  $Q_0$ ; after the supply shift they are  $P_1$  and  $Q_1$ .

The total benefit from the research-induced supply shift is equal to the area beneath the demand curve and between the two supply curves (TS = area  $I_0abI_1$ ). This area may be thought of as being equal to the sum of two parts (a) the cost saving on the original quantity (the area between the two supply curves to the left of  $Q_0$  — area  $I_0acI_1$ ), and (b) the economic surplus due to the increment to production and consumption which is equal to the triangular area abc (the total value of the increment to consumption — area  $Q_0abQ_1$  — less the total cost of the increment to production — area  $Q_0cbQ_1$ ).

Alternatively, we can consider the total benefit as comprising the sum of benefits to consumers in the form of the change in consumer surplus ( $\Delta CS = \text{area } P_0 \text{ab} P_1$ ) and the benefits to producers in the form of the change in producer surplus ( $\Delta PS = \text{area } P_1 \text{bI}_1$  minus area  $P_0 \text{aI}_0$ ). Under the special assumption of a parallel supply shift (so that the vertical difference between the two curves is constant) area  $\text{dcI}_1 = \text{area } P_0 \text{aI}_0$  and therefore the change in producer surplus is simply equal to the producer surplus on the increment to production

Harberger (1971 p. 785 suggested that three basic postulates should be accepted as providing a conventional framework for applied welfare economics. The postulates are: "(a) the competitive demand price for a given unit measures the value of that unit to the demander; (b) the competitive supply price for a given unit measures the value of that unit to the supplier; (c) when evaluating the net benefits or costs of a given action (project, program, or policy), the costs and benefits accruing to each member of the relevant group (e.g. a nation) should be added without regard to the individual(s) to whom they accrue."



from  $Q_0$  to  $Q_1$  (area  $P_1$ bcd) that would be measured off either supply curve.

We can express these effects algebraically as follows:

$$\Delta CS = P_0 Q_0 Z (1 + 0.5 Z \eta) \tag{1}$$

$$\Delta PS = P_0 Q_0 (K - Z) (1 + 0.5 Z \eta)$$
 (2)

$$\Delta TS = \Delta CS + \Delta PS = P_0 Q_0 K (1 + 0.5 Z \eta)$$
 (3)

where K is the vertical shift of the supply function expressed as a percentage of the initial price,  $\eta$  is the absolute value of the elasticity of demand,  $\varepsilon$  is the elasticity of supply, and  $Z = K\varepsilon/(\varepsilon+\eta)$  is the percentage reduction in price due to the supply shift.

Typically we would have in mind annual supply and demand functions for the product. Thus the benefits shown in Figure 1 are annual flows of benefits accruing to producers and consumers. To get total benefits we would need to aggregate over time. The comparative statics abstract from the issues of dynamic supply and demand responses, lengths of run, and lags in research, development, and adoption of technology. Perhaps less obviously, this model abstracts from the question of market level. This aspect will be considered next in the context of disaggregation of benefits.

# 2.2 Disaggregating Benefits and Costs

The model in Figure 1 might be used to measure research benefits in terms of supply and demand defined at the farm level, retail level, or some intermediate stage of the marketing system. The measurement of total benefits is not affected by the choice of where to measure benefits in the marketing chain: the total producer and consumer surplus (or total change in surplus) is the same at all market

levels. What is affected by this choice is whose benefits are included in producer surplus and whose are included in consumer surplus<sup>2</sup>.

When we measure research benefits at the retail level, producer surplus includes quasi-rents to all factors employed in producing the retail product including marketing, distribution and processing that takes place beyond the farm level as well as quasi-rents to farming inputs; consumer surplus measures the surplus of consumers who buy at retail. When we measure research benefits at the farm level, producer surplus includes only the quasi-rents accruing to inputs used in farming; quasi-rents accruing to marketing inputs are included along with final consumer surplus in "consumer surplus" measured at the farm level. Thus, by choosing the market level for the analysis we make an implicit choice about the aggregation of surplus among owners of productive factors and final consumers — i.e. the vertical aggregation.

Implicit choices about *horizontal* aggregation of surpluses are also made in the basic model. At a given market level we have aggregated all suppliers together and all demanders together. For some purposes it is desirable to disaggregate suppliers or demanders into subcategories according to geopolitical boundaries (e.g. domestic or foreign), according to income classes, or according to their business characteristics (e.g. small or large farms, adopters or non-adopters of the new technology).

In addition to these choices, some more subtle questions of aggregation arise when we choose whether to use general equilibrium or partial equilibrium definitions of the supply and demand curves. When we use a general equilibrium definition (allowing for the feedback effects when other product prices adjust), the surplus measures reflect welfare effects in other commodity markets as well as the one being studied. When we use a partial equilibrium definition (assuming that other commodity prices are constant) we may have to do some further analysis to compute the effects of any induced price changes in related markets that feed back into the market of interest.

In the pages that follow, choices about the aggregation of surpluses among these various agents are

made explicitly. The case of a single product (in partial equilibrium) produced with two factors in fixed proportions is discussed first. Then we proceed to variable proportions and multiple factors, and, after that, to multiple products and general equilibrium issues.

# 3. Vertical Market Relationships

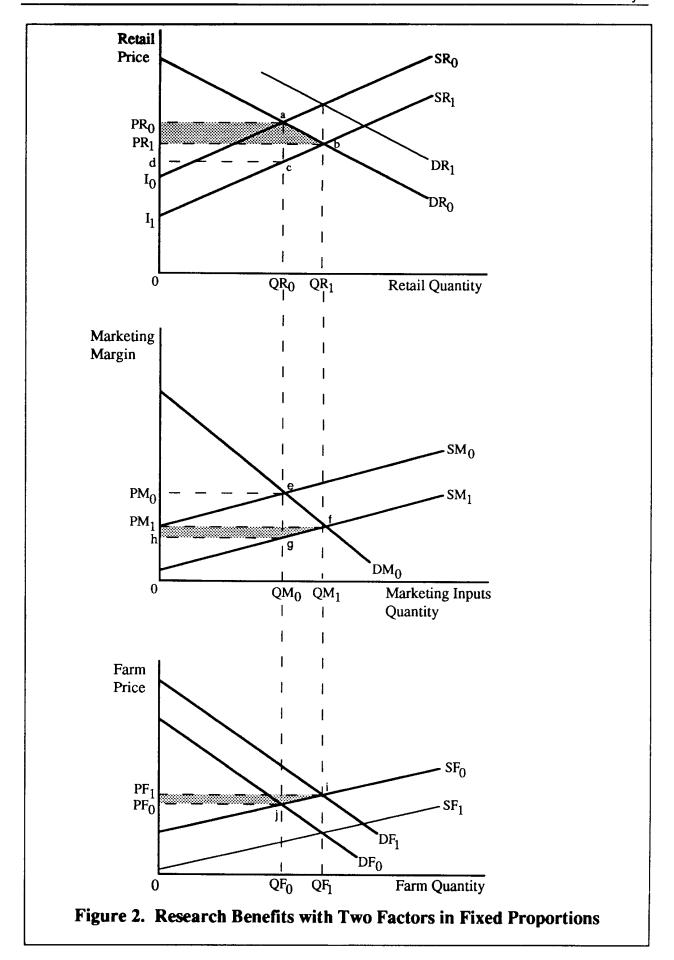
To study vertical market relationships in multistage production systems, we abstract from the temporal ordering of the stages of production and treat the different stages as if they occur at one time. The participants in different stages of the production system are represented as input suppliers and their welfare is reflected in the distribution of economic surplus among inputs. The multi-stage nature of input decisions can be reflected in constraints on substitutability among inputs through separability assumptions.

#### 3.1 Two Factors with Fixed Factor Proportions

The simplest case we can consider is when two factors of production are used in fixed proportions to produce a homogeneous product. The case of derived factor demand and output supply with two factors and fixed factor proportions is illustrated by Friedman (1976) with the example of knives, blades and handles. That model can be used to show market equilibrium and surplus distribution between, for example, two farming inputs (say land and other inputs) used to produce a farm product. Alternatively, we may use the same approach to analyze a multi-stage production system — say when a farm product and marketing inputs are used to produce a retail product.

Figure 2 represents the markets for a farm product and a composite marketing input that are used in fixed proportions to produce a retail food product. The market situation is defined by (a) the technology of production (i.e. the fixed amounts of the two factors used to produce a unit of the retail product), (b) the supply conditions for the factors of production (the farm product supply is SF<sub>0</sub> and the supply of marketing inputs is SM<sub>0</sub> with the units of factor

For a discussion of this point and proofs, see Just and Hueth (1979) or Just, Hueth and Schmitz (1982).



quantities defined per unit of the retail product), and (c) the demand function for the retail product (DR<sub>0</sub>). Because the factors are used in fixed proportions it is straightforward to derive the retail supply and factor demand equations. The retail supply function (SR<sub>0</sub>) is given as the vertical sum of the underlying factor supply functions (SF<sub>0</sub> and SM<sub>0</sub>) so that the marginal cost of a quantity of the retail product is equal to the sum of the marginal costs of the corresponding factor quantities. The derived demand function for the farm product  $(DF_0)$ is given by the vertical difference between the retail demand and the supply of marketing inputs. Similarly, the derived demand for marketing inputs  $(DM_0)$  is given by subtracting the supply function for the farm product (vertically) from the retail demand function.

The initial equilibrium in the product market is defined by the intersection of retail supply and demand at price  $PR_0$  and quantity  $QR_0$ . Equivalently, equilibrium may be defined in terms of one of the factor markets: equilibrium of supply and demand of the farm product is at price  $PF_0$  and quantity  $QF_0$ ; supply and demand for marketing inputs are in equilibrium at price  $PM_0$  and quantity  $QM_0$ .

Now, suppose the supply function for marketing inputs shifts down in parallel from  $SM_0$  to  $SM_1$ . This shift affects the equilibrium in all three markets. The supply of the retail product shifts down (by the same absolute amount per unit) from  $SR_0$  to  $SR_1$ . The demand for the farm product shifts up in parallel (also by the same absolute amount per unit) from  $DF_0$  to  $DF_1$ . All quantities increase in proportion (to  $QR_1$ ,  $QM_1$ , and  $QF_1$ ). The prices of the marketing input and the retail product fall (to  $PM_1$  and  $PR_1$ ) and the price of the farm product rises (to  $PF_1$ ).

As a consequence of these changes there is a total welfare gain of  $I_0abI_1$  comprising a change in consumer surplus ( $\Delta$ CS) of  $PR_0abPR_1$  and a change in producer surplus ( $\Delta$ PS) of  $PR_1bcd$ . The change in producer surplus comprises a change in surplus to suppliers of marketing inputs ( $\Delta$ MS =  $PM_1fgh$ ) and a change in surplus to suppliers of the farm product ( $\Delta$ FS =  $PF_1ijPF_0$ ). We can express these effects algebraically in the same form as we did for

the basic model as follows:

$$\Delta CS = PR_0QR_0Z(1 + 0.5Z\eta) \tag{1'}$$

$$\Delta PS = PR_0 QR_0 (K - Z)(1 + 0.5Z\eta)$$
 (2')

$$\Delta TS = \Delta CS + \Delta PS = PR_0 QR_0 K(1 + 0.5 Z\eta)$$
 (3')

where K is now the vertical shift of the marketing inputs supply function expressed as a percentage of the initial retail price (PR<sub>0</sub>),  $\eta$  is the absolute value of the elasticity of demand at retail,  $\varepsilon$  is the elasticity of supply to retail, and  $Z = K\varepsilon/(\varepsilon + \eta)$  is the percentage reduction in retail price due to the supply shift. The components of the change in producer surplus are:

$$\Delta FS = PF_0QF_0(K - Z)(\epsilon/\epsilon_f)(1 + 0.5Z\eta)$$

$$= \Delta PS(\epsilon PF_0/\epsilon_f PR_0)$$
(4)

$$\Delta MS = PM_0QM_0(K - Z)(\varepsilon/\varepsilon_m)(1 + 0.5Z\eta)$$

$$= \Delta PS(\varepsilon PM_0/\varepsilon_m PR_0)$$
(5)

and  $\Delta PS = \Delta MS + \Delta FS$ 

Equivalently, we could measure the total benefits in the market for marketing inputs as the area beneath the demand curve (DM<sub>0</sub>) between the two supply curves  $(SM_0 \text{ and } SM_1)$ . This area comprises "producer surplus" (i.e.  $\Delta MS = PM_1fgh$ ) and "consumer surplus" in the market for marketing inputs  $(PM_0efPM_1 - which includes \Delta CS to final con$ sumers and  $\Delta FS$  to suppliers of the farm product). Alternatively, we could measure the total benefits and their distribution in the market for the farm product: the total benefits in this case are equal to the area between the two demand curves and above the supply curve. The increase in "producer surplus" in the market for the farm product is benefits to producers of the farm product ( $\Delta$ FS) and the increase in "consumer surplus" reflects benefits to final consumers and suppliers of marketing inputs (i.e.  $\Delta CS + \Delta MS$ ).

This set of results may be extended to any arbitrary number of factors of production. Considering individual factors, in any factor market the "producer surplus" refers to surplus to suppliers of that factor while the "consumer surplus" refers to surplus of both final consumers and suppliers of all other factors. Alternatively, we can consider surplus in markets for intermediate products. At any market level, the "producer surplus" is the sum of quasi-rents accruing to all factors used in production of the intermediate good (i.e. factors used up to that market level). The "consumer surplus" is the sum of final consumer surplus and the quasi-rents accruing to all factors used in conjunction with the intermediate good (i.e. beyond that market level).

Another feature of the results warrants emphasis: the distribution of benefits is entirely independent of which of the curves shifts. That is, the total benefit and distribution of benefits would be the same from a shift down of the farm product supply function by the same amount per unit—i.e. to SF<sub>1</sub> (or, for that matter, from a shift up of the final demand function by the same amount per unit—i.e. to DR<sub>1</sub>), so long as the shifts are parallel. Thus, in this setting, farmers could afford to be indifferent both about where new technology applies in the production and marketing system and about where a levy to fund research is collected: maximizing total benefits will maximize farmer benefits.

In this part we have treated technical change in terms of either a shift of the supply of the marketing inputs or a shift of the supply of the farm product. An alternative type of technical change would be a change in the production function that combines the raw materials (the farm product and marketing inputs). The change could be neutral (reducing the amount of both inputs required to produce a unit of the product but maintaining factor proportions), biased (reducing the amount of only one of the inputs required per unit of the product), or some combination of biased and neutral changes (changing the proportions and amounts of both inputs required per unit of the product). Figure 3 shows the effects of a biased (marketing inputs saving) technical change in the context of the market model described in Figure 2. The technical change reduces — in proportion — the amount of marketing inputs used per unit of the farm product and per unit of the retail product. This amounts to a proportional shift down of the supply of marketing inputs (where the input quantities are expressed per unit of the final product) from SM<sub>0</sub> to SM<sub>1</sub> from the point of view of the producers of the retail product (equivalently, a percentage reduction in the cost of

supplying "efficiency units" of the marketing input).

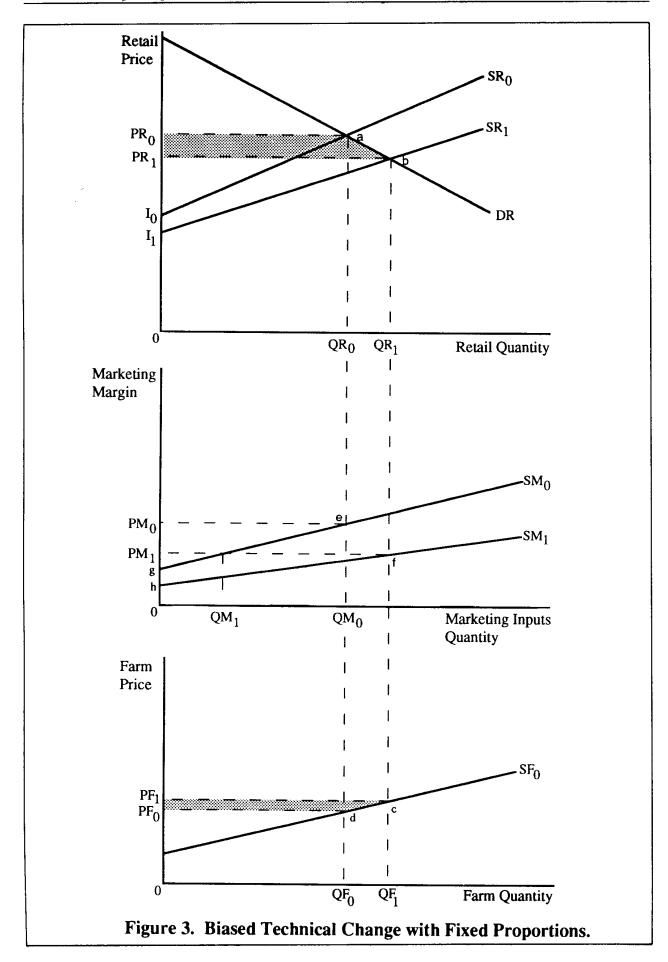
The welfare effects are slightly more complicated in this case. For the retail product, consumer surplus is increased by  $\Delta CS = PR_0abPR_1$  and total surplus is greater by  $\Delta TS = I_0 ab I_1$ . For suppliers of the farm product, producer surplus increases by  $\Delta FS = PF_1cdPF_0$ . For the marketing input, the number of "efficiency units" is greater, but actual use of marketing inputs may be smaller. The effect of the technical change on surplus accruing to marketing inputs is equal to the difference  $\Delta MS =$ PM<sub>1</sub>fh - PM<sub>0</sub>eg. This difference may be positive or negative depending, primarily, upon the elasticity of final demand: a sufficient condition is that it will be negative when final demand is inelastic. Thus farmers and consumers necessarily benefit from a biased (marketing inputs saving) technical change; marketing input suppliers may gain or lose. Technical change biased against the farm product could be modeled in the same way by switching the roles of the farm product and marketing inputs in Figure 3. By analogy, then, farmers may gain or lose from a farm product-saving technical change in the food industry. Notice that biased technical change has effects that are similar to those of a proportional downward shift of the factor supply function<sup>3</sup>.

With a neutral technical change, it is relatively easy to show that — when both factor supply functions slope up — both inputs will benefit when demand for the product is elastic (in which case total expenditure on both inputs rises with an increase in output) and conversely both will lose when demand for the product is inelastic. These issues are more easily addressed as a special case in the context of the model of technical change with variable factor proportions that will be developed next.

# 3.2 Two Factors with Variable Factor Proportions

The assumption of fixed factor proportions has been surprisingly popular in the agricultural eco-

<sup>&</sup>lt;sup>3</sup> As shown by Mullen, Wohlgenant and Farris (1988), an  $X_1$ -saving biased technical change may be modeled as an "equivalent" shift in the supply of  $X_1$ . It will not be equivalent in all senses, however, and care must be exercised in assuming equivalence.



nomics literature, especially in the context of the analysis of marketing margins (e.g. see Tomek and Robinson 1981). Clearly, the extent of input substitution possibilities is an empirical matter. The assumption that the elasticity of substitution between a pair of inputs is zero is an extreme one, and there is some empirical support for using a less restrictive assumption that allows the possibility of substitution between farm inputs or substitution between farm products and marketing inputs (e.g. Wohlgenant 1989). We saw above that the analysis of research benefits and their distribution for any number of inputs (or stages of production) is quite straightforward under the assumption of fixed proportions. Allowing for variable proportions makes it difficult to get useful algebraic results for more than three factors of production<sup>4</sup>.

Edwards and Freebairn (1981) provided an initial analysis of distribution of research benefits among factors of production in a multi-stage production system in the context of their study of Australian rural research. In an extension of that approach, Freebairn, Davis and Edwards (1982) analyzed the distribution of research benefits in a three-stage model with fixed proportions between purchased farming inputs, a farm product and marketing inputs. They illustrated their results with an application to the U.S. hog industry. Their key points were that — with parallel supply shifts — (a) innovation at any stage of a multi-stage production process confers positive benefits on consumers and producers in all stages (i.e. factors) of production, and (b) the distribution of benefits is independent of where the innovation applies in the system.

In a comment on Freebairn, Davis and Edwards (1982), Alston and Scobie (1983) illustrated that the distribution of research benefits among factors (or stages) of production depends crucially upon the elasticity of substitution. Their approach to modeling benefits from technical change has subsequently been adopted, adapted, and extended in several studies<sup>5</sup>. The approach owes its origin to Muth (1964) who presented an elegant, simple model of equilibrium displacement in a two-factor model of supply and factor demand in a competitive industry<sup>6</sup>. First a slightly modified version of Muth's (1964) model of market equilibrium displacements is presented. Then the welfare economic effects of research-induced technical changes

are considered.

Following Muth (1964), we can model the market equilibrium of a competitive industry producing a homogeneous product using two factors of production in terms of six equations:

Consumer Demand 
$$Q = f(P)$$
 (6)

Production 
$$Q = Q(X_1, X_2)$$
 (7)

Factor Demand 
$$W_1 = PQ_1$$
 (8)

$$W_2 = PQ_2 \tag{9}$$

Factor Supply 
$$X_1 = g(W_1)$$
 (10)

$$X_2 = h(W_2)$$
 (11)

The endogenous variables in the model are industry output (Q), the amounts of the two factors used by the industry  $(X_1 \text{ and } X_2)$ , the price per unit of the final product (P), and the factor prices  $(W_1 \text{ and } W_2)$ . Equation (6) is the demand for the industry's output, equation (7) is the production function, equations (8) and (9) are factor demand equations with each factor being paid the value of its marginal product  $(Q_i \text{ for factor } i)$ , and equations (10) and (11) are the factor supply equations. Constant returns to scale is assumed at the industry level<sup>7</sup>. When technical change causes a small shift from an initial equilibrium, changes in prices and quantities may be approximated linearly by totally differentiating equations (7) through (11) and converting

<sup>&</sup>lt;sup>4</sup> For two factors the results are fairly transparent (e.g. see Alston and Scobie 1983); for three factors the analytics are quite cumbersome and the transparency is reduced (see Holloway 1989). Numerical rather than algebraic solutions are likely to be necessary for studies involving three or more factors. Wohlgenant (1982) has provided a general solution for the case of one output and N factors of production.

<sup>&</sup>lt;sup>5</sup> Examples include Mullen, Wohlgenant and Farris (1988), Mullen, Alston and Wohlgenant (1989), Mullen and Alston (1989), Lemieux and Wohlgenant (1989), and Holloway (1989a). <sup>6</sup> Gardner (1975) used a very similar model to analyse marketing margins. Miedema (1976) clarified the connection between the Muth (1964) and Gardner (1975) models. More recently, Gardner (1987) has applied the same type of modeling approach to a range of agricultural policy issues.

<sup>&</sup>lt;sup>7</sup> See Diewert (1981) for a discussion of this assumption.

<sup>&</sup>lt;sup>8</sup> Muth (1964) shows how to do this for the case being considered here. Mullen, Alston and Wohlgenant (1989, pp. 44-45) show the steps involved in this transition for the three-factor case, approaching the problem from the dual side (i.e. using a cost function rather than a production function).

them to elasticity form<sup>8</sup>. Differentiating equations (7) through (11) and adding exogenous shocks yields the following system of logarithmic differential equations — expressed in terms of relative changes and elasticities:

$$EQ = -\eta[EP - \alpha] \tag{6'}$$

$$EQ = s_1 E X_1 + s_2 E X_2 + \delta$$
 (7')

$$EW_1 = EP - (s_2/\sigma)EX_1 + (s_2/\sigma)EX_2 + \delta + \gamma$$
 (8')

EW<sub>2</sub> = EP + 
$$(s_1/\sigma)EX_1 - (s_1/\sigma)EX_2 + \delta$$
  
-  $(s_1/s_2)\gamma$  (9')

$$EX_1 = \varepsilon_1 [EW_1 + \beta_1] \tag{10'}$$

$$EX_2 = \varepsilon_2 [EW_2 + \beta_2] \tag{11'}$$

where E denotes relative changes (i.e. Ey = dy/y = dlny),  $\eta$  is the elasticity of demand,  $\alpha$  is a vertical shift in the demand function reflecting an *increase* in demand,  $s_i$  is the cost share of factor i ( $s_i$  =  $W_i X_i / PQ$ ) and  $s_1 + s_2 = 1$ ,  $\delta$  is a (neutral) upward shift in the production function,  $\gamma$  is a biased ( $X_2$ -saving) technical change  $^9$ ,  $\sigma$  is the elasticity of substitution between  $X_1$  and  $X_2$ ,  $\epsilon_i$  is the elasticity of supply of factor i, and  $\beta_i$  is a vertical shift down in the supply of factor i reflecting an *increase* in supply.

The exogenous shift parameters ( $\alpha$ ,  $\beta_1$ ,  $\beta_2$ ,  $\delta$ , and γ) express equilibrium displacements relative to an initial equilibrium. Thus, for instance, setting  $\alpha =$ 0.1 would imply a 10 per cent increase in consumers' willingness to pay for the initial quantity of the product. While the shift of demand is expressed as a percentage of the initial price, it cannot be presumed that there has been a proportional shift of demand. Rather, a measures the vertical shift in demand at a point, locally, for any type of demand shift (e.g. proportional, parallel, pivotal). Similarly,  $\beta_1$  measures the shift down of the supply of factor X<sub>1</sub> with the magnitude of the reduction in marginal cost (at the point of approximation, the initial equilibrium) being expressed relative to the initial price of the factor. These shifts are shown in Figure 4 which is a diagramatic representation of the model in equations (6') through (11').

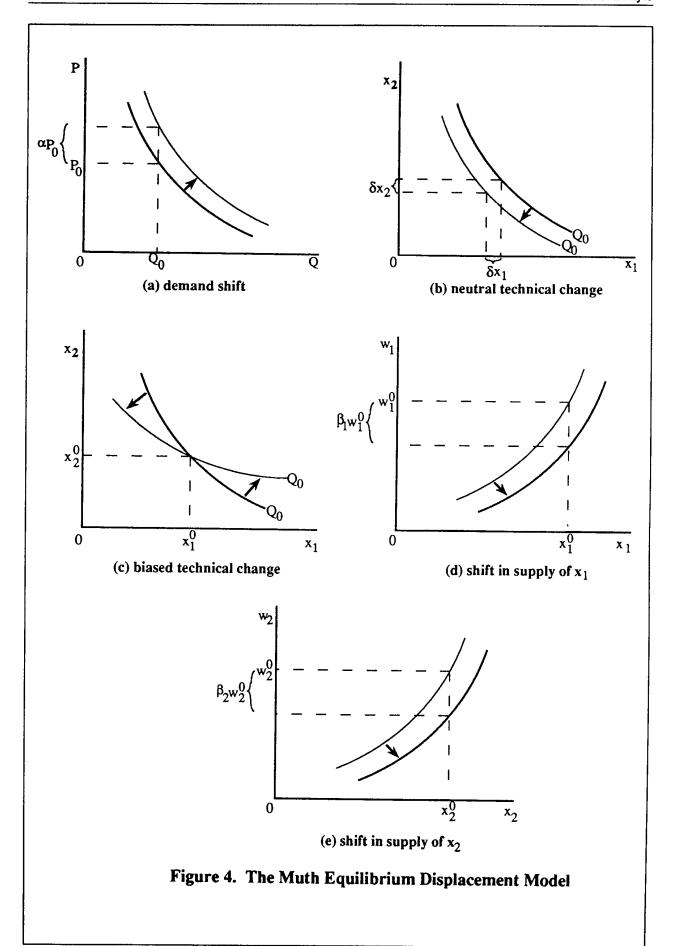
Nothing is presumed in this specification about the

magnitude of the supply shift at other points along the supply curve. The nature of the shift (e.g. proportional, parallel, or pivotal) is treated as a separate question from the amount of the shift relative to the initial equilibrium. For the most part parallel shifts of supply and demand are assumed but those shifts are expressed relative to initial prices and quantities. Further, for the most part it is assumed that all supply and demand curves are linear — at least in the relevant range of the equilibrium displacement<sup>10</sup>. But these assumptions are not necessarily implied by the specification of the equations of the equilibrium displacement model. At the same time, they are wholly consistent with the equations of the model when we make clear that we are using the model to approximate the consequences of parallel displacements of linear supply and demand equations.

The assumptions of (approximately) linear supply and demand with parallel shifts are required for the economic surplus measures that are used below. In cases when it is preferred to use a different assumption about the functional forms or nature of supply and demand shifts, it still may be convenient to use the linear elasticity equilibrium displacement model to estimate changes in prices and quantities — and for small shifts it is likely to be a good approximation. However, the surplus formulas below are correct only for parallel shifts of linear supply and demand; non-parallel shifts require different equations to compute changes in economic surpluses.

<sup>&</sup>lt;sup>9</sup> Freebairn, Davis and Edwards (1983) objected to the Muth (1964) specification of biased technical change and suggested an alternative treatment in which only one factor demand equation is affected. This objection is primarily terminological: Muth (1964) claimed correctly that any technical change could be modeled as a combination of his biased component (twisting the isoquant —  $\gamma$ ) and a neutral component (either relabelling isoquants, or relabelling the axes —  $\delta$ ). Neither of these treatments allows the possibility of a technical change that would alter the elasticity of substitution (i.e. the curvature of the isoquants).

<sup>10</sup> Much has been written about the implications of functional forms of supply and demand, elasticities of supply and demand, and the nature of research-induced supply shifts for the size and distribution of research benefits (e.g. Lindner and Jarrett 1978 and Norton and Davis 1981). In relation to total benefits, functional forms and elasticities are relatively unimportant compared to the nature of the supply shift. In relation to distribution of benefits, functional forms are relatively unimportant compared to the sizes of elasticities and nature of the supply shift. The assumption of a parallel shift is very important. With parallel shifts, Alston and Wohlgenant (1990) have shown that functional form choices have little effect on either the size or distribution of benefits.



### **Table 1: Reduced Form Solutions to the Muth Model**

$$EQ = [ \eta \{ \varepsilon_1 \varepsilon_2 + \sigma(s_1 \varepsilon_1 + s_2 \varepsilon_2) \} \alpha + s_1 \varepsilon_1 \eta(\sigma + \varepsilon_2) \beta_1 + s_2 \varepsilon_2 \eta(\sigma + \varepsilon_1) \beta_2 + \eta \{ \sigma(1 + s_1 \varepsilon_1 + s_2 \varepsilon_2) + \varepsilon_1 \varepsilon_2 + s_1 \varepsilon_2 + s_2 \varepsilon_1 \} \delta + s_1 \sigma \eta(\varepsilon_1 - \varepsilon_2) \gamma ] / D$$
(12)

EP = 
$$[\eta(\sigma+s_2\varepsilon_1+s_1\varepsilon_2)\alpha - s_1\varepsilon_1(\sigma+\varepsilon_2)\beta_1 - s_2\varepsilon_2(\sigma+\varepsilon_1)\beta_2 +$$

$$- \{\sigma(1+s_1\varepsilon_1+s_2\varepsilon_2)+\varepsilon_1\varepsilon_2+s_1\varepsilon_2+s_2\varepsilon_1\}\delta - s_1\sigma(\varepsilon_1-\varepsilon_2)\gamma\}/D$$
(13)

$$EX_{1} = [ \eta \varepsilon_{1}(\sigma + \varepsilon_{2})\alpha + \{ \eta \sigma + (s_{2}\sigma + s_{1}\eta)\varepsilon_{2} \} \varepsilon_{1}\beta_{1} - s_{2}(\sigma - \eta)\varepsilon_{2}\varepsilon_{1}\beta_{2} + (\sigma + \varepsilon_{2})(1 - \eta)\varepsilon_{1}\delta + \varepsilon_{1}\sigma(\varepsilon_{2} + \eta)\gamma ]/D$$
(14)

$$EX_2 = \{ \eta \varepsilon_2(\sigma + \varepsilon_1)\alpha - s_1(\sigma - \eta)\varepsilon_1\varepsilon_2\beta_1 + \{ \eta \sigma + (s_1\sigma + s_2\eta)\varepsilon_1 \}\varepsilon_2\beta_2 +$$

$$-(\sigma+\epsilon_1)(1-\eta)\epsilon_2\delta - (s_1/s_2)\epsilon_2\sigma(\epsilon_1+\eta)\gamma]/D$$
 (15)

$$EW_1 = [ \eta(\sigma + \varepsilon_2)\alpha - (s_1\sigma + s_2\eta + \varepsilon_2)\varepsilon_1\beta_1 - s_2(\sigma - \eta)\varepsilon_2\beta_2 +$$

$$-(\sigma+\epsilon_2)(1-\eta)\delta + \sigma(\epsilon_2+\eta)\gamma]/D \tag{16}$$

$$EW_2 = [\ \eta(\sigma + \epsilon_1)\alpha - s_1(\sigma - \eta)\epsilon_1\beta_1 - (s_2\sigma + s_1\eta + \epsilon_1)\epsilon_2\beta_2 +$$

$$-(\sigma + \varepsilon_1)(1-\eta)\delta - (s_1/s_2)\sigma(\varepsilon_1+\eta)\gamma]/D \tag{17}$$

where D = 
$$\sigma(\eta + s_1 \varepsilon_1) + s_2 \varepsilon_2 + \eta(s_2 \varepsilon_1 + s_1 \varepsilon_2) + \varepsilon_1 \varepsilon_2$$
, and

$$D > 0$$
 for  $\eta > 0$ ,  $\sigma > 0$  and  $\varepsilon_1$  and  $\varepsilon_2 > 0$ 

Variable or Parameter	Definition	
Endogenous Variables		
Q	Quantity of product	
P	Price of product	
$X_i$	Quantity of factor i (for $i = 1,2$ )	
$W_{\mathbf{i}}$	Price of factor i (for $i = 1,2$ )	
$Q_{\mathbf{i}}$	Marginal product of factor i (for $i = 1,2$ )	
Market Parameters		
η	Absolute value of the elasticity of final demand	
$\epsilon_{ m i}$	Elasticity of supply of factor i (for $i = 1,2$ )	
s <sub>i</sub>	Cost share of factor i (for $i = 1,2$ )	
σ	Elasticity of factor substitution	
Exogenous Shift Variables		
α	Relative increase in demand (shift up in the price dintion)	
$eta_{\mathbf{i}}$	Relative increase in supply of factor i (shift down in price direction)	
γ	Relative increase in marginal product of factor $X_1$ to an $X_2$ -saving biased technical change, holding out constant	
δ	Relative increase in output and marginal products both factors due to a neutral technical change	

As we can see in the equations of the model (i.e., equations (6') through (11')) or in Figure 4, mutually consistent changes in prices and quantities of factors and products may arise from shifts of the final demand ( $\alpha$ ), either factor supply function ( $\beta_1$  or  $\beta_2$ ), a neutral technical change ( $\delta$ ), or a biased technical change ( $\gamma$ ). Solutions may be obtained by a sequence of substitutions (as by Muth 1964) or by matrix algebra methods.

The reduced form solutions are shown in Table 1 as equations (12) through (17)<sup>11</sup>. The parameters and variables in equations (12) through (17) are defined in Table 2. These equations differ slightly from Muth's (1964, p. 233) in the signs on expressions because here, to aid interpretation, the parameters are all defined as being positive and the shift variables are defined so that, when they have positive values, the relevant quantity increases.

# 3.3 Surplus Measures With Input Substitution

To measure the surplus changes associated with the equilibrium displacements described by the two factor model above it is necessary to define the functional forms of the factor supply and demand functions and the nature of the shifts induced by the various changes. As Lindner and Jarrett (1978) and others have shown in the context of the "basic model", the functional form and nature of supply shift have important implications for measures of benefits — the nature of the shift is especially important.

The model in equations (6) through (17) does not involve any explicit or implicit assumptions about the functional forms of supply and demand. It is a local approximation to unknown functions; the approximation is linear in logarithmic differentials (i.e. relative changes) and elasticities; it is not assumed that the elasticities are constant<sup>12</sup>.

In the work that follows it is assumed that supply and demand functions are approximately linear in the region of interest and that the curves shift in parallel as a result of exogenous factors  $(\alpha, \beta_i, \gamma,$  and  $\delta)$ . Under these assumptions the benefits accruing to consumers ( $\Delta$ CS) and factors of production ( $\Delta$ PS<sub>i</sub> for i=1,2) may be measured — in terms of the changes in factor and product prices

and quantities from equations (12) through (17)—using:

$$\Delta CS = -P_0 Q_0 (EP - \alpha)(1 + 0.5EQ)$$
 (18)

$$\Delta PS_i = W_i X_i (EW_i - \beta_i)(1 + 0.5EX_i)$$
 (19)

$$\Delta TS = \Delta CS + \sum_{i} \Delta PS_{i}$$
 (20)

Mullen, Alston and Wohlgenant (1989) present equivalent formulas for the case of three factors<sup>13</sup>. Equations (18) through (20) may be used for an arbitrary number of factors (or stages of production) to estimate total benefits and the distribution of those benefits from equilibrium displacements under the assumptions being used here. They can be used to examine the effects of a combination of displacements (which add linearly) or individual changes in isolation.

The qualitative results considering individual exogenous shifts in isolation are shown in Table 3. With the exception of the biased technical change, consumers always gain from the displacements associated with positive values for any of the exogenous shift variables: they either shift demand up ( $\alpha$ ) or shift final market supply down ( $\beta_i$ ,  $\delta$ ). In the case of a biased (X<sub>2</sub>-saving) technical change, consumers will benefit only when the elasticity of supply of  $X_1$  is greater than that of  $X_2$  (i.e.  $\varepsilon_1 > \varepsilon_2$ )<sup>14</sup>. Factor suppliers gain from a parallel shift down of their own supply function (i.e. surplus to producers of  $X_1$  increases with positive values of  $\beta_1$  and surplus to producers of X<sub>2</sub> increases with positive values of  $\beta_2$ ). However, the cross-effects of factor supply shifts may be positive or negative, depending upon whether the two factors are gross substitutes or gross complements.

<sup>&</sup>lt;sup>11</sup> These equations are derived from Muth (1964, p. 233) but the notation is slightly different and we have corrected the error in his equation (24) as noted by Freebairn, Davis and Edwards (1983).

<sup>&</sup>lt;sup>12</sup> For instance, it is perfectly valid to use this type of model to analyze the effects of parallel shifts in the case of linear supply and demand functions. Alston and Wohlgenant (1990) have shown that this type of linear elasticity model is exactly correct for linear supply and demand and only approximately correct for constant elasticity functions.

<sup>&</sup>lt;sup>13</sup> Freebaim, Davis and Edwards (1983) present formulas for surplus changes that correspond to equations (18) and (19) after substitution of terms from equations (12) to (17).

<sup>&</sup>lt;sup>14</sup> Freebaim, Davis and Edwards (1983) suggested an alternative specification of biased technical change that they found more plausible and which avoided this ambiguity.

Table 3: Incidence of Benefits from Technical Change in the Muth-model				
	Interest groups			
Type of Change in Technology	Suppliers of X <sub>1</sub>	Suppliers of X <sub>2</sub>	Consumers	
Demand Increase $(\alpha > 0)$	+	+	+	
Increase in Supply of $X_1$ ( $\beta_1 > 0$ )	+	σ<η	+	
Increase in Supply of $X_2$ ( $\beta_2 > 0$ )	σ<η	+	+	
$X_2$ -saving $(\gamma > 0)$	+	<del>-</del>	$\varepsilon_1 > \varepsilon_2$	
Neutral ( $\delta > 0$ )	η > 1	η > 1	+	

Notes: Entries denote conditions under which interest groups benefit.

- + indicates that benefits are positive under all conditions.
- indicates there are no conditions under which benefits are positive.

All entries are subject to the assumptions that  $\eta$ ,  $\sigma$ ,  $\varepsilon_1$ , and  $\varepsilon_2 \ge 0$ .

Entries in the row for  $X_2$ -saving technical change assume  $\sigma$  is strictly positive. When  $\sigma = 0$ , there are no effects from biased technical change as defined by Muth (1964); all the entries in that row become zeros.

When the elasticity of substitution is *less* than the absolute value of the demand elasticity ( $\sigma < \eta$ ) the two factors are gross complements (i.e. the crossprice elasticity of factor demand is negative so that a fall in price of either factor will increase the demand for the other factor). In this case, both factors benefit when either factor supply function shifts down. In the extreme case of fixed proportions ( $\sigma$ =0) the distribution of benefits is independent of which factor supply function shifts.

When the elasticity of substitution is greater than the absolute value of the demand elasticity  $(\sigma > \eta)$  the two factors are gross substitutes (i.e. the crossprice elasticity of factor demand is positive so that a fall in price of either factor will reduce the demand for the other factor). In this case, suppliers

of  $X_1$  lose when the supply function for  $X_2$  shifts down ( $\beta_2>0$ ) and suppliers of  $X_2$  lose when the supply function for  $X_1$  shifts down ( $\beta_1>0$ ).

Both factors gain from a neutral technical change ( $\delta > 0$ ) when demand is elastic ( $\eta > 1$ ); both factors lose when demand is inelastic. Factor  $X_1$  benefits from a biased ( $X_2$ -saving) technical change and factor  $X_2$  loses unless we have fixed proportions ( $\sigma = 0$ ) in which case there is no effect on quantity or price of output and no effect on quantity or price of either factor.

Alston and Scobie (1983) and Freebairn, Davis and Edwards (1983) considered the distribution of benefits of these various types of technical change in the two factor case (between a farm product and

marketing inputs). They concluded that, in contrast to the case of fixed factor proportions, when there is input substitution the distribution of benefits depends on the nature of the research-induced technical change. They also suggested that the model can be used to measure the incidence of costs of a levy to fund research. When there is input substitution the incidence of a research levy on the farm product will be different from the incidence of benefits from research, other than research directed at shifting the farm product supply function. These issues have been explored further in empirical models. One issue that has not been resolved in this literature is how best to model biased technical change 15.

# 3.4 Models with More than Two Factors of Production

Several studies have provided numerical estimates of the size and distribution of research benefits across three (or more) factors of production (e.g. Mullen, Alston and Wohlgenant 1989). However, the only published analytic solutions are those of Holloway (1989). Those results serve, among other things, to illustrate how quickly the analysis becomes intractable when the number of stages of production increases. Holloway (1989) extended the two-factor case studies by Alston and Scobie (1983) to a three-factor case (a farm product with two marketing stages, processing and distribution).

Holloway's (1989) key results are summarized below. He showed (p. 341) that farmers always gain from increases in final demand or from biased technical change that is farm product using (i.e. distribution or processing services saving technical change). Conditions for farmers to gain from other types of research are:

- 1. Increase in supply of
  - (a) "distribution services":  $\eta > \sigma_d$
  - (b) "processing services":  $(\sigma_d \sigma_p)(\epsilon_d + \eta) >$

$$s_i(\sigma_d-\eta)(\epsilon_d+\sigma_p)$$

2. Neutral technical change in

- (a) "distribution":  $\eta > 1$
- (b) "processing":  $(\sigma_d-1)(\varepsilon_d+\eta) > s_i(\sigma_d-\eta)(\varepsilon_d+1)$
- 3. Primary-input-saving technical change in
  - (a) "distribution":  $\eta > \sigma_d$
  - (b) "processing":  $(\sigma_d \sigma_p)(\varepsilon_d + \eta) > s_i(\sigma_d \eta)(\varepsilon_d + \sigma_p)$

where  $\sigma_d$  = the elasticity of input substitution in the distribution industry,  $\sigma_p$  = the elasticity of input substitution in the processing industry,  $\varepsilon_d$  = the elasticity of supply of distribution services,  $\eta$  is the absolute value of the elasticity of final demand, and  $s_i$  is the cost share of the intermediate input.

Muth (1964), Gardner (1975), Perrin (1980, 1981), and Holloway (1989) all tackled the problem from the primal side (specifying production functions). Wohlgenant (1982) suggested using a dual approach (specifying a cost function instead), and he used it to illustrate solutions for the case of N factors <sup>16</sup>. In this approach, the equations of the model in the case when N factors are used to produce a single product are specified in logarithmic differential form as:

Final Demand: 
$$EQ = -\eta[EP - \alpha]$$
 (21)

Market Clearing: 
$$EP = \sum_{i} s_{i}EW_{i}$$
 (22)

Factor Supply: 
$$EX_i = \varepsilon_i [EW_i + \beta_i]$$
 (23)

Factor Demand:  $EX_i = \sum_i \eta^*_{ii} EW_i + EQ + \delta_i$  (24)

<sup>15</sup> Muth (1964) suggested one approach — shifting both factor demand curves, in effect twisting the isoquant to change the ratio of marginal products but holding output constant; Freebaim, Davis and Edwards (1983) criticised that approach and offered an alternative — incorporating a shift variable in only one factor demand equation. Mullen, Wohlgenant and Farris (1988) suggested that a biased (X<sub>2</sub>-saving) technical change (of the type defined by Muth 1964) could be modeled as an "equivalent" shift of the factor supply functions (i.e. there is some combination of values for b<sub>1</sub> and b<sub>2</sub> that has effects equivalent to those from a particular value of g). It is not completely clear in what sense(s) the shifts will be "equivalent", however.

<sup>&</sup>lt;sup>16</sup> Several studies have followed that suggestion including Mullen, Wohlgenant and Farris (1988), Mullen and Alston (1989) and Mullen, Alston and Wohlgenant (1989), for example.

This system consists of 2N+2 simultaneous equations in which the variables are as previously defined (i.e. Wi is the price of factor i, P is the final product price, X<sub>i</sub> is the quantity of factor i, and Q is the quantity of the product). The parameters are (i) the absolute value of the elasticity of final demand  $(\eta > 0)$ , (ii) elasticities of factor supply ( $\varepsilon_i$  i=1,...N), (iii) output constant own-and cross-price elasticities of factor demand  $(\eta^*_{ij})$ , and (iv) factor cost shares (s<sub>i</sub>). The exogenous shift variables are a final demand shift  $(\alpha)$ , shifts of factor supply functions  $(\beta_i)$ , and shifts of factor demand functions  $(\delta_i)$ . This specification has used the assumption of constant returns to scale of the industry production function. The elasticities of factor demand may be expressed in terms of cost shares and Allen partial elasticities of factor substitution (i.e.  $\eta^*_{ij} = s_j \sigma_{ij}$  for  $i \neq j$ ). Restrictions on the parameters can be derived from assumptions of symmetry of the cost function ( $\sigma_{ii}$ =  $\sigma_{ji}$ ) and homogeneity of the cost function in the factor prices ( $\sum_{i} \eta^*_{ij} = 0$ ). Using these restrictions, the full set of N<sup>2</sup> (output constant) factor demand elasticities can be represented by N-1 factor shares and N(N-1)/2 elasticities of substitution<sup>17</sup>.

When the interest is only in the factor markets, equations (21) and (22) can be eliminated by substituting (22) into (21) and then substituting the result from that into (24). The result is:

Factor Supply: 
$$EX_i = \varepsilon_i [EW_i + \beta_i]$$
 (23)

Factor Demand: 
$$EX_i = \sum_j \eta_{ij} EW_j + \eta \alpha + \delta_i$$
 (25)

where the factor demand equations are now Marshallian (rather than Hicksian or output constant) demands so that the elasticities are Marshallian own- and cross-price elasticities:  $\eta_{ij} = s_j(\sigma_{ij} - \eta) = \eta^*_{ij} - s_j\eta$ . Equations (23) and (25) contain all of the information that was included in equations (21) through (24). A system of equations such as (23) and (25) (or equations (21) through (24)) can be used to solve numerically for the price and quantity effects of a range of types of technical changes in the case where a single product is produced using a variety of factors of production. Then the size and distribution of the total benefits from research may be computed by substituting the results into equations (18) through (20):

$$\Delta CS = -P_0Q_0(EP - \alpha)(1 + 0.5EQ)$$
 (18)

$$\Delta PS_i = W_i X_i (EW_i - \beta_i) (1 + 0.5EX_i)$$
 (19)

$$\Delta TS = \Delta CS + \sum_{i} \Delta PS_{i}$$
 (20)

# 4. Horizontal Market Relationships

The discussion so far has referred strictly to the case of a homogeneous product being sold in a single market. Now we consider three extensions to that analysis: (a) multiple markets for a single product, (b) multiple products, and (c) quality change.

# 4.1 Multiple Markets for a Single Product

Sometimes it is of interest to disaggregate welfare "horizontally" at a particular market level (e.g. to distinguish among consumers according to their income class or where they live; or to distinguish among suppliers of a particular factor of production or product according to where they live or whether they adopt new technology).

One case that has arisen frequently is the case of an internationally traded good when the interest is in domestic research benefits (usually in the exporting country). In this case the economic welfare of domestic producers and consumers must be separated from the total measures of benefits. This is reasonably straightforward: the "rest-of-the-world" (ROW) can be represented by an excess (export) demand function that faces the exporting country and the economic surplus under that demand function is distinct from the measures of domestic producer and consumer benefits measured off domestic supply and demand. Market clearing is enforced by equating excess supply (the difference between domestic supply and demand) and excess demand18.

More generally, one can have in mind an arbitrary number of groups of suppliers (N) and an arbitrary number of groups of consumers (M) with market

<sup>&</sup>lt;sup>17</sup> Further restrictions upon elasticities of substitution may be invoked using separability assumptions to represent multi-stage decision processes as by Holloway (1989).

<sup>&</sup>lt;sup>18</sup> Examples of this approach include Edwards and Freebairn (1981, 1982, 1984), Alston, Edwards and Freebairn (1988) and Mullen, Alston and Wohlgenant (1989).

clearing conditions on quantities and prices. For instance, the model below assumes that total quantity demanded and total quantity supplied are equal, and prices are set competitively with zero transport costs among markets but with a domestic price wedge (an ad valorem consumption tax) in each market:

Supply: 
$$Q_{s,i} = f_i (P_{s,i}, B_i)$$
 (26)

Demand: 
$$Q_{dj} = g_j (P_{d,j}, A_j)$$
 (27)

Market Clearing: 
$$\sum_{i=1}^{N} Q_{s,i} = \sum_{i=1}^{M} Q_{d,i}$$
 (28)

$$P_{s,i} = (1-\tau_i)P_{d,i} \text{ and } P_{s,i} = P_{s,j} \forall i,j$$
 (29)

where  $P_{s,i}$  is the supply price,  $Q_{s,i}$  is the quantity supplied and  $B_i$  is a supply shift variable for the ith group of suppliers (or producers) and  $P_{d,j}$  is the demand price,  $Q_{d,j}$  is the quantity demanded,  $A_j$  is a demand shift variable for the jth group of demanders (or consumers), and  $\tau_i$  is the consumption tax in market i.

This model can be solved either using specific functional forms for the supply and demand equations or by taking an approximation<sup>19</sup>. In logarithmic differential form, the model in equations (26) through (29) may be approximated as follows<sup>20</sup>:

Supply: 
$$E(Q_{s,i}) = \varepsilon_i [E(P_{s,i}) + \beta_i]$$
 (26')

Demand: 
$$E(Q_{d,i}) = -\eta_i [E(P_{d,i}) - \alpha_i]$$
 (27')

Market Clearing: 
$$\sum_{i=1}^{N} ss_i E(Q_{s,i}) = \sum_{j=1}^{M} ds_j E(Q_{d,j})$$
 (28')

$$E(P_{s,i}) = E(P_{d,i}) + E(1-\tau_i)$$
 and  
 $E(P_{s,i}) = E(P_{s,j}) \forall i,j$  (29')

In equation (28') the share weighted sum of relative changes in quantities supplied equals the share weighted sum of relative changes in quantities demanded, where the ith supply share is  $s_i = Q_{s,i}/[\sum_i Q_{s,i}]$  and the jth demand share is  $ds_j = Q_{d,j}/\sum_j Q_{d,j}$ .

The system of equations can be solved for the endogenous relative changes in prices and quantities as functions of the elasticities of supply and demand, shares, and exogenous shift variables. For example, considering the case where there are no price wedges  $(\tau_i = 0 \ \forall \ i$ , so that  $E(P_{d,i}) = E(P_{s,j}) = EP \ \forall \ i,j)$  and there are no demand shifters  $(\alpha_i = 0 \ \forall \ i)$ , the relative change in price is:

$$E(P_{d,i}) = E(P_{s,j}) = EP = -\sum_{i} ss_{i}\beta_{i}/[\sum_{i} (ss_{i}\epsilon_{i} + ds_{i}\eta_{i})]$$

$$= -\sum_{i} ss_{i}\beta_{i}/(\epsilon_{w} + \eta_{w})]$$
(30)

where  $\varepsilon_w$  and  $\eta_w$  are the aggregate (world) elasticities of supply and demand.

For large numbers of distinct suppliers and demanders, with price wedges in the model, it might not be sensible to obtain analytic solutions; numerical solutions are easy to compute. Gross annual research benefits may be computed using:

$$\Delta CS_i = -P_{d,i}Q_{d,i}[E(P_{d,i}) - \alpha_i][1 + 0.5E(Q_{d,i})](18')$$

$$\Delta PS_i = P_{s,i}Q_{s,i}[E(P_{s,i}) - \beta_i][1 + 0.5E(Q_{d,i})]$$
 (19')

$$\Delta TS = \sum_{i} [\Delta CS_{i} + \Delta PS_{i}]$$
 (20')

Equation (20') measures global benefits. Benefits to any sub-aggregate of consumers and producers may be computed using the relevant components from equations (18') and (19'). For instance, benefits to "country" i could be computed as  $\Delta TS_i = \Delta PS_i + \Delta CS_i$ . If there were any tax wedges we would need to augment the measures with changes in government revenues as well.

Consider a case where the subscripts "i" denote different countries. In the model above, a supply (or demand) shift in any one country will affect the price, quantity, and economic surpluses, in every other country. Thus research-induced technical changes in one country have effects that are not confined to the country where the innovation takes place. It is easy in this model to show the intuitively reasonable result that, when a subset of producers

<sup>19</sup> Edwards and Freebairn (1981, 1982, 1984) and Davis, Oram and Ryan (1987) assumed linear supply and demand, for example. Alston and Wohlgenant (1990) have shown that the logarithmic differential (linear elasticity) approximation approach is a very good approximation for small changes with constant elasticity supply and demand and exactly correct with linear supply and demand.

<sup>&</sup>lt;sup>20</sup> Perrin and Scobie (1981) use this type of horizontal multimarket model to analyze Colombian food policy. They present general algebraic solutions as well as numerical results.

adopt an improved technology, all consumers benefit and producers who do not adopt the innovation lose (e.g. see Edwards and Freebairn 1982, 1984). This is one type of spillover: the pecuniary effect when innovation by one group of producers affects prices received by another group of producers. Another type of spillover than can occur is when technology developed by one country (or group of producers) can be adopted elsewhere<sup>21</sup>. In the context of the model above this "leakage" of research results can be analyzed by treating the supply shifters ( $\beta_i$ ) as mutually dependent rather than independent so that there is a shift in supply in one region as a consequence of a shift in another region<sup>22</sup>.

The multiple market (or trade) model described here is essentially the same as those of Edwards and Freebairn (1981, 1982, 1984) and Davis, Ryan and Oram (1987)<sup>23</sup>. There is one important difference, however. It is possible to combine this multimarket model with the multi-factor model described in the previous section so that we can consider jointly, and in a theoretically consistent manner, multiple markets (i.e. international trade) and multiple factors (i.e. multi-stage production)<sup>24</sup>.

### 4.2 Multiple Products — Some General Issues

There have been few studies dealing formally with the incidence of R&D among industries. Changes in technology in one industry (affecting one commodity) will affect industries producing different commodities that are related either in consumption or production to the one where the innovation occurs. For example, an improvement in technology in the chicken meat industry will result in a shift in supply of chicken and a reduction in the price of chicken. A second round effect of this change will be a reduction in demand for beef (a substitute in consumption) and, at the same time, a reduction in supply of beef due to increased feedgrain costs (assuming the change in technology was not feed saving), because beef and chicken compete for feed grains. Alternatively, as a second example, improvements in technology of feedgrain production will lower costs of both chicken and beef, but the effects may differ in size and the net effects on either may be unfavourable due to substitution in consumption.

A third alternative multi-product situation is where the two goods are related in production directly (rather than through a shared factor) such as in the Australian wool, sheepmeats and wheat industries, or in the dairy processing industry that uses milk to produce a range of products. In these two examples the various products will be related in three ways: (a) substitution in consumption (say, between butand cheese), (b) substitution (or complementarity) in production (say, between butter and skim milk powder), and (c) competition (or substitution) between products in the use of specialized factors (say, between using milk to produce butter and skim milk powder and using milk to produce cheese). A change in technology might be of the kind that affects the joint product relationships as well as the kinds that affect factor use (the neutral and biased technical changes) considered already in the single-product model.

The welfare measures taken in the context of a single market model may reflect the welfare changes in related markets. Consider the first example, an improvement in technology of chicken production. A conventional partial equilibrium analysis of welfare changes in the chicken market will reflect induced shifts in demand for beef but will still be correct so long as the beef price is exogenous. It will be incorrect, however, if the price of beef is af-

<sup>&</sup>lt;sup>21</sup> The implications of inter-regional or international "leakage" of research results has received attention from various studies including Edwards and Freebairn (1981, 1982, 1984), Davis, Oram and Ryan (1987), and Mullen, Alston and Wohlgenant (1989), for example.

<sup>&</sup>lt;sup>22</sup> For instance, suppose a technology developed by country 1 that reduces costs by 1 per cent is adopted at the same time by country 2 but is not adopted by country 3. This could be analyzed by setting the shift parameters as  $b_1 = b_2 = 0.01$  and  $b_3 = 0$ . The qualitative results are that "leakage" of results will (i) increase global total benefits, (ii) increase global producer benefits in total, (iii) increase global consumer benefits in total, and (iv) reduce benefits to producers in country 1 and country 3. Country 2 clearly benefits from leakage. International leakage of results from country 1 to country 2 may involve a net benefit or a net cost in countries 1 and 3. This is an empirical question, the answer to which will depend on the characteristics of the markets and trade in the affected commodity.

<sup>&</sup>lt;sup>23</sup> That is, it deals with a single commodity; research shifts a linear supply function down in parallel; and there may be spillover effects in other supply functions; to be sure, it does not contain all the explicit details of exchange rates and so on, but they could easily be incorporated.

<sup>&</sup>lt;sup>24</sup> This was done, for example, by Mullen, Alston and Wohlgenant (1989) in their study of international incidence of benefits and costs of Australian investments in R&D applicable to various stages of the world wool industry.

fected; and it will be complicated further when there is feedback from the beef market into supply or demand in the chicken market. A natural impulse would be to add up the areas of welfare changes in the beef and chicken markets, but adding up effects across markets could involve double counting.

Alternatively, a general equilibrium definition of the supply and demand equations for chicken could be used (allowing prices to adjust in related markets in response to shifts in chicken supply). The ordinary (ceteris paribus ) demand curve shows how consumption of a good responds to changes in its own price, holding constant all other prices and money income. A general equilibrium (mutatis mutandis) demand curve shows how consumption of a good responds to changes in its own price, allowing prices of related goods to adjust in response to the own-price changes, and allowing the ceteris paribus demand for the good to shift in response to these induced changes in prices of related goods. The elasticity of this type of general equilibrium demand curve corresponds to the "total elasticity" concept introduced by Buse (1958). The supply side counterpart allows for induced changes in the prices of related products to feed back into the supply curve of interest. The welfare measures taken off those supply and demand equations will reflect welfare changes in the beef market (and any other related markets) as well as the welfare of the participants in the chicken market. Once again, as in the case of multiple stages, the single market model can measure the full effects. Just, Hueth and Schmitz (1982, p. 192) put it succinctly: "...net social welfare effects over the economy as a whole of intervention in any single market can be measured completely in that market using equilibrium supply and demand curves of sufficient generality."

There are two alternative correct ways to measure welfare effects when there are multiple price changes and cross-price effects induced by a supply shift in one market<sup>25</sup>. The first is to add up effects across markets using the welfare measures taken off *ceteris paribus* supply and demand curves in all of the affected markets, all of which may shift as a consequence of an exogenous supply (or demand) shift. The second is to use the *mutatis mutandis* supply and demand curves for the commodity of interest,

in which case there are no *endogenous* shifts of any of the curves and there is no need to add up effects across related markets. When a "general equilibrium" model is used to measure the price and quantity changes caused by a supply shift in one market, those measured responses will reflect feedback from related markets. If those responses were used to compute welfare changes, it would be double counting to add them up across markets. The main point is that it is important to be aware of the dangers of double counting (and partial counting) and to ensure that the welfare changes are computed in a fashion that is consistent with the structure of the model.

Two difficult questions remain. First, how do we disentangle the effects among the related commodity markets and, from there, among factor suppliers and consumers? Second, what can be done when two or more exogenous displacements occur simultaneously?

Some recent work by Thurman (1990) provides a partial answer to the first question. Thurman (1990) explores the welfare significance (and non-significance) of general equilibrium supply and demand curves. He considers two goods that may be related through substitution in consumption, substitution in production, or both. He verifies the results of Just, Hueth and Schmitz (1982, p. 192) — that, if care is taken, the total welfare effects in both markets can be measured in the context of a single market. In addition, he points out that the areas behind the general equilibrium supply and demand curves for a commodity may have no welfare significance taken separately (in that they may not measure welfare of an identifiable group) although they do taken together (as a measure of the total welfare change). This is because the conventional welfare areas reflect welfare changes in related markets (for example, the area that conventionally represents changes in consumer surplus for beef may now contain some components due to changes

<sup>&</sup>lt;sup>25</sup> The arguments here are literally correct only when using compensated demand curves as done by Just, Hueth and Schmitz (1982) and Thurman (1991). They are approximately correct when the ordinary (uncompensated) demand curves are good approximations of the compensated ones which will be true when income effects are small. This qualification applies generally when Marshallian surplus measures are used to approximate compensating variation measures of welfare changes.

in producer surplus for chicken). The most useful result that he obtains — for the present purpose — is to show that when there is only one source of feedback (i.e. when the goods are related either through consumption or production but not both), the conventional measures of welfare change taken off the general equilibrium supply and demand curves do have significance (i.e. they do measure changes in welfare of identifiable groups)<sup>26</sup>.

These results mean that, with only one source of general equilibrium feedback, it is possible to measure the total welfare change — and its incidence due to a displacement in one market, taking account of general equilibrium adjustments. In applications to research-induced market displacements we have a remaining problem of defining the impact of technical change as a displacement of a general equilibrium supply or demand function. What is the percentage shift of a general equilibrium supply curve for chicken in response to a K percent reduction in costs of chicken production? The implications of Thurman's (1990) results are clearest for either (a) a supply shift when there is feedback to demand through substitution in consumption, or (b) a demand shift when there is feedback to supply through substitution in production. In these cases we can use the conventional (partial equilibrium) measures of research-induced displacements.

Regarding the second question above, it seems clear that welfare measurement with only one displacement is difficult enough in a general equilibrium setting with only one source of feedback. To allow two or more changes to occur simultaneously would be very difficult, even with limited options for feedback. It is not difficult to model price. quantity, and revenue changes in multiple factor and product markets with general equilibrium feedback and multiple displacements occurring simultaneously (e.g. see Mullen, Alston and Wohlgenant 1989). The difficulty is to measure the welfare consequences. For that reason, in all of the welfare analysis that follows attention is restricted to a single source of market displacement and only a single source of general equilibrium feedback.

#### 4.3 Multiple Products Related in Consumption

The simplest case of multiple products is a case

where two (or more) products are substitutes in consumption, but entirely unrelated in production (i.e. the technologies of production are independent and there are no specialized factors in common — any factors that are used in both products are perfectly elastically supplied to both industries). An example may be beer and wine. A supply and demand model for this case with N products could be written as:

Supply: 
$$QS_i = f_i(P_i, B_i)$$
 (31)

Demand: 
$$QD_i = g_i(P_1,...,P_N, A_i)$$
 (32)

Supply of each product (i) depends only upon its own price and exogenous supply shifters  $(B_i)$  but demand for each product depends on the prices of all the products and exogenous demand shifters  $(A_i)$ . Let us assume competitive market clearing  $(P_{s,i} = P_{d,i} = P_i)$  and  $Q_{s,i} = Q_{d,i} = Q_i$ . In logarithmic differential form, in the case of two products (i.e. for i = 1,2), the system of supply and demand equations may be written as:

Supply: 
$$E(Q_i) = \varepsilon_i [E(P_i) + \beta_i]$$
 (31')

Demand: 
$$E(Q_i) = \eta_{i1}E(P_1) + \eta_{i2}E(P_2) - \eta_{i1}\alpha_i(32)$$

In these equations the parameter definitions are slightly different from those used in the single product model. The elasticity of supply of product i  $(\epsilon_i)$  is as before, but the own-price elasticities and cross-price elasticities of demand  $(\eta_{ij})$  are the natural values rather than absolute values so that own-price elasticities are negative  $(\eta_{ii} < 0 \text{ i=1,2})$ . The solutions for relative changes in price are:

$$E(P_1) = -[(\eta_{11}\alpha_1 + \varepsilon_1\beta_1)(\varepsilon_2 - \eta_{22}) + \eta_{12}(\alpha_2 - \beta_2)]/D'$$
(33a)

<sup>&</sup>lt;sup>26</sup> For instance, consider the case when the demand curve for good 1 reflects equilibrium adjustments of prices for good 2 but changes in the good 2 market do not affect the supply of good 1 (i.e. there is feedback of price changes in good 2 into consumption of good 1 but not into production of good 1). Then the consumer surplus taken off the demand curve for good 1 measures consumer surplus changes in both markets plus producer surplus for good 2 while the producer surplus taken off the supply curve for good 1 is simply producer surplus for good 1. Similarly, when the goods are related in production (but not related in consumption) the "producer surplus" for good 1 measures producer surplus for both goods plus consumer surplus for good 2.

$$E(P_2) = -[(\eta_{22}\alpha_2 + \varepsilon_2\beta_2)(\varepsilon_1 - \eta_{11}) + \eta_{21}(\alpha_1 - \beta_1)]/D'$$
(33b)

where D' = 
$$(\varepsilon_1 - \eta_{11})(\varepsilon_2 - \eta_{22}) - \eta_{12}\eta_{21}$$
.

Notice that, when the two goods are independent in consumption as well as production (i.e. when the cross-elasticities of demand are zero), the equations for relative changes in prices revert to those that would apply for a single product analyzed in isolation:

$$E(P_i) = -(\eta_{ii}\alpha_i + \varepsilon_i\beta_i)/(\varepsilon_i - \eta_{ii})$$
 (33c)

The equations to be used to evaluate gross annual economic welfare changes are of the same form as equations (18') and (19'), but the interpretation here is that the subscripts denote different commodities rather than the same commodity in different countries:

$$\Delta CS_{i} = -P_{i}Q_{i}[E(P_{i}) - \alpha_{i}][1 + 0.5E(Q_{i})]$$
 (34a)

$$\Delta PS_{i} = P_{i}Q_{i}[E(P_{i}) - \beta_{i}][1 + 0.5E(Q_{i})]$$
 (34b)

In addition, and in contrast to the single product case, it is not appropriate simply to add up these measures across i's (now commodities rather than countries) to get a measure of total welfare change. As suggested by the quote above from Just, Hueth, and Schmitz (1982) the total welfare changes due to a supply (or demand) shift in the ith market are reflected in the general equilibrium measures of consumer and producer surplus changes in that market alone. Because the welfare measures in equations (34a) and (34b) are based on general equilibrium changes in prices and quantities, they are general equilibrium welfare measures. Adding up the welfare effects of a particular supply (or demand) shift across markets would lead to double counting. To measure the incidence of a change we may have to look across markets in a disaggregated fashion.

For example, consider the case of an increase in supply of good 1 ( $\beta_i > 0$ ) with no other exogenous shifts ( $\alpha_1 = \alpha_2 = \beta_2 = 0$ ). The correct measures of welfare change taken in the market for good 1 (assuming a parallel shift) are:

$$\Delta CS* = -P_1Q_1E(P_1)[1 + 0.5E(Q_1)]$$
 (34c)

$$\Delta PS_1 = P_1Q_1[E(P_1) - \beta_1][1 + 0.5E(Q_1)]$$
 (34d)

$$\Delta TS = \Delta CS * + \Delta PS_1 \tag{34e}$$

where  $\Delta CS*$  is the change in consumer surplus measured off the general equilibrium demand curve for good 1 and comprises consumer surplus from both goods plus producer surplus on good 2 ( $\Delta CS* = \Delta CS_1 + \Delta CS_2 + \Delta PS_2$ ). To disaggregate these measures further we can use:

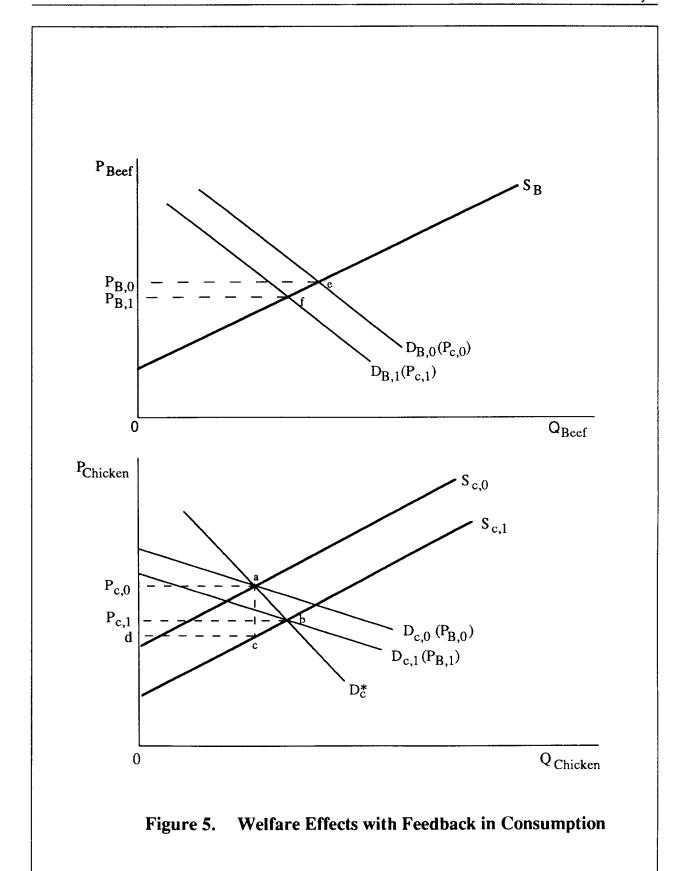
$$\Delta PS_2 = P_2 Q_2 E(P_2)[1 + 0.5E(Q_2)]$$
 (34f)

$$\Delta CS = \Delta CS * - \Delta PS_2 \tag{34g}$$

To clarify these points, consider Figure 5. In Figure 5, the lower panel represents the market for one good (say, chicken meat) and the upper panel represents the market for a substitute (say, beef). The initial demand curves (D<sub>C,0</sub> and D<sub>B,0</sub>) are defined in the usual way as conditioned on the price of the other good being constant at its initial value (P<sub>C,0</sub> or P<sub>B,0</sub>). When the supply curve for chicken meat shifts (from S<sub>C.0</sub> to S<sub>C.1</sub>), a series of general equilibrium type adjustments take place in both markets: a fall in the chicken price causes a fall in demand for beef (because they are substitutes) and the subsequent fall in beef price causes a fall in demand for chicken and so on. Ultimately, a new equilibrium is achieved at prices P<sub>B,1</sub> and P<sub>C,1</sub> with corresponding demand curves D<sub>C,1</sub> and D<sub>B,1</sub>.

The curve D\*C is the "general" equilibrium demand curve for chicken that traces out the demand response to price changes in the chicken market holding constant the supply curve for beef. The usual treatment — holding constant the price of beef — is a special case that applies when the supply curve for beef is perfectly elastic. The solutions from the equilibrium displacement model for relative changes in prices and quantities reflect these "general" equilibrium type responses. This fact is important when using the model solutions to compute welfare changes.

In Figure 5, the full welfare consequences of the shift of chicken supply can be measured as the area beneath the demand curve D\*<sub>C</sub> between the two



supply curves ( $S_{C,0}$  and  $S_{C,1}$ ). This area comprises "consumer surplus" of area  $P_{C,0}abP_{C,1}$  and (with parallel supply shifts) "producer surplus" equal to area  $P_{C,1}bcd$ . In this case the change in "consumer surplus" comprises changes in consumer surplus from consumption of both beef and chicken and changes in beef producer surplus. These components could be disentangled with a little effort: note that the fall in beef producer surplus is given by area  $P_{B,0}efP_{B,1}$ .

One question to consider is what are the consequences of ignoring the effects in the related market? We can consider that question by comparing equations (33a) and (33b) with (33c) in the case where only one supply function shifts (i.e.  $\beta_2 = \alpha_1 = \alpha_2 = 0$ ). Under these conditions (33a) and (33b) reduce to:

$$E(P_1) = -\varepsilon_1 \beta_1(\varepsilon_2 - \eta_{22})D' \qquad (33a')$$

$$E(P_2) = -\eta_{21}\beta_1/D' \tag{33b'}$$

Under the same set of conditions but erroneously assuming no price effects in the market for the other good (i.e. either the cross-elasticities are zero or the other good is perfectly elastically supplied ( $\varepsilon_2 = \infty$ )) equation (33c) becomes:

$$E(P_1) = -\varepsilon_1 \beta_1 / (\varepsilon_1 - \eta_{11}); E(P_2) = 0.$$
 (33c')

The welfare measures obtained by substituting equations (33a') and (33b') into equations (34c)-(34g) will be correct. Using equations (33c') instead will lead to errors, depending on the nature of substitution effects. When the two goods are gross substitutes ( $\eta_{21} > 0$ ), there will be an understatement of the declines in prices of both goods (and an understatement of the consumer benefits) and an overstatement of producer benefits (an overstatement of the producer surplus gain to producers of good 1 and an understatement of the loss of producer surplus for producers of good 2). For complements, the errors will differ.

#### 4.4 Multiple Products Related in Production

So far we have considered interaction among products only through substitution in consumption. Now we consider cases where multiple products are related either through their production technology or through factor use. Clearly in some cases production processes are interdependent among products, such as in the Australian cropping-grazing industry or in the dairy processing industry. An alternative way for product markets to interact is when they share the use of a specialized factor of production — i.e. a factor of production whose price is affected by the industries' output and factor use<sup>27</sup>.

Mullen, Wohlgenant and Farris (1988, pp. 247-9) presented a two-product, two-input model that they applied to the U.S. beef processing sector. In this model two products are produced using two specialized factors. The products are related in production and through factor markets but (surprisingly) not in consumption. The key simplifying assumptions are that the production function underlying the model is (a) characterised by constant returns to scale and (b) separable between inputs and outputs. Their model is as follows. The production function has the form:

$$Q(Y_1, Y_2) = F(X_1, X_2)$$
 (35a)

Because Q is linearly homogeneous in  $X_1$  and  $X_2$ , the cost function is separable:

$$C = h(W_1, W_2)Q$$
 (35b)

Corresponding output constrained input demand functions are obtained by application of Shephard's Lemma to this equation; that is,

$$X_1 = h_1(W_1, W_2)Q$$
 (35c)

$$X_2 = h_2(W_1, W_2)Q$$
 (35d)

The second part of the problem is to maximize revenue subject to a constrained level of inputs, F. Here, homogeneity conditions result is a separable revenue function:

<sup>&</sup>lt;sup>27</sup> Clear examples are (a) when livestock industries (e.g. hogs and chickens) affect feed-grain prices the supply functions of livestock products will be related, and (b) the use of milk in production of various dairy products (e.g. Perrin 1980). In these examples the products also are related in consumption and, perhaps, through technology of production.

$$R = r(P_1, P_2)F$$
 (35e)

Corresponding input constrained output supply functions are (by Hotelling's Lemma):

$$Y_1 = r_1(P_1, P_2)F$$
 (35f)

$$Y_2 = r_2(P_1, P_2)F$$
 (35g)

The system of logarithmic differential equations describing equilibrium becomes:

Final Demand:

$$EY_1 = -\eta_1 [EP_1 - \alpha_1]$$
 (36a)

$$EY_2 = -\eta_2[EP_2 - \alpha_2]$$
 (36b)

Constrained Output Supply (Transformation) and Factor Demand (Substitution):

$$EY_1 - EY_2 = \tau[EP_1 - EP_2]$$
 (36c)

$$EX_1 - EX_2 = \sigma[EW_1 - EW_2]$$
 (36d)

Market Equilibrium:

$$m_1EP_1 + m_2EP_2 = s_1EW_1 + s_2EW_2$$
 (36e)

$$m_1EY_1 + m_2EY_2 = s_1EX_1 + s_2EX_2$$
 (36f)

Factor Supply:

$$EX_1 = \varepsilon_1[EW_1 + \beta_1] \tag{36g}$$

$$EX_2 = \varepsilon_2[EW_2 + \beta_2] \tag{36h}$$

In these equations (with some slight changes from the notation used by Mullen, Wohlgenant and Farris 1988) the parameters and variables are mostly as defined above but there are some differences. The quantity of product i is  $Y_i$  and its price is  $P_i$ , the price of factor i is  $W_i$  and its quantity is  $X_i$ , the fraction of revenue accounted for by product i is  $m_i$ , the fraction of cost accounted for by factor i is  $s_i$ , the absolute value of the demand elasticity for product i is  $n_i$ , the supply elasticity for factor i is  $s_i$ , the elasticity of product transformation is t, and the elasticity of factor substitution is t.

This model includes only two types of equilibrium displacements — those due to shifts of final demand and those due to shifts of factor supply (the shift of demand for product i is  $\alpha_i$ , and the shift of supply for factor i is  $\beta_i$  — and it does not allow for the products to interact in consumption. Mullen, Wohlgenant and Farris (1988) show how to obtain numerical solutions to this model using matrix algebra. The solution is a vector of values for the relative changes in prices and quantities of the factors and products. Measures of welfare changes can then be computed using the following formulas:

$$\Delta CS_i = -P_i Y_i [E(P_i) - \alpha_i] [1 + 0.5E(Y_i)]$$
 (37a)

$$\Delta PS_i = W_i X_i [E(W_i) - \beta_i] [1 + 0.5E(X_i)]$$
 (37b)

$$\Delta CS = \sum_{i} \Delta CS_{i}$$
 (37c)

$$\Delta PS = \sum_{i} \Delta PS_{i} \tag{37d}$$

$$\Delta TS = \Delta PS + \Delta CS \tag{37e}$$

where equation (37a) measures the change in consumer surplus in consumption of good i, equation (37b) measures the change in producer surplus in supplying factor i, equation (37c) measures the change in consumer surplus across both products, equation (37d) measures the change in producer surplus on all factors, and equation (37e) measures the total welfare change.

In the equilibrium displacement model it would be straightforward to allow for substitution in consumption between the two goods. This would entail simply augmenting each demand equation with a cross-price term (in equation (36a) add " $+\eta_{12}EP_2$ " and in equation (36b) add " $+\eta_{21}EP_1$ "). However, the problem of computing surplus effects becomes more awkward once substitution is allowed in consumption. One must address the issues raised in the previous section in relation to avoiding double-counting consumer surplus effects but incorporating general equilibrium feedback effects.

It would also be reasonably easy to extend the model to involve more factors (say N) and more products (say M). The equations for final demand and factor supply could be of the exact form shown

in equations (36a,b,g and h). The market equilibrium conditions would need to be extended to include (share weighted) quantities (36e) and prices (36f) of all factors and products. The constrained output supply and factor demand equations are expressed in difference form in equations (36c) and (36d) and these would have to be replaced with the following (undifferenced) equations:

Output Supply:

$$EY_i = \sum_{j=1}^{M} m_j \tau_{ij} EP_j + EF$$
 (36c')

Input Demand:

$$EX_i = \sum_{k=1}^{N} s_k \sigma_{ik} EW_k + EQ$$
 (36d')

In these equations  $\tau_{ij}$  = the partial elasticity of transformation between product i and product j and  $\sigma_{ik}$  = the partial elasticity of substitution between factor i and factor k. Adding up and symmetry restrictions apply both to elasticities of transformation and to elasticities of substitution. The other variables and parameters are as defined above. To close the model we use EF = EQ.

### 4.5 Changes in Product Quality

A recurring problem in the analysis of effects of new technology is the question of whether the changes in technology involve changes in product quality characteristics as well as changes in factor use for a given product. For example, the mechanical tomato harvester required that the tomatoes be sufficiently robust to withstand the process; higher yielding wheat varieties may have lower protein content. In most cases it seems likely that technological changes will involve some changes in product characteristics, and sometimes these changes will be very important. However, for the most part agricultural economists have begged the question of modeling jointly technological changes and associated product quality changes. Several recent studies have made some inroads into this area including Unnevehr (1986, 1990), Lemieux and Wohlgenant (1989), Macagno (1990), and Voon and Edwards (1990a, 1990b, 1991). The study by Ulrich, Furtan and Schmitz (1987) is pertinent as well.

One approach to this problem is to use a multiproduct model of the type described in the previous section(s) and either to treat product characteristics as products (so that "quality" is continuously variable) or treat different qualities of products as being different products (discrete variation in "quality"). The latter approach may be more restrictive but also probably more practicable. An example is the study of wheat varieties by Brennan, Godyn, and Johnson (1989).

The approach most commonly used in the literature is to introduce an *ad hoc* shift in demand for the product induced by changes in quality. Technical change that leads to a change in product quality is a change in supply conditions *not* demand conditions, and it would be better to model it as such<sup>28</sup>. It is not known what conditions are required in order that a change in supply of product quality can be treated legitimately as an *equivalent* change in demand; nor is it known in what senses the changes will be equivalent. These are topics for further work.

On the other hand, using a multi-product modeling approach to deal with quality changes may not be easy because the substitution effects between different qualities of a particular product are likely to be large and highly important and they are also likely to be very difficult to measure — especially for ex ante studies where the different qualities might not exist when the analysis is being undertaken. In addition, substitution effects in production (say, between two varieties of wheat) are likely to be too important to dismiss when dealing with various qualities of a particular product and these, too, are difficult to quantify. Thus, to model quality changes formally may require using a model with multiple sources of general equilibrium feedback. We have seen above that measuring welfare changes may be intractable in such a setting, even if we were confident about the measures of substitution effects in production and consumption among quali-

<sup>&</sup>lt;sup>28</sup> For instance, the development of technology for filtered cigarettes may have had gross effects similar to those from an increase in demand for the aggregate good "cigarettes" (i.e. greater sales at a higher price), but it might at the same time have led to a reduction in demand for tobacco per cigarette with an ambiguous net effect on demand for tobacco. Modeling this change simply as an increase in demand for cigarettes would lead to an erroneous conclusion that demand for all inputs used in cigarettes had increased.

ties. However, in some cases, where product quality change is important, a formal attempt to measure its effects in a logically consistent fashion may be worthwhile.

# 5. Conclusion

This article is about measuring the benefits from research, and their distribution, in a multiple market setting. Just, Hueth, and Schmitz (1982) have shown that the total effects of supply and demand shifts can be measured in the market in which they occur so long as one uses supply and demand curves of sufficient generality. Thus, to a great extent, the issues about measurement of total benefits that arise in a multi-market setting have been covered adequately in previous (single market) studies of research benefits. The remaining issues to be addressed concern the distribution of benefits from R&D among groups of producers, consumers, factor suppliers, regions, or nations. The bulk of work in this paper has concerned how to disaggregate the measures of research benefits that would be obtained from treating R&D in a single market context.

Two general theoretical issues have been raised throughout this paper. First, how do we represent technical changes — and model their effects on equilibrium displacements — in a theoretically consistent manner? Second, how do we translate those equilibrium displacements into welfare measures? We can answer some parts of these questions completely, but we can answer the complete questions only in part. In addition, there are the practical and empirical questions of how to measure the relevant parameters and shift variables (the latter representing the effects of the exogenous technical changes). We do not attempt to answer these questions here, but they do impinge on the choice of practicable solutions to the theoretical issues.

A single modeling approach, following Muth (1964) and Alston and Scobie (1982) has been considered. Using a linear elasticity model it is possible to represent a variety of technical changes and to show effects among multiple factor and product markets while retaining strong links to economic theory. Clear algebraic solutions may be obtained for a single product with two factors or for two

products with a single factor. Once we go beyond these extreme cases, disaggregating research benefits analytically among factors or products quickly becomes intractable and we must settle for numerical solutions only.

The primal approach used by Muth (1964) is advantageous for seeing clearly the nature of the technical change in the model, but is difficult to use for more than two factors; the dual approach suggested by Wohlgenant (1982) is more tractable to solve for larger numbers of factors (and products) but how to incorporate technical change in the model (especially biased technical change) is a vexed issue. Most studies with more than two factors have not confronted this question and instead have settled for using a factor supply shift to represent an "equivalent" biased technical change. It seems that there is potential for error here and that in general it would be better to model the technical change in question directly rather than as some partial "equivalent". Similar comments apply to the treatment of quality change as an "equivalent" demand shift.

There have been many studies of distribution of research benefits among multiple markets for a single product (i.e. among countries). The analytics are relatively clear and empirical results abound. On the other hand, there have not been many studies of distribution among different commodities. Areas that warrant further work include cases when multiple products are related (a) only in consumption, (b) only in production, (c) only through factor markets, or (d) in some combination of multiple products being related in production, consumption and factor markets. Multiple product models may be a way to get at research-induced quality changes as well.

In relation to measuring the welfare effects from a given set of displacements the issues are primarily: (a) What is the size of the initial displacement? (b) What are the functional forms of supply and demand? (c) What is the functional form of the supply (or demand) shift? and (d) What are the feed-back effects (in a general equilibrium sense)<sup>29</sup>? In the

<sup>&</sup>lt;sup>29</sup> Thurman and Wohlgenant (1989) show some implications of general equilibrium feedback for empirical estimation of supply and demand equations.

model used here we have a range of types of parametric equilibrium displacements. We have used linear elasticity approximations to general supply and demand functions and assumed parallel shifts. The linear elasticity approximation is good for small displacements with parallel shifts and it is easy to apply to obtain measures of changes in prices and quantities for the general class of multifactor multi-product models that has been considered in this article. The remaining issue is the one about general equilibrium feed-back effects: that is, how do we translate the changes in prices and quantities into measures of welfare change?

The economic theory of welfare measurement in a general equilibrium context seems to be uncontroversial. Just, Hueth and Schmitz (1982, Chapter 9) provide a good exposition. All that remains is to apply that received theory competently to the particular cases of general equilibrium, equilibrium displacements that are of interest. Thurman (1990) makes clear, however, that disaggregating surpluses is difficult when there is more than one source of general equilibrium feedback. Thus the work here has been restricted to single sources of general equilibrium feedback with single exogenous displacements occurring in isolation. It is still not clear how to measure the distributional consequences of research-induced technical change in the more general case of multiple simultaneous exogenous market displacements with multiple paths of general equilibrium feedback.

Further refinements might be made in a number of directions. These include: (a) allowing for imperfect competition<sup>30</sup>; (b) taking explicit account of government intervention in the commodity and factor markets of interest<sup>31</sup>; (c) allowing explicitly for dynamic responses over time (in terms of both adoption of innovations and evolution of supply and demand responses to price changes); and (d) allowing for stochastic elements (and uncertainty) in supply and demand response functions. Some of these topics have received some attention recently in the literature on returns to research.

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<sup>&</sup>lt;sup>30</sup> Extreme cases such as pure monopoly or pure monopsony may be reasonably straightforward but models involving monopolistic competition or oligopoly are likely to be very complicated or very restrictive (e.g. see Holloway 1991).

<sup>&</sup>lt;sup>31</sup> This could include price policies (e.g. Alston, Edwards and Freebaim 1988) and measures used to raise revenues to fund R&D (e.g. Alston, Mullen and Ridley 1988) — or other market distortions (e.g. externalities from agricultural chemicals).

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