Optimal Oyster Management in Chesapeake Bay Incorporating Sanctuaries, Reserves, Aquaculture and Externalities

Nikolaos Mykoniatis
Department of Agricultural Economics Sociology and Education
The Pennsylvania State University
University Park, PA 16802 USA
Tlf: 814-441-9501
E-mail: nzm132@psu.edu

Richard Ready
Department of Agricultural Economics Sociology and Education
The Pennsylvania State University
University Park, PA 16802 USA

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1. Introduction

The Eastern oyster (*Crassostrea Virginica*) has been a keystone species for the Chesapeake Bay (Bay hereafter) from both a commercial and an ecological point of view. Its peak in terms of employment and harvest was during the 1880s, when about 50,000 oystermen were in business harvesting about 17 million bushels per year (CBF Report, 2010 p.4). Moreover, historical annual harvest from 1920 to 1969 ranged from 3.6 to 5.8 million bushels, with an average of 4.9 million bushels (Lipton, 2008) and an average economic impact of $207 million a year for the states of Maryland and Virginia (CBF Report, 2010 p.6). In addition, oysters are filter-feeders, with an adult oyster[1] able to filter up to fifty gallons of water a day (CBF Report, 2010 p.10), removing nitrogen as well as phosphorus from the Bay.

Oysters remove nutrients in two ways. First, nutrients contained in the phytoplankton consumed by oysters become part of oysters' shell and meat[2] and removed from the Bay when oysters are harvested. For the direct nitrogen removal through harvesting, reports indicate that 700 to 5,500 pounds of total nitrogen are removed annually per one millions market-sized oysters (Chesapeake Bay TMDL, Appendix U). Second, oysters remove nutrients while still in water by accelerating denitrification in sediments (Newell, 2004). Therefore, if present at large numbers, oysters have the potential to significantly improve the water quality and clarity of the Bay. The ecological role of native oysters has been further acknowledged in the filter-feeder component calculation part of Total Maximum Daily Load (TMDL), taken into account by the Chesapeake Bay Water Quality and Sediment Transport Model. Lastly, oyster reefs provide habitat, shelter against predation at early stages of life and other microorganisms, as sources of food, for numerous species such as fishes and blue crabs (CBF Report, 2010 p.14).

Due to overharvesting between the 1890s and 1930 oyster landings dropped, from the reported 1880s figure of 17 million bushels, by two-thirds and remained stable until 1950. Then two parasite-driven diseases, MSX and Dermo made their appearance and coupled with

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[1] An adult oyster is currently defined as of at least three inches. The species grows approximately one inch per year with its rate of growth generally being dependent on water temperature, salinity, disease, food quality and habitat

[2] This process is known as nitrogen (or phosphorus) assimilation (Stephenson, 2009; Chesapeake Bay TMDL).

overharvesting and habitat degradation[3] they devastated oyster population in the Bay. Nowadays, harvests have declined to less than one percent of their historic levels in the late 19th century (Wilberg et al., 2011), with cumulative monetary losses of more than $4 billion for the economies of Maryland and Virginia (CBF Report, 2010 p.6) and a dramatic impact to local employment in the fishery. With oyster employment, harvest and income showing decreasing trends as of 2010, Chesapeake Bay Foundation Report (2010) highlights the link between Bay's boost in economic growth and restoration of native oyster populations.

An important aspect of current oyster management is spatial segregation of the Bay into four distinct types of areas: oyster sanctuaries, harvest reserves, privately-leased aquaculture areas and areas that are open for harvest to any licensed boat. Oyster sanctuaries are no-harvest zones. They are created not only to protect the existing stock from harvest, but also to facilitate restoration activities and in particular oyster reef construction. While protecting the existing stock, sanctuaries provide the proper substrate for successful oyster larval settlement and survival. Besides their adequate bottom condition, sanctuaries are created in areas, where the existing stock has already survived disease. By eliminating fishing pressure sanctuaries ensure the maintenance of a disease-resistant stock. Moreover, greater oyster abundance associated with sanctuaries promotes biodiversity along with water quality and clarity boosting tourism, swimming and other recreational activities (Brumbaugh and Toropova, 2008). Nonetheless, in 2009 both Maryland and Virginia, along with the U.S Army Corps of Engineers, as part of their efforts to restore oyster stock in the Bay, rejected the idea of imposing a total moratorium on harvest. This is so because the permanent closure of oyster grounds greatly harms watermen's income. For that reason harvest reserves (reserves hereafter) are relevant. These areas are chosen based on adequate bottom conditions and low-to-moderate disease mortality (CBP, 2004). Reserves are areas that are closed to harvest for a period of time until the majority of the stock reaches a specified biomass threshold[4]. Once oysters have reached the desired size, the area is available to licensed harvest for only a very short period of time, typically days.

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[3] One of the harvesting techniques as we will see below, which was massively used in the period of excess harvesting is power dredging. This method progressively " flattens" oyster reefs destroying the habitat for future oysters, which need hard bottom surfaces for their settlement and survival (CBF Report, 2010 p.14).

[4] This threshold is usually measured by the length of oysters' shell. The most common threshold is three to four inches, which is the shell length for commercially harvestable oysters.

Commercial oyster aquaculture takes place in privately-leased areas and consists of two types: contained off-bottom and on-bottom. Contained off-bottom aquaculture uses either cages or floating rafts, whereas on-bottom utilizes spat-on-shell, a method that involves spreading seed oysters on shell at the bottom and allow them to grow (Wieland, 2008). Even though its off-bottom counterpart is emerging, on aggregate, on-bottom lease is the most common type of aquaculture[5] in the Bay (Webster D.W. and Wieland R. personal communication). Contrary to Maryland, Virginia has had a long and commercially successful history of oyster aquaculture, coupled with an efficient institutional framework to manage the industry (Wieland, 2008). Nonetheless, recently Maryland is trying to catch up by updating its laws and allowing a greater percentage of oyster bars for leasing.

The areas open to harvest to any licensed boat consist the remaining segment of the Bay. This segregation is what the literature refers to as an open access fishery. The primary harvesting methods at the Bay's public fishery are: diving, power dredging, shaft (or hand) tonging and patent tonging (Wieland, 2006). The above gear types have different efficiency levels and are applicable at different depths. Furthermore, they have been observed to change with varying stock of oysters. In particular, declining oyster populations imply that less efficient methods, such as shaft tonging become less important (Wieland, 2006).

2. Research questions and their significance

The present study addresses three research questions. Below each one is presented along with their significance. The first research question can be summarized as follows: How long should reserves stay closed between each pulse of harvest? To understand the importance of this research question several aspects should be pointed out. First, even though Bay is segregated into distinct areas the four systems are really close to each other. Second, oyster larvae[6] are "free swimmers" and they can passively move from one area to another depending on water currents

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[5] The "most common type" of oyster aquaculture is measured in terms of water area devoted to on-bottom type and profitability (Webster. D. W. and Wieland R. personal communication).

[6] Oysters that have hatched from their eggs but have not developed their shell yet.

(CBP, 2004). Of course, in a given area, this randomness is reduced depending on the quantity and quality of its substrate upon which larvae settle (CBF Report, 2010). Nonetheless, the point that needs to be made is that larval dispersal constitutes an important link between all areas and can be viewed as the first type of externality this paper addresses. We will often refer to this externality as stock spillover. Third, as mentioned in the introduction, oysters contribute to denitrification and nitrogen removal through harvest, both of which can be viewed as environmental spillovers with important economic and non-market benefits. It is now that the importance of our research question becomes clear. On the one hand, by closing the reserve for a specified period of time the stock is allowed to grow allowing both stock and environmental spillovers to occur. On the other hand the opening of the reserve results in pulsed harvest that instantaneously boosts watermen's income at the expense of externalities. Therefore, at any instant of time the opening of the reserve brings to light the tradeoff between leaving the stock in the water versus harvesting it.

The second research question can be stated as follows: What is the optimal combination of management strategies? This question is important because its answer would determine whether the current area allocation is indeed optimal[7]. Public oyster areas receive stock spillovers from neighboring areas, while they are subject to continuous harvest making their contribution to denitrification and stock externalities small. On the contrary, sanctuaries provide significantly greater stock recruitment and nutrient removal at the expense of watermen's income. In addition, sanctuaries involve restoration activities and creation costs. In between the two extremes just mentioned, the reserves are the middle ground providing spillovers only for a specified period, while instantaneously boosting watermen's income. However, oyster seed, shell and other restoration activities are also included in the creation of reserves (CBF, Report 2010). Moreover, both sanctuaries and reserves involve enforcement and monitoring costs[8]. All such costs are not only limited, but also competing resources, for both management systems. It might be therefore optimal to devote all funds to sanctuaries, stop operating the reserves and devote more space to public harvest or aquaculture.

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[7] Actual acres devoted to each management regime will be presented latter.

[8] Sanctuaries and reserves, for the time they remain closed, should be protected against poaching. In addition, during the harvest pulse in reserve, monitoring takes place to ensure that certain rules, such as duration of harvest and stock-size limits are followed. The significance of explicitly modeling such interactions is addressed later.

The third and last research question examined in this paper is the following: How important is oysters' interaction with a second species, namely the blue crab, and how important a role can oysters play in meeting water quality goals? The impact of oysters on blue crabs constitutes the third type of externality associated with enhanced biodiversity the increased oyster stock promotes. We will often refer to this externality as biodiversity spillover. As we saw above, oyster densities promote biodiversity by providing food and shelter to various species. The reason we associate oyster abundance with the blue crab stock is due to the commercial importance and long history of the latter. Moreover, besides including the value of clean water in social welfare, we examine the importance of allowing oysters to contribute towards the achievement of TMDL targets versus not.

To answer our research questions we develop, solve and explore numerically an optimal control model that tracks oyster stocks in each management area, along with the blue crab stock and nitrogen pollutant[9] in the Bay. The model allows nitrogen to be removed not only through oyster management, but also through direct pollution control, namely Best Management Practices (BMPs). We examine the sensitivity of our results based on two different costs of BMPs taken from the literature (Nelson, 2005; Wieland et al., 2009). In addition, the opening of the reserve causes a "jump" in area's stock and this effect is captured by our model. Our methodology addresses the consequences of such a jump on the public stock, on Bay's stock, on the amount of nitrogen reduced via BMPs and on the stock of blue crabs. Moreover, we allow social welfare to include the value of clean water implied by meeting the TMDL target for nitrogen. Finally, all spillover effects are explicitly modeled. Control variables include the amount of area dedicated to each management system, the duration of closure in the reserves, as well as harvest effort for oysters and crabs.

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[9] Our methodology can be also readily applied to phosphorus.

3. Literature Review

A considerable part of the literature deals with a marine reserve or sanctuary, both defined as permanent no-harvest areas, located near an open access fishery. Holland and Brazee (1996) use a dynamic model of an inshore single species fishery incorporating age-class dynamics, along with a stock recruitment relationship. The authors examine what the optimal size of the reserve should be by maximizing the present value of harvest rather than maximization of sustainable yield (Hannesson, 1998). An important assumption is that effort is fixed and able to be displaced due to increased rents associated with the creation of reserve. Among their main findings is that the benefits the reserve provides depends on two things: their effect on public fishery and the discount rate. In particular, as short-run harvest loss due to reserve goes up, the smaller the reserve benefits and therefore the smaller their optimal size should be. Same argument holds for the discount rate. The empirical part demonstrates that the more exploited the fishery is, in terms of devoted effort, the bigger the reserve size should be in order to achieve maximum value for the fishery.

Later on, Holland (2000) developed an empirical spatially explicit age-structural bioeconomic model, integrated with a discrete choice, random utility nested-logit model of fleet dynamics. In addition, the model includes more than one species and total space is divided into main and subareas with species' seasonal movements and interactions defined. Holland investigates how the different sizes, shapes and spatial configuration of reserves affect the distribution of stock, rents and effort and how the latter feedbacks into stock abundance. Effort is endogenously determined. The main finding is that if the pre-reserve effort is too high, reserves have the potential to boost both harvest and income (Holland, 2000). However, it is pointed out that reserves are second-best solutions compared to more direct controls on effort and harvest. The general conclusion that in a deterministic setting, if catch and effort can be perfectly and permanently controlled make the creation of reserves essentially needless is also supported by other studies (Holland and Brazee, 1996; Anderson, 2002).

Both no-harvest and open access areas are also examined by Anderson (2002) using different methodology, while examining the effect of the reserve area on the total stock size[10], as well as the stock in reserve. The author sets up a discrete model assuming homogenous productivity in the entire area (both reserve and public) and the logistic function for the stock. Harvest occurs during the first part of every period during which the stock neither grows nor migrates. The reserve enables the protected fish to migrate to the neighboring area during the second part of every period. Among the main findings is that migration coefficient and effective carrying capacity are important factors for whether a positive minimum total stock size will be achieved, as well as whether the equilibrium total stock size and stock size in the reserve will increase or not (Anderson, 2002). Hannesson (1998) compares the performance of three regimes: a combination of reserve with a public fishing area next to it, open access in the entire area and an optimal fishing regime defined in terms of maximized annual sustainable rents. Similar to Anderson, the author uses the logistic growth function but sets up a discrete as well as a continuous model. Hannesson's findings are in accordance with previous ones regarding the efficiency of reserves relative to cost-effective regulation of fishing effort and capacity. His findings also indicate that if migration rate is too high the protected stock will be overharvested despite the existence of reserve. Furthermore, the increased catchable stock from reserve relative to its counterpart from the entire-open-access regime would still lead to dissipation of economic rents. The most striking result is that reserves have to be a big proportion of the entire area (approximately 70-80%), in order to achieve stock and harvest levels similar to the optimal-fishing system. However, even in that case, if effort and capacity are not properly controlled, all benefits will vanish.

Moving from the research questions of optimal reserve size and relative short run increase of stock and rents, Pezzey and colleagues (2000) ask the important question of whether reserve creation will increase both aggregate sustainable biomass, as well as aggregate sustainable harvest compared to the pre-reserve regime. In their model, the entire reserve management area (a reef) is treated as being spatially homogenous. In addition, adult fish are assumed to be completely immobile, while their eggs and larvae are uniformly dispersed from the reserve to the area of open access. The authors keep unit cost of effort and fish price constant

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[10] Total stock size refers to the stock of both the reserve and public fishing area.

while ignoring non-consumptive and ecological benefits from reserves. Their main analytical finding is that if the initial density[11] prior to reserve is less than half, its creation will increase aggregate sustainable catch. What Pezzey and colleagues stress as a limitation, i.e. not modeling ecological benefits, it is addressed by others (Kar and Chakraborty, 2009) in terms of ecotourism, while taking also into account predator-prey dynamics.

The same research question is posed by Sanchirico and Wilen (2001). The authors call the potential join increase of sustainable biomass and harvest, induced by reserve creation, a "double-payoff" or a win-win situation for both stock and fishermen, acknowledging the political and economic difficulties of directly controlling (perfectly and permanently) catch and effort. In their important work they consider a patchy and heterogeneous management area with biophysical linkages, connecting patches and non-uniform neither perfect and instantaneous dispersal mechanics (Sanchirico and Wilen 2001). Economic behavior is modeled as costless entry and exit into the fishery, with agents corresponding to emerging profit opportunities by moving across space. Three systems are analyzed in detail: a closed one, a sink-source and a density-dependent using two patches in each system as an example. The main result is that besides in the case of a closed system, where closing one of the patches yields only increased biomass, in the other two systems there are chances of achieving the desired win-win outcome. The success of this depends upon the nature of the initial conditions of the patch under closure, its type (whether it is a sink or a source), the relative cost-price ratio, dispersal intensity and the intrinsic growth rates.

In a subsequent paper and adopting the same methodology, Sanchirico and Wilen (2002) show that placing a reserve next to an overexploited area managed by input and license controls, could result in the "double-payoff" outcome between aggregate biomass and license price[12]. Moreover, the authors demonstrate that when purely biological criteria are used in favor of reserve creation, fishermen participation will be limited.

Another short but relevant to our research literature is the interactions between aquaculture and public fisheries, along with modeling approaches of the first. Mikkelsen (2007)

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[11] Defined as the ratio of fish population to its carrying capacity (Pezzey et al., 2000).

[12] License price is an indicator of fishermen's participation in the industry. A drop of licensed price signals limited participation and vice versa (Sanchirico and Wilen 2002).

explicitly models such interactions as negative spillovers from aquaculture's volume on carrying capacity, intrinsic growth rate (ecological effect), as well as on the catchability coefficient (fishing efficiency effect). The spillovers are addressed in terms of steady-state values of stock, fishing effort, yield and rents under open access and sole-ownership (Mikkelsen, 2007). Moreover, results are analyzed under two different management regimes, namely primary property rights and social optimum. On the contrary, Karp and colleagues (1986) examine three decisions relevant to aquaculture namely optimal harvest, optimal time for restocking and optimal level of farmed stock. Two aspects from their approach are relevant to our study: The concept of optimal time and the fact that harvest is modeled as "jump" in the stock biomass[13]. Impulsive control theory has made use of both concepts in a variety of important applications such as partial aquaculture harvesting (Yu and Leung, 2006); simultaneous management of moose and trees (Bergland et al., 2006) and deciding between a continuous tree growth versus clear-cut harvest regime as extension to the traditional Faustmann rotation model (Termansen, 2007).

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[13] In addition to harvest, restocking is also modeled as jump in the stock biomass in the paper by Karp and colleagues but we do not model such economic decision here.

4. A Model of Eastern Oysters, Blue Crabs and Externalities

We begin setting up the model with some notation and basic assumptions. Let a_j being the unit of area devoted to a particular management system with $j = S, A, R, W$ indexing them. An important assumption is that the area available for harvest to any licensed boat can be perfectly regulated. We are therefore indexing this area by the subscript "W" denoting a regulated public harvest regime, or what we will refer to as the "public area"[14]. In should be clear that this management system is different from an unregulated open-access fishery. In addition, the four oyster areas do not overlap with each other ($a_S \cap a_R \cap a_R \cap a_W = \emptyset$) and the second species, namely the blue crab, is uniformly distributed in the entire area. The model does not distinguish between small and market-sized stock but rather it deals only with the harvestable stock. Moreover, Bay's stock per unit area (O) is defined as the weighted average of populations in the four areas. With $\mathrm{O_j}$ being the oyster stock per unit of given area, the total stock per unit area is:

$$O = \frac{\sum_j a_j O_j}{\sum_j a_j} = \frac{\sum_j a_j O_j}{a_{BAY}}, \mathrm{j = S, R, A, W}$$

where $a_{BAY} = \sum_j a_j$ stands for the total area devoted to the four management systems.

We now turn to the nitrogen dynamics. Let $\bar{L}$ be the loadings of nitrogen ending up in the water body. Nitrogen loadings include point, non-point and atmospheric deposition of the pollutant. For simplicity, we ignore nitrogen recycling and focus only on elements of nitrogen dynamics that can be affected by management. We further assume that some amount of nitrogen, say R is reduced through Best Management Practices, at the constant unit cost of l. Given the exogenous level of $\bar{L}$ let d be the natural decay of nitrogen excluding nitrogen assimilation in oysters' biomass, as well as the amount of nitrogen removed through denitrification. The amount of nitrogen in oysters' shell and meat, which is removed through aggregate harvest (H_O) is denoted by s. Furthermore, it is assumed that denitrification is a linear function of the Bay's stock with ρ being the constant denitrification rate per adult oyster present in all areas. We can then write the state equation for nitrogen as:

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[14] The rest of the notation clearly stands for sanctuary, aquaculture and reserve respectively.

$$\dot{N} = \overline{L} - R - \rho a_{BAY} O - sH_O - dN$$

The effect of parameters ρ and s on nitrogen dynamics constitutes the environmental spillover related to Bay's aggregate stock and harvest respectively. Finally, we assume that there exist other commercial and non-use benefits related to clean water from which society derives utility. Therefore, we denote the value of clean water related to nitrogen by $\mathrm{U(N)}$. This is a term in the social welfare function and it is measured in monetary terms. In particular, $\mathrm{U(N)}$ reflects society's willingness to pay for reduction in Bay's water ambient nitrogen. It is conjectured that $U'(N) < 0$ and $U''(N) \geq 0$. The initial amount of nitrogen in Bay's water body is $N(0) = N_0$.

We now start the description of biology and economics of each area by oyster aquaculture. It is assumed that private leased areas do not have a particular biology that requires explicit modeling. We therefore do not account for aquaculture stock dynamics, but rather take the stock per unit area O_A fixed at its maximum sustainable level. That is $O_A = K_O/2$ with K_O being the carrying capacity of oysters per unit area in the Bay. The economic decisions in aquaculture are modeled as a simple production volume model with quadratic costs. With P_O, y and H_A being the exogenous price, cost parameter and volume of oyster production measured in per unit area terms, rents in aquaculture are given as

$$\Pi_A = P_O a_A H_A - y a_A H_A^2$$

which yields a maximized level of total harvest equal to $H_A^* = a_A \frac{P_O}{2y}$ and a total cost of $a_A \frac{{P_O}^2}{4y}$. The social planner knowing that private leased operators maximize their rents in the preceding manner, will always set harvest at H_A^* in the social welfare function, incurring the associated cost[15].

Before the sanctuary and the public area are described, a few important assumptions about the biology of the remaining three areas need to be pointed out. To begin with, all areas are taken to be homogenous in the sense that there is no difference in stock productivity coming from different bottom conditions[16]. Moreover, stock dynamics in each of the remaining areas are

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[15] We implicitly assume here that social planner plays at every period a Stackelberg game with private leased operators.

[16] Differentiating the productivity in each area would imply modeling the supply of oyster seed and shell, both of which determine the quality of bottom condition.

decomposed in two elements: (i) the somatic growth of area's adult stock $F(O_n)$, $n = S, W, R$ and (ii) the recruitment from Bay's aggregate stock per unit area $I(O)$. The last term captures the stock spillover, which is a function of the total stock per unit area. Area-specific recruitment is therefore not modeled explicitly but we rather assume that adult oysters, from all areas, release their eggs which are "mixed" and returned back as larvae in a random and unknown manner, partially defined by Bay's hydrological conditions. $I(O)$ increases the adult stock of a specific area over time, even in the absence of any adult stock at $t = 0$. We can now write the state equation for the stock per unit area as

$$\dot{O_n} = F(O_n) + I(O), \qquad n = S, W, R \quad (*)$$

Finally, for simplicity we assume that stock's productivity is not affected by nitrogen levels in the Bay. It is conjectured that $\frac{\partial \mathrm{F}}{\partial \mathrm{O_n}} \geq 0$ for $\mathrm{O_n} \leq \overline{\mathrm{O}} = K_O/2$ and $\frac{\partial \mathrm{F}}{\partial \mathrm{O_n}} < 0$ for $\mathrm{O_n} > \overline{\mathrm{O}} = K_O/2$ with $\overline{\mathrm{O}}$ being the oyster density that yields the maximum growth. Moreover, $\frac{\partial \mathrm{I}}{\partial \mathrm{O}} \geq 0$.

Turning to the description of the sanctuary and reserve, the stock dynamics of the first are given by (*)[17] since harvest is not permitted in the area. There are no economic decisions to be made in the sanctuary. Nonetheless, there are creation costs per unit area. We denote them by C_S^{cr} and are assumed to occur only once, at the beginning of the planning horizon. As for the enforcement and monitoring costs, even though they occur at every instant of time they will not be modeled because they are not significantly different among areas[18].

The stock dynamics of the public area should include the effect of harvest, which we denote by H_W. Adopting the catch-per-unit-effort assumption (Clark, 1990) with q_O and E_W being the catchability coefficient and effort per unit area respectively, expression (*) is rewritten as

$$\dot{O_W} = F(O_W) + I(O) - q_O E_W O_W$$

Watermen in the public area are price takers and with v being the unit cost of effort, rents per unit area are

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[17] After replacing the subscript "n" with "S".

[18] As Maryland's Natural Resource Police unit informed us (Lt. Kirkwood, pers. communication) the time a typical officer spends monitoring each of the four management regimes is the same. Therefore, monitoring and enforcement costs will not be included in either sanctuary or reserve.

$$\Pi_W = P_O q_o E_W O_W - vE_W$$

E_W is a control variable, while both H_W and vE_W are elements of the social welfare function[19]. Before addressing the reserve and its associated jumps it would be useful to describe the second species affected by oyster management, the blue crabs denoted by C. Blue crabs are uniformly distributed in the entire area and their population is taken to be positively affected by oyster abundance. Therefore, it is logical to model aggregate oyster stock having a positive effect on either blue crabs' intrinsic growth rate, carrying capacity or both. Such an effect can be generically presented in the crab state equation as

$$\dot{\mathrm{C}} = F_C(\mathrm{C}, \mathrm{O}) - \mathrm{q_C E_C C}$$

where F_C represents the growth of blue crabs (including recruitment) as a function of the species' stock, as well as the Bay's aggregate stock. The dependence of F_C on O represents the third spillover this study models and that is related to biodiversity. In is conjectured that $\frac{\partial \mathrm{F_C}}{\partial \mathrm{C}} \geq 0$ for $C \leq \bar{C} = K_C/2$ and $\frac{\partial \mathrm{F_C}}{\partial \mathrm{C}} < 0$ for $> \bar{C} = K_C/2$, as well as $\frac{\partial \mathrm{F_C}}{\partial \mathrm{O}} > 0$, $\frac{\partial^2 \mathrm{F_C}}{\partial \mathrm{O}^2} \leq 0$ with K_C being crabs' carrying capacity. Watermen are price takers ($\mathrm{P_C}$) and with m being the unit cost of effort, rents are

$$\Pi_\mathrm{C} = \mathrm{P_C q_C E_C C - mE_C}$$

Similar to oystering in the public area, $\mathrm{E_C}$ is a control variable, while both $H_C = \mathrm{q_C E_C C}$ and $m\mathrm{E_C}$ are elements of the social welfare function.

Let us now turn to the description of reserve and denote its opening time by T_R^i, with $i = 1, \dots, k$ being the number of openings and $k \to \infty$. The opening of reserve at T_R^i will attract a significant amount of effort causing the stock in the area to exhibit a jump. The magnitude of the jump is given as

$O_R\left(T_R^{i+}\right) - O_R\left(T_R^{i-}\right) = \left[\frac{v}{P_O q_O} - O_R\left(T_R^{i-}\right)\right] d_R(T_R^i)$, with $d_R\left(T_R^i\right) = 1$ at $i = 1, \dots, k$ with $k \to \infty$.

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[19] As we have pointed out, if effort was determined privately we would have the well-known open-access result of having effort entering the fishery till rents are dissipated.

In the literature of impulsive optimal control theory (Seierstad and Sydsæter, 1987) $O_R(T_R^{i+})$ and $O_R(T_R^{i-})$ denote the right and left hand limits respectively of $O_R(T_R^i)$. When the reserve opens, effort (E_R) enters constantly for a very short time period, say τ[20] and reduces the amount of the harvestable stock to the point where revenues per unit effort are equal to its cost, an identical outcome to the open access regime. Therefore, adopting the same notation from the public area and assuming non-negative level and unit cost of effort, fishing effort ceases at the point where

$$P_O q_O O_R(T_R^i + \tau) = P_O q_O O_R(T_R^{i+}) = v \text{ or}$$

$$O_R(T_R^{i+}) = \frac{v}{P_O q_O}$$

Having defined the magnitude of the jump, we need to specify an expression for harvest effort and its cost during the pulse. While stock dynamics before the reserve opening are identical to (*), during the pulse are given as

$$\dot{O}_R = -q_O E_R O_R$$

Integrating over the harvest interval we have

$$\int_{T_R^i}^{T_R^i+\tau} \frac{\dot{O}_R}{O_R} dt = \int_{T_R^i}^{T_R^i+\tau} -q_O E_R dt$$

Given that effort is constant during the pulse, the last expression simplifies into

$$O_R(T_R^i + \tau) = O_R(T_R^i) e^{-q_O E_R \tau}$$

and solving for effort per unit area we get[21]

$$E_R = \frac{1}{q_O} ln\left[\frac{O_R(T_R^i)}{O_R(T_R^i + \tau)}\right] = \frac{1}{q_O} ln\left[\frac{O_R(T_R^{i-})}{O_R(T_R^{i+})}\right]$$

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[20] The variable τ is typically measured in days. It is used here only for notational purposes without further application.

[21] $O_R(T_R^i)$ is essentially the same as $O_R(T_R^-)$.

The cost of effort per unit area is therefore given as

$$vE_R = \frac{v}{q_O} ln\left[\frac{O_R\left(T_R^{i-}\right)}{O_R\left(T_R^{i+}\right)}\right] = \frac{v}{q_O} ln\left[\frac{O_R\left(T_R^{i-}\right)}{v/P_O q_O}\right] \quad (**)$$

Nonetheless, the jump in the stock of reserve will trigger jumps in a number of other state variables. In particular, optimal management of the reserve might imply for the regulatory agency to set harvest at such a rate so that crabs also experience a jump. With b_{ci} the percentage of the crab stock harvested in the ith opening, the magnitude of the jump in crabs is

$C\left(T_R^{i+}\right) - C\left(T_R^{i-}\right) = -b_{ci}C\left(T_R^{i-}\right)$, at $i = 1, \dots, k$ with $k \to \infty$ and $b_{ci} = [0,1]$.

In the literature of optimal control with jumps in state variables, b_{ci} is referred to as jump parameter and is followed by its own necessary condition *(*Seierstad and Sydsæter, 1987) as we will see in the next section. To derive the cost of crab effort during the pulse we make use of (**), making the necessary notational changes in stock, unit cost effort, catchability coefficient, as well as the equation for the magnitude of the jump. With these changes the cost of crab effort during the pulse is

$$\frac{m}{q_C} ln\left[\frac{C\left(T_R^{i-}\right)}{C\left(T_R^{i+}\right)}\right] = \frac{m}{q_C} ln\left[\frac{1}{(1-b_{ci})}\right]$$

Moreover, oysters in the public area will experience a jump. Similar to crabs, if we define the jump parameter b_{wi} as the percentage of the public oyster stock harvested in the ith opening, the magnitude of the jump will be

$O_W\left(T_R^{i+}\right) - O_W\left(T_R^{i-}\right) = -b_{wi}O_W\left(T_R^{i-}\right)$, at $i = 1, \dots, k$ with $k \to \infty$ and $b_{wi} = [0,1]$.

Following the same approach, the cost per unit area of harvesting the public stock during the pulse can be shown to be

$$\frac{v}{q_o} ln\left[\frac{O_W\left(T_R^{i-}\right)}{O_W\left(T_R^{i+}\right)}\right] = \frac{v}{q_o} ln\left[\frac{1}{(1-b_{wi})}\right]$$

In addition, it is logical to model nitrogen exhibiting a jump, since some amount of it is rapidly removed through total harvest from both reserve and the public area. For the magnitude of the jump of nitrogen we have

$N(T_R^{i+}) - N(T_R^{i-}) = s\left[a_R\left(O_R(T_R^{i-}) - \frac{v}{P_O q_O}\right) + a_W b_{wi} O_W(T_R^{i-})\right] d_N(T_R^i)$, with $d_N(T_R^i) = 1$ at $i = 1, \dots, k$ with $k \to \infty$.

While Rl at any time $t \neq i$ represents the cost of BMPs, at $t = i$ there exists a reduction in BMP cost associated with nitrogen removal through harvest. The Net Present Value of total instantaneous benefits (IB), related to each jump can be expressed as

$$IB = \sum_{i=1}^{k}\Bigg\{P_O\left[a_R\left(O_R(T_R^{i-}) - \frac{v}{P_O q_O}\right) + a_W b_{wi} O_W(T_R^{i-})\right] - a_R\frac{v}{q_O} ln\left(\frac{O_R(T_R^{i-})}{v/P_O q_O}\right)$$
$$- a_W\frac{v}{q_o} ln\left[\frac{1}{(1-b_{wi})}\right] + P_C b_{ci} C(T_R^{i-}) - \frac{m}{q_C} ln\left[\frac{1}{(1-b_{ci})}\right]$$
$$+ s\left[a_R\left(O_R(T_R^{i-}) - \frac{v}{P_O q_O}\right) + a_W b_{wi} O_W(T_R^{i-})\right] l\Bigg\} e^{-\delta T_R^i}$$

where δ is the discount rate and the last term represents BMP cost savings[22]. The model will be solved in two stages. In the first, we allow the social planner at the beginning of the planning horizon to choose the amount of areas allocated to each management system. The second stage finds what the optimal rotation lengths in reserve and effort levels for public stock and crabs should be at every time instant, given the initial area allocation. The analytical results, presented in the next section, assume the respective areas as given and their actual choice is explored numerically latter. We can now specify our objective function. With SW being the Social Welfare of Chesapeake Bay, and the areas as given, the problem can fully expressed as

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[22] Note that the amount of nitrogen reduction associated with each jump is multiplied by the unit cost of BMPs, l converting the respective quantity in monetary terms.

$$\max_{T_R^i, E_W, E_C} SW = -a_S C_S^{cr} + \int_0^\infty \left\{ P_O \left[a_W q_O E_W O_W + a_A \frac{P_O}{2y} \right] + P_C q_C E_C C + U(N) - Rl - a_W v E_W - a_A \frac{{P_O}^2}{4y} - m E_C \right\} e^{-\delta t} dt \text{ for all } t \neq i \text{ and}$$

$$IB = \sum_{i=1}^{k} \left\{ P_O \left[a_R \left(O_R(T_R^{i-}) - \frac{v}{P_O q_O} \right) + a_W b_{wi} O_W(T_R^{i-}) \right] - a_R \frac{v}{q_O} ln \left(\frac{O_R(T_R^{i-})}{v/P_O q_O} \right) - a_W \frac{v}{q_o} ln \left[\frac{1}{(1-b_{wi})} \right] + P_C b_{ci} C(T_R^{i-}) - \frac{m}{q_C} ln \left[\frac{1}{(1-b_{ci})} \right] + s \left[a_R \left(O_R(T_R^{i-}) - \frac{v}{P_O q_O} \right) + a_W b_{wi} O_W(T_R^{i-}) \right] l \right\} e^{-\delta T_R^i} \text{ for all } t = i$$

S.t.

$$\dot{\mathrm{O}}_\mathrm{S} = \mathrm{F}(\mathrm{O}_\mathrm{S}) + \mathrm{I}(\mathrm{O}) \text{ for all } t$$

$$\dot{\mathrm{O}}_\mathrm{W} = \mathrm{F}(\mathrm{O}_\mathrm{W}) + \mathrm{I}(\mathrm{O}) - \mathrm{q}_\mathrm{O}\mathrm{E}_\mathrm{W}\mathrm{O}_\mathrm{W} \text{ for all } t \neq i$$

$$\dot{\mathrm{O}}_\mathrm{A} = K_O/2 \text{ for all } t$$

$$\dot{\mathrm{O}}_\mathrm{R} = \mathrm{F}(\mathrm{O}_\mathrm{R}) + \mathrm{I}(\mathrm{O}) \text{ for all } t \neq i$$

$$\dot{\mathrm{C}} = F_C(\mathrm{C}, \mathrm{O}) - \mathrm{q}_\mathrm{C}\mathrm{E}_\mathrm{C}\mathrm{C} \; t \neq i$$

$$\dot{\mathrm{N}} = \bar{L} - R - \rho a_{BAY} O - s \left(a_W q_O E_W O_W + a_A \frac{P_O}{2y} \right) - dN \text{ for all } t \neq i$$

$$\text{with } O = a_S O_s + a_R O_R + a_A O_A + a_W O_W$$

5. Analytical Results

5.1 The continuous case

The current-value Hamiltonian is

$$\begin{aligned} H = P_O\left[a_W q_O E_W O_W + a_A\frac{P_O}{2y}\right] + P_C q_C E_C C + U(N) - lR - a_W v E_W - a_A\frac{{P_O}^2}{4y} - mE_C \\ + \lambda_S[F(O_S) + I(O)] + \lambda_R[F(O_R) + I(O)] + \lambda_W[F(O_W) + I(O) - q_O E_W O_W] \\ + \lambda_C[F_C(C,O) - q_C E_C C] \\ + \lambda_N\left[\bar{L} - R - \rho a_{BAY} O - s\left(a_W q_O E_W O_W + a_A\frac{P_O}{2y}\right) - dN\right] \end{aligned}$$

The first order condition, for the amount of nitrogen reduced through best management practices is

$$\frac{\partial H}{\partial R} = -l - \lambda_N = 0 \rightarrow \lambda_N = -l \quad (1)$$

λ_N is a negative number as it represents the future value, in current terms, of increasing nitrogen in the Bay by one unit at some point in time. It can be viewed as the disutility, in current monetary terms, of increasing nitrogen sometime in the future and we will often refer to it as the future value of nitrogen. Equation (1) simply states that the future value of nitrogen should equal the constant cost of removing it. It therefore holds that $\dot{\lambda}_N = 0$.

The first order condition for crab harvest effort is:

$$\frac{\partial \mathrm{H}}{\partial E_C} = \{P_C q_C C - m - \lambda_C q_C C \leq 0, E_C \geq 0, [P_C q_C C - m - \lambda_C q_C C]E_C = 0\} \rightarrow P_C - \frac{m}{q_C C} \leq \lambda_C$$

The LHS of the above inequality is the net benefit from a unit crab, whereas the RHS is the current value of harvesting one crab sometime in the future. λ_C is known as the shadow value or the user cost of stock. Whenever λ_C exceeds $P_C - \frac{m}{q_C C}$ crab stock should be expanded by ceasing all crab harvest. Whenever $P_C - \frac{m}{q_C C} > \lambda_C$ crab effort should be employed at the rate of our technological capability. The process will continue until $P_C - \frac{m}{q_C C} = \lambda_C \quad (2)$. Expression (2) is the singular solution and constitutes a standard result in the literature of renewable resources.

The adjoint equation for crab stock becomes:

$$\dot{\lambda_C} = \delta\lambda_C - \frac{\partial H}{\partial C} = \delta\lambda_C - P_C q_C E_C - \lambda_C F_C' + \lambda_C q_C E_C = \lambda_C(\delta - F_C') - P_C q_C E_C + \lambda_C q_C E_C$$

where $F_C' = \frac{\partial F_C(C,O)}{\partial C}$. Differentiating (2) with respect to time and using the state equation for the crab stock yields

$$\dot{\lambda_C} = \frac{m}{q_C C^2}\dot{C} = \frac{m}{q_C C^2}[F_C(C,O) - q_C E_C C] \quad (3)$$

Replacing λ_C from (2) into the adjoint equation for crab stock and simplifying we get

$$\dot{\lambda_C} = \left(P_C - \frac{m}{q_C C}\right)(\delta - F_C') - \frac{mE_C}{C} \quad (4)$$

Equating (3) and (4), simplifying and rearranging yields

$$\delta = F_C' + \frac{mF_C(C,O)}{C[P_C q_C C - m]} \quad (5)$$

Given, O expression (5) is an implicit equation that solves for C^* and such a stock level is called the singular solution for the crab control problem (Conrad and Clark, 1987 p.75). Expression (5) is known as "fundamental equation of renewable resources" (Conrad, 1999 p.14) and states that at all times the marginal net growth rate plus the marginal stock effect (the two known as resource's internal rate of return) should equal the discount factor. As for the marginal stock effect, it is the ratio of the marginal value of the stock $mF_C(C,O)/C$ relative to the marginal value of effort $P_C q_C C - m$. Equation (5) is another standard result and will serve as the baseline for comparison with a similar, but extended expression, for the public oyster stock.

The first order condition for effort in the public oyster area is (for the singular solution)

$$\frac{\partial H}{\partial E_W} = P_O a_W q_O O_W - v a_W - \lambda_W q_O O_W - \lambda_N s a_W q_O O_W = 0$$

After rearranging we get

$$P_O - \frac{v}{q_O O_W} - s\lambda_N = \frac{\lambda_W}{a_W} \quad (6)$$

Expression (6) states that the net benefit from a public-caught oyster plus the marginal impact of its current nitrogen removal, through harvest on the future value of nitrogen, equals its user cost per unit area. The adjoint equation for public oysters becomes

$$\dot{\lambda}_W = \delta\lambda_W - \frac{\partial H}{\partial O_W}$$

$$= \delta\lambda_W - \left[P_O a_W q_O E_W + \lambda_S \frac{\partial I}{\partial O}\frac{\partial O}{\partial O_W} + \lambda_R \frac{\partial I}{\partial O}\frac{\partial O}{\partial O_W} + \lambda_W\left(F'(O_W) + \frac{\partial I}{\partial O}\frac{\partial O}{\partial O_W} - q_O E_W\right)\right.$$
$$\left. + \lambda_C \frac{\partial F_C}{\partial O}\frac{\partial O}{\partial O_W} - \lambda_N\left(\rho a_{BAY}\frac{\partial O}{\partial O_W} + s a_W q_O E_W\right)\right]$$

For practical purposes and where there will be no confusion we will denote

$\frac{\partial I}{\partial O}\frac{\partial O}{\partial O_i} = \frac{\partial I}{\partial O_i}$ for $i = S, R, W$ where $\frac{\partial O}{\partial O_i} = \frac{a_i}{a_{BAY}}$ for $j = S, R, W, A$ and

$\frac{\partial F_C}{\partial O}\frac{\partial O}{\partial O_i} = \frac{\partial F_C}{\partial O_i}$ for $i = S, R, W$.

With these notational simplifications we rewrite the above equation as

$$\dot{\lambda}_W = \delta\lambda_W - \left[P_O a_W q_O E_W + \lambda_S \frac{\partial I}{\partial O_W} + \lambda_R \frac{\partial I}{\partial O_W} + \lambda_W\left(F'(O_W) + \frac{\partial I}{\partial O_W} - q_O E_W\right) + \lambda_C \frac{\partial F_C}{\partial O_W}\right.$$
$$\left. - \lambda_N\left(\rho a_{BAY}\frac{\partial O}{\partial O_W} + s a_W q_O E_W\right)\right]$$

which simplifies into

$$\dot{\lambda}_W = \lambda_W\big(\delta - F'(O_W)\big) - \frac{\partial I}{\partial O_W}(\lambda_S + \lambda_R + \lambda_W) - P_O a_W q_O E_W + \lambda_W q_O E_W - \lambda_C \frac{\partial F_C}{\partial O_W}$$
$$+ \lambda_N(a_W \rho + s a_W q_O E_W) \quad (7)$$

The adjoint equation for the stock in the reserve area is

$$\dot{\lambda}_R = \delta\lambda_R - \frac{\partial H}{\partial O_R}$$

$$= \delta\lambda_R - \left[\lambda_S \frac{\partial I}{\partial O_R} + \lambda_R[F'(O_R) + \frac{\partial I}{\partial O_R}] + \lambda_W \frac{\partial I}{\partial O_R} + \lambda_C \frac{\partial F_C}{\partial O_R} - \rho a_{BAY}\lambda_N \frac{\partial O}{\partial O_R}\right]$$

$$= \lambda_R\big(\delta - F'(O_R)\big) - \frac{\partial I}{\partial O_R}(\lambda_S + \lambda_R + \lambda_W) - \lambda_C \frac{\partial F_C}{\partial O_R} + a_R\rho\lambda_N \quad (8)$$

Similarly, the adjoin equation for the stock in the sanctuary becomes

$$\dot{\lambda}_S = \lambda_S\big(\delta - F'(O_S)\big) - \frac{\partial I}{\partial O_S}(\lambda_S + \lambda_R + \lambda_W) - \lambda_C \frac{\partial F_C}{\partial O_S} + a_S\rho\lambda_N \quad (9)$$

Expressions (8) and (9) make intuitive sense. They state that the rate of change in the value of $O_i, i = R, S$ (in situ value of one oyster in the reserve and/or sanctuary), at any time should equal the extent to which the opportunity cost of future stock value exceeds the marginal impact of current stock on its future in situ value, adjusted by three terms: a) the marginal impact of current stock on the future in situ value of all areas affected by stock spillover, b) the marginal impact of current crab stock on its user cost attributed to the biodiversity spillover and c) the marginal impact of current denitrification rate on the future value of nitrogen.

We have not interpreted equation (7) yet but we will do so, once we express the adjoint equation for nitrogen. That is

$$\dot{\lambda}_N = \delta\lambda_N - \frac{\partial H}{\partial N} = \delta\lambda_N - U'(N) + d\lambda_N = (\delta + d)\lambda_N - U'(N) \quad (10)$$

We have shown that $\dot{\lambda}_N = 0$, so (10) can be rewritten as

$$\lambda_N = \frac{U'(N)}{(\delta + d)} = -l \quad (11)$$

Similar to λ_N, $U'(N)$ is negative, representing people's willingness to pay (WTP) for reduced nitrogen. Expression (11) states that the user cost of nitrogen equals people's WTP for ambient nitrogen reductions accounting for the discount rate and the natural decay of the pollutant. Optimal management requires the unit cost of nitrogen reduction from best management practices to be set at that level.

Turning now to equation (7) we will try to give it a form similar to (5). Differentiating (6) with respect to time and keeping in mind that $\dot{\lambda}_N = 0$ yields

$$\dot{\lambda}_W = a_W \frac{v\dot{O}_W}{q_O O_W^2}$$

And given that $\dot{O}_W = F(O_W) + I(O) - q_O E_W O_W$ we get

$$\dot{\lambda}_W = a_W \frac{v}{q_O O_W^2}[F(O_W) + I(O) - q_O E_W O_W] \quad (12)$$

Substituting λ_W from (6) into (7) and simplifying we get

$$\begin{aligned}\dot{\lambda}_W = \left[P_O a_W - \frac{a_W v}{q_O O_W} - a_W s \lambda_N\right][\delta - F'(O_W)] - \frac{\partial I}{\partial O_W}(\lambda_S + \lambda_R) \\ - \frac{\partial I}{\partial O_W}\left[P_O a_W - \frac{a_W v}{q_O O_W} - a_W s \lambda_N\right] - a_W \frac{v E_W}{O_W} - \lambda_C \frac{\partial F_C}{\partial O_W} + a_W \rho \lambda_N \quad (13)\end{aligned}$$

Equating (12) with (13) simplifying and rearranging (details are provided in the Appendix) yields.

$$\begin{aligned}\delta = \left[F'(O_W) + \frac{\partial I}{\partial O_W}\right] + \frac{v}{O_W[P_O q_O O_W - v - s\lambda_N q_O O_W]}[F(O_W) + I(O)] \\ + \frac{q_O O_W}{a_W[P_O q_O O_W - v - s\lambda_N q_O O_W]}\left\{\frac{\partial I}{\partial O_W}(\lambda_S + \lambda_R) + \lambda_C \frac{\partial F_C}{\partial O_W} - a_W \rho \lambda_N\right\} \quad (14)\end{aligned}$$

which for $\lambda_N = -l$ results into

$$\begin{aligned}\delta = \left[F'(O_W) + \frac{\partial I}{\partial O_W}\right] + \frac{v}{O_W[P_O q_O O_W - v + slq_O O_W]}[F(O_W) + I(O)] \\ + \frac{q_O O_W}{a_W[P_O q_O O_W - v + slq_O O_W]}\left\{\frac{\partial I}{\partial O_W}(\lambda_S + \lambda_R) + \lambda_C \frac{\partial F_C}{\partial O_W} + a_W \rho l\right\} \quad (14)'\end{aligned}$$

Expressions (14) and (14)’ are equivalent and they hold at any point in time provided that solutions for λ_S, λ_R and their respective stocks (in the $I(.)$ function) have been found. Moreover, equation (14) is an extended version of (5). The marginal net growth rate of the stock ($F'(O_W)$) now captures the effect of the area’s stock on its own recruitment. Given the way we have defined $F(O_n)$ i.e. to capture somatic growth excluding recruitment, the first RHS term in (14) and $(14)'$ is the same with F_C' from (5). The second RHS term, as in (5) represents the marginal stock effect i.e. the marginal value of the stock ($\frac{v[F(O_W)+I(O)]}{O_W}$) relative to the marginal value of effort ($P_O q_O O_W - v - s\lambda_N q_O O_W$). The difference is that the marginal value of the stock now incorporates spillovers in recruitment coming from the whole Bay (stock spillover). Moreover, the marginal value of effort captures the rate at which the value of nitrogen is affected by nitrogen removal through harvest.

The third RHS term constitutes additional marginal stock effects. Note that the marginal value of effort (the denominator) is the same. The term $q_O O_W \left\{ \frac{\partial I}{\partial o_W}(\lambda_S + \lambda_R) + \lambda_C \frac{\partial F_C}{\partial o_W} - a_W \rho \lambda_N \right\}$ in (14) captures the marginal impact of current stock on the in situ values of the stock in neighboring areas due to stock spillovers, on the future value of crabs due to the biodiversity spillover (change in oyster abundance), as well as the marginal impact of current stock on the in situ value of nitrogen through denitrification. All three marginal stock effects are positive providing incentive to keep oysters longer in the water, increasing their opportunity cost (δ).

5.2 Optimization at the jump points

In what follows we adopt the approach by Seierstad and Sydsæter (1987) from their 7th theorem regarding the necessary conditions when there are jumps in state variables[23]. At the jumps points, the necessary condition for the stock in reserve becomes

$$\lambda_R(T_R^{i+}) - \lambda_R(T_R^{i-}) = -\frac{\partial IB}{\partial O_R(T_R^{i-})} - \lambda_R(T_R^{i+}) \frac{\partial \left[\left[\frac{v}{P_O q_O} - O_R(T_R^{i-}) \right] \right]}{\partial O_R(T_R^{i-})}$$

which simplifies into

$$\lambda_R(T_R^{i-}) = a_R \left[P_O + sl - \frac{v}{q_O O_R(T_R^{i-})} \right] \quad (15)$$

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[23] The model assumes that there is only one reserve, which is harvested at discrete points in time. In practice, there will be many different reserve areas, each of which will be opened for harvest at different times.

Equation (15) states that the shadow value of the stock right before the jump should equal the net revenue, including cost savings in BMPs, at that point in time. The necessary condition for crabs at the jump points is

$$\lambda_C(T_R^{i+}) - \lambda_C(T_R^{i-}) = -\frac{\partial IB}{\partial C(T_R^{i-})} - \lambda_C(T_R^{i+})\frac{\partial[-b_{ci}C(T_R^{i-})]}{\partial C(T_R^{i-})}$$

The above expression simplifies into

$$[1-b_{ci}]\lambda_C(T_R^{i+}) + P_C b_{ci} = \lambda_C(T_R^{i-}) \quad (16)$$

Equation (16) has the economic interpretation that at all jumps points, the value of the unharvested portion of the stock $(1-b_{ci})$ right after the jump along with the revenue from the harvested one (b_{ci}), should equal the value of the stock right before the jump. Similarly, the necessary condition for public oyster at the jump points yields

$$[1-b_{wi}]\lambda_W(T_R^{i+}) + P_O a_W b_{wi} = \lambda_W(T_R^{i-})$$

Interestingly, the necessary condition for nitrogen gives us.

$\lambda_N(T_R^{i+}) - \lambda_N(T_R^{i-}) = 0$ or $\lambda_N(T_R^{i+}) = \lambda_N(T_R^{i-})$. Therefore, the costate variable for nitrogen is continuous. This is so because both the instantaneous benefit function and magnitude of the jump in nitrogen are independent of N. We turn now into the necessary condition about the jump parameters. Following condition 75 from Seierstad and Sydsæter (1987 p.197) for b_{ci} we have.

$$\frac{\partial \mathrm{IB}}{\partial b_{ci}} + \lambda_C(T_R^{i+})\frac{\partial[-b_{ci}C(T_R^{i-})]}{\partial b_{ci}} = 0$$

which simplified and rearranged yields

$$\left[[P_C - \lambda_C(T_R^{i+})]C(T_R^{i-}) - \frac{m}{q_C(1-b_{ci})}\right] = 0 \quad (17)$$

Solving (17) for b_{ci} and plugging back into (16), upon simplification yields

$$P_C - \frac{m}{C(T_R^{i-})q_C} = \lambda_C(T_R^{i-}) \quad (18)$$

Expression (18) simply confirms what one might expect. The net benefit from catching and selling a crab should equal its user cost at all times. A comparison with equation (2) confirms that. The steps for the public stock are identical with those in crabs obtaining

$$\left[P_O - \frac{v}{q_o O_W\left(T_R^{i-}\right)} - s\lambda_N\right] = \frac{\lambda_W\left(T_R^{i-}\right)}{a_W} \quad (19)$$

Finally we have the expression for the optimal rotation length in reserve. That is given by

$$H\left(T_R^{i+}\right) - H\left(T_R^{i-}\right) - \delta IB = 0$$

This is the arbitrage condition that states that all instantaneous and future benefits from delaying the opening of reserve $H\left(T_R^{i+}\right)$ less the opportunity cost of such a decision should equal the benefits (instantaneous and future) from not delaying the opening of reserve.

6. Numerical analysis and specification of cases and scenarios

To simulate the system, specific functional forms for oyster as well as blue crab dynamics must be chosen. First, for oyster growth equation in all areas except aquaculture, we assume that the stock grows logistically having a linear additive term representing the stock spillover:

$$F(O_n) + I(O) = r_o O_n\left(1 - \frac{O_n}{K_O}\right) + \varphi O, \;\; n = S, W, R$$

The parameters r_o and K_O are oysters' intrinsic growth rate and carrying capacity respectively, while φ represents the effect Bay's stock per acre has on recruitment in each area through larvae dispersal. Next, blue crabs also grow logistically, while allowing oyster stock relative to its own carrying capacity to impose a positive spillover on crabs' carrying capacity:

$$F_C(C, O) = r_C \mathrm{C}\left[1 - \frac{C}{K_C\left(1 + \zeta\frac{O}{K_O}\right)}\right]$$

The parameters r_C and K_C stand for blue crabs' intrinsic growth rate and carrying capacity respectively, while ζ represents the percentage increase in species' carrying capacity with an increase of Bay's aggregate stock per acre relative to its own carrying capacity.

The nonuse value from water quality, $U(N)$ is difficult to parameterize. Instead, we assume that the optimal singular solution for N, N^* is known. We set N^* equal to the equilibrium level of nitrogen that would occur in the Bay if nitrogen loadings were equal to the TMDL target level of 185.9 million pounds per year (EPA, Chesapeake Bay TMDL, Executive Summary). We assume that this is the level of N where $U^{'}(N)$ is equal to the marginal cost of reducing nitrogen loadings. Because the optimal solution sets $N = N^*$ at all points in time, we do not need to know the shape of $U(N)$ for other values of N. Nonetheless, we need to know the value of $U(N^*)$ to calculate the social welfare function, though its specific value will not affect the optimal values of control variables. The choice of a numerical value for $U(N^*)$ is described below.

Table 1 summarizes all parameter values and their definitions[24]. For all areas we impose the same initial condition on the oyster stock, equal to $\frac{v}{P_O q_O}$,which is the open access stock level. Similarly, the initial stock for blue crabs is given by $m/P_C q_C$. Numerical simulations are performed for 200 years with the time step being 0.1 or 1/10th of a year.

[Table 1]

We now turn to the specification of cases and scenarios for our simulations. We examine two broad cases: one with cost of BMPs equal to \$22.1 and one with \$5. Each case has the same scenarios, the first of which is the baseline. The baseline scenario in each case (l = 22.1 or 5) is defined as having the total available area in the Bay[25] equally divided among the different management systems. Moreover, the baseline scenarios in both cases will serve as our starting point to answer the research question regarding the optimal rotation length in reserves and how sensitive this answer is in the different costs of BMPs. The rest of scenarios in each case represent different combinations of area allocations. The amount of aquaculture is held constant in all scenarios, because our data on costs and returns from aquaculture is limited. For each case,

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[24] A detailed presentation of parameter calculation, calibration and source of origin (wherever applicable) would consume too much space. This information is available on request.

[25] Following our model, this parameter is denoted as a_{BAY} in Table 1 and equals 637197.2 acres of quality oyster habitat.

all scenarios will be compared and the question of optimal area allocation and simultaneous existence among regimes will be answered. Table 2 below illustrates, for each case, the allocation scenarios, as percentage of the total oyster area in the Bay.

Table 2: Simulation Scenarios and NPV for each case

	a_S	a_R	a_W	a_A	NPV for l=22.1	NPV for l=5
Baseline	0.25	0.25	0.25	0.25	0	0
Scenario 1	0.25	0.5	0	0.25	-806,569,056	-594,360,986
Scenario 2	0.25	0	0.5	0.25	771,999,964	595,857,956
Scenario 3	0	0.5	0.25	0.25	1,687,072,048	2,546,866,536
Scenario 4	0	0.25	0.5	0.25	2,496,344,736	3,151,193,838
Scenario 5	0	0	0.75	0.25	3,263,224,799	3,748,787,287

The final set of results will be a comparison between the baseline scenarios of case 1 (l =22.1) versus the same scenario but eliminating the environmental and biodiversity spillovers. In that way we can have interesting insights regarding the effects of oysters on nitrogen and crab dynamics by exploring the difference in the cost of the BMPs, optimal rotation lengths and the singular solution in the public.

7. Simulation Results

Starting with the baseline scenario of case 1, we plotted the stock dynamics in the reserve and the public area, both of which are presented in figures 1 and 2 below

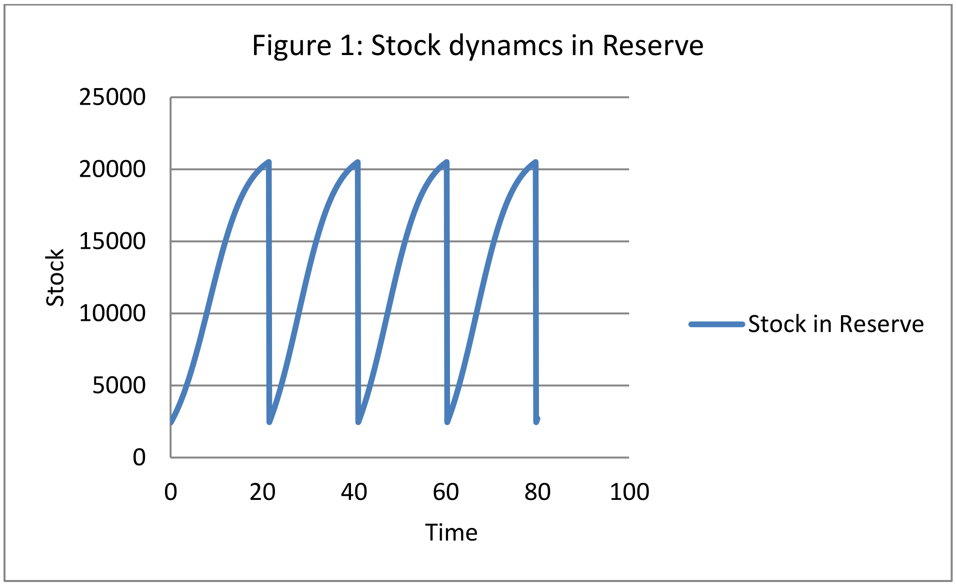


Figure 1: Stock dynamcs in Reserve

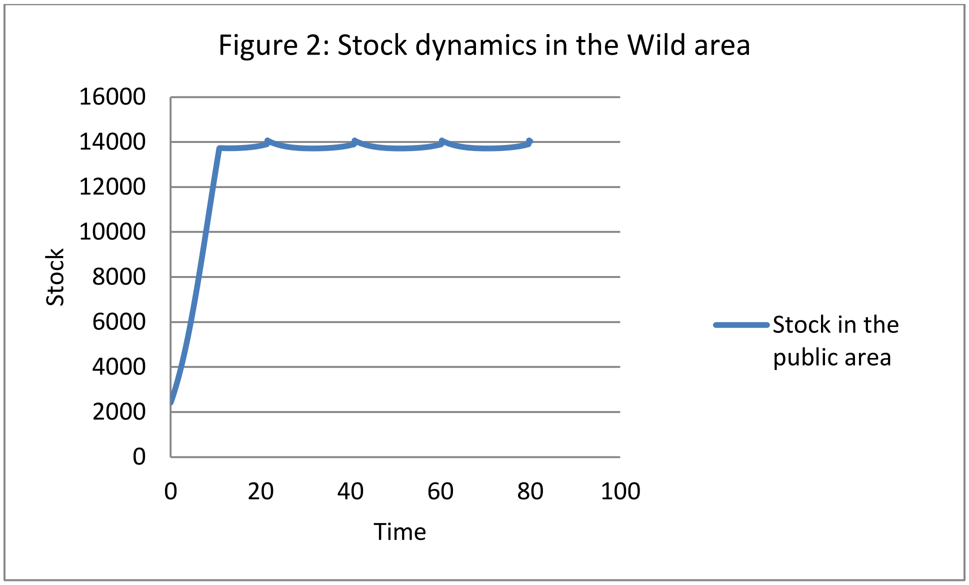
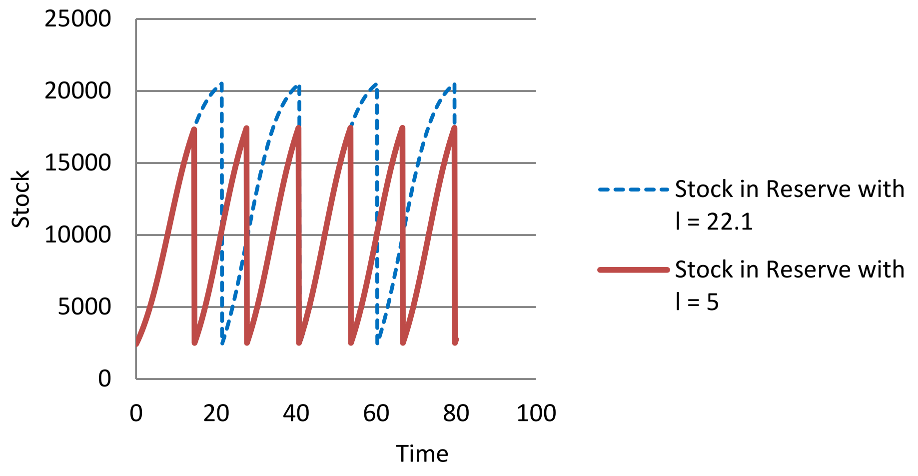


Figure 2: Stock dynamics in the Wild area

Contrary to the rotation cycles of the stock in reserve, its counterpart in the public exhibits considerably smaller fluctuations. The baseline simulation of case 1 reveals that the optimal rotation length in reserve becomes stabilized at 19.4 years after one jump period. This is so due to the low initial condition that we imposed, as it takes some time for the stock to be stabilized. Moreover, with rotation length at 19.4 years the stock in the public is stabilized around 13,700 oysters per acre. The results of the baseline scenario in case 2 (l = 5) reveal the interesting result that optimal rotation lengths should be 13 years. This is a considerable difference of 6.4 years between the two baseline scenarios and depicted in Figure 3.

Figure 3: Stock dynamcs in Reserve



An intuitive explanation for this result is that when the cost of BMPs is high, the stock should stay unharvested longer, allowing the stock to reach higher levels resulting in more denitrification by oysters. When BMPs are cheap the stock should be more frequently harvested since denitrification has lost its relative attractiveness.

We now turn to address the research questions regarding the efficiency tradeoffs of having one management system versus a combination of them or whether all regimes can coexist. Our criterion of choice is maximization of Net Present Value (NPV) of the social welfare function. For each case, $U(N^*)$ is set at that level so that the NPV of the social welfare function is equal to zero for the baseline scenario. In this way, all other scenarios are measured relative to the baseline for that case. Table 2 summarizes this information in dollar amounts, across all scenarios for both cases. Several findings are interesting. To begin with, under expensive BMPs (case 1), NPV is maximized under the "corner solution" of having the total area, other than aquaculture, allocated to public oysters (scenario 5). Additionally, having half of the Bay public, yields considerably higher NPV when reserve is operated instead of the sanctuary versus the reverse (compare scenario 4 with 2). In fact, this scenario can be viewed as the "second best" when the solution of having the entire Bay as a public area, given aquaculture, cannot be achieved. However, when more area is allocated to reserves relative to the public, welfare drops (compere scenario 4 with 3). Furthermore, in all scenarios, shifting area from sanctuaries or reserves to the public improves the NPV of the system. Finally, having some area in the public yields always higher NPV versus not (compare scenario 1 with the rest). Our results remained robust under the second case (l = 5).

Next we present a comparison using the baseline scenario of case 1 with versus without environmental and biodiversity spillovers to see how some of the above results might change. First, we explore how large of a contribution oysters might provide towards meeting TMDL goals. When the baseline scenario with environmental spillovers was simulated we found the NPV of BMP costs to be $120.4 billion. However, when environmental spillovers (denitrification and nitrogen removal through harvest) are assumed not to occur, the corresponding figure is $130.5 billion. This difference of $10.1 billion is about an 8 percent cost savings in BMPs towards achieving the TMDL nitrogen target. Moreover, the optimal rotation length when denitrification and nitrogen assimilation are ignored was found to be 11.9 years, a considerable difference of 7.5 years compared with the complete baseline scenario. Moreover, the singular oyster stock in the public area is considerably higher when environmental spillovers are accounted for than when they are assumed away (Figure 4). The difference is about 3100 oysters per acre. The above results demonstrate that environmental spillovers induce longer rotation cycles and higher oyster stocks. As before, denitrification seems to dominate nitrogen removal through harvest providing the incentive to leave the stock longer in the water and resulting in higher stock for the singular solution than when these externalities are ignored.

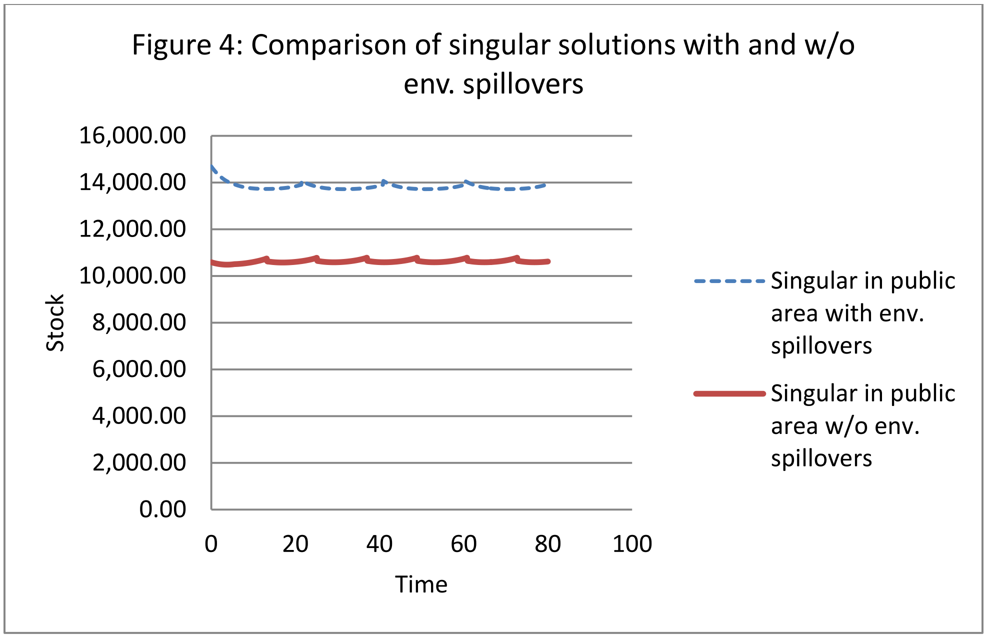


Figure 4: Comparison of singular solutions with and w/o env. spillovers

Finally, the baseline scenario was modified to exclude the biodiversity spillover by oysters on crabs. Here, accounting for the biodiversity spillover has a much smaller impact on the optimal management of oysters. In this case, simulation results revealed an optimal rotation length of 19.3 years, a 0.1 (or 1/10th of a year) difference compared to the baseline in the presence of the externality. Furthermore, the singular solution for the public area with the biodiversity spillover is about 40 oysters per acre, higher than the singular solution without the biodiversity spillover. Even though the differences are small, the results make intuitive sense. If managers account for the fact that oysters positively affect blue crabs, they should leave the reserves unharvested longer and leave more oysters in the public areas.

8. Discussion of Results and Conclusion

The Eastern oyster has both ecologic and commercial significance for the Chesapeake Bay. Habitat degradation, overharvesting and disease have resulted in dramatic declines of the stock calling for restoration and more efficient management strategies. Current oyster management has resulted in spatial segregation of the Bay into four distinct types of areas: sanctuaries, reserves, aquaculture and areas available for harvest to any licensed boat. Incorporating stock, environmental and biodiversity externalities, we develop a bioeconomic model and ask several questions regarding the optimal rotation length in reserves, optimal area allocation, combination and significance of simultaneous existence of the four management systems, as well as the importance of oysters on clean water and another commercially important species, the blue crab.

Our results can be summarized as follows. When the cost of BMPs is low, the optimal rotation length of reserves is shorter than when BMPs are more expensive. This is because relatively cheap BMPs reduce the importance of denitrification resulting in more frequent harvesting of the stock. We define as relatively "cheap" cost of BMPs, the figure of $5 per pound reduction of nitrogen found in literature (Nelson, 2005) versus the more conservative figure of $22.1 found elsewhere (Wieland et al., 2009).

Second, regardless of the cost of BMPs, the system that generates the highest net benefits is public oyster harvest, where the grounds are open for continuous harvest but effort is regulated at the efficient level. This is the first best system.

Third, our empirical results demonstrate that oysters can play an important role in meeting nitrogen reduction objectives, but that they cannot come close to being the only solution. Optimal management of Bay oysters can result in cost savings of 8%. Because much of the nitrogen reduction through oysters occurs as a result of denitrification, accounting for the impact of oysters on nitrogen reduction induces us to harvest oysters less aggressively than we would if oysters did not filter nutrients.

Fourth, accounting for both biodiversity spillovers produces slightly longer rotation cycles for the reserves and higher singular solutions for the public stock. Because the relative change, compared to the situation where biodiversity spillover is ignored, is small these results require further research as well as more information about the precise effect of oysters on blue crabs. Nonetheless, it should be pointed out that the choice of blue crabs was primarily for their historic and commercial importance and our theory and simulation approach is applicable to other species, as long as they can be positively affected be oyster abundance.

While studies have modeled economic interactions and spillovers from either a sanctuary or a reserve (both defined as no-harvest zones) to the neighboring public harvest area (Holland and Brazee, 1996; Hannesson, 1998; Holland, 2000; Anderson, 2002; Pezzey et al., 2000; Sanchirico and Wilen, 2001, 2002) or from aquaculture to open-access area (Mikkelsen, 2007), to the best of our knowledge, no study has analyzed the four areas simultaneously and specifically for the Bay region. Our results are based on a number of assumptions and simplifications. An important assumption is that effort in the public areas can be perfectly regulated. Holland (2000) finds that reserves, defined as no-harvest zones, are second best solutions compared to more direct and efficient controls on fishing effort and harvest. Our results of optimal area allocation are in exactly the same spirit. Overall, the idea that in a deterministic setting, like the one in this paper, if effort can be perfectly and permanently controlled makes the no-harvest zone needless is also supported by others (Holland and Brazee, 1996; Hannesson 1998; Anderson, 2002). An important question for future research is to examine how our results would be different if effort in the public area cannot be optimally regulated. If this cannot be achieved, sanctuaries could play a more important role in the optimal solution.

Similar to Hannesson (1998), our model compares different management scenarios, but uses as choice criterion the Net Present Value instead of maximizing annual sustainable rents. Furthermore, our model differs from previous ones in the way the main stock under examination is characterized. Oysters are attached to their substrate and they are therefore immobile. Consequently, watermen in neighboring areas cannot harvest grown and protected stock that has migrated (Anderson, 2002) due to density difference between patches (Holland and Brazee, 1996) or other sink-source mechanisms (Sanchirico and Wilen, 2001). Due to fish high migration coefficient, present in his model, Hannesson (1998) demonstrates the striking result that overharvesting is at such an extent, that approximately 70-80 percent of the entire area should be dedicated to the protected area. We believe that it is the assumptions of complete regulation of effort coupled with the inherent immobility of oysters that make our results different from his.

Our model has some caveats and it is not free from criticism. For instance the optimal rotation lengths in reserves are very long. A typical reserve in Maryland depending on disease intensity and salinity is open to harvest approximately every 4-5 years (Cambell, E. pers communication). Our model ignores restoration activities, such as oyster seeding and creation of oyster habitat using shell. Such activities are likely to have resulted in shorter rotation lengths and will be addressed in the future. A more profound reason for having longer rotation lengths is the intrinsic growth rate parameter r_o. Our calculation of this parameter is 0.2, which is less than previous reported values (0.239-0.275) for market-sized oysters (Wieland and Kasperski, 2008; Kasperski and Wieland). Even though the above values of the intrinsic growth rate are used to answer different research questions, it would worth to examine the sensitivity of our results on these. Another limited assumption in our model is the homogeneity of all areas. While such supposition is convenient and has been made before (Pezzey et al., 2000) it is rather unlikely that Bay's bottom is fully homogeneous. Areas like sanctuaries, receive more attention than simply our initial cost of creation assumes raising the important issues of shell and seed supply necessary for oyster bar restoration. Last but not least, the way we model aquaculture is simplistic. We acknowledge its importance and allow a constant volume to be produced given its area allocation. Nonetheless, from the introduction it should be clear that the different types of aquaculture along with the different regulatory approaches the states adopt make this system more complicated. Because our model of aquaculture is simplistic, our results should not be used to evaluate whether aquaculture should be expanded or contracted in the Bay. All these topics represent excellent candidates for future research.

Table 1. Parameter Values

Symbol	Value	Definition
Biological Parameters		
$\bar{L}$	356,000,000.00	long-run average annual loadings of N (in pounds)
ρ	0.001653	denitrification rate (in pounds per market-sized oyster)
s	0.001146	nitrogen removal through harvest (in pounds per-market-sized oyster)
d	0.9	Annual nitrogen decay
r_0	0.2	Oysters' intrinsic growth rate
K_0	18935.64	Per acre Oysters' carrying capacity
φ	0.034	Stock spillover from larvae dispersal
r_c	0.52	blue crabs' intrinsic growth rate
K_C	550,000,000.00	blue crabs' carrying capacity
ζ	0.3	Biodiversity spillover
N(0)	250,497,792.00	Initial condition for nitrogen
N*	200,000,000.00	Nitrogen in the Bay implied by TMDL target
Economic Parameters		
δ	0.03	Discount rate
a_{BAY}	637197.2	Total amount of quality oyster habitat in Bay (acres)
P_O	0.5	Unit price of market-sized oyster
v	186	Cost per acre per unit effort in oyster harvest
q_O	0.153830645	Catchability coefficient for oysters
P_C	0.3	Unit price of age-1+ Blue crab
m	305	Cost per unit effort in blue crab harvest
q_C	2.88917E-05	Catchability coefficient for blue crabs
y	0.000131579	Cost parameter is aquaculture
l	5 or 22.1	cost of one pound of nitrogen reduction by BMPs
U(N)	350,000,000.00	Value of clean water for the entire Bay
C^{cr}_s	10000	Per acre creation cost of sanctuaries

Mathematical Appendix

Derivation of equation (14):

Equating (12) with (13) we have:

$$\begin{aligned} & a_W \frac{v}{q_O O_W^2}\left[F(O_W) + I(O) - q_O E_W O_W\right] \\ &\qquad = \left[P_O a_W - \frac{a_W v}{q_O O_W} - a_W s \lambda_N\right]\left[\delta - F'(O_W)\right] - \frac{\partial I}{\partial O_W}(\lambda_S + \lambda_R) \\ &\qquad - \frac{\partial I}{\partial O_W}\left[P_O a_W - \frac{a_W v}{q_O O_W} - a_W s \lambda_N\right] - a_W \frac{v E_W}{O_W} - \lambda_C \frac{\partial F_C}{\partial O_W} + a_W \rho \lambda_N \end{aligned}$$

$$\begin{aligned} & a_W \frac{v}{q_O O_W^2}\left[F(O_W) + I(O)\right] - a_W \frac{v q_O E_W O_W}{q_O O_W^2} \\ &\qquad = \left[P_O a_W - \frac{a_W v}{q_O O_W} - a_W s \lambda_N\right]\left[\delta - F'(O_W)\right] - \frac{\partial I}{\partial O_W}(\lambda_S + \lambda_R) \\ &\qquad - \frac{\partial I}{\partial O_W}\left[P_O a_W - \frac{a_W v}{q_O O_W} - a_W s \lambda_N\right] - a_W \frac{v E_W}{O_W} - \lambda_C \frac{\partial F_C}{\partial O_W} + a_W \rho \lambda_N \end{aligned}$$

The terms $a_W \frac{v q_O E_W O_W}{q_O O_W^2}$ and $a_W \frac{v E_W}{O_W}$ cancel out so that

$$\begin{aligned} & a_W \frac{v}{q_O O_W^2}\left[F(O_W) + I(O)\right] \\ &\qquad = \delta\left[P_O a_W - \frac{a_W v}{q_O O_W} - a_W s \lambda_N\right] - F'(O_W)\left[P_O a_W - \frac{a_W v}{q_O O_W} - a_W s \lambda_N\right] \\ &\qquad - \frac{\partial I}{\partial O_W}(\lambda_S + \lambda_R) - \frac{\partial I}{\partial O_W}\left[P_O a_W - \frac{a_W v}{q_O O_W} - a_W s \lambda_N\right] - \lambda_C \frac{\partial F_C}{\partial O_W} + a_W \rho \lambda_N \end{aligned}$$

$$\begin{aligned} & \delta\left[P_O a_W - \frac{a_W v}{q_O O_W} - a_W s \lambda_N\right] \\ &\qquad = F'(O_W)\left[P_O a_W - \frac{a_W v}{q_O O_W} - a_W s \lambda_N\right] + \frac{\partial I}{\partial O_W}\left[P_O a_W - \frac{a_W v}{q_O O_W} - a_W s \lambda_N\right] \\ &\qquad + \frac{v}{q_O O_W^2}\left[F(O_W) + I(O)\right] + \left\{\frac{\partial I}{\partial O_W}(\lambda_S + \lambda_R) + \lambda_C \frac{\partial F_C}{\partial O_W} - a_W \rho \lambda_N\right\} \end{aligned}$$

Dividing through by $\left[P_O a_W - \frac{a_W v}{q_O O_W} - a_W s \lambda_N\right]$ we get

$$\delta = F'(O_W) + \frac{\partial I}{\partial O_W} + \frac{v}{O_W(P_O q_O O_W - v - q_O O_W s \lambda_N)}\left[F(O_W) + I(O)\right] + \frac{q_O O_W}{a_W(P_O q_O O_W - v - s\lambda_N q_O O_W)}\left\{\frac{\partial I}{\partial O_W}(\lambda_S + \lambda_R) + \lambda_C \frac{\partial F_C}{\partial O_W} - a_W \rho \lambda_N\right\}$$

which is equation (14).

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