Dynamic Cross-Hedge Ratios: An Application of Copula Models

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Abstract

In this study, we propose a new approach to estimating optimal dynamic cross-hedge ratios. In particular, we apply copula models to discuss the use of corn futures contracts to cross hedge grain sorghum, and the use of Kansas wheat futures contracts to cross hedge barley. Hedge (or cross-hedge) ratios are generally estimated by using the variances of cash and futures returns and the correlation between these returns. We compute the time-varying variances of cash and futures returns by applying the Error Correction Model (ECM) with GARCH error terms. The time-varying correlation term in the dynamic cross-hedge ratio is obtained from eight copula models – two elliptical copulas (Gaussian and Student's-t) and six Archimedean copulas (Clayton, rotated Clayton, Gumbel, rotated Gumbel, Frank, and Plackett). We use maximum likelihood estimation techniques to estimate the copula models and compare the performance of these copula models by their maximum likelihood values. Results confirm the significant linkages between these markets and demonstrate the effectiveness of cross-hedging as a mechanism for managing price risks.

1. Introduction

In financial markets, hedging has become a popular way to control risks and offset losses. A hedge can be constructed across many different financial instruments, such as stocks, futures, options, and swaps. The basic idea of hedging is to reduce the risk of an investment by investing in another asset with adverse price movements. A cross hedge is to hedge one position by taking an offsetting position in another good whose price is highly correlated. In agricultural economics, investigating the optimal hedge ratio has important impacts on controlling risk for agricultural commodity prices. For example, for goods that have no futures market, a cross hedge would be an effective way to reduce price risks (Anderson and Danthine 1981).

Existing research on cross-hedge ratios that focuses on the dependence between returns has been addressed using the Pearson correlation. A limitation of the Pearson correlation is that it is based on the assumption of normality, or more precisely it is only a measure of dependence in the elliptical family of distributions. On the other hand, it only measures linear dependence and therefore misses nonlinear correlations between returns. One way to overcome the shortage of the Pearson correlation is to measure more flexible types of dependence using copulas. Copula models have been widely discussed as a more effective tool to model more flexible types of dependence (or correlation) between variables either jointly or separately from their marginal distributions.

In this study, we discuss the use of corn futures contracts to cross hedge grain sorghum, and the use of Kansas wheat futures contracts to cross hedge barley by applying copula models. Investigating optimal cross-hedge ratios has important implications for managing risks among prices in the grain sorghum and barley markets. Grain sorghum is used as a substitute for corn in some industries; while wheat is considered as a good substitute for barley, which is generally thinly traded in futures markets. Grain sorghum lacks an explicit futures market, which again leads to a dependence on corn futures as a risk management instrument. Time-varying cross-hedge ratios are obtained from the dependence derived from copula models using cash and futures returns, and the variances computed from the Error Correction Model (ECM) with GARCH error terms. Two elliptical copulas (Gaussian and Student's-t) and four Archimedean copulas (Clayton, Rotated Clayton, Gumbel, Rotated Gumbel) are utilized to estimate the degree and behavior of dependence. We compare the performance of these copula models by their maximum likelihood values and the AIC criterion. Our results confirm the significant linkages between these markets and demonstrate the effectiveness of cross-hedging as a mechanism for managing price risks.

2. Previous Research

An extensive literature has examined the methodology of estimating optimal risk-minimizing hedge ratios. Traditional hedging theory emphasizes that holders of an asset can protect themselves against the loss from a cash price decline by selling futures contracts with an amount equal to the size of the long cash position. Working (1953), an early contributor to the study of hedging, argued that most hedging was done in expectation of a change in the correlation of spot and futures prices and the traditional hedging theory failed to explain how futures markets work. Combining the concept of risk avoidance from the traditional hedging theory and the idea that hedge depends on the expected change from Working, Johnson (1960) constructed a model that applied the basic portfolio theory to clarify the concept of hedging. This model is the first to apply both expectation of return and variance of return to the analysis of hedging.

Early research on the optimal hedge ratio (e.g., Ederington 1979; Hills and Schneeweis 1982) applied the technique of regressing cash prices on futures prices and used the estimate of slope as a measure of the optimal hedge ratio. However, many subsequent studies (e.g., Myers and Thompson 1989; Cecchtti, Cumby and Figlewski 1988) have shown that the estimate of coefficient (or slope) is not satisfactory. Researchers then continued to investigate the dynamic optimal hedge ratio. The first study of the dynamic hedging model in agricultural economics was discussed by Karp (1987).

Since the 1990s, GARCH models have been extensively applied in studies of hedging. Baillie and Myers (1991) were the first to use the GARCH model to estimate time-varying hedge ratios, using conditional variances and covariance of cash and futures prices. The advantage of using GARCH models is that they permit the second moments of the conditional distributions of returns to change over time, and they also allow for leptokurtosis in the unconditional distribution of returns. Furthermore, when extending to multivariate GARCH models, we can also examine the time-varying dependence from the conditional variance-covariance matrices. To improve the performance of GARCH models for estimating dynamic hedge ratios, Kroner and Sultan (1993) developed a bivariate error correction model with a GARCH error structure to deal with the problem of long-run cointegrating relationships among financial assets. This model has become generally accepted as a basis for most recent research on dynamic hedge ratios. Subsequent research has been extended to the application of out-of-sample forecasting techniques (e.g., Lien 2005).

Cross hedging has become an important part of the research on hedging due to its great impact on price risk controlling, especially for goods that have no futures market. Among early studies, Anderson and Danthine (1981) provided a theoretical model of cross hedging in futures markets and explained the important role of cross hedging for risk management. Due to the profound impact of cross hedging, a number of studies have been done on agricultural commodities (e.g., Hayenga and DiPietre 1982; Blake and Catlett 1984; Vukina and Anderson 1993; Haigh and Holt 2000; Franken and Parcell 2003). Most of the more recent applications in agricultural markets have been conducted by using the GARCH-type models. For example, Haigh and Holt (2000) investigated time-varying hedge ratios by applying MGARCH, the OLS and SUR methodologies and explained how international grain traders could reduce price risks by selecting combinations of different cash and futures contract positions. Research on cross hedging remains attractive, and most of them are mainly focusing on financial and foreign exchange markets (e.g., Chang and Wong 2003; Wong 2007).

3. Methodology

3.1 Hedging Model: Bivariate ECM with GARCH Error Terms

Suppose an investor has a fixed long cash position of one unit of a commodity and a short position of $-b$ units in a futures market at time $t-1$. Let s_t and f_t be the changes in the spot and futures prices (or returns of these prices) at time t. The return on holding this portfolio of cash and futures positions between $t-1$ and t, x_t , can be denoted by

$$x_t = s_t - b_{t-1} f_t \, .$$

Assume that the utility function for the investor is the mean-variance expected utility,

$$EU(x) = E(x) - \gamma Var(x), \;\; \gamma > 0,$$

where γ is the degree of absolute risk aversion. Thus, the investor's decision problem under the information at $t-1$ can be expressed as

$$\max_{b_{t-1}} E_{t-1} U(x_t) = E_{t-1}(s_t) - b_{t-1} E_{t-1}(f_t) \\ - \gamma \{ Var_{t-1}(s_t) + b_{t-1}^2 Var_{t-1}(f_t) - 2 b_{t-1} cov_{t-1}(s_t, f_t) \} \, .$$

The first-order condition implies that the optimal value of b_{t-1} is

$$b_{t-1}^{*} = \frac{-E_{t-1}(f_t) + 2\gamma cov_{t-1}(s_t, f_t)}{2\gamma Var_{t-1}(f_t)}.$$

If the futures price (F_t) follows a martingale, which means $E_{t-1}(F_t) = F_{t-1}$ or $E_{t-1}(f_t) = 0$, then b_{t-1}^{*}, the optimal hedge ratio, becomes

$$b_{t-1}^{*} = \frac{cov_{t-1}(s_t, f_t)}{Var_{t-1}(f_t)}\text{, or}$$

$$b_{t-1}^{*} = corr_{t-1}(s_t, f_t)\sqrt{\frac{Var_{t-1}(s_t)}{Var_{t-1}(f_t)}} \quad (3.1).$$

Kroner and Sultan (1993) proposed a bivariate error correction model (ECM) with GARCH error terms to solve for the dynamic optimal hedge ratio, b_{t-1}^{*}, which is derived as follows.

$$s_t = \beta_{0s} + \beta_{1s}(S_{t-1} - \delta F_{t-1}) + \varepsilon_{st} \quad (3.2)$$

$$f_t = \beta_{0f} + \beta_{1f}(S_{t-1} - \delta F_{t-1}) + \varepsilon_{ft} \quad (3.3)$$

$$\begin{bmatrix}\varepsilon_{st}\\ \varepsilon_{ft}\end{bmatrix} \Big| \Psi_{t-1} \sim N(0, H_t)$$

$$H_t = \begin{bmatrix} h_{st} & \rho\sqrt{h_{st}h_{ft}} \\ \rho\sqrt{h_{st}h_{ft}} & h_{ft} \end{bmatrix}$$

$$h_{st} = \alpha_{0s} + \alpha_{1s}\varepsilon_{s,t-1}^{2} + \alpha_{2s}h_{s,t-1} \quad (3.4)$$

$$h_{ft} = \alpha_{0f} + \alpha_{1f}\varepsilon_{f,t-1}^{2} + \alpha_{2f}h_{f,t-1} \quad (3.5),$$

where S_{t-1} and F_{t-1} are spot and futures prices at $t-1$, and Ψ_{t-1} is the information set at $t-1$. The dynamic optimal hedge ratios, b^{*}, are then calculated by the estimates of h_{st}, h_{ft}, and ρ.

In equation (3.2) and (3.3), the term $(S_{t-1} - \delta F_{t-1})$ is called the error correction term. This term imposes the long-run relationship between S_t and F_t into a short-run model. In the long-run, S_t and F_t could share the same stochastic trend (i.e. S_t and F_t are cointegrated). Therefore, the ignorance of the term $(S_{t-1} -$

δF_{t-1}) could lead to a statistical bias in b_{t-1}^{*}.

Kroner and Sultan assumed that the conditional GARCH residuals are normally distributed. However, a great amount of empirical evidence suggests that most financial returns may not follow a normal distribution. Thus, for the distribution assumptions of the conditional GARCH residuals, we extend to Student's-t, and skewed-t, which are commonly used in recent studies for distributions of returns in financial analysis. The density functions of these distributions are showed as follows:

(1) Normal distribution (with zero mean):

$$f_{Z_t}(z_t) = \frac{1}{\sqrt{2\pi}} exp\left(-\frac{z_t^2}{2}\right) \quad (3.6),$$

where z_t is ratio of the GARCH residual and its conditional variance, or $z_t = \varepsilon_t / h_t$.

(2) Student's-*t* distribution with degrees of freedom (DoF) υ:

$$f_{Z_t}(z_t; \upsilon) = \frac{\Gamma\left(\frac{\upsilon+1}{2}\right)}{\Gamma\left(\frac{\upsilon}{2}\right)\sqrt{\upsilon\pi}\left(1+\frac{z_t^2}{\upsilon}\right)^{\left(\frac{\upsilon+1}{2}\right)}} \quad (3.7)$$

where $\Gamma(\cdot)$ is the gamma function which is defined as $\Gamma(x) = \int_0^\infty t^{x-1} e^{t} dt$, and if x is an integer, then $\Gamma(x) = (x-1)!$.

(3) Skewed-*t* distribution:

$$f_{Z_t}(z_t; \xi, \upsilon) = \begin{cases} \frac{2}{\xi + \frac{1}{\xi}} g(\xi z_t | \upsilon),\ z_t < 0 \\ \frac{2}{\xi + \frac{1}{\xi}} g\left(\frac{z_t}{\xi} \middle| \upsilon\right),\ z_t \geq 0 \end{cases}$$

where $g(\cdot\,|\upsilon)$ is the symmetric (unit variance) Student's-*t* density (equation 3.7) with DoF υ, and ξ measures the degree of asymmetry.

3.2 Estimating Optimal Hedge Ratios by Using Copula Models

In this study, instead of a bivariate GARCH model, we estimate the correlation term in equation (3.1), $corr_{t-1}(s_t, f_t)$, by applying copula models, and obtain the time-varying variances, $Var_{t-1}(s_t)$ and $Var_{t-1}(f_t)$, by estimating the two univariate GARCH models (equation (3.2) and (3.4), and (3.3) and (3.5)) with different distribution assumptions.

The application of copula models has become a significant improvement in modeling conditional dependence. The idea of copula, which was first proposed by Sklar (1959), states that any N-dimensional joint distribution function can be decomposed into N marginal distributions and a copula function, and the copula function can properly describe the dependence between the N variables. By Sklar's theorem, for a 2-dimensional joint distribution function $G(x, y)$ with continuous marginal cumulative density functions $F_X(x)$ and $F_Y(y)$, there exists a copula function $C(\cdot)$ that satisfies

$$G(x,y) = C\big(F_X(x), F_Y(y)\big).$$

This equation implies that marginal distributions and the dependence described by the copula can be separated. Thus, copula functions allow greater flexibility in measuring dependence and modeling joint distributions.

The bivariate joint CDF of the conditional GARCH residuals, ε_{st} and ε_{ft}, can be rewritten as

$$G_t\big(\varepsilon_{st}, \varepsilon_{ft} \big| \Psi_{t-1}\big) = C_t\left(F_{\varepsilon_{st}}(\varepsilon_{st}), F_{\varepsilon_{ft}}(\varepsilon_{ft}) \middle| \Psi_{t-1}\right)$$

And therefore the dependence of the two returns, s_t and f_t, conditional on the information at $t-1$ can be measure by a copula function with marginal CDFs of ε_{st} and ε_{ft}.

Assume that $G_t(\cdot)$ is twice differentiable. Then the conditional joint PDF becomes

$$g_t\big(\varepsilon_{st}, \varepsilon_{ft} \big| \Psi_{t-1}\big) = \frac{\partial G_t\big(\varepsilon_{st}, \varepsilon_{ft} \big| \Psi_{t-1}\big)}{\partial \varepsilon_{st} \partial \varepsilon_{ft}}$$

$$= c_t\left(F_{\varepsilon_{st}}(\varepsilon_{st}), F_{\varepsilon_{ft}}(\varepsilon_{ft}) \middle| \Psi_{t-1}\right) \times f_{\varepsilon_{st}}(\varepsilon_{st} | \Psi_{t-1}) \times f_{\varepsilon_{ft}}\big(\varepsilon_{ft} \big| \Psi_{t-1}\big)$$

(3.9).

From equation (3.9), the log-likelihood function for all parameters is

$$log\left(g_t(\varepsilon_{st},\varepsilon_{ft}|\Psi_{t-1},\theta)\right)$$
$$= logc_t(u_t,v_t|\Psi_{t-1},\theta_c) + logf_{\varepsilon_{st}}(\varepsilon_{st}|\Psi_{t-1},\theta_s)$$
$$+ logf_{\varepsilon_{ft}}(\varepsilon_{ft}|\Psi_{t-1},\theta_f)\,,$$

where θ's represent parameter sets in the copula density and marginal densities, $u_t = F_{\varepsilon_{st}}(\varepsilon_{st})$, and $v_t = F_{\varepsilon_{ft}}(\varepsilon_{ft})$. One approach to estimate the parameters is the two-step estimation. In this approach, parameters in the copula function and those in the density functions of ε_{st} and ε_{ft} are estimated separately, which implies

$$\hat{\theta}_s = arg\max_{\theta_s}\sum_{t=1}^{T} logf_{\varepsilon_{st}}(\varepsilon_{st}|\Psi_{t-1},\theta_s)$$

$$\hat{\theta}_f = arg\max_{\theta_f}\sum_{t=1}^{T} logf_{\varepsilon_{ft}}(\varepsilon_{ft}|\Psi_{t-1},\theta_f)$$

$$\hat{\theta}_f = arg\max_{\theta_c}\sum_{t=1}^{T} logc_t(u_t,v_t|\Psi_{t-1},\theta_c)\,.$$

From equation (3.9),

$$c_t\left(F_{\varepsilon_{st}}(\varepsilon_{st}),F_{\varepsilon_{ft}}(\varepsilon_{ft})\middle|\,\Psi_{t-1}\right) = \frac{g_t(\varepsilon_{st},\varepsilon_{ft}|\Psi_{t-1})}{f_{\varepsilon_{st}}(\varepsilon_{st}|\Psi_{t-1})\times f_{\varepsilon_{ft}}(\varepsilon_{ft}|\Psi_{t-1})}\,. \quad (3.10)$$

By the invariance property of copulas, a monotonic transformation of variables ε_{st} and ε_{ft} will not affect the form and parameters of the copula, so that dependence of variables deriving from that copula will not change (See Nelson 1999, Theorem 2.4.3). Thus, the dependence measured from $C_t\left(F_{\varepsilon_{st}}(\varepsilon_{st}),F_{\varepsilon_{ft}}(\varepsilon_{ft})|\Psi_{t-1},\rho_t\right)$ is the same as that from $C_t\left(F_{\tilde{\varepsilon}_{st}}(\tilde{\varepsilon}_{st}),F_{\tilde{\varepsilon}_{ft}}(\tilde{\varepsilon}_{ft})|\Psi_{t-1},\rho_t\right)$, where $\tilde{\varepsilon}_{it}$ is the monotonic transformation of ε_{it}. For example, in the Gaussian copula, the monotonic transformation can be $\tilde{\varepsilon}_{it} = \Phi^{-1}\left(F_{\varepsilon_{it}}(\varepsilon_{it})\right)$, where $\Phi^{-1}(\cdot)$ is the inverse of the standard normal CDF (Cumulative Density function), and $i = s, f$. In the student's-t copula,

$\tilde{\varepsilon}_{it} = T^{-1}\left(F_{\varepsilon_{it}}\left(\varepsilon_{it}\right), \nu\right)$, where $T^{-1}(\cdot)$ is the inverse of the student's-t CDF with DoF ν.

Performing the monotonic transformation of the GARCH residuals simplifies the estimation of the copula density $c_t\left(F_{\varepsilon_{st}}\left(\varepsilon_{st}\right), F_{\varepsilon_{ft}}\left(\varepsilon_{ft}\right) \middle| \Psi_{t-1}\right)$ by estimating $c_t\left(F_{\tilde{\varepsilon}_{st}}\left(\tilde{\varepsilon}_{st}\right), F_{\tilde{\varepsilon}_{ft}}\left(\tilde{\varepsilon}_{ft}\right) \middle| \Psi_{t-1}\right)$ without affecting the dependence of the GARCH residuals. That is, to simplify the estimation of $c_t\left(F_{\varepsilon_{st}}\left(\varepsilon_{st}\right), F_{\varepsilon_{ft}}\left(\varepsilon_{ft}\right) \middle| \Psi_{t-1}\right)$, we can estimate the dependence from

$$c_t\left(F_{\tilde{\varepsilon}_{st}}\left(\tilde{\varepsilon}_{st}\right), F_{\tilde{\varepsilon}_{ft}}\left(\tilde{\varepsilon}_{ft}\right) \middle| \Psi_{t-1}\right) = \frac{g_{\tilde{\varepsilon}_{st}, \tilde{\varepsilon}_{ft}}\left(\tilde{\varepsilon}_{st}, \tilde{\varepsilon}_{ft} \middle| \Psi_{t-1}\right)}{f_{\tilde{\varepsilon}_{st}}\left(\tilde{\varepsilon}_{st} \middle| \Psi_{t-1}\right) \times f_{\tilde{\varepsilon}_{ft}}\left(\tilde{\varepsilon}_{ft} \middle| \Psi_{t-1}\right)}.$$

The invariance property of copula also explains why we could still use elliptical copulas to measure the dependence between the two GARCH residuals even if the distributions of these residuals deviate from the marginal distributions derived from the copula models.

3.3 Measuring Dependence

Copula models can be used to measure dependence or association among random variables. There are various ways to measure dependence between two random variables, for example, Kendall's τ and Spearman's ρ. Let X and Y be two continuous random variables whose copula is C. Then the (population version of) Kendall's τ for X and Y is given by

$$\tau_{X,Y} = 4\iint_{\mathrm{I}^2} C(u,v)\,dC(u,v) - 1.$$

And the (population version of) Spearman's ρ is given by

$$\rho_{X,Y} = 12\iint_{\mathrm{I}^2} C(u,v)\,du\,dv - 3.$$

Another important measure we will discuss in this section is the tail dependence, which measures the dependence between the two variables in the upper-right quadrant and in the lower-left quadrant of I^2. Let X and Y be two continuous random variables with CDF F_X and F_Y, respectively. The upper tail dependence parameter

λ_U and the lower tail dependence parameter λ_L are defined as

$$\lambda_U = \lim_{t \to 1^-} P\left[Y > F_Y^{-1}(t) \middle| X > F_X^{-1}(t) \right] \quad (2.3.11)$$

$$\lambda_L = \lim_{t \to 0^+} P\left[Y \le F_Y^{-1}(t) \middle| X \le F_X^{-1}(t) \right] \quad (2.3.12)$$

Let C be the copula of X and Y, if the limits in equation (2.3.11) and (2.3.12) exist, then

$$\lambda_U = 2 - \lim_{t \to 1^-} \frac{1 - C(t,t)}{1-t}$$

$$\lambda_L = \lim_{t \to 0^+} \frac{C(t,t)}{t}.$$

3.4 Elliptical Copulas and Archimedean Copulas

In this study, we apply two elliptical copulas (Gaussian and Student's-*t*), and six Archimedean copulas (Clayton, rotated Clayton, Gumbel, rotated Gumbel, Frank, and Plackett) to measure the dependence between s_t and f_t, and compare the performance of these copulas by the AIC criterion and their maximum likelihood values.

For elliptical copulas, the Gaussian (or normal) copula and the student's-t copula are popular in applications due to their convenience for computation. The Gaussian copula has no tail dependence, while the student's-t copula allows different degrees of symmetric tail dependence (DoF). A smaller DoF implies greater tail dependence. As the DoF goes to infinity, the student's-t copula converges to the Gaussian copula. For these two elliptical copulas, there exists a bivariate joint density function $g_{MN}(m,n)$ that satisfies

$$C(u,v) = \int_{-\infty}^{F_M^{-1}(u)} \int_{-\infty}^{F_N^{-1}(v)} g_{MN}(m,n) dn dm \,,$$

where $u = F_X(x)$, $v = F_Y(y)$, and $F_M(\cdot)$ and $F_N(\cdot)$ are marginal CDFs derived from $g_{MN}(m,n)$. In a Gaussian copula, $g_{MN}(m,n)$ is the density function of a bivariate standard normal distribution, and $F_M(\cdot)$ and $F_N(\cdot)$ are standard normal

CDFs. In a student's-t copula, $g_{MN}(m,n)$ is the probability density function of a bivariate student's-t distribution with DoF υ, and $F_M(\cdot)$ and $F_N(\cdot)$ are univariate student's-t CDF with DoF υ.

From the above discussion, a Gaussian copula is,

$$C_t^G(u_t, v_t|\rho_t) = \int_{-\infty}^{\Phi^{-1}(u_t)} \int_{-\infty}^{\Phi^{-1}(v_t)} \frac{1}{2\pi\sqrt{|R_t|}} exp\left(-\frac{U'R_t^{-1}U}{2}\right) dndm$$

where $u_t = F_{Z_{st}}(z_{st}),\ z_{st} = \varepsilon_{st}/h_{st},\ v_t = F_{Z_{ft}}(z_{ft}),\ z_{ft} = \varepsilon_{ft}/h_{ft},\ U = [m \quad n]'$, $\Phi^{-1}(\cdot)$ is the inversed standard normal cumulative density function (CDF), and R_t is the correlation matrix defined as

$$R_t = \begin{bmatrix} 1 & \rho_t \\ \rho_t & 1 \end{bmatrix}.$$

From equation (3.10), the Gaussian copula density function is

$$c_t^G(u_t, v_t|\rho_t) = \frac{1}{\sqrt{1-\rho_t^2}} \exp\left(-\frac{\tilde{\varepsilon}_{st}^2 + \tilde{\varepsilon}_{ft}^2 - 2\rho_t \tilde{\varepsilon}_{st}\tilde{\varepsilon}_{ft}}{2(1-\rho_t^2)} + \frac{\tilde{\varepsilon}_{st}^2 + \tilde{\varepsilon}_{ft}^2}{2}\right),$$

where $\tilde{\varepsilon}_{st} = \Phi^{-1}(u_t) = \Phi^{-1}\left(F_{\varepsilon_{st}}(\varepsilon_{st})\right)$, and $\tilde{\varepsilon}_{ft} = \Phi^{-1}(v_t) = \Phi^{-1}\left(F_{\varepsilon_{ft}}(\varepsilon_{ft})\right)$.

A student's-t copula with DoF ν is

$$C_t^T(u_t, v_t|\nu, \rho_t) = \int_{-\infty}^{T_\nu^{-1}(u_t)} \int_{-\infty}^{T_\nu^{-1}(v_t)} \frac{\Gamma\left(\frac{\nu+2}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)\sqrt{(\pi\nu)^2|R_t|}\left(1+\frac{U'R_t^{-1}U}{\nu}\right)^{\frac{\nu+2}{2}}} dndm$$

where $U = [m \quad n]'$, and $T_\nu^{-1}(\cdot)$ is the inversed t CDF with DoF ν.

The student's-t copula density function is

$$c_t^T(u_t, v_t|\nu, \rho_t) = \frac{\Gamma\left(\frac{\nu+2}{2}\right)\Gamma\left(\frac{\nu}{2}\right)\left(1+\frac{\bar{\varepsilon}_{st}^2}{\nu}\right)^{\left(\frac{\nu+1}{2}\right)}\left(1+\frac{\bar{\varepsilon}_{ft}^2}{\nu}\right)^{\left(\frac{\nu+1}{2}\right)}}{\sqrt{(1-\rho_t^2)}\left(1+\frac{\bar{\varepsilon}_{st}^2 + \bar{\varepsilon}_{ft}^2 - 2\rho_t\bar{\varepsilon}_{st}\bar{\varepsilon}_{ft}}{(1-\rho_t^2)\nu}\right)^{\frac{\nu+2}{2}}\left(\Gamma\left(\frac{\nu+1}{2}\right)\right)^2}$$

where $\bar{\varepsilon}_t = \left[\bar{\varepsilon}_{st} \quad \bar{\varepsilon}_{ft}\right]'$, and $\bar{\varepsilon}_{it} = T_\nu^{-1}\left(F_{\varepsilon_{it}}(\varepsilon_{it})\right), i = s, f$.

Figure 1 presents the contour plots for these elliptical copulas. Parameter ρ_t in these two elliptical copulas will be used to estimate the correlation in the optimal cross-hedge ratio (equation (2.3.8)).

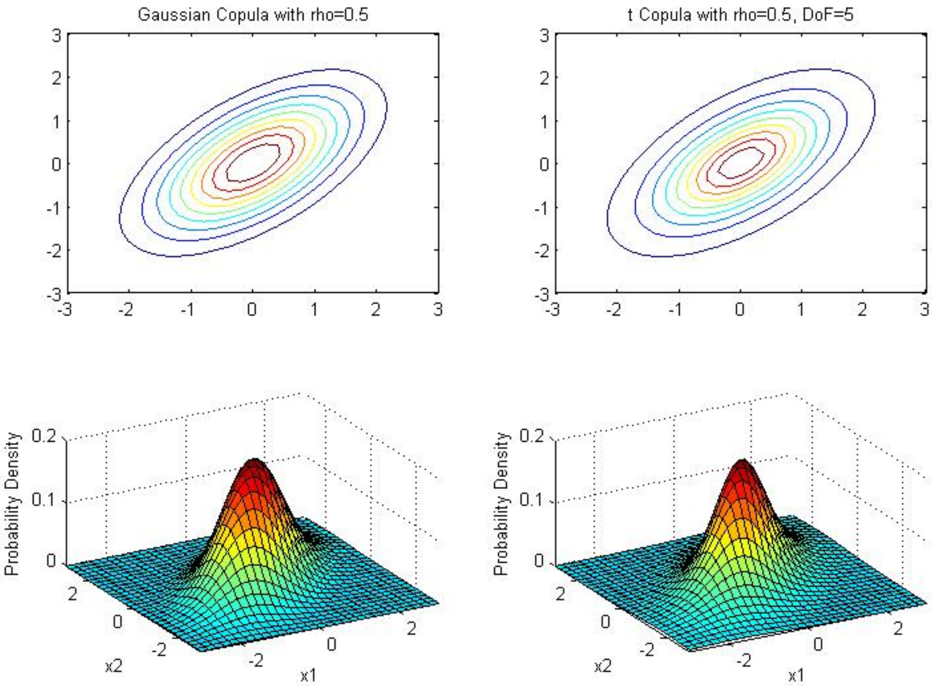


Figure 1. Contour Plots of Selected Elliptical Copulas.

In contrast of elliptical copulas, Archimedean copulas, although can only be used to measure positive dependence, allow asymmetric tail dependence in some cases. For example, the Clayton and the rotated Gumbel copulas allow for lower tail dependence but zero upper tail dependence, while the rotated Clayton and the Gumbel copulas allow for upper tail dependence but zero lower tail dependence. A copula is called Archimedean copula if it satisfies

$$\mathrm{C}(u_1, \dots, u_d) = \Psi^{-1}\big(\Psi(u_1) + \cdots + \Psi(u_d)\big),$$

where $\Psi^{-1}(\cdot)$ is called generator and $\Psi^{-1}(\cdot)$ with range $[0,1]$ is continuous and nonincreasing on $[0, \infty)$. The generators of six Archimedean copulas (Clayton, rotated Clayton, Gumbel, rotated Gumbel, Frank, and Plackett), and the CDFs and PDFs of these copulas (all in bivariate cases) are shown as follows.

1. Clayton copula

$$\Psi_C^{-1}(t) = (1+t)^{-1/\theta}\ ,\quad \theta \in (0,\infty)$$

$$C_t^C\left(u_t, v_t \middle| \theta_t\right) = \left[u_t^{-\theta_t} + v_t^{-\theta_t} - 1\right]^{-1/\theta_t}$$

$$c_t^C\left(u_t, v_t \middle| \theta_t\right) = \left(u_t v_t\right)^{-1-\theta_t}\left(1+\theta_t\right)\left[u_t^{-\theta_t} + v_t^{-\theta_t} - 1\right]^{(-1-2\theta_t)/\theta_t}$$

2. Rotated Clayton copula

$$C_t^{RC}(u_t, v_t|\theta_t) = u + v - 1 + C_t^C(1-u_t, 1-v_t|\theta_t)$$

$$c_t^{RC}(u_t, v_t|\theta_t) = c_t^C(1-u_t, 1-v_t|\theta_t)$$

3. Gumbel copula

$$\Psi_{Gb}^{-1}(t) = \exp\left(-t^{1/\theta}\right),\quad \theta \in (1,\infty)$$

$$C_t^{Gu}\left(u_t, v_t \middle| \theta_t\right) = \exp\left\{-\left[\left(-\log u_t\right)^{\theta_t} + \left(-\log v_t\right)^{\theta_t}\right]^{1/\theta_t}\right\}.$$

$$c_t^{Gu}\left(u_t, v_t \middle| \theta_t\right) = \frac{\exp\left\{-\left[\left(-\ln u_t\right)^{\theta_t} + \left(-\ln v_t\right)^{\theta_t}\right]^{1/\theta_t}\right\}\left[\ln u_t \ln v_t\right]^{\theta_t-1}\left\{\left[\left(-\ln u_t\right)^{\theta_t} + \left(-\ln v_t\right)^{\theta_t}\right]^{1/\theta_t} + \theta_t - 1\right\}}{u_t v_t\left[\left(-\ln u_t\right)^{\theta_t} + \left(-\ln v_t\right)^{\theta_t}\right]^{2-1/\theta_t}}$$

4. Rotated Gumbel copula

$$C_t^{RGu}(u_t, v_t|\theta_t) = u + v - 1 + C_t^{Gu}(1-u_t, 1-v_t|\theta_t)$$

$$c_t^{RGu}(u_t, v_t|\theta_t) = c_t^{Gu}(1-u_t, 1-v_t|\theta_t)$$

5. Frank copula

$$\Psi_F^{-1}(t) = -\frac{\ln\left(1-[1-\exp(-\theta)]\exp(-t)\right)}{\theta}$$

$$C_t^F(u_t, v_t|\theta_t) = -\frac{1}{\theta_t} log\left\{1 - \frac{\left(1-\exp(-\theta_t u_t)\right)\left(1-\exp(-\theta_t v_t)\right)}{1-\exp(-\theta_t)}\right\}$$

$$c_t^F(u_t, v_t|\theta_t) = \frac{\theta_t\left(1-\exp(-\theta_t)\right)\exp[-\theta_t u_t - \theta_t v_t]}{\left\{1-\exp(-\theta_t) - \left(1-\exp(-\theta_t u_t)\right)\left(1-\exp(-\theta_t v_t)\right)\right\}^2}$$

6. Plackett copula

$$C_t^P(u_t, v_t|\theta_t) = \begin{cases} \frac{1}{2}\frac{1}{(\theta_t - 1)}A - \sqrt{A^2 - 4u_t v_t \theta_t(\theta_t - 1)} & if \quad \theta > 0 \; and \; \theta \neq 1 \\ u_t v_t & if \;\; \theta = 1 \end{cases}$$

where $A = 1 + (\theta_t - 1)(u_t + v_t)$.

$$c_t^P(u_t, v_t|\theta_t) = \frac{\theta_t[1 + (\theta_t - 1)(u_t - 2u_t v_t + v_t)]}{\{[1 + (\theta_t - 1)(u_t + v_t)]^2 - 4u_t v_t \theta_t(\theta_t - 1)\}^{3/2}}.$$

Figure 2 presents selected contour plots for these elliptical copulas. For these Archimedean copulas, we will use Spearman's ρ in equation (2.3.10) to measure the correlation in equation (2.3.8).

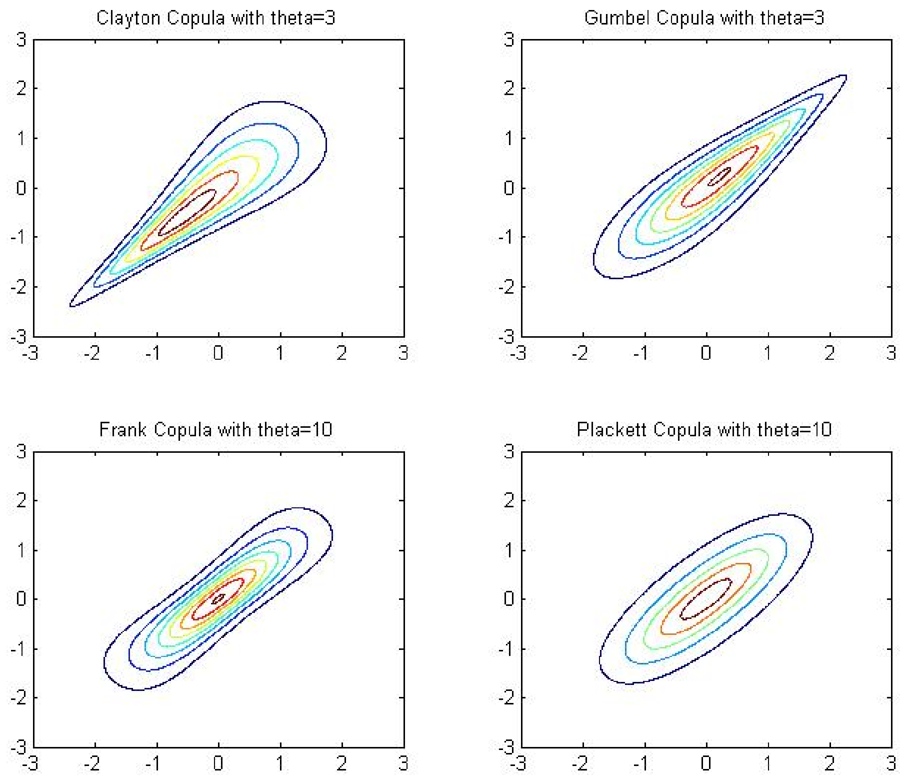


Figure 2. Contour Plots of Selected Elliptical Copulas.

3.5 Estimating the Time-Varying Dependence (or Correlation) for Copulas

Following Patton's (2006) work which extends the standard copula concept to model time-varying conditional heteroskedasticity, we impose some restrictions on the parameters in the copula models to estimate the time-varying. We assume that ρ_t in the elliptical copulas (or θ_t in the Archimedean copulas), depends on the previous cumulative probabilities u_{t-1}, v_{t-1}, …, u_{t-N}, v_{t-N}, and $1 \leq N \leq t-1$. Thus, ρ_t can be rewritten as a function of $\rho_t(u_{t-1}, v_{t-1}, \dots, u_{t-N}, v_{t-N} | \Theta)$, where Θ is a set of parameters. In this study, spot and futures prices have very strong positive correlation, so we assume that ρ_t should be positive ($0 < \rho_t < 1$). In the two elliptical copula cases, ρ_t is defined as

$$\rho_t = \frac{1}{1 + \exp\{\beta_0 + \sum_{i=1}^{N} (\beta_{i1} u_{t-i}^2 + \beta_{i2} v_{t-i}^2 + \beta_{i3} u_{t-i} v_{t-i})\}}.$$

Therefore, estimating ρ_t is equivalent to estimate the parameters of β_0, β_{i1}, β_{i2} and β_{i3} $(i = 1,2, \dots, N)$. Similarly, for the six Archimedean copulas, θ_t can be defined as

$$\theta_t(u_{t-1}, v_{t-1}, \dots, u_{t-N}, v_{t-N} | \Theta).$$

According to the properties of those parameters and the correlation between spot and futures prices, we define

$$\theta_t = \exp\left\{\beta_0 + \sum_{i=1}^{N} (\beta_{i1} u_{t-i}^2 + \beta_{i2} v_{t-i}^2 + \beta_{i3} u_{t-i} v_{t-i})\right\}$$

for Clayton, rotated Clayton, Frank, and Plackett copulas, and

$$\theta_t = 1 + \exp\left\{\beta_0 + \sum_{i=1}^{N} (\beta_{i1} u_{t-i}^2 + \beta_{i2} v_{t-i}^2 + \beta_{i3} u_{t-i} v_{t-i})\right\}$$

for Gumbel and rotated Gumbel copulas.

4. Data

The data we use in this study consist of weekly average cash prices of grain sorghum (or milo) and barley, and weekly average futures prices of corn and Kansas wheat from 1/3/2003 to 11/1/2010. We use weekly average prices because daily cash prices of sorghum or barley are often constant for a week or even longer, which complicates analysis since it leads to an excess of zero return observations. Futures prices of corn and Kansas wheat are weekly average closing prices from the Chicago Board of Trade (CBOT) and the Kansas City Board of Trade (KCBT). These futures prices are obtained from the nearby contracts. To avoid expiration effects (i.e., liquidation bias), we roll over to the next nearest contract one month prior to expiration of the futures contract. The cash price of grain sorghum is on the basis on the Gulf Coast, and the cash price of barley is on the basis of Lethbridge.

As with most studies in commodity prices, it is reasonable to base the inference on natural logarithms of prices. The cash and the futures returns are calculated by percentage changes in prices:

$$r_t = 100 \times \{ln(p_t) - ln(p_{t-1})\},$$

where p_t is the cash or futures price at time t. Figure 3 and Figure 4 illustrate the natural logarithms of the four prices. Similar patterns of trends can be found for prices of sorghum and corn, and for prices of barley and Kansas wheat.

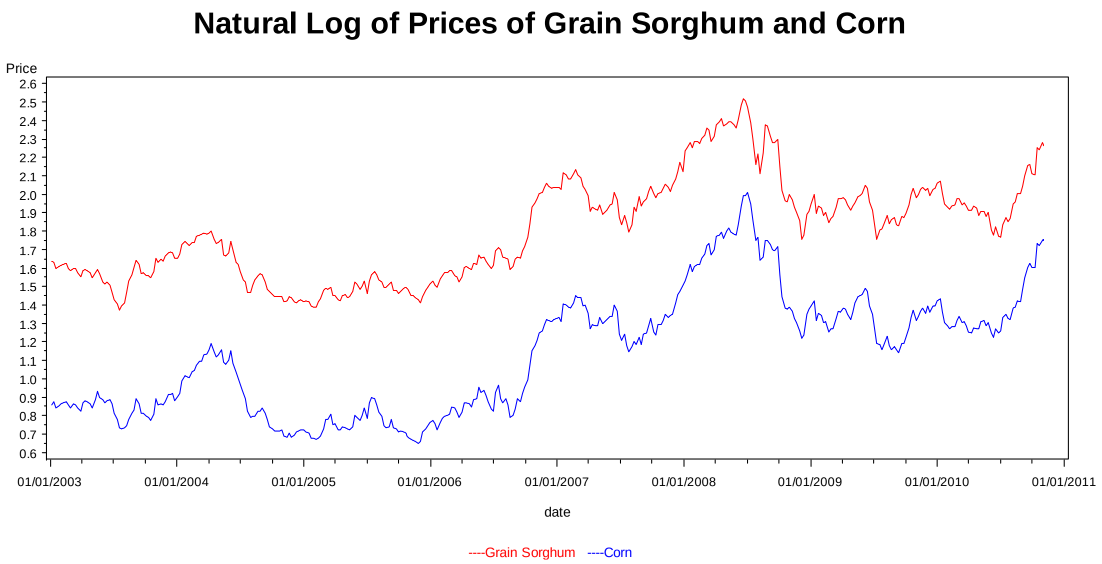


Figure 3. Cash Price of Sorghum and Futures Price of Corn (in Natural Logarithms)

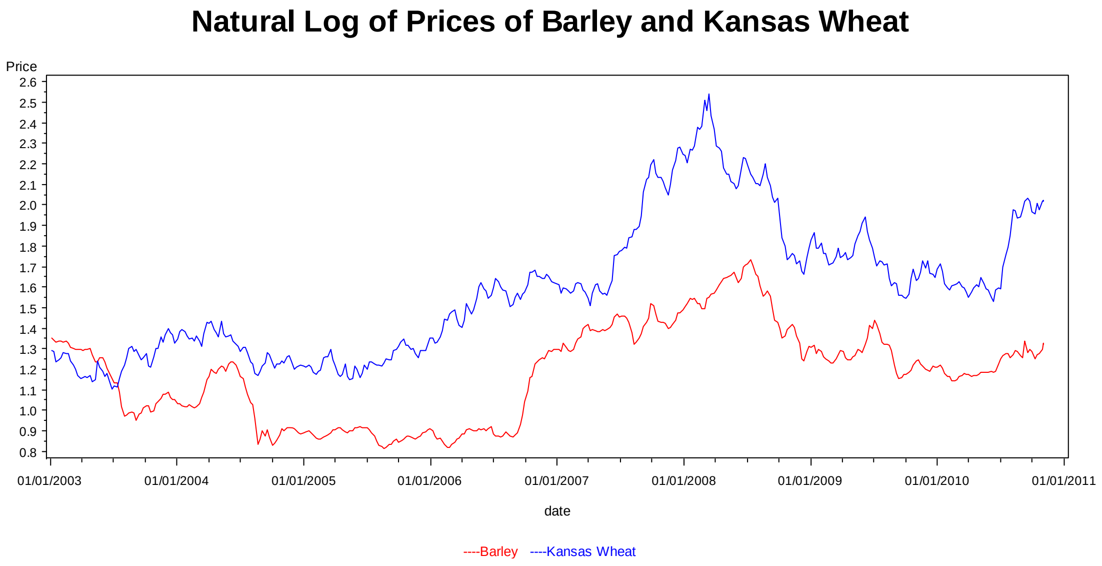


Figure 4. Cash Price of Barley and Futures Prices of Kansas Wheat (in Natural Logarithms)

Table 1 reports descriptive statistics for the four returns. Table 2 shows the results of unit root and cointegration tests in price levels. The cash price of sorghum and the futures price of corn (in natural logarithms) are nonstationary and have common stochastic trend. Same results have been found for barley and Kansas wheat. Thus, the error correction terms in equation (3.2) and (3.3) are necessary for this analysis. From Table 2, when $\delta = 0.8$ for the corn and sorghum case or $\delta = 0.7$ for the Kansas wheat and barley case, the error correction term $(S_{t-1} - \delta F_{t-1})$ becomes stationary. for the case of using corn futures to cross hedge sorghum and the case of using wheat futures to cross hedge barley. (The value 0.8 and 0.7 are chosen from the beta vector of VECM models.) Thus, we impose $\delta = 0.8$ for the analysis of using corn futures contracts to cross hedge grain sorghum, and $\delta = 0.7$ for the analysis of using Kansas wheat futures contracts to cross hedge barley. Table 3 shows the correlations and dependence between the returns.

Table 1. Summary Statistics for returns (r_t)

	sorghum	corn	barley	Kansas wheat
Observations	408	408	408	408
Mean	0.15557	0.21910	-0.00654	0.17908
Standard Deviation	3.85451	3.80475	2.33105	3.63875
Minimum	-14.65557	-15.11906	-12.57179	-10.49357
Maximum	15.15956	13.45274	7.96618	12.78522
Skewness	-0.11047	-0.31493	-0.44891	0.35037
Kurtosis	1.87783	1.37600	2.91553	0.77565
ADF Tau (single mean)	-13.19***	-12.83***	-10.76***	-12.57***

Note: (1) *** refers to the rejection of the null hypothesis of a unit root at 1%.

Table 2. ADF test and cointegration test for prices (cash and futures)

(a)

	ADF Test	
Variable	Single Mean	Trend
sorghum	-1.34	-2.39
corn	-0.94	-2.07
barley	-1.51	-2.15
Kansas wheat	-1.20	-1.96

(b)

	Cointegration Test	
	H0:rank=0 Ha:rank>0	H0:rank=1 Ha:rank>1
sorghum and corn	34.4919	2.4426
barley and Kansas wheat	20.6285	1.8167

Note: (1) Price data are in natural logarithms.
(2) The 5% critical value for cointegration test is 15.34 when H0: rank=0, and is 3.84 when H0:rank=1.

(c) $\left(S_{t-1}-\delta F_{t-1}\right)$ when $\delta=0.8$ or $\delta=0.7$

	ADF Test		
	Zero Mean	Single Mean	Trend
sorghum and corn	***	-3.62***	-3.62**
barley and Kansas wheat	-2.99***	-3.14**	-3.32*

Note: (1) ***, and ** refer to the rejection of the null hypothesis of a unit root at 1% and 5%.
(2) No unit root in levels is found.

Table 3 Correlations for Unconditional Returns (Cash and Futures)

	Pearson	Spearman	Kendall Tau	Hoeffding
sorghum and corn	0.893***	0.890***	0.726***	0.436***
barley and Kansas wheat	0.229***	0.224***	0.155***	0.016***

Note: *** indicates the correlations are significantly different from zero at 1% significance level.

5. Results

We estimate the GARCH models with three distribution assumptions – normal, student's-*t*, and skew-*t*. Results of the parameter estimates are shown in Table 4 and Table 5. The normal distribution is nested in (a special case of) the student's-t distribution and the student's-t distribution is nested in the skew-t distribution. Therefore, we can choose the optimal distribution by testing the significance of relative parameters.

In the grain sorghum case, the skewness parameter is not significantly different from 1, which means the distribution can be reduced to a student's-t. The inverse of DoF in the student's-t case is not significantly different from 0, which means the upper bound of the 90% (or 95%) confidence interval for the estimate of DoF is the positive infinity. Thus, we choose normal distribution for the grain sorghum case in the following analysis. In the corn case, the skewness parameter is also not significantly different from 1. The 90% confidence interval for the estimate of DoF is $[5.0, 62.4]$, and the upper bound of the 95% confidence interval for this estimate is the positive infinity. So, we also choose normal distribution for corn in the following analysis. Similar results could be found in the Kansas wheat case, and the normal distribution is also the optimal fit in this case. Finally, in the barley case, the skewness parameter in the skew-t distribution is significantly different from 1, so we choose skew-t distribution for the barley case.

Table 4. GARCH Model Results for the Grain Sorghum and the Corn Cases

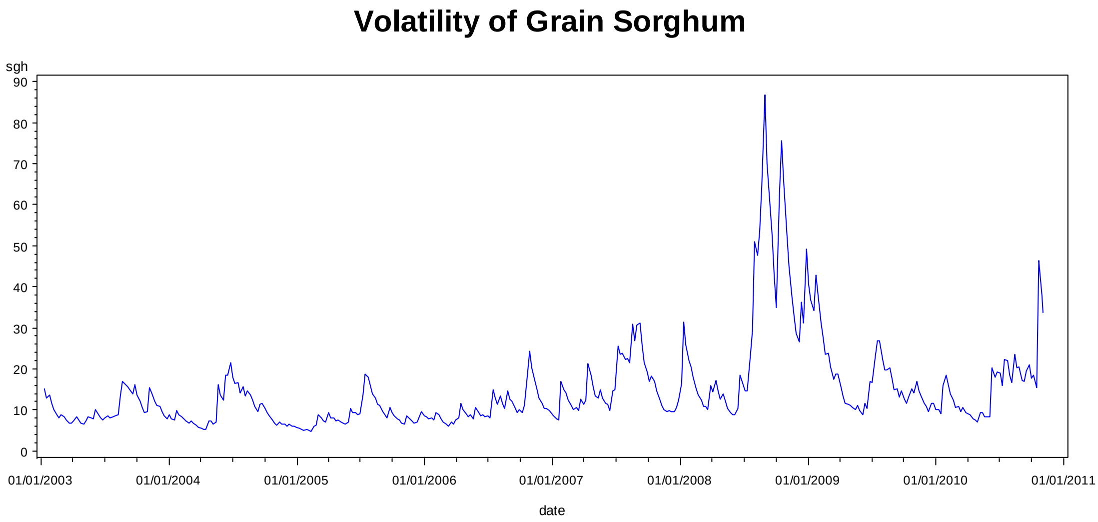
	Grain Sorghum		
	normal	*t*	skew-*t*
Intercept	-7.2482	-5.8792	-5.7413
	(2.8812)**	(8.9240)**	(3.0942)*
scd1	8.2186	6.7064	6.5402
	(3.2139)**	(3.1983)**	(3.4368)*
ARCH0	0.8381	0.8836	0.8793
	(0.3663)**	(0.4815)*	(0.5329)*
ARCH1	0.1528	0.1553	0.1603
	(0.0458)***	(0.0549)***	(0.0551)***
GARCH1	0.7947	0.7896	0.7889
	(0.0535)***	(0.0673)***	(0.0720)***
inverse of df	-	0.0886	-
	-	(0.0539)	-
df	-	-	10.7277
	-	-	(5.6940)*
skewness	-	-	0.9753
	-	-	(0.0706)***

	Corn		
	normal	*t*	skew-*t*
Intercept	-12.9049	-12.6220	-12.5313
	(2.8725)***	(8.9092)***	(3.0080)***
scd1	14.6627	14.3653	14.2466
	(3.1988)***	(3.1951)***	(3.3425)***
ARCH0	0.5465	0.6946	0.6863
	(0.3299)*	(0.5339)	(0.4699)
ARCH1	0.0887	0.0807	0.0807
	(0.0333)***	(0.0394)**	(0.0374)**
GARCH1	0.8762	0.8721	0.8750
	(0.0447)***	(0.0629)***	(0.0546)***
inverse of df	-	0.1083	-
	-	(0.0561)*	-
df	-	-	8.8878
	-	-	(3.8814)**
skewness	-	-	0.9788
	-	-	(0.0727)***

Table 5. GARCH Model Results for the Barley and the Kansas Wheat Cases

	Barley		
	normal	*t*	skew-*t*
Intercept	0.2845	0.2845	0.1854
	(0.0817)***	(0.0817)***	(0.1040)*
scd1	-1.7723	-1.7723	-1.6012
	(0.4816)***	(0.4816)***	(0.5019)***
ARCH0	0.6289	0.6289	0.4293
	(0.1565)***	(0.1565)***	(0.2016)**
ARCH1	0.4416	0.4416	0.4001
	(0.0865)***	(0.0865)***	(0.1086)***
GARCH1	0.5035	0.5035	0.5939
	(0.0742)***	(0.0742)***	(0.0915)***
inverse of df	-	0.0000	-
	-	(0.0000)***	-
df	-	-	6.0875
	-	-	(1.9634)***
skewness	-	-	0.9531
	-	-	(0.0711)***

	Kansas Wheat		
	normal	*t*	skew-*t*
Intercept	0.2780	0.2186	0.2728
	(0.1974)	(0.1841)	(0.1798)
scd1	-0.9977	-1.3362	-0.5021
	(1.0801)	(1.0651)	(1.0312)
ARCH0	0.5605	0.3959	0.1153
	(0.3802)	(0.3577)	(0.1795)
ARCH1	0.0650	0.0607	0.0459
	(0.0284)**	(0.0301)**	(0.0227)**
GARCH1	0.8928	0.9102	0.9479
	(0.0499)***	(0.0490)***	(0.0315)***
inverse of df	-	0.0838	-
	-	(0.0434)*	-
df	-	-	23.3459
	-	-	(26.1912)
skewness	-	-	1.3456
	-	-	(0.1276)***



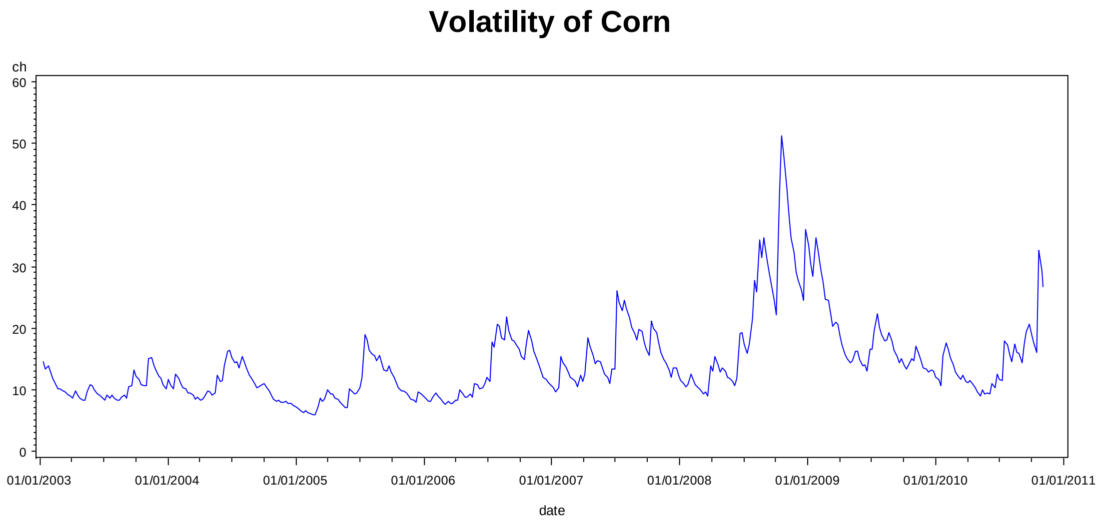


Figure 5. Price Volatilities of Grain Sorghum and Corn

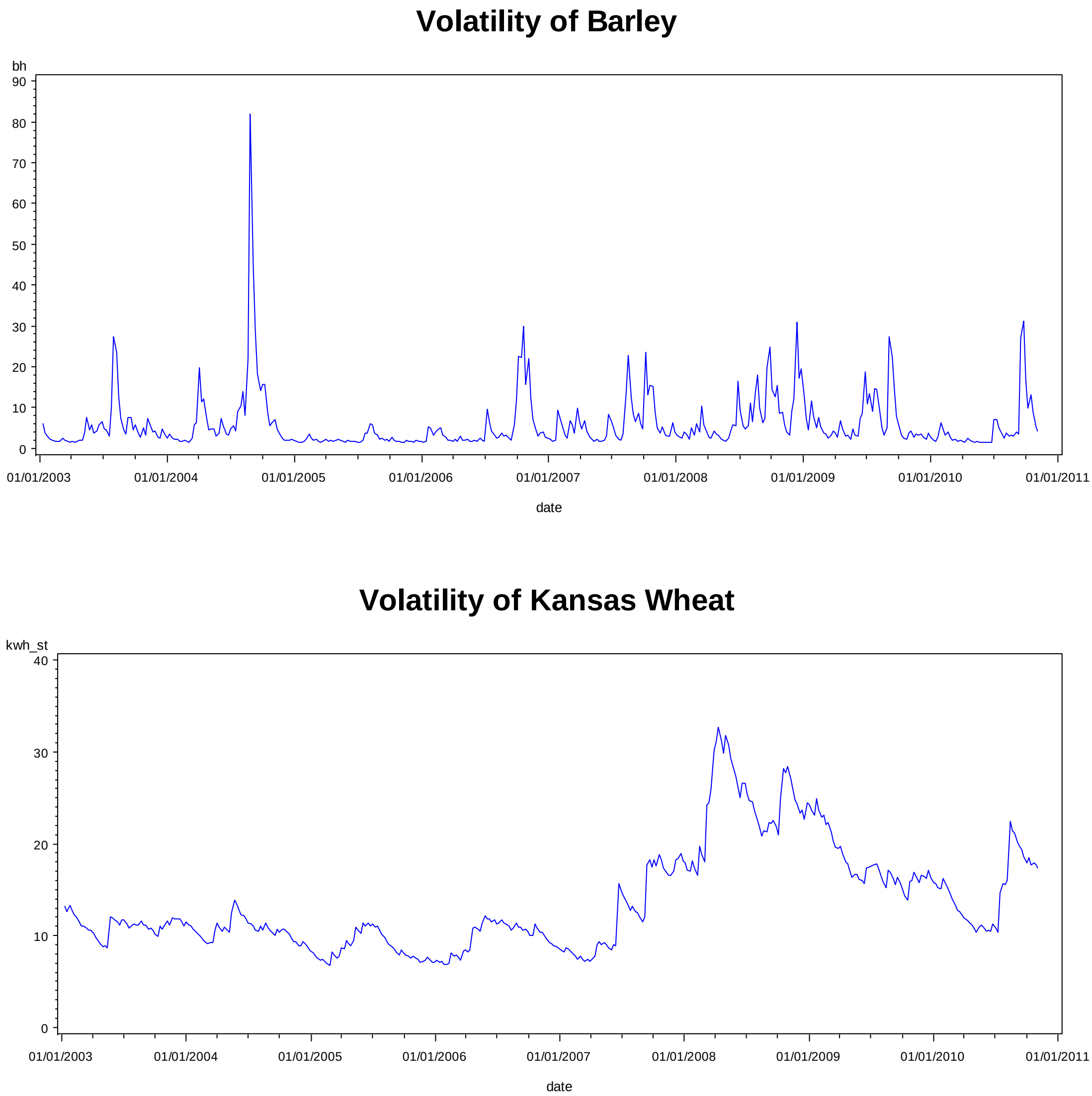


Figure 6. Price Volatilities of Barley and Kansas Wheat

Figure 5 and 6 show the price volatilities of grain sorghum, corn, barley, and Kansas wheat that are derived from the GARCH model. Similar patterns can be found in these time series plots.

In the next step, we estimate the correlation term in equation (2.1) by applying copula models using the conditional GARCH residuals, z_t's. Figure 7 and 8 illustrate the histograms and scatter plots of standardized GARCH residuals z_t's. It is obvious that the cash returns of grain sorghum and the futures returns of corn have very strong positive correlation. For the case of barley and Kansas wheat, correlation is not as strong as in the first case.

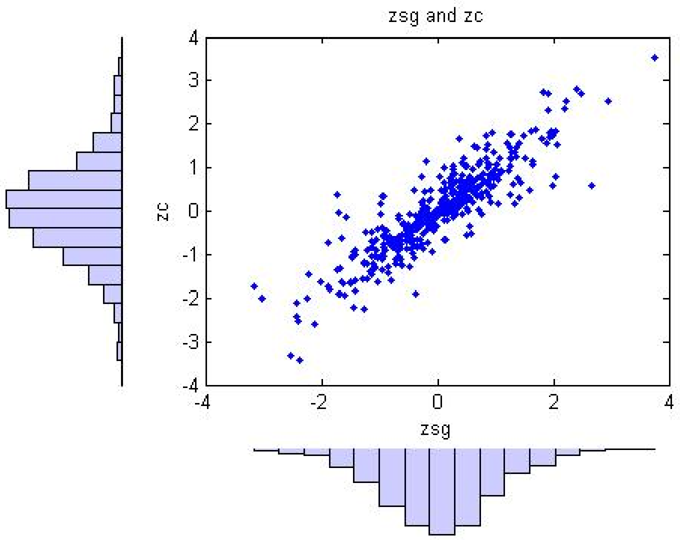


Figure 7. Scatter plots and Histograms for z_t's of grain Sorghum and Corn

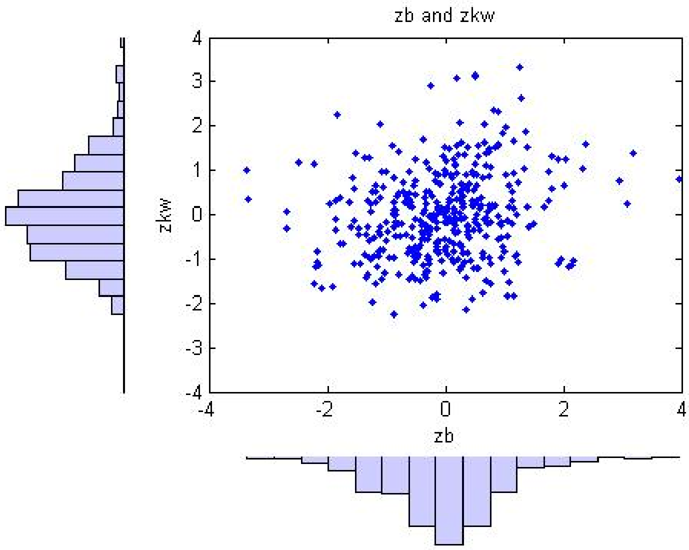


Figure 8. Scatter plots and Histograms for z_t's of Barley and Kansas Wheat

The next step is to estimate the copula models and obtain the time-varying correlation from the estimates of these copula models. Results of the estimates of the eight copula models are shown in Table 6 through Table 9. We first treat the dependence parameters as constant then estimate the dependence with dynamic forms.

Figure 9, 10, 14 and 15 illustrate the dynamic optimal hedge ratios derived from the copula models and the GARCH models. Figure 11, 12, 16 and 17 show the dynamic correlations. Figure 13 and 18 show the changes in the tail dependences.

Table 6. Results for Copula Models in the Grain Sorghum and Corn Case (1)

	Gaussian	t	Frank	Plackett
constant rho	0.8887	0.8998	13.3473	53.5288
	(0.0117)***	(0.0101)***	(0.7978)***	(6.0977)***
df (in constant rho)	-	4.7833	-	-
	-	(1.3197)***	-	-
Log Likelihood	318.0778	331.8257	329.7112	346.8238
b1	-2.3179	-2.4622	2.6549	4.1696
	(0.1735)***	(0.1855)***	(0.0956)***	(0.1798)***
b2	8.9531	10.1031	-6.3590	-9.5544
	(2.5807)***	(3.3562)***	(2.3570)***	(3.1872)***
b3	7.6460	10.0524	-5.8627	-10.2878
	(1.9168)***	(2.6463)***	(2.0462)***	(2.1066)***
b4	-16.5455	-20.0351	12.4012	19.8953
	(4.2890)***	(5.8281)***	(4.3433)***	(5.1255)***
df (in dynamic rho)	-	5.2267	-	-
	-	(0.0024)***	-	-
Log Likelihood	331.7318	341.0738	339.4480	354.5630
AIC	-655.4636	-672.1476	-670.8960	-701.1260

Table 7. Results for Copula Models in the Grain Sorghum and Corn Case (2)

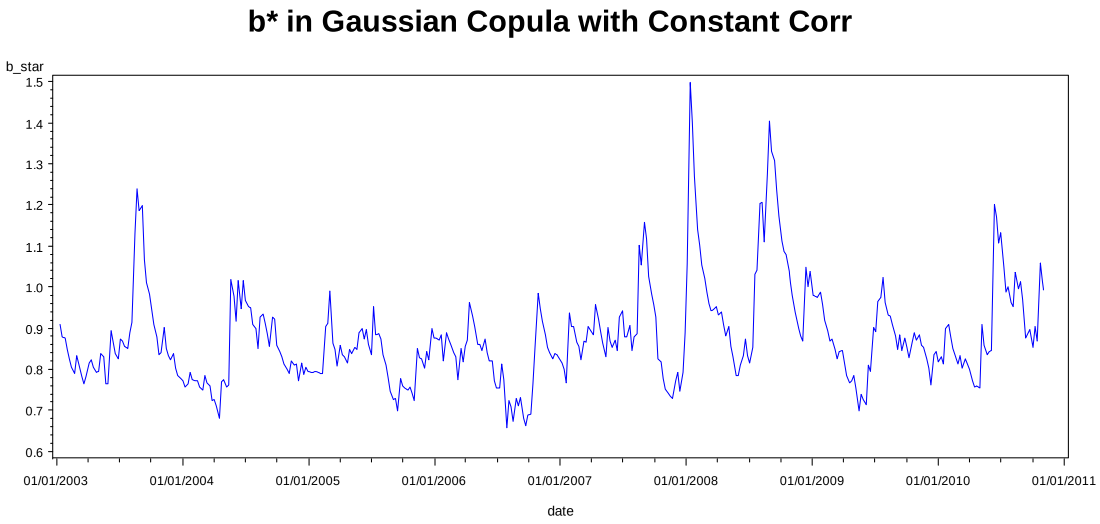
	Clayton	Rotated Clayton	Gumbel	Rotated Gumbel
constant rho	2.9460	2.9679	3.2298	3.2137
	(0.3383)***	(0.2939)***	(0.1779)***	(0.1919)***
Likelihood	236.6830	247.3485	311.9990	305.6471
b1	1.1987	1.4094	1.0568	0.9093
	(0.1680)***	(0.1532)***	(0.1223)***	(0.1335)***
b2	-5.6688	-7.7584	-8.2604	-5.6856
	(4.2596)	(1.7925)***	(2.0108)***	(2.9828)*
b3	-6.9206	-6.9001	-7.4429	-6.3440
	(4.0239)*	(1.7440)***	(1.8434)***	(2.5297)**
b4	12.6309	14.2936	15.4725	12.0664
	(8.2117)	(3.2857)***	(3.6999)***	(5.4236)**
Likelihood	242.3167	264.6100	326.8187	312.7585
AIC	-476.6334	-521.2200	-645.6374	-617.5170

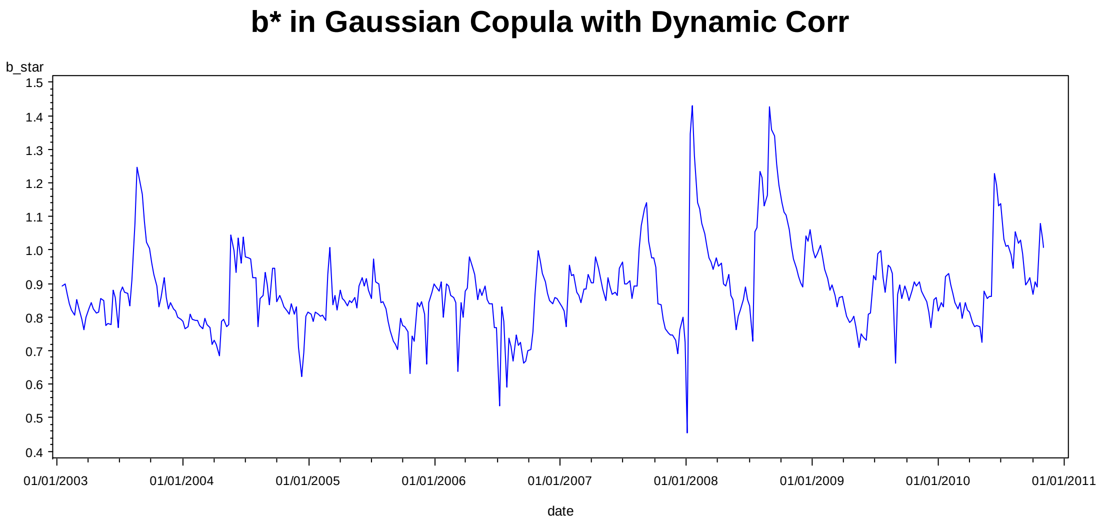
Table 8. Results for Copula Models in the Barley and Kansas Wheat Case (1)

	Gaussian	t	Frank	Plackett
constant rho	0.1879	0.1898	1.3659	1.9774
	(0.0456)***	(0.0458)***	(0.3147)***	(0.2958)***
df (in constant rho)	-	95.3416	-	-
	-	(3.2951)	-	-
Log Likelihood	8.2384	8.2643	10.0490	10.2483
b1	-0.2082	-0.2075	1.5619	1.1053
	(0.3959)	(0.3947)	(0.2276)***	(0.2242)***
b2	18.6186	18.2027	-16.4661	-1.9075
	(9.3242)**	(8.9877)**	(8.4068)*	(0.8855)**
b3	8.3138	8.0649	-6.4202	-1.5513
	(4.4759)*	(4.3444)*	(3.7731)*	(1.0489)
b4	-20.0386	-19.3395	17.2959	2.4022
	(11.8274)*	(11.4135)*	(10.0586)*	(1.8388)
df (in dynamic rho)	-	216.1254	-	-
	-	(0.0000)***	-	-
Log Likelihood	15.6181	15.5805	17.9671	13.1982
AIC	-23.2362	-21.1610	-27.9342	-18.3964

Table 9. Results for Copula Models in the Barley and Kansas Wheat Case (2)

	Clayton	Rotated Clayton	Gumbel	Rotated Gumbel
constant rho	0.1136	0.2791	1.1543	1.0824
	(0.0549)**	(0.0841)***	(0.0443)***	(0.0349)***
Likelihood	3.0280	9.2137	8.8979	4.7208
b1	-0.6026	0.2233	-0.4335	-0.9290
	(0.4755)	(0.3889)	(0.3716)	(0.4229)**
b2	-18.2091	-13.2052	-13.9401	-19.9293
	(8.1490)**	(6.5652)**	(7.4789)*	(8.7073)**
b3	-4.9866	-13.9463	-12.2292	-7.0095
	(2.2715)**	(5.4854)**	(5.9614)**	(3.2568)**
b4	15.3248	24.6211	22.9904	19.3063
	(6.9964)**	(11.6259)**	(12.0304)*	(9.2629)**
Likelihood	8.8202	16.7819	16.1863	10.1461
AIC	-9.6404	-25.5638	-24.3726	-12.2922





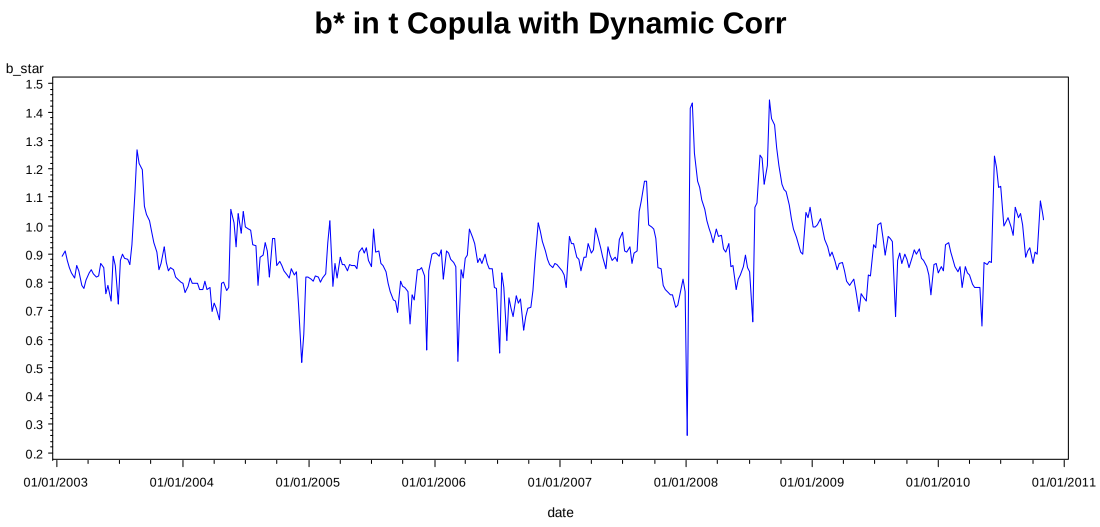
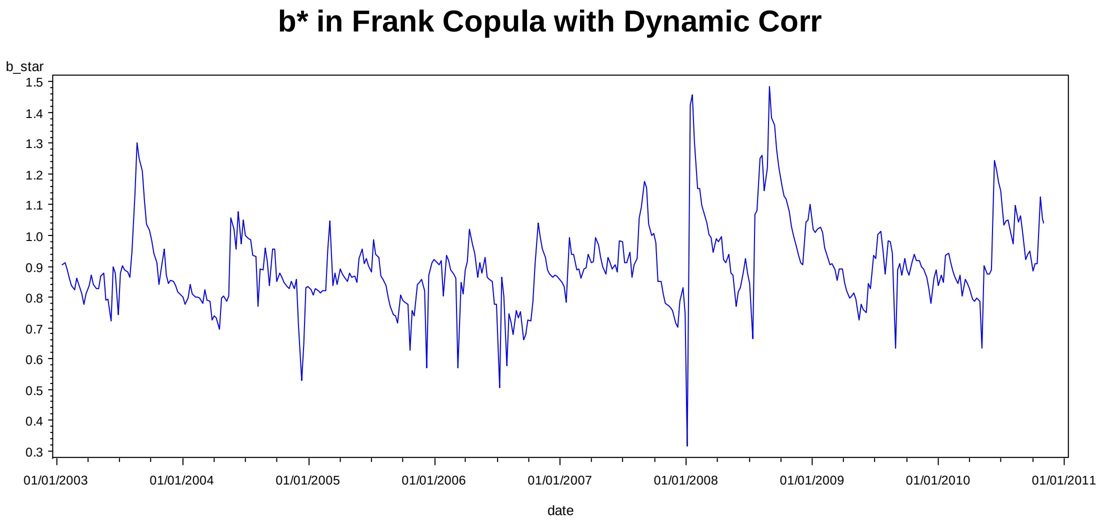
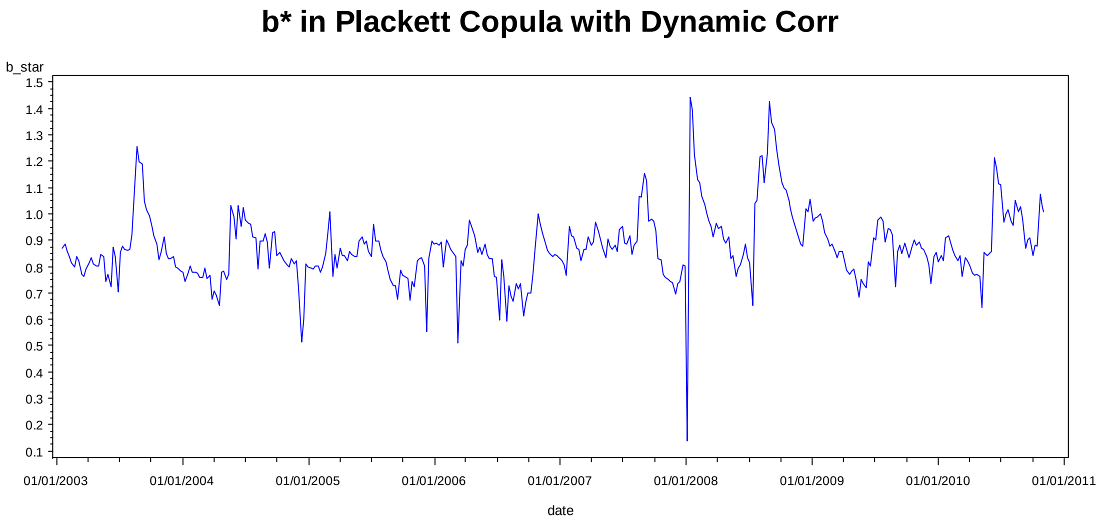


Figure 9. b^* in the Grain Sorghum and Corn Case





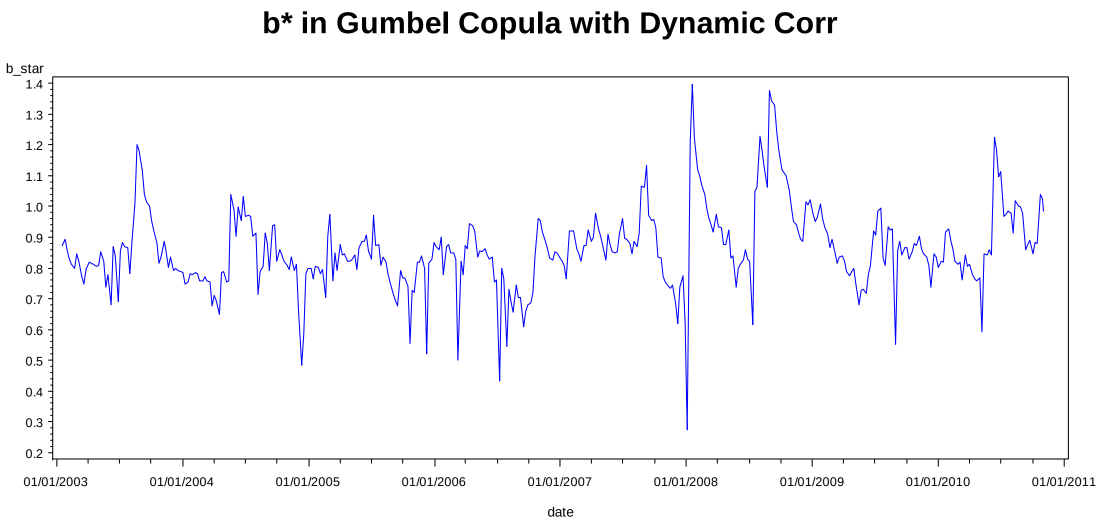


Figure 10. b^* in the Grain Sorghum and Corn Case

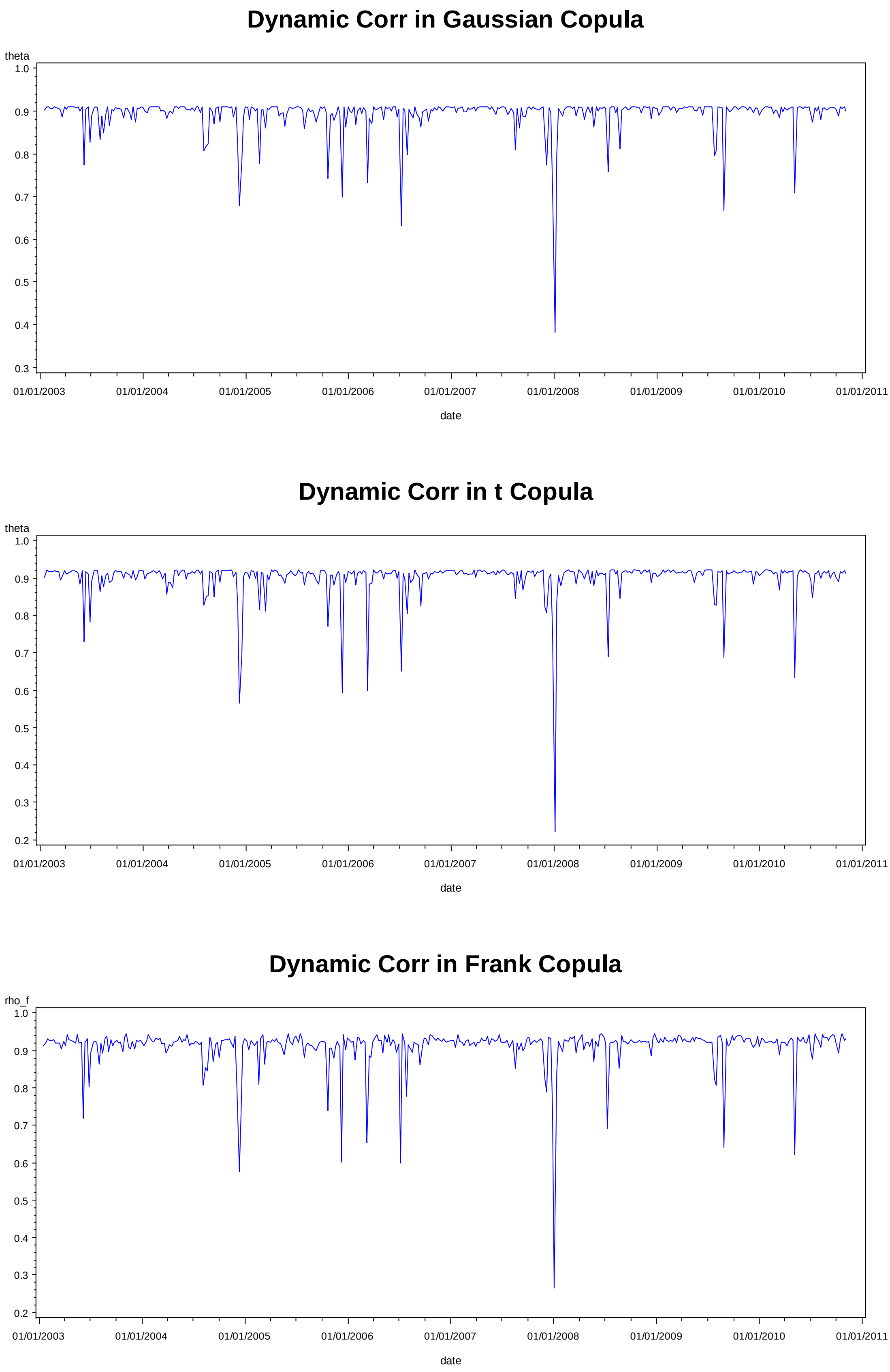
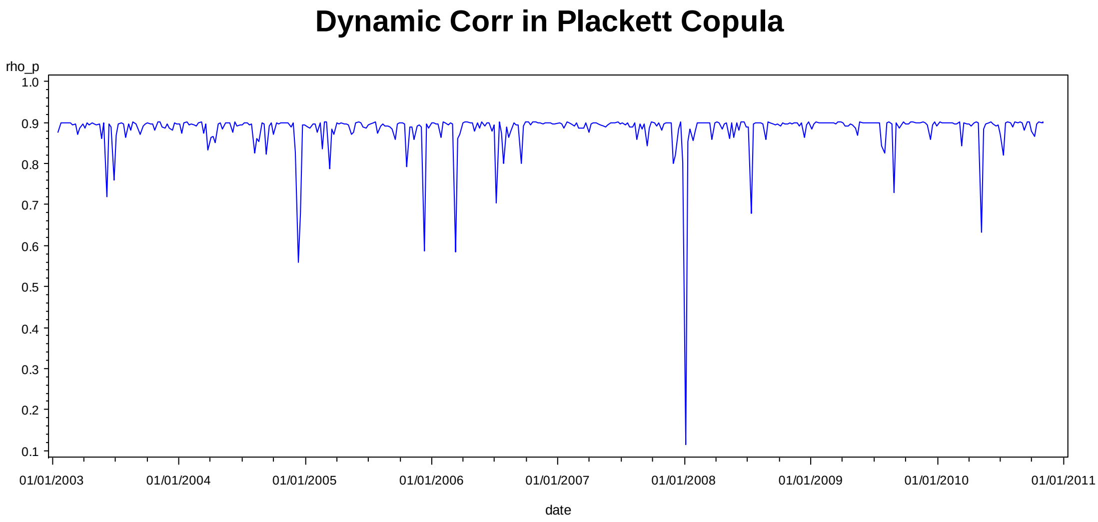


Figure 11. Dynamic Correlation (ρ_t) in the Grain Sorghum and Corn Case



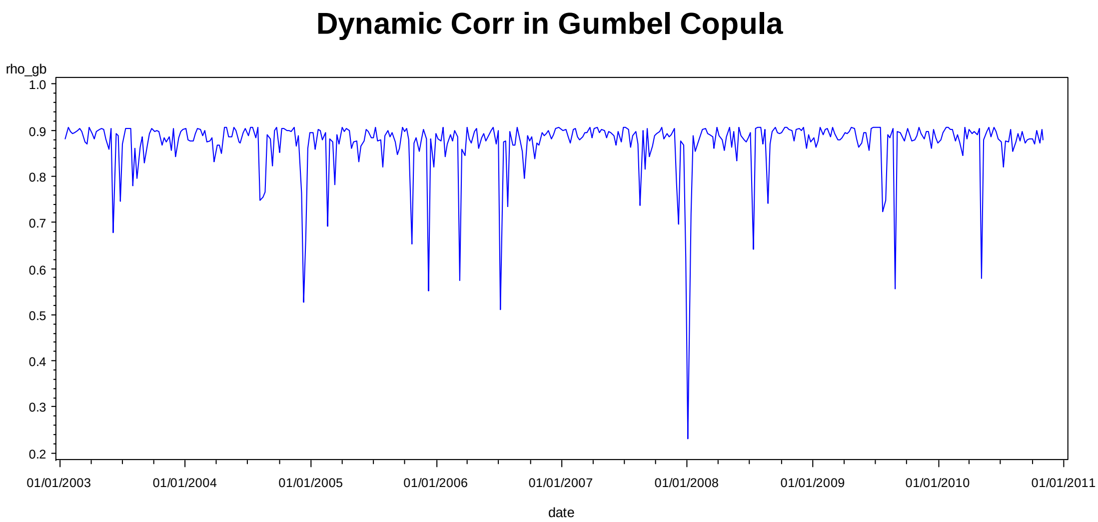
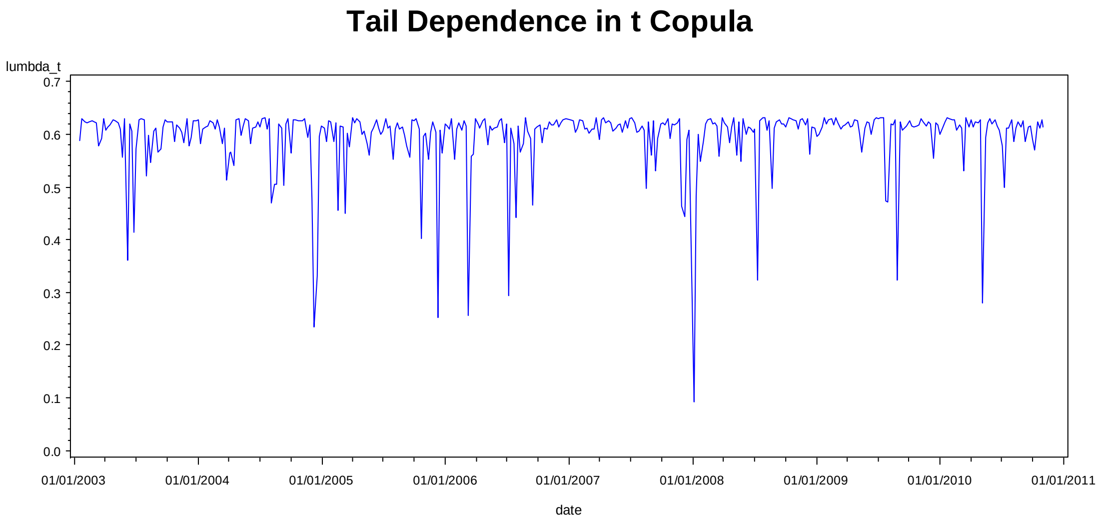


Figure 12. Dynamic Correlations (ρ_t) in the Grain Sorghum and Corn Case



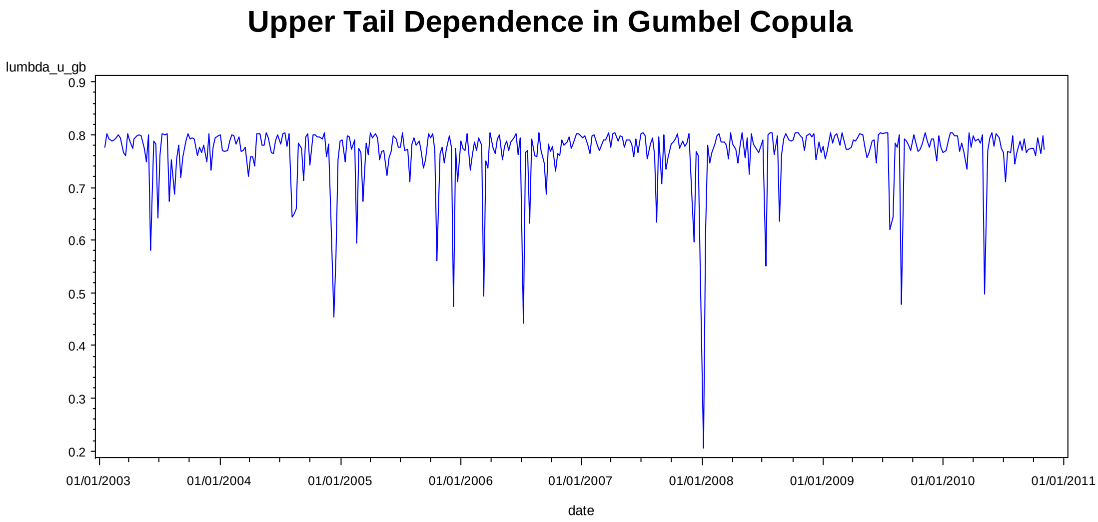


Figure 13. Tail dependences in the Grain Sorghum and Corn Case

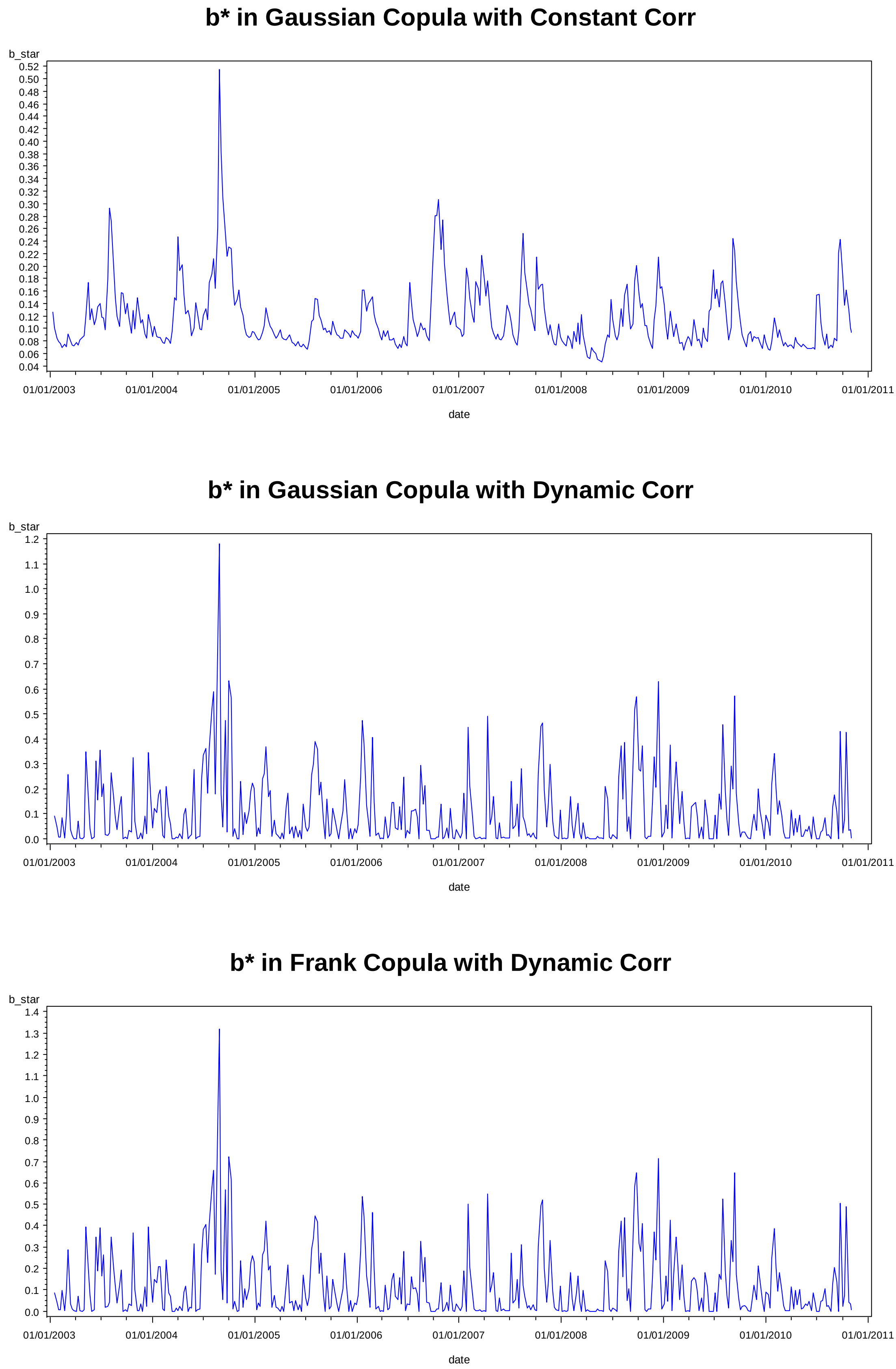
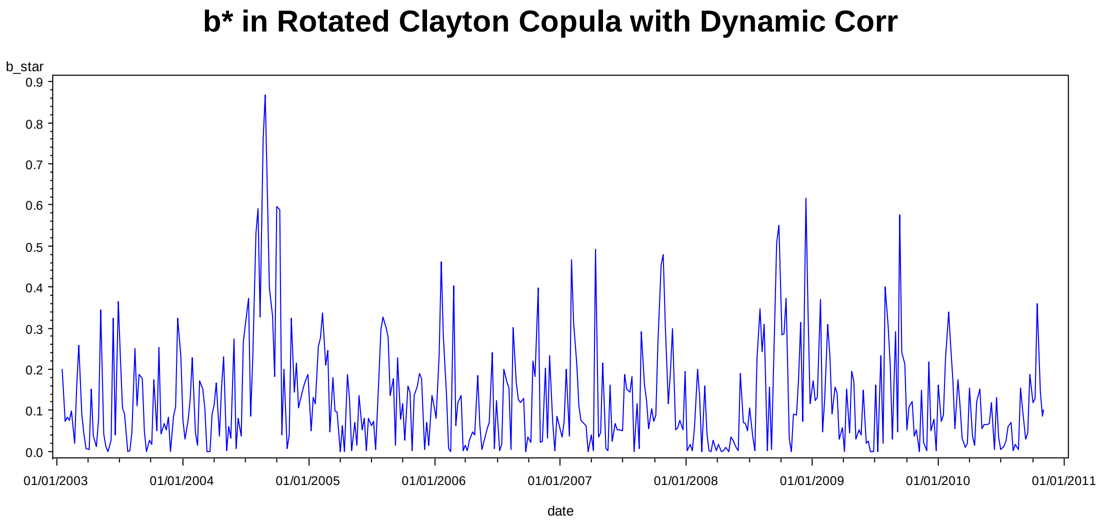


Figure 14. b^* in the Barley and Kansas Wheat Case



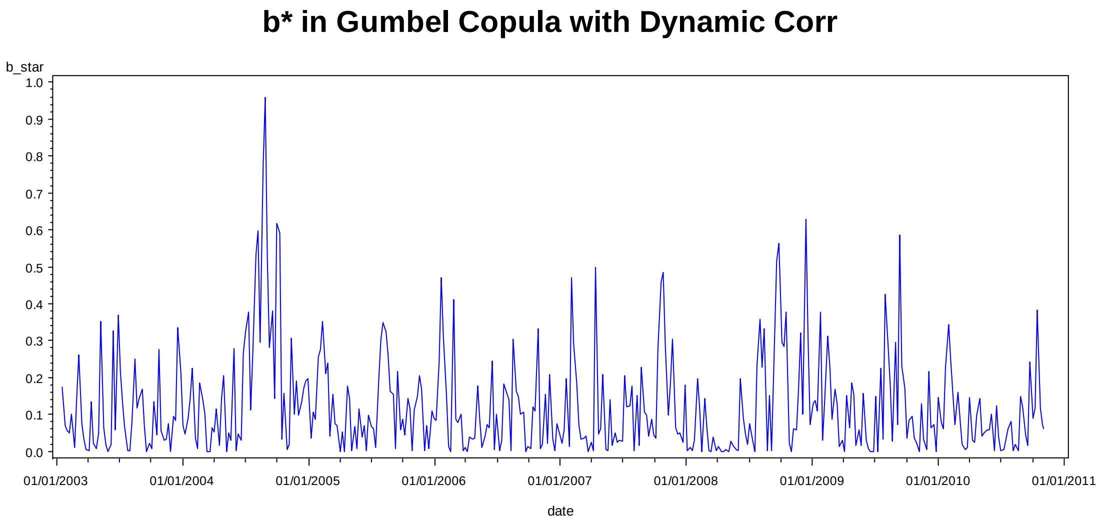
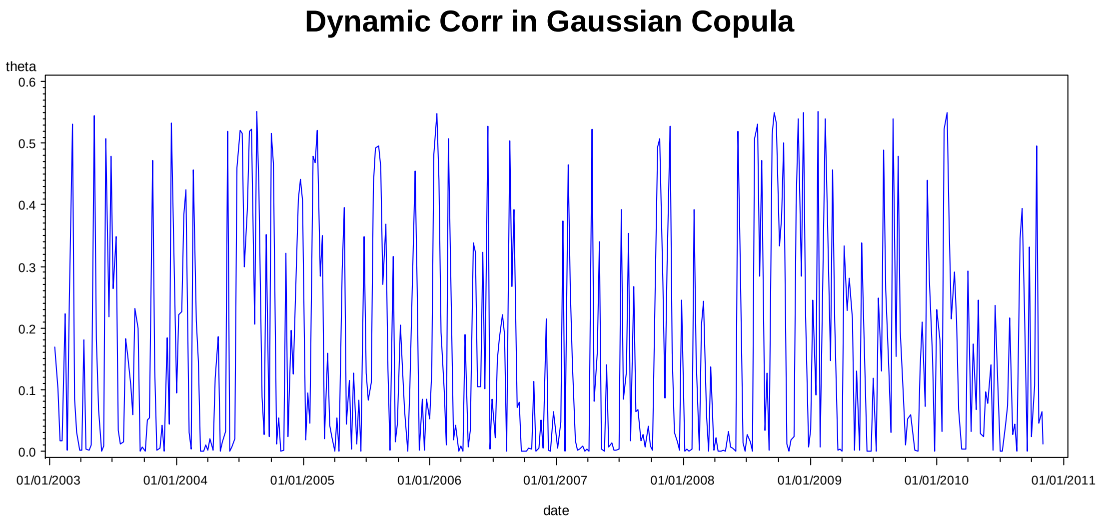


Figure 15. b^* in the Barley and Kansas Wheat Case



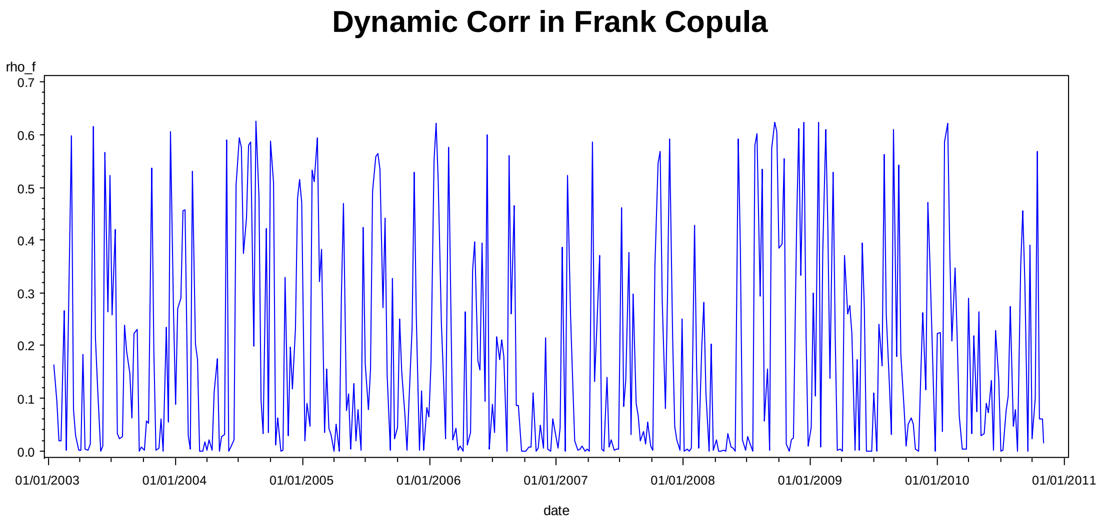
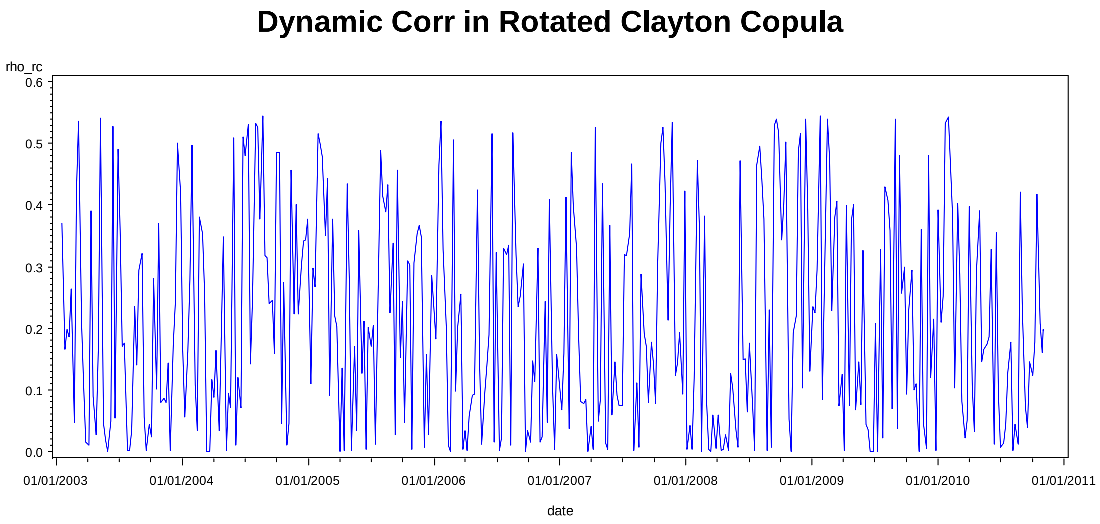


Figure 16. Dynamic Correlation (ρ_t) in the Barley and Kansas Wheat Case



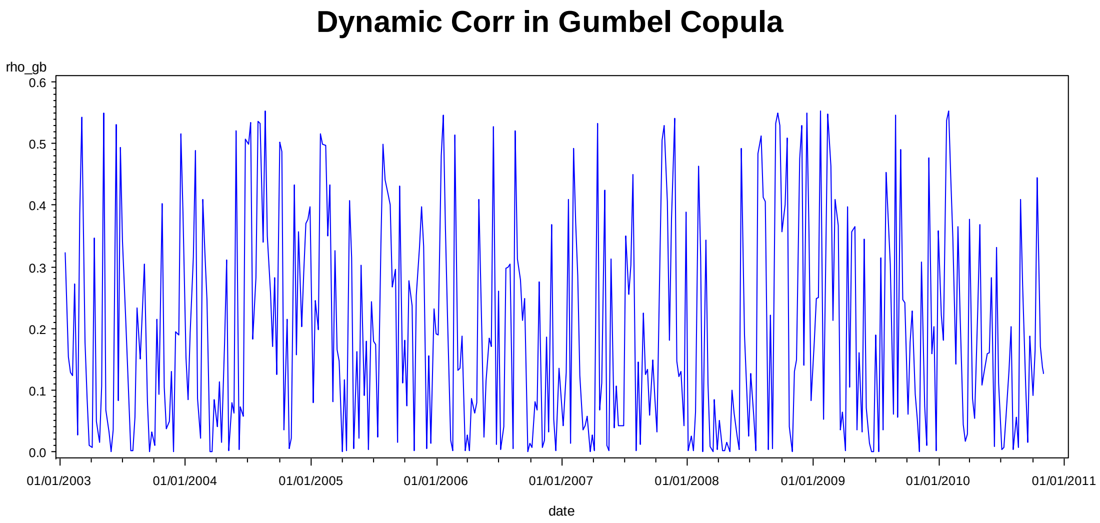
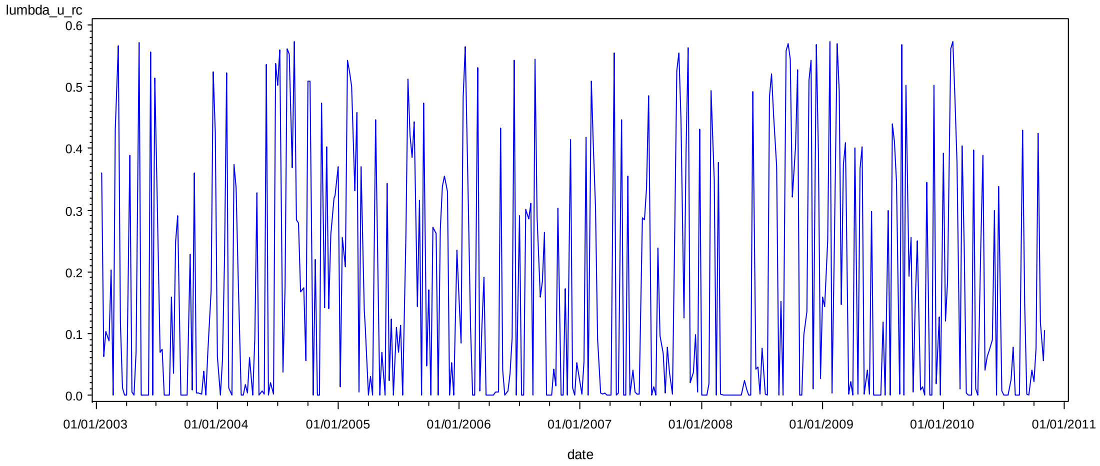


Figure 17. Dynamic Correlation (ρ_t) in the Barley and Kansas Wheat Case

Upper Tail Dependence in Rotated Clayton Copula



Upper Tail Dependence in Gumbel Copula

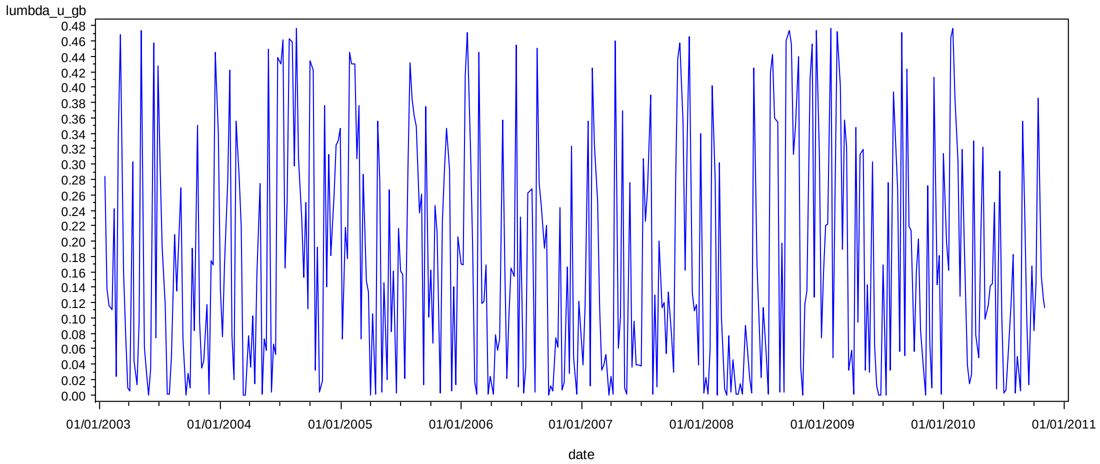


Figure 18. Tail Dependences in the Barley and Kansas Wheat Case

6. Conclusion

In this paper, we discuss the application of copula models to estimate dynamic cross-hedge ratios. Results confirm the significant linkages between these markets and demonstrate the effectiveness of cross-hedging as a mechanism for managing price risk. Further, the results reveal that common approaches to the derivation of optimal hedging positions may suffer from important specification biases that could imply financial losses. Implications for cross hedging in the barley and grain sorghum markets are highlighted in our conclusions. Grain sorghum has become increasingly important as a substitute for corn because of very high corn prices. Our results are particularly timely in this regard.

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