Estimating Gravity Equation Models in the Presence of Sample Selection and Heteroskedasticity

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Abstract:

Gravity models are widely used to explain patterns of trade. However, two stylized features of trade data, sample selection and heteroskedasticity, challenge the estimation of gravity models. We propose a Two-Step Method of Moments (TS-MM) estimator that deals with both issues. Monte-Carlo experiments show that, under certain qualifications, the TS-MM model outperforms the Poisson Pseudo Maximum Likelihood model, the Heckman model, and the E.T.-Tobit model. Moreover, we suggest a model selection strategy to guide the selection of estimators in practice. A re-examination of world trade in 1990 illustrates the usefulness of the TS-MM estimator and the model selection strategy.

Keywords: gravity equation, heteroskedasticity, zeros, sample selection, Two-Step Method of Moments, intensive margin, extensive margin, market access

JEL classification: F10, C10

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1. Introduction

The gravity equation model has been a long-time workhorse in international trade since Tinbergen (1962). It posits that the bilateral trade flow from one country to another can be explained by the two countries' income levels, geographic distance, and various other factors (such as import tariffs, non-tariff regulations, contiguity condition, historical colonial relationship, and religion similarity between the trading partners) that could affect the cost of trade. In addition to its empirical success in fitting trade data reasonably well (Baldwin and Taglioni, 2006), the gravity equation model has recently received more recognition because of the development of its microeconomic foundations.[1] Following Anderson (1979), Anderson and van Wincoop (2003) derive a full specification of the gravity equation model with trade costs from the utility maximization behavior of a representative consumer with Constant Elasticity of Substitution (CES) preferences. Most importantly, they emphasize the role of countries' multi-lateral trade resistance terms in a cross-sectional gravity equation analysis. Novy (2010) innovates a gravity equation under a general equilibrium framework with a translog demand system. Markusen (1986) and Bergstrand (1985, 1989) introduce non-homothetic preferences in gravity equation models and shed light on the impacts of per-capital income on trade patterns. Deardorff (1998) shows that a gravity equation can emerge from a Heckscher-Ohlin setting as well. Evenett and Keller (2002) report that both the Heckscher-Ohlin theory and the monopolistic-competition trade theory can lead to the gravity equation and that each provides unique insights to the international variation of production and trade patterns. In a comprehensive review, Feenstra, Markusen, and Rose (2001) examine how

[1] Interested readers are referred to Anderson (2010) for a survey on the theoretical and empirical developments of the gravity equation approach to trade.

various trade theories are linked to the gravity equation approach and provide evidence in favor of the monopolistic-competition theory and the reciprocal-dumping theory. Following the new trade theory of heterogeneous firms, Helpman, Melitz, and Rubinstein (2008) (HMR hereafter) build up a generalized gravity equation with firms facing fixed costs of exporting. Their model predicts that only the most productive firms are able to overcome the fixed cost of trade and penetrate foreign markets, and that trade liberalization induces more firms to participate in the world market.

Despite the rapid development of the microeconomic functions for the gravity equation model, there is no consensus in the literature on how to statistically estimate a gravity equation in the presence of the two stylized features of trade data: sample selection and heteroskedasticity. On the one hand, zeros are commonly found in trade data, which could give rise to the classical sample selection issue. For example, zeros can take up nearly 50% of all bilateral trade records at the national level (e.g., HMR). Even with panel data covering more recent years in agricultural sectors, zeros easily account for 30% of all the observations (Sun and Reed, 2010; Grant and Boys, 2012). The treatment of these frequent zeros is an important concern in the analysis of trade policies for at least two reasons. First, from a statistical viewpoint, the omission or mis-treatment of zeros could lead to the sample selection bias, as defined by Heckman (1979), unless the zeros correspond to "missing at random."[2] Second, from an economic perspective, the modeling of zeros directly speaks to the question whether trade polices improve or deteriorate market access for sporadic traders who frequently opt out of the world market. Such market access effect is of particular importance when the policies of interest play a

[2] Interested readers are referred to Little and Rubin (1987) for a classification of missing data problems.

major role in determining the cost of trade for smallholder exporters from the developing world. For instance, Shepherd (2010) shows that the reduction in export costs, tariffs, and transport costs can encourage developing countries to export to more destinations. Besedes and Prusa (2011) argue that developing countries are more likely to experience long-run export growth if new entrants to world market have a better chance to survive beyond the first year. Bergin and Lin (2008) demonstrate that currency unions facilitate international trade predominantly through increasing the number of exporting firms and the number of traded products.

On the other hand, trade data often exhibit heteroskedasticity. The data sample of a gravity equation analysis usually consists of bilateral trade flows collected from multiple countries, which naturally gives ground to heteroskedasticity. In general, heteroskedasticity is less a concern as long as the model is correctly specified because it does not undermine the consistency of estimates. In a gravity equation analysis, however, heteroskedasticity challenges the common practice of logarithmic transformation. As Santos Silva and Tenreyro (2006) (SST hereafter) show, if the true gravity equation model is in its multiplicative form and heteroskedasticity is present, estimates from the log-linearized gravity equation models can be severely biased. Arguably, the above two features of trade data, sample selection and heteroskedasticity, warn against the use of the Ordinary Least Square technique. As various new estimators for the gravity equation model are being proposed, two camps emerge in the literature.

One camp in the debate focuses on the economics of zero trade flows. The new trade theory, pioneered by Melitz (2003) and later developed by several others such as Chaney (2008) and HMR, posits that the absence of trade can be attributed to firms' self-selection behavior: zero trade flow is observed when none of the firms in the potential exporting country is productive enough to overcome the fixed costs imposed by the destination market. Therefore, zeros can be seen as generated from a selection process, which gives grounds to the Heckman sample selection model (Heckman, 1979), or, to a less degree, the E.T.-Tobit model (Eaton and Tamura, 1994). In a Heckman sample selection model, the selection equation fully captures zeros and explains why trade takes place at all, while the outcome equation characterizes the volume of the trade conditional on trade occurring. The E.T.-Tobit model treats zeros as censored outcomes and assumes that there is minimal threshold to jump if trade flows are to be observed. Besides well connected with the new trade theory, both the Heckman sample selection model and the E.T.-Tobit model deliver rich comparative statics. Specifically, one can decompose the effect of trade liberalization into the intensive margin (the intensification of pre-existing trade) and the extensive margin (the creation of new trade partnership).[3] Nevertheless, built upon the log-linearized version of the gravity equation, the Heckman sample selection model or the E.T.-Tobit model may deliver biased estimates when trade data exhibits heteroskedasticity in levels.

The other camp in the debate suggests specifying the gravity equation in its multiplicative form and estimating it via some variants of count data models. In particular, SST propose the Poisson Pseudo Maximum Likelihood (PPML) estimator to accommodate heteroskedasticity in trade data. By estimating trade flows in levels, as opposed to in logs, the PPML estimator permits zeros and has been shown to be robust to

[3] Throughout the paper, we refer to the extensive margin of trade as new trade partnership at national level. Alternatively, the extensive margin can refer to the newly entered firms (HMR), or the newly traded varieties (Hummels and Klenow, 2005), or the newly reached consumers (Arkolakis, 2010). We omit these dimensions because our data is at national level.

a wide range of heteroskedastic patterns. However, Martin and Pham (2008) note that the PPML estimates are severely biased when zeros are not random outcomes.[4] Some variants of the PPML estimator are also proposed. For example, Burger, Linders, and Oort (2009) consider the Negative Binomial Pseudo Maximum Likelihood estimator (NBPML), the Zero Inflated Poisson Pseudo Maximum Likelihood estimator (ZIPPML), and the Zero Inflated Negative Binomial Pseudo Maximum Likelihood estimator (ZINBPML). Although with merits of their own (such as permitting over-dispersion and excessive zeros), none of the above variants is robust to a change of the unit of measurement of the dependent variable (e.g., different estimates result when trade flows are measured in thousands of dollars instead of dollars). Such a defect arguably prevents NBPML, ZIPPML, and ZINBPML from being widely adopted.

We contribute to the estimation of gravity equation models in two important ways. First, we propose a Two-Step Method of Moments (TS-MM) estimator that simultaneously deals with sample selection and heteroskedasticity. The estimator works as follows. In the first step, we characterize the binary decision of trade or no trade by a selection process and predict the probability of trade accordingly. As a result, we can explain the absence of trade and evaluate determinants of market access. In the second step, we capture positive trade flows by a gravity equation in its multiplicative form, with the potential sample selection bias corrected. By estimating the gravity equation via the method of moments approach and constructing the heteroskedasticity-resistant standard errors (White, 1980), we are able to obtain consistent point estimates and conduct statistical inferences correspondingly. Our Monte-Carlo experiments confirm that the

[4] In a reply, Silva and Tenreyro (2011) show that the PPML estimator is able to accommodate high frequency of zeros, without fully addressing the sample selection issue.

proposed TS-MM estimator strictly dominates the Heckman, PPML, and E.T.-Tobit models under certain qualifications.

Second, we suggest a model selection strategy that allows one to choose the most appropriate estimator in practice. Our proposed strategy utilizes both economic theory and statistical tests. Guided by the new trade theory, we argue that, in the presence of sample selection and heteroskedasticity, the Heckman sample selection model, the TS-MM model, and the PPML model are three competing estimators to choose from. We employ the MacKinnon-White-Davidson test (MacKinnon, White, and Davidson, 1983) to differentiate the Heckman sample selection model and the TS-MM model. The survivor of the MWD test is considered the most preferred estimator if evidence of sample selection bias is found. Otherwise, we use the Theil's inequality coefficient (Theil, 1961), as a measure of goodness of fit, to further compare the estimator surviving the MWD test with the PPML estimator. We illustrate how the proposed estimator and model selection work by re-examining the bilateral world trade in 1990.

The rest of the article is organized as follows. Section 2 introduces the TS-MM estimator and discusses its properties. Section 3 uses a set of Monte-Carlo experiments to assess the performance of various estimators. Section 4 presents the model selection strategy. Section 5 applies the TS-MM estimator and the model selection strategy to the data set in SST. Section 6 concludes.

2. The Gravity Equation and the Two-Step Method of Moments Estimator

The gravity equation approach to trade posits that country j's import from country i, M_{ij}, can be characterized by

(1) $M_{ij} = Y_i^{\phi} Y_j^{\varphi} \Big/ D_{ij}^{\gamma} \equiv \exp(X_{ij}\beta)$,

where Y_i and Y_j denote country i and j's characteristics (e.g., GDP, population, remoteness to the rest of the world); D_{ij} includes any trade cost terms that are specific to the country pair (e.g., applied tariff rates, geographic distance, contiguity condition, historical colonial relationship, religion similarity, and the existence of preferential trade agreements);[5] ϕ, φ, and γ are parameters to be estimated. Simple algebra leads to the last term in equation (1), where X_{ij} is a row vector containing all explanatory variables in their log scales and β is a column vector stacking all parameters. To take the gravity equation to practice, one needs to specify the stochastic version of (1), which we pursue next.

Motivated by the new trade theory, we explicitly account for the absence of trade by introducing a selection process. Specifically, we set up the stochastic gravity equation model as follows:

(2a) $M_{ij}^{*} = \exp(X_{ij}\beta) + \mu_{ij}$,

(2b) $d_{ij}^{*} = Z_{ij}\alpha + \upsilon_{ij}$,

(2c) $d_{ij} = I(d_{ij}^{*} > 0)$,

(2d) $M_{ij} = d_{ij} M_{ij}^{*}$.

[5] Interested readers are referred to Anderson and van Wincoop (2004) for a detailed discussion of trade costs.

M_{ij}^{*} is the *notional* trade flow from country i to country j in the absence of fixed cost of trade.[6] d_{ij}^{*} is the latent variable for the binary trade decision d_{ij} which equals one if country j imports from country i, and 0 otherwise. Z_{ij} contains all factors that potentially affect the fixed cost of trade between the two countries, and α is the associated vector of parameters. M_{ij} is the *observed* trade flow, which is a product of the binary decision and the notional trade flow. As in Heckman (1979), we assume that μ_{ij} and υ_{ij} are two idiosyncratic terms following a bivariate normal distribution.[7] Specifically, $\begin{pmatrix} u_{ij} \\ \upsilon_{ij} \end{pmatrix} \rightarrow N\left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_{11ij} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{pmatrix}\right]$, where $\sigma_{12} = \sigma_{21}$. The correlation between the two idiosyncratic terms accounts for omitted variables that affect both the fixed and variable costs of trade. Noticeably, heteroskedasticity is allowed because σ_{11ij} varies across countries.

The model setup, (2a)-(2d), is appealing in three aspects. First, the theoretical gravity equation, (2a), is expressed in its multiplicative form, thus is free from the bias due to logarithmic transformation. Furthermore, as elaborated below, consistent estimates of β can be derived even if heteroskedasticity is present in (2a). Second, (2b)-(2c) captures the absence of trade and allows investigating determinants of international market access. In fact, in addition to all variables in X_{ij}, Z_{ij} can contain extra variables

[6] The concept of the notional trade is similar to the desired amount of trade as defined by Ranjan and Tobias (2007).

[7] Alternatively, the approach of instrumental variable can be used to address the sample selection issue (e.g., Chang and Kott (2008)).

that exclusively affect the fixed cost of trade.[8] Lastly, the characterization of observed trade flows in (2d) facilities the decomposition of the extensive and intensive margins of trade. For instance, the elimination of tariffs promotes international trade, M_{ij}, either by improving market accessibility, d_{ij}, or by enhancing pre-existing trade, M_{ij}^*, or both.

Following Maddala (1986), we estimate system (2a)-(2d) via a two-step procedure. In the first step, we estimate (2b)-(2c) using a standard Probit model. Mathematically, the probability that country j imports from country i can be derived as:

(3a) $\Pr(d_{ij}=1)=\Phi(Z_{ij}\theta)$,

where $\theta=\alpha/\sqrt{\sigma_{22}}$. Defining the extensive margin of trade as the probability of trade in its logarithmic scale, we can compute the marginal effect through the extensive margin by differentiating (3a). For instance, a change in a trade determinant, z_{ij}, would lead to a change in the extensive margin as follows:

(3b) $\partial\ln(\Pr(d_{ij}=1))\big/\partial z_{ij}=\hat{\theta}_z\,\lambda_{ij}$,

where $\lambda_{ij}=\phi(Z_{ij}\hat{\theta})\big/\Phi(Z_{ij}\hat{\theta})$ is the Inverse Mill's Ratio as in Heckman (1979).

In the second step, we characterize the volume of trade conditioning on trade taking place. Taking advantage of the bivariate normality of μ_{ij} and υ_{ij}, we can derive the conditional trade volume as:

(4a) $E(M_{ij}\,|\,d_{ij}=1)=\exp(X_{ij}\beta)+\omega\lambda_{ij}$.

where $\omega=\sigma_{12}/\sqrt{\sigma_{22}}$. Intuitively, (4a) states that the observed trade follows a gravity equation augmented by an additional term correcting for the potential sample selection

[8] For example, HMR examine how institutional factors such as "days to start business" can affect firms' decision to trade.

bias. In the extreme case where $\sigma_{12} = 0$, (4a) reduces to the specification proposed by SST.[9]

We estimate the second-stage equation, (4a), via the method of moments (MM) and construct the heteroskedasticity-consistent variance covariance estimates as in White (1980). Specifically, the point estimates of $[\beta', \omega']'$ satisfy the following system of equations:

$$(M_p - \exp(X_p \beta) - \omega \lambda_p) \otimes \Omega = 0,$$

where M_p is column vector stacking all positive trade flows, X_p and λ_p are subsets of X an λ where positive trade flows are observed, and $\Omega = [X_p, \lambda_p]'$. The MM method has two major advantages. First, the resulting estimates are consistent as long as (4a) is correctly specified. Therefore, the MM estimates are robust to heteroskedasticity.[10] Second, when endogeneity is a concern, the MM technique can be easily extended to generalized methods of moments (GMM) in practice.

Defining the intensive margin of trade as the conditional trade volume in its logarithmic scale, we can compute the marginal effect through the intensive margin by differentiating (4a). For instance, a change in a trade determinant, x_{ij}, leads to a change in the intensive margin as follows:

(4b) $$\partial \ln(E(M_{ij} \mid d_{ij} = 1)) \big/ \partial x_{ij} = \beta_x + \omega \cdot \frac{\theta_x(\phi'(Z_{ij}\theta) \big/ \Phi(Z_{ij}\theta) - \lambda_{ij}^2) - \beta_x \lambda_{ij}}{\exp(X_{ij}\beta) + \omega \lambda_{ij}},$$

[9] However, even in this extreme case, (4a) suggests that the PPML technique can be only applied to the truncated sample with positive trade flows.

[10] In fact, the unknown heteroskedastic pattern pre-excludes the characterization of the higher moments or the full distribution of trade flows.

where $\varphi'(\cdot)$ is derivative of the normal density function. Intuitively, (4b) states that a change in market conditions or trade policies affects the volume of trade via two channels. Besides the direct impact through β, there is an indirect impact through altering the self-selection behavior, as represented by the second term on the right hand side of (4b). The overall marginal effect, if the factor of interest affects both the fixed and variable costs of trade, is the sum of its effect through the extensive margin, (3b), and the intensive margin, (4b). Or, the overall marginal effect is computed as

$$\partial \ln E(M_{ij}) / \partial x_{ij} = \partial \ln(\Pr(d_{ij} = 1)) / \partial x_{ij} + \partial \ln(E(M_{ij} \mid d_{ij} = 1)) / \partial x_{ij} .$$

We now compare the proposed TS-MM estimator with the alternative estimators in the literature, i.e., the Heckman model, the E.T.-Tobit model, and the PPML estimator.[11] The treatment of zeros in the TS-MM estimator is similar to that in the Heckman sample selection model or the E.T.-Tobit model: all three models attribute zeros to countries' self-selection to not trade. However, the TS-MM model differs from the Heckman or the Tobit model in that it characterizes the volumes of trade in levels, as opposed to in logs. Therefore, when the true trade data generating process is in levels and heteroskedasticity is present, the TS-MM model is more likely to deliver consistent estimates (as shown in Section 3 below). Additionally, the TS-MM estimates are more stable than the Heckman estimates because the identification of the TS-MM model does not require an excluded variable.[12] Compared to the PPML estimator which uses one single process to explain both positive and zero trade flows, the TS-MM model

[11] We exclude NBPML, ZIPPML, and ZINBPML because of their vulnerability to re-scaling of the dependent variable, as mentioned earlier.

[12] The near linearity of the Inverse Mills' Ratio often makes the second stage of the Heckman procedure unidentifiable, unless a variable can be excluded in the second stage. See Puhani (2000) for more discussions.

accommodates zeros in a way that is consistent with the new trade theory and addresses the sample selection issue. Practically, while the PPML model is muted about the market access effect, the TS-MM model allows disentangling the extensive margin from the intensive margin of trade.

3. The Monte-Carlo Experiments

In this section, we conduct a set of Monte-Carlo experiments to assess the performance of the proposed TS-MM estimator and the alternative estimators (the PPML model, the Heckman procedure, and the E.T.-Tobit model), under the hypothesis that the system (2a)-(2d) is the underlying data generating process. We expect the TS-MM estimator to outperform the alternatives because it simultaneously deals with sample selection and heteroskedasticity.

For simplicity, we introduce only one explanatory variable, x, to the data generating process. Specifically, x is a drawn from a normal distribution with mean 1 and variance 0.1, i.e., $x \sim N(1, 0.1)$. One can think of x as the importing country's income, which presumably affects both the volume of trade and the propensity to trade. We let $\beta_1 = 1$ and $\alpha_1 = 0.05$ be the coefficients of x in (2a) and (2b) respectively, so that the variable of interest affects trade primarily through the intensive margin. We set $\beta_0 = -1$ for the intercept in (2a). As to the intercept in (2b), we consider two scenarios: (a) $\alpha_0 = 0.05$, in which case we have relatively few zeros; and (b) $\alpha_0 = -0.05$, in which case we have many zeros. In particular, if we let $\sigma_{22} = 0.005$, the proportion of zeros is about 15% in case (a) and 50% case (b).

To allow heteroskedasticity, we consider three functional forms for σ_{11k}, where k denotes a specific observation in the simulated sample: (i) homoskedastic errors, or $\sigma_{11k} = 0.01$; (ii) heteroskedastic errors when the variance is proportional to the mean, or $\sigma_{11k} = 0.01 m_k$, where $m_k = \exp(\beta_0 + \beta_1 x_k)$; (iii) super-heteroskedastic errors when the variance is a quadratic functional form of the mean, or $\sigma_{11k} = 0.01(m_k + m_k^2)$. Lastly, we set $\sigma_{12} = 0.005$, so that the correlation coefficient of two idiosyncratic terms is about 0.7 in case (i), 0.7 in (ii), and 0.5 in case (iii).

In summary, to investigate how sample selection affects the performance of estimators, we consider two scenarios: (a) few zeros and (b) many zeros. To assess the impact of heteroskedasticity, we construct three scenarios: (i) homoscedasticity, (ii) heteroskedasticity, and (iii) super-heteroskedasticity. Therefore, a total of six cases emerge from the Monte Carlo experiments. In each case, we generate a sample of 1000 observations ($k = 1, 2, ...1000$) and apply each estimator to the sample. We iterate the procedure for 1000 times and report the biases, variances, and the mean square errors of $\hat{\beta}_1$ in Table 1.

Table 1. Simulation results in six cases

Estimator	Few zeros (15%)			Many zeros (50%)		
	Bias	Var.	MSE	Bias	Var.	MSE
Homoskedasticity						
PPML	0.085	0.012	0.019	0.452	0.158	0.362
Heckman	0.158	36.00	36.03	0.200	127.0	127.0
Tobit	-0.892	0.013	0.808	-0.978	0.001	0.956
TS-MM	-0.022	0.002	0.003	-0.071	0.007	0.012
Heteroskedasticity						
PPML	0.090	0.011	0.019	0.418	0.180	0.355
Heckman	0.209	17.84	17.88	0.709	80.23	80.73
Tobit	-0.899	0.009	0.817	-0.978	0.000	0.958
TS-MM	-0.017	0.003	0.003	-0.072	0.006	0.011
Super-heteroskedasticity						
PPML	0.088	0.012	0.020	0.430	0.179	0.364
Heckman	-0.035	28.08	28.08	0.470	174.6	174.8
Tobit	-0.862	0.042	0.785	-0.968	0.001	0.938
TS-MM	-0.017	0.005	0.005	-0.065	0.018	0.022

Note: Bias, Var., and MSE refer to the Monte Carlo bias, variance, and mean square error of $\hat{\beta}_1$ respectively.

We discuss the performance of each estimator in turn. As shown in Table 1, the PPML estimate of β_1 is biased upward by 9% when zeros are few, and by more than 40% when zeros are prevalent. The reason is that, without differentiating the extensive margin from the intensive margin, the PPML estimate co-finds the effect through β_1 and the effect through α_1. The problem becomes more evident when the portion of zeros increases, as the extensive margin of trade plays a greater role. This finding echoes Martin and Pham (2008) in that the PPML estimates can be severely biased when trade data is limitedly dependent and zeros are frequent. Nevertheless, the PPML estimates are fairly stable across different heteroskedastic patterns, as claimed in SST.

Three features are worth noting in the Heckman estimates. First, the Heckman estimates are generally biased, due to the logarithmic transformation of trade flows. The magnitude of the bias ranges from -4% in the case of few zeros and super-heteroskedasticity to over 70% in the case of many zeros and heteroskedasticity. Second, the Heckman estimates are not robust to heteroskedasticity. In either the case of few zeros or many zeros, the Heckman estimate varies a lot as the variance structure of the error term changes. Thirdly, the variances of the Heckman estimates are large in all cases, illustrating the identification problem of the Heckman model in the absence of an excluded variable. A glance at the E.T.-Tobit models reveals that the associated estimates are severely biased in all scenarios, as found in SST.

Now we discuss the performance of the proposed TS-MM estimator. Table 1 suggests that the TS-MM estimate is reasonably accurate, with the bias around 2% when zeros are few, and 7% when zeros are many. In other words, the TS-MM estimator satisfactorily addresses the issue of sample selection. Moreover, the TS-MM estimate is robust to various degrees of heteroskedasticity, as evidenced by the stability of bias and variance across different heteroskedastic patterns. In fact, by the criteria of either the magnitude of bias or mean squared error, the TS-MM estimator strictly dominates the PPML estimator, the Heckman sample selection model, or the E.T.-Tobit model.

Several robustness checks are warranted for the Monte Carlo experiments. One legitimate question is whether the TS-MM estimator is robust to heteroskedasticity in the stage of selection as well. To address this concern, we conduct another set of Monte-Carlo experiments in which we replace $\sigma_{22k} = 0.005$ with $\sigma_{22k} = 0.01 m_k$ (so that the variance of the error term in the selection equation increases with x). The associated results, reported in Appendix 1A, suggest that the TS-MM estimator again outperforms the alternatives. Another interesting scenario worth considering is when the two margins of trade work in opposite directions. For example, one can think of technical barriers, which might increase the market shares of larger and capital-abundant exporters, while driving out smallholder exporters who can barely meet the regulations. In this case, we expect the PPML estimates, which co-find the two margins, to be biased downward. To test the hypothesis, we conduct another set of experiments in which we set $\alpha_1 = -0.05$.[13] The associated results, reported in Appendix 1B, confirm that the PPML model delivers attenuated results, while the TS-MM model remains outperforming all other alternatives.

We conclude from the Monte Carlo experiments that the proposed TS-MM model outperforms the alternatives when the underlying data generating process follows the system (2a)-(2d), because it simultaneously deals with both sample selection and heteroskedasticity.

[13] To maintain the same proportions of zeros, we set $\alpha_0 = 0.125$ and $\alpha_0 = 0.05$ for the case of few zeros and many zeros respectively.

4. The Model Selection Strategy

In practice, however, the true data generating process is barely known to researchers. Therefore, one has to explore whether sample selection is a concern and to what degree heteroskedasticity matters for a particular application. To guide applied work, we suggest a model selection strategy that allows one to choose the most appropriate estimator.

Our proposed model selection strategy starts with screening various estimators based on their economic and statistical properties. Specifically, we focus on each estimator’s capability in dealing with zeros, accommodating heteroskedasticity, and addressing sample selection. It is worth noting that the concern of heteroskedasticity is closely related to the functional form in which the gravity equation is specified, i.e., whether trade flows ought to be characterized in levels, or in their logarithmic scales. Given the right specification, heteroskedasticity is less of a concern for statistical inferences if we use the heteroskedasticity-consistent standard errors. Therefore, the issue with heteroskedasticity translates into the choice between the specification in levels and the one in logs. The sample selection issue, in the context of trade, is closely related to the identification of the two margins of trade. That is, the two margins of trade can be told apart only when the sample selection is properly addressed.

Table 2 summarizes the strengths and weaknesses of commonly used estimators for the gravity equation model. We argue that one can eliminate the Truncated OLS estimator and the E.T.-Tobit model from the pool of candidate estimators. First, the Truncated OLS estimator is inferior to other alternatives because it fails to accommodate zeros at all. Second, the E.T.-Tobit model is dominated by the Heckman sample selection model. The reason is that, although similar to the Heckman model in many aspects (as shown in Table 2), The E.T.-Tobit model imposes a common threshold for all countries to jump (Eaton and Tamura, 1994), which is at odds with the fact that fixed costs of trade vary a lot across countries (Anderson and van Wincoop, 2004). Therefore, the evaluation of the economic and statistical properties of estimators leads to a candidate pool of three competing estimators: the PPML estimator, the Heckman sample selection model, and the TS-MM estimator.

Table 2. Advantages and disadvantages of various estimators

Estimator	Zeros?	In levels? (robust to heteroskedasticity)	Two margins? (sample selection)
Trun-OLS	no	no	no
PPML	yes	yes	no
Heckman	yes	no	yes
E.T.-Tobit	yes	no	yes
TS-MM	yes	yes	yes

The second-round selection involves two statistical tools. First, we focus on the sample selection issue and compare the Heckman model with the TS-MM model. While both models correct for the potential sample selection bias, the TS-MM model differs from the Heckman model in that, in the second-stage estimation, it characterizes trade in levels, as opposed to in logarithmic scales. The MacKinnon-White-Davidson (MWD) test can be used to choose between the specification in levels and the one in logs (MacKinnon, White, and Davidson, 1983). Intuitively, the MWD test works as follows.[14] We fit both the TS-MM model and the Heckman model and generate predicted trade flows in the second stage. Denoting the series of predicted trade from the TS-MM model and the Heckman model as $\hat{M}$ and $\hat{N}$ respectively, we run the second stage of the TS-MM model again with an additional explanatory variable $\ln(\hat{M}) - \hat{N}$. We reject the null hypothesis that the TS-MM model is correctly specified if the auxiliary variable is statistically significant.[15] Therefore, the MWD test results enable us to choose a preferred model between the Heckman model and the TS-MM model.

The sample selection bias, as revealed by the model that survives the MWD test, may or may not be statistically significant. In case the sample selection is an issue indeed, we conclude from the model selection strategy that the model wins the MWD test is the most appropriate model. On the other hand, if the sample selection bias is insignificant, we need to further compare the model that wins the MWD test with the PPML model. The reason is that, in the absence of sample selection, the PPML model might perform

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[14] Interested readers are referred to MacKinnon, White, and Davidson (1983) and Gujarati (2004) for more discussions.

[15] One can also test the Heckman model against the TS-MM model by fitting the second stage of Heckman with an additional variable $\hat{M} - \exp(\hat{N})$. The null hypothesis that the Heckman model is correctly specified is rejected if the auxiliary variable is statistically significant.

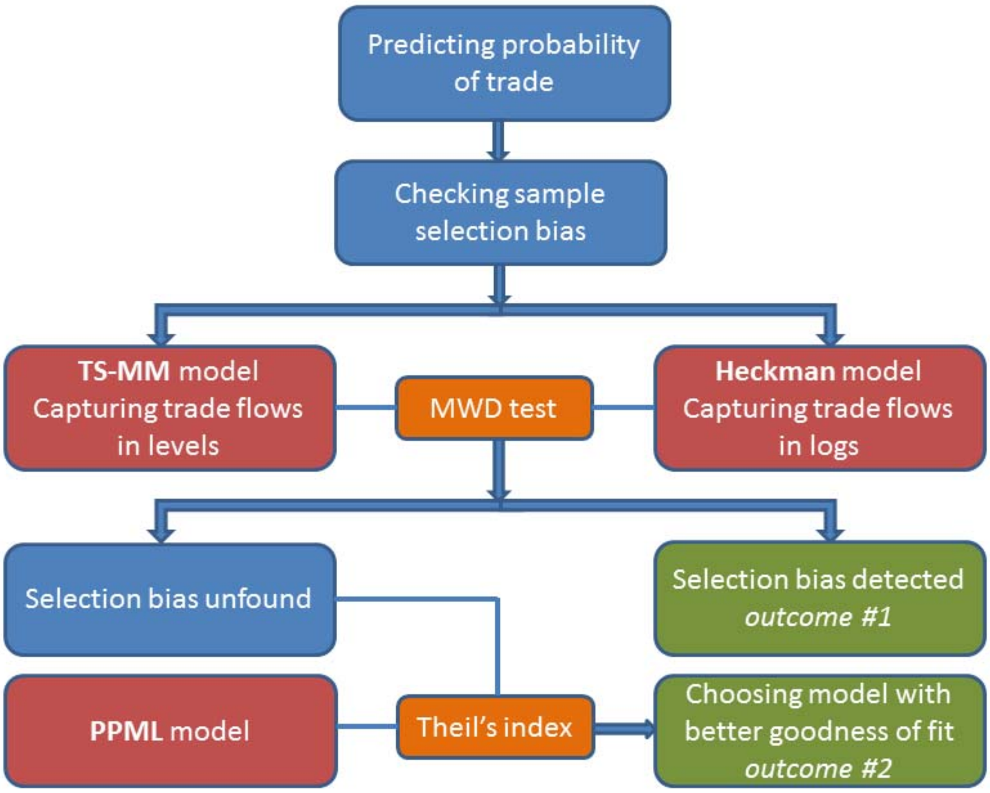
well in estimating the overall marginal effects. We use the Theil's inequality coefficient, as a measure of goodness-of-fit, to compare the two models. Specifically, the Theil's inequality coefficient is computed as

$$TU = \sqrt{\sum_i (\hat{y}_i - y_i)^2} \Big/ \left(\sqrt{\sum_i \hat{y}_i^2} + \sqrt{\sum_i y_i^2}\right),$$

where y and $\hat{y}$ denote the observed and predicted trade flows respectively.[16] The Theil's inequality coefficient lies between 0 and 1, with a smaller value indicating a better goodness-of-fit. Therefore, in the absence of sample selection, the most appropriate model is either the PPML model or the model that wins the MWD test, depending on which fits the data better. The decision tree in Figure 1 summarizes the model selection strategy.

[16] For the PPML model, the predicted trade is computed as $\exp(X\hat{\beta})$. For the TS-MM model, the predicted trade is computed as $\Phi(Z\hat{\theta})\cdot(\exp(X\hat{\beta})+\hat{\omega}\lambda)$. For the Heckman model, the predicted trade is computed $\Phi(Z\hat{\theta})\cdot\exp(X\hat{\beta}+\hat{\omega}\lambda)$.

Figure 1. Decision tree for the model selection strategy



5. An Empirical Application

We illustrate how the proposed TS-MM estimator and the model selection strategy work by investigating world trade in 1990. The data come from SST. We have aggregate bilateral trade records among 136 countries in the year 1990. Among the 18360 (=136*135) observations, 48% are zeros. The explanatory variables include the geographic distance, the border dummy variable, the common language dummy variable, the colonial tie, and the FTA dummy variable. As in Anderson and van Wincoop (2003), we include both the importers' fixed effects and the exporters' fixed effects in the regression analysis to control for the multi-lateral trade resistance terms.[17]

Following the model selection strategy, we restrict our attention to the three competing estimators: the PPML model, the Heckman sample selection model, and the TS-MM model. We expect two countries further apart to trade less. On the other hand, we speculate that the volume of trade is larger if the two countries share a country border, or use a common language, or had a colonial relationship, or engage in a regional trade agreement. Table 3 presents the econometric results from all three models.

[17] Due to the cross-sectional nature of the data, all country-specific characteristics (such as income, population, and remoteness) are subsumed into the exporters' and importers' fixed effects.

Table 3. Regression results from the PPML, Heckman, and TS-MM models

	PPML	Heckman[a] 2nd stage Inten. Margin	1st stage Exten. Margin	TS-MM[a] 2nd stage Inten. Margin
(1)	(2)	(3)	(4)	(5)
ln(*dist*) [b]	-0.75***	-1.35***	-1.12***	-0.77***
	(0.04)	(0.03)	(0.06)	(0.04)
border	0.37***	0.16	0.14	0.35***
	(0.09)	(0.12)	(0.12)	(0.09)
language	0.38***	0.41***	0.42***	0.42***
	(0.09)	(0.06)	(0.05)	(0.09)
colony	0.08	0.67***	0.30***	0.04
	(0.13)	(0.07)	(0.06)	(0.13)
FTA	0.38***	0.29***	1.46***	0.38***
	(0.08)	(0.10)	(0.18)	(0.08)
IMR [c]	n.a.	0.09	n.a.	-933.86
		(0.06)		(849.66)
importers' fixed effects	yes	yes	yes	yes
exporters' fixed effects	yes	yes	yes	yes

Note: a. The Heckman model and the TS-MM model share the same 1st stage estimation; b. To facilitate the identification of the Heckman model, the distance variable is expressed in levels, instead of logs, in the 1st stage estimation; c. Inverse Mill's Ratio is calculated from the 1st stage estimation. Heteroskedasticity-resistant standard errors are in parenthesis. *, **, and *** denote the significance levels at 10%, 5%, and 1% respectively.

The PPML estimates, shown in Column (2) of Table 3, replicate the results reported in SST. All explanatory variables bear the expected signs and all are statistically significant except for the colonial tie dummy variable. Noticeably, since the PPML model co-finds the two margins of trade, the estimated raw coefficients can be interpreted as the overall marginal effects. Column (4) of Table 3 reports the first-stage estimation from the Probit model, which is shared between the Heckman model and the TS-MM model. Instead of presenting the estimated raw coefficients, we report the marginal effects on the extensive margins of trade, as defined in (3b), after fitting the Probit model. Interestingly, while all other trade determinants affect the propensity to trade in ways we anticipate, a common country border does not seem to increase the likelihood of trade significantly. The second-stage estimation of the Heckman model, as shown in Column (3) of Table 3, reinforces this finding by showing contiguity does not matter for the size of trade either. Additionally, since the sample selection bias is not statistically significant in the Heckman model, the estimated raw coefficients in the second stage directly translate into the marginal effects on the intensive margins of trade.

Turning to the second-stage estimation of the TS-MM model, or Column (5) of Table 3, we find that the sample selection bias is not statistically significant either. Hence, the estimated raw coefficients can be interpreted as the marginal effects on the intensive margins of trade, as defined by (4b). Compared to the Heckman estimates, the results from the second-stage TS-MM estimation suggest that countries sharing borders trade more, but that countries with historical colonial ties do not trade significantly more (although they are more likely to trade).

The difference in statistical and economic inferences across three models calls for diagnostic analysis. Following the proposed model selection strategy, we first deal with the sample selection issue and choose one between the Heckman model and the TS-MM model. Specifically, we conduct two WMD tests to guide the choice of the specification for the gravity equation (i.e., whether trade flows should be modeled in levels or in their logarithmic scales). As shown in Table 4, the first WMD test is under the hypothesis that the Heckman model is correctly specified, or, the logarithmic transformation can be taken to the gravity equation; whereas the second one tests the TS-MM model against the Heckman model. The associated P values of the WMD tests suggest that the TS-MM model is preferred over the Heckman model.

Table 4. MWD test results and Theil's indices

The MWD tests	**P value**
H_0: the 2nd stage of Heckman is correctly specified	0.00
H_0: the 2nd stage of TS-MM is correctly specified	0.16
Goodness of fit	**Theil's inequality coefficients**
PPML	0.14
TS-MM	0.14

However, the insignificance of the sample selection bias in the TS-MM model compels us to further compare the TS-MM model with the PPML model. Coincidently, the associated Theil's inequality coefficients in Table 4 suggest that the TS-MM model and the PPML model fit the data equally well. Nevertheless, we consider the TS-MM model weakly preferred over the PPML model because it sheds light on the two margins of trade.

In summary, applying the proposed model selection strategy, we find that the TS-MM model is the most appropriate estimator. Next, we discuss the economic implications of the TS-MM estimates. The elasticity of distance is of the magnitude -1.89(=-1.12-0.77), more than doubling the effect reported in SST.[18] In terms of the border effect, our finding reinforces Anderson and van Wincoop (2003) in that a shared border enlarges the size of trade by nearly 30%. While the colonial tie fosters trade primarily through the extensive margin, a common language increases both the chance and size of trade. In addition, regional trade agreements not only enhance pre-existing trade by 38% (which is compatible with the result reported by Baier and Bergstrand (2007)), but also significantly improves market access.[19]

6. Conclusion

A vexing issue in the gravity equation model is its statistical estimation in the presence of two stylized features of trade data: sample selection and heteroskedasticity. We contribute to empirical applications of the gravity equation model in two important ways. First, we propose a Two-Step Method of Moments (TS-MM) estimator that deals

[18] Nevertheless, the distance effect we find is within the range reported by Disdier and Head (2008).
[19] Similarly, Felbermayr and Kohler (2006) show that the WTO membership facilitates international trade primarily via the extensive margin.

with both issues. The novel estimator works as follows. In the first step, the estimator explains why trade takes place at all and sheds light on the extensive margin of trade. In the second step, the volumes of trade are characterized, in levels, by an augmented gravity equation with correction for the sample selection bias. The method of moments technique delivers consistent estimates regardless of heteroskedastic patterns. Our second contribution is the provision of a model selection strategy, which allows one to choose the most appropriate estimator in practice. In particular, we show how economic theories and statistical tools can be used together to guide the estimation of a gravity model.

Several extensions are worth attempting for future research. For instance, the identification of different sources of zeros is of great relevance: while some zero trade records are due to the inability to trade, others may reflect missing data entries. Further, the TS-MM estimator can be applied to other constant-elasticity models, such as the Mincer's earnings model (Mincer, 1974), where sample selection and heteroskedasticity might be of concern.

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Appendix 1A

Table 1A. Simulation results under six scenarios when heteroskedasticity is present in the selection process

Estimator	Few zeros (15%)			Many zeros (50%)		
	Bias	Var.	MSE	Bias	Var.	MSE
Homoskedasticity						
PPML	-0.019	0.022	0.023	0.277	0.174	0.250
Heckman	0.106	236.1	236.1	-0.492	385.4	385.6
Tobit	-0.949	0.002	0.903	-0.982	0.000	0.965
TS-MM	-0.018	0.001	0.002	-0.057	0.016	0.019
Heteroskedasticity						
PPML	-0.021	0.021	0.022	0.286	0.165	0.247
Heckman	-0.043	155.6	155.6	1.322	337.2	339.0
Tobit	-0.946	0.006	0.901	-0.980	0.002	0.963
TS-MM	-0.020	0.001	0.002	-0.064	0.007	0.011
Super-heteroskedasticity						
PPML	-0.030	0.025	0.026	0.291	0.177	0.262
Heckman	0.530	250.0	250.3	1.522	791.4	793.7
Tobit	-0.948	0.003	0.901	-0.974	0.001	0.950
TS-MM	-0.021	0.003	0.003	-0.057	0.012	0.015

Note: Bias, Var., and MSE refer to the Monte Carlo bias, variance, and mean square error of $\hat{\beta}_1$ respectively.

Appendix 1B

Table 1B. Simulation results under six scenarios when the variable affects two margins in opposite directions

Estimator	**Few zeros** (15%)			**Many zeros** (50%)		
	Bias	Var.	MSE	Bias	Var.	MSE
			Homoskedasticity			
PPML	-0.199	0.021	0.060	-0.640	0.154	0.564
Heckman	-0.214	55.27	55.31	-0.682	130.0	130.5
Tobit	-0.955	0.006	0.919	-1.001	0.000	1.003
TS-MM	-0.003	0.001	0.001	-0.026	0.006	0.007
			Heteroskedasticity			
PPML	-0.187	0.019	0.054	-0.651	0.152	0.575
Heckman	0.246	43.27	43.33	-0.188	198.5	198.6
Tobit	-0.957	0.005	0.921	-1.001	0.000	1.003
TS-MM	-0.001	0.001	0.001	-0.022	0.002	0.002
			Super-heteroskedasticity			
PPML	-0.194	0.022	0.060	-0.650	0.151	0.573
Heckman	-0.157	42.37	42.39	-0.230	183.0	183.1
Tobit	-0.951	0.005	0.910	-1.001	0.000	1.003
TS-MM	0.001	0.002	0.002	-0.025	0.003	0.004

Note: Bias, Var., and MSE refer to the Monte Carlo bias, variance, and mean square error of $\hat{\beta}_1$ respectively.