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Abstract

In the last decade, the Murray-Darling Basin (MDB), Australia faced a severe drought which affected its agriculture production. Sustainable diversion limits as proposed in the Australian Government's basin plan together with climate change is expected to impact on future agriculture production and development in the MDB. We developed a biophysical-economic mathematical model calibrated against the observed multi-period land use data utilising the positive mathematical programming (PMP) approach to evaluate the impacts on agricultural production activities of a range of climate events and policy options. This is an extension of our previous work where the model was calibrated against a single year and focus was on the southern MDB only. The multi-period calibrated model has strong predictive capacity as it matches simulated irrigated area, water use and gross value of irrigated agricultural product (GVIAP) well with the observed irrigated land, water use and GVIAP for all the crops in all the regions of the MDB across the highly variable climatic conditions from 2005 to 2009.
The approach will be useful in assessing economic impacts of climate change on irrigation, farmers' adaptation options and/or water policies including water markets and irrigation efficiency improvement.

Keywords

Integrated hydrology and economic model, multi-period calibration, climate change, drought, agriculture, positive mathematical programming

1. Introduction

In most of the first decade of the 21st century the Murray-Darling Basin (MDB) in Australia has faced severe drought and reduction in rainfall. Inflows into the Murray River in the last decade (except year 2010-11) were about half the historic average (MDBA, 2009) while those in 2006-7 were considerably lower (i.e. less than 10%) than the previous historic minimum (Kirby et al., 2012). Subsequently, the volume of water held in many major storages has also fallen to record low levels (i.e. less than 20% of many of the storages' full capacity) and water available for diversion and allocations of irrigation water were significantly lower than the licensed entitlements in most regulated river valleys of the MDB (MDBA, various reports). Future climate change is expected to cause a greater reduction in rainfall and irrigated water allocation in many parts of Australia with consequent effects on surface water availability in the MDB.

CSIRO (2008) assessed water availability under a range of climate scenarios for the MDB and found reductions in surface water availability of between 3% and 21% in some catchments

(CSIRO, 2008). Further, requirements for greater and more secure environmental flows and rapidly evolving water policy are changing water sharing between the irrigation sector and the environment. Reduction in water availability could have a significant but varying impact in terms of crop yields, input costs and agricultural profitability across regions of the MDB. The understanding of economic implications of reduced rainfall and water allocations and the capacity of farmers to adapt to less irrigation water will be of particular interest of water resource managers and policy makers.

A multi-period calibrated model is warranted that can be used to assess impacts of future climate change scenarios and water policies on individual sectors in different catchments of the MDB. Mathematical programming approaches despite their analytical power struggled with poor tracking records of observed behaviour (Heckelie and Britz, 2005) and faced the problem of overspecialisation in agricultural production (Howitt et al., 2010). The problem of overspecialisation is more severe in aggregate models due to a number of reasons (Howitt, 1995):

- the number of empirically justified constraints relative to the number of observed production activities is smaller compared to the farm level;
- data, time and computational restrictions often do not allow specifying relevant non-linearity in aggregate technology that would force more production activities into the solution; and
- output price endogeneity and risk behaviour which can cause diversification are often not incorporated into the objective function.

Positive mathematical programming (PMP) approaches solve the problem of overspecialisation (faced using linear programming models) by assuming a profit-maximising equilibrium in the reference period (Howitt, 1995). Based on an assumption of unobserved information, the PMP approach recovers additional information from observed activity levels and specifies a non-linear objective function. This consequently results in the model exactly producing the observed behaviour of farmers (Cortignani and Severini, 2009) without introducing artificial constraints ((Heckelie and Britz, 2005) and making it a widely accepted method for policy analysis (Griffin, 2006; Howitt et al., 2010, Merel and Bucaram, 2010; Qureshi et al., paper in review).

Despite their wide use for policy analysis, especially related to agriculture, water and climate change policy, conventional PMP models have come under scrutiny, partly due to their reliance on single observation of the cropping pattern to calibrate model parameters directly controlling supply responses and their inability to reproduce robust and realistic supply responses (Heckelie and Britz, 2005; Merel and Bucaram, 2010). This raises the question of whether a generally calibrated PMP model is capable of capturing the behavioural response of farmers to changing economic conditions, such as impact of price change (Heckelie and Britz, 2005). The need for incorporating prior information regarding the responsiveness of activities to price changes into PMP models that rely on one observation has been acknowledged in the recent literature (Heckelie, 2002; Heckelei and Britz, 2005). This is because PMP models, particularly positive quadratic programming models, are under-identified. Additional information on (price and) supply elasticities can minimise the under-identification problem (Merel and Bucaram, 2010). The key issue with a single calibration observation is that it is not sufficient to infer the value of model parameters that directly control the way the model responds to changes in price conditions (Heckelei and Britz, 2000, 2005).

In countries like Australia, which face great variation in rainfall and as a result irrigation water availability, it becomes crucial to assess future climate change impacts against an appropriate and a representative base case. Further, the magnitude of reduction in rainfall and water allocations depends on whether we are comparing against a base case 'year' with high, long term average or low rainfall and water allocation year. In this study, we extend our previous single year calibration model (Qureshi et al. paper in review) to a multi-period calibrated model. We use multi-period information of commodity price, yield, water use and water allocation to inform the changes in marginal incentives if one moves away from the observed allocation and to avoid extremely unreasonable supply responses. This has allowed estimation of the model parameters underlying the observed response behaviour of procedures in respective years varying from wet to dry years of rainfall and water allocation and use as well as crop yield and price. Later, we used an average of these non linear cost function parameters and assessed drought impact in the individual years. The model is found to be robust against observed irrigated area, water use and GVIAP and as such provides a more robust basis for the estimation of future climate or policy on different agricultural sectors across the regions of the MDB.

Our paper is structured as follows. In this section we outlined the background, basis and rationale for the PMP modelling approach. In Section 2 we provide a brief overview of the case study and data collection procedure. Along with introducing the MDB as a case study, we describe the steps taken in collecting and adjusting sectoral and regional biophysical and economic data. In Section 3 we present our multi-period calibrated PMP modelling approach including calibration and flexibility. Results are presented in Section 4. We conclude the paper with a discussion of the approach and a brief summary of the opportunities for further research.

2. Study area and data collection procedure

This study is focussed on the Murray-Darling Basin (MDB) which accounts for about 1/3rd of Australia's gross value of agricultural production (ABS, 2010). We require data on rainfall and irrigation water use, agricultural land uses, gross value of production, and commodity prices across the MDB regions for which we parameterise our model. Irrigation area and water use per hectare by crop and NRM (natural resources management) region data were obtained from the ABS catalogue 46180 series (ABS, 2008). Gross value of irrigated agricultural product (GVIAP) and crop price data were obtained from catalogue 46100 (ABS, 2011). Rainfall has a significant impact on returns to irrigation. In the current analysis, the ABS water per hectare is assumed water applied after accounting for crop effective rainfall in individual catchments across the MDB. Effective rainfall data of different crops grown in the MDB were estimated while considering their growing cycle.

Nine major categories of agricultural crops (or commodities) which occupy most of the irrigated areas in 17 major regions of the MDB are considered in the analysis (cereals, cotton, rice, pasture for dairy, beef and sheep production, fruits and nuts, grapes, and vegetables). Irrigation area, water use, GVIAP, and price data were obtained for these commodities for 2005-06 to 2008-09 (i.e. multi-period four years data). For example, ABS provides data for about two dozen crops including various cereal crops and fruits and nuts. For simplicity, in the analysis, all the cereal crops which produce grains are categorised as cereals and their average values are considered appropriate. Similarly, the average values of all the fruits and nuts data and vegetables data are used to represent all the fruits and nuts and vegetables, respectively. Crop yield data which is critical for modelling agriculture production and supply and simulating the impact of different water allocation scenarios and policies were not

available for the nine individual crops for the multi-periods. We estimated crop yield values (i.e. tonnes per hectare or t/ha) first by dividing GVIAP by the individual crop irrigated area and then dividing per hectare gross value by price).

Crop yields across regions and for different years were more variable than we expected. In regions and for those years where the crop in question had a relatively smaller or larger crop yield (t/ha), it was attributed to a possibly small sample size and under/over representation of the population (i.e. great standard errors of population inferences from the survey samples). For example, the data standard deviation in some crops was 4 times greater than the data mean values. For these crops, an adjustment was made using the procedure as follows. First we created two thresholds (i.e. 10th and 80th percentiles). The value less than 10th percentile was assigned 15th percentile value while the value greater than 80th percentile was assigned 75th percentile value. This procedure removed the extreme values along with providing the missing values. Second we multiplied the revised yield values by crop price and area and estimated gross values of the individual crops as well as of regions for the four years.

To quantitatively assess whether the adjusted gross values match the ABS observed values, we applied one of the six simple but useful quantitative approaches or indices (i.e. Nash-Sutcliffe model efficiency coefficient) for assessing a model's skill or its predictive power (Stow et al., 2003; 2009).[1] We found that except cereals, fruits and vegetables, the Nash-Sutcliffe efficiencies ranged from 0.51 to 0.98 indicating the adjusted GVIAP for these crops as accurate as the mean of the ABS estimated GVIAP. The low efficiencies for cereals, fruits and vegetables indicate that the ABS adjusted mean values are better predictor for these crops than the estimated GVIAP. The efficiency test was also performed to compare the basin level adjusted GVIAP and the observed GVIAP and found the efficiency of 0.74 indicating the adjusted values for these crops are as accurate as the mean of the ABS estimated GVIAP of the whole basin. Next we noticed that some regions had irrigation areas data but their GVIAP values were missing. We multiplied the adjusted crop yield value by crop irrigated area and estimated these missing values of irrigation water use (ML/ha). Some of these values were more variable than we expected. By using the similar procedure described above, we removed the extreme water use values and found the missing values. The final adjusted catchment-wise and crop-wise irrigated area, irrigation water use per ha, crop yield and price data for 2006 to 2009 are available on request.

3. PMP Model formulation

The model objective is to maximise gross value *MaxGV* in each year *t* for the whole MDB after accounting for operating or variable costs subject to available land and water. The objective function of maximising GV in each year is expressed algebraically in Equation (1).

$$MaxGV_t = \sum_r \sum_j Price_{rj} \, x \, Yield_{rj} \, x \, Area_{rj} - \sum_r \sum_j VC_{rj} \, x \, Area_{rj} \qquad (1)$$

[1] In the Nash-Sutcliffe modelling efficiency test, a value near one indicates a close match between observations and model simulated values; a value of zero indicates that the model simulates individual observations no better than the average of the observations; and values less than zero indicate that the observation average would be a better predictor than the model simulated results (Stow et al., 2009).

where r, irrigation region; j, cropping activities; VC, operating or variable costs; Price, price of each activity in dollars per tonne ($/t); Yield, crop yield in tonnes per hectare (t/ha); Area, production activity levels or irrigated area (ha) allocated to each activity in each region.

The model is subject to the following irrigation water use or availability (2), irrigated land (3) and positive activity (4) constraints

$$\sum_r \sum_j IW_{rj} \times AREA_{rj} \leq TWat_{rj} \qquad \forall r \tag{2}$$

$$\sum_r \sum_j AREA_{rj} \leq TArea_r \qquad \forall r \tag{3}$$

$$Area_{rj} \geq 0 \tag{4}$$

Given our data sets described above are in some cases averages of various activities or based on sectoral and regional averages, the solution of this problem is bound to be overspecialised in the mathematical programming approach, as discussed above. In particular, this is because the number of empirically justified (or available) resource constraints is well below the number of observed agricultural activities (Heckelei and Britz, 2005; Howitt *et al*., 2010). A positive mathematical programming or PMP approach (Howitt, 1995) is applied assuming a profit-maximising equilibrium for each year (i.e. reference period) to address the problem of overspecialisation in agricultural production.

Based on the assumption of unobserved or missing information, the PMP approach recovers the additional (or missing) information from observed activity levels and specifies a non-linear objective function so that the model exactly produces the observed behaviour of farmers.

The approach required three steps, including

1. Extending and reformulating the GV maximisation model as a constrained non-linear programming model and specifying the parameters of a nonlinear objective function in such a way that the model calibrates almost exactly to the observed activity levels;
2. Estimating a quadratic variable cost function to capture all farming conditions not modelled in an explicit way; and
3. Formulating a quadratic programming model and including the variable cost function in the objective function.

For reasons of computational simplicity and following Heckelei and Britz (2000) we used the quadratic cost function (TC) shown in Equation (5).

$$TC = \alpha x + \frac{1}{2}\gamma x^2 \tag{5}$$

where x represents production activity level and α and γ are parameters to be estimated.

A simple one activity example of the PMP approach is given in Appendix A to show how the given information is used to calculate the nonlinear cost function coefficients and how once

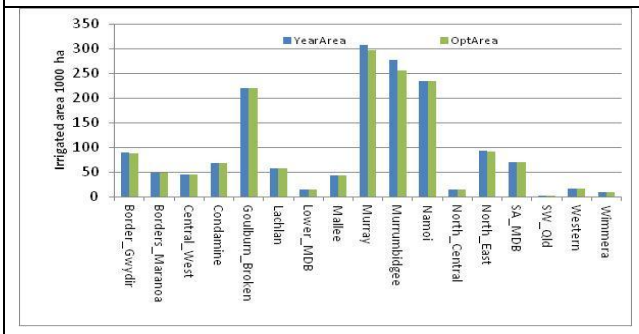
added in the objective function results in the activity level (area) that is equal to the observed area. With some derivations, γ is calculated by equation (6) and λ is the difference between marginal revenue or value and average cost.

$$\gamma = (2 * \lambda) / x \qquad (6)$$

In this sense the PMP approach is assumed to capture information on all farming conditions influencing the distribution of returns that are not modelled in an explicit way with normal linear programming. We used the information contained in dual variables of the non-linear programming problem constrained to the observed activity levels by calibration constraints specified the non-linear objective function such that the observed activity levels are reproduced by the optimal solution of the new programming problem without constraints. For each observed year (i.e. 2006, 2007, 2008 and 2009), we calculated parameters of the quadratic cost function (i.e. alpha α, lambda λ and gamma γ) and based on the PMP assumptions of missing information using the observed areas by crop, crop water use, crop yield and commodity price for the individual year. By including the cost function and removing the above mentioned constraints we specified the model against the individual year's irrigated land use for each region of the MDB.

Figure 1 shows that the model calibrates well against the observed land and water use when an individual year's calculated non linear cost function coefficients are used (year 2006 and 2008 are only shown for comparison). Except a small variation in the Murrumbidgee, the model allocated irrigated land perfectly matches with the respective year's observed irrigated land use in all regions. Figure 1 also shows that Murray, Murrumbidgee, Namoi and Goulburn-Borken regions have the highest irrigated land use, respectively. Murray and Murrumbidgee are also the first and the second highest irrigation water use regions respectively while Goulburn-Borken and Namoi are respectively the third and the fourth highest irrigation water use regions. The model allocated land also matches with the observed irrigated land for each crop in the MDB. Pasture for dairy production has the highest irrigated land area followed by cereals, beef and cotton, respectively. Cotton is the highest irrigation water use activity followed by rice, dairy and cereals, respectively. As also shown in Figure 1, except cereals where irrigated area increased by about 20%, all the activities reduced their irrigated land in 2008. These changes are discussed further in Kirby et al. (2012).

2006



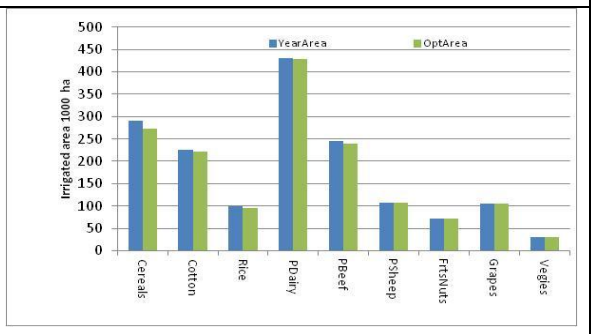


Figure 1 Region and crop-wise model simulated and observed irrigated land and water use in 2006 and 2008

4. Model results - reliability for future predictions

Given the uncertainty and high hydro-climatic variability in the MDB (which may likely to increase considering climate change projections), the model must be capable of assessing the impact on agricultural production across a wide range of the future climate scenarios (i.e. dry, medium or wet year), and of a range of possible future) water policies, with reasonable accuracy.

Assuming 2006 was similar to long term average rainfall and water allocation, we initially used the coefficients of the cost function (i.e. lambda, alpha and gamma) obtained for year 2006 to simulated irrigated land and water and economic impacts for all the four years.[2] As

[2] We used individual year' crop yield and price data to assess gross value of irrigated agricultural product in the calibration process. However, for future scenario assessments it

shown in Table 1, as expected, the PMP approach simulated land and water use with great accuracy for 2006 observed values. The change from the observed values is less than one percent. This provides a sample single year calibrated base model against which we can compare the performance of a multi-period calibration approach. As can be seen the simulated values do not match the observed irrigated land and water use with reasonable accuracy for the other three years. For example, in 2008, Border_Gwyder and Borders_Maronoa, there is about 34% and 32% decline in simulated area while in Murray and Namoi, there is increase in their simulated area of 21% and 32%, respectively. Even greater variation is found in the simulated areas of crops compared to their observed areas for the three years. In 2008, the simulated irrigated areas in 2008 for rice and beef (for example) are 431% and 203% of the reported areas respectively. That is there appears to be a significant discrepancy between the modelled PMP parameters and reported areas for years other than the calibration year.

Table 1 Catchment-wise change in simulated area and water from reported data using a single calibration year (2006)

	2006 (Calibration year)		2007		2008		2009	
	Area	Water use	Area	Water use	Area	Water use	Area	Water use
Border_Gwydir	-0.01%	-0.01%	-1.82%	0.00%	-34.00%	0.00%	-25.11%	0.00%
Borders_Maranoa	0.00%	0.00%	8.75%	0.00%	-32.27%	-6.61%	-42.15%	-31.64%
Central_West	0.00%	0.00%	-9.84%	0.00%	-20.90%	0.00%	-13.75%	0.00%
Condamine	-0.01%	-0.02%	-0.86%	0.00%	-5.32%	0.00%	-30.27%	0.00%
Goulburn_Broken	-0.28%	0.00%	5.20%	0.00%	13.42%	0.00%	7.27%	0.00%
Lachlan	-0.01%	-0.01%	-13.63%	0.00%	-19.63%	-9.42%	3.00%	0.00%
Lower_MDB	-0.01%	0.00%	-8.95%	-6.02%	5.06%	0.00%	0.77%	0.00%
Mallee	0.00%	0.00%	-19.07%	-20.94%	7.84%	0.00%	0.81%	0.00%
Murray	-0.04%	-0.03%	22.01%	0.00%	21.13%	0.00%	18.56%	0.00%
Murrumbidgee	-0.02%	-0.02%	1.10%	0.00%	-16.04%	0.00%	-17.38%	0.00%
Namoi	0.00%	0.00%	4.12%	0.00%	33.86%	0.00%	3.74%	0.00%
North_Central	0.00%	0.00%	-4.24%	0.00%	-3.50%	0.00%	5.14%	0.00%
North_East	-0.15%	-0.06%	-2.85%	0.00%	-19.59%	0.00%	-24.62%	0.00%
SA_MDB	1.12%	0.00%	-8.10%	-7.88%	1.37%	-1.12%	-5.87%	-16.36%
SW_Qld	0.00%	0.00%	-0.07%	0.00%	10.67%	0.00%	20.40%	0.00%
Western	-0.19%	-0.06%	4.77%	0.00%	-19.61%	0.00%	9.90%	0.00%

will be more appropriate to use the average of the four years crop yields, and to set prices according to specific assumptions relevant to the scenario being modelled.

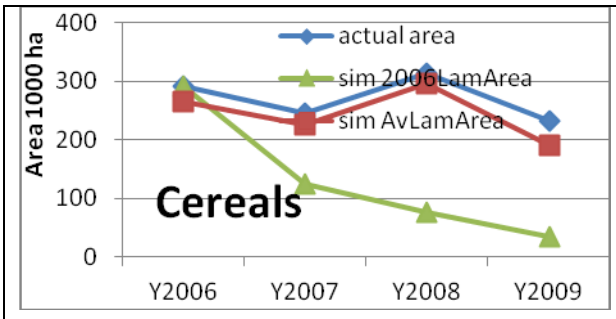
Wimmera	0.00%	0.00%	-27.79%	-21.17%	-3.27%	0.00%	-4.18%	0.00%

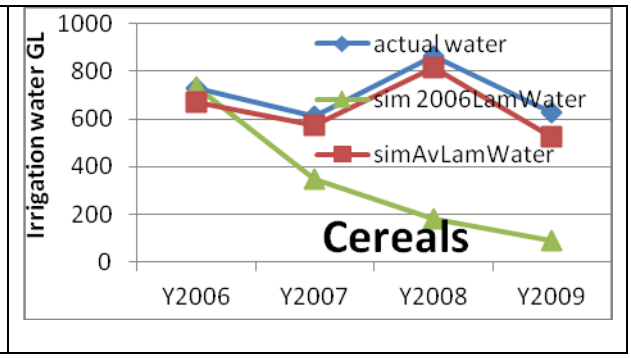
Table 2 Crop-wise change in simulated area and water from reported data using a single calibration year (2006)

	2006		2007		2008		2009	
	Area	Water use	Area	Water use	Area	Water use	Area	Water use
Cereals	0.46%	0.44%	-49.07%	-43.49%	-75.78%	-78.83%	-85.51%	-85.64%
Cotton	-0.01%	-0.01%	1.22%	0.87%	109.59%	108.76%	18.66%	19.17%
Rice	0.17%	0.17%	-9.80%	-7.39%	430.89%	457.69%	20.45%	22.40%
PDairy	-0.19%	-0.21%	16.40%	14.41%	-15.34%	-23.11%	-4.71%	-3.85%
PBeef	-0.04%	-0.03%	57.23%	47.09%	203.57%	145.27%	184.62%	191.73%
PSheep	-0.02%	-0.02%	53.80%	43.41%	100.04%	74.68%	39.39%	16.12%
FrtsNuts	-0.35%	-0.27%	-5.43%	-0.16%	-23.11%	-22.80%	0.60%	-4.62%
Grapes	-0.47%	-0.46%	-26.73%	-29.16%	5.24%	4.78%	-18.26%	-18.02%
Vegies	-0.08%	-0.09%	14.88%	14.90%	20.80%	21.91%	35.75%	32.39%

To overcome the limitations of a single calibration year and improve the predictive capacity of the model we used average lambda values that we estimated from separate single year calibrated models for each of the four years for which we have data. This approach resulted in a significant reduction between simulated and actual values. As shown in Figure B1 of Appendix B, except 2006, use of the average lambda values has improved for all years and for all the regions and crops.

In Figure 2 the improved simulation performance is demonstrated graphically for cereals and cotton. Use of a single 2006 calibration lambda (green line) diverges from observed land use in future years (blue line), while use of average lambda values across the four have closely replicates observed areas and water use (red line). We also estimated GVIAP and compared them with the ABS adjusted observed GVIAP and found little variation in the estimated and observed values for all the crops and regions of the MDB. As shown in Figure 3, significant variation in 2008 for cereals and cotton (for example) has been reduced when the average lambda values are used instead of using 2006 lambda values. These findings illustrate the apparent robustness of the multi-period calibrated PMP approached.





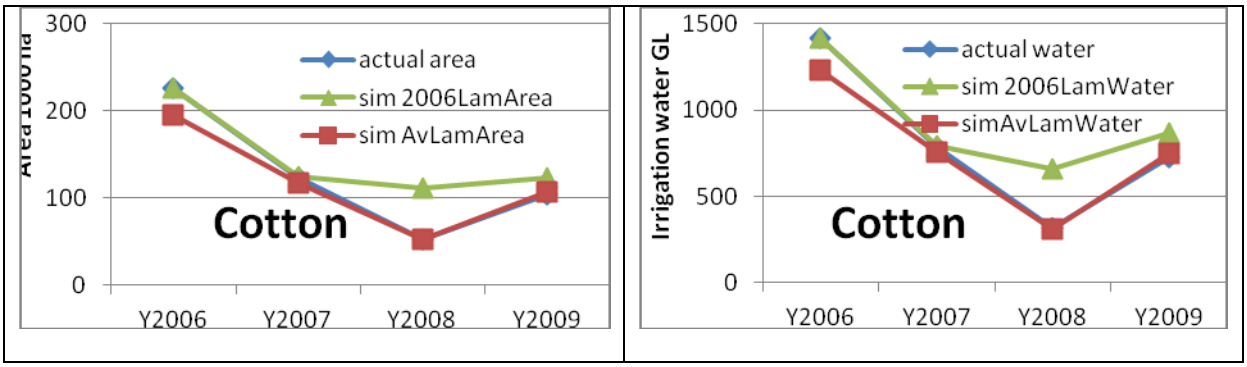


Figure 2 Model simulated area and water use versus observed area and water use using 2006 use lambda values and temporal average lambda values

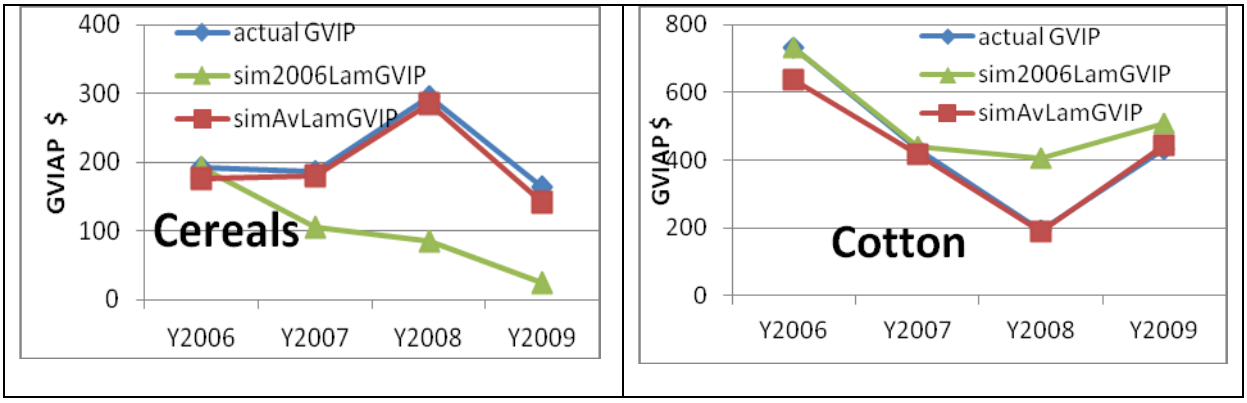


Figure 3 Model simulated GVIAP versus observed GVIAP using 2006 use lambda values and temporal average lambda values

We tested our visual conclusion that average lambda values were providing a better model than single year calibration using the modelling efficiency (or Nash-Sutcliffe model efficiency coefficient) to compare our simulated irrigated areas, water use and GVIAP across all the crops (when 2006 lambda and average lambda values are used) with the ABS estimated irrigated areas, water use and GVIAP of these crops. Table 3 shows the Nash-Sutcliffe efficiencies of cereals and cotton (for example) for three variables, i.e. irrigated area, water use and GVIAP. As shown in Table 3, when the average lambda values are used, the efficiencies of three variables of the two crops range from 0.33 to 0.94 compared to the efficiencies varying from -23.68 to 0.78 when 2006 lambda values are used. The efficiencies obtained using the average lambda values are close match between the model simulated and ABS observed values indicate the improvement (especially for cereals where the negative efficiencies associated with the average lambda values indicate that the observed value average would be a better predictor than the model simulated results) in the model performance and demonstrate its predictive capability.

Table 3 Nash-Sutcliffe efficiencies (N-S Efficiency) of simulated and observed irrigated area, water use and gross values for selected crop (cereals and cotton)

	Irrigated area N-S Efficiency	Water Use Efficiency	GVIAP Efficiency

2006 lambda values			
Cereals	-23.68	-19.44	-5.91
Cotton	0.76	0.78	0.65
Average lambda values			
Cereals	0.33	0.58	0.91
Cotton	0.94	0.94	0.93

5. Conclusions and discussion

The PMP approach provides a neat way to calibrate programming models to observed behaviour and results in more realistic smooth aggregate supply response relative to a linear programming model. However, the PMP has come under scrutiny to reproduce robust and realistic responses when there is uncertainty in physical and economic conditions. We extended our previous PMP model which was calibrated against a single year land use observation and applied to southern part of the MDB.

In this paper we propose an innovative way of calibrating the model across multiple periods by using the data specific to individual years to capture information based on the PMP assumptions about individual landholder behaviour. The theoretical basis for PMP models suggests that alpha and gamma values are likely to vary from year to year, as they are in part dependent on the observed conditions in that particular year. Lambda in contrast can be viewed as a fixed variable in the short term (for example across the period for which the aggregate capital infrastructure can be viewed as fixed *and* for which no major structural change in industry conditions occurs). Hence, we use individual year point estimates of lambda to estimate an average lambda across for four years. We anticipate that the average lambda will dampen some of the individual year variations that may be caused by inadvertent capture of exogenous impacts on cropping, such as highly with variable hydro-climatic conditions .

The resultant multi-period PMP model generated considerably improved predictive capacities relative to a single period calibrated model. We argue that the multi-period calibrated approach is therefore likely to be more accurate when used to evaluate the impact of reduced water availability through interventions and/or climate change and to explore judicious water management/allocation options to minimize negative impacts on production. The approach will be useful in assessing climate change impacts (e.g. reduced crop effective rainfall and water allocations), farmers' adaptation options and/or water policies including water markets and trade and irrigation efficiency improvement. The approach can also be applied in other water basins in the world which face extreme variation in their rainfall and irrigation water allocations.

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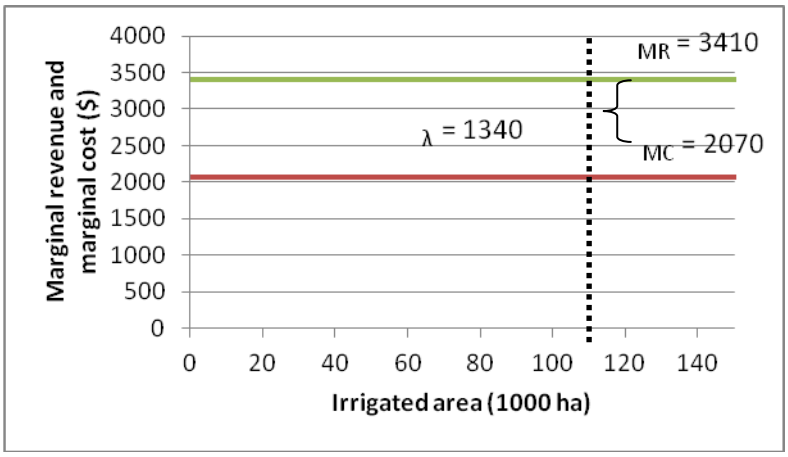
Appendix A

Here, a simple example of one region with one agricultural activity (rice) for demonstration purpose only is presented. The observed irrigated area (A) of rice is 115000 ha. The yield of rice is 10 tonnes. The price of rice is $341 per tonne. Rice uses 9 ML per ha and the price of water is $230/ML.

Total revenue or TR	*= Price x Yield x Area*
Marginal revenue or MR	*= Price x Yield*
	= $341 x 10 = 3410
Total cost or TC	*=$230 x 9 x Area*
TC	*=2070 x Area*
Marginal cost or MC	*= $2070*
Profit (or π) $_{Area\ =115}$	*=($3410 – $2070)* $_{Area\ =115\ ha}$



Assuming a simple hypothetical (quadratic) TC function:

$$TC = \alpha x + \frac{1}{2}\gamma x^2$$

$$\lambda = 1340$$

$$= 3410 - 2070$$

$$\lambda = MR - AC \qquad but\ MR = MC$$

$$\lambda = MC - AC \qquad (1)$$

$$MC = \alpha + \gamma x$$

$AC = \alpha + \frac{1}{2}\gamma x \quad = 2070 \qquad (2)$

$Therefore\ (1)\ implies\ \lambda = \alpha + \gamma x - \alpha - \frac{1}{2}\gamma x$

$\lambda = \frac{1}{2}\gamma x$

$\gamma x = 2\lambda$

$\gamma = \frac{2\lambda}{x} = 2 * \frac{1340}{115} = 23.30$

$\gamma = 23.30$ (this is slope of the cost function)

$(2) \Rightarrow \quad \alpha + \frac{1}{2} * 23.30 * 115 = 2070$

$\alpha + 1340 = 2070$

$\alpha = 2070 - 1340 = 730$

$Intercept\ of\ the\ function = \alpha = 730$

$Slope\ of\ the\ function = \gamma = 23.30$

$Max\ \pi = TR - TC$

$= 3410x - \alpha x - \frac{1}{2}\gamma x^2$

$= 3410x - 730x - \frac{1}{2} * (23.30) * x^2$

$\pi = 2680x - \frac{1}{2}(23.30)x^2$

$\frac{\partial \pi}{\partial x} = 2680 - (23.30)x = 0$

$23.30x = 2680$

$x\ or\ Area\ of\ rice = 115$

Appendix B

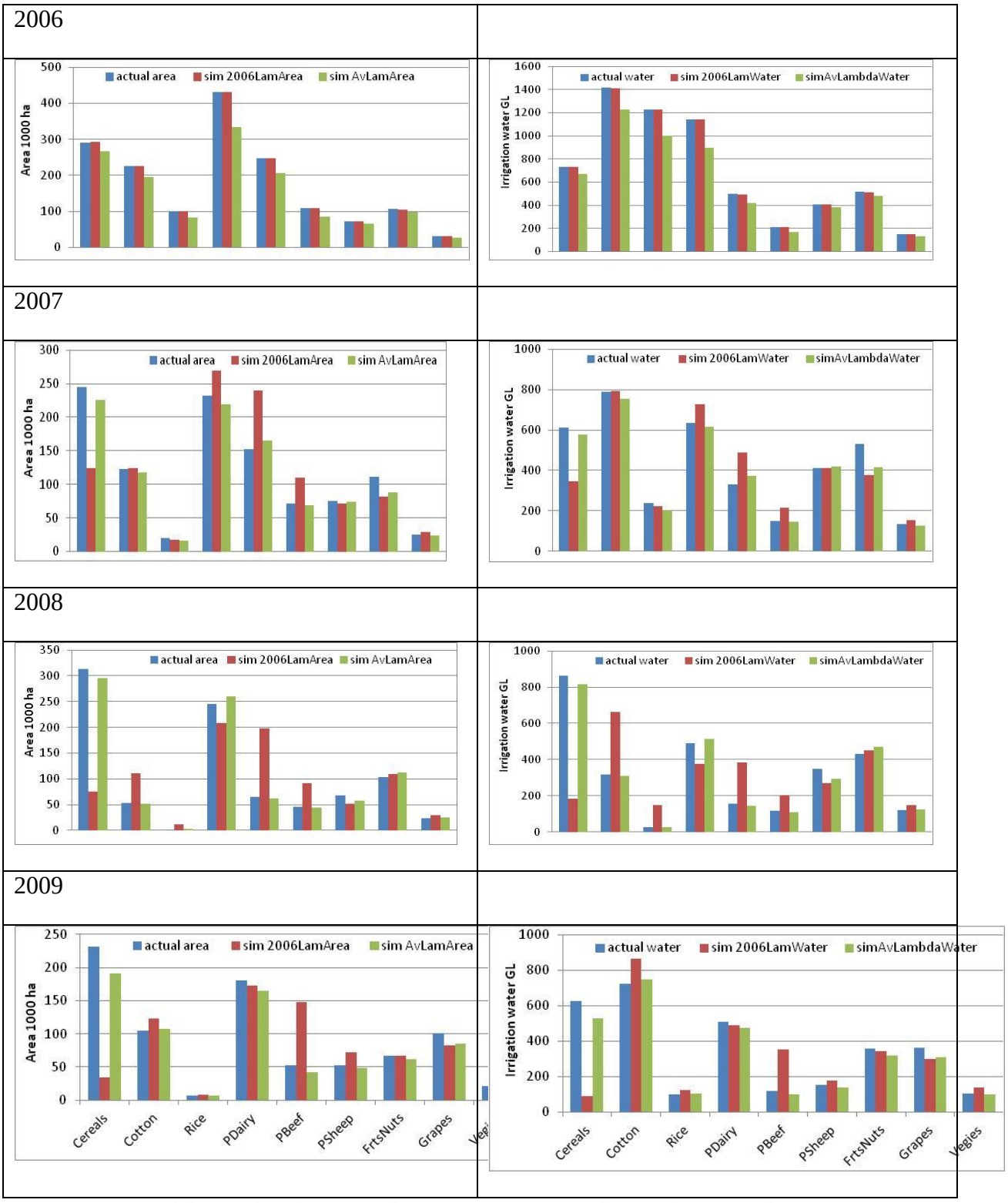


Figure B1 Model simulated area and water use versus observed area and water use using 2006 lambda values and temporal average lambda values

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