A Comparison of Traditional and Copula based VaR with Agricultural portfolio

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Abstract

Mean-Variance theory of portfolio construction is still regarded as the main building block of modern portfolio theory. However, many authors have suggested that the mean-variance criterion, conceived by Markowitz (1952), is not optimal for asset allocation, because the investor expected utility function is better proxied by a function that uses higher moments and because returns are distributed in a non-Normal way, being asymmetric and/or leptokurtic, so the mean-variance criterion cannot correctly proxy the expected utility with non-Normal returns. Copulas are a very useful tool to deal with non standard multivariate distribution. Value at Risk (VaR) and Conditional Value at Risk (CVaR) have emerged as a golden measure of risk in recent times. Though almost unutilized so far, as agriculture becomes more industrialized, there will be growing interest in these risk measures. In this paper, we apply a Gaussian copula and Student's t copula models to create a joint distribution of return of two (Farm Return and S&P 500 Index Return) and three (Farm Return, S&P 500 Index Return and US Treasury Bond Index) asset classes and finally use VaR measures to create the optimal portfolio. The resultant portfolio offers better hedges against losses.

Keywords: Portfolio Choice, Downside Risk Protection, Value at risk, Copula.

JEL Classification: C52, G11, Q14.

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1. Introduction:

Potential investors in the agricultural market have a range of entry points to consider, with many large institutions allocating capital to real assets such as agricultural land in an effort to provide shelter for capital and to underwrite the value of investment portfolios. Anecdotal evidence suggests that investments in the agricultural sector are most attractive to those seeking specific characteristics, for example long-term growth driven by basic fundamental trends such as population expansion and resource scarcity, and a hedge against the effect of inflation on the capital value of assets within a portfolio. It has been argued that real assets within the agricultural sector display characteristics that are attractive to investors seeking to align the performance of their portfolio with inflation, preserve capital during downturn markets, and generate income streams that are not wholly dependent upon the performance of traditional equity markets.

Depending on whether investing in an investment fund with an agricultural theme, acquiring and managing/leasing agricultural land, taking a position within a listed agricultural business, or actually owning and operating a farm business, the characteristics of agricultural investments vary from strategy to strategy and from market to market. For example, agricultural land is an asset-backed investment that offers capital preservation, income, and inflationary growth, whilst being relatively illiquid, whereas investing in a listed agricultural business allows the investor liquidity whilst sacrificing the low volatility associated with land investments. However, risk-return imbalances often characterize farm asset returns. As the competition for capital allocation grows within alternative modes of investment, effective investment and portfolio planning methods for investors, whether farmers or outside investors, are needed to improve the risk-return efficiency of farm investments.

This study analyzed the composition of agricultural investment portfolios. The basis for portfolio optimization in agricultural finance is not a new research area, and a vast range of literature has emanated from various applications of the standard Markowitz model. Our study makes three novel contributions:

1) It compares the results of optimum portfolio allocation utilizing two modern pillars of portfolio optimization theory: a Power-Log utility function to account for downside risk protection and, in the role of a benchmark, the classic Markowitz Mean-Variance optimization. The Power-Log utility function combines long-term portfolio growth maximization with the behavioral implications of prospect theory, treating portfolio losses and gains asymmetrically. Maximizing long-term portfolio growth can be taken as a key objective for agricultural investors. Therefore the first part of the study derived allocation of optimum portfolio using the Power-Log utility function with a broad range of downside protections and compared the results with the equivalent portfolios obtained using Markowitz portfolio theory. Comparisons were based on asset weights and risk-adjusted returns. The Sharpe Ratio, which compares risk-adjusted ratios, was used to make a final comparison of the portfolios.

2) It introduces the concept of Value at Risk (VaR) in portfolio context and compares VaRs obtained by three classical methods (historical simulation, Monte Carlo simulation, and variance-covariance approach). While VaR has recently been criticized for several

bounding assumptions (normal return of the portfolio, sub-additivity, etc.), it is regarded as the gold standard of risk management in modern finance. It gained supreme status as an effective risk management tool after the collapse of Barings Bank (1995) and some other financial institutions. For a given portfolio, probability, and time horizon, VaR is used as a threshold value such that the probability that the mark-to-market loss on the portfolio over the given time horizon exceeds this value (assuming normal markets and no trading in the portfolio) is the given probability level. Application of the VaR in the agricultural portfolio context has been limited. One notable agricultural application of VaR was done by Katchova and Barry (2005) on a portfolio of Illinois farms where CreditMetrics and KMV models of credit quality used to estimate default VaRs.

3) It introduces another important instrument in modern finance in terms of Copula Theory, a way of formalizing dependence structures of random vectors. Although copulas have existed for a long time, they were rediscovered relatively recently in applied sciences (biostatistics, reliability, biology, etc.). In finance, they have become something of a standard tool with broad applications: multi-asset pricing (especially complex credit derivatives), credit portfolio modeling, risk management, etc. To our knowledge, copulas have not been used previously in construction of agricultural investment portfolios, despite the fact that they offer several advantages in obtaining multivariate distributions rather than being dependent on draws from more constrained marginal probability distributions. After testing for appropriateness in relation to return structures within our data set, we utilized two basic copula models, Gaussian and student-t, which treat a normal and a student-t distribution, respectively, as the joint distribution of returns from multi-asset portfolios. The results obtained using these were then compared across the optimum portfolios constructed in the first part of the study.

Background: Rise of VaR as Risk Measure:

Value-at-risk (VaR) measures are used to estimate the probability of a portfolio of assets loses more than a specified time period due to adverse movements in the underlying market factors of a portfolio. Recent explosion of interest in value-at-risk stems from its use in risk disclosure and risk reporting. The history of VaR can be traced back to as early as 1995 when in the wake of several well publicized derivatives debacles, such as the Barings Bank failure in 1995, several regulatory bodies have recommended or mandated the reporting of VaR estimates by firms (e.g. large trading banks) that maintain large derivatives positions in order to provide a clear, forward looking measure of a firm’s downside risk potential associated with derivatives positions. The recent trading loss of $2 billion by JP Morgan & Chase has again raised questions of risk management by big financial institutions.Most notably, in January of 1997 the Securities and Exchange Commission (SEC) established rules for the quantitative and qualitative reporting of risks associated with highly market sensitive assets (i.e. derivatives positions) of reporting firms. Value-at-risk was one of only 3 quantitative risk reporting methods approved for use in SEC disclosures. Similarly, futures exchanges use VaR to measure the probability of default by clearing members. Due to VaR’s emphasis on downside risk, it is considered by many to be a more intuitive measure of risk and more easily understood by top level managers and outside investors who may or may not be well trained in statistical methods. As a result of the interest in value-at-risk, an entire industry has evolved devoted to the implementation and use of VaR. Much of this can be attributed to JP Morgan's publications of their Risk Metrics system for developing Value-at-Risk measures. By doing this, JP Morgan has attempted to position their estimation methodology as the industry standard for computing VaR. VaR has also been suggested for firm level risk management. VaR could be beneficial in making hedging decisions, managing cash flows, setting position limits and overall portfolio selection and allocation.

Though VaR has been used extensively in financial risk management, however, there are some criticisms to VaR. The major drawback of using VaR is that it is not coherent. VaR has been shown to be not sub-additive which means the VaR of a portfolio of two securities may be greater than the VaR of each individual security (see Artzner et al 1999; Dowd 2005). VaR has also shown to estimate erroneous results when the data is not normally distributed (Jorion, 2005). VaR has further been criticized as it led to excessive risk-taking and leverage at financial institutions, focused on the manageable risks near the center of the distribution and ignored the tails (Artzner, 1999).

Copula methods

Copulas are a way of formalizing dependence structures of random vectors. Although they have been known about for a long time (Sklar, 1959), they have been rediscovered relatively recently in applied sciences (biostatistics, reliability, biology etc.). In finance, they have become a standard tool with broad applications: multi asset pricing (especially complex credit derivatives), credit portfolio modeling, risk management, etc. Some examples are Li (1999), Patton (2001) and Longin and Solnik (1995). The literature on copulas is growing very fast. They are a very general tool for describing dependence structures and have been successfully applied in many cases. Some vital facts about copulas to be kept in mind:

In general, it is quite difficult to estimate copulas from data. Of course, this stems in particular from the generality of copulas. In many multivariate distributions it is quite well understood how to estimate the dependence structure, so a sensible statistical estimation of a copula will entail consideration of a particular parameterization, i.e. stemming from a multivariate family and estimation of the parameters will typically be much easier. Some copulas stem from multivariate distributions, for example the Gaussian and the t-copula. Based on the fact that copula marginals and the dependence structure can be disentangled, the dependence structure can be handled independent of the marginals. Obviously the Gaussian copula can be applied to t-marginals and vice versa. This results in new multivariate distributions with different behavior. This or similar procedures may often be seen in applications of copulas. However, the use of this procedure is questionable and the outcomes should be handled with care. On the other hand, various copulas which do not stem from multivariate distributions, such as Archimedean copulas, take their form mainly because of mathematical tractability. Therefore their applicability as natural models for dependence should be verified in each case. Finally, copulas apply to a static concept of dependence, while many applications, especially in finance, are concerned with time series and therefore a dynamic concept of dependence is needed. Despite this critique, it is remarkable that the use of copulas has greatly improved the modeling of dependencies in practice. For example, in contrast to linear correlation, the use of copulas avoids typical pitfalls and therefore leads to a mathematically consistent modeling of dependence.

2. Portfolio Optimization with Downside Protection

Kale (2006) first proposed the use of a power-log utility function to derive optimum portfolio weights in the event of downside loss. This framework had been further utilized by Lagerkvist (2007) to derive optimum portfolio weight of 3 asset based portfolio (farm asset, S&P Stock Index & US Treasury bills Index). This paper is built on the framework built by Lagerkvist (2007) with further inclusion of E-V Framework and Value at Risk measures for risk analysis.

Property of Power-Log Utility Functions:

Mean-variance framework for portfolio selection was developed by Markowitz in 1952. This is a one-period model and probably the most widely used technique till today for portfolio selection. However, there are other models too. Many authors have discussed the multi period portfolio theory which is based on log and power utility functions. Since both log and utility functions have the 'myopic' property, so the multiperiod portfolio optimization problem can be converted into a one period optimization problem (Kale, 2009). Prospect Theory derived from behavioral finance provides another perspective on investor decision. This was first proposed by Kahneman and Tversky (1979). The so called s-shaped function had three essential characteristics, namely reference dependence, loss aversion and diminishing sensitivity. Reference dependence refers to the fact that the carriers of value are gains and losses defined relative to a reference point. S-shaped function is steeper in the negative region than the positive region signifying that losses are more accounted for than the gains. Also the marginal value of both gains and losses decreases with their size. All these properties give rise to the S-shaped utility function which is concave above the reference point and convex below it. Kale (2006) developed the power-log utility function taking clues from the prospect theory to represent the normative behavior of investors. The main characteristics are as follows:

i. Power-log utility function combines some of the properties of the S-shaped utility function with multiperiod portfolio theory. It balances growth maximization with downside protection.
ii. The log utility function is used when the goal is to maximize portfolio growth over time and this is given by: $U = \ln(1+r) = \frac{1}{\gamma}(1+r)^{\gamma}$

where *r* is the portfolio return and ln is the natural log function.

The expected utility criterion leads to the one-period optimization problem: Maximize $E(U) = \sum p_s U_s$ where s is the scenario s and the summation is over all scenarios, $p_s$is the probability of scenario s and Us is the utility in scenario s based on the portfolio return in the scenario r_s where the utility function in equation 1. The portfolio return in scenario R_s is calculated as a weighted average of the returns to the assets in the portfolio,

$$R_s = \sum w_i \ r_{is}$$

where i is a given asset and the summation is over all assets in the portfolio, w_i is the investment weight of asset *i* in the portfolio and r_{is} is the return to asset *i* in scenario s.

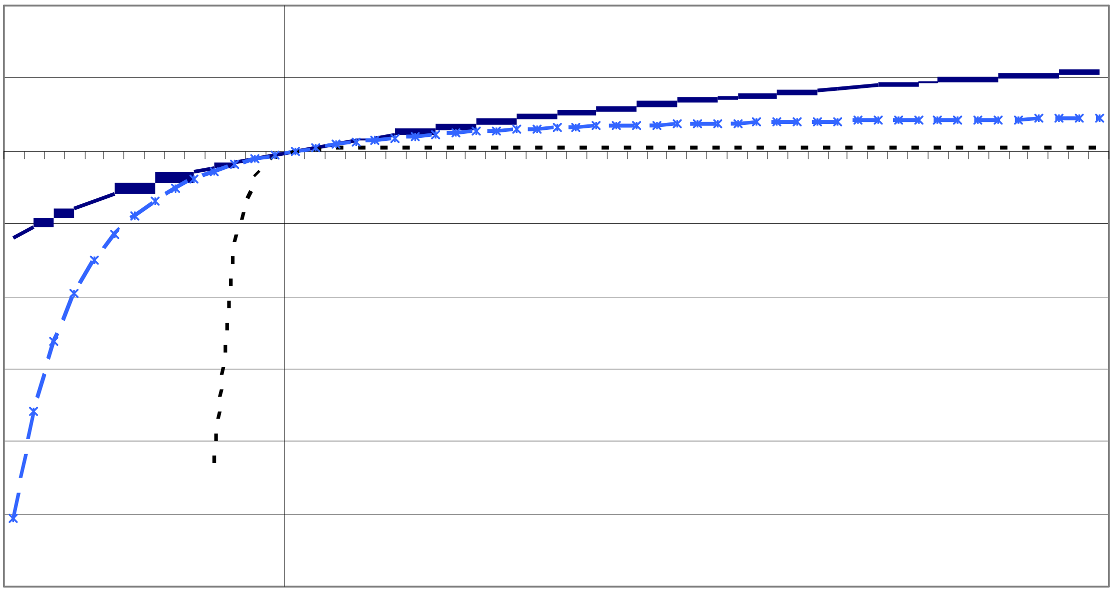


Figure 1: Log and power utility functions. Power utility (Source: Lagerkvist, 2007)

Power Utility function is

i. When the return is zero, these utility functions have a value of zero.
ii. For gains or positive returns, the utility is positive and for losses or negative return the utility is negative.
iii. The power utility function with gamma equals to zero is the growth portfolio and gamma equals to one is the risk neutral portfolio.
iv. All power utility functions with power less than 1 are risk averse utility functions including the log utility function.
v. With the downside power increasing the slope become steeper. So the penalty for losses increases more quickly than it does for the log utility function as the size of the losses increases. Thus it provides more downside protection than log utility function. This is a desirable property to represent an investor whose aversion to losses is greater than the aversion to losses as represented by the log utility function.

a. Characteristics of Power-log utility functions are:

i. Characterized by utility that increases as returns increase and marginal utility that decreases as returns increase.
ii. Gains and losses are valued with different utility functions when the downside power is less than zero.
iii. These utility functions allow investors who are more loss averse to increase the penalty for losses while leaving the utility of gains unchanged.
iv. Steeper in negative region than in the positive region, conforming to Kahneman and Tversky postulates of reference dependence and loss aversion.
v. Also represent characteristics of <u>diminishing sensitivity for gains</u>, since the marginal utility of the log function decreases as the size of the gain increases, represented by a concave function.

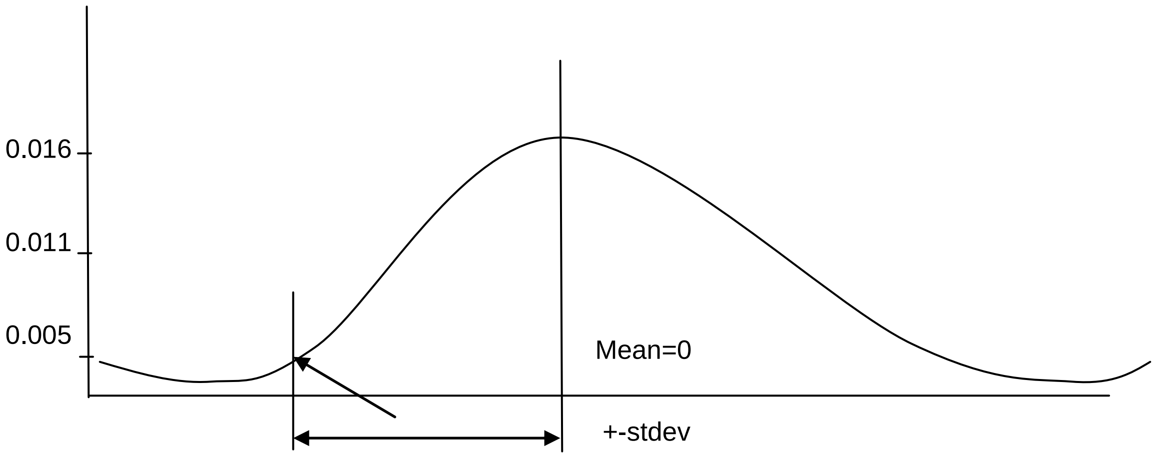
vi. However, they do not conform to Kahneman and Tversky postulate of diminishing sensitivity for losses. Rather, they represent an increasing sensitivity of losses as the size of losses increase, which represents risk averse behavior that is represented by a concave function for losses.

vii. Experiments in support of prospect theory are designed as gambles where only a small amount of wealth is at stake. Other studies of investor behavior tend to be for very short periods of time where holding period is only a day or less.

viii. These utility functions are continuously differentiable in the entire region, they do not have a kink at a return of zero. Slopes of both log and power utility functions for return zero are the same.

3. On VaR Methodology:

Value at Risk (VaR) has become the standard measure that financial analysts use to quantify this risk. VaR represents maximum potential loss in the value of a portfolio of financial instruments with a given probability over a certain horizon. In other words, it is a number that indicates how much an investor can lose with probability 'X' over a given time horizon. Value at risk is usually defined for some given confidence level $\alpha \in (0,1)$, and the VaR of a given portfolio is then given by the smallest number l such that the probability that the loss L exceeds l is no longer than (1-α) (McNeil, 2005). Hence: $VaR_\alpha = \inf\{l \in \Re : P(L > l) \le 1 - \alpha\} = \inf\{l \in \Re : F_L(l) \le \alpha\}$.



Approx Normal Curve representing Value at Risk

Figure 2: Illustration of Value at Risk (Source: Jorion, 2005)

Methods to Calculate VaR:

Currently, there are three ways to compute VaR of a portfolio:

A. Historical Simulation

The Historical simulation method is a popular method of estimating VaR. It involves using past data in a very direct way as a guide to what might happen in the future. We apply the current weights to the historical asset returns by going back in time such as over the last 100 days. The current portfolio weights are computed using standard mathematical optimization.

$$R_p = \sum_{i=1}^{N} h_i R_i$$

A distribution of portfolio returns is obtained. These portfolio returns are then sorted and depending on the target probability the corresponding quantile of the distribution is taken. This gives us the 1-day VaR using Historical Simulation method. Hypothetical portfolios can also be generated using the current portfolio weights and the historical asset returns.

Historical simulation method is relatively simple to implement if the past data is readily available for estimating Value-at-Risk. Historical simulation method allows non linearities and non normal distribution by relying on the actual prices. It does not rely on underlying stochastic structure of the market or any specific assumptions about valuation models. Historical simulation method does not rely on valuation models and is not subjected to the risk that the models are wrong. The Historical Simulation method assumes the availability of sufficient historical price data. This is a drawback because some of the assets may have a short history or in some cases no history at all. There is also an assumption that the past represents the immediate future which is not always true. The Historical Simulation method quickly becomes cumbersome for large portfolios with complicated structures.

B. Monte Carlo Based Simulation

The Monte Carlo simulation method can be briefly summarized in two steps. In the first step, a stochastic process is specified for the financial variables. In the second step, fictitious price paths are simulated for all financial variables of interest. Each of these "pseudo" realizations is then used to compile a distribution of returns from which a Value-at-Risk (VaR) figure can be measured.

The Monte Carlo method can incorporate nonlinear positions, non normal distributions, implied parameters, and even user-defined scenarios. As the price of computing power continues to fall, this method is bound to take on increasing importance. The biggest disadvantage of the Monte Carlo method is its computational time. If 1000 sample paths are generated with a portfolio of 1000 assets, the total number of valuations amounts to 1 million. In addition, if the valuation of assets on the target date involves itself a simulation, the method requires "simulation within a simulation." Therefore, computer and data requirements are much higher than that required by the other approaches. The method is the most expensive to implement in terms of systems infrastructure. Another potential weakness of the Monte Carlo method is that it is subject to the risk that the models are wrong. The Monte Carlo method relies on specific stochastic processes for the underlying risk factors as well as the pricing models for securities such as options or mortgages. Simulation results should be complemented with some sensitivity analysis to check if the results are robust to changes in the model.

C. Variance-Covariance Approach

In this approach, it is assumed that the underlying market risk factor has a normal distribution. Under this assumption value of risk of the portfolio is simply given by:

- VaR (daily VaR) (X%) = $Z_{X\%} * \sigma$
    - VaR(X %): X% Probability VaR
    - $Z_{X\%:}$ Critical z-value for X probability
    - σ : standard deviation (volatility) of the asset (or portfolio) of daily returns

- Portfolio VaR
- $\sigma_{portfolio} = \surd\ w_a^{2}*\sigma_a^{2} + w_b^{2}*\sigma_b^{2} + 2*w_a*w_b* \sigma_a* \sigma_b* \sigma_{ab}$;
- $VaR_{portfolio}$ (daily VaR) (in %) = $\surd\ w_a^{2} *(\%VaR_a)^2 + w_b^{2} *(\%VaR_b)^2 + 2*w_a*w_b* (\%VaR_a)* (\%VaR_b)* \sigma_{ab}$

The var-covar method is easy to implement because it involves a simple matrix multiplication. It is also computationally fast, even with a large number of assets, because it replaces each position by its linear exposure. Portfolios that are linear combinations of normally distributed risk factors are themselves normally distributed. It only requires the market values and exposures of current positions, combined with risk data. Also, in many situations, this method provides adequate measurement of market risks. As a parametric approach, VaR is easily amenable to analysis, since measures of marginal and incremental risk are a by-product of the VaR computation. This method is important not only for its own sake but also because it illustrates the “mapping” principle in risk management. Another problem is that the method inadequately measures the risk of nonlinear instruments, such as options or mortgages. Under the var-covar method, options positions are represented by their “deltas” relative to the underlying asset. Asymmetry in the distribution of options is not captured by the delta-normal VaR.

4. Copulas

Dependence between random variables can be modeled by copulas. A copula returns the joint probability of events as a function of the marginal probabilities of each event. This makes copulas attractive, as the univariate marginal behavior of random variables can be modeled separately from their dependence. For a random vector X of size n with marginal cumulative density functions F_i, the copula with cdf C(.) gives the cumulative probability for the event x: $P(X \le x) = C(F_1(x_1),.....;F_n(x_n))$.

A copula is a multivariate cumulative distribution function defined on the n-dimensional unit cube [0,1] n with the following properties:

1. The range of C (u1, u2,….un) is the unit interval [0,1]
2. $\left(u_1, u_2, .. u_N\right) = 0$ if any $u_i = 0$ for $i = 1, 2 \ldots n$.
3. C(1,…1,u1,1,….,1)=ui, for all ui Є [0,1]

Going by the definition above, an infinite number of copulas can be generated. However, there is one particular family of copulas that has been used extensively in the field of risk management is the Archimedean copula (Hennessy and Lapan, 2002). The Archimedean copula has been used extensively for these applications because of the relative ease of calculating the copula.

Sklar's Theorem

Although the application of copulas to statistical problems is relatively recent, the theory behind copulas was developed in 1959 (Sklar 1959). Sklar's Theorem states (Nelson, 2006):

Let H be a joint distribution function with marginal distributions of F and G. Then there exists a copula C such that for all x, y in $\overline{R}$, H(x,y)=C(F(x),G(y)). If F and G are continuous, then the copula function C is unique. If F and G are not continuous, then C is uniquely determined on RanF*RanG. In addition, if C is a copula and F and G are distribution functions, then the function H is a joint distribution function with marginal distributions F and G.

Gaussian Copula

Gaussian Copula is basically an extension of the multivariate normal distribution. The convenience of the Gaussian copula is that it can be used to model multivariate data that may exhibit non-normal dependencies and fat tails. The Gaussian Copula is formally defined as:

$C(u_1, ... u_n; \Sigma) = \Psi^K(\Psi^{-1}(u_1), ..., \Psi^{-1}(u_n); \Sigma)$.....(1) The copula function $C(u_n)$ is defined by the standard multivariate normal distribution (Φ^K) and the linear correlation matrix (Σ). When n=2, equation 1 can be rewritten in the as:

$$C(u_{1,} u_2) = \int_{-\infty}^{\Phi^{-1}(u)} \int_{-\infty}^{\Phi^{-1}(u)} \frac{1}{2\pi\sqrt{1-\rho^2}} \exp\left\{ \frac{-(r^2 - 2\rho rs + s^2}{2(1-\rho^2)} \right\} drds$$

Where ρ is the linear correlation between the two variables.

The copula density function is now derived in the following manner:

$$c(\Psi(x_1), ..., \Psi(x_n)) = \frac{f^{Gaussian}(x_1, ..., x_n)}{\prod_{i=1}^{n} f^{Gaussian}(x_i)}$$

This can be written down using the definitions of the Gaussian functions:

$$= \frac{\frac{1}{(2\pi)^{n/2} |\Sigma|^{1/2}} \exp(-\frac{1}{2} x' \Sigma^{-1} x)}{\prod_{I=1}^{n} \frac{1}{\sqrt{2\pi}} \exp(-\frac{1}{2} x_i^2)}.$$

Student's t Copula:

This is closely related to the Gaussian copula with cdf: $C_n^{\psi}(u;\Omega^{\psi},v) = \psi_n(\psi^{-1}(u_1;v),.....,\psi^{-1}(u_n;v);\Omega^{\psi},v)$, where ψ_n denotes the cdf of an n-variate Student's t distribution with correlation matrix Ω^{ψ} and degrees of freedom parameters v>2 and ψ^{-1} is the inverse of the cdf for the univariate Student's t distribution with mean zero, dispersion parameter equal to one and degrees of freedom v. The Gaussian and Student's t copula belong to the class of elliptic copulas. A higher value for v decreases the probability of tail events. As the Student's t copula converges to the Gaussian copula for v $\rightarrow \infty$, the Student's t copula assigns more probability to tail events than the Gaussian copula. Moreover, the Student's t copula exhibits tail dependence (even if correlation coefficients equal to zero).

Goodness of fit measures:

To determine which copula fits the best, there are a couple of measures one can use:

1. Maximum Log-Likelihood
2. Distance based tests like Anderson-Darling and Kolmogorov-Smirnoff Tests.
3. BIC/AIC Criteria.

Once the dependence between the returns has been estimated, a joint distribution function can be estimated.

5. Data:

Three distinct asset classes are considered in this study: Farm Return, S&P 500 Index and US Treasury Index. Farm data are from the Southwestern Minnesota Farm Business Management Association records. This data is online which can be downloaded through FINBIN at http://www.finbin.umn.edu/. The time series initially collected covers the period 1993 through 2011 and includes approximately 250 farm operations only sole proprietors are included. We have used both upper and lower 20% of Return on Assets (ROA) with agricultural farms where crop was the main output. Common Stock returns are based upon the Standard and Poor's Composite Index (S&P 500). The bond return includes coupon payments (interest) and appreciation and is based upon the 10 year to maturity U.S. Government Index. They were obtained from Federal Bank of St. Louis website at http://research.stlouisfed.org/fred2/.

6. Results & Interpretation

To implement the model as discussed above model, few simplifying assumptions are made. First, it is assumed that investor does not face any transaction costs, so there are no costs for rebalancing or for short selling. Also, it is assumed that the investor/ asset manager has to invest an amount of money at the beginning of the period (that, for simplicity is taken equal to 1) and does not receive or disinvest anything until the end. Portfolio weights have been chosen using both maximizing power-log utility functions and using mean-variance (Markowitz portfolio selection) approaches. Further, two more cases have been elaborated where in the first there has no constraint was imposed on the farm asset weight and in the later the farm asset was asked to be more than 70% (representing a farmer's choice). Details are given in tables 3A and 3B.

Table 1: Summary Statistics

Statistic	Farm Return	Stock Return	Bond Return
Mean	0.006521	0.15877	0.05967
Minimum	-0.0100	-0.1188	0.3757
Maximum	0.1600	0.3757	0.0992
Std.Dev	0.04471	0.1501	0.0190

Asset Correlations

Data Range: Farm return, S&P 500 Index and US Govt. Treasury Bills Index from 1983 to 2005.

Table 2: Pearson's Correlation

	Farm Return	S&P 500 Index	US Govt. T-Bills Return
Farm Return	1	-0.15939	-0.06632
S&P 500 Index	-0.15939	1	0.18110
US Govt. T-Bills Return	-0.06632	0.18110	1

Table 2 reveals that Farm return is negatively correlated with either or both the stock and bond returns. Hence, the only two copulas we can use are the Gaussian/ normal and student's t copula.

Optimization Scheme to calculate optimum portfolio weights:

An optimization scheme was outlined to calculate the optimum portfolio weights comprising of farm asset, S&P Stock Index and Treasury bond indices:

1. For each downside protection level, calculate the optimum portfolio weight.
2. Calculate the rate of return using this optimum portfolio weights obtained.
3. Plug-in this rate of return as the expected return in the mean-variance (E-V) framework.
4. Subsequently, find out the corresponding optimum portfolio weights in the E-V framework.
5. Compare the portfolio weights obtained in step 1 and 4.
6. Calculate another set of optimum portfolio weights adding one more constraint in the optimization scheme of 1 with farm asset now being greater or equal to 70% in

weight. This step was performed to take care of a farmer's intention of retaining his/her farming business.

7. Calculate the corresponding optimum portfolio weights using the E-V framework.
8. Finally, compare the weights obtained in steps 6 and 7.

Table 3A: Portfolio Weights using Power-Log Utility Functions and Mean-Variance Approach under Normal Scenario

		Power-Log Utility			Mean-Variance		
Downside Power	Exp Return	Farm Weight	Bond Weight	S&P Weight	Farm Weight	S&P Weight	Bond Weight
0	13,98	0,1	0,1	0,8	0,211	0,789	0
-0,5	13,98	0,1	0,1	0,8	0,211	0,789	0
-2	13,98	0,1	0,1	0,8	0,211	0,789	0
-4	2,88	0,1	0,1	0,8	0,104	0,796	0,1
-6	4,37	0,219	0,1	0,681	0,223	0,677	0,1
-8	5,98	0,347	0,1	0,553	0,35	0,55	0,1
-10	8,53	0,315	0,233	0,452	0,459	0,441	0,1
-12	9,75	0,282	0,337	0,381	0,423	0,37	0,207
-14	10,56	0,258	0,413	0,329	0,383	0,319	0,298
-16	11,18	0,241	0,47	0,289	0,352	0,28	0,368
-18	11,66	0,227	0,516	0,257	0,328	0,249	0,423
-20	8,42	0,216	0,552	0,232	0,308	0,225	0,467
-25	12,74	0,197	0,617	0,186	0,273	0,18	0,548
-30	13,2	0,186	0,659	0,155	0,249	0,149	0,602
-35	13,52	0,179	0,689	0,132	0,232	0,128	0,641
-40	13,75	0,175	0,71	0,115	0,219	0,111	0,67
-45	13,81	0,172	0,725	0,103	0,19	0,1	0,71
-50	13,62	0,169	0,731	0,1	0,147	0,1	0,753

From Table 3A it is interesting to see that the farm weight initially rises with increasing downside protection in the range of -6 to -20; however; it starts to fall with higher ranges between -25 and higher. It might be intuitive to conclude that a downside protection range of higher than -20 would not fit the profile of a rational investor.

Table 3 B: Portfolio Weights using Power-Log Utility Functions and Mean-Variance Approach when Farm Asset is > 70%

		Power Utility Functions			Mean-Variance		
Downside Power	Exp. Return	Farm Weight	S&P_Weight	Bond_Weight	Farm Weight	S & P weight	BondWeight
0	8,399	0,7	0,2	0,1	0,7	0,198	0,102
-0,5	8,399	0,7	0,2	0,1	0,7	0,198	0,102
-2	8,399	0,7	0,2	0,1	0,7	0,198	0,102
-4	8,399	0,7	0,2	0,1	0,7	0,198	0,102
-6	8,399	0,7	0,2	0,1	0,7	0,198	0,102
-8	8,399	0,7	0,2	0,1	0,7	0,198	0,102
-10	8,399	0,7	0,2	0,1	0,7	0,198	0,102
-12	8,399	0,7	0,2	0,1	0,7	0,198	0,102
-14	8,399	0,7	0,2	0,1	0,7	0,198	0,102
-16	8,399	0,7	0,2	0,1	0,7	0,198	0,102
-18	8,399	0,7	0,2	0,1	0,7	0,198	0,102
-20	8,399	0,7	0,2	0,1	0,7	0,198	0,102
-25	8,225	0,7	0,182	0,118	0,7	0,181	0,119
-30	7,915	0,7	0,151	0,149	0,7	0,15	0,15
-35	7,691	0,7	0,129	0,171	0,7	0,127	0,173
-40	7,52	0,7	0,111	0,189	0,7	0,11	0,19
-45	7,408	0,7	0,1	0,2	0,7	0,1	0,2
-50	7,408	0,7	0,1	0,2	0,7	0,1	0,2

From Table 3B, the portfolio weights obtained in both Power-Log and E-V framework are almost same. No further intuition gained on asset performance using E-V framework.

Computation of Value at Risk or VaR:

VaR at 95% confidence interval had been calculated for the above mentioned 18 portfolios using 5 different methodologies as described above; i.e., Var-Covar or delta normal VaR, VaR by historical simulation, VaR by Monte Carlo Simulation and finally VaR by using two copulas (Gaussian and student's t).

Table 4A: Traditional Portfolio VaRs at 95% C.I. of 18 Portfolios of Table 3A as constructed using Power-Log Utility Function under Normal Scenario

Downside Power	Exp. Return	Hist. Sim	Mont. Car	Var-Covar
0	13,98	0,019	0,028	0,024
-0,5	13,98	0,019	0,028	0,024
-2	13,98	0,019	0,028	0,024
-4	13,98	0,019	0,028	0,198
-6	12,88	0,022	0,027	0,168
-8	11,68	0,032	0,035	0,137
-10	10,67	0,051	0,014	0,113
-12	9,94	0,057	0,014	0,096
-14	9,41	0,053	0,015	0,084
-16	9	0,055	0,020	0,075
-18	8,68	0,054	0,024	0,069
-20	8,42	0,052	0,026	0,063
-25	7,95	0,050	0,028	0,053
-30	7,64	0,048	0,031	0,047
-35	7,41	0,047	0,033	0,043
-40	7,24	0,047	0,034	0,040
-45	7,11	0,049	0,034	0,038
-50	7,08	0,049	0,034	0,038

Table 4B:

Traditional Portfolio VaRs at 95% C.I. of 18 Portfolios of Table 3A as constructed using Mean-Variance Optimization under Normal Scenario

Downside Power	Exp. Return	Hist. Sim	Mont. Car	Var-Covar
0	13,98	0,059	0,062	0,194
-0,5	13,98	0,059	0,062	0,194
-2	13,98	0,059	0,062	0,194
-4	13,98	0,076	0,277	0,197
-6	12,88	0,079	0,179	0,167
-8	11,68	0,078	0,087	0,136
-10	10,67	0,078	0,149	0,111
-12	9,94	0,090	0,097	0,095
-14	9,41	0,109	0,127	0,083
-16	9	0,117	0,121	0,074
-18	8,68	0,124	0,134	0,067
-20	8,42	0,129	0,167	0,062
-25	7,95	0,139	0,178	0,052
-30	7,64	0,136	0,161	0,046
-35	7,41	0,154	0,173	0,042
-40	7,24	0,158	0,179	0,040
-45	7,11	0,164	0,211	0,038
-50	7,08	0,020	0,056	0,038

Table 4C:

Traditional Portfolio VaRs at 95% C.I. of 18 Portfolios of Table 3B as constructed using Power-Log Utility Function when Farm Asset >=70%

Downside Power	Exp. Return	Hist. Sim	Mont. Car	Var-Covar
0	13,98	0,045	0,087	0,073
-0,5	13,98	0,045	0,087	0,073
-2	13,98	0,045	0,087	0,073
-4	13,98	0,045	0,087	0,073
-6	12,88	0,045	0,087	0,073
-8	11,68	0,045	0,087	0,073
-10	10,67	0,045	0,087	0,073
-12	9,94	0,045	0,087	0,073
-14	9,41	0,045	0,087	0,073
-16	9	0,045	0,087	0,073
-18	8,68	0,045	0,087	0,073
-20	8,42	0,045	0,087	0,073
-25	7,95	0,048	0,069	0,071
-30	7,64	0,052	0,071	0,067
-35	7,41	0,047	0,082	0,065
-40	7,24	0,043	0,075	0,064
-45	7,11	0,040	0,071	0,063
-50	7,08	0,040	0,082	0,063

Table 4D:

Traditional Portfolio VaRs at 95% C.I. of 18 Portfolios of Table 3B as constructed using Mean-Variance Optimization when Farm Asset >=70%

Downside Power	Exp. Return	Hist. Sim	Mont. Car	Var-Covar
0	13,98	0,038	0,085	0,073
-0,5	13,98	0,038	0,085	0,073
-2	13,98	0,038	0,085	0,073
-4	13,98	0,038	0,085	0,073
-6	12,88	0,038	0,085	0,073
-8	11,68	0,038	0,085	0,073
-10	10,67	0,038	0,085	0,073
-12	9,94	0,038	0,085	0,073
-14	9,41	0,038	0,085	0,073
-16	9	0,038	0,085	0,073
-18	8,68	0,038	0,085	0,073
-20	8,42	0,038	0,085	0,073
-25	7,95	0,048	0,072	0,070
-30	7,64	0,052	0,086	0,067
-35	7,41	0,047	0,076	0,065
-40	7,24	0,042	0,071	0,064
-45	7,11	0,040	0,071	0,063
-50	7,08	0,040	0,071	0,063

Simulation Steps for Copula Based VaR:

1. Collect the historical data.
2. Compute relative changes.
3. Choose marginal distribution functions.
4. Estimate parameters of marginal distributions.
5. Choose joint distribution function.
6. Estimate parameter(s) of joint distribution function.
7. Compute Value at Risk of a given quintile (95% here)

Table 5: Goodness of Fit measure between Gaussian and Student's t-copula

Copula	Log likelihood(*ll*)	BIC	K-S
Gaussian	1622.1	-3061.7	0.77
Student's t	1912.4	-3205.1	0.68

From Tables 4A-D, it can be seen that Student's t copula gives a better fit as it gives the highest log-likelihood function and the lowest Bayesian Information Criterion (BIC) and lowest K-S (Kolmogorov-Smirnov) test Statistic.

Table 6: Copula Based VaR

Downside Power	Exp. Return	Gaussian Copula	Student's-t Copula
0	13,98	0,021	0,019
-0,5	13,98	0,021	0,019
-2	13,98	0,021	0,019
-4	13,98	0,179	0,164
-6	12,88	0,153	0,142
-8	11,68	0,149	0,137
-10	10,67	0,125	0,123

Interpretation of the Result

Tables 3A represent respectively the optimum portfolios obtained through maximizing power-log utility function with a downside risk protection ranging from 0 to a power of -50. Table 3B repeats the same optimization scheme with an added constraint of farm asset >= 70% of weight of the portfolio. Results show that farm asset is not competitive unless the farmer becomes quite risk averse. For a downside protection of range between -6 to -20, it is modestly competitive compared to other financial assets such as stocks or bonds. Table 4A represent the value at risk at 95% confidence interval for 18 portfolios as of table 3A which were constructed through maximizing Power-Log Utility function. Table 4B represent the same VaR amount for the 18 portfolios which were constructed using Mean-Variance optimization. Tables 4C & 4D represent the VaR amounts of portfolios constructed through two optimization schemes with added constraint of farm asset >=70% in weight. Here portfolios with either low downside risk (from range 0 to 4) and with high downside risk (over 20) perform better with low amount of VaR. This result indicate that adopting VaR as the standard risk measure, portfolios will perform better against possible losses either at low or high end of downside risk protection powers. Table 5 represent the goodness of fit measures for using either a Gaussian or student's t-distribution as the joint distribution prior to estimate the copula VaR. As per the statistical evidence of distance based tests, student's t is a better fit for our data as the joint distribution of the asset returns. Table 5 represent the VaR amounts again at 95% confidence interval. The amounts are comparable to the values obtained earlier.

8. Conclusion

The findings of this study can be used to diversify an agricultural investment portfolio during financial turmoil such as the recent global economic crisis, where investors found agricultural commodities an ideal mode of diversification due to the slight or negative correlation with other classes. To our knowledge, this is the first attempt to rigorously investigate the nature of farm asset return and risk-return efficiency utilizing several tools from modern finance theory. The results may be of benefit to a range of practitioners, from agricultural investors to academia. The results obtained so far highlight the lack of competitiveness of farm asset when conjugated with other financial assets such as stock or bonds. The optimum portfolios obtained either through Power-Log Utility Maximization or Mean-Variance framework do not carry more than 20% of farm asset in a broad downside protection range of 0 to -50. Moreover, when we apply VaR as the standard risk measurement tool, portfolios constructed at the low or high downside risks become more attractive with lower VaRs compared to portfolios in the mid-range of downside risk of -6 to -20 where farm asset picked up a little bit of more weight compared to other two assets of stocks & bonds. Use of copula based VaR provides a refinement to the traditional VaR methodology, however, results provide the same behavior for farm asset.

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