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The Relationship Between the Economic Surplus and Production Function Approaches for Estimating Ex-Post Returns to Agricultural Research

Jeff Davis*

The objective of this paper is to develop the conceptual linkage between the production function and economic surplus approaches used to evaluate the returns to agricultural research. First it is established that, for the Cobb-Douglas functional form generally used, an increase in research expenditure will result in a pivotal divergent shift in the supply function. It is then shown that the measure of this change in economic surplus is approximately the same as the value marginal product. Finally it is concluded that if the other possible types of supply shifts, that is, proportional divergent, parallel or convergent, are appropriate, then the production function approach will under-estimate the benefits and therefore rate of return to research expenditure.

1 Introduction

The economic surplus (or index number) approach and the production function (or marginal product) approach are the two main *ex post* procedures that have been used to evaluate agricultural research. While applications of these two approaches have been extensive, there has been little attempt to discuss the conceptual relationship between them.[1] The only published discussion has been by Peterson (1971, pp. 149-153), who emphasized the average versus marginal rate of return difference which may exist.

The objective of this paper is to discuss briefly this conceptual relationship and to determine whether similar results could be expected from the two approaches. The paper is divided into two parts. First, a more exact interpretation of the production function approach benefits is developed and compared with the conventional value marginal product (*VMP*) measure. Second, these measures are compared with those used in the economic surplus approach.

2 Comparison of the Two Approaches

2.1 Introduction

In discussing the two approaches Peterson (1971) pointed out that the economic surplus approach can be used to give either an average or a marginal internal rate of return for agricultural research expenditures. He showed, with the aid of an illustration, that the marginal rate was considerably higher than the average rate. Peterson also noted that the production function approach — through its use of the value marginal product of research to find an internal rate of return — has estimated only a marginal rate. This average versus marginal aspect of the benefit estimation process is not the main focus of this paper, however it is appropriate to briefly list a few critical points.

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* Special Economist, New South Wales Department of Agriculture, Sydney. The author wishes to acknowledge the useful comments of Col Gellatly, Garry Griffith, David Godden and in particular the *Review* referees.

[1] For a recent review of these approaches see Norton and Davis (1981) and for a summary table of estimated rates of return see Ruttan (1978, pp. 15-16).

(i) Peterson's conclusion about the relative size of the average and marginal rates of return is specific to the data used in his analysis. In particular, as he pointed out, he used the period 1910 to 1967 to estimate the average rate and 1957 to 1967 to estimate the marginal rate. It is more appropriate to use the same period of time for the distribution of benefits with total research expenditure for the average rate and the incremental unit for the marginal rate. With these measures, in general, we would expect the average rate to be larger than the marginal rate. That is, the more beneficial projects would be looked at first.

(ii) In terms of the production function approach it is possible to calculate an average rate of return by using the average rather than marginal product. Certainly in this case for stage two of the production process the average product is always greater than the marginal product. This can be seen from equation (5).

The important issue considered in this paper is; if the marginal rate is found using both approaches, are they in fact conceptually attempting to measure the same thing?

To facilitate a comparison of the marginal rates found using each approach it is necessary to have a common unit of measure for the benefits[2]. The appropriate common measure is the change in economic surplus[3]. To date benefits measured by the production function approach have not been presented in terms of economic surpluses[4]. To permit a comparison of the two approaches, then, the production function approach will be re-formulated in a supply and demand dimension. Once the appropriate economic surplus measure has been derived it will be compared with the usually estimated value marginal product. Finally, these measures will be compared with those adopted by the economic surplus approach.

2.2 An Economic Surplus Interpretation of the Production Function Approach

The basis for including research expenditure in an aggregate production function is founded on the argument that this research creates technical change which in turn causes a change in the production process. The same change will cause a shift in the aggregate supply function. The implications of this are best illustrated with an example. Most production function approach studies have used a Cobb-Douglas

[2] Since a marginal rate is of interest the cost for both approaches is the same unit increase in research expenditure.

[3] "Appropriate" is used here in terms of best for the purposes of providing a common measure. The debate in the literature as to the theoretical appropriateness of economic surplus as a measure of societal benefits has emphasized the potential deficiencies of this measure. See for example Currie, Murphy and Schmitz (1971) and Willig (1976).

[4] Although under a set of rigid implicit assumptions the *VMP* is a measure of the change in this economic surplus. These assumptions are discussed in the next section.

functional form, although a few have used a C.E.S. function. Here the following is used[5]:

(1) $$Q = A\, X_1^b\, X_2^c\, R^d$$

where: Q is aggregate output
X_i for i = 1, 2 are the conventional inputs
R is the research expenditure
b, c and d are the respective production coefficients
A is a shift factor

If it is assumed that R is determined exogenously[6], that the objective of all producers is to maximise profits, and that all producers face competitive market conditions, then equation (1), the expansion path and the cost equation can be used to derive the following supply relation (S)[7]:

(2) $$Q = Z\, R^{\frac{d}{1-b-c}}\, P^{\frac{b+c}{1-b-c}}$$

where: $$Z = A^{\frac{1}{1-b-c}}\, b^{\frac{b}{1-b-c}}\, c^{\frac{c}{1-b-c}}\, G_1^{\frac{-b}{1-b-c}}\, G_2^{\frac{-c}{1-b-c}}$$

G_i for i = 1, 2 are the prices of the conventional inputs
P is the output price

The impact of a small change in research expenditure (R) on supply is found from equation (2) as

(3) $$\frac{\partial Q}{\partial R} = \frac{d}{1-b-c}\,\frac{Q}{R}$$

Making use of this it can be seen that the aggregate supply after this change (S') now becomes

(4) $$Q = Z\, R^{\frac{d}{1-b-c}}\, P^{\frac{b+c}{1-b-c}}\, \{1 + \frac{d}{1-b-c} R^{-1}\}$$

[5] A critical implicit assumption is that we ignore all the aggregation problems associated with these aggregate production functions.

[6] This assumption seems reasonable because even if farmers do have some influence on the level of research expenditure, there is a lag before output will be affected.

[7] For more detail of the computational procedure used see, for example, Heady *et al* (1961, pp. 12-13).

It is possible to illustrate using Figure 1 the influence of this shift in supply and the associated change in economic surplus. From the above discussion it is seen that an increase in research expenditure results in a pivotal divergent shift in supply from S to S'. At output price P_1 the impact of this research on Q, as given by equation (3), is equivalent to $Q_4 - Q_1$ or AE. If the demand is assumed to be perfectly elastic then the resultant increase in economic surplus is the area AEO. More realistically, if demand is less than perfectly elastic, say DD, then the increase in economic surplus is ADO, the shaded area in Figure 1. If the assumptions underlying this example are accepted then this latter area is the most appropriate measure of the benefits (economic surplus) resulting from an increase in research expenditure.

Studies using the production function approach, however, have not attempted to estimate ADO as the benefits of increased research expenditure. Instead they have used the value marginal product derived from the production function. For the example used above, the marginal product is found from equation (1) as:

$$MP = d\frac{Q}{R}$$

If the original output level is Q_1 then the value marginal product is the MP by the product price, P_1, or

$$VMP = \frac{dQ_1P_1}{R}$$

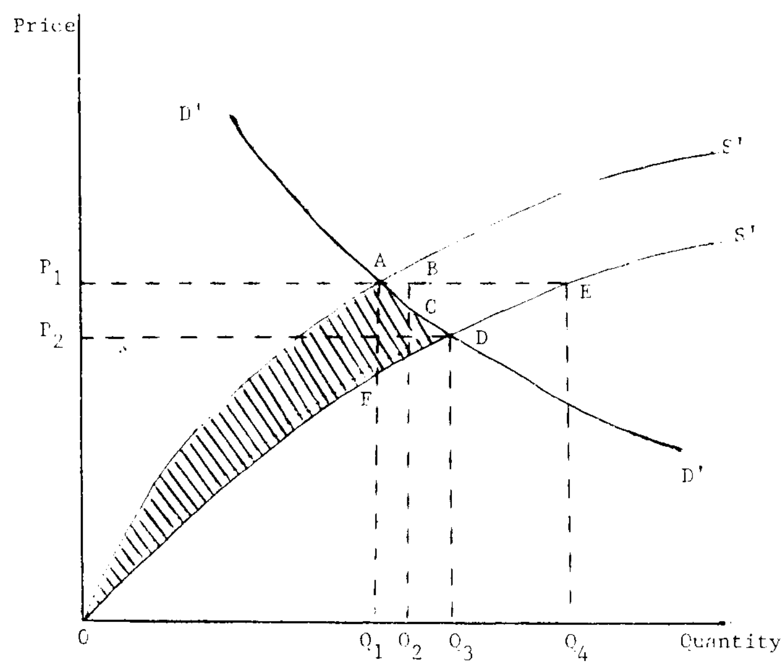

Figure 1

There are two important implicit assumptions underlying this use of the value marginal product. First, it is assumed that the level of use of all other inputs remain the same. The implication of this assumption is that the very short run is being considered

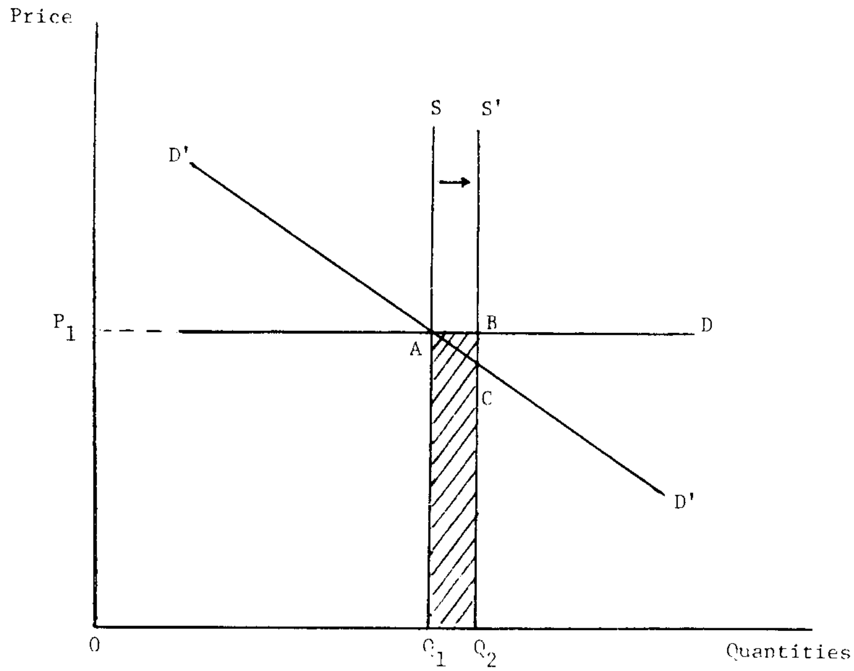

Figure 2

and therefore the aggregate supply is perfectly inelastic. The marginal increment of research expenditure will, therefore, cause a parallel shift of the aggregate supply to the right, as shown in Figure 2. Second, it is also assumed that the change in output has no impact on the product price, in other words demand is perfectly elastic (DD in Figure 2). On the basis of these assumptions the value marginal product measures the shaded area in Figure 2, that is, ABQ_2Q_1.

It is unlikely that the aggregate demand will be perfectly elastic, in which case the value marginal product will over-estimate the returns by an area similar to ABC in Figure 2. However, since only a marginal change is being considered this area will be relatively small.

More critical, however, is the assumption that all other inputs are fixed. The impact of increased research expenditure takes several years to have its full effect, so it is doubtful if the assumption of the very short-run is realistic. If this assumption is relaxed then as discussed above the appropriate measure of the benefits becomes AEO in Figure 1. If both assumptions are relaxed the appropriate area is $\dot{A}\bar{D}\tilde{O}$.

The important question becomes: What is the relationship between the areas $\dot{A}\bar{D}O$ and ABQ_2Q_1 in Figure 1? From equations (3) and (5) it is seen that if $0<(b+c)<1$ then the marginal product will be smaller than the shift in supply Q_1Q_4, if some inputs are not fixed[8]. However, it is not clear from a diagramatical illustration that the relative sizes of the quantity changes reflect the relative sizes of the respective benefit measures. A more rigorous comparison is required.

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[8] It is of interest to note that most empirical results for the United States indicate that the sum of the conventional input coefficients, ($b + c$ in this case) are in fact greater than one. In reality, however, the implications of this for the economic surplus measure are avoided via the fact that it is unlikely that all inputs together can be regarded as variable. It is likely then that the sum of the coefficients of the variable inputs will be less than one.

2.3 Benefit Measures for Both Interpretations

2.3.1 *The Economic Surplus Interpretation*

The elasticity of supply, α, can be found from equation (2) as

$$\alpha = \frac{b + c}{1 - (b + c)} ,$$

which by re-arranging gives

$$(b + c) = \frac{\alpha}{1 + \alpha} \qquad (6)$$

If in addition a constant elasticity demand function *(D)*

$$Q = B\, P^{-\delta} \qquad (7)$$

is assumed, where B represents all of the appropriate shift factors, then by substituting equation (6) where appropriate, the inverse forms of the supply (S and S) and demand (D) functions used above become respectively

$$P = Z^{-1/\alpha}\, R^{\frac{-d(1 + \alpha)}{\alpha}}\, Q^{\frac{1}{\alpha}} , \qquad (8)$$

$$P = Z^{-1/\alpha}\, R^{\frac{-d(1 + \alpha)}{\alpha}}\, Q^{\frac{1}{\alpha}} \{1 + d(1 + \alpha)R^{-1}\}^{\frac{-1}{\alpha}} , \qquad (9)$$

$$P = B^{\frac{1}{\delta}}\, Q^{\frac{-1}{\delta}} \qquad (10)$$

It is possible to find an expression for Q_1 by substituting P_1 in equations (8) and (10) and solving for the equilibrium situations depicted in Figure 2, that is

$$Q_1 = B^{\frac{\alpha}{(\delta + \alpha)}}\, Z^{\frac{\delta}{(\delta + \alpha)}}\, R^{\frac{\delta d(1 + \alpha)}{(\delta + \alpha)}} \qquad (11)$$

Similarly, using equations (9) and (10) it is possible to find an expression for Q_3,

$$Q_3 = B^{\frac{\alpha}{(\delta + \alpha)}}\, Z^{\frac{\delta}{(\delta + \alpha)}}\, R^{\frac{\delta d(1 + \alpha)}{(\delta + \alpha)}} \{1 + d(1 + \alpha)R^{-1}\}^{\frac{\delta}{(\delta + \alpha)}} \qquad (12)$$

With the above set of information it is now possible to derive a quantitative expression for the economic surplus interpretation of the production function approach. The change in economic surplus as represented by the shaded area in Figure 1 is given by

$$\Delta ES = \int_{o}^{Q_1} S\, d\, Q + \int_{Q_1}^{Q_3} D\, d\, Q - \int_{o}^{Q_3} \underline{S}'\, d\, Q$$

which by expansion and collecting terms can be shown to give

$$\Delta ES = \frac{\alpha}{\alpha + 1}\, Z^{\frac{-1}{\alpha}}\, R^{\frac{-d(1+\alpha)}{\alpha}} \left\{ Q_1^{\frac{(1+\alpha)}{\alpha}} - Q_3^{\frac{(1+\alpha)}{\alpha}} \{1 + d(1+\alpha)R\}^{\frac{-1}{\alpha}} \right\}$$

$$+ \frac{\delta}{\delta - 1}\, B^{\frac{1}{\delta}} \left\{ Q_3^{\frac{\delta - 1}{\delta}} - Q_1^{\frac{\delta - 1}{\delta}} \right\}$$

Now by substitution of Q_1 and Q_3 from equations (11) and (12) and a laborious collection of terms this reduces to

(13)

$$\Delta ES = B^{\frac{(1+\alpha)}{(\delta+\alpha)}}\, Z^{\frac{(\delta-1)}{(\delta+\alpha)}}\, R^{\frac{d(\delta-1)(\alpha+1)}{(\delta+\alpha)}} \left[\left\{ \{1 + d(1+\alpha)R^{-1}\}^{\frac{(\delta-1)}{\delta+\alpha}} - 1 \right\} \left\{ \frac{(\delta+\alpha)}{(\delta-1)(\alpha+1)} \right\} \right]$$

Recall that for the binominal series

$$f(x) = (1 + x)^p,$$

where p is not a positive integer and $-1<x<1$, that as Maxwell (1958, p.52) has shown a good approximation is given by

(14)

$$(1 + x)^p = 1 + px + \frac{p(p-1)\, x^2}{2}$$

Considering the following component of equation (13)

$$\left[1 + d(1 + \alpha)R^{-1} \right]^{\frac{(\delta - 1)}{(\delta + \alpha)}},$$

it is seen that for any normal values of the parameters the above conditions hold.

Therefore substituting this component into equation (14) gives the following expansion

$$1 + \frac{(\delta - 1)(1 + \alpha)}{(\delta + \alpha)} \frac{d}{R} + \frac{1}{2} \frac{(\delta - 1)}{(\delta + \alpha)} \left[\frac{(\delta - 1)}{(\delta \quad \alpha)} - 1 \right] (1 + \alpha)^2 \frac{d^2}{R^2}$$

Substituting this into equation (13) and collecting terms gives

$$\text{(15)} \qquad \Delta ES = \frac{d}{R} B^{\frac{(1 + \alpha)}{(\delta + \alpha)}} Z^{\frac{(\delta - 1)}{(\delta + \alpha)}} R^{\frac{d(\delta - 1)(1 + \alpha)}{\delta + \alpha}} \left\{ 1 - \frac{d}{R} \frac{(1 + \alpha)^2}{(\alpha + \delta)} \right\}$$

which is the expression for the economic surplus interpretation of the production function approach.

2.3.2 *The Value Marginal Product Interpretation*

The value marginal product interpretation can be expressed in terms of similar parameters. As above it is assumed that the marginal change occurs at equilibrium price, P_1, and quantity, Q_1. Multiplying the marginal product, equation (5), by the product price gives

$$\text{(16)} \qquad VMP = d \cdot \frac{Q_1}{R} \cdot P_1$$

Substituting for P_1 from equation (10) and for Q_1 from equation (11) and collecting terms gives

$$\text{(17)} \qquad VMP = \frac{d}{R} B^{\frac{(1 + \alpha)}{(\delta + \alpha)}} Z^{\frac{(\delta - 1)}{(\delta + \alpha)}} R^{\frac{d(1 + \alpha)(\delta - 1)}{(\delta + \alpha)}}$$

A comparison of equations (15) and (17) reveals that since $\alpha \geq 0$ and $\delta \geq 0$, $VMP \geq \Delta ES$

Since the most likely values of d/R are very small it can be concluded that both interpretations give approximately the same measure of the benefits from an increase in research expenditure, at the margin[9]. Given the assumptions underlying this illustration the magnitudes of the supply and demand elasticities have little impact on the difference between the two interpretations[10].

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[9] Since a Cobb-Douglas production function has been assumed it is noted that the research production elasticity estimate, d, will be scale free, that is the level of research expenditure can be scaled to any unit and the parameter estimated will not change. Despite this the likely values of d/R should be small. For most previous studies estimates of d have been of the order of 0.06. Research expenditure levels used have been of the order of millions of dollars. If this variable were scaled such that $d/R > 1$ the unit of measure would most probably be a billion dollars. A one unit change would then be beyond most conceptions of a marginal change.

[10] It can be seen that as $\delta \to \infty$, $\Delta ES \to VMP$.

2.4 Comparison with the Economic Surplus Approach

Lindner and Jarrett (1978) provided a useful summary of the types of research benefits measured by the economic surplus approach. They classified the types of shifts in supply due to technical change as pivotal or proportional divergent, parallel and convergent. It was shown that the change in economic surplus was lower for a pivotal divergent shift than for any of the other types of supply shifts. Although Rose (1980) pointed out some errors in Lindner and Jarrett's quantification of those differences he still showed that, for example, a parallel shift results in double the change in economic surplus of a pivotal divergent shift. All production function approach studies have assumed functional forms which imply a pivotal divergent supply shift. There is therefore potential for considerable differences in the measures of benefits using the two approaches.

The discussion so far has been restricted to potential differences between research benefit estimates due to different conceptual relationships which in turn are dependent upon the underlying assumptions of the measurement procedures. In addition, however, there is considerable potential for differences due to empirical problems. These differences may occur within a particular measurement procedure or between them. For example, in the case of the economic surplus approach the recent debate by Rose (1980), Wise and Fell (1980) and Lindner and Jarrett (1980) indicate the range of these issues. Also for the production function approach Davis (1981) has shown that the internal rate of return computational procedure adopted has varied considerably between past studies and this can effect the estimates considerably.

Between the two measurement procedures empirical problems can also be a source of differences in benefit estimates, a good example of which is estimation of the shift in supply due to research expenditure. The production function approach assumes a specific impact via the functional form chosen. The economic surplus approach uses a shift parameter, k, which has not been estimated on the basis of any explicit relationship between the parameter and the research expenditure level. Often this k has been estimated more in line with an average rate of return than a marginal rate.

In summary then under certain assumptions the economic surplus and production function approaches will be conceptually measuring the same benefits from a change in research expenditure. However, because of significant potential empirical problems for the majority of applications to date this occurrence will have been co-incidental. Unfortunately, only a few of these studies have estimated the benefits for the same commodity, the same time period and with both approaches. Peterson and Fitzharris (1977) and Lu and Cline (1977) provided rate of return estimates for four time periods between 1930 and 1970 for aggregate U.S. agriculture. On average the economic surplus approach rates reported by Peterson and Fitzharris were 1.7 times larger than the production function approach rates given by Lu and Cline. An important difference between the two studies is that the former assumed parallel shifts in supply while the latter implicitly assumed a pivotal divergent shift.

On the other hand, Peterson (1967) used both approaches to evaluate poultry research in the U.S. He found that the economic surplus approach rates were only slightly larger than the production function approach results. For the economic surplus approach a proportional divergent shift in supply was assumed.

Despite the fact that this very limited empirical comparison appears to confirm the conclusions of the analysis in this paper, the scope for discrepancies due to the empirical problems is too great to place much emphasis on these results. The point remains however, that careful consideration should be given to this conceptual relationship when choosing the approach to use in *ex post* research evaluation.

3 Conclusions

In this paper it has been shown that benefits to an increase in research expenditure as measured by the value marginal product are approximately equal to the change in economic surplus associated with the increase. However, it was also shown that the functional forms used by most production function studies implicitly assume that the resultant supply shift is pivotal divergent. In comparison, applications of the economic surplus approach have assumed pivotal divergent, proportional divergent or parallel shifts in supply due to research effort. It was pointed out that if either of the latter two are appropriate then production function approach estimates may be up to half the value they should be.

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