The Optimal Season Length for a Sugar Mill in the Australian Sugar Industry[1]

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Abstract

In this paper we present a model of the optimal crushing season length for sugar cane. The approach taken is to view the optimal season length problem as an optimal stopping problem for both the Mill and a representative grower. We formulate the optimal stopping problems for both Mill and grower based on "real option" theory using Ito calculus. Because the interests of the Mill and growers do not coincide this results in a stochastic differential game of optimal stopping. We solve the model numerically using a finite difference algorithm.

Keywords Sugarcane Harvesting, Seasonality, Real Option Theory, Optimal Stopping, Ito Calculus, Stochastic Differential Games.

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1 Introduction

Canegrowers have responded positively to the progressive relaxation over the past decade of the constraints on the area planted to sugarcane. The area under cane has increased by about 30 percent during that time while production has expanded to record levels. Milling capacity has not kept pace with the increase in farm production. Sugar mills were able to take advantage of an approximate 30 percent increase in crushing capacity for a relatively small capital outlay by converting to continuous crushing (7 days per week crushing instead of five). In spite of this increase in milling capacity, there has been a tendency for the length of the crushing season to be extended, particularly in years like 1997 when there was a very large tonnage of cane to process. This has been due in part to the relatively high yields obtained in recent years from newly released varieties of cane, better seasonal conditions, and the conversion to trash retention farming by growers in many areas.

The cane harvesting and crushing season has traditionally been conducted over a 20-week period starting in June and which was normally completed by mid- to late-November. This allowed sufficient time for ratoon crops to become established prior to the start of the rainy season some time in January or February. However, with the larger tonnages to crush in recent years, the harvesting season has extended beyond 23 weeks in some cases. The problem with extending the crushing season into mid- or even late-December is that their growing season of the subsequent ratoon crop is potentially reduced if these crops are not to be harvested late again the following year. When the adverse effects of exposing the young ratoon crops to early onset of the wet season are added, growers can experience quite poor yielding crops from late-cut ratoons.

With a cropping cycle that normally includes a plant cane crop and four or more ratoon crops, growers are unable to manage their harvesting operations to avoid harvesting any cane that is to be ratooned during the last month of the season. If it was practical to do so, growers would avoid this problem by reserving cane that is to be ploughed out and replanted for harvesting during this period but it is not possible to do this. Adverse weather early in the harvesting season will often cause growers to harvest crops that are destined to be ploughed out when it does not matter if the field gets damaged during harvesting.

The whole question of optimum season length is a controversial issue with the two principal parties having quite different interests. The milling companies would like to extend the crushing season because it allows them to run a smaller capacity plant and thereby reduce fixed costs. Growers, on the other hand, want a crushing season that is decidedly shorter than the millers so that it is possible to maximise payments for cane and allow the subsequent ratoon crop a growing season that is as long as possible.

Of interest is the question who is correct, growers or mills, and whether or not their interests diverge. In order to address this issuer we study a stochastic differential game of the optimal season length for sugarcane harvesting in the Australian sugar industry. the problem falls in to a class of game theoretic problems known as games of timing. Although these are usually and somewhat idiosyncratically static games they need not be static. We have chosen to model the problem in continuous-time because as was pointed out earlier both harvesting and crop growth occur in continuous-time and not discrete time. Mills run 24 hours a day seven days a week during the harvest season. Thus our justification for using continuous-time is essentially the same as that used to justify the use of continuous-time models in finance, if anything the case for a continuous-time approach to the present problem is in fact stronger than the case for their use in financial applications.

For previous discussions of both stochastic differential games and games of timing we would refer the reader to Baser and Olsder (1982) and Dresher (1961). the seminal paper on stochastic differential games is by Friedman (1971).

The season length problem is viewed from the perspective of a representative mill using real option theory as a problem of whether to continue crushing cane or to stop[2]. The problem is thus an optimal stopping problem. Growers on the other hand, although they also view the problem of season length from the perspective of optimal stopping, may disagree with the mill as to the value of options of continuing crushing/harvesting or stopping.

Previous work on the optimal season length in sugar cane has not attempted such a detailed analysis. The Boston Consulting Group (1996) developed a simple model of the optimal season length based on competitive market assumptions. Their model can be seen as a simple parametric bugeting exercise. The model presented here generalises that approach considerably by taking into account seasonal fluctuations in CCS, the impact of season length on cane yield uncertainty and the strategic nature of the relationship between growers and a representative mill.

The model should not be considered as a completely accurate depiction of the real rela-

[2]For a discussion of real option theory see Dixit (1993) and Dixit and Pindyck (1994).

tionship between growers and a Mill, as we abstract from a number of important features. For example, we ignore the difference in yield between different ratoon crops, spatial aspects of the problem and consider only a single representative grower. The model should be viewed as a first approximation to what is a quite complex problem. Our intent in this paper is first to develop the methodology as a first step to solving a more general and more realistic problem.

2 The Model

Harvestable cane yields are modelled as a lagged Ito process whereby the mean harvestable yield at any point in time is a function of season length.

$$dy = \left[\alpha y(t-\tau)(1-\frac{y(t-\tau)}{y_{\max}}) - h(t)\right] dt + \sigma dw$$

where y is the available yield at time t, $h(t)$ the amount harvested, α the intrinsic growth rate of sugar cane, T the maximum length of the growing season, τ the amount by which the growing season is reduced, i.e. the length of the harvest season. Note that this equation is a stochastic delay-differential equation[3]. We assume that all of the cane harvested is ratoon cane of equal age. If we were to drop this assumption, the dynamics of cane supply would have to be represented using an age-structured formulation with different "starting dates" for different crops.

We use a logistic growth function for cane yields that depends on the length of the growing season $t-\tau$. In the following we make the substitution $h(t) = y(t)$ where necessary.

The pool price of sugar p_s is assumed to be constant and the mills profit is defined by

$$\Pi_m = p_s ccs \xi y - C(y) - c_2 \tau$$

where ccs is the sugar content of cane as a percentage, ξ a parameter to convert this to decimals[4], c_2 the marginal time cost of lengthening the season and C(y) the variable crushing costs of the mill.

The sugar content of cane CCS is assumed to fluctuate seasonally, depending on when cane is harvested. The change in CCS is therefore represented as follows by a seasonally forced stochastic differential equation.

$$dccs = cos(\theta\tau)dt + \hat{\sigma} dz$$

We assume that cane yields y and CCS are uncorrelated.

If the mill were to stop crushing cane then it would incur during the idle period the discounted costs of maintaining fixed capital $-rKe^{-\delta t}$ plus the opportunity cost of not crushing cane which is the same as the foregone profit $-e^{-\delta t}\Pi$. These fixed costs are not decision relevant as they are incurred whether the Mill crushes or not.

The Mills objective function in time τ is given by[5]

$$F(\Pi_m, y, ccs, \tau) = \max\left\{0, \Pi_m(y, ccs, \tau) + (1+\delta d\tau)^{-1} EF(y+dy, ccs+dccs, \tau+d\tau)|y, ccs\right\}$$

[3]For a discussion of delay-differential equations see Kuang (1993).
[4]This is a necessary addition if one whishes to avoid an additional application of Ito's lemma.
[5]See (1994) for the derivation of the objective function.

Because of the dependency of the Wiener noise terms $w(t)$ and $z(t)$ on t and not τ, it is simpler to work at first in terms of t. This is because the derivatives $\frac{dw(t)}{dt}$ and $\frac{dz(t)}{dt}$ do not exist making it impractical to reparameterise the noise terms in terms of τ. Thus we reparameterise the objective function in terms of t.

$$F(\Pi_m, y, ccs, t) = \max\left\{0, \Pi_m(y, ccs, t) + (1 - \delta dt)^{-1} EF(y + dy, ccs + dccs, T - t - dt)|y, ccs\right\}$$

This leads to the following partial differential equation.

$$-F_t + (\alpha y(2t - T)(1 - \frac{y(2t - T)}{y_{\max}}) - y(t))F_y + cos(\theta\tau)F_{ccs} + \delta F(\Pi_m, y, ccs, t) + \Pi_m(y, ccs, t)$$

$$+\frac{1}{2}\hat{\sigma}^2 F_{ccsccs} + \frac{1}{2}\sigma^2 F_{yy} = 0$$

This needs to be solved in harvest time τ, thus we reparameterise by substituting everywhere for t to obtain the following partial differential equation in τ.

$$F_\tau + (\alpha y(T-2\tau)(1-\frac{y(T-2\tau)}{y_{\max}}) - y(T-\tau))F_y + cos(\theta\tau)F_{ccs} + \delta F(\Pi_m, y, ccs, \tau) + p_s ccs\xi y(T-\tau) - C(y(T-\tau)) - c_2\tau$$

$$+\frac{1}{2}\hat{\sigma}^2 F_{ccsccs} + \frac{1}{2}\sigma^2 F_{yy} = 0$$

The problem for a representative grower is similar, although profit is defined differently.

$$\bar{\Pi} = p_c(ccs)y - c(y)$$

where $p_c = 0.009p_s((ccs - 4) + 0.0575)$.
Growers face the same yields as millers.

$$dy = \alpha y(t - \tau)(1 - \frac{y(t - \tau)}{y_{\max}})dt + \sigma dw$$

The optimisation problem for growers is given by

$$G(y, ccs, \tau) = \max\left\{0, \bar{\Pi} + (1 + \delta d\tau)^{-1} EG(y + dy, ccs + dccs, \tau + d\tau)\right\}$$

Reparameterising this by replacing τ with t gives.

$$G(y, ccs, t) = \max\left\{0, \bar{\Pi} + (1 - \delta dt)^{-1} EG(y + dy, ccs + dccs, T - t - dt)\right\}$$

This leads to the following partial differential equation in t.

$$-G_t + (\alpha y(2t - T)(1 - \frac{y(2t - T)}{y_{\max}}) - y(t))G_y + cos(\theta\tau)G_{ccs} + \delta G(\bar{\Pi}_g, y, ccs, t) + \bar{\Pi}_g(y, ccs, t)$$

$$+\frac{1}{2}\hat{\sigma}^2 G_{ccsccs} + \frac{1}{2}\sigma^2 G_{yy} = 0$$

Reparameterising this by replacing t with τ gives.

$$G_\tau + (\alpha y(T-2\tau)(1-\frac{y(T-2\tau)}{y_{\max}}) - y(T-\tau))G_y + cos(\theta\tau)G_{ccs} + \delta G(\bar{\Pi}_g, y, ccs, \tau) + p_c(ccs)y(T-\tau) - c(y(T-\tau))$$

$$+\frac{1}{2}\hat{\sigma}^2 G_{ccsccs} + \frac{1}{2}\sigma^2 G_{yy} = 0$$

Simultaneous solution of the partial differential equations gives a solution to the game in terms of the value matching and smooth pasting conditions. Note that the decision problems of the mill and the grower are linked by the stochastic differential equation for cane yield, both players benefit from the cane harvest it is this linkage which leads to the problem being a game in the sense of (1971). The game differs from most stochastic differential games in that the strategy set is time. The game is thus what is termed in game theory a "game of timing" the game is further complicated by the need for two different time scales, growing period and harvesting period.

3 Numerical Solution of the Stochastic Differential Game of Optimal Switching

In order to numerically solve the optimal switching game between harvesting-crushing and laying idle a finite difference method is employed utilising a fixed step in the y, ccs and τ directions.

The pair of partial differential equations is then solved simultaneously. Value matching and smooth pasting conditions are then applied to determine the optimal point in time τ^* at which the mill and the grower would switch strategies. This is the optimal crushing season length.

3.1 The Finite Difference Algorithm

The finite difference method is a standard method of solving option pricing problems numerically (See Hull (1997)) and indeed for solving partial differential equations and systems of partial differential equations generally(Burden et al. 1981).

The technique involves approximating the derivatives in the equation to be solved by "finite differences", hence the name.

The partial differential equation for the Mill's payoff may be written in finite difference form as

$$F_{\tau+1} = F_\tau + \Delta\tau \left[\left[\alpha y(T-2\tau)(1-\frac{y(T-2\tau)}{y_{\max}}) - y(T-\tau) \right] \frac{F_{y+1}-F_y}{\Delta y} + cos(\theta\tau)\frac{F_{ccs+1}-F_{ccs}}{\Delta ccs} + \right.$$

$$\delta F(y,ccs,\tau) + p_s ccs\xi y(T-\tau) - C(y(T-\tau)) + \frac{1}{2}\hat{\sigma}^2 \frac{F_{ccs+1}-2F_{ccs}+F_{ccs-1}}{\Delta ccs^2}$$

$$\left. + \frac{1}{2}\sigma^2 \frac{F_{y+1}-2F_y+F_{y-1}}{\Delta y^2} \right]$$

where step sizes are represented by Δ terms.

The partial differential equation for the growers payoff may be written in finite difference form as.

$$G_{\tau+1} = G_\tau + \Delta \left[\left[\alpha y(T-2\tau)(1-\frac{y(T-2\tau)}{y_{\max}}) - y(T-\tau) \right] \frac{G_{y+1}-G_y}{\Delta y} + cos(\theta\tau)\frac{G_{ccs+1}-G_{ccs}}{\Delta ccs} \right.$$

$$+\delta G(y,ccs,\tau) + p_c(ccs)y(T-\tau) - c(y(T-\tau)) + \frac{1}{2}\hat{\sigma}^2 \frac{F_{ccs+1}-2F_{ccs}+F_{ccs-1}}{\Delta ccs^2} +$$

$$\frac{1}{2}\sigma^2\frac{G_{y+1}-2G_y+G_{y-1}}{\Delta y^2}\Big]$$

It can be implemented in spreadsheets for cases up to and including three (3) independent variables including time. For higher order equations it is advisable to use either a specialist package or a higher level programming language. In our case although we have three independent variables in each equation y, ccs and τ, the lag terms make spreadsheet implementation more difficult. Therefore we have implemented the algorithm in C++. The source code is available from the authors on request.

4 Results

The results of the model depend critically on the mills crushing costs. This was to be expected. Unfortunately data on this is confidential and one can only guess at the likely magnitude of this factor. In a number of runs of the model it was found that for low crushing costs growers favoured a season length much shorter than the mill. For large crushing costs this situation was reversed.

The following parameter values were used in the model

$y_{max} = 10$ measured in units of 10 tonnes

$\theta = 0.01$

$T = 1$

$p_s = 300$

$\xi = 0.01$

$\delta = 0.06$

max grid value yield 10

max grid value ccs 15

max grid value time 10

$\sigma = 0.01$

$\hat{\sigma} = 0.1$

$\alpha = 0.05$

variable production cost of grower 15

boundary values 10

Note that we varied the size of c_2 between 0 and 10000. At high values around \$10000 per unit τ the mill began to lose money very quickly, i.e. during the first iteration in the time direction and at high CCs and yield values. If one reduced c_2 to the other extreme, growers profit always remained positive up to a τ of 0.8. The programme then terminated due to lack

Profit	τ	Time index	CCS index	yield index
0.97	0.7	7	10	8
0.97	0.7	7	10	9
0.97	0.7	7	10	10
0.87	0.7	7	11	7
0.932	0.7	7	11	8
0.932	0.7	7	11	9
0.932	0.7	7	11	10
0.53	0.8	8	9	8
0.53	0.8	8	9	9
0.53	0.8	8	9	10
0.58	0.8	8	10	1
0.58	0.8	8	10	2
0.58	0.8	8	10	3
0.58	0.8	8	10	4
0.58	0.8	8	10	5
0.58	0.8	8	10	6
0.44	0.8	8	10	7

Table 1: Profit, Harvest duration and Grid Position

of memory[6]. The model produces approximately 10,500 numbers per run (1500 iterations) making it's memory requirements quite demanding.

Nevertheless grower profit did drop considerably although never quite going negative. The following table gives an indication of the grid positions at which grower profit was at a minimum.

These figures for τ of 0.7 and 0.8 translate to a crushing season length of about 36 and 41 weeks as opposed to currrently 23 weeks.

In comparing our results with the (1996) study it should be noted that they summed profits for the whole mill area obtaining results of between about 26 and 31 weeks. On proceeding in the same manner we obtain very similar figures. However summing profits involves introducing an additive welfare function for the mill area that may not be compatible with individual incentives, thus mill and growers may prefer season lengths that diverge from the joint profit solution.

5 Conclusion

In our model the optimal length of the harvest season appears to be primarily determined by the mills marginal cost of increased season length rather than by the shorter growing seasons experienced by growers in subsequent seasons. The model does not support growers claims that the crushing season is at 23 weeks too long, rather it suggests that the crushing season could be considerably lenghthened if the mill so desires. It should however be noted that longer crushing seasons are likely to reduce grower profits although it would still pay growers to accept the longer season length. The model takes growers objectives to be profit

[6]We ran the model using the GNU C++ compiler running under Linux Redhat 5.2 on a Pentium II with 333 Mhz and 64 Mb of RAM.

maximisation. The reason for growers current concerns may however have less to do with profit maximisation and more to do with their demand for leisure. It would be a relatively simple matter to extend the model to incorporate utility maximisation and leisure. This modification may explain growers concerns and lead to different results to those found here.

The model might also be extended to incorporate more than one grower. Mathematically this is relatively simple but it would greatly increase tthe computational requirements. As mill areas typically have around 200 growers such problems would require parallel computation and supercomputing facilities.

Another possible extension of the model would be to incorporate crop classes into the model with crops of different starting dates. This would lead to some very complicated mathematics, as it necessitates the optimal control of a stochastic partial differential equation. Our intent with this paper was simply to illustrate one possible approach to the solution of the optimal season length problem and to do this for the simplified case of identical ratoons. Extending the model to incorporate different crop classes would appear desirable but at this stage must be left for another paper.

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