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International Environmental Agreements with Mixed Strategies and Investment

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Abstract

We modify a canonical participation game used to study International Environmental Agreements (IEA), considering both mixed and pure strategies at the participation stage, and including a prior cost-reducing investment stage. The use of mixed strategies at the participation stage reverses a familiar result and also reverses the policy implication of that result: with mixed strategies, equilibrium participation and welfare is higher in equilibria that involve higher investment.

Keywords: International Environmental Agreement, climate agreement, participation game, investment, mixed strategy.

JEL classification numbers C72, H4, Q54

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1 Introduction

Limiting increases in Greenhouse Gas (GHG) stocks requires an international effort. The global nature of GHG pollution and the fact of national sovereignty complicate the usual problems of eliciting the provision of a public good, in this case, reduction of GHG emissions. International Environmental Agreements (IEAs) attempt to alleviate the free riding problem associated with transboundary pollution. Although there are many examples of successful IEAs, these do not involve problems on the same scale as climate change. In the absence of historical analogies, economic theory plays an especially important role in helping to think about ways of addressing the climate policy problem.

The literature on IEAs uses both non-cooperative and cooperative game theory. A major non-cooperative strand uses the definition of cartel stability in a participation game (D'Aspremont et al., 1983). In this game, countries first make a binary decision, whether to join or stay out of an IEA, and then the resulting IEA decides on the level of abatement. Early papers include Carraro and Siniscalco (1993) and Barrett (1994); Wagner (2001) and Barrett (2003) review this literature. IEA members internalize the effect of their actions on other members, while non-members ignore the externalities. A stable IEA is both internally stable (members do not want to defect) and externally stable (outsiders do not want to join); that is, the equilibrium at the participation stage is Nash in the binary participation decision.

These models are generally pessimistic about the equilibrium level of participation, and they suggest that IEAs are especially ineffective when potential benefits of cooperation are large (Ioannidis et al. 2000, Finus 2001, Barrett 2003). Hoel (1992) and Barrett (1997) show that IEAs may not be able to improve substantially upon the non-cooperative outcome even when countries are heterogeneous. Karp and Simon (2011) reconsider these issues using a non-parametric setting. Barrett (2002) and Finus and Maus (2008) show that coalitions with modest ambitions may be more successful than those that try to fully internalize damages. Kosfeld et al. (2009), Burger and Kolstad (2009) and Dannenberg et al. (2009) use laboratory experiments to test the predictions of this participation game.

A different non-cooperative strand replaces the Nash assumption at the participation stage with a more sophisticated notion of stability; agents form beliefs about how their provisional decision to join or leave a coalition would affect other agents' participation decisions (Chwe, 1994, Ecchia and Mariotti, 1998, Xue, 1998 and Ray and Vohra, 2001). These agents are said to be "farsighted". Diamantoudi and Sartzetakis (2002), Eyckmans (2001), de Zeeuw (2008) and Osmani and Tol (2009) apply this notion of stability to IEA models. A distinct line of research uses cooperative games and the theory of the core to model IEAs (e.g. Chander and Tulkens (1995, 1997)). In one such setting, a country believes that if it defects from a coalition, other countries would abandon any form of cooperation and act as singletons.

This paper contributes to the strand of IEA literature based on the noncooperative Nash equilibrium to the participation-game. We examine a mixed rather than a pure strategy equilibrium to this participation game, and we include a prior stage at which countries individually decide whether to invest in a public good that reduces abatement costs. For most (but not all) levels of abatement cost, a mixed strategy at the participation stage leads to a lower level of expected participation, and lower expected welfare, compared to the pure strategy equilibrium. In this respect, the model with mixed strategies is even more pessimistic than the model with pure strategies. However, for sufficiently low abatement costs (large enough benefits of cooperation), the mixed strategy equilibrium has high participation and nearly solves the free riding problem. Mixed strategies create endogenous risk; risk aversion increases the equilibrium probability of participation. In these respects, the mixed strategy equilibrium yields a more diverse set of outcomes compared to the pure strategy equilibrium.

The two participation games, corresponding to either mixed or pure strategies, induce two investment games. In general, there are multiple pure strategy equilibria to both of these investment games, but they have markedly different characteristics and different policy implications. With pure strategies at the participation stage, as emphasized by the canonical model, higher investment leads to lower participation and potentially lower welfare for all agents. With mixed strategies at the participation stage, we obtain conditions under which higher investment leads to higher participation and higher welfare for all agents. Due to the existence of multiple Pareto-ranked equilibria, there is a coordination problem in both of the investment games. In the game with pure strategies, society would like to coordinate on the low-investment equilibrium, and in the game with mixed strategies, society would like to coordinate on the high-investment equilibrium.

Golombek and Hoel (2005, 2006, 2008) consider the effect of investment with technological spillovers when membership in the IEA is exogenous; in our setting, investment affects IEA membership. Muuls (2004) and Bayramoglu (2010) consider firms' incentives to invest in technology, when firms anticipate that countries will bargain over the level of transboundary pollution; the participation problem is absent and investment is a private good in their models. Barrett (2006) studies the situation where countries choose investment non-cooperatively prior to the participation game, at which stage countries use pure strategies. He points out that opportunities to invest may not help to solve the participation problem. Hoel and de Zeeuw (2009) assume that the IEA chooses both investment and the abatement level after the participation stage, and they allow for the possibility that investment can drive the abatement cost to a level below the threshold at which abatement is a dominant strategy for individual countries; we exclude that possibility and we adopt the same timing as in Barrett (2006), but we emphasize mixed strategies at the participation stage.

Kohnz (2006) considers a mixed strategy in an IEA game with a first stage that determines the minimum number of members needed for ratification, as in Carraro et al. (2009); this first-stage decision may commit the IEA to behave sub-optimally, conditional on the number of members. In contrast, in our model, the minimum number of members needed for the IEA to decide to abate is determined by the optimization of the IEA; here, the IEA's decision maximizes the total welfare of its members, instead of being determined at an earlier stage. Sandler and Sargent (1995) and McGinty (2010) use mixed strategies in one-stage coordination games, but the issue of minimum participation does not arise in their settings.

Our paper is most closely related to Dixit and Olson (2000), which has a similar structure to Palfrey and Rosenthal (1984). Dixit and Olson rely primarily on numerical examples to characterize the mixed strategy equilibrium, whereas we emphasize analytic results. More importantly, the two papers have different objectives. Dixit and Olson's primary point is that Coasian bargaining does not solve the participation problem; they demonstrate this by showing that in general participation is low even though the resulting members of the agreement engage in Coasian bargaining. We find that in special circumstances Coasian bargaining *is* consistent with high participation, and that investment in abatement technology may lead to exactly these circumstances. As we noted above, under pure strategies this parametric model implies that investment lowers or leaves unchanged the level of participation.

2 Model basics and rationale for mixed strategies

We first review a canonical IEA model, then discuss the use of pure and mixed strategies at the participation stage, and then explain our modification to the canonical model.

2.1 A canonical IEA model

Barrett (1999) first proposed the following IEA model, which has the advantage of a simple analytic solution with clear intuition; see also Barrett (2003, Chapter 7), Ulph (2004), Burger and Kolstad (2009) and Kolstad (2011, Chapter 19). There are N identical countries, each of which has two sequential binary decisions: participation in an IEA and abatement. The cost and benefit of abatement are both linear, so in equilibrium each country abates at the level 0 or at capacity, normalized to 1. Given linearity, there is no additional loss in generality in assuming that the abatement decision is binary. Abatement is a global public good. By choice of units, the benefit of each unit of abatement, to each country, equals 1. Each country's abatement cost is c, with $1 < c < N$. The first inequality means that it is a dominant strategy for a country acting alone not to abate and the second inequality means that the world is better off when a country abates. In view of the normalization of benefit, c also equals the private cost-benefit ratio of abatement. Figure 1 shows the two stages of the game. In the first stage, each country decides whether to participate in an IEA. In the second stage, the IEA instructs all members whether to abate, and non-members make their own individual decisions on abatement. The IEA maximizes the total welfare of its members. We study a subgame perfect equilibrium.

In the abatement stage, non-members' dominant strategy is to not abate. In a subgame with m members in the IEA, one unit of abatement creates benefits of m units for the IEA and costs c. The optimal behavior of the IEA is to instruct its

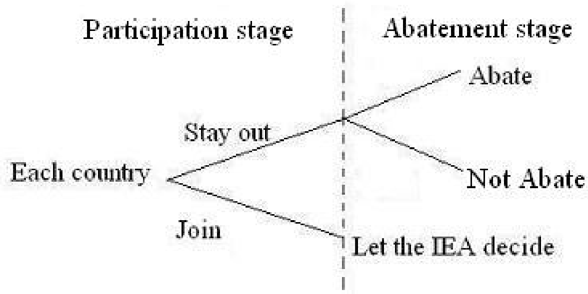

Figure 1: A canonical IEA model

members to abate if and only if $m - c \geq 0$. An IEA with $f(c)$ members, where the function $f(x)$ returns the smallest integer not less than x, is the "minimally successful IEA" under the assumption that the IEA optimizes conditional on membership.

Under the tie-breaking assumption that an agent who is indifferent between joining and not joining will join, the unique pure strategy Nash equilibrium at the participation stage consists of $f(c)$ members. For example, with $c = 3.2$, there are four members in the pure strategy equilibrium. An outcome with more than $f(c)$ members is not internally stable because the "extra" members would want to leave: each member's defection leaves unchanged other members' equilibrium action, resulting in a net benefit $c - 1 > 0$ to the defector. An outcome with $f(c)$ members is internally and externally stable; defection by a single member causes the IEA to instruct remaining members to not abate, causing a net loss to the defector $f(c) - c \geq 0$; a non-member loses $c - 1$ by joining. An outcome with $f(c) - 1$ members is not externally stable because by joining, a nonmember induces other members to abate and obtains a net benefit $f(c) - c \geq 0$. By the tie-breaking assumption, outcomes with fewer than $f(c) - 1$ members are also not externally stable.

Equilibrium global welfare here is $(N - c) f(c)$. *A small reduction in c can lead to a discrete reduction in equilibrium membership, consequently reducing global welfare.* The potential benefits of cooperation are $(N - c) N$. The fraction of benefits that are realized, $\frac{f(c)}{N}$, is weakly increasing in the abatement cost, c, and thus weakly decreasing in the potential benefits of cooperation. This conclusion reflects a pessimistic consensus of the literature: the IEA mechanism is especially ineffective when benefits of cooperation are large. Mixed strategies can reverse this conclusion.

2.2 Mixed strategies

Our contributions rely on the assumption that countries use mixed strategies, whereas most previous papers assume that countries use pure strategies. Here we discuss the rationale for using mixed strategies. With pure strategies, the Nash equilibrium selects the number but not the identity of the participants; ex ante identical agents behave differently. With (symmetric) mixed strategies, ex ante identical agents use the same strategy, and the number of participants is the realization of a random variable. A coordination problem arises with pure but not with mixed strategies: How does the group choose which agent participates and which stays outside the agreement under pure strategies? The coordination difficulty motivates the use of mixed strategies in the literature of corporate takeovers (Bagnoli and Lipman, 1988; Holmstrom and Nalebuff, 1992) and wars of attrition (Maskin, 2003).

A pure strategy equilibrium does not capture the uncertainty about the ability of nations to agree on a meaningful treaty; a mixed strategy equilibrium captures exactly this uncertainty. Benedick (2009, page xv), in discussing his experience as a US negotiator in many IEAs, objects to what he considers academics' tendency to view the negotiating process as mechanistic, yielding a deterministic outcome. He emphasizes the contingency of the process, and the possibility of surprises. The empirical literature that attempts to explain participation in IEAs (Murdoch, Sandler and Vijverberg, 2003) of course uses a probabilistic model.

Harsanyi (1973) points out that a mixed strategy equilibrium of a game with complete information can be interpreted as a pure strategy Bayesian Nash Equilibrium of a closely related game with a small amount of incomplete information. Makris (2009) suggests that restricting attention to the symmetric mixed-strategy equilibrium of the game with no uncertainty provides a shortcut for a model where there is very small uncertainty about the players' preferences and the expected group-size is large. Crawford and Haller (1990) consider games in which identical players have identical preferences among multiple equilibria, but must learn to coordinate over which equilibrium to play by repeatedly playing the game. They show that in finite time the equilibrium converges to a strategy profile in which all players obtain equal expected payoffs. In the IEA game only the symmetric mixed-strategy equilibrium has this property.

Finally, given that much of the IEA theory focuses on pure strategies, it is at least worthwhile looking at how insights might change due to the use of another equilibrium concept.

2.3 Our modification

We alter the model in Subsection 2.1 in two respects. First, we consider the symmetric mixed strategy equilibrium at the participation stage. Countries choose a participation probability, understanding that the IEA will instruct its members to abate if and only if the number of members is at least $f(c)$. Second, we include a stage prior to participation in which countries individually decide whether to invest in a public good that reduces abatement costs. In this model, agents have three sequential binary decisions: investment, participation, and abatement; there are two public goods: investment and abatement.

At the participation stage c is given, but previous investment determines its level. If k countries invested, $c = c(k)$, a decreasing function. The cost of investment is ε, which might be either large or small. Countries have rational expectations, and understand that their investment decision affects c and thereby affects equilibrium expected participation. We are especially interested in situations where $c(N)$ is only slightly larger than 1. In this case, if N countries invest, equilibrium membership in the pure strategy equilibrium is 2, and few of the potential benefits of cooperation are realized.

We have in mind investment in developing a technology that can be easily replicated. Patent law enables investors to capture some of the benefit of their investment, but in many cases there are significant spillovers. Our emphasis on the situation where investment is a public good captures an important element of the problem. Modeling investment as a public good means that agents that did invest in the first stage and those that did not invest have the same abatement costs at the participation stage. Consequently, we can consider symmetric strategies at the participation stage, a fact that greatly simplifies the analysis. The public good nature of investment also means that for the world as a whole, investment has a high return.

3 The participation stage

We characterize the equilibrium mixed strategy, $p = p(c, N)$, the probability that any country joins the IEA at the participation stage. We emphasize the risk neutral case. Here, for most but not all of parameter space, the model with mixed strategies is more pessimistic than the model with pure strategies. This result is interesting as a robustness check to one of the major insights of this literature: optimal behavior on the part of the IEA does little to resolve the problem of providing a global public good. We then note that risk aversion increases equilibrium participation probabilities.

In the trivial case where $N = f(c)$ there exists no strictly mixed strategy equilibrium. In the rest of this paper, we focus on the nontrivial case $N > f(c)$. The IEA instructs its members to abate if and only if there are at least $f = f(c)$ members. Therefore, a country is pivotal if and only if exactly $f(c) - 1$ other countries join. If fewer than $f(c) - 1$ other countries join, then there would still be too few members to elicit abatement even if an additional country joins. If more than $f(c) - 1$ other countries join, then membership by an additional country is (from its own standpoint) superfluous: by joining it has no effect on other countries' abatement but it incurs the net cost $c - 1 > 0$.

With the probability of each country joining equal to p, the probability that a country is pivotal is

$$g(p) = \frac{(N-1)!}{(f-1)!\,(N-f)!} p^{f-1} (1-p)^{N-f}.$$

The probability that at least f other countries join is

$$G(p) = \sum_{i=f}^{N-1} \frac{(N-1)!}{i!\,(N-1-i)!} p^{i} (1-p)^{N-1-i}.$$

The net expected benefit of joining is the benefit of joining when the country is pivotal, $f - c$, times the probability that it is pivotal, g, minus the loss when the IEA would have abated even had this country not joined, $c - 1$, times the probability of that event, G. In a mixed strategy equilibrium, p must be such that a country is indifferent between joining and not joining. The equilibrium condition for p is therefore

$$(f - c)\, g(p) = (c - 1)\, G(p). \tag{1}$$

Remark 1 *The right side of equation (1) is continuous in c. The left side is discontinuous in c at integers; as c approaches an integer (greater than* 1*) from below, the left side approaches* 0*; moreover, the left side equals* 0 *when c is an integer. For $c > 1$ the right side equals 0 if and only if $p = 0$. Thus, as c approaches an integer greater than* 1 *from below, the equilibrium p approaches* 0*, and $p = 0$ is the only mixed strategy equilibrium when c is an integer.*

For non-integer values of $c > 1$, there are two solutions to equation (1), $p = 0$ and $0 < p < 1$. We are interested in the latter. To study non-integer values of c we rearrange equation (1) to obtain

$$\frac{f - c}{c - 1} = \frac{G(p)}{g(p)}. \tag{2}$$

When $p \neq 1$ and $p \neq 0$, the right hand side of equation (2) is well defined because $g(p) \neq 0$. We simplify the right side of equation (2) using

$$\frac{G(p)}{g(p)} + 1 = (f - 1) \int_0^1 (1 - t)^{f-2} \left(1 + t\frac{p}{1 - p}\right)^{N-f} dt \equiv \sigma(p; N, f). \tag{3}$$

The appendix contain the derivation of this equation and proofs of some propositions.

Equation (3) leads to a simple demonstration of the following:[1]

Proposition 1 *(i) When c is a non-integer, there is a unique nontrivial symmetric mixed strategy equilibrium $p(c, N) > 0$. (ii) Over any interval of c for which f is constant, the equilibrium mixed strategy p is a decreasing function of c.*

Figure 2 illustrates the comparative statics of p with respect to c, holding $f(c)$ constant. The monotonic curves graph the right side of equation (1) and the hump shaped curves graph the left side. The solid curves correspond to $c = 1.2$ and the dashed curves correspond to $c = 1.4$, both with $N = 20$. The figure shows that an increase in c, holding $f(c) = 2$, decreases the left side and increases the right side of the

[1] In Palfrey and Rosenthal's (1984) "refund case", the probability of joining is monotonically decreasing in the cost of contribution (Corollary 5.1). In their model, the benefit of being pivotal is fixed at 1 while in our model the benefit of being pivotal is $f - c$. When $f = c$, the benefit is 0 and thus $p = 0$. However, as c increases slightly, $f - c$ jumps to almost 1. This difference explains why the relation between p and c is monotone in their model whereas it is a saw-tooth in our model.

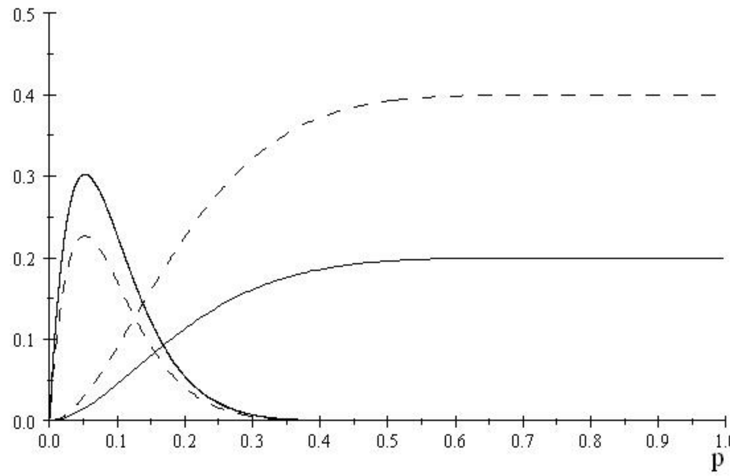

Figure 2: Monotonic curves show $(c-1)G$ and hump-shaped curves show $(f-c)g$. Solid curves correspond to $c = 1.2$ and dashed curves correspond to $c = 1.4$. $N = 20$ for both pairs of curves.

equilibrium condition, thereby reducing the equilibrium value of p. Intuitively, with $f(c)$ being fixed, an increase in c decreases the benefit of joining when the country is pivotal, $f - c$, increases the loss of joining when the IEA would have abated even had this country not joined, $c - 1$, and consequently makes joining the IEA less attractive; as a result, p is lowered in equilibrium.

Remark 1 and Proposition 1 imply that the graph of p as a function of c is a saw-tooth, with upward jumps occurring at integers. Figure 3 illustrates this relation for $N = 20$ and $c \in (1.01, 6)$. In the pure strategy equilibrium m denotes the deterministic number of participants; here we use m to denote the *expected* number of participants: $m \equiv pN$. Because m is proportional to p, expected membership is also a saw-toothed function of c.

The next result compares the equilibrium expected membership under mixed strategies with membership in the pure strategy equilibrium:

Proposition 2 *(i) In the interval $c \in (j-1, j]$ for any integer $j \geq 3$, the expected membership in the mixed-strategy equilibrium, m, is less than j (the membership of the corresponding pure-strategy equilibrium). (ii) In the interval $c \in (1, 2]$, m is greater than 2 (the membership of the corresponding pure-strategy equilibrium) for c sufficiently small. Moreover, as c approaches its lower bound 1, p converges to 1, and m converges to N.*

Table 1 summarizes the results in Proposition 2.

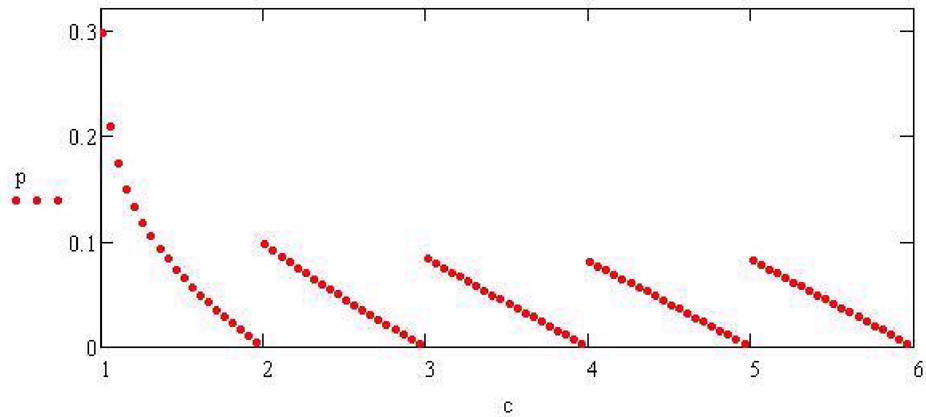

Figure 3: The relation between p and c with $N = 20$ and $c \in (1.01, 6)$.

c	Pure Strategy	Mixed Strategy
$c > 2$	$m = f(c)$	$m < f(c)$
$1 < c \leq 2$	$m = 2$	$m \to N$ when $c \to 1$

Table 1: Comparison of (expected) membership under mixed and pure strategies

As noted above, an important implication of the pure strategy equilibrium in this game is that the fraction of countries that participate in equilibrium, $\frac{f(c)}{N}$, is high if and only if the potential global benefit of cooperation, $N(N-c)$, is small; this result arises because equilibrium membership equals the size of the minimally successful IEA, which is weakly increasing in c and is always less than $c+1$. The saw-toothed relation between p and c under mixed strategies implies that higher costs have an ambiguous effect on expected participation when countries use mixed strategies. When c increases, holding $f(c)$ constant, the probability of participation and expected participation decrease; however, if c increases from slightly below to slightly above an integer, the probability of participation and expected participation both increase. Moreover, we find that for $c \geq 2$ expected participation is lower under mixed strategies compared to pure strategies. In this respect, the model with mixed strategies is even more pessimistic than the model with pure strategies.

An important difference is that when abatement costs are sufficiently low, expected participation is high when countries use mixed strategies. The explanation is straightforward. If c is close to 1, a country's loss when it joins the coalition when the IEA would have abated even had this country not joined, $c-1$, is close to 0. However, it loses nearly one unit $(2-c)$ in the case when the membership does not reach the critical size of 2 due to its non-participation. Therefore, when c is close to 1, the benefit of joining looms larger than its cost for moderate values of p; a country is indifferent between joining and staying out, only if it is very confident that the IEA will reach its critical size, i.e. when p is close to 1.

The equilibrium p is close to 1 only as the cost-benefit parameter, c, gets close to the threshold 1, below which abatement is a dominant strategy. For example, for $N = 10$, in equilibrium $p > 0.5$ if and only if $c - 1 < 0.018$, and $p > 0.75$ if and only if $c - 1 < 1.04 \times 10^{-4}$; for $N = 20$, in equilibrium $p > 0.5$ if and only if $c - 1 < 3.6 \times 10^{-5}$, and $p > 0.75$ if and only if $c - 1 < 2.1 \times 10^{-10}$. If one interprets this model literally, the conclusion is that for almost all parameter space the outcome with pure strategies provides an optimistic upper bound to the outcome when agents use mixed strategies. Thus, the results here provide a robustness check for the pessimistic result that optimal behavior by the IEA does little to solve the problem of providing a global public good.

It may be a mistake to adopt a literal interpretation of a model that is intended to provide qualitative insights. The pure strategy equilibrium has a discontinuity at $c = 1$: if c is slightly greater than 1, the IEA has only 2 members; if c is slightly less than 1 it is a dominant strategy for countries to abate and there is no need for an IEA. The new insight here is that mixed strategies eliminate the discontinuity at $c = 1$: as c approaches 1 from above, optimal behavior by the IEA essentially solves the problem of providing the public good. This result reverses the pessimistic conclusion in the literature that the IEA is especially ineffective when the potential benefit of cooperation is large. Moreover, the real interest in mixed strategies is that they produce qualitatively different insight about the role of investment, as we discuss in Section 4.

In the wake of the negotiations at Copenhagen 2009, some people believe that a smaller group of countries will have a better chance of achieving a successful climate agreement. Of course, the model with homogenous agents ignores the heterogeneity that actually exists among countries, one of the most salient features of the climate problem. Nevertheless, it is interesting to note that an increase in the number of agents does reduce the equilibrium probability that any country will participate, and has ambiguous effects on the expected level of participation. The following proposition summarizes these comparative statics.[2]

Proposition 3 *(i) The equilibrium probability of joining, p, decreases with the number of countries, N. (ii) For N sufficiently small,* $\frac{dm}{dN} < 0$*, and for N sufficiently large* $\frac{dm}{dN} > 0$*. (iii) When c converges to* 1 *from above,* $\frac{dm}{dN} > 0$*. (iv) When c converges to f from below,* $\frac{dm}{dN} < 0$*.*

It can be shown that, for a given p, the ratio of the probability that a country is pivotal over the probability that more than $f - 1$ other countries join, $\frac{g(p)}{G(p)}$, decreases with N. The intuition for the first part of Proposition 3 is that, for a given p, as N increases, it becomes relatively less likely that a country will be pivotal and relatively more likely that the additional membership of this country will be superfluous. It is

[2] To conserve space, we relegate the proofs of Propositions 3 and (4) to the working paper Hong and Karp (2011).

thus less attractive for a country to join. In order to maintain indifference, the value of p must decrease. It is not surprising that expected membership, pN, is non-monotonic in N: a larger N increases one factor in this expectation and decreases the other factor.

The next proposition gives the formula for the equilibrium expected participation as $N \to \infty$.[3]

Proposition 4 *As $N \to \infty$, (i) $p \to 0$ and (ii) $m \to \lambda$, where λ is determined by*

$$\frac{1}{c-1} = \int_0^1 (1-t)^{f-2} e^{t\lambda} dt; \tag{4}$$

and (iii) membership follows a Poisson distribution with parameter λ.

The right side of equation (4) increases in λ while the left side decreases in c. Therefore, over a region where f remains constant, the equilibrium λ decreases in c. Where c passes through an integer value from below, the right side jumps down for given λ so the equilibrium λ must jump up. Thus, in the limit as $N \to \infty$ we have the same kind of saw tooth pattern as in the case with finite N.

In moving from pure to mixed strategies with no exogenous uncertainty, we create *endogenous* risk about participation. The model above assumes that the payoff equals the expected value of abatement minus abatement costs, i.e. it assumes risk neutrality. Our working paper (Hong and Karp, 2011) shows that the equilibrium participation probability increases with the level of risk aversion: concave transformations of the risk neutral payoff increase expected participation. Examples show that for moderate levels of constant relative risk aversion, countries are likely to join the IEA when environmental damage is large relative to the baseline level of income (when no country abates).

4 The investment stage

Here we investigate how, under risk neutrality, the use of pure or mixed strategies in the participation game affects the equilibrium in the investment game, and overall equilibrium welfare. By incurring the investment cost ε, a country reduces the abatement cost in the next stage. We emphasize the situation where countries use pure strategies at the investment stage.

The two participation games (with pure strategies and with mixed strategies) induce two investment games. In both games, there may be multiple equilibria, but the nature of the multiplicity is quite different. When countries use pure strategies at the participation game, the multiplicity at the investment stage arises from the non-monotonicity of payoffs in costs. In this game, simulations show that the equilibrium with higher investment leads to lower welfare, exclusive of investment costs. Because

[3] Myerson (1998a, 1998b, 2005) uses the Poisson limit theorem to study the limiting behavior in binary action games when the number of players approaches infinity.

the countries that invest incur the investment cost, all countries have lower welfare in the high-investment equilibrium, so the low-investment equilibrium is Pareto dominant.

The investment game when countries use mixed strategies at the participation stage is more interesting. The fact that p, and therefore $m = pN$, is a saw-toothed function of c means that the reduction in c caused by higher investment might lower equilibrium participation and hence lower welfare, just as with pure strategies. However, Proposition 2.ii implies that if the investment lowers abatement costs sufficiently, the benefit of investment may be high. In this case, the investment decisions may be strategic complements. Consider matters from the standpoint of a country that has not yet decided whether to invest. If that country believes that few other countries will invest, it may rationally calculate that its own investment would lower expected participation in the next stage and could be unprofitable even if investment costs are negligible. However, if it believes that many other countries will invest, the additional decrease in abatement costs caused by its investment might then increase expected membership and be profitable. In this case, the mixed strategy at the participation game induces a coordination game at the investment stage, with high and low investment equilibria. Here, the high investment equilibrium is likely to be Pareto-dominant.

The non-monotonicity of payoffs, and the fact that they are defined only over integer values of k, complicates the analysis. However, the basic conclusions are intuitive, and we illustrate these using a numerical example. Additional simulations confirm that the reported results are robust.

4.1 Pure strategies at the participation stage

Investment is a pure public good, so a country's investment decision does not distinguish that country at the participation stage. As in Barrett (2006) and Rubio and Ulph (2007), we therefore assume that all countries expect to face the same probability that they will be one of the participants in the next stage, regardless of whether they invested. Therefore, the probability that any country joins the IEA in the pure strategy equilibrium is simply the number of members, $f(c)$, divided by the total number of countries, N. Let $\varpi(k)$ be the next-stage expected payoff given investment level k:

$$\varpi(k) = f(c(k)) - \frac{f(c(k))}{N} c(k).$$

The benefit of investment for a country given that $k-1$ other countries invest is the change in its next-stage expected payoff:

$$\phi(k-1) = \varpi(k) - \varpi(k-1).$$

Given the belief that $k-1$ other countries invest, a country will invest if and only if $\phi(k-1) \geq \varepsilon$.

If $\phi(0) < \varepsilon$, then there is a corner equilibrium in which no country invests; if $\phi(N-1) \geq \varepsilon$, there is a corner equilibrium in which every country invests. If there exists a K, with $0 < K < N$ and $\phi(K-1) > \varepsilon > \phi(K)$, then there is an interior equilibrium in which K countries invest and $N-K$ countries do not invest.

Global welfare as a function of k is

$$\Phi(k) = (N - c(k)) f(c(k)) - k\varepsilon.$$

A larger value of k lowers abatement costs but it increases total investment costs and might decrease participation; therefore, global welfare may be non-monotonic in k. The difference in global welfare under zero investment and under a positive value of k is

$$\Delta(k) \equiv \Phi(0) - \Phi(k) = (N - c(0))[f(c(0)) - f(c(k))] - f(c(k))[c(0) - c(k)] + k\varepsilon.$$

Remark 2 *When investment changes the level of participation (i.e. $f(c(0)) - f(c(k)) > 0$), $\Delta(k) > 0$ for sufficiently large N. In this situation, global welfare is lower under $k > 0$ compared to $k = 0$.*

The following example illustrates some of the possibilities:

$$c(k) = \frac{5.99}{0.248k + 1},\ N = 20,\ \varepsilon = 0.11 \tag{5}$$

With this example, unit abatement costs fall from 5.99 to 1.005 and participation in the pure strategy equilibrium falls from 6 members to 2 members as the amount of investment, k ranges from 0 to 20.

Table 2 shows values of the net benefit of investment, $\phi(k-1) - \varepsilon$, and global welfare, Φ, for several values of k; the asterisks identify the equilibrium values of k. For this example $k = 0$ and $k = 2$ are the only two equilibria. If no other countries invest, it does not pay any country to invest ($\phi(0) - \varepsilon < 0$). If one other country invests, it pays any country to invest ($\phi(1) - \varepsilon = 0.09 > 0$), but once two (or more) countries invest, further investment yields a negative payoff. Thus, $k = 2$ is the only interior equilibrium. In this example, both investors and non-investors have a higher payoff when 0 rather than 2 countries invest, so the non-investment equilibrium is Pareto-dominant.

k	$\phi(k-1) - \varepsilon$	$c(k)$	m	$\Phi(k)$	$\varpi(k)$
$0*$	NA	5.99	6	84.1	4.20
1	-0.51	4.80	5	75.9	3.8
$2*$	0.09	4.004	5	79.8	4.00
3	-0.80	3.43	4	65.9	3.31
4	-0.02	3.01	4	67.5	3.40

Table 2: Investment, membership and welfare with pure strategies at the participation stage. Equilibrium k denoted by $*$.

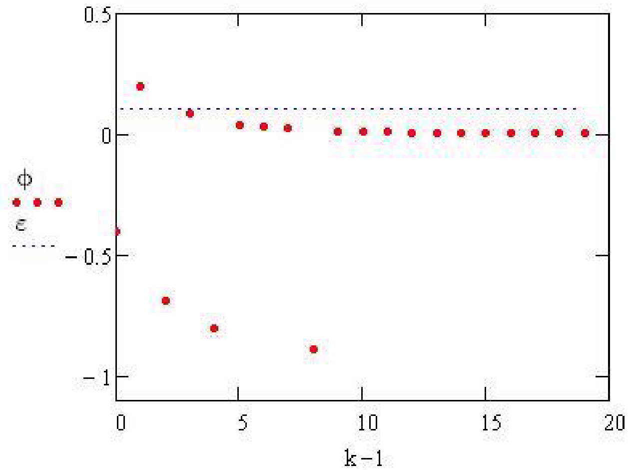

Figure 4: The individual benefit of investment under pure strategies

This example also shows that the multiplicity of equilibrium is due to the non-monotonicity of the payoffs in c, and disappears for slightly larger investments costs. Here, for $\varepsilon > 0.11 + 0.09 = 0.2$ there is a unique equilibrium, $k = 0$. The example also shows that if investment were determined by a social planner who maximizes world welfare (rather than as a non-cooperative equilibrium), the planner would choose 0 investment.

Figure 4 shows the benefit to investment as a function of $k-1$. The function is non-monotonic; however, because an interior equilibrium k requires $\phi(k-1) > \varepsilon > \phi(k)$, only values of k for which $\phi(k-1) > 0$ are candidates for an interior equilibrium. For the set where $\phi(k-1) > 0$, the function $\phi(k-1)$ is decreasing in k. This fact means that there is at most one interior equilibrium, and as we increase ε from 0, the interior equilibrium has a smaller k; for ε above a critical level (0.2 in this example), the interior equilibrium disappears.

4.2 Mixed strategies at the participation stage

The assumption that investment is a public good means that both non-investors and investors have the same probability of participation. The equilibrium p is a function of c, which depends on the level of investment, k, so the equilibrium p is a function of k. For given k, a country's expected welfare equals the probability that it participates times its expected payoff conditional on participation, plus the probability that it does not participate times its expected payoff conditional on non-participation. In equilibrium, these two expectations (conditioned on the different actions) are equal, so the expected payoff equals

$$w(k) = \sum_{i=f}^{N-1} \frac{(N-1)!}{i!\,(N-1-i)!} p^i (1-p)^{N-1-i}\, i, \tag{6}$$

which is the expected abatement when at least one country does not join. When $k-1$ other countries invest, the benefit to a country of also investing is

$$\Omega(k-1) = w(k) - w(k-1).$$

If $\Omega(0) < \varepsilon$, then there is a corner equilibrium in which no country invests; if $\Omega(N-1) \geq \varepsilon$, there is a corner equilibrium in which every country invests; again, there may be interior equilibria.

As c varies between adjacent integers (so that f is fixed), the left side of equation (2) is continuous in c, and the right side is continuous in p, by equation (3); moreover, the derivative of the right side with respect to p is bounded away from 0 for $p \in (0,1)$. Therefore, the equilibrium $p(c)$ is continuous on the domain $c \in (1,2)$. Remark 1 and Propositions 1 and 2 state that over this domain, p decreases in c, implying that the range of p is $(0,1)$. Consequently, for any $c(0) > 1$ there exists a $\hat{c} \in (1,2)$ that solves $p(\hat{c}) = \frac{f(c(0))}{N}$, i.e. $\hat{c} = p^{-1}\left(\frac{f(c(0))}{N}\right)$, a weakly decreasing step function in $c(0)$. The following proposition considers the case where a high level of investment leads to large cost reductions, which may then lead to relatively high participation and abatement. In this situation, a country's expected payoff (exclusive of investment costs) is monotonic in investment for sufficiently high levels of investment, and is maximized when all countries invest.

Proposition 5 *Assume that N is sufficiently large and c decreases sufficiently rapidly in k that there exists a smallest integer $\hat{k} \leq N$ at which $c(\hat{k}) \leq \hat{c}$. In this case, for any $k_1 \geq \hat{k}$ and all $k_2 < k_1$, $w(k_1) > w(k_2)$: $w(k)$ is therefore maximized at $k = N$.*

The sufficient condition $c(N) \leq \hat{c}$ is equivalent to the condition

$$\begin{aligned} Np(c(N)) &\geq f(c(0)) \Longleftrightarrow c(N) \leq p^{-1}\left(\frac{f(c(0))}{N}\right) \\ &\Longleftrightarrow \frac{c(0)}{c(N)} \geq \frac{c(0)}{p^{-1}\left(\frac{f(c(0))}{N}\right)} \equiv \alpha(c(0)). \end{aligned}$$

The function $\alpha(c(0))$ is increasing for non-integer values of $c(0)$ and has a discontinuous upward jump as $c(0)$ passes through an integer value from below. Thus, the sufficient condition (for aggregate welfare excluding investment costs to be higher when all countries invest) is equivalent to the condition that the factor by which investment by all countries reduces abatement costs, $\frac{c(0)}{c(N)}$, exceeds a critical level, $\alpha(c(0))$. This condition is intuitive: investment increases welfare whenever investment lowers abatement costs sufficiently. Figure 5 graphs this critical value for $N = 20$ and $c(0) \in (2,7)$. For much of this domain, the graph of the critical value is between 2 and 6.

In the investment game, interior equilibria might be eliminated by increasing ε, for the same reason as with pure strategies. For example, if $\varepsilon \geq w(k)$ for any k such that $c(k) > \hat{c}$, then $\varepsilon > w(k) - w(k-1)$; in this case, there can be no interior equilibria

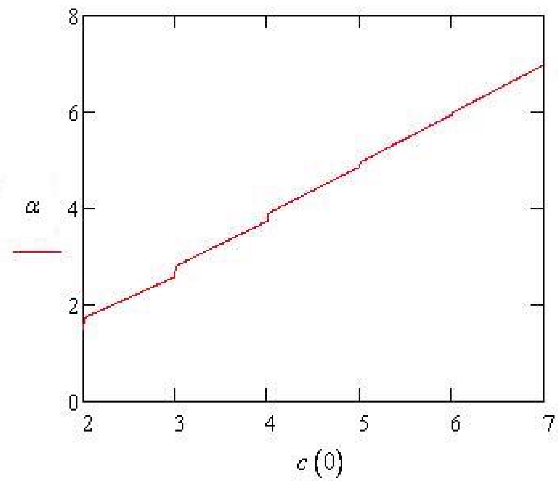

Figure 5: The critical ratio of $\frac{c(N)}{c(0)}$ for $N = 20$ and $c(0) \in (2, 7)$.

with investment less than $\hat{k}$. By eliminating equilibria, a large value of ε eases the coordination problem. Of course, an extremely large value of ε eliminates all equilibria with positive investment. However, if $\hat{k} < N$, and

$$\Omega(N-1) > \varepsilon > \Omega(k-1) \text{ for all } k < \hat{k}$$

then there are two equilibria, at which $k = 0$ and $k = N$. (There may be other equilibria with investment $\hat{k} \leq k < N$.) If it is individually rational for a country to invest, then that investment also increases global welfare. We saw that with pure strategies at the participation stage, it might be individually rational to invest in cases where this lowers social welfare. The expected global welfare as a function of k is

$$\Psi(k) = Nw(k) - k\varepsilon.$$

Using example (5), Figure 6 shows that (under second stage mixed strategies) the benefit to investment, although non-monotonic, increases rapidly for large k. The figure also shows horizontal lines corresponding to three possible values of ε. An interior equilibrium requires $\Omega(k-1) > \varepsilon > \Omega(k)$. Increases in ε eliminate the smaller equilibria, but not the boundary equilibrium $k = N$ until ε becomes large.

Table 3 shows levels of welfare and expected participation for several values of k, including all of the equilibrium values when $\varepsilon = 0.11$, marked by an asterisk. There are four equilibria, with $N = 20$ giving the highest level of welfare both for investors and non-investors. Increasing ε to greater than $0.11 + 0.15 = 0.26$ removes the $k = 4$ equilibrium, and increasing ε to greater than $0.11 + 0.28 = 0.39$ removes the $k = 8$ equilibrium. Zero investment is always an equilibrium, and $k = 20$ is an equilibrium provided that $\varepsilon \leq 0.11 + 1.98 = 2.09$.[4]

[4] We also examined the equilibria of the investment game under logarithmic utility (contant relative

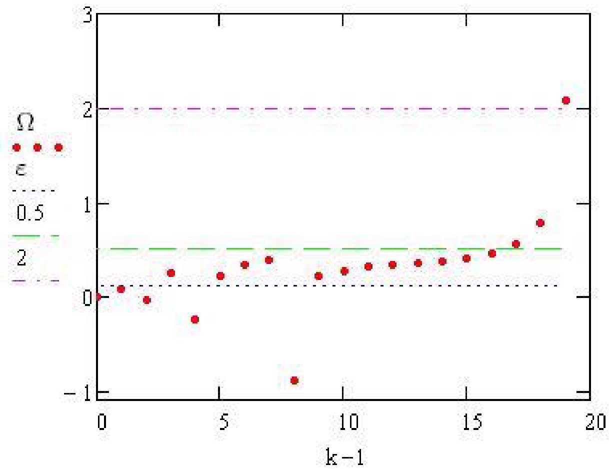

Figure 6: The benefit to investment $\Omega(k-1)$ and three levels of costs, $\varepsilon \in \{0.11, 0.5, 2\}$

k	$\Omega(k-1)-\varepsilon$	$c(k)$	m	$\Psi(k)$	$w(k)$
$0*$	NA	5.99	0.018	1.28×10^{-12}	6.4×10^{-14}
1	-0.11	4.80	0.34	-0.11	6.1×10^{-5}
$4*$	0.15	3.007	1.68	5.61	0.302
5	-0.36	2.67	0.58	0.54	0.054
$8*$	0.28	2.007	1.94	18.86	0.987
9	-2.01	1.85	0.34	0.72	0.086
$20*$	1.98	1.005	6.54	121.81	6.21

Table 3: Investment, expected membership and welfare with mixed strategy participation. Equilibrium k denoted by $*$

4.3 Comparison

It is obvious that if investment cost are sufficiently low, and if a sufficiently high level of investment can drive abatement costs below the threshold at which abatement is a dominant strategy ($c = 1$), then in both models above there is an equilibrium in which all countries invest and the first-best outcome is achieved. When the lowest possible abatement cost, $c(N)$, remains (slightly) above 1 (a case of main interest to us), however, the choice between pure and mixed strategies at the participation stage induces two types of investment games: a discontinuity again arises at $c = 1$ with pure strategies played at the participation stage but not with mixed strategies. In the

risk aversion) over business-as-usual income (zero abatement) plus abatement benefits minus abatement costs. For sufficiently low investment costs, ε, there is an equilibrium in which every country invests. Expected membership (abatement level) and welfare at the participation stage are maximized in the full investment equilibrium.

former case, when there are two investment equilibria (a boundary equilibrium $k = 0$ and an interior equilibrium), the 0-investment equilibrium has higher membership, and with N sufficiently large, higher welfare for both investors and non-investors. In contrast, when agents use mixed strategies at the participation stage, for sufficiently low investment costs, there is a boundary equilibrium at which *all* countries invest, as well as interior equilibria and the boundary equilibrium at which no country invests. Expected participation and welfare are higher at the equilibria with high investment that drives abatement costs close to 1.

The two types of equilibria in the participation game therefore lead to different policy implications regarding investment. The pure strategy equilibrium, emphasized by the previous literature, leads to a game in which even if investment might occur in equilibrium, that investment would likely lower abatement and welfare. This result occurs because the pure strategy participation equilibrium equals the minimally successful IEA, which is weakly increasing in abatement costs. The mixed strategy equilibrium, which we emphasize, leads to a game in which again investment might occur in equilibrium, but here it increases abatement and welfare.

Unless the abatement cost can be reduced to a level *extremely* close to the threshold $c = 1$, the implications of the game with mixed strategies – like those of the game with pure strategies – are pessimistic. In our example, expected participation in the mixed strategy when all countries invest (6.54) is only slightly higher than participation in the pure strategy when no countries invest (6). Expected welfare is much higher in the former case (122 compared to 84) due to the lower abatement cost resulting from investment. However, even the larger level is only a fraction of the potential global welfare, equal to $(20 - 1.005 - 0.11)\, 20 = 377.7$, so both models are pessimistic about being able to capture the benefits of cooperation. We also found examples where expected participation under mixed strategies is lower but expected welfare higher, compared to the levels under pure strategies. Thus, when costs are endogenous, determined at the investment stage, the model with mixed strategies may be less pessimistic than the model with pure strategies.

In the interest of completeness, we also examined the use of mixed strategies at the investment stage. For the example in equation (5), we found that the unique mixed strategy investment equilibrium, given that countries use pure strategies at the participation stage, involves a zero probability of investment. When countries use mixed strategies at both stages, the levels of expected membership and welfare are between the highest and the lowest equilibrium levels corresponding to the scenario (reported in Table 3) in which countries use pure strategies at the investment stage and mixed strategies at the participation stage.

5 Summary

Most previous theory about IEAs (within the strand of literature that we follow) uses a participation game with a pure strategy equilibrium. These models (typically) imply that the equilibrium participation is low and that investment that reduces abatement costs also reduces the incentives to join the IEA, resulting in lower equilibrium membership. Under risk neutrality and predetermined abatement costs, the use of mixed rather than pure strategies tends to reinforce the conclusion that optimal behavior by the IEA does not solve the problem of providing a global public good. For low abatement costs, however, the use of mixed strategies overturns this conclusion.

More significantly, the participation games with mixed and with pure strategies have opposite policy implications with regard to investment. In both of the investment games induced by the two types of strategies at the participation stage, there may be multiple Pareto-ranked (pure strategy) investment equilibria. Under pure strategies at the participation stage, society would like to coordinate on a low-investment equilibrium, and under mixed strategies at the participation game, society would like to coordinate on the high-investment equilibrium. The overall policy conclusion is that when nations use mixed strategies, the promotion of investment that lowers abatement cost can ameliorate the problem of climate change, even though it is unlikely to provide a complete solution to that problem.

A Appendix

This appendix derives equation (3), and proves Propositions 1, 2, and 5.

Derivation of equation (3) To ease notation, we define $n = N - 1$, the number of "other" countries and $q = f - 1$, the number of "other members" needed for a country to be pivotal; because $N > f$, we have $n \geq q + 1$. Using these definitions we have

$$\frac{G(p)}{g(p)} = \frac{\sum_{i=q+1}^{n} \frac{n!}{i!(n-i)!} p^i (1-p)^{n-i}}{\frac{n!}{q!(n-q)!} p^q (1-p)^{n-q}}$$
$$= \frac{\sum_{i=q}^{n} \left(\frac{n!}{i!\,(n-i)!} p^i (1-p)^{n-i} \right) - \left(\frac{n!}{q!\,(n-q)!} p^q (1-p)^{n-q} \right)}{\frac{n!}{q!\,(n-q)!} p^q (1-p)^{n-q}}$$
$$= \frac{\sum_{i=q}^{n} \left(\frac{n!}{i!\,(n-i)!} p^i (1-p)^{n-i} \right)}{\frac{n!}{q!\,(n-q)!} p^q (1-p)^{n-q}} - 1.$$

Define

$$\sigma = \frac{\sum_{i=q}^{n} \left(\frac{n!}{i!\,(n-i)!} p^i \left(1-p\right)^{n-i} \right)}{\frac{n!}{(q)!\,(n-q)!} p^q \left(1-p\right)^{n-q}} = \text{hypergeom} \left(\left[1, -n+q\right], \left[1+q\right], \frac{p}{-1+p} \right) \tag{7}$$

where hypergeom $(\cdot)$ is the hypergeometric function. Using the definition of the hypergeometric function, and the fact that the gamma function $\Gamma(x+1) = x!$ when x is an integer, we write[5]

$$\begin{aligned} \sigma &= \tfrac{\Gamma(1+q)}{\Gamma(1)\Gamma(1+q-1)} \int_0^1 \tfrac{t^{1-1}(1-t)^{1+q-1-1}}{\left(1-t\frac{p}{p-1}\right)^{q-n}} dt \\ &= \tfrac{\Gamma(1+q)}{\Gamma(q)} \int_0^1 \tfrac{(1-t)^{q-1}}{\left(1-t\frac{p}{p-1}\right)^{q-n}} dt = q \int_0^1 \tfrac{(1-t)^{q-1}}{\left(1+t\frac{p}{1-p}\right)^{q-n}} dt \\ &= q \int_0^1 (1-t)^{q-1} \left(1 + t\tfrac{p}{1-p}\right)^{n-q} dt. \end{aligned} \tag{8}$$

[5] To check this result we also derived equation **(3)** without using the hypergeometric function and instead using the identity

$$\sum_{i=q}^{n} \left(\frac{n!}{i!\,(n-i)!} p^i \left(1-p\right)^{n-i} \right) = \frac{n!}{(q-1)!\,(n-q)!} \int_0^p t^{q-1} \left(1-t\right)^{n-q} dt$$

from Wang (1994, page 116); details are availble on request.

Proof. (Proposition 1) Part (i) The fact that t is defined over a positive interval implies

$$\frac{d\left(1+t\frac{p}{1-p}\right)}{dp}=\frac{t}{\left(-1+p\right)^2}>0.$$

Using this expression, we have

$$\frac{d\frac{G(p)}{g(p)}}{dp}=\frac{d\sigma}{dp}=q\int_0^1\left(1-t\right)^{q-1}\left(n-q\right)\left(1+t\frac{p}{1-p}\right)^{n-q-1}\frac{t}{\left(-1+p\right)^2}dt>0. \tag{9}$$

Define $x=\frac{p}{1-p}$. By using the last equality of equation (8), we have

$$\sigma\left(0\right)=\int_0^1 q\left(1-t\right)^{q-1}dt=1,^{6}$$

$$\frac{d\sigma}{dx}=q\int_0^1\left(1-t\right)^{q-1}\left(n-q\right)\left(1+tx\right)^{n-q-1}tdt>0,$$

and

$$\frac{d^2\sigma}{dx^2}=q\int_0^1\left(1-t\right)^{q-1}\left(n-q\right)\left(n-q-1\right)\left(1+tx\right)^{n-q-2}t^2dt\geq 0.$$

for any $n\geq q+1$. Because σ is increasing and convex in x, the domain of x is $[0,\infty)$ and $\sigma\left(0\right)=1$, we know that the range of σ is $[1,\infty)$. Thus $\sigma-1$ is increasing and convex in x and the range of $\sigma-1$ is $[0,\infty)$. When c is not an integer, $\frac{f-c}{c-1}$ is positive. Therefore, $\sigma-1$ will cross $\frac{f-c}{c-1}$ once and only once. There is a unique solution to the equation $\frac{f-c}{c-1}=\sigma-1$. By the one-to-one correspondence between p and x, there exists a unique p that satisfies the equilibrium condition.

Part (ii) Equation (9) shows $\frac{d\frac{G(p)}{g(p)}}{dp}>0$. Consider an interval of c over which f is constant. Differentiating equation (2) with respect to p, treating c as a function of p we have

$$\begin{aligned}\frac{d\left(\frac{f-c}{c-1}\right)}{dc}\frac{dc}{dp} &= -\frac{f-1}{\left(c-1\right)^2}\frac{dc}{dp}=\frac{d\frac{G(p)}{g(p)}}{dp}\Rightarrow\\ \frac{dc}{dp} &= -\frac{d\frac{G(p)}{g(p)}}{dp}\frac{\left(c-1\right)^2}{f-1}<0.\end{aligned}$$

■

Proof. (Proposition 2) *The first step.* When $c\in(j-1,j]$, $f\left(c\right)=j$. The limit of the left hand side of equation (2) as c converges to $j-1$ from the right is:

$$\lim_{c\to(j-1)^+}\frac{f-c}{c-1}=\frac{1}{j-2}.$$

[6] $q\left(1-t\right)^{q-1}$ is the density function of $Beta(1,q)$.

When $j=2$, $\lim_{c\to 1^+}\frac{f-c}{c-1}=\frac{1}{2-2}=\infty$; when $j\geq 3$, $\lim_{c\to (j-1)^+}\frac{f-c}{c-1}=\frac{1}{j-2}\leq 1$. Since the left hand side of equation (2) decreases in c, the limit $\frac{1}{j-2}$ is the supremum of the left hand side over the interval $c\in(j-1,j]$. For the equality to hold, $\frac{1}{j-2}$ must also be the supremum of the right hand side of equation (2).

The second step. For any integer $j\geq 3$, the supremum of the right hand side of equation (2), $\frac{1}{j-2}$, is not more than 1. We now show the right hand side of equation (2), $\frac{G(p)}{g(p)}$, evaluated at $p=\frac{j}{N}$ exceeds 1. Given that this is true, the inequality $\frac{d\frac{G(p)}{g(p)}}{dp}>0$ implies that the equilibrium p must be strictly less than $\frac{j}{N}$ and therefore m must be less than the level in the pure-strategy equilibrium. To confirm that the right hand side of equation (2), evaluated at $\frac{j}{N}$ exceeds 1, we evaluate the numerator and the denominator separately.

With $f(c)=j$, the first term in the sum in the numerator on the right hand side of equation (2) evaluated at $p=\frac{j}{N}$ equals

$$\begin{aligned}
&= \frac{(N-1)!}{j!\,(N-1-j)!}\left(\frac{j}{N}\right)^{j}\left(1-\frac{j}{N}\right)^{N-1-j}\\
&= \frac{(N-1)!}{(j-1)!\,(N-1-j)!}\left(\frac{j}{N}\right)^{j-1}\left(1-\frac{j}{N}\right)^{N-1-j}\left(\frac{j}{N}\frac{1}{j}\right)\\
&= \frac{(N-1)!}{(j-1)!\,(N-1-j)!}\left(\frac{j}{N}\right)^{j-1}\left(1-\frac{j}{N}\right)^{N-1-j}\frac{1}{N}.
\end{aligned}$$

The denominator on the right hand side of equation (2), evaluated at $\frac{j}{N}$, equals

$$\begin{aligned}
&\frac{(N-1)!}{(j-1)!\,(N-j)!}\left(\frac{j}{N}\right)^{j-1}\left(1-\frac{j}{N}\right)^{N-j}\\
&= \frac{(N-1)!}{(j-1)!\,(N-1-j)!}\left(\frac{j}{N}\right)^{j-1}\left(1-\frac{j}{N}\right)^{N-j-1}\frac{1}{N-j}\left(1-\frac{j}{N}\right)\\
&= \frac{(N-1)!}{(j-1)!\,(N-1-j)!}\left(\frac{j}{N}\right)^{j-1}\left(1-\frac{j}{N}\right)^{N-j-1}\frac{1}{N}.
\end{aligned}$$

These two intermediate results imply

$$\begin{aligned}
&\frac{(N-1)!}{j!\,(N-1-j)!}\left(\frac{j}{N}\right)^{j}\left(1-\frac{j}{N}\right)^{N-1-j}\\
&= \frac{(N-1)!}{(j-1)!\,(N-j)!}\left(\frac{j}{N}\right)^{j-1}\left(1-\frac{j}{N}\right)^{N-j}.
\end{aligned}$$

Thus

$$
\begin{aligned}
&\sum_{i=j}^{N-1} \frac{(N-1)!}{i!\,(N-1-i)!}\left(\frac{j}{N}\right)^{i}\left(1-\frac{j}{N}\right)^{N-1-i} \\
= \;& \frac{(N-1)!}{j!\,(N-1-j)!}\left(\frac{j}{N}\right)^{j}\left(1-\frac{j}{N}\right)^{N-1-j} \\
&+\sum_{i=j+1}^{N-1} \frac{(N-1)!}{i!\,(N-1-i)!}\left(\frac{j}{N}\right)^{i}\left(1-\frac{j}{N}\right)^{N-1-i} \\
= \;& \frac{(N-1)!}{(j-1)!\,(N-j)!}\left(\frac{j}{N}\right)^{j-1}\left(1-\frac{j}{N}\right)^{N-j} \\
&+\sum_{i=j+1}^{N-1} \frac{(N-1)!}{i!\,(N-1-i)!}\left(\frac{j}{N}\right)^{i}\left(1-\frac{j}{N}\right)^{N-1-i} \\
\geq \;& \frac{(N-1)!}{(j-1)!\,(N-j)!}\left(\frac{j}{N}\right)^{j-1}\left(1-\frac{j}{N}\right)^{N-j}.
\end{aligned}
$$

Therefore, we have

$$
\frac{1}{j-2} \leq 1 \leq \frac{\sum_{i=j}^{N-1} \frac{(N-1)!}{i!(N-1-i)!}\left(\frac{j}{N}\right)^{i}\left(1-\frac{j}{N}\right)^{N-1-i}}{\frac{(N-1)!}{(j-1)!(N-j)!}\left(\frac{j}{N}\right)^{j-1}\left(1-\frac{j}{N}\right)^{N-j}}. \tag{10}
$$

We conclude that the right hand side of equation (2) evaluated at $\frac{j}{N}$, given by the right hand side of inequality (10) exceeds the supremum of the left hand side of equation (2) when $j \geq 3$. Since the right hand side of equation (2) increases in p, p must be always less than $\frac{j}{N}$, and thus m is always less than j when $j \geq 3$.

The third step. By Proposition 1, p decreases in c in the interval of $1 < c \leq 2$. So p approaches its maximum value (denoted by p_1^*) as c converges to 1 from above. Because the left hand side of equation (2) approaches infinity as c approaches 1, the right hand side of equation (2) must also approach infinity. Thus, in the interval of $1 < c \leq 2$, as p approaches p_1^*, the right hand side of equation (2) goes to infinity. Since the right hand side of equation (2) increases in p and it is finite when $p = \frac{2}{N}$, we must have $p_1^* > \frac{2}{N}$. As p decreases with c, we must have $p > \frac{2}{N}$ when c is small enough. And when $p > \frac{2}{N}$, we have $m > 2$. Therefore, for sufficiently small c, expected membership in the mixed strategy equilibrium exceeds membership in the pure strategy equilibrium.

The right hand side of equation (1) approaches zero as c approaches 1. The left side of equation (1) approaches zero only if p approaches either 0 or 1. We know that p approaches p_1^* and $p_1^* > \frac{2}{N} > 0$. Therefore, p approaches 1 as c approaches its lower bound 1. ■

Proof. (Proposition 5) Because $k_1 > k_2$, we have $c(k_1) < c(k_2)$. We consider the case of $c(k_1) < c(k_2) < 2$ and the case of $c(k_1) < 2 < c(k_2)$ separately.

(a) $c(k_1) < c(k_2) < 2$. We have $f(c(k_1)) = f(c(k_2)) = 2$. Given $f = 2$, we can write equation (6) as

$$\begin{aligned} w &= \sum_{i=2}^{N-1} \frac{(N-1)!}{i!(N-1-i)!} p^i (1-p)^{N-1-i} i \\ &= (N-1)p - (N-1)p(1-p)^{N-2} \\ &= (N-1)p\left[1-(1-p)^{N-2}\right], \end{aligned}$$

which increases with p. By Proposition 1, we have $p(k_2) < p(k_1)$. Therefore $w(k_1) > w(k_2)$.

(b) $c(k_1) < 2 < c(k_2)$. Proposition 2 shows that $p(c) < \frac{f(c)}{N}$ when $c > 2$. Because $c(0) \geq c$ for any c, we have $p < \frac{f(c(0))}{N}$ when $c > 2$, which implies that $p(k_2) < \frac{f(c(0))}{N}$. By Proposition 1, $p(k_1) \geq \frac{f(c(0))}{N}$ because $c(k_1) \leq \hat{c}$. Thus, we have $p(k_2) < \frac{f(c(0))}{N} \leq p(k_1)$. Moreover, $2 = f(c(k_1)) < f(c(k_2))$ because $c(k_1) < 2 < c(k_2)$. We have

$$\begin{aligned} w(k_2) &= \sum_{i=f(c(k_2))}^{N-1} \frac{(N-1)!}{i!(N-1-i)!} (p(k_2))^i (1-p(k_2))^{N-1-i} i \\ &< \sum_{i=f(c(k_1))}^{N-1} \frac{(N-1)!}{i!(N-1-i)!} (p(k_2))^i (1-p(k_2))^{N-1-i} i \\ &< \sum_{i=f(c(k_1))}^{N-1} \frac{(N-1)!}{i!(N-1-i)!} (p(k_1))^i (1-p(k_1))^{N-1-i} i \\ &= w(k_1) \end{aligned}$$

The last inequality comes from the fact that w is increasing in p when $f = 2$, as shown in case (a). ■

B Referee's appendix: Proofs of Propositions 3 and 4

Proof. (Proposition 3) To prove part (i) we differentiate the equilibrium condition

$$\frac{f-c}{c-1} = \sigma - 1 = q\int_0^1 (1-t)^{q-1}\left(1+t\frac{p}{1-p}\right)^{n-q} dt - 1$$

with respect to n to obtain

$$\frac{dp}{dn} = -\frac{\int_0^1 (1-t)^{q-1}\left(1+t\frac{p}{1-p}\right)^{n-q} \ln\left(1+t\frac{p}{1-p}\right) dt}{\int_0^1 (1-t)^{q-1}(n-q)\left(1+t\frac{p}{1-p}\right)^{n-q-1} \frac{t}{(1-p)^2} dt} < 0.$$

Since $n = N - 1$, we have $\frac{dp}{dN} = \frac{dp}{dn} < 0$.

To prove the next three parts of the proposition we substitute $m = p(n+1)$ into the last equality of (8) to obtain

$$\sigma = q \int_0^1 (1-t)^{q-1} \left(1 + t\frac{m}{n+1-m}\right)^{n-q} dt. \tag{11}$$

Differentiating m with respect to n gives

$$\begin{aligned}
\frac{dm}{dn} &= -\frac{A}{B} \text{ with} \\
A &\equiv \int_0^1 (1-t)^{q-1} e^{(n-q)\ln\left(1+t\frac{m}{n+1-m}\right)} \left(\ln\left(1+t\frac{m}{n+1-m}\right) - (n-q)\frac{\frac{tm}{(n+1-m)^2}}{1+t\frac{m}{n+1-m}}\right) dt \\
B &\equiv \int_0^1 (1-t)^{q-1} e^{(n-q)\ln\left(1+t\frac{m}{n+1-m}\right)} \left((n-q)\frac{\frac{t(n+1)}{(n+1-m)^2}}{1+t\frac{m}{n+1-m}}\right) dt
\end{aligned}$$

The denominator is positive, so the sign of $\frac{dm}{dn}$ is the opposite of the sign of

$$\int_0^1 (1-t)^{q-1} e^{(n-q)\ln\left(1+t\frac{m}{n+1-m}\right)} \left(\ln\left(1+t\frac{m}{n+1-m}\right) - (n-q)\frac{\frac{tm}{(n+1-m)^2}}{1+t\frac{m}{n+1-m}}\right) dt,$$

or equivalently

$$\int_0^1 (1-t)^{q-1} e^{(n-q)\ln\left(1+t\frac{p}{1-p}\right)} \left(\ln\left(1+t\frac{p}{1-p}\right) - \frac{n-q}{n+1}\frac{tp}{(1-p)^2\left(1+t\frac{p}{1-p}\right)}\right) dt.$$

Now define $s(p,t) = \ln\left(1+t\frac{p}{1-p}\right)$, $r(p,t) = \frac{tp}{(1-p)^2\left(1+t\frac{p}{1-p}\right)}$. Since $\frac{n-q}{n+1} = 1 - \frac{f}{N}$, a sufficient condition for $\frac{dm}{dN} < 0$ is, for any $t \in (0,1)$, $\frac{s(p,t)}{r(p,t)} > 1 - \frac{f}{N}$. A sufficient condition for $\frac{dm}{dN} > 0$ is, for any $t \in (0,1)$, $\frac{s(p,t)}{r(p,t)} < 1 - \frac{f}{N}$.

To prove part (ii), we have

$$\frac{s(p,t)}{r(p,t)} = (1-p+tp)\frac{\ln\left(1+t\frac{p}{1-p}\right)}{t\frac{p}{1-p}}. \tag{12}$$

Each term in the right hand side of (12) is positive and less than 1, so $0 < \frac{s(p,t)}{r(p,t)} < 1$. Also we know that $1 - \frac{f}{N}$ is increasing in N, $\lim_{N\to f} 1 - \frac{f}{N} = 0$, and $\lim_{N\to\infty} 1 - \frac{f}{N} = 1$. Therefore, for sufficiently large N, we have $\frac{s(p,t)}{r(p,t)} < 1 - \frac{f}{N}$ for any t, which is sufficient for $\frac{dm}{dN} > 0$; and for sufficiently small N we have $\frac{s(p,t)}{r(p,t)} > 1 - \frac{f}{N}$ for any t, which is sufficient for $\frac{dm}{dN} < 0$.

To prove part (iii) we use Proposition 2, which shows that p converges to 1 as c converges to 1. Because $\lim_{p\to 1} s = \lim_{p\to 1} r = \infty$, we can apply the L'Hopital's Rule:

$$\begin{aligned}
\lim_{p\to 1}\frac{s}{r} &= \lim_{p\to 1}\frac{s_p}{r_p} \\
&= \lim_{p\to 1}\frac{\frac{\frac{t}{(1-p)^2}}{1+t\frac{p}{1-p}}}{\frac{t(1-p)(1-p+tp)+tp(2(1-p+tp)-t)}{(1-p)^2(1-p+tp)^2}} \\
&= \lim_{p\to 1}\frac{(1-p)(1-p+tp)}{(1-p)(1-p+tp)+p(2(1-p+tp)-t)} \\
&= \frac{0}{0+t} = 0 < 1-\frac{f}{N}.
\end{aligned}$$

Therefore when c converges to 1 from the right, $\frac{s}{r}$ is less than $1-\frac{f}{N}$ for any t, which is sufficient for $\frac{dm}{dN} > 0$. To prove part (iv), we first note that Remark 1 establishes that $\lim_{c\to f^-} p = 0$.

Second, we use $\lim_{p\to 0} s = \lim_{p\to 0} r = 0$. Again, we apply the L'Hopital's Rule.

$$\begin{aligned}
\lim_{p\to 0}\frac{s}{r} &= \lim_{p\to 0}\frac{s_p}{r_p} \\
&= \lim_{p\to 0}\frac{(1-p)(1-p+tp)}{(1-p)(1-p+tp)+p(2(1-p+tp)-t)} \\
&= \frac{1}{1} = 1 > 1-\frac{f}{N}.
\end{aligned}$$

Hence, as c converges to f from the left, p converges to 0, and $\frac{s(p,t)}{r(p,t)} > 1-\frac{f}{N}$ for any t, which is sufficient for $\frac{dm}{dN} < 0$. ■

Proof. (Proposition 4) **Part (i)**. To establish that the limiting value of $p \to 0$ as $N \to \infty$ we use the equilibrium condition

$$\frac{f-c}{c-1}+1 = \sigma = (f-1)\int_0^1 (1-t)^{f-2}\left(1+t\frac{p}{1-p}\right)^{N-f} dt. \tag{13}$$

If p remained strictly positive as $N \to \infty$, the right side of equation (13) would approach infinity while the left side remains constant. Therefore, p must approach 0 as $N \to \infty$.

Part (ii). We prove this claim in a series of steps. Step 1 shows that $m \equiv Np$ converges to a finite constant, defined as λ, as $N \to \infty$. Step 2 provides an intermediate result used in a subsequent calculation, and Step 3 shows that λ satisfies equation (4).

Step 1 Equation (13) implies that $\left(1+t\frac{p}{1-p}\right)^{N-f} < \infty$ as $N \to \infty$; consequently, $(N-f)\ln\left(1+t\frac{p}{1-p}\right) < \infty$ as $N \to \infty$. By Part (i) of the proposition, $\lim_{N\to\infty} t\frac{p}{1-p} = 0$.

Therefore $\ln\left(1+t\frac{p}{1-p}\right)\sim t\frac{p}{1-p}$ as $N\to\infty$. Thus we have $t\frac{N-f}{N}\frac{m}{1-p}=(N-f)\,t\frac{p}{1-p}<\infty$ as $N\to\infty$, implying that the limiting value of m (as $\to\infty$) is finite. Moreover, Proposition 3ii shows that $\frac{dm}{dN}>0$ for N sufficiently large. Thus we know that as $N\to\infty$, m increases in N but is finite, implying that $\lim\limits_{N\to\infty} m$ must exist. We define $\lim\limits_{N\to\infty} m=\lambda$.

Step 2 Let $n=N-1$, and $q=f-1$. We want to show that $\lim\limits_{n\to\infty}\left(1+t\frac{m}{n+1-m}\right)^{n-q}=e^{t\lambda}$. To this end, let $R(n)=\left(1+\frac{tm}{n+1-m}\right)^{\frac{n+1-m}{tm}}$ and $S(n)=tm\frac{n-q}{n+1-m}$. By Step 1, $\lim\limits_{n\to\infty}\frac{tm}{n+1-m}=0$. Making use of the property that $\lim\limits_{x\to 0}(1+x)^{\frac{1}{x}}=e$, we have $\lim\limits_{n\to\infty}R(n)=e$. Also, we have

$$\lim_{n\to\infty}S(n)=t\lim_{n\to\infty}m\left(\frac{1-\frac{q}{n}}{1+\frac{1}{n}-p}\right)=t\left(\lim_{n\to\infty}m\right)\left(\lim_{n\to\infty}\frac{1-\frac{q}{n}}{1+\frac{1}{n}-p}\right)=t\lambda,$$

where the second equality makes use of the algebraic limit theorem that $\lim\limits_{n\to c}(a(n)\,b(n))=\left(\lim\limits_{n\to c}a(n)\right)\left(\lim\limits_{n\to c}b(n)\right)$ if both $\lim\limits_{n\to c}a(n)$ and $\lim\limits_{n\to c}b(n)$ exist.

We have

$$\begin{aligned}&\lim\nolimits_{N\to\infty}R(n)^{S(n)}=\lim\nolimits_{N\to\infty}e^{S(n)\ln R(n)}=e^{\lim_{N\to\infty}(S(n)\ln R(n))}=e^{(\lim_{N\to\infty}S(n))(\lim_{N\to\infty}\ln R(n))}\\&=e^{(\lim_{N\to\infty}S(n))(\ln\lim_{N\to\infty}R(n))}=\left[e^{\ln\lim_{N\to\infty}R(n)}\right]^{\lim_{N\to\infty}S(n)}=\left(\lim\nolimits_{N\to\infty}R(n)\right)^{\lim_{N\to\infty}S(n)}.\end{aligned}\tag{14}$$

The second inequality in equation (14) uses the property that $\lim\limits_{n\to c}d^{R(n)}=d^{\lim_{n\to c}R(n)}$, where d is any positive real number; the third equality is by the algebraic limit theorem. The fourth equality uses the fact that $\lim\limits_{n\to c}\ln z(n)=\ln\left(\lim\limits_{n\to c}z(n)\right)$ for any $z(n)$. To confirm this property, define $y(n)=\ln z(n)$. We have $\lim\limits_{n\to c}z(n)=\lim\limits_{n\to c}e^{y(n)}=\lim\limits_{n\to c}e^{\ln e^{y(n)}}=e^{\lim_{n\to c}\left(\ln e^{y(n)}\right)}=e^{\lim_{n\to c}(\ln z(n))}$; taking logs of each part of this chain of equalities implies $\ln\left(\lim\limits_{n\to c}z(n)\right)=\lim\limits_{n\to c}(\ln z(n))$.

Thus we have

$$\lim_{n\to\infty}\left(1+t\frac{m}{n+1-m}\right)^{n-q}=\lim_{N\to\infty}[R(n)]^{S(n)}=\left(\lim_{n\to\infty}R(n)\right)^{\lim_{n\to\infty}S(n)}=e^{t\lambda}$$

where the second inequality uses equation (14).

Step 3 We now show that equation (4) implicitly defines λ, the limiting value of m. Using equations (2) and (11) we write the equilibrium condition as

$$\frac{1}{c-1}=\int_0^1(1-t)^{q-1}\left(1+t\frac{m}{n+1-m}\right)^{n-q}dt.\tag{15}$$

We have

$$\begin{aligned} \tfrac{1}{c-1} &= \lim_{n\to\infty} \int_0^1 (1-t)^{q-1} \left(1+t\tfrac{m}{n+1-m}\right)^{n-q} dt \\ &= \int_0^1 \left[\lim_{n\to\infty} (1-t)^{q-1} \left(1+t\tfrac{m}{n+1-m}\right)^{n-q}\right] dt \\ &= \int_0^1 (1-t)^{q-1} e^{t\lambda} dt. \end{aligned} \tag{16}$$

The first equality comes from taking limits of both sides of equation (15); the third equality comes from step 2; the second equality, where we pass the limit operator through the integral, is nontrivial, and requires that we use the "dominated convergence theorem". This theorem states that the second equality in equation (16) is valid if we can find a function $l(t)$ that is independent of n and integrable (that is, $\int_0^1 l(t)\, dt < \infty$) and "dominates" $h_n(t)$ (that is, $|h_n(t)| \leq l(t)$), where

$$h_n(t) = (1-t)^{q-1} \left(1 + t\frac{m}{n+1-m}\right)^{n-q}$$

is the integrand in the second line of equation (16). We construct the function $l(t)$ in a separate lemma, available on request, thus establishing the validity of the second equality in equation (16).

Part (iii). When N is finite, membership of the mixed strategy equilibrium follows a binomial distribution. Parts (i) and (ii) of the proposition show that $p \to 0$ and $m \to \lambda$ when $N \to \infty$. Then by Poisson Limit Theorem, the membership of the mixed strategy equilibrium follows a Poisson distribution with parameter λ, when $N \to \infty$. ■

We include the final lemma, used in the proof of Proposition 4 for the sake of completeness.

Lemma 1 *It is valid to pass the limit operator through the integral, i.e.*

$$\lim_{n\to\infty} \int_0^1 (1-t)^{q-1} \left(1 + t\frac{m}{n+1-m}\right)^{n-q} dt = \int_0^1 \left[\lim_{n\to\infty} (1-t)^{q-1} \left(1 + t\frac{m}{n+1-m}\right)^{n-q}\right] dt.$$

Proof. (Lemma 1) We rewrite the integrand $h_n(t)$ as

$$(1-t)^{q-1} \left[\left(1 + \frac{tm}{n+1-m}\right)^{\frac{n+1-m}{tm}}\right]^{\frac{tm}{n+1-m}(n-q)}. \tag{17}$$

We now construct the function $l(t)$ that satisfies the conditions of the dominated convergence theorem (integrability and dominance), using Steps (a) and (b) as follows.

Step (a): We first show that for any $x > 0$, $\left(1 + \frac{1}{x}\right)^x$ increases in x, i.e. we show that the derivative of $\left(1 + \frac{1}{x}\right)^x$ is positive:

$$\begin{aligned} \frac{d\left(1+\frac{1}{x}\right)^x}{dx} &= \frac{de^{x\ln\left(1+\frac{1}{x}\right)}}{dx} = e^{x\ln\left(1+\frac{1}{x}\right)} \left[\ln\left(1+\frac{1}{x}\right) + \frac{x\left(-\frac{1}{x^2}\right)}{1+\frac{1}{x}}\right] \\ &= e^{x\ln\left(1+\frac{1}{x}\right)} \left[\ln\left(1+\frac{1}{x}\right) - \frac{1}{x+1}\right] > 0. \end{aligned}$$

Note that $\lim_{x\to 0}\left[\ln\left(1+\frac{1}{x}\right)-\frac{1}{x+1}\right]=\infty$, $\lim_{x\to\infty}\left[\ln\left(1+\frac{1}{x}\right)-\frac{1}{x+1}\right]=0$, and

$$\frac{d\left[\ln\left(1+\frac{1}{x}\right)-\frac{1}{x+1}\right]}{dx}=\frac{-\frac{1}{x^2}}{1+\frac{1}{x}}+\frac{1}{(x+1)^2}=\frac{1}{x+1}\left(\frac{1}{x+1}-\frac{1}{x}\right)<0.$$

Thus $\ln\left(1+\frac{1}{x}\right)-\frac{1}{x+1}$ is always positive. As a result, $\frac{d\left(1+\frac{1}{x}\right)^x}{dx}$ is always positive.

Step (b): In this step, we construct the function $l(t)$. Using the definition $x=\frac{n+1-m}{tm}=\frac{1}{t}\left(-1+\frac{1}{p}\right)$, we have $\left(1+\frac{tm}{n+1-m}\right)^{\frac{n+1-m}{tm}}=\left(1+\frac{1}{x}\right)^x$. As n increases, p decreases (by Proposition 3(i), thus x increases, and $\left(1+\frac{1}{x}\right)^x$ increases by Step (a). Also we know that $\lim_{n\to\infty}\left(1+\frac{1}{x}\right)^x=e$. Hence, $\left(1+\frac{1}{x}\right)^x<e$, and we have

$$h_n(t)<(1-t)^{q-1}\,e^{t\left[\frac{m(n-q)}{n+1-m}\right]}. \tag{18}$$

By the algebraic limit theorem $\lim_{n\to\infty}\frac{m(n-q)}{n+1-m}=\left(\lim_{n\to\infty}m\right)\left(\lim_{n\to\infty}\frac{1-\frac{q}{n}}{1+\frac{1}{n}-p}\right)=\lambda$. Given any positive ε_2, we can find n^* such that $\left|\frac{m(n-q)}{n+1-m}-\lambda\right|<\varepsilon_2$ for any $n\geq n^*$. Note that m depends on n. Thus n^* depends on ε_2 only, independent of n. Let $l(t)=(1-t)^{q-1}e^{tv}$ where $v=\max\{\max_{n<n^*}\frac{m(n-q)}{n+1-m},\lambda+\varepsilon_2\}$, which is independent of n. Thus $l(t)$ is independent of n. We have

$$0<h_n(t)<(1-t)^{q-1}\,e^{t\left[\frac{m(n-q)}{n+1-m}\right]}\leq l(t). \tag{19}$$

Thus $|h_n(t)|<l(t)$, i.e., $h_n(t)$ is dominated by $l(t)$. Also, since m is finite, $\frac{m(n-q)}{n+1-m}$ is finite, v is finite, and thus $l(t)$ is finite, so $l(t)$ is integrable. As a result, by dominated convergence theorem, the second equality of (16) holds. ■

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