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An Optimal Control Model of Deforestation in the PDR of Laos

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Abstract: *The People's Democratic Republic of Laos has thus far retained a large proportion of its natural forests relative to other South-East Asian countries. This disproportionally high level of forestation, along with current regional political and economic factors acting to reduce regional timber supply, may create an increase in the demand for Lao timber. In particular, if the Lao government maintains control of natural timber harvesting and sets sustainable rates, then the increase in demand may flow into Laos' illegal timber market. This paper uses a simple optimal control model to analyse the impact of an increase in the demand for illegal timber. It argues that the illegal timber market damages both the environment and economy.*

1 Introduction

Laos, unlike many other Asian countries, has so far escaped the severe pressures of over population. In the absence of such pressures, Laos has maintained around 47 percent of its natural forests. While Laos maintains highly forested relative to other Asian countries, it is still experiencing some deforestation pressures (Domoto 1997). In 1940 Laos had 17 million hectares of natural forest. This had fallen to 12.7 million hectares in 1973, to 11.2 by 1981 and now remains at around 11 million hectares.

Based on current policies it appears that Laos' Government has adopted a relatively sustainable approach to forestry management (Tookey 1997). Forests are currently logged on a quota system. AusAid (1996) reported that quotas stabilised at around 275,000m^3, while the World Bank estimated the sustainable extraction rate at 288,000m^3. Illegal logging, however, was estimated at between 100,000m^3 and 150,000m^3 and therefore undermined the sustainability of the quota system. Other factors undermining the sustainable extraction rates include slash and burn agriculture and non-commercial timber extraction, most notably for firewood.

Illegal logging is not only the most significant threat to Lao natural forests, but economic and political conditions in Laos and it's neighbours indicate that illegal logging may be of growing concern. In 1988 floods claimed 350 lives in Thailand and in 1998 floods in China claimed more than 3,000. The devastating impact of such floods were significantly amplified by the lack of water retention due to deforestation (Lombardini 1994, Kan 1998, Eckholm 1998a,b). While floods and deforestation are not new to SE Asia (Jinchang 1982, Smil 1984), after respective floods both countries reacted by banning logging.

The logging bans in China and Thailand imply shortages in domestic supply. If decreasing domestic supply of timber in Thailand and China can not be replaced through the legal markets in other countries, then it is possible that the unfulfilled demand may spill into Laos' illegal market. While little quantitative evidence is available, it is believed that exports of illegally harvested timber to deforested neighbours is already a significant problem (Library of Congress 1995, p192).

This paper uses control theory to analyse illegal logging in a small forested economy. The small economy is inspired by Laos, but in reality the model is meant to represent a specific problem rather than a specific country. Explicitly, the problem is, given there exists both a legal and illegal forestry sector, how does a model economy react to increased external demand for timber.

2 Natural Forests and Optimal Control Theory

A large proportion of optimal control models have been applied to problems of renewable resources. The two main areas of such work are ocean and forest exploitation. Forest exploitation models have tended to examine plantation timber. Such models examine optimal rotation and harvest times and commonly examine a trade-off between other uses such as agriculture (Swallow *et al* 1990), foliage (Stienkamp and Betters 1991) or more general alternatives (Hartman, 1976).

Models analysing deforestation are less common. Hassan and Hertzler (1988) use optimal control to examine deforestation in Sudan, and more generally arid and semi-arid regions. In this model the primary reason for deforestation is cutting wood for use as firewood. The paper argues that the common property nature of Sudan's forests leads to firewood being undervalued and over consumed.

Ehui *et al* (1990) proposes a two-sector control model analysing agricultural productivity and deforestation in the tropics. In this model, once natural forests are harvested they are converted to agricultural land and never allowed to regenerate. Ehui's theoretical model maximises discounted utility over an infinite time frame. A motive to deforest is provided by agricultural yields while rental forest services provides a conservation motive. The model concludes that higher agricultural yields result in greater deforestation early in the process of deforestation and less later. Ehui and Hertel (1989) apply a similar a model to Cote d'Ivoire. However, they do not attempt to quantify rental forest services.

A significant assumption in both models is that there are either little or no property rights or enforcement of property rights. In others words the forests are effectively exploited as open access. This assumption is certainly appropriate in Laos (Tookey 1997 p 4). Clark *et al* (1993) analyse the role of property rights enforcement in managing forests. Clark *et al* develop a model where forest manager's choose legal harvest rates and expenditure on enforcing property rights. In the model illegal harvest rates are determined by an illegal timber supply curve that depends on enforcement expenditure. Clark *et al* derive the supply curve by endogenising the probability of detection.

3 Analytical Framework

The model presented in this paper maximises community profits, where profits depend wholly on profits generated by illegal timber harvesting and rental forest services. As distinct from Clark *et al* (1993), government profits from legal logging are not included in the maximisation problem. In Clark's model a forest manager controls the forest for the good of the economy. Here the community controls illegal harvest rates and government profits are assumed independent. Clark's approach is more general. In this paper, however, the model is hoped to analyse the illegal timber market's response to increased demand. The government is excluded from the maximisation problem as it is argued that an environmentally responsible, price taking government will be unaffected by demand increases.

The direct financial rewards from harvesting timber provides a motivation to deforest. The forests are harvested both legally and illegally, by the government and the community respectively. The government, over each time frame, assesses the remaining forest stock and issues quotas to log the forest at a strongly sustainable rate. That is they take only as much as will regrow in the time period (Turner 1993, p11). In calculating the sustainable rate of timber harvesting, the government continually assumes no illegal timber harvesting. Thus, over each time frame the rate of deforestation is dictated purely by illegal loggers.

The assumption that the government sets a strongly sustainable rate is a simplification. The level of quotas are constantly changing, and while it may appear that quotas are set with sustainability in mind they are certainly not set at a strongly sustainable rate. AusAid (1996) reports a United Nations estimate of 2,000 hectares of annual regrowth compared to quotas totalling around 275,000m^3 which equates to roughly 185,000 hectares.

The government's other role in this model economy is to control illegal logging. The model economy is based on a poor sparsely populated country with poor infrastructure. These attributes make controlling the problem a difficult task. In this model the probability of making a profit from illegal logging is given by an endogenous but unspecified function: $g(x, h)$. The probability of making profits is assumed to increase as forest stock increases, but as a decreasing rate. As harvest rates increase, the probability of making profits are assumed to decrease but at an increasing rate.

A conservation motive is provided by rental forest services. These are benefits associated with having and intact forest system. In Laos these benefits include hunting, collecting fruit and herbal medicine. Such services continue to be important to the predominately rural dwelling Lao economy (Library of Congress 1995, p86,160). Rental forest services are specified as a linear function of a price indicator (P_f). The price values the average dollar benefit from a hectare of forest over time $(t, t + dt)$.

Given the analytical framework specified above, the problem becomes that of maximising the following:

$$\underset{h}{W} = \int_0^\infty ((hP_i g(x,h) + xP_f)\, e^{-rt} dt \tag{1}$$

subject to:

$$\dot{x} = -h \tag{2}$$

$$x(0) = x_0 \tag{3}$$

$$x - h \leq 0 \tag{4}$$

$$h \geq 0 \tag{5}$$

where h refers to the quantity of timber illegally harvested in hectares, P_i the average profit per hectare of illegally harvested timber, $g(x, h)$ the probability of making a profit from illegal logging, x the remianing stock of natural forest in hectares, P_f an implied per hectare profit from forest rental services, and r an exogenous discount rate. All variables have an implicit time subscript. The subscript has been ignored for ease of notation.

3.1 Steady-State Analysis

Given the equations outlined above, the current value Hamiltonian becomes:

$$\widetilde{H} = hP_i g(x, h) + xP_f - \psi h \tag{6}$$

A Lagrangian is used to constrain the model. Equation (7) shows the Lagrangian as the current value Hamiltonian with the appropriate constraint attached. The term λ acts as the multiplier:

$$L = hP_i g(x, h) + xP_f - \psi h + \lambda(x - h) \tag{7}$$

The necessary conditions associated with the Lagrangian are the first order condition (8); the costate equation (9) and the Kuhn-Tucker condition (10):

$$(P_i g - P_i g_h - \psi - \lambda)\, h = 0 \tag{8}$$

$$\dot{\psi} = r\psi - P_i h g_x - P_f + \lambda \tag{9}$$

$$\lambda(x - h) = 0 \tag{10}$$

Solving the first order condition for ψ, then totally differentiating with respect to time gives:

$$\dot{\psi} = \left(\frac{P_i\left(g(x,h) + 3hg_h + h^2 g_{hh}\right)}{h} - \frac{P_i h g(h,x) + P_i h^2 g_h - x\lambda}{h^2} \right) \dot{h} + \left(\frac{P_i h g_x + P_i h^2 g_{xh} - \lambda}{h} \right) \dot{x} \tag{11}$$

Setting (11) equal to the costate equation (9), substituting in $\dot{x}$, solving for $\dot{h}$ and collecting the Lagrangian gives:

$$\dot{h} = \frac{-\lambda(r+1)}{2P_i g_h + h g_{hh}} + \frac{P_i(h^2 g_{hx} + rg(h,x) + rhg_h) - \gamma P_f}{P_i(2g_h + hg_{hh})} \tag{12}$$

Given it is not prices as such, but rather the ratio between prices that effects deforestation, (12) can be further refined by defining $\gamma = P_i/P_f$. Therefore $P_i = \gamma P_f$. Substituting this into (12) shows:

$$\dot{h} = \frac{-\lambda(r+1)}{2P_i g_h + h g_{hh}} + \frac{h^2 g_{hx} + rg(h,x) + rhg_h - \gamma}{2g_h + hg_{hh}} \tag{13}$$

Equations (13) and (2) define the dynamic model economy. However, the final form in which the equations are applied changes depending on which constraints bind. Given there are two constraints ($x \geq h$ and $h \geq 0$) there are four possible combinations of constraints. These combinations are considered in turn.

Case 1: $x \geq h$ and $h < 0$. In Case 1 the $h \geq 0$, constraints binds. This implies that $\lambda \neq 0$ and $h = 0$. Solving (13) for λ and substituting in $h = 0$ shows:

$$\dot{h} = 0 \tag{14}$$

Case2: $x < h$ and $h \geq 0$. In Case 2 the harvest rates exceed the available stock. The $h < x$ constraint therefore binds implying $\lambda \neq 0$ and $x = h$. The relevant equations for $\dot{x}$ and $\dot{h}$ becomes:

$$\dot{x} = -x \tag{15}$$

$$\dot{h} = -h \tag{16}$$

Case 3: $x \geq h$ and $h \geq 0$. In Case 3 neither of the constraints are binding. Therefore λ equals zero and equation (13) becomes (17):

$$\dot{h} = \frac{h^2 g_{hx} + rg(h,x) + rhg_h - \gamma}{2g_h + hg_{hh}} \tag{17}$$

Case 4: $x < h$ and $h < 0$. In the fourth case both constraints bind and equations (15) and (16) are relevant.

3.2 Phase-Diagram Analysis

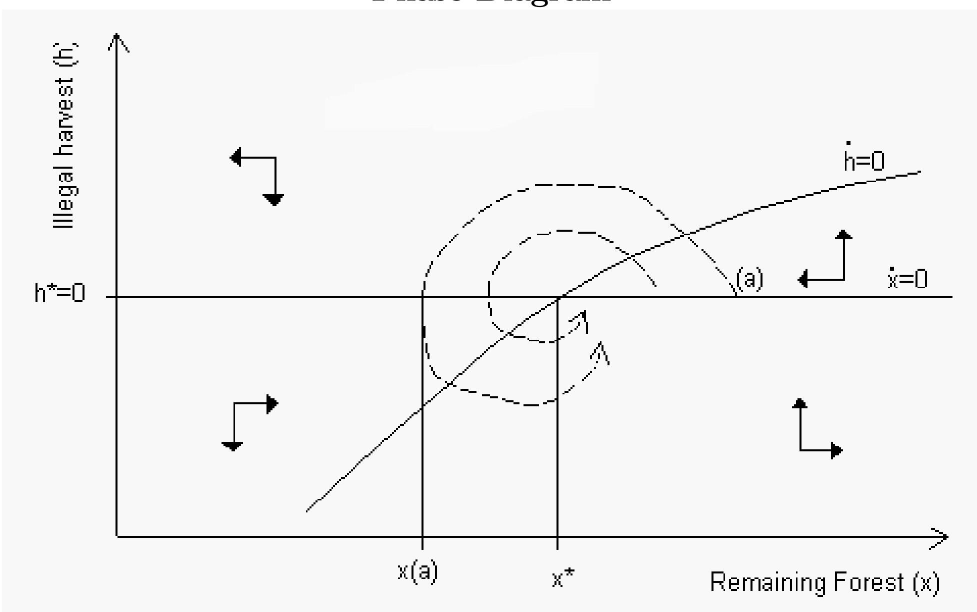
Since the probability function is unspecified, a phase diagram cannot be constructed quantitatively. However, qualitative information is enough to determine local stability of the model. The model economy is shown to contain an asymptotically stable spiral point. This is an analytical conclusion based on the trace and Δ of the Jacobian matrix (B), as defined in Beavis and Dobbs (1990). The steps taken in reaching this a conclusion are detailed in Appendix 1.

Two lines of demarkation are derived to show points where $\dot{x}= 0$ and $\dot{h}= 0$. The equation $\dot{x}= -h$ is equal to zero when h is equal to zero. Therefore $\dot{x}= 0$ equals zero along the line $h = h^* = 0$.

Because the unspecified probability of making a profit depends on harvest rates, the line indicating $\dot{h}= 0$ can only be hypothesised. Intuitively, if forest stocks increase, so to will the volume of illegal harvests. The demarkation line is assumed to have a positive slope and, assuming the steady state level of forestation is positive, cross the line $\dot{x}= 0$ at some positive value. Given that the probability of making a profit from illegal logging is assumed to increase at a decreasing rate as the remaining forest stock increases, the $\dot{h}= 0$ demarkation curve be can assumed to be concave.

Figure 1 shows the demarkation lines along with a spiraling asymptotically stable equilibrium. From any disequilibrium point (x, h) on Figure 1, the equilibrium (x^*, h^*) is approached but over shot. Eventually illegal harvest rates converge to h^* and forest stock to x^*.

Figure 1
Phase Diagram



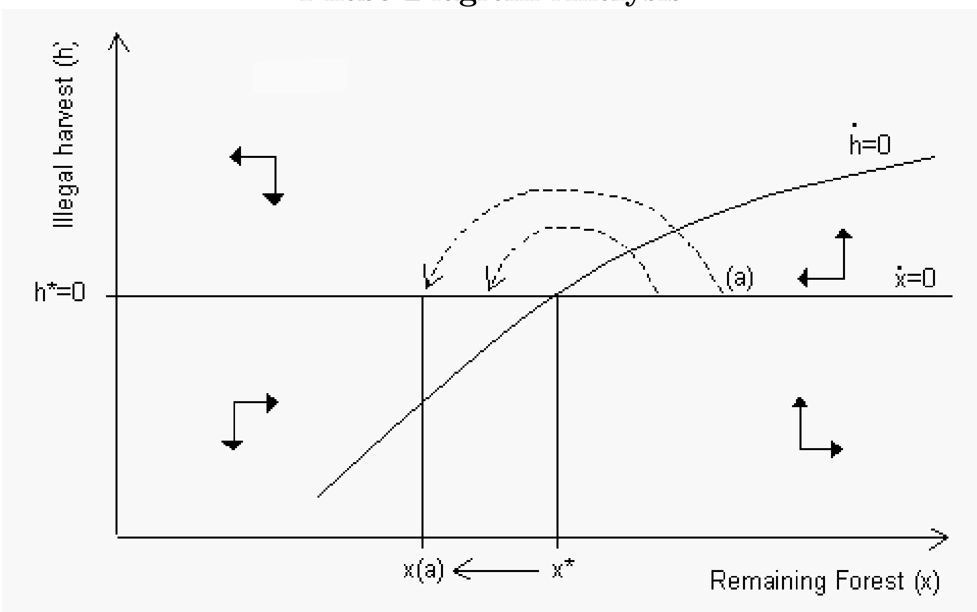
Points along $\dot{h}=0$ show points where there is neither incentive to illegally harvesting more or less timber. At any point below $\dot{h}=0$ there is incentive to harvest more timber. Alternatively points above $\dot{h}=0$ show an incentive to harvest less.

Points along $\dot{x}=0$ show sustainable timber harvesting. As stated above this implies no illegal logging. For any values above $\dot{x}=0$ forest stock will decrease, while for values below $\dot{x}=0$ forest stock will increase.

Negative harvest rates, of course, are not plausible. If they were plausible then the trajectories would follow those specified by Figure 1. In this model economy, however, trajectories follow those of Figure 1 only as far as $\dot{x}=0$ at which point the constraints bind. Forest stocks are irreversibly bounded at the intersection of respective trajectories and $\dot{x}=0$.

Figure 2 shows two trajectories where harvests are constrained to be positive. Trajectory a has a starting point where there is incentive to harvest more. The line $\dot{h}=0$ is crossed and the incentive to harvest is lessened as deforestation decreases the probability of making a profit. Incentives continue to fall until eventually no timber is illegally harvested (x^a). This leaves the forest stock $x^* - x^a$ less than its unconstrained equilibrium.

Figure 2
Phase Diagram Analysis

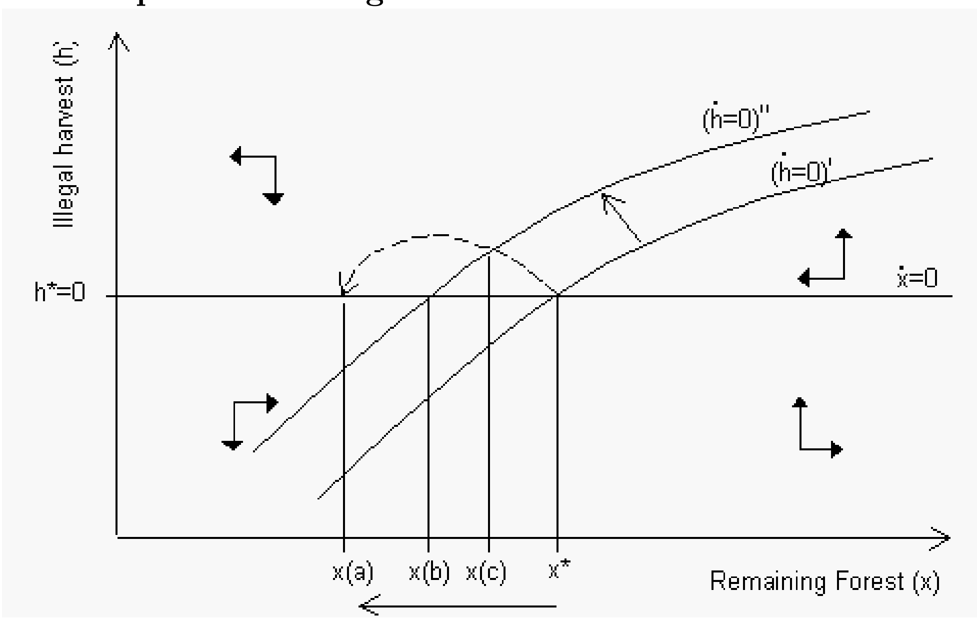


4 The Impact of Increased External Timber Demand

The system specified above is based on simplifying assumptions. In addition the model's characteristics are determined using qualitative techniques and an unspecified endogenous probability function. Therefore, the hypothesis regarding the impact of Laotion forests to increased external timber demand can only be analysed theoretically.

Assume potential trading partners of the model economy increase their demand for timber such that it can neither be meet either domestically or through their trading partners' legal markets. This implies the extra demand may spill into the model economy's illegal market. The increased demand will increase timber prices. A one-off increase in the price of timber is shown on Figure 3.

Figure 3
Impact of an Exogenous Increase in Timber Prices



The price increase moves the curve $(\dot{h}=0)'$ on Figure 3 leftwards to $(\dot{h}=0)''$. Given this leaves the harvest rate associated with x^* less than that implied by the new line $(\dot{x}=0)''$, there is incentive to harvest move. As harvest rates increase above the sustainable rate of 0, as shown along $\dot{x}=0$, the forest stock decreases. The shrinking forest stock lowers the probability of making a profit, and therefore eventually there is no longer any motive to increase illegal harvest rates. The profit of the residents would now be maximised at $x(b)$. However, given the irreversibility of the harvesting decision, the forest stock has a new upper bound of $x(a)$. The stability of the model shows illegal harvesters do self manage but ultimately will leave the economy worse off.

Discounting

Several authors discuss the ideological problems associated with discounting environmental decisions (Gowdy and O'Hara 1997, Gowdy 1994, Tisdell 1991). The model presented above highlights the problem. In the model economy, where planners do discount the future, an increase in the discount rate, *ceteris paribus*, increases the motivation to deforest. Starting from an equilibrium point illegal (and therefore unsustainable) harvesting increases and does not begin to fall until the level of deforestation is less than the new lower unconstrained equilibrium level of forestation.

5 Conclusions

The optimal control model developed by the paper allows a community to choose optimal harvest rates in order to maximise profits. A forest conservation motive is provided by rental forests services while timber sales provide an incentive to deforest. In the model the government is active. However, assuming they harvest at a strongly sustainable rate and that their harvest does not provide any profits to the community, the effect of the government falls out of the model.

The model's primary purpose is to provide a theoretical framework from which to analyse the impact of increased timber demand on deforestation in Laos. The model has been simplified to focus on this problem. The simplifications, in particular the treatment of the government, may deem the model too restrictive for alternative interpretations. However, theoretical conclusions can be draw regarding the specific problem.

The main conclusion of the model is that not only does increased demand for timber have a negative impact on Lao forests, but by arguing that the dynamic system contains a spiralling asymptotically stable equilibrium, the negative impact of illegal harvesting is exaggerated by the irreversibility of harvesting decisions and the sluggish manner in which illegal harvesters adjust their harvests.

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Appendix 1

The asymptotically stable nature of the model is based on the assumption that both the trace and Δ of the model's Jacobian matrix (as defined in Beavis and Dobbs 1990), take on negative values. The Jacobian matrix (B) is defined as:

$$B = \begin{bmatrix} \frac{\partial \dot{h}}{\partial h} & \frac{\partial \dot{h}}{\partial x} \\ \frac{\partial \dot{x}}{\partial h} & \frac{\partial \dot{x}}{\partial x} \end{bmatrix} = \begin{bmatrix} \frac{\partial \dot{h}}{\partial h} & \frac{\partial \dot{h}}{\partial x} \\ -1 & 0 \end{bmatrix} \tag{A1}$$

The *trace* of B is defined as:

$$trace(B) = \frac{\partial \dot{h}}{\partial h} - \frac{\partial \dot{x}}{\partial x} = \frac{\partial \dot{h}}{\partial h} \tag{A2}$$

The trace, therefore, cancels to equal the derivative of the harvest rate's motion equation with respect to harvest rates. This is negative intuitively. As h increases the unspecified probability of making a profit decreases. Therefore the motion of harvest rates will be negative as h increases, all else remaining equal.

The term Δ is defined as:

$$\Delta = trace(B)^2 - 4\,|B| \tag{A4}$$

The determinate of B cancels as follows:

$$|B| = \frac{\partial \dot{h}}{\partial h} . \frac{\partial \dot{x}}{\partial x} - \frac{\partial \dot{x}}{\partial h} . \frac{\partial \dot{h}}{\partial x} = \frac{\partial \dot{h}}{\partial x} \tag{A5}$$

Given (A2) and (A5), Δ becomes:

$$\Delta = \left(\frac{\partial \dot{h}}{\partial h} \right)^2 - 4 . \frac{\partial \dot{h}}{\partial x} \tag{A6}$$

The asymptotically stable nature of the model is based on (A6) taking a negative value. The first term in (A6) will be positive given it is a square. The derivative $\partial \dot{h} / \partial h$ from the second term in (A6), therefore, must not only be positive but also greater than a quarter of the square of $\partial \dot{h} / \partial h$. This condition is shown in (A7):

$$\frac{\partial \dot{h}}{\partial x} > \frac{\left(\frac{\partial \dot{h}}{\partial h} \right)^2}{4} \tag{A7}$$

Unfortunately this condition is hard to prove mathematically. Again one must rely on intuition. The derivative $\partial \dot{h} / \partial h$ is harvest rate's motion equation with respect to forest sock. As forest stock increases so to does the unspecified probability of making a profit. All else remaining equal, if forest stock is increasing the motion of illegal harvest rates will never be negative.

Finally the issue of magnitude must be addressed. As stated above, $\partial \dot{h} / \partial x$ must be greater than a quarter of the square of $\partial \dot{h} / \partial h$. While the magnitude criterion must be assumed, the plausibility of the assumption can be demonstrated using three simple (local) scenarios:

1. if $\partial \dot{h} / \partial h$ equals $\partial \dot{h} / \partial x$ condition (A7) is valid up to the point that $\partial \dot{h} / \partial h$ equals 4;

2. if $\partial \dot{h} / \partial h$ equals unity then $\partial \dot{h} / \partial x$ need be between $^1/_4$ and infinity to satisfy (A7); and

3. if $\partial \dot{h} / \partial x$ equals unity then $\partial \dot{h} / \partial h$ need be between -2 and 2 to satisfy (A7);

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