# Modelling Farm Households in a Spatial Context:  Vietnamese Agriculture

by

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## Introduction

For a decade, Vietnam has been implementing economic reforms in relation to agricultural production. These new government policies have had significant impacts on agricultural production as well as overall economic growth. Vietnam has attained self-sufficiency in rice production and has become the third largest rice exporter to the world market. However, the impacts of a range of agricultural policies on household incomes through changes in input and output pricing rules, land use policies (the land market and land holding ceilings), credit provision and taxation at the farm level are important questions that have had very limited investigation.

Modelling frameworks to assist in the analysis of such complex questions also still need development. This paper is about the development of a modelling framework which integrates household modelling, risk, and spatial trading systems at the micro level. The issue of asset transfers and the relationship of assets to credit are also integral issues under consideration.

## Background

The share of agriculture in the GDP of the Vietnam has fallen from 36 per cent in 1986 to 27 per cent in 1997 (World Bank 1998). This reflects the significant structural changes under way in the country. Rice production has risen from 11.6 million tonnes in 1980 compared to 27 million tonnes in 1997 as a result of the reforms already undertaken. However, as the World Bank (1997) reports there is still extensive poverty particularly in rural areas along with growing rural-urban income disparities. The poor, in particular, are more vulnerable to the changes in the economy and the world rice market. Thus, a capacity to model and analyse the impacts of existing and potential government policy changes at the household and/or local level are important in the development of policy. The work outlined in this paper is designed to progress the theoretical foundation for a project funded by the Australian Centre for International Agricultural Research on "Impacts of Alternative Policy Options on the Agricultural Sector in Vietnam." One of the key aspects of the project is the modelling of the transactions between groups of households or villages. The approach might be referred to as 'village modelling' although at this stage the approach must be considered as in very early stages of development.

## Issues for Model Construction

Models used for policy analysis need to have a strong theoretical foundation and also the possibility of quantification. However, in relation to the problem of analysing policy changes at the level of household groups in Vietnam there are a number of more specific issues which are particularly important. These issues are briefly outlined in the following paragraphs.

(a) Effects of land consolidation

Following the reforms of the 1980s when land-use rights were assigned to households, often in numerous small parcels, the issue of land consolidation and the transfer of land-use rights is raised, In addition, there are mandatory ceilings to the size of agricultural holdings designed to avoid landlessness and the concentration of land ownership. With the economic development of Vietnam it can be expected that the opportunity value of labour and other inputs will rise. If this happens then the effective costs of farming dispersed small parcels of land will also rise. In addition, there are the costs of physically moving between parcels of land and moving the tools and equipment. With land consolidated into a few larger parcels or a single parcel there is also a

possibility of larger-scale farming techniques being applied and the possible economic benefits from doing so. There are clearly important social and economic consequences of land consolidation.

The possible exchange of land-use rights in a land-use rights market is therefore one scenario which will need to be simulated so that the effects of land consolidation can be evaluated. To do so requires a model to include multiple households linked to a number of parcels of land which have the potential, under different policy scenarios, of amalgamation to form different operating unit sizes.

(b) Input/output pricing policies and land use

Vietnam has a range of policies which relate to production management for various products, controls and specified preference to state-owned enterprise for the allocation of credit and various approaches to the management of input prices through import controls for fertiliser and seeds (World Bank 1998). Thus the model needs to allow for experimentation with the effects of changes in input and output prices and also include various scenarios in relation to the opportunity cost of family labour.

Consideration will also need to be given to including risk in constructing the models and particularly price and production risk. Work by Hazell and Norton (1986), Paris and Easter (1985) and Saha and Stroud (1994) provide a guide to some of the alternatives. These include mathematical programming models involving the effects of price risk and risk in the technical production system.

(c) Gender issues

As in many developing countries the roles generally taken by different members of the household can have policy implications. The structure of the household components of therefore need to have the possibility of including gender specific consumption and time allocation following the work of de Janvry, Fafchamps, Raki and Sadoulet (1992) on a computable household model and the use of cross-sectional data by Sawit and O'Brien (1995) and Adulavidhaya, Quoroda, Lau and Yotopoulos (1992).

(d) Production technology

Wherever there is the possibility of changes in farm size and adjustment to policy changes then there is also the possibility of technical change also. Thus flexible production systems are needed where economies of size in the use of inputs is possible.

The use of nonlinear production functions in mathematical programming farm models and their solutions has been illustrated by Fox (1986, pp 71-79) using Minos (Murtagh and Saunders 1983).

(e) Impact of credit and tax policies

Credit and tax policies in Vietnam are complex. However, as indicated by the World Bank (1998) they are likely to have a significant effect on the development of Vietnamese agriculture. The favouring of state owned enterprises, the limited ability to use land-use transfer rights as collateral for credit and limitations on the activities of foreign banks are all important. Thus the simulation

of alternative credit scenarios is essential. These may simply reflect different interest rates and different availabilities of credit.

In addition, the nature of the tax system is complex and in some cases expressed in terms of rice production. The possibility for examining different taxation arrangements will also be needed.

(f)  Household responses and land-use flexibility

Land-use flexibility implies the ability to change the crops grown and the use of land for various agricultural purposes. With the reforms of the 1980s came the possibility of greater choice of production methods and mix of enterprises but there is still a considerable degree of central management of agriculture with through the use of plans at the provincial level and thus limitations on the flexibility with which enterprises can be changed. By using models with a considerable range of activities, such as alternative crops, animal production and vegetable growing and imposing various types of restrictions on the ability to change enterprises it will be possible to evaluate the relaxation of these restrictions. Implementation of different rates of change may one approach to this issue.

 (g)  Interactions of policies

The effects of price support policies (such as, regulated prices and the effects of limitations or restrictions on rice exports causing a lowering of domestic prices), on the extent and nature of land consolidation processes will be investigated (see International Food Policy Research Institute 1996). The assumption to be used is that the land consolidation will take place as an endogenous process within the model and the ways in which this will change under various input and output price scenarios will be investigated.

An examination will also be made of the impacts that different taxing and credit policies have on the way in which land may be used and consolidated. From the rules for collateral in mortgaging a land use right it is likely that there may be incentives to leave land in small and distributed bundles. Also, since the land use tax is based on rice yields it is apparent that the interaction of land type, yields and tax rates will have an impact on how land is used.

Given that it is currently difficult to change land use, this stickiness needs to be simulated and the alternative of a much more flexible system of adjustment to price changes examined. The related set of experiments is focussed on the idea that with the rice industry opening to the world market, as Vietnam is now a major exporter of rice, then there is likely to be a benefit to a flexible system of land use and adjustment of cropping patterns. Thus the implications of the flexibility in land use in response to patterns of price changes that might be expected from the world market will need investigation.

**Basic Household Model**

There is an extensive literature on the area of household models. The standard development of household economics is given in Nakajima (1986) and Nakajima (1970) and based on the earlier work of Chayanov (1966). Ellis (1993) also provides a general overview of the various forms of household models and Singh, Squire and Straus (1986) provide and extensive treatment of a wide variety of forms of the model.

To provide a perspective on the essential structure, the model developed by Barnum and Squire (1979) and summarised in Ellis (1993) is outlined below. The model included the assumptions that there was a labour market, that household labour could be employed or the household could hire labour, that the land available to the household is fixed, that the household produces Z-goods (goods produced within the household from other goods by using household time), and that a significant choice for the household is between own consumption of output and sale of output to earn cash income so as to purchase other goods. Risk effects are ignored. The model can be expressed as:

(1)     $U = f(T_Z, C, M)$

Where U is a well-behaved utility function, T is time for the production of Z-goods and leisure, C is home consumption of output, and M is purchased goods. The household is subject to a production function:

(2)     $Q = f(A, L, V)$

where Q is the household's production, A is land used for production, L is total labour available (household and hired) and V is the set of variable inputs used in production. The household is also subject to a time constraint as follows:

(3)     $T = T_Z + T_F + T_W$

where $T_Z$ is time allocated to Z-goods and leisure, $T_F$ is time allocated to farming, and $T_W$ is the time (positive or negative) allocated to wage based employment. Finally, the household is subject to an income constraint:

(4)     $p(Q - C) + w T_W - v V = m M$

where the production sold is Q-C, p is the output price, w is the labour wage rate, v is the price of variable inputs V and m is the average price of purchased goods. Equations (3) and (4) may be combined into

(5)     $w T_Z + p C + m M = \prod + w G$

where G is the household's own time or $T_Z + T_F$ and $\prod$ is the net farm income. The constraint implies that the opportunity cost of the time spent producing Z goods plus the market value of home consumption and the value of market purchases must equal the farm profit plus the value of total household time valued at the market wage rate w.

The important equilibrium conditions for the solution to the model relate to production and consumption. For production the marginal value product of labour equals the wage rate and the marginal value product of the other variable inputs equal their average price. For consumption the marginal rates of substitution between each pair of goods in the utility function must equal the price ratios between them.

The important results from such a model relate to the trade offs between the use of time on farm for production and non-farm activities for earning cash income, choices on hiring in and hiring out labour and choices on home consumption versus marketed surplus. The effects of changes in relative prices can also be observed.

Many extensions to this basic model have been proposed. The area of risk is of particular interest and examples of efforts to include risk in models include Roe and Graham-Tomasi (1986), Fabella (1989), Fafchamps (1993), Saha (1994), Saha and Stroud (1994) among others. In the following section a dynamic model of the household is developed which models equilibrium in land, labour and consumption decisions under both production and price risk.

**Dynamic Model of Household Decisions under Risk**

Hertzler (1991) developed a stochastic dynamic programming model for decisions by agricultural households. This model will be specialised for small farms that are part of a village and may be isolated from commodity markets. The model itself is deliberately kept small to demonstrate the principles of including risk into household decisions. Households are assumed to behave as if they maximise their expected utility subject to a budget constraint for the change in wealth.

subject to

$$(6) \qquad J(0, W_o) = Max \ E\left[ \int_0^T e^{-\rho t} U(Q) dt + J(T, W_T) \right]$$

$$(7) \qquad dW = \delta(W, Q, D) dt + \sigma(Q, D) dZ; \qquad W_0 \ \text{is given}.$$

A household's satisfaction is assumed to be summarized by expected utility, $J$. Satisfaction is derived from the utility of consumption, $U(Q)$, integrated over all years, $t$, and discounted at the rate of time preference, $\rho$. Satisfaction also includes expected utility of wealth at the end of the planning horizon, $T$. Starting from time zero, initial wealth, $W_0$, increases with changes in wealth, $dW$. A change in wealth has an expected change $\delta dt$, where $\delta$ is the instantaneous mean, and an error term $\sigma dZ$, where $\sigma$ is a vector of instantaneous standard deviations and $dZ$ is a vector of Weiner increments. The mean and standard deviations are functions of wealth, $W$, consumption, $Q$, and decision variables, $D$, chosen at time $t$ to apply over a decision interval of length $dt$.

Maximising expected utility over a household's time horizon is equivalent to maximising the Hamilton-Jacobi-Bellman equation in each decision interval. The Hamilton-Jacobi-Bellman equation (8) is a partial differential equation in time and wealth, subject to a boundary condition at time $T$.

$$(8) \qquad \frac{\partial J}{\partial t} + Max \left[ U + \frac{\partial J}{\partial W} \delta + \frac{1}{2} \frac{\partial J}{\partial W^2} \sigma \Omega \sigma' \right] - \rho J = 0 ;$$

In the boundary condition, expected utility at the terminal time equals the discounted utility of wealth, $e^{\rho(T-t)} V(W_T - \ddot{W})$, if wealth is above a subsistence level, $\ddot{W}$, and equals zero if wealth is below the subsistence level. This is the overall constraint affecting the viability of a household. It is the only inequality constraint in the model but even it is not a hard constraint. Going below a subsistence level is feasible, just not pleasant. Other constraints such as problems in obtaining credit are not included. Borrowing is almost always feasible if the household is viable. However, borrowing may be expensive.

The expression in brackets to be maximised is discounted current utility of consumption $U$, plus the marginal utility of wealth, $\partial J/\partial W$, multiplied by the instantaneous mean, $\delta$, plus one-half the derivative of the marginal utility of wealth, $\frac{1}{2}\partial^2 J/\partial W^2$, multiplied by the instantaneous covariance, $\sigma\Omega\sigma'$, which equals the standard deviation squared. Optimality conditions are the derivatives set equal to zero.

$$(10) \quad \frac{\partial U/\partial Q}{\lambda} + \frac{\partial\delta}{\partial Q} - \frac{1}{2}R\frac{\partial(\sigma\Omega\sigma')}{\partial Q} = 0;$$

$$\frac{\partial\delta}{\partial D} - \frac{1}{2}R\frac{\partial(\sigma\Omega\sigma')}{\partial D} = 0;$$

$$\text{where} \quad \lambda(W) = \frac{\partial J}{\partial W}; \text{ and } R(W) = -\frac{\partial^2 J/\partial W^2}{\partial J/\partial W}.$$

The first optimality condition is for consumption and the second is for production and investment decisions. The marginal utility of consumption is normalised by the expected marginal utility of wealth, $\lambda$. The terms containing $R$ are marginal risk premiums. To simplify notation, $R$ is defined as the coefficient of absolute risk aversion. It measures the curvature of expected utility with respect to current wealth. It is distinguished from an Arrow-Pratt coefficient of risk aversion that measures the curvature of utility with respect to terminal wealth. Marginal utility of wealth and the risk aversion coefficient encapsulate all of the information about the future. If their current values can be measured, optimal decisions in a single period are also dynamically optimal.

The instantaneous mean and covariance required in equation (10) are determined by the stochastic differential equation for wealth. Hertzler (1991) provided a method for deriving this equation. The household's wealth, as the sum of assets and liabilities, is differentiated to give the change in wealth under risk. Wealth changes with capital gains, depreciation, revenues and costs. Here we present and interpret a simplified model of a household on a small farm. The instantaneous mean is:

$$(11) \quad \delta(W,Q,D) = \delta_W W + h(\delta_h - \delta_W)H + \left(y_1(\delta_{y_1} - \delta_W - \delta_{S_1}) + p_1 - s_1\right)S_1$$
$$+ \left(y_1(1+\delta_{y_1})Y_1(L_1,X_1) - x_1 X_1\right)H_1 + \left(y_2(1+\delta_{y_2})Y_2(L_2,X_2) - x_2 X_2\right)(H - H_1)$$
$$+ l(L - L_1 H_1 - L_2(H - H_1)) - y_1(1+\delta_{y_1})Q_1 - q_3 Q_3.$$

On the left hand side, the instantaneous mean is a function of wealth, $W$, consumption, $Q$, and decision variables, $D$. On the right-hand side, the first line includes investment; the second line includes production of commodities; and the third line includes expenditures.

Upper case letters denote quantities. Three different commodities are denoted by subscripts. Commodity 1 is both produced and consumed; commodity 2 is produced but not consumed; and commodity 3 is consumed but not produced. For example, $Q_1$ and $Q_3$ are consumption of commodities 1 and 3. Yields, $Y_1$ and $Y_2$ are production of commodities 1 and 2. Yields are functions of labour, $L$, and variable inputs, $X$. Other decision variables include hectares of land, $H$, and storage, $S$.

Lower case letters denote prices. Prices include the price of a hectare of land, *h*, commodity prices, *y*, a seasonal premium, *p*, variable input prices, *x*, an opportunity cost of labour, *l*, a storage cost, *s*, and consumption prices, *q*. Assumptions must be made for a farmer's expectations about prices. For example, the commodity price at the beginning of a production season may be *y*, the actual price at the end of the season will be *y* + *dy*, but the price that the farmer expects is *y* + *E*{*dy*}. Both actual and expected prices can be modeled by stochastic differential equations. Although these equations can be arbitrarily complex, it is typical to assume log-normal distributions. In other words, the percentage change in prices is normally distributed and prices themselves are always positive.

$$db = b\delta_W dt;$$
$$dh = h\delta_h dt + h\sigma_h dz_h;$$
$$dy = y\delta_y dt + y\sigma_y dz_y;$$
$$dp = \sigma_p dz_p.$$

Expected rates of change are terms like $\delta dt$ and errors are terms like $\sigma dz$. The price of a risk-free investment, *b*, has a known rate of return and no error term. If the risk-free investment is simply to hoard money, then the rate of return is zero. The land price, *h*, and commodity prices, *y*, have both expected rates of change and error terms. The seasonal price, *p*, has no expected change, only an error term. Because the seasonal premium can be positive or negative, it is assumed to be normally distributed.

Assumptions are also required for the farmer's expectations about yields and storage.

$$dY = Y\sigma_Y dZ_Y;$$
$$dS = S\delta_S + S\sigma_S dZ_S.$$

Yields, *Y*, have no expected change, only error terms. Storage, *S*, has both an expected rate of physical degradation and an error term.

In equation (11), wealth and hectares of land are investment decisions. Current wealth, *W*, could be invested at a risk-free rate of return of $\delta_W$. Hectares, *H*, are valued at price *h*, and attract a return above the risk free rate of $h(\delta_h - \delta_W)$. Expected revenues equal the expected price at the end of the season, $y(1 + \delta_y)$, multiplied by yield, *Y*. Subtracting variable input costs, *xX*, gives a gross margin per hectare. Gross margins multiplied by the number of hectares, *H*, gives net returns per farm. Labour has opportunity costs either as leisure or off-farm employment and generates income, *lL*, which is reduced by the amount of labour used on the farm. Commodities go into storage, *S*, at the commodity price, *y*, with a return on investment above the risk-free rate of $y(\delta_y - \delta_W)$. Stored commodities will physically degrade at the rate $\delta_S$, with a degradation cost of $y\delta_S$. Commodities sold between harvests will attract a seasonal premium, *p*, and storage will cost *s* dollars per unit. Consumption, *Q*, for commodity 3 is purchased at price *q*. However, consumption of commodity 1 reduces the quantity that can be sold and has an opportunity cost equal to the expected selling price.

The error terms for the change in wealth are:

(12) $\quad \sigma(Q,D)dZ = hH\sigma_h dz_h + S_1\sigma_{p_1} dz_{p_1} - y_1 S_1\sigma_{S_1} dZ_{S_1}$

$$y_1\left(Y_1(L_1,X_1)H_1 + S_1 - Q_1\right)\sigma_{y_1} dz_{y_1} + y_2 Y_2\left(L_2,X_2\right)\left(H - H_1\right)\sigma_{y_2} dz_{y_2}$$

$$+y_1 Y_1(L_1,X_1)H_1\sigma_{Y_1} dZ_{Y_1} + y_2 Y_2\left(L_2,X_2\right)\left(H - H_1\right)\sigma_{Y_2} dZ_{Y_2}.$$

The first line of this equation includes investment and storage risks; the second line includes price risks; and the third line includes yield risks. The variances required in equation (10) are found by squaring the errors.

(13) $\quad \sigma(Q,D)\Omega\sigma'(Q,D) = h^2 H^2\sigma_h^2 + S_1^2\sigma_{p_1}^2 + y_1^2 S_1^2\sigma_{S_1}^2$

$$+y_1^2\left(Y_1(L_1,X_1)H_1 + S_1 - Q_1\right)^2\sigma_{y_1}^2 + y_2^2 Y_2^2\left(L_2,X_2\right)\left(H - H_1\right)^2\sigma_{y_2}^2$$

$$+y_1^2 Y_1^2(L_1,X_1)H_1^2\sigma_{Y_1}^2 + y_2^2 Y_2^2\left(L_2,X_2\right)\left(H - H_1\right)^2\sigma_{Y_2}^2.$$

There are seven variances for the seven sources of risk. In some cases, covariances between prices and yields may be important. These will affect the mean as well but will not change the results significantly.

**Optimal Decisions**

More specific versions of the optimality conditions in equation (10) are derived using the mean in equation (11) and the variances in equation (13). Decisions about variable inputs, labour, land, storage and consumption are:

*Variable Inputs, $X_1$ and $X_2$*

(14) $\quad y_1\left(1 + \delta_{y_1}\right)\dfrac{\partial Y_1}{\partial X_1} - Ry_1^2\left(Y_1 H_1 + S_1 - Q_1\right)\sigma_{y_1}^2 \dfrac{\partial Y_1}{\partial X_1} - Ry_1^2 Y_1 H_1\sigma_{Y_1}^2 \dfrac{\partial Y_1}{\partial X_1} = x_1.$

(15) $\quad y_2\left(1 + \delta_{y_2}\right)\dfrac{\partial Y_2}{\partial X_2} - Ry_2^2 Y_2\left(H - H_1\right)\sigma_{y_2}^2 \dfrac{\partial Y_2}{\partial X_2} - Ry_2^2 Y_2\left(H - H_1\right)\sigma_{Y_2}^2 \dfrac{\partial Y_2}{\partial X_2} = x_2.$

*Labour, $L_1$ and $L_2$*

(16) $\quad y_1\left(1 + \delta_{y_1}\right)\dfrac{\partial Y_1}{\partial L_1} - Ry_1^2\left(Y_1 H_1 + S_1 - Q_1\right)\sigma_{y_1}^2 \dfrac{\partial Y_1}{\partial L_1} - Ry_1^2 Y_1 H_1\sigma_{Y_1}^2 \dfrac{\partial Y_1}{\partial L_1} = l.$

(17) $\quad y_2\left(1 + \delta_{y_2}\right)\dfrac{\partial Y_2}{\partial L_2} - Ry_2^2 Y_2\left(H - H_1\right)\sigma_{y_2}^2 \dfrac{\partial Y_2}{\partial L_2} - Ry_2^2 Y_2\left(H - H_1\right)\sigma_{Y_2}^2 \dfrac{\partial Y_2}{\partial L_2} = l.$

*Land, $H$ and $H_1$*

(18) $\quad y_2\left(1 + \delta_{y_2}\right)Y_2 - x_2 X_2 - lL_2 - Rh^2 H\sigma_h^2 - Ry_2^2 Y_2^2\left(H - H_1\right)\sigma_{y_2}^2 - Ry_2^2 Y_2^2\left(H - H_1\right)\sigma_{Y_2}^2$

$$= h\left(\delta_W - \delta_h\right)$$

(19) $\quad y_1\left(1 + \delta_{y_1}\right)Y_1 - x_1 X_1 - lL_1 - Ry_1^2\left(Y_1 H_1 + S_1 - Q_1\right)Y_1\sigma_{y_1}^2 - Ry_1^2 Y_1^2 H_1\sigma_{Y_1}^2$

$$= y_2\left(1 + \delta_{y_2}\right)Y_2 - x_2 X_2 - lL_2 - Ry_2^2 Y_2^2\left(H - H_1\right)\sigma_{y_2}^2 - Ry_2^2 Y_2^2\left(H - H_1\right)\sigma_{Y_2}^2.$$

*Storage, $S$*

$$(20) \quad y_1 \delta_{y_1} + p_1 = s_1 + y_1 (\delta_W + \delta_S) + R(S_1 \sigma_{p_1}^2 + y_1^2 S_1 \sigma_{S_1}^2) + R y_1^2 (Y_1 H_1 + S_1 - Q_1) \sigma_{y_1}^2 = 0.$$

*Consumption, Q₁ and Q₃*

$$(21) \quad \frac{\partial U / \partial Q_1}{\lambda} + R y_1^2 (Y_1 H_1 + S_1 - Q_1) \sigma_{y_1}^2 = y_1 (1 + \delta_{y_1})$$

$$(22) \quad \frac{\partial U / \partial Q_3}{\lambda} = q_3.$$

Although the optimality conditions seem complex, they are easy to interpret. Demands are on the left-hand sides of the equations and supplies are on the right hand sides. Terms beginning with *R* are marginal risk premiums. If a household is risk neutral, the marginal risk premiums are zero and the optimality conditions collapse to familiar conditions from production, investment and consumption theory. For example, the optimality conditions for variable inputs and labour would equate the marginal value products to input prices. Investment in land would equate the returns per hectare to the interest costs of investment above expected capital gains. All land would be allocated to the most profitable crop with no diversification. Crops may be rotated because of biological interactions, although these interactions have not been included in the yield specifications. Demand for storage would depend upon the increase in the selling price and supply of storage would include marginal storage, interest and degradation costs. The optimality conditions for consumption would equate marginal utility of consumption, normalised by the marginal utility of wealth, to prices.

If the household is averse to risk, demand and supply curves will include marginal risk premiums. The demands for variable inputs and labour in equations (14) through (17) equal the expected marginal value products minus marginal risk premiums for price and yield risks. Investment in land and allocation among crops in equations (18) and (19) become a portfolio selection problem. The supply of storage in equation (20) also includes marginal risk premiums. Perhaps most unusually, in equation (21) price risk increases the consumption demand for the produced commodity. Consumption itself becomes a hedge because price risk only affects the quantity remaining for sell. In equation (22), risk does not directly affect the consumption of a purchased commodity.

**Illustration of the Household Model**

The optimality conditions are a non linear system of equations that must be solved simultaneously. First, however, functions for yield and the utility of consumption are required.

*Yield, Y₁ and Y₂*

$$(23) \quad Y_1(L_1, X_1) = a_1 L_1^{b_1} X_1^{u_1} e^{-v_1 X_1}.$$

$$(24) \quad Y_2(L_2, X_2) = a_2 L_2^{b_2} X_2^{u_2} e^{-v_2 X_2}.$$

*Consumption, Q₁ and Q₃*

$$(25) \quad U(Q_1, Q_3) = \alpha (Q_1 - \gamma_1)^{\beta_1} (Q_3 - \gamma_3)^{\beta_3}$$

Yields are assumed to be Cobb-Douglas functions in labour and transcendental functions in variable inputs. Utility of consumption is assumed to be a Stone-Geary function in both commodities.

An illustrative set of parameters is listed in Table 1.

**Table 1: Parameter Values for the Model.**

| | | Commodity Specific | | |
| --- | --- | --- | --- | --- |
| | Non-Specific | 1 | 2 | 3 |
| $R$ | 0.00002 | | | |
| $\lambda$ | 0.006 | | | |
| $L$ | 7000 | | | |
| $H$ | 10 | | | |
| $l$ | 6 | | | |
| $y$ | | 50 | 40 | |
| $x$ | | 90 | 80 | |
| $p$ | | 30 | | |
| $s$ | | 5 | | |
| $q$ | | | | 40 |
| $\delta_W$ | 0.1 | | | |
| $\delta_h$ | 0.1 | | | |
| $\delta_y$ | | 0.1 | -0.1 | |
| $\delta_S$ | | 1.1 | | |
| $\sigma_h$ | 0.4 | | | |
| $\sigma_y$ | | 0.5 | 0.3 | |
| $\sigma_p$ | | 10 | | |
| $\sigma_Y$ | | 0.3 | 0.4 | |
| $\sigma_S$ | | 0.4 | | |
| $a$ | | 8 | 5 | |
| $b$ | | 0.4 | 0.5 | |
| $u$ | | 1.2 | 1.5 | |
| $v$ | | 0.2 | 0.3 | |
| $\alpha$ | 1 | | | |
| $\beta$ | | 0.5 | 0.4 | |
| $\gamma$ | | 50 | 20 | |

For illustration, the non-linear system is solved using Solver in Excel. The baseline solution of the model is shown in Table 2.

**Table 2:  Baseline Solution.**

|  | Commodity | | | | |
| --- | --- | --- | --- | --- | --- |
|  | 1 | 2 | 3 | Off-farm | Price |
| *Y* | 254.74 | 346.68 | | | |
| *X* | 5.78 | 4.86 | | | |
| *L* | 529.19 | 773.42 | | | |
| *H* | 6.67 | 3.33 | | | 11,526.74 |
| *S* | 390.25 | | | | |
| *Q* | 795.49 | | 600.03 | | |
| *YH* | 1693.04 | 1153.62 | | | |
| *XH* | 37.89 | 16.71 | | | |
| *LH* | 3530.99 | 2573.63 | | 895.38 | |

*Commodity Supply under Risk*

Equations (14) through (16) define the demands for inputs into production.  These can be rearranged to show that the expansion path for inputs is unaffected by risk.  For example, equation (14) can be divided by equation (16).

$$(26) \qquad -\frac{\partial Y_1/\partial X_1}{\partial Y_1/\partial L_1} = -\frac{x_1}{l} = \frac{dL_1}{dX_1}.$$

Taking the total derivative of yield and setting it to zero along an isoquant, shows that the slope of an isoquant equals the negative of the input price ratio.  These points of tangency define the expansion path as shown in Figure 1.
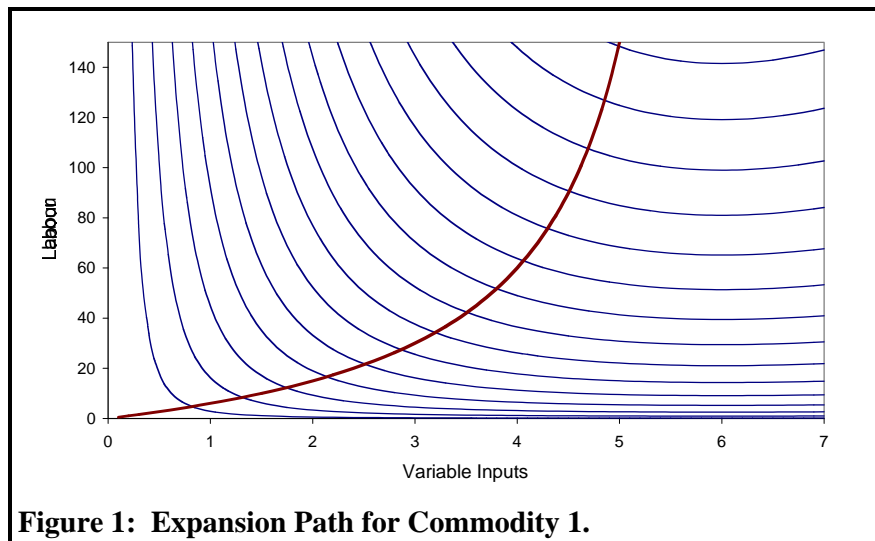


**Figure 1:  Expansion Path for Commodity 1.**

The points on the expansion path define the least-cost combinations of inputs.  In the usual manner, these least cost combinations give the total cost curve as a function of yield, which will

be denoted by $c(Y)$. The derivative of total costs is marginal costs. As a result, the two optimality conditions for variable inputs and labour are equivalent to a dual condition in which yield is the decision variable.

$$(14\ \&\ 16)\qquad y_1\big(1+\delta_{y_1}\big)=\frac{\partial c_1}{\partial Y_1}+Ry_1^2\big(Y_1H_1+S_1-Q_1\big)\sigma_{y_1}^2+Ry_1^2Y_1H_1\sigma_{Y_1}^2.$$

$$(15\ \&\ 17)\qquad y_2\big(1+\delta_{y_2}\big)=\frac{\partial c_2}{\partial Y_2}+Ry_2^2Y_2\big(H-H_1\big)\sigma_{y_2}^2+Ry_2^2Y_2\big(H-H_1\big)\sigma_{Y_2}^2.$$

On the left-hand side is marginal revenue and on the right-hand side is the supply curve under risk, including marginal risk premiums for price and yield risk. Assuming that storage, consumption and the hectares of land remain at the levels in Table 2, the supply curve for commodity 1 is shown in Figure 2.
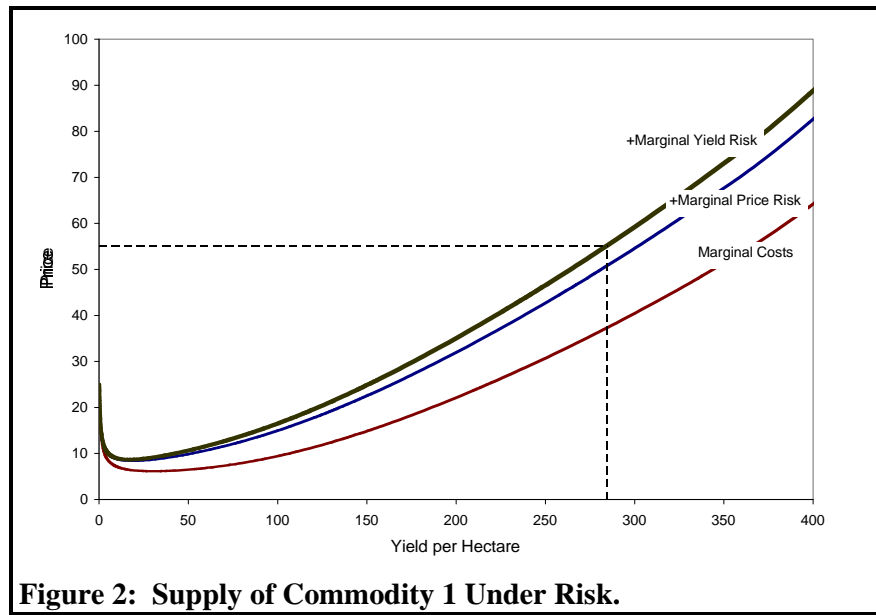


**Figure 2: Supply of Commodity 1 Under Risk.**

The supply curve is the top curve in which marginal risk premiums for price and yield risk are added to the usual marginal cost curve. The expected marginal revenue of 55 intersects the supply curve under risk at an optimal yield per hectare of 285. As the household becomes less averse to risk, the marginal risk premiums collapse. A risk neutral household would produce 375 where expected marginal revenue intersects the marginal cost curve.

*Demand for Commodity 1*

Interestingly, the household can hedge against price risk by consuming commodity 1. Consumption decreases the marginal risk premium. Assuming that yield, hectares, storage and the consumption of commodity 3 remain as in Table 2, the demand for commodity 1 is shown in Figure 3.
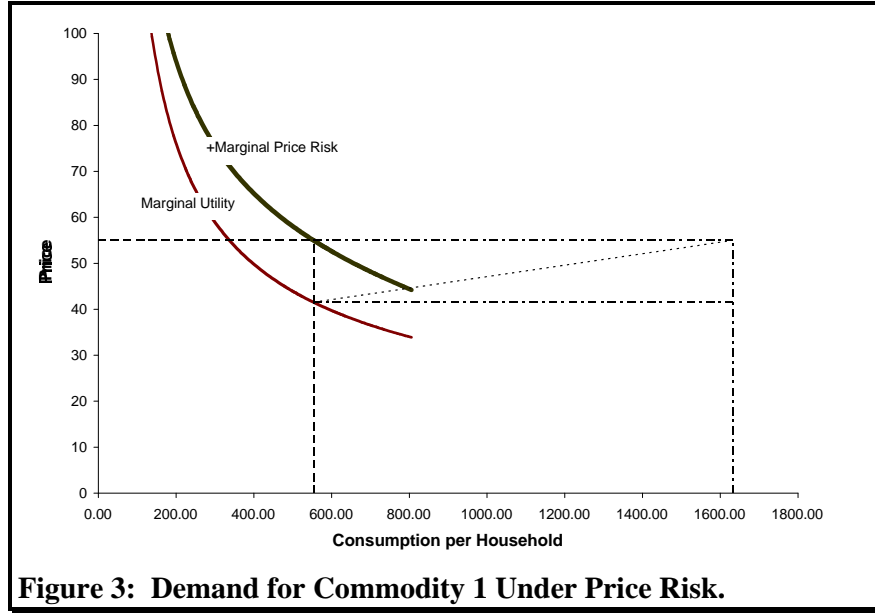
**Figure 3: Demand for Commodity 1 Under Price Risk.**

The demand curve equals marginal utility plus the marginal risk premium. As the household becomes less averse to risk, the marginal risk premium collapses and the demand curve approaches the marginal utility curve. The opportunity cost of consumption is the expected marginal revenue of 55 and intersects the demand curve at consumption of 555.

The vertical distance between the demand curve and the marginal utility curve is the marginal risk premium of 13.5. The total production plus storage equals 1630. This leaves 1630-555 or 1075 subject to price risk. On the graph, this unhedged quantity of 1075 is translated into the marginal risk premium of 13.5 by the slope of the dotted line in the figure. From the optimality condition for consumption in equation (21), the slope equals the coefficient of risk aversion multiplied by the variance of the price.

$$(27) \qquad R y_1^2 \sigma_{y_1}^2 = \frac{y_1\left(1 + \delta_{y_1}\right) - \dfrac{\partial U / \partial Q_1}{\lambda}}{\left(Y_1 H_1 + S_1 - Q_1\right)}.$$

If marginal utility could be observed, this equation would give a simple way to calculate a household's coefficient of risk aversion. Unfortunately marginal utility can't be observed and a household must have some other hedging alternative, such as a fixed price contract, before the degree of risk observation will be revealed by actual decisions.

*Demand for Land*

As a final example, we will solve the model for several different sizes of farms to see what might happen to a household if land can be transferred. The demand for land is defined by equation (18). Rearranging this equation gives the shadow price of land.

$$(28) \qquad h = \frac{y_2\left(1 + \delta_{y_2}\right)Y_2 - x_2 X_2 - l L_2 - R h^2 H \sigma_h^2 - R y_2^2 Y_2^2\left(H - H_1\right)\sigma_{y_2}^2 - R y_2^2 Y_2^2\left(H - H_1\right)\sigma_{Y_2}^2}{\left(\delta_W - \delta_h\right)}.$$

Varying the size of the farm and solving the model several times traces out the demand for land in Figure 4.



**Figure 4: Demand for Land.**
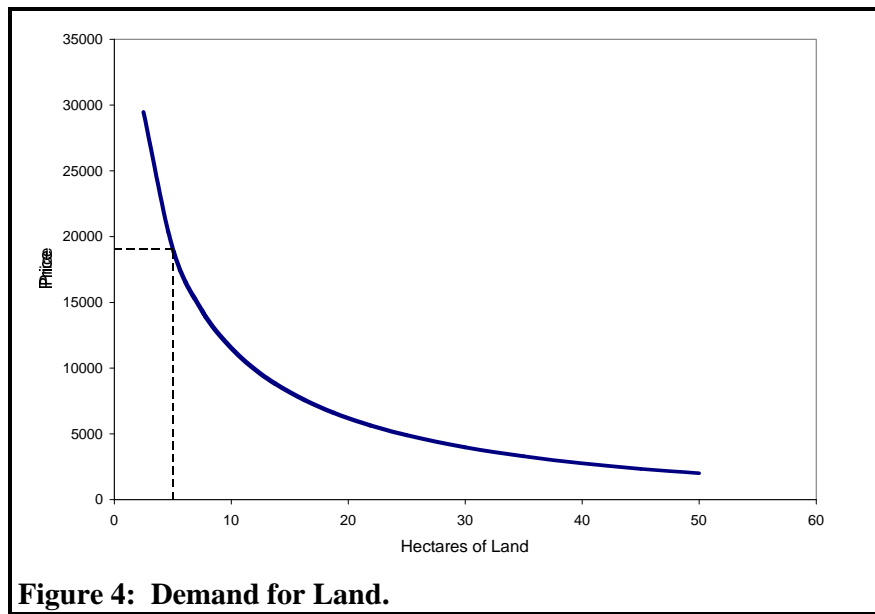
A 5 hectare farm will be willing to pay 11,527 per hectare of land.

The mix of hectares devoted to each commodity is an optimal portfolio problem as defined by equation (19). As the size of the farm changes the proportion of land in each commodity remains fixed, as shown in Figure 5.



**Figure 5: Portfolio of Hectares in Each Commodity.**
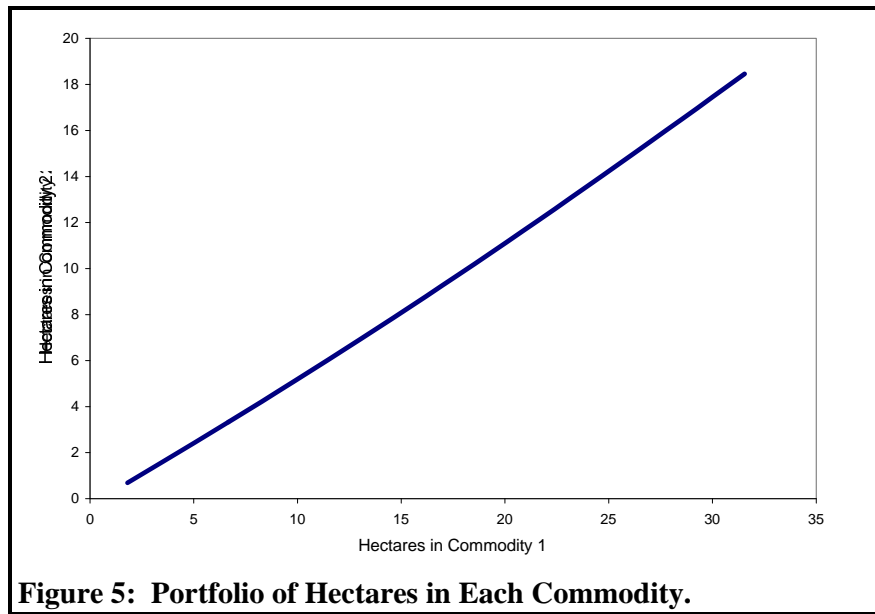
This result follows because hectares enter the production process linearly. Crop rotations and economies of scale could be modelled by including hectares non linearly. In this case, the proportion of the farm devoted to each commodity would change with farm size.

Finally, changes in farm size have implications for the demand or supply of labour as shown in Figure 6.



**Figure 6: Off-farm Labour as Farm Size Changes.**
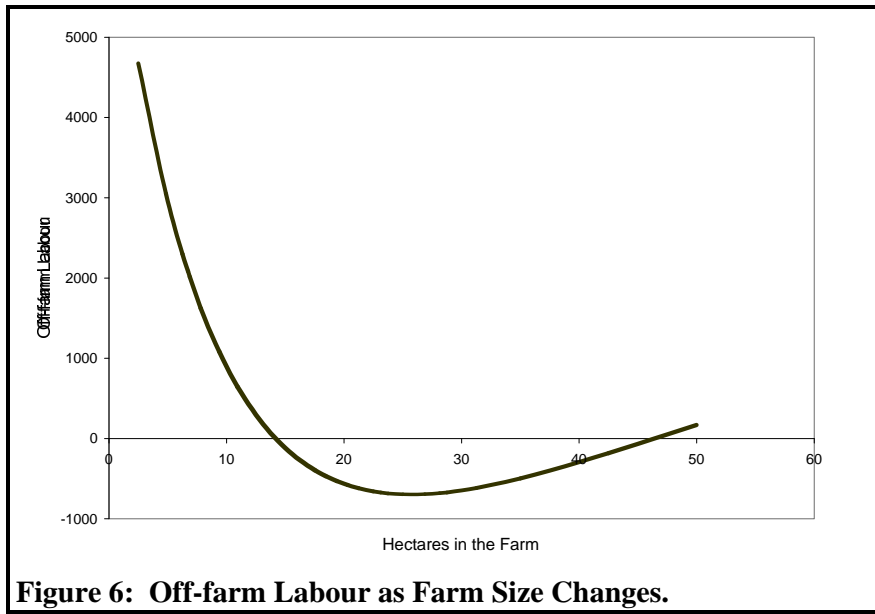
Until the farm reaches 15 hectares in size, the household supplies off-farm labour. Above 15 hectares, the household demands labour from other households. Surprisingly, at 47 hectares, the farm ceases demanding labour from others and again has excess labour to supply off-farm. The reason for this curious behaviour is increased risk exposure and no possible hedges against risk. Figure 7 shows the supply of commodity 1 for a farm of 25 hectares.



**Figure 7: Supply of Commodity 1 by a 25-Hectare Farm.**
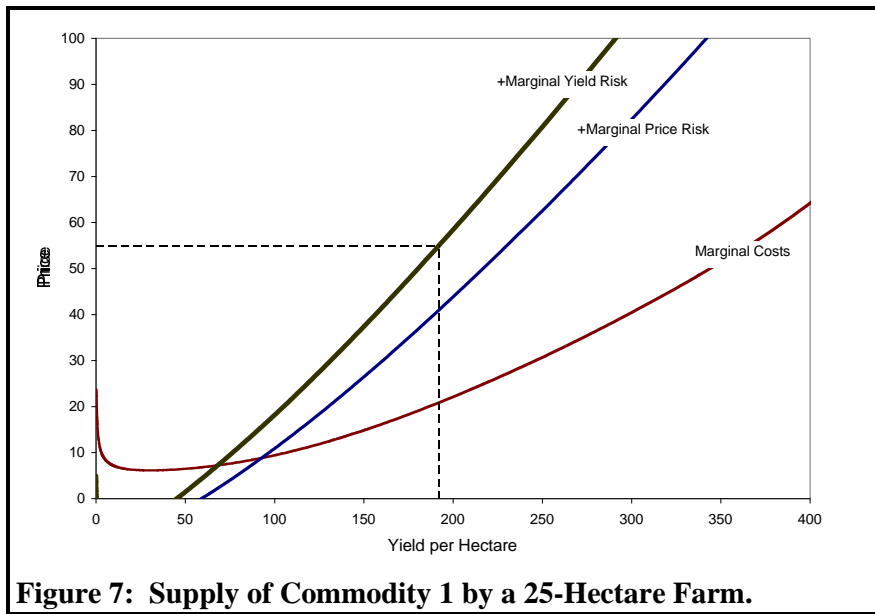
Yield has dropped to 192 per hectare as the household avoids risk by reducing inputs and, hence, outputs. If households are risk averse, then risk will limit the size of farms that are viable. This was shown previously in Figure 4 for the demand for land. Small farms will pay

more per hectare because they have a low exposure to risk. Farm size is determined by efficiency in production and economies of scale, but also by willingness and ability to bear risks.

**Spatial Trading of Land**

Spatial trading models are a way to put transaction costs into market transactions. For this reason spatial models have been used extensively in international trade. However, every market, no matter how small, is spatially segregated. Every trade must pay transaction costs. To demonstrate how the model of a single agricultural household can be aggregated into a spatial trading model, we will consider trading land between a less risk-averse household and a more risk-averse household, as shown in Figure 8.
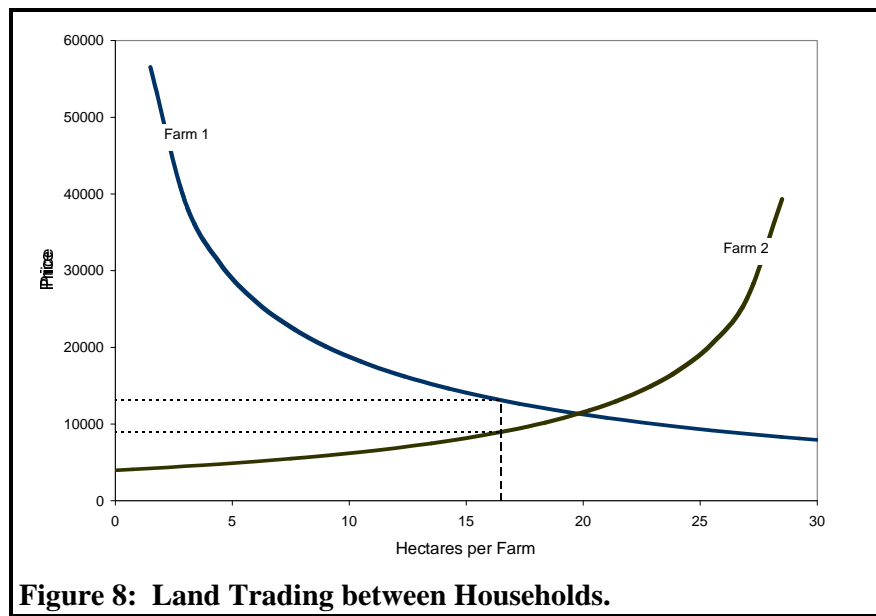


**Figure 8: Land Trading between Households.**

A total of 30 hectares is available. The size of Farm 1 increases from left to right and the size of Farm 2 increases from right to left. The household of Farm 1 is less risk averse than the household of Farm 2 and more willing to pay for land. Farm 2 is the same farm as shown in previous figures. If Farm 2 transfers land to Farm 1, but there are transaction costs of 4,136, then, at equilibrium, the price of land paid by Farm 1 will be 13,102 and the price received by Farm 2 will be 8,966. The size of Farm 1 will be 16.5 hectares and the size of Farm 2 will be 14.5 hectares.

In a similar way, the model of a single household can be aggregated into markets for commodities, inputs, and labour. All of the prices, which are parameters in Table 1, will become endogenous variables in the spatial trading model. If the marginal utility of wealth and coefficient of risk aversion are held constant, the spatial trading model will be a single period model. Trading, however, will alter wealth and, in turn, alter marginal utility of wealth and the coefficient of risk aversion. For these to be endogenous requires that the stochastic dynamic programming model in equation (6) be solved for each household.

**Concluding Comments**

The set of issues outlined at the beginning of the paper that were in need of consideration in the Vietnamese context were:

    (a)  Effects of land consolidation
    (b)  Input/output pricing policies and land use
    (c)  Gender issues
    (d)  Production technology
    (e)  Impact of credit and tax policies
    (f)  Household responses and land-use flexibility
    (g)  Interactions of policies

The issues directly incorporated in the model discussed above are (a), (b), (d), (f) and (g). With some modifications and adjustment each of the others can be incorporated in an expanded formulation. As well, the important issue of risk is fully incorporated into the household model and it has been demonstrated using artificial data that such a model can be solved and that it can be integrated into a spatial trading system. The next challenge is to devise means of obtaining reasonable estimates of the required parameters and also to devise ways to carry out the appropriate sensitivity analysis with such data.

In relation to risk it is noted that the "risk preferences" of decision makers are often discussed. Strictly speaking, preferences about risk don't exist. The concept of "risk aversion" is so inexact as to be misleading. Risk affects decisions whenever the world is non linear. Behaviour under risk can be explained by the type of non linearity which caused it. Non linearities fall into the following categories:

- Functions
    - Utility of consumption
    - Utility of terminal wealth
    - Production and cost functions
    - Endogenous prices
- Probability distributions
    - Non normal distributions
    - Co variances
- Asymmetries
    - Technical infeasibilities
    - Options and insurance
    - Subsistence or bankruptcy

In the model of agricultural households, utility functions are non linear as one source of "risk aversion". Production functions are non linear in variable inputs and labour, but these inputs are not risky and input decisions are the same as without risk. Prices are exogenous in a single household model, but endogenous in a spatial trading model. Endogenous prices are variables which multiply other variables and introduce non linearity.

Only normal probability distributions are defined by linear differential equations. Any other distribution is non linear. In the household model, distributions are assumed to be log normal. As a consequence, risk exposure increases with the size of the farm. This is the source of the behaviour in Figure 4 in which larger farms will pay less per hectare of land. Co variances can easily be included in the household model. Co variances are a source of non linearity because they alter means and variances, similar to endogenous prices.

Asymmetries are various forms of non linearity which alter the probabilities of events. Sometimes events simply can't happen and we use inequality constraints to exclude infeasible outcomes. In many cases, other approaches may be more realistic. Two examples are financial options and yield insurance. These eliminate downside risks for the household even though the downside events still happen. Options and insurance affect other decisions by shifting demand and supply curves. For an agricultural household, avoiding the risk of falling below subsistence is similar to purchasing insurance. To "purchase" this insurance, a household alters its production, investment and consumption decisions.

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