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# **Livestock Dynamics: An Old Problem and Some New Tools**

by

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## **Livestock Dynamics: An Old Problem and Some New Tools\***

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A problem encountered in the econometric estimation of dynamic models of livestock systems is dynamic instability during simulation of models. In the case of systems where economic decisions are involved in the breeding and the slaughter of animals it is possible for parameters to be obtained which generate dynamic instability. In the paper, a simple representation of the dynamics of a livestock system is examined using techniques of dynamic analysis. The dynamic stability of such models depends fundamentally on the parameters determining the births and deaths and thus the relative flows in and out of the stock of animals. In some situations very narrow stability ranges for the values of the parameters are observed.

The motivation for this paper derives from four observations: first, a number of researchers have discovered that during the construction of econometric models for

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various beef industries their models have been dynamically unstable, for example, Foote (1954), Crom (1970) and Yver (1971). It is suspected that many others have had such problems but they have not been reported in the literature. Second, only in a limited number of cases have the dynamic properties of beef models been examined (for example, Foote (1954) for the feed-livestock economy, Reeves, Longmire and Reynolds (1980) and Freebairn and Rausser (1975)). Third, in major model building projects the costs incurred in searching for a model structure and coefficient estimates which give dynamically stable simulation results can be very considerable. Work carried out at Agriculture Canada was a case in point (Agriculture Canada 1980). Finally, the relationships between the structure and parameters of a beef model and its characteristic roots do not appear to have been examined.

Given these observations and the fact that large complex models do not provide a suitable means for examining the relationships between the characteristic roots (that is, dynamic properties of a model) and the coefficients of a system, it was apparent that some form of abstraction was necessary. Thus, the objective for this paper is to report on the stability properties of a small model designed to capture some of the key elements of beef industry models and to consider some of the factors that influence the dynamic stability of such a model. From such results it is always difficult to draw generalisations for larger more complex models but some general principles are likely to become apparent. First, however, the question of dynamic stability must be addressed.

### **Partial Models and Instability**

In constructing dynamic econometric models, analysts, on occasion, are faced with the issue of whether or not a partial equilibrium system should be accepted as satisfactory if it is dynamically unstable. Although observations on the real world would seem to indicate that dramatic changes can occur from time to time these changes can often be attributed to a single cause (for example, effects of the oil price rises during the energy crisis, rapid shifts in exchange rates, disease outbreaks, etc.) or a set of causes. These changes can be labelled as 'shocks' and for a partial equilibrium system can be generally considered as exogenous. (In the context of this paper partial equilibrium is taken to mean that some of the major influences on a system are treated as exogenous.) Thus, in partial models some fluctuations will have exogenous sources. This in no way resolves the question of whether or not a partial system should be accepted if it is internally unstable. Foote (1954, p. 59) in considering the dynamics of the United States feed-livestock economy appeared to imply that dynamically unstable models may be acceptable but warned that the system would '... become inapplicable to the facts well before any such explosive tendencies became apparent to the observer'. Pindyck and Rubinfeld (1976, p. 345) have also suggested that if the dynamic instability of a model is not severe the model may be adequate for the purposes of policy analysis.

Observation on the real world would also seem to indicate that the effects of shocks on our economic system tend to dissipate as time passes, rather than intensify. This seems reasonable on the grounds that usually decisions can be made and then actions

taken which will alleviate some of the undesirable effects. Thus, much of our economic system as a whole will be dynamically stable even though it may be stable around economically, politically or socially undesirable states from time to time.

In a partial model of an economic system, some of the linkages to other sectors of the economy are usually specified as exogenous or assumed to be of no importance. Thus, a one-way causality is implied from the general economic system to the particular sector of the economy being modelled. Since part of the feedback is not specified in such models it would seem possible that a partial model of an economic system might be dynamically unstable when treated on its own but when fully integrated into an economic system the whole is dynamically stable (in general equilibrium systems the condition of gross substitution is required for perfect stability (Hicks, 1939 and Metzler, 1945)). If such was the case then it would seem reasonable to conclude that the bounds of the model were improperly specified. The corollary, of course, is that observed stability of a complete system (such as the food sector) may not imply stability of all the components of the system (such as a beef industry). The consequence of this is that it is vital to examine a model which is to be used for policy purposes for dynamic stability and also to have some a priori notions about the dynamic stability of an industry.

Various forms of misspecification may also affect the dynamic stability of models. Use of linear functions when non-linear would be theoretically correct and the effects of missing data or missing inter-sectoral links may all be important. Specification of expectation mechanisms without observed data and the making of assumptions about lagged decision responses may all have a bearing on the dynamic stability of a model. Thus, it must be concluded that specification of models will be an integral part of determining whether or not economic models are dynamically stable.

### **Approaches to Modelling the Beef Sector**

The beef sector has been one of the most frequently modelled agricultural sectors. A wide variety of different approaches has been used for the specification of models of the sector but on the supply side there appear to have been only two broad approaches. The first is one in which a slaughter function is estimated containing an inventory variable and then a separate inventory demand function is estimated (for example, Freebairn and Rausser 1975, the Wharton Agricultural Model reported in Chen 1976, and Haack, Martin and MacAulay 1978). The second is one in which the interaction between slaughter and inventory is taken into account more explicitly so that the inventory is the outcome of marketing and replacement decisions plus, of course, births and deaths (Crom 1970; Yver 1971; Nores 1972; Jarvis 1974; Bain 1977; Reeves, Longmire and Reynolds 1980; Ospina and Shumway). It is worth noting that mathematically these two approaches can be shown to be equivalent but that econometrically the two approaches may provide quite different results.

Although over time there has been a considerable improvement in the understanding of how to model the decision processes in the beef sector, more would seem to be required if models are to become sufficiently reliable for consistent and continuous

use by policy advisers. Closer attention might be given to the physical and biological links within the beef system. This is likely to be particularly true for quarterly models where the natural link between cohorts and age-groups is lost compared with annual models. In quarterly models it would seem to be necessary to specify the way in which cattle flow from one age group to the next. Behavioural equations can be developed for such a specification. The advantages of doing so are that such models are more credible to industry participants and therefore more useful for policy purposes, that the demographic relationships will tend to constrain the model so that some of the potential sources of instability are eliminated and that effects of technological change on fertility rates, death rates and rates of growth etc. can be taken into account.

From reviewing the various approaches to modelling the beef sector it is possible to distil out a relatively simple model which captures some of the key features and which carries sufficient theoretical richness to be useful. Emphasis in this paper will be on the second type of model discussed above.

### **A Simplified Beef Model**

The basic relationships in the simplified model developed here for didactic purposes consist of a demand function representing a retail level demand (either the price or quantity dependent form could be used without affecting the results), a slaughter function based on a farm level price, a price transmission equation linking the two levels together, an inventory-slaughter balancing identity and a calves-born equation. The model can be viewed as either an annual or quarterly model. The various equations may be written as follows (Greek symbols are used for model parameters and only lags are indicated in brackets with all other variables referring to the current period):

a) Retail demand

$$(1) \quad Q = \alpha - \beta PR$$

b) Slaughter function

$$(2) \quad Q = \theta + \delta PF(-1) + \phi I(-1)$$

c) Price transmission

$$(3) \quad PF = \mu + \sigma PR(-1)$$

d) Inventory-slaughter balance

$$(4) \quad I + Q = I(-1) + B$$

e) Calves born

$$(5) \quad B = \varepsilon I(-1)$$

where:

Q is the quantity demanded and slaughtered ( in animal equivalents);

PR is the retail price of beef;

PF is the farm price of cattle;

I is the inventory of animals on hand at the beginning of a period;

B is the number of calves born.

This model captures the essential demand, slaughter and inventory relationships assuming that cattle are not subdivided into various categories but treated as homogeneous. The model thus ignores: (a) animal classes; (b) deaths and other losses; (c) carcass weight effects; (d) inter-regional trade; (e) numerous shifter variables including any price response on the calves born equation (to be considered later). The price transmission equation implies a mixed, fixed and/or proportional margin between the farm and retail levels and a delay in the transmission of price from farm to retail levels. A diagrammatic representation of the model is given in Figure 1.

The size of the parameters in the model are hypothesised to satisfy the following relationships indicated in Table 1.

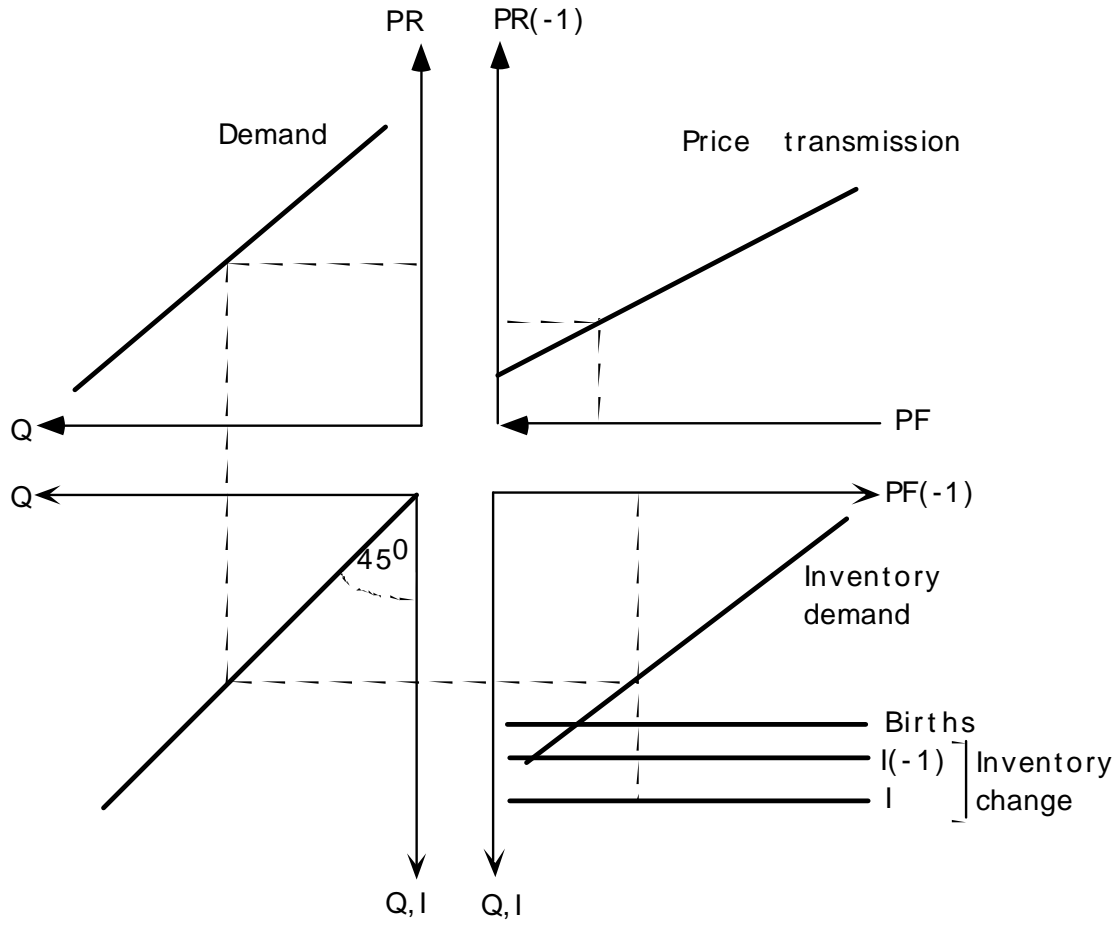


Figure 1 Diagrammatic representation of a simplified beef model

Table 1  
*Parameter Ranges for the Specified Model*

Coefficient and range	Comment
$\beta > 0$	Demand slope coefficient assumed to be downward sloping
$-\infty < \delta < \infty$	Short-run price effect on slaughter, likely to be negative
$0 < \phi < 1$	Slaughter rate from inventory the size of which will depend on the definition of the inventory but for all cattle might be of the order of 0.3 to close to 1.0 for a category such as steers
$\sigma \approx 1$	Coefficient of price transmission likely to be approximately 1.0
$0 < \varepsilon < 1$	Calving rate will depend on the definition of the inventory category. If it is for female cattle it would be about 0.85 otherwise less than this value

By substituting equation (5) into (4) the system may be reduced in size by one variable and written in matrix form as:

$$(6) \quad \begin{bmatrix} -1 & -\beta & 0 & 0 \\ -1 & 0 & 0 & 0 \\ -1 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} Q \\ PR \\ PF \\ I \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & \delta & \theta \\ 0 & 0 & 0 & 1+\varepsilon \\ 0 & \sigma & 0 & 0 \end{bmatrix} \begin{bmatrix} Q(-1) \\ PR(-1) \\ PF(-1) \\ I(-1) \end{bmatrix} = \begin{bmatrix} -\alpha \\ -\theta \\ 0 \\ -\mu \end{bmatrix}$$

This system of equations may be written more compactly as:

$$(7) \quad Ju + Kv = d$$

where J and K are the respective matrices of coefficients in equation (6) and u, v and d the respective vectors of the variables in the system.

Having represented the beef sector model in a compact form it is now possible to analyse its dynamic properties. Following Chiang (1984, pp. 554-6; pp. 608-12) it is possible to find the equilibrium solution to this system using the particular integrals and a set of starting values. The time path is the sum of the particular integral and the complementary function. It is also possible to examine the dynamic properties of the system using the characteristic equation (details given in the Appendix).

In the case of the model given in equation (6) the characteristic equation is:

$$(8) \quad b^4 + (\phi - \varepsilon - 1) b^3 + \frac{\sigma\delta}{\beta} b^2 - \frac{\sigma\delta}{\beta} (1 + \varepsilon) b = 0.$$

A common factor of  $b$  may be taken out of equation (8) so that it is necessary to find the roots of a third degree equation rather than a fourth degree equation. Even with one of the solutions of  $b = 0$  the equation proved too complex to solve algebraically so as to obtain information on how the parameter values affect the value of the roots of the system.

### Some Observations on the Characteristic Equation

a) The discriminant

If all the parameters of the model are positive then the discriminant of the characteristic equation will also be positive. This implies that the characteristic equation will have one real root and two complex conjugate roots. It also happens that, if the parameters are positive, all the roots will be greater than unity.

b) Zero values for  $\sigma$  or  $\delta$

Equation (8) may be redefined as:

$$(9) \quad b^4 + a_1 b^3 + a_2 b^2 + a_3 b = 0,$$

where:

$$a_1 = \phi - \varepsilon - 1,$$

$$a_2 = \frac{\sigma\delta}{\beta},$$

$$a_3 = -\frac{\sigma\delta}{\beta} (1 + \varepsilon).$$

It is clear that if  $\sigma$  or  $\delta$  equal zero then equation (9) becomes:

$$(10) \quad b^3(b + a_1) = 0.$$

The roots of this equation are  $b = 0$  or  $b = -a_1$ .

The implication is that if there is no price effect in the slaughter equation, and/or a constant margin between the retail level and the farm level, then the roots of the system are simply a function of the difference between the herd slaughter rate and the calving rate. Since  $-a_1$  is the calving rate less the slaughter rate plus one then the root

can only have a value of less than one if the slaughter rate is greater than the calving rate. At first sight this is a strange result. However, it is more reasonable if one considers what would be required for a cycle of constant amplitude. For this to occur the root would have to have a value of exactly 1.0 and for such a value the calving rate would have to exactly equal the slaughter rate. From a strictly biological point of view this makes sense. If the calving rate was greater than the slaughter rate (analogous to the death rate in some biological systems) then numbers should expand in the longer term. If the calving rate is less than the slaughter rate then numbers will fall in the longer term.

c) Small values for  $\sigma$  or  $\delta$  or large  $\beta$

In a similar way, as  $\sigma$  or  $\delta$  become small, or  $\beta$  large, then the coefficients  $a_2$  and  $a_3$  tend to zero. As this happens the absolute value of the dominant root of the system tends towards the value of  $a_1$  (that is,  $\phi - \varepsilon - 1$ ).

d) Negative values for  $\delta$

In most beef models the short-term price effect for slaughter is found to be negative (Jarvis 1974). If this is the case then the signs of the terms  $a_2$  and  $a_3$  are reversed and therefore their effects on the roots of the equation are also reversed. With signs reversed and the value of  $(1 + \varepsilon)$  in  $a_3$  greater than 1.0, since the calving rate must be positive, then as the value of  $\beta$  increases to a large value,  $a_3$  will have a larger positive effect than the negative effect of  $a_2$ . Because of the difficulty of deciding what the overall effects on the roots would be, it was decided to carry out a set of simulation experiments.

### **Some Simulation Experiments**

In an effort to obtain more information about the nature of the roots, a computer program was developed to calculate the roots of equation (8) for a range of values of the parameters. The results can be summarised in the following diagrams for reasonable ranges of the parameters.

a) Zero values for  $\sigma$  or  $\delta$

If  $\sigma$  or  $\delta$  are zero then the parameters  $\phi$  and  $\varepsilon$  will determine the roots as in Figure 2.

From Figure 2 it can be seen that if there were a constant margin, or no slaughter price response, then for the model to be stable the slaughter rate coefficient,  $\phi$ , in the slaughter equation has to exceed the calving rate coefficient,  $\varepsilon$ . In econometrically estimated beef models this is not likely to be the case, nor is it likely to be the case in reality.

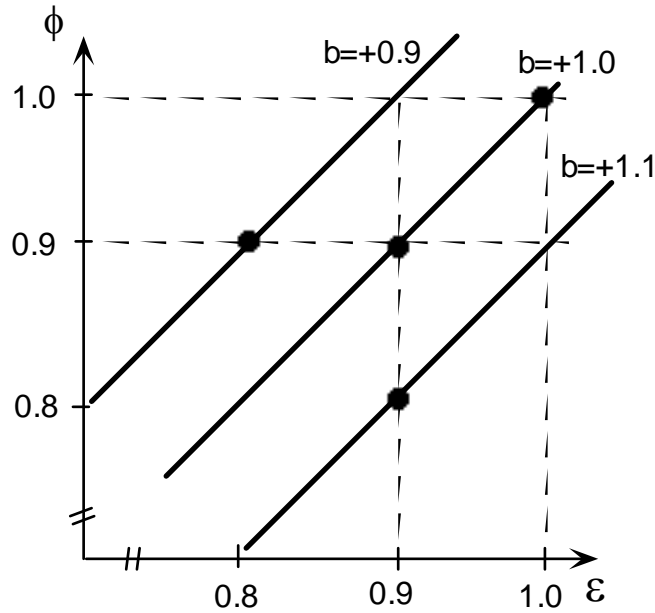


Figure 2 Combinations of  $\phi$  (the slaughter rate) and  $\epsilon$  (the calving rate) for values of the dominant root of 0.9, 1.0 and 1.1 given either  $\sigma$  or  $\delta$  are zero (or  $\beta$  very large)

b) Price effects included

By choosing values for  $\phi$ ,  $\sigma$  and  $\epsilon$  the combinations of  $\delta$  (slaughter price coefficient) and  $\beta$  (demand coefficient) were varied in a grid pattern (for example, see Table 2). Using the grid search it was possible to construct Figure 3 approximately ( $d^*$  refers to the absolute value of the dominant root which is the modulus in the complex case). The most significant conclusion to be drawn from Figure 3 is that for such a beef model to be dynamically stable the coefficient  $\delta$  on the price variable, PF, in the slaughter equation must be negative.

Table 2  
*Absolute Values of the Dominant Root for a Given Set of Parameter Values<sup>a</sup>*

$\beta$ value	$\delta$ value										
	-10	-8	-6	-4	-2	0	2	4	6	8	10
1.0	3.22	2.93	2.6	2.2	1.64	1.00	2.92	2.5	2.86	3.16	3.43
2.0	2.41	2.20	1.95	1.65	1.22	1.00	1.68	2.02	2.28	2.50	2.69
3.0	2.04	1.86	1.21	1.38	1.02	1.00	1.53	1.81	2.02	2.20	2.36
4.0	1.81	1.64	1.46	1.22	0.89	1.00	1.43	1.68	1.87	2.02	2.16
5.0	1.64	1.50	1.32	1.11	0.80	1.00	1.37	1.59	1.76	1.90	2.02
10.0	1.22	1.11	0.97	0.80	0.54	1.00	1.22	1.37	1.49	1.59	1.68

Values of the coefficients other than  $\beta$  (demand slope) and  $\delta$  (price coefficient in slaughter function) were set as follows:  $\phi = 0.9$ ,  $\sigma = 0.8$ ,  $\varepsilon = 0.90$ . Values in the table are the modulus of the complex conjugate roots where complex roots occur otherwise they are the absolute value of the dominant root.

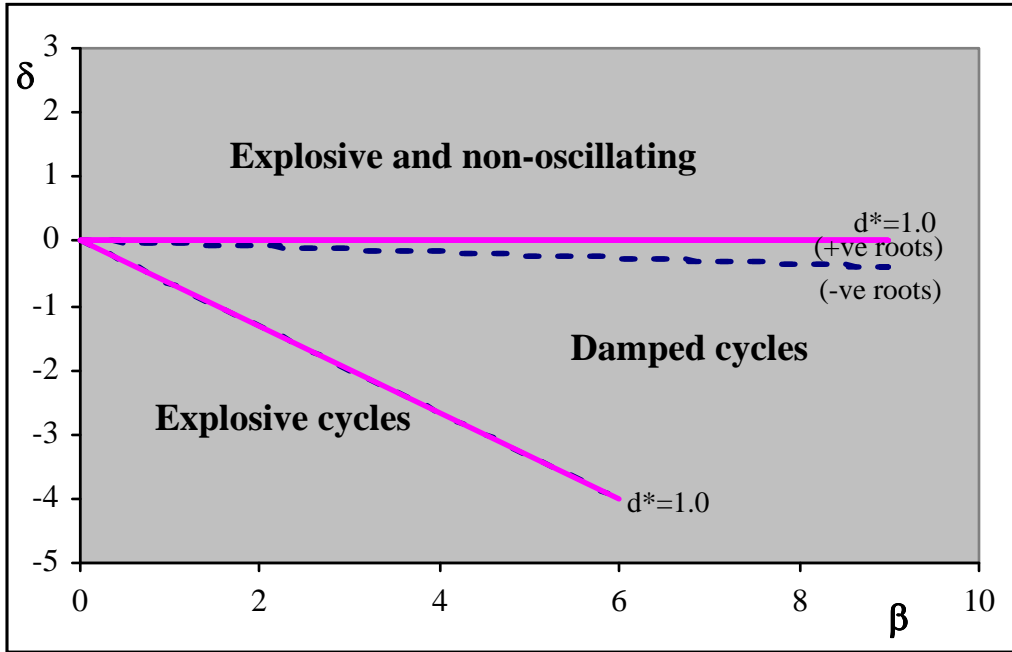


Figure 3 Combinations of  $\beta$  and  $\delta$  for which the dominant root has a modulus of 1.0 with  $\phi = 0.9$ ,  $\delta = 0.8$  and  $\varepsilon = 0.9$

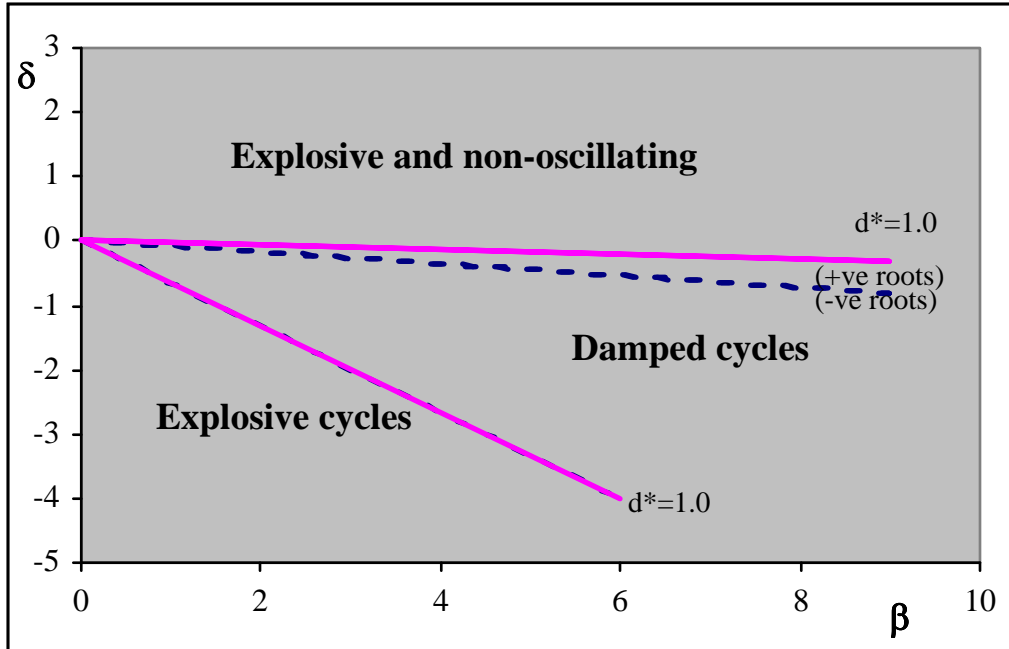


Figure 4 Combinations of  $\beta$  and  $\delta$  for which the dominant root has a modulus of 1.0 with  $\phi = 0.9$ ,  $\delta = 0.8$  and  $\varepsilon = 0.95$

Changing the scenario to  $\phi = 0.9$ ,  $\sigma = 0.8$  and  $\varepsilon = 0.95$  (that is, the calving rate is greater than the off-take rate) changes the diagram so that the dominant root at  $\delta = 0$  has an absolute value of 1.05 (that is  $\phi - \varepsilon - 1 = 0.9 - 0.95 - 1 = -1.05$ ).

In the situation illustrated in Figure 4 the area of damped cycles has been tipped slightly downwards. Thus as the off-take rate is smaller than the calving rate, more negative coefficients are needed on the price variable in the slaughter equation to maintain dynamic stability.

If the scenario is again changed so that the off-take rate is greater than the calving rate then the zone of damped cycles is tipped upwards with small positive coefficients on the price variable in the slaughter equation being possible.

It is also possible to change the width of the band for which the model is dynamically stable by changing the value of  $\sigma$ , the price transmission coefficient. As the value of this coefficient is decreased the stability band widens given fixed values for the other coefficients. Very large values make the band narrower.

By inserting the values of the various coefficients in a set of equations similar to equations (1) to (5), the model was simulated through time. A limited number of the parameter combinations were tested and found to confirm the above results.

### An Alternative Specification

To observe the effect of a slight change in specification on the dynamic stability of the model equation (5) was modified to include a price response. The basis for this was that the breeding decision is likely to be affected by expected prices represented in a simple way as a lagged price. Thus:

$$(11) \quad B = \varepsilon I(-1) + \lambda PF(-1), \quad \lambda > 0.$$

The addition of the price variable leads to the following characteristic equation:

$$(12) \quad b^4 + a_1 b^3 + a_2 b^2 + a_3 b = 0$$

where;

$$a_1 = \phi - \varepsilon - 1,$$

$$a_2 = \frac{\sigma\delta}{\beta},$$

$$a_3 = \frac{\sigma}{\beta}(\lambda\phi - \delta(1 + \varepsilon)).$$

In this case the coefficients are the same as in the earlier version of the model except for  $a_3$ . This coefficient now contains the product of the coefficient on the price variable in the births equation,  $\lambda$ , and the coefficient on the inventory variable in the slaughter equation,  $\phi$ .

The effects of changing the model as indicated are illustrated in Figure 5 for different values of the price coefficient in equation (11). The change in the structure of the model has made it possible to have positive values on the short-run price effect in the slaughter equation (2). The stability region has been shifted upwards particularly on the left-hand side of the diagram. The movement upwards is greater the larger the value for  $\lambda$ . The significance of this change in the model is that a positive coefficient on lagged price in the births equation can compensate to some extent for a positive short-run effect of price on slaughter. Thus, one means of raising the chances of a stable model of the type illustrated is to make sure that price effects are included on the inflow (births) side of the model as well as on the outflow or slaughter side. This would be particularly so if the short-term price effect was found to be positive.

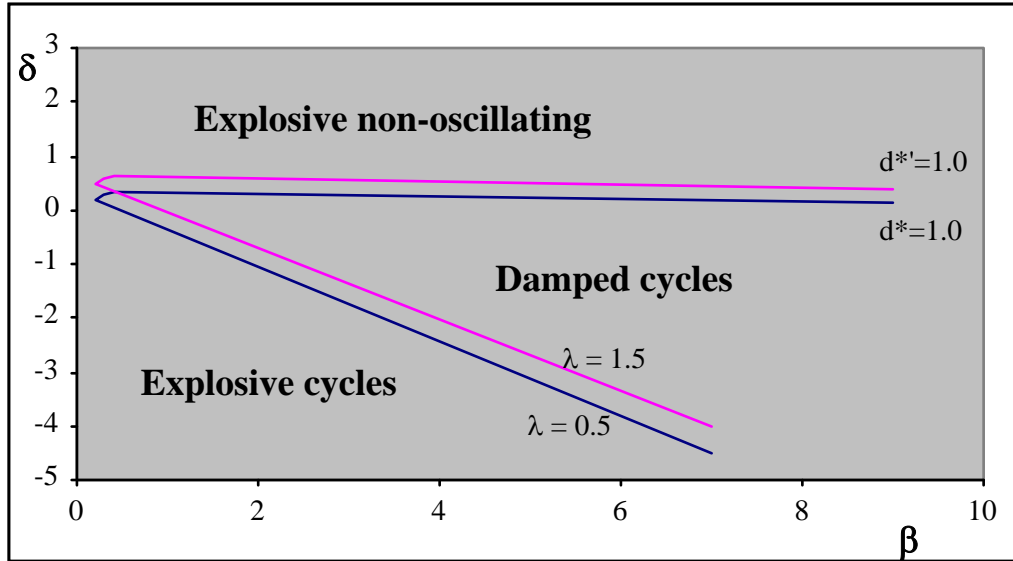


Figure 5 Combinations of  $\beta$  and  $\delta$  for which the dominant root has a modulus of 1.0 with  $\phi = 0.9$ ,  $\delta = 0.8$  and  $\varepsilon = 0.95$  and  $\lambda = 0.5$  and 1.5

### Concluding Comments

From these results it is not surprising that models of the beef sector are often unstable. The possible combinations of parameters for which the roots lie within the unit circle would appear to be somewhat limited. With data errors, estimation errors, inappropriate methods of estimation, and difficulties in defining econometric structures, the chance of all the coefficients being within the appropriate set of ranges is not likely to be high when the ranges are narrow.

Increasing the demand coefficient, ensuring the slaughter function has a negative price coefficient, and ensuring that the off-take rate is greater than or close to the calving rate, are changes in a model which may lead to improved chances of dynamic stability. As well, ensuring price effects on the inflow side of the model are captured as well as on the outflow side may ensure a greater chance of stability.

A particular structure was chosen for the beef model used as an illustration and changes to the model are likely to change the nature of the characteristic equations. However, some general principles have been indicated and by using the methods illustrated here it is possible to carry out tests on representative structures for any type of model which is being considered for construction and testing.

It is possible that using stochastic analysis further insight in the nature of the stability of such models could be obtained. However, it is most likely that by taking into account stochastic variation the parameter bands would be reduced slightly. Significant changes would not be expected.



## APPENDIX

### Particular Integrals and the Characteristic Equation

To find the particular integrals of an equation, such as equation (6), let all lags collapse to zero so that:

$$Q_{t+1} = Q_t = \bar{Q}$$

$$PR_{t+1} = PR_t = \bar{PR} \quad ,$$

$$PF_{t+1} = PF_t = \bar{PF} \quad ,$$

$$I_{t+1} = I_t = \bar{I} \quad .$$

In effect:

$$(A.1) \quad u = v = \begin{bmatrix} \bar{Q} \\ \bar{PR} \\ \bar{PF} \\ \bar{I} \end{bmatrix}$$

Equation (6) thus reduces to:

$$(A.2) \quad [J + K] \begin{bmatrix} \bar{Q} \\ \bar{PR} \\ \bar{PF} \\ \bar{I} \end{bmatrix} = d$$

If an inverse exists for  $[J + K]$  then the particular integrals can be expressed as:

$$(A.3) \quad \begin{bmatrix} \bar{Q} \\ \bar{PR} \\ \bar{PF} \\ \bar{I} \end{bmatrix} = [J + K]^{-1} d$$

The particular integrals are of little interest in determining the dynamic characteristics of the system. The complementary functions are central to the determination of the dynamic properties of the system. By trial solutions for  $u$  and  $v$ , then:

$$(A.4) \quad u = \begin{bmatrix} mb^{t+1} \\ nb^{t+1} \\ ob^{t+1} \\ pb^{t+1} \end{bmatrix} = \begin{bmatrix} m \\ n \\ o \\ p \end{bmatrix} b^{t+1} \quad \text{and} \quad v = \begin{bmatrix} mb^{t+1} \\ nb^{t+1} \\ ob^{t+1} \\ pb^{t+1} \end{bmatrix} = \begin{bmatrix} m \\ n \\ o \\ p \end{bmatrix} b^{t+1}$$

When substituted into the equation,

$$(A.5) \quad Ju + Kv = 0,$$

the result is:

$$(A.6) \quad J \begin{bmatrix} m \\ n \\ o \\ p \end{bmatrix} b^{t+1} + K \begin{bmatrix} m \\ n \\ o \\ p \end{bmatrix} b^t$$

After multiplying through by  $b^t$  (a scalar) and factoring, then:

$$(A.7) \quad [bJ + K] \begin{bmatrix} m \\ n \\ o \\ p \end{bmatrix} = 0$$

From equation (A.7) the values of  $b$ ,  $m$ ,  $n$ ,  $o$  and  $p$  can be found using trial solutions so as to make the latter determinate.

So as to avoid trivial solutions it is necessary that:

$$(A.8) \quad [bJ + K] = 0$$

This is the characteristic equation of the system, written in determinantal form, the characteristic roots of which should lie within the unit circle for dynamic stability.

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