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A Dynamic Optimisation Model of Weed Control

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Abstract

It is argued in this paper that static approaches to weed management, where the benefits and costs are only considered within a single season, are inappropriate for assessing the economic benefits of weed control technologies. There are carryover effects from weed management as weeds that escape control in one season may reproduce and replenish weed populations in following seasons. Consequently, it is appropriate to view weed control in the context of a resource management problem where the goal is to determine the optimal inter-temporal level of weed control that maximises economic benefits over some pre-determined period of time.

A dynamic optimisation model for weed control is presented. Using the tools of comparative static analysis and Pontryagin's maximum principle, the conditions for optimal input use (ie weed control) are compared for static and dynamic situations. It is shown that a higher level of input use for a given weed population is optimal using a dynamic framework than would be derived under a static framework. The analysis is further extended by the incorporation of uncertainty and shows that the optimal level of weed control is also affected by uncertainty in herbicide efficacy and the survival of weed seeds produced. A case study of the optimal long-term management under deterministic and stochastic conditions of an annual cropping weed, *Avena fatua*, is presented.

Introduction

Weed management has historically aimed to control weeds through herbicide treatments and/or tillage operations, primarily to reduce yield losses through competition. Consequently, a range of weed control decision making frameworks, such as the economic threshold (Auld, Menz and Tisdell 1987), have been developed to maximise economic returns in the current season or year.

It is possible that taking a short-term, or static, decision making horizon results in less than optimal weed control decisions and economic returns. It is hypothesised in this paper that incorporating the carryover effects of weed management through a dynamic economic framework leads to a greater level of weed control for a given weed population and higher economic returns in the long term.

Defining the Economic Problem

An important concept in determining the response of agricultural systems to both fixed and variable factors of production is the production function. For instance, crop yield is determined by factors such as varietal type, soil, rainfall, temperature, pests, diseases etc. The production function is generally written as

(1) $Y = f(X_1, X_2, X_3, .. , X_n)$

The quantity of the product produced per time period is dependant upon the quantities of inputs $X_1,..,X_n$. If all factors of production but one are held constant, it is possible to trace the response in yield from variations in the parameter values of this factor through the production function. For instance, yield is expected to decline with an increase in weed density, all other factors held constant.

Weeds invading agricultural crops and pastures directly reduce farm income for the following reasons. First, weeds compete with crops and pastures for nutrients, water and light thereby reducing yield. In the case of pastures the reduced yield leads to lower livestock carrying capacities and consequently lower income. Second, weeds can contaminate agricultural produce (eg. grain contamination, vegetative fault in wool, milk tainting in dairy cows), thereby incurring a penalty and reducing the on-farm price. Third, weeds result in increased production costs as a result of control measures being undertaken (eg. herbicides, tillage). Finally, weeds can impact upon the management of farm resources. A weed population may increase to a level whereby it is no longer profitable to produce a preferred enterprise and a farmer is forced to switch to a less profitable alternative, eg rotating from wheat to a pasture or fallow phase due to a chronic weed infestation.

For simplicity of exposition assume that the only form of weed control is from the application of a herbicide. If the initial weed population is denoted as x, the herbicide dose rate is denoted as u, and all other factors of production are fixed and denoted by z, the production function can be re-written as

(2) $Y = f(x,u,z)$

The effect of x through the production function is to directly reduce yield while the effect of u is to ameliorate the yield loss effect of the variable x. Therefore, equation (2) can be dissagregated into separate equations for estimating the weed free yield (Y_0) and the yield loss (Y_L) associated with weed density and weed control.

(3) $Y_0 = f(z)$

(4) $Y_L = f(x,u)$

(5) $Y = Y_0(1-Y_L)$

Where Y_L is measured as percentage yield lost because of weed competition. Cousens (1985) has argued that the appropriate damage function that best describes yield loss as a function of weed density is a rectangular hyperbola. The biological grounds for this argument are that at a low weed density weeds are most competitive to crops and cause a maximum marginal reduction in yield, hence the effect of an increase in weed numbers at low densities is additive. However, when the density is high increased intra-specific weed competition tends to reduce the marginal yield loss. The rectangular hyperbola function derived is

(6) $$Y_L = \frac{ID}{1 + \frac{ID}{A}}$$

Where D is the weed density influencing yield[1], I is the percentage yield loss per unit weed density as weed density approaches zero, and A is an estimate of the maximum yield loss of a weedy crop relative to the yield of a weed-free crop.

Weed density that affects crop yield (D) is a function of initial weed density and the proportion of weeds killed by herbicide application (γ).

(7) $D = x(1-\gamma)$

[1] Note that D differs from the initial weed population, x, as it represents the residual weed population after weed control.

The kill function for herbicide dose response must be bounded by 0 and 1. Various functional forms for γ have been proposed, with the logit and probit functions (Finney 1971) the preferred for herbicide dose response. The variable γ may also be a parameter determined from an integrated weed management (IWM) strategy. An IWM strategy may include various mixtures of cultural management tactics (eg. fertiliser, stocking rates, crop and pasture rotations, spray grazing, hay or silage, cultivation, fallow, delayed sowing, seed collection), chemical management tactics (eg. pre- and post-emergent herbicides, spray-topping, crop-topping, chemical fallow) and biological control tactics. In some systems (eg. grazing systems) herbicides may not even feature in an economically optimal IWM strategy.

The profit function for an optimal herbicide dose problem is defined as

(8) $\pi = P_y Y(x,u) - P_u u - C_1 - C_2$

Where π is profit, P_y is the output price per unit of the commodity, P_u is the per unit cost of weed control, C_1 is the constant application costs for the weed control input (machinery and labour), and C_2 is the constant cost of production of the remaining factors of production, z. The first term of the equation (P_yY) is the total revenue and is determined not only by the level of the control variable but also by the initial weed density x. Thus, the total revenue for any variation in u will be specific to the initial value of x.

A Specific Weed Problem

Wild oats (*Avena* spp.) is an important weed of winter grain crops in southern Australia as it competes vigorously with crops, resulting in yield loss, and can produce large numbers of seed. The population dynamics of this weed are given in the following equations.

(9) $$S_{it} = p_i \sum_{i=1}^{3} \delta S B_t$$

(10) $$D_{it} = \sum_{i=1}^{3} (1-\phi_i)(1-\gamma_i) S_{it}$$

(11) $$R_{it} = \sum_{i=1}^{3} \exp\left[\alpha_i \log D_{it} / \left(\mu_i + \varepsilon_i \log D_{it}\right)\right]$$

(12) $$N_t = \sum_{i=1}^{3} \kappa R_{it} - \eta + \xi$$

(13) $$SB_{t+1} = N_t + (1-\psi)[(1-\delta)SB_t]$$

Three population cohorts are included in the model, represented by the parameter *i*. SB_t is the weed seed bank at the beginning of year *t*, S_{it} is seedlings of the *i*th cohort in year *t*, D_{it} is the density of mature plants, R_{it} is seed resulting from the reproduction of wild oats, N_t is new seed added to the seed bank, δ is the annual germination of wild oats seed, p_i is the proportion of germination corresponding to the *i*th cohort, ϕ_i is mortality resulting from tillage or use of a knockdown herbicide to control weed seedlings prior to sowing, γ_i is the herbicide induced mortality of seedlings, α_i, μ_i and ε_i are regression coefficients (Medd et al 1995) in the fecundity equation (12), κ is the survival rate of new seed, η is seed export such as the removal of seeds at harvest, ξ is the import of seeds (e.g. through sowing), and ψ is the death rate of dormant seed. The parameter values for *p*, δ, ϕ, α, μ, ε, κ, η, ξ and ψ applicable to wild oats (Jones and Medd 2000) and used in this study are given in Table 1. The calculation of R_{it} was constrained to zero when D_{it} < 0.5 plants m^{-2} because, due to the nature of the functional form used, the result of equation (11) degenerates to infinity as D_{it} approaches zero.

Pandey (unpublished) estimated a dose response function for diclofop-methyl to control wild oats. The effect of climate upon herbicide efficacy was accounted for by including a soil moisture index variable, with soil moisture condition being rated as 1, 2 or 3 for dry, average and good conditions. The dose response relationship was estimated using a logit model and the following function was determined.

(14) $$\log\left[\frac{\gamma}{(1-\gamma)}\right] = -1.323 + 1.496\log(u) + 0.945SM + 0.895A$$

Where γ is the proportion of weeds killed, *u* is the quantity of herbicide applied (litres ha^{-1}), *SM* is an index of soil moisture conditions up to the date of spray, and *A* is a dummy variable (*A* = 1 if adjuvant is added, *A* = 0 otherwise). Reformulating this function and setting *A* = 1, the following equation for the proportion of weeds killed is obtained.

(15) $$\gamma = \frac{\exp[-0.428 + 1.496\log(u) + 0.945SM]}{1 + \exp[-0.428 + 1.496\log(u) + 0.945SM]}$$

A weed free yield (Y_0) of 3.9 tonnes ha^{-1} was estimated from a production function for wheat in southern New South Wales (G.M. Murray, personal communication). The function used was

(16) $$Y_0 = 4.21 + 0.0095RF - 0.028SD$$

Where *RF* is rainfall over the period April to October and *SD* is the number of days up to the optimum sowing date of 26 April (ie. 116 days). As the sowing date variable is a constant in this analysis the function becomes Y_0 = 0.962 + 0.0095*RF*. Solving this function for an average winter rainfall of 310 mm results in a weed free crop yield of 3.9 tonnes ha^{-1}.

Yield loss from the phytotoxic effects of herbicide use (Y_p) was estimated from Pandey (unpublished) at Y_p=0.01333*u*. The yield loss equation parameters (equation 6) were derived from a study by Martin et al (1987) and calculated at *I* = 1.044 and *A* = 81.96. Actual yield was thus estimated from the following equation.

(17) $$Y = Y_0(1-Y_L)(1-Y_p)$$

An Overview of Alternative Model Frameworks for Determining Optimal Herbicide Use

A number of alternative economic frameworks for determining the optimal rate of herbicide for a given initial weed density are assessed. Economic models can be defined as being static, dynamic, deterministic or stochastic. In a generalised order of increasing complexity an individual model may be static and deterministic, static and stochastic, dynamic and deterministic, or dynamic and stochastic. Given the diverse range of model types and weed problems it is pertinent to ask what is an appropriate modelling framework for weed management. This will depend upon the particular management problem and the questions being asked. Outlined below are the main distinguishing features of static and dynamic models.

Static model

Assuming a farmer's objective is to maximise profit, the goal is to determine the rate of herbicide that maximises π for a given initial weed seedling density, *S* (equation 9). Increasing the rate of herbicide dosage will increase total revenue (P_yY) as well as the total cost from control ($P_u u + C_1$) in the profit function. The rate of herbicide that maximises profit will occur when the marginal benefit (MB) of *u* is equal to the marginal cost (MC). If MB exceeds MC then an increase in u will contribute more to total revenue than total cost, justifying an increase in the input level. Likewise, if MC exceeds MB then an increase in u will contribute more to total costs than total revenue and it would pay to reduce the level of herbicide.

Varying the level of u has a direct effect upon π through equations (15), (7) and (17). Determination of the optimal rate of herbicide can be obtained from the application of comparative-static analysis. The first-order conditions for profit maximising input use require that u be used until its cost equals the value of marginal product. Taking the derivative of the profit function with respect to herbicide dose rate we obtain

(18) $\partial\pi/\partial u = P_y(\partial Y/\partial u) - P_u = 0$

(19) $P_y(\partial Y/\partial u) = P_u$

This equation states that at the optimum herbicide dose, u^*, the cost of the last increment of herbicide input equals the value of the extra output obtained. In other words, the revenue from a marginal change in weed reduction equals the marginal cost of herbicide. Assuming that the function is concave and that diminishing returns apply the second-order condition for a maximum ($\partial^2\pi/\partial u^2 < 0$) is automatic (Dillon, 1968). Further transforming equation (19) we obtain

(20) $\partial Y/\partial u = P_u/P_y$

This equation states that for an initial seedling density, S, the optimal level of the herbicide input u occurs when the marginal product of the herbicide input equals the inverse price ratio P_u/P_y (Dillon 1968). Since P_y and P_u must be non-negative this equation implies $\partial Y/\partial u$ can never be negative at the level of herbicide rate that maximises profit. A number of important features of the optimal herbicide rate can be determined from equation (20). An increase in P_u will decrease the optimal level of u, while increases in Y or P_y will increase the optimal level of u.

A dynamic model

A static approach to weed management, where the benefits and costs are only considered within a single season, may be an inappropriate framework given that there are often carryover effects associated with weed control. Weeds that escape control in one season may reproduce and replenish the weed population in following seasons. Consequently, it may be more appropriate to consider weed control in the context of a resource management problem where the goal is to determine the optimal inter-temporal level of weed control.

The processes of growth and decay of a resource are often summarised by an equation of motion which explicitly states how the resource stock changes over time. For renewable resources the equation of motion is equal to the growth rate per period less the amount removed by harvest, destruction and natural depletion. Dynamic problems also involve issues of resource stocks, where stocks represent the level, amount or quality of the resource. The level of stocks can influence optimal resource use over time, eg because of the stocks' influence upon growth rates.

Viewing weeds a resource stock involves a modification to the economic framework for valuing the benefits from weed control. At issue is how much of the weed stock to consume or deplete in the current period and how much should be left *in situ* for the future (McInerney 1976). From an economic perspective a weed can be viewed as a renewable resource (Conrad and Clark 1987; Clark 1990) with the seed bank representing the stock of this resource. The size of the seed resource stock changes through time due to depletion by weed management and new seed stocks being created via the process of self renewal through seed production. The change in the seed bank from one period to the next is described by the function:

(21) $x_{t+1} = x_t - S_t - M_t + N_t$

Where x_t represents the state variable of initial density of seeds in the soil (ie. seed bank) in year t, S_t is seedling recruitment, M_t is the seed loss due to predation and natural mortality and N_t is new seed added to the seed bank either from reproduction or importation through natural spread or operations such as harvesting and sowing. Equation (21) represents a summary of the more detailed population dynamics equations given in equations (9) to (13).

The seed bank can be indirectly regulated by changing weed control inputs that target the mortality or vigour of plants (eg. cultivation, herbicides) or directly through targeting reproduction and seed rain processes (eg. selectice spray-topping, crop-topping, seed catching, windrowing) or through losses via seed mortality (eg. cultivation, stubble burning, seed predation).

In a dynamic setting the objective of the farmer is to determine the level of depletion of the stock of the seed resource (x) from herbicide application (u) in each season or year that maximises profit over a period of T years. The objective function can be formally stated as

(22) $$\max J = \sum_{t=0}^{T} \beta^t \pi(x_t, u_t)$$

subject to

(23) $$x_{t+1} - x_t = g(x_t, u_t)$$

Where J is the net present value (NPV) of cumulative profits over the planning horizon T, x denotes the weed seed bank state variable, u is the control variable (ie herbicide dose rate), π is a measure of annual farm profit which is a function of the state and control variables, β is the discount factor, and g is the rate of change in the seed bank from application of a herbicide at rate u. The equation of motion (23) represents the change in the state variable from one period to the next. Equation (21) represents an example of an equation of motion.

Optimal control theory can be used to determine the annual rates of herbicide that maximise the objective function. An important component of the dynamic problem is the costate variable, denoted by λ, which is akin to a Lagrange multiplier. The means through which the costate variable enters the optimal control problem is the Hamiltonian function. Note that π in equation (22) is identical to the profit function (8), hence the current-value Hamiltonian for the herbicide dose rate problem is

(24) $$H_t = P_y Y(x_t, u_t) - P_u u_t - C_1 - C_2 + \beta\lambda_{t+1} g(x_t, u_t)$$

The Hamiltonian function is the net profit obtained from an existing level of the state and control variables plus the value of any change in the stock of the state variable valued at the costate variable, λ_{t+1}. The costate variable thus represents the shadow price of a unit of the stock of the seed bank and is also referred to as the user cost (or benefit) from stock depletion. In the last term on the right hand side of (24), the $g(x_t,u_t)$ function indicates the rate of change of the seed bank corresponding to herbicide dose u. When the function is multiplied by the costate variable, λ_{t+1}, it is converted to a monetary value and represents the rate of change of the economic value of the seed bank corresponding to herbicide dose u. In effect this term can be viewed as the future profit effect of weed population changes. The dynamic maximisation problem presented in equation (24) thus differs to the static maximisation in equation (8) in that the future income effects from current period decisions are explicitly included in the current period return.

The first order conditions for this problem, as developed by Pontryagin et al (1962), are

(25) $$\frac{\partial H}{\partial u_t} = P_y \frac{\partial Y}{\partial u_t} - P_u + \beta\lambda_{t+1} \frac{\partial g}{\partial u_t} = 0$$

(26) $$\beta\lambda_{t+1} - \lambda_t = -\frac{\partial H}{\partial x_t} = -P_y \frac{\partial Y}{\partial x_t} - \beta\lambda_{t+1} \frac{\partial g}{\partial x_t}$$

(27) $$\frac{\partial H}{\partial \lambda_t} = x_{t+1} - x_t = g(x_t, u_t)$$

Equation (25) is the maximum principle, the standard condition for maximisation with respect to u_t; equation (26) is the adjoint equation and denotes the rate of change of the shadow price over time; and (27) is a re-statement of the equation of motion. This set of equations allows the solution of the three unknown optimal trajectories x_t^*, u_t^* and λ_t^*. These trajectories depend critically on the initial state of the system and, although x_0 is generally given, λ_0 is unknown and an additional condition, known as the transversality condition, is required to obtain a unique solution. In this particular problem, where terminal time, T, is given and the terminal state, x_T, is free, the transversality condition is $\lambda_T = 0$.

The fact that the initial costate value is unknown complicates the numerical solution of the problem. Solution, for a given x_0, starts with an arbitrary value of λ_0 and the numerical integration of the system (25)-(27). Depending on the resulting value of λ_T the value of λ_0 is adjusted, this process continues until the transversality condition is satisfied. This was the procedure used to solve the wild oat problem, with (27) represented by the weed population dynamics (equations 9 to 13) rather than a single equation of motion.

To gain further insight into the difference between static and dynamic solutions rearrange (25) to obtain

(28) $$\frac{\partial Y}{\partial u_t} = \frac{P_u - \beta\lambda_{t+1} \frac{\partial g}{\partial u_t}}{P_y}$$

As in the static case, this condition states that optimal weed management occurs when the marginal product of herbicide application equals the ratio of input price to output price, but now input price is decreased by the value of its beneficial future effect (the second term in the numerator). This will result in a higher herbicide application rate than in the static case. To understand why, note that $\lambda_t \leq 0$ (ie. weeds have a negative effect on profit), and $\partial g/\partial u \leq 0$ (ie. herbicide application decreases weed population growth and reproduction), since $\beta > 0$ the second term in the numerator is either negative or zero. When the strict inequalities apply, the right hand side (RHS) of (28) is smaller than the RHS of the static (equation 20) solution. Given that $\partial Y/\partial u > 0$ and $\partial^2 Y/\partial u^2 < 0$, the value of u would have to be increased to obtain a lower marginal product (left-hand side) and thus satisfy condition (28).

Note that the transversality condition $\lambda_T = 0$ means that at time T-1 the dynamic solution is the same as the static solution, as the second term in the numerator of (28) vanishes. This result occurs because the optimal control model (22) does not contain a terminal value, ie. it is assumed that the final weed seed population does not affect the value of the land. If this assumption is relaxed and a final value $F(x_T)$ is included in (22), the transversality condition will become $\lambda_T = F'(x_T)$ and the dynamic solution will not converge to the static solution so long as the final weed population has a negative effect on land value.

In summary, an increase in the number of seeds (and weeds) reduces profits, up to the maximum weed population possible, hence $\lambda_t \leq 0$. Due to the beneficial effect of the current level of control on future profits, a higher level of optimal weed control occurs than when only current profits are maximised. Therefore, including the inter-temporal effects of weed control into our decision-making framework will, for a given size of the seed bank, result in a greater level of weed control and a higher economic return than if control decisions are based solely on the current period effects. Consequently, a dynamic economic framework is theoretically preferred to static frameworks for either undertaking economic analysis of weed control technologies or providing a framework for decision support systems.

Another important issue is the existence of a steady-state equilibrium and whether it can be reached from the initial state. The study of this issue requires the solution of the infinite-time problem and is out of the scope of this paper.

The Effects of Uncertainty

Various studies have found that pesticides are risk reducing (Carlson 1984; Feder 1979; Olson and Eidman 1992), which means that pesticides lead to a lower income variance and, consequently, farmers will increase pesticide applications. Pannell (1995) has argued that these studies only consider uncertainty about pest density and pesticide effectiveness and that if other sources of uncertainty were considered the pesticides can be risk increasing.

Pannell (1990b) estimated the effect of variability on the herbicide rate that maximises profit under the assumption of risk-neutrality. As the variance of initial weed density increases, the herbicide rate that maximises expected profit decreases. An important factor in this result is the shape of the yield-loss function. Auld and Tisdell (1987) showed that uncertainty about weed density reduced expected yield loss because of the convexity of the relationship between weed density and crop yield, thereby reducing the marginal productivity of herbicide. Pannell (1990b) found increasing the variability of weed competitiveness reduced the optimal herbicide rate while variability about the efficacy of weed kill from herbicide was ambiguous in terms of its effect upon the optimal dose rate. An increase in the variability of weed kill increased expected weed survival which increased optimal herbicide rate, and uncertainty about weed density decreased optimal herbicide rate.

Pannell (1995) found that for variability in weed density, increasing herbicide application was more attractive for risk-averse farmers. For variability in weed mortality Pannell (1995) found that the effect of risk-aversion to be ambiguous, however, concluded that increased herbicide use would reduce weed variance and lead to an increase in optimal herbicide rate under risk aversion. Deen et al (1993) supported the findings of Pannell (1990b) for the effects of variability for risk-neutral farmers. Under the assumption of expected utility maximisation and uncertainty in weed density, Deen et al (1993) found that optimal herbicide dose increased with uncertainty for extreme risk aversion.

Application of Model Approaches

Population model

A simulation model of the population dynamics of wild oats was developed to determine the effects of varying the control variable, herbicide dose rate. This model explicitly traced the changes in the seed bank through equations (9) to (13) and the dose response function (equation 15) using the parameter values reported in Table 1.

The model was simulated for seed banks ranging from 0 to 2,000 seeds m^{-2} and for herbicide dosage varying from 0 to 4 litres ha^{-1} of diclofop-methyl. The model derived changes in the seed bank for each state and control variable combination. When no herbicide is applied, regardless of the initial seed population there is a dramatic increase in the size of the seed bank in the following period (Figure 1). As anticipated, as herbicide is applied the magnitude in the seed population increase is diminished. Further increases in dosage will lead to a herbicide rate from where reductions in the seed bank population in period t+1 can be achieved, ie the population curve lies below the line $x_t = x_{t+1}$. From Figure 1 this would appear to occur at rates of 1.5 litres ha^{-1} and above.

A stochastic version of the population dynamics model was developed by incorporating probability distributions for both rainfall and seed survival into the model. The proportion of weeds killed from herbicide (equation 15) and new seed added to the seed bank (equation 12) were represented as random variables in the model. A normal probability distribution for κ (mean 50% and standard deviation 5%) was derived from Medd and Jones (1996). Historical rainfall records for Wagga Wagga for the period 1898 to 1996 were used to determine the probability distribution for season type so as to estimate herbicide mortality from the dose response function. Rainfall for the period April to October was the determinant of season type, with a dry season being rainfall in the lower 25th percentile (RF < 255mm) and a wet season being rainfall in the top 25th percentile (400mm < RF). Therefore an average season was rainfall between 255mm and 400mm. Rainfall for the period April to October has a normal distribution with mean 335mm and standard deviation 105mm. It is expected that there would be a high correlation between these two variables and this was incorporated into the Monte Carlo sampling process.

The model was solved for each state and control variable combination using a monte carlo sampling procedure. The sampling process involved 5,000 iterations of the population model, resulting in a set of state values for the following period from which the state transition probabilities were derived. A transition probability ($P^k_{i,j}$) reflects the probability of moving from the ith to the jth state for the kth decision. The seed bank was divided into 20 state intervals and examples of the transition probabilities derived for two control variable values (u = 1.5 and u = 3.0) are given in Table 2.

Static optimum model

The level of u which maximised π for a given seed bank was estimated from the following function

(29) $\max \pi = P_y Y(x,u) - P_u u - C_1 - C_2$

subject to

(30) $x = a$

Where $0 \le a \le 2000$. Solution of the static optimum (SO) problem is best determined numerically, consequently, a non-linear mathematical programming model was developed to determine the optimal herbicide dose. The objective function was the maximisation of current period profit for a single control variable, herbicide dose. The model explicitly included the dose response function (equation 15) and the yield loss function (equation 6).

Numerical optimal control model

A numerical optimal control (NOC) model of the weed management problem was developed based upon equations (24) to (27). The solution procedure used was based upon that given by Cacho (1998). The equation of motion used a population dynamics model represented by equations (9) to (13). The model was solved for values of x_0 ranging from 0 to 2000 seeds m^{-2}.

Dynamic programming model

The application of a NOC model requires that the Hamiltonian, which includes the equation of motion, to be continuously differentiable. In many agricultural and natural resource problems, including weed management, there are discontinuities in the equation of motion and the calculation of current period profit. In such situations it is difficult to apply a NOC model and an alternative approach is required. Dynamic programming (DP) is an alternative dynamic optimisation technique that has had widespread application in agriculture and natural resources research (Kennedy 1986; 1988). When using DP for the weed management problem the Hamiltonian in equation (24) is replaced by the following recursive equation.

(31) $V_t(x_t) = \underset{u_t}{Max}\left[\pi_t\left(x_t, u_t\right) + \beta V_{t+1}\left(x_{t+1}\right)\right]$

Where V_t is the optimal current period return in period t, β is the discount factor, π is the current period stage return, and x and u are the state and control variables as previously defined. The problem is solved by backward recursion, subject to the equation of motion defined by equations (9) to (13), and gives the optimal decision policy for any given state and stage combination.

One disadvantage of the DP approach is that it is unable to directly provide an equivalent to the costate variable, λ_{t+1}, in the standard solution. Therefore, it misses in providing an important set of economic information, ie the shadow price on the rate of change of the state variable from one period to the next. Kennedy (1988), however, has demonstrated an approach for deriving the equivalent to the costate variable from the DP solution. Another disadvantage of this approach is the 'curse of dimensionality' problem (Bellman 1957), where there is an explosion in the size of the model (and computer time and memory requirements) as the total number of states increases.

The optimal dose rates were calculated from a DP model with the objective function being the maximisation of profit over a 10 year period. Given that dynamic programming does not have to rely upon continuously differentiable functions for the equation of motion and stage return, the population model was solved for each state and control variable combination to provide the actual state transformation and stage return values for the DP solution.

Stochastic dynamic programming

One major advantage of DP over NOC is that stochastic effects can be readily incorporated into the framework. The equation of motion is replaced by the concept of transition probabilities of moving from the *i*th to the *j*th state for the *k*th decision ($P^k_{i,j}$) and the problem becomes one of maximising expected returns. The recursive equation becomes

$$V_t(x_t) = \underset{u_t}{Max}\left[\pi_t\left(x_t, u_t\right) + \beta E V_{t+1}\left(x_{t+1}\right)\right] \tag{32}$$

Where E is an expectations operator. A stochastic dynamic programming (SDP) model was developed which used the transition probabilities derived from the stochastic population dynamics model.

Results

The optimal herbicide rates, u^*, for each state variable value derived by the alternative models are given in Figure 2. For any given state variable, application of either the NOC or DP models resulted in a significantly higher u^* than the SO model. This analysis indicated that there was very little difference in u^* derived by the two dynamic models. A stochastic model for the static optimum scenario was not developed in this paper as there is sufficient evidence from earlier studies (Pannell 1990b, 1995; Deen et al 1993) of the effect of uncertainty upon u^*. For a risk-neutral scenario the u^* from the SO model is expected to be decreased from that given in Figure 2. Inclusion of uncertainty in the rate of seed kill and herbicide efficacy in the dynamic model had a similar result as obtained by Pannell (1990b) and Deen et al (1993) for static cases. The u^* determined by the SDP model was lower than that obtained by the deterministic DP model. Despite the u^* obtained by the SO and SDP models being similar, no direct comparisons of these two approaches can be made as a stochastic static scenario was not calculated to compare with the SDP model. This may be an area for further research. The results obtained from the SDP model were for a risk-neutral scenario and it is anticipated that solution of a risk-averse scenario would derive higher u^* values than given by the SDP model in Figure 2.

The costate variable values determined by the NOC model for each level of the state variable are given in Figure 3. As previously described, the costate variable is negative implying that the seed bank has a detrimental effect upon future income. The results reported in Figure 3 show that there is a significantly greater effect of increases in the state variable at low seed banks than at the upper range reported. For instance, at seed banks between 0 and 50 seeds m^{-2} an increase in the seed bank by 1 seed m^{-2} will decrease future income by between \$2 to \$3 seed^{-1}. Alternatively, at a seed bank of 2000 seeds m^{-2} the reduction in income is less than \$0.25 seed^{-1}. This reinforces the results given in Figure 1 showing there is a diminishing marginal change in the seed population increase as the seed bank increases.

The u^* and x^* obtained from the SO, NOC and DP models were simulated over a 10 year period for an initial seed bank of 500 seeds m^{-2}. There were negligible differences each year in u^* for the NOC and DP models (Figure 4). The annual u^* derived from the SO model were lower than determined by the dynamic models for the first three years of the simulation, but thereafter the dynamic models resulted in significantly lower u^*. The change in x^* (Figure 5) over the simulation period was identical for the NOC and DP models. The dynamic models resulted in a significantly greater decline in the seed bank compared to that obtained by the SO model. The seed bank was depleted to 0 seeds m^{-2} by year 7 under the dynamic models whereas under the SO model the seed bank was still greater than 100 seeds m^{-2} in year 10.

The optimal Hamiltonian values for two costate variable values, $\lambda_t = 0.0$ and $\lambda_t = -1.0$, for $x_0 = 500$ seeds m^{-2} are given in Figure 6. The first scenario corresponds to the SO solution, and shows that the dose rate that maximises the Hamiltonian is around 2 litres ha^{-1}. This is identical to the result given in Figure 2. The second scenario indicates that the Hamiltonian is maximised at a much higher dosage when the costate variable is considered. This reinforces the result presented in Figure 2 indicating that the NOC model gives a higher u^* for a given x than the SO model.

The NPVs obtained from the three models are given in Figure 7, which indicates that the two dynamic models are economically superior to the SO model. This economic dominance would be greater if a longer simulation period had been considered.

Discussion

Varying the herbicide dose rate is not always possible as in some Australian states legislation enforces the use of recommended label dose rates for herbicide usage (Pannell 1990a). This fact, however, does not invalidate the findings of this analysis ie that the optimal level of weed control that maximises economic returns is significantly higher when a dynamic as opposed to a static framework is used for assessing the benefits from weed control. From a weed management perspective the challenge then is to determine a mix of weed control options, including herbicides at the recommended label dose rates, which achieve this level of control. This is the goal of integrated weed management.

Consequently, the temporal aspects of managing weeds are determined to be important in any model to evaluate the benefits from weed management. However, incorporation of the stochastic effects of input use also has a major influence upon the optimal level of weed control. In this study incorporating uncertainty in herbicide dose response and seed survival resulted in a lower optimal herbicide dose rate than for the deterministic DP or NOC solutions. Given the importance of climate and seasonal effects upon weed population dynamics it would appear prudent to incorporate seasonal variability and dynamic responses into any modelling framework of weed management systems.

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Figure 1. Change in seed bank for various herbicide rates (seeds m^{-2})

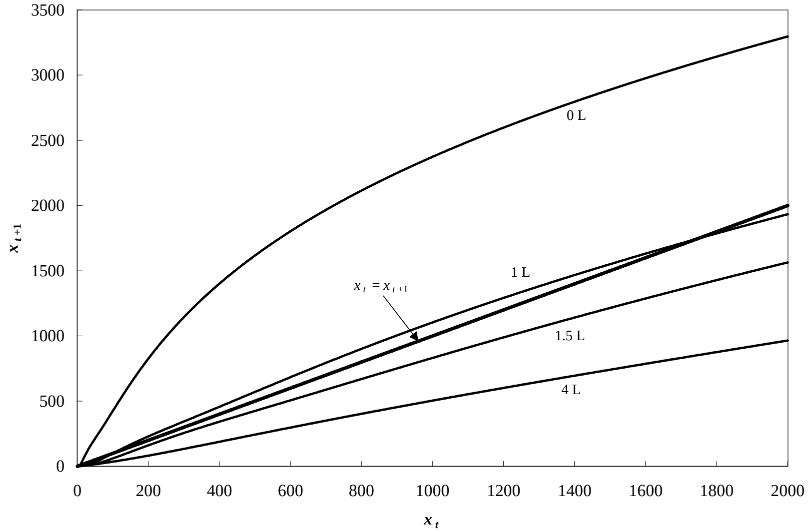

Figure 3. Costate variable (λ_{t+1}) values from NOC model

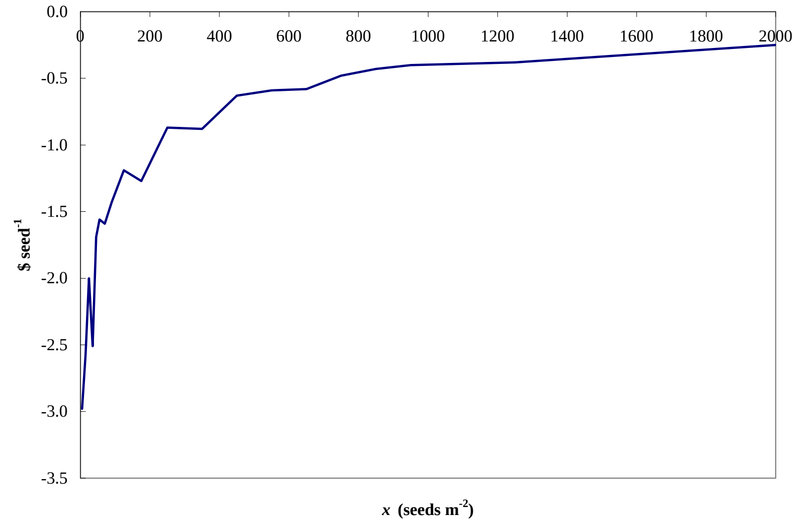

Figure 2. Optimal herbicide dose rates (litres ha^{-1})

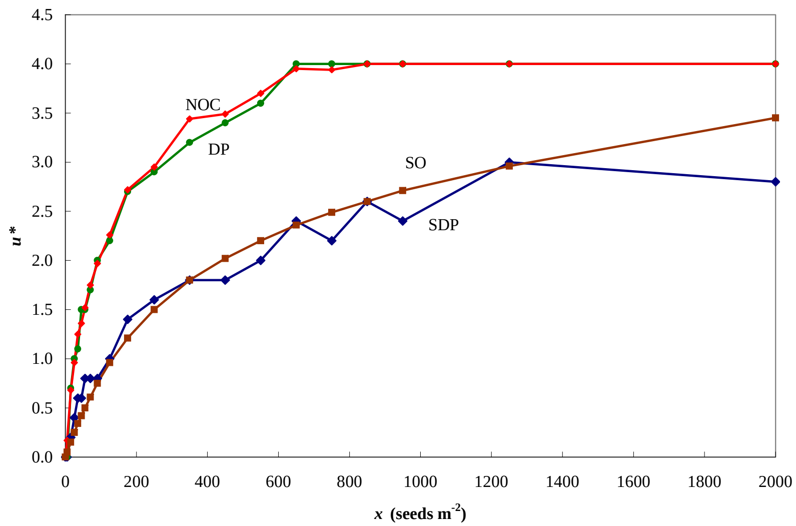

Figure 4. Optimal herbicide rate for 10 year simulation (litres ha^{-1})

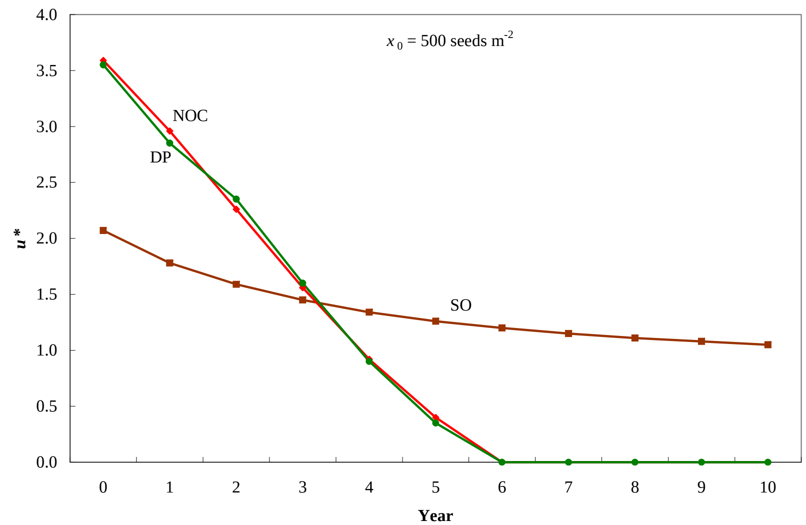

Figure 5. Optimal seed bank from 10 year simulation (seeds m^{-2})

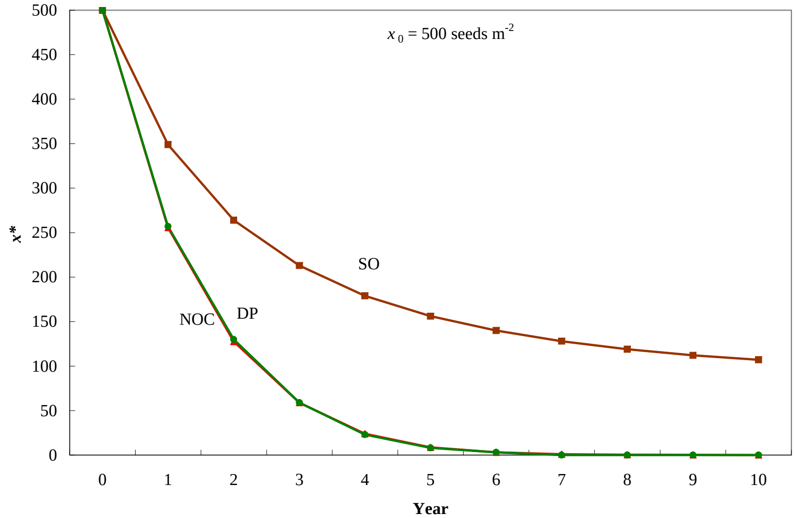

Figure 6. Hamiltonian values for $\lambda_t = 0.0$ and $\lambda_t = -1.0$

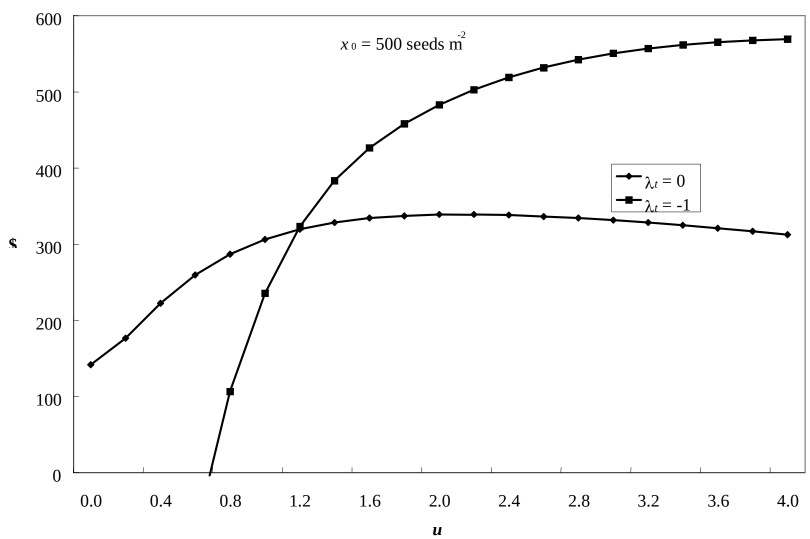

Figure 7. Net present values

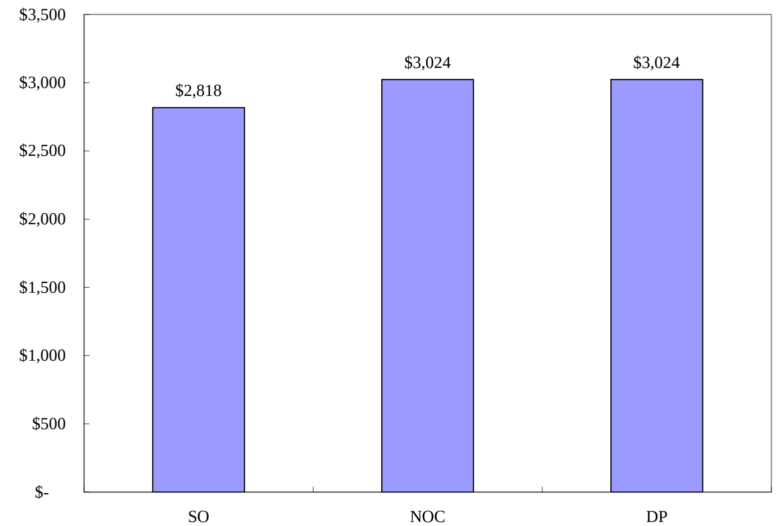

Table 1. Parameter values

Population dynamic parameters			
δ (%)		50	
ψ (%)		50	
η (seeds m^{-2})		0	
ξ (seeds m^{-2})		0	
κ (%)		50	
	Cohort 1	Cohort 2	Cohort 3
p_i (%)	30	60	10
ϕ_i (%)	100	0	0
α_i	8.60	7.60	6.80
μ_i	0.74	1.20	2.00
ε_i	0.88	0.80	0.67
Economic parameters			
P_y ($ tonne^{-1})		165.00	
P_u ($ litre^{-1})		22.50	
Y_0 (tonnes ha^{-1})		3.90	
C_1 ($ ha^{-1})		2.22	
C_2 ($ ha^{-1})		166.24	

Table 2. Probability transition matrix ($P^k_{i,j}$)

										i										
j	0-10	11-20	21-30	31-40	41-50	51-60	61-80	81-100	101-150	151-200	201-300	301-400	401-500	501-600	601-700	701-800	801-900	901-1000	1001-1500	1500-
u = 1.5 litres ha^{-1}																				
0-10	1.000	1.000	1.000	0.789																
11-20				0.211	1.000	0.792														
21-30						0.208	0.776													
31-40							0.224	0.765												
41-50								0.026	0.538											
51-60								0.209	0.222											
61-80									0.019	0.524										
81-100									0.221	0.274										
101-150										0.202	0.771	0.048								
151-200											0.067	0.621	0.067							
201-300											0.162	0.331	0.72	0.532	0.072					
301-400													0.213	0.276	0.708	0.549	0.113			
401-500														0.192	0.022	0.227	0.495	0.522		
501-600															0.198	0.071	0.176	0.246	0.298	
601-700																0.153	0.159	0.013	0.232	
701-800																	0.057	0.219	0.244	
801-900																			0.003	
901-1000																			0.135	0.511
1001-1500																			0.088	0.361
1501-																				0.128
u = 3.0 litres ha^{-1}																				
0-10	1.000	1.000	1.000	1.000																
11-20					1.000	1.000														
21-30							1.000													
31-40								0.771												
41-50								0.229	0.774											
51-60																				
61-80									0.226	0.776										
81-100											0.013									
101-150										0.224	0.776	0.384								
151-200											0.211	0.398	0.413							
201-300												0.218	0.326	0.538	0.451					
301-400													0.261	0.269	0.340	0.532	0.439			
401-500														0.193	0.209	0.254	0.322	0.530		
501-600																0.214	0.044	0.236	0.516	
601-700																	0.195	0.123	0.292	
701-800																		0.111	0.001	
801-900																			0.076	0.285
901-1000																			0.115	0.204
1001-1500																				0.511
1501-																				