Taxes and Quality: Theoretical Results from a Market-Level Analysis

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The typical analysis of agricultural policy assumes that the commodity of interest is homogeneous, and that the nature of the commodity does not change as a result of the implementation of the policy. In many cases, these may be appropriate assumptions. In a number of instances, though, commodities are quite heterogeneous. Most agricultural commodities have at least two characteristics, such as their agronomic variety and time of year when they are available, which vary over different units of production. In fact, many products have several relevant characteristics, including producer, geographic region, appearance, distance from market, size, ripeness, freshness, uniformity, grade, and packaging. Each of these objectively-defined characteristics can be thought of as a dimension of product heterogeneity, some of which are defined discretely, while others are defined continuously. If there are n characteristics, then it may be helpful to think of a unit's position in the n-dimensional characteristic space as reflecting that unit's quality.

An assumption that all units of a commodity are of identical quality is quite restrictive. While most economists recognize that commodities are not perfectly homogeneous, the assumption is rarely mentioned or justified in studies of commodity policy. In some cases, the homogeneity assumption may be based on the belief that quality effects are not important, so that modeling the market for a commodity as if it were homogeneous closely approximates reality. In other cases it may be believed that quantities and prices are properly aggregated to reflect the distribution of quality, or some "representative" quality, and that this is sufficient to accommodate quality issues.

Whether the assumption is one of homogeneity or one of perfect aggregation methods, representing a commodity as if it were homogeneous fails to account for changes in the distribution of quality (or average quality) that may occur as a result of the implementation of policy. Alchian and Allen (1964) and Barzel (1976) make persuasive arguments for the existence of such quality changes. Furthermore, many instances may be found where quality varies and has responded to policy. Nevertheless, quality responses have yet to be incorporated formally into the analysis of agricultural commodity policy. Rather, economists have presumed implicitly that explicitly modeling quality responses to policy is unnecessary. However, it is impossible to determine how closely a homogeneous goods model approximates the actual policy impacts until a more complete model has been developed and implemented.

This paper accounts for quality responses in a very simplified framework, where a commodity is available in two qualities: high and low. The focus here is on taxes, though the results may be easily re-interpreted as applying in the context of production quotas and subsidies. Subsidies, of course, are the equivalent of negative taxes, so that the effects of per unit and ad valorem subsidies are simply the negatives of the effects of per unit and ad valorem taxes, respectively. The effects of a per unit tax are directly analogous to those of a quota, provided that it is freely transferable. As is the case with a per unit tax, quota rent per unit is the difference between the consumer price and the marginal cost of production at the quota quantity. Here, to represent a quota, the quota rent per unit generated from a fixed quota quantity is specified, rather than the quantity itself.

1 Quality Responses

The Alchian and Allen theorem was introduced in 1964 as an heuristic example of the law of demand, but has since been coined as the third law of demand (Bertonazzi, Maloney, and McCormick 1993). The theorem postulates the effects of transportation costs on the relative consumption of high-quality and low-quality goods. The original example given by Alchian and Allen (1964) concerned "good" and "bad" grapes grown in California, with good grapes selling for a higher price than bad grapes. They noted that the cost of transporting grapes to, say, New York is the same for all shipments of grapes, regardless of their quality. From an individual consumer's perspective, prices are fixed so that the price of each quality of grapes increases by the same amount for consumers in New York. Thus, good grapes become relatively cheaper for a consumer in New York, and hence, a New Yorker will consume a larger proportion of good grapes relative to a person in California with identical preferences.

While the Alchian and Allen theorem is intuitively appealing, it is theoretically provable only under very restrictive conditions. Gould and Segall (1968) showed that when the same per unit cost is added to the prices of high- and low-quality goods, the individual consumer unequivocally increases relative consumption of the high-quality good only in a two-good world with no income effects. Introduction of either an income effect or a third good renders the change ambiguous. Borcherding and Silberberg (1978) argued that while it is possible for the Alchian-Allen theorem to be negated with the introduction of a third good, unless the high- and low-quality products have very different consumption relationships with the third good, the standard Alchian-Allen result will hold.

The typical per unit cost introduced to produce the Alchian-Allen effect is a transportation cost. The hypothesized increase in the consumption share of high-quality goods could occur as a result of many other types of per unit costs. Umbeck (1980) discussed the nature of these costs, and pointed out that many examples proposed by Borcherding and Silberberg (1978) to demonstrate the Alchian-Allen effect fail to do so because of the nature of the per unit costs considered. The primary criteria for a per unit cost to generate the Alchian-Allen result are that the cost does not change the good itself, and that it does not have any inherent economic value in and of itself–i.e., it acts just like a per unit tax. Because the analysis is at the individual consumer level, though, prices are exogenous. When either transportation costs or per-unit taxes are introduced at the market level, these costs are shared by consumers and producers, and theory has little to say about the relative changes in production and consumption of low- and high-quality commodities. While the Alchian-Allen effect has not been proven theoretically at the market level, it is a convincing empirical regularity (Bertonazzi, Maloney, and McCormick 1993). This suggests that the Alchian-Allen effect may be more compelling as a conjecture about market-level relationships than it is regarding individual behavior.

Barzel (1976) addressed a similar phenomenon at the market level in his alternative approach to taxation. Barzel noted that every commodity is more or less a bundle of characteristics. Because an ad valorem tax applies to the commodity's entire value, it essentially taxes all of its characteristics. In contrast, if a per unit tax is imposed, the tax statute will use a subset of characteristics to define the commodity, assuming that an exhaustive description is either impossible or very costly. As a result, the per unit tax is actually taxing the defining characteristics. In maximizing their profits subject to the tax constraint, producers may alter the characteristics included in their units of production in order to minimize their costs of the per unit tax. Barzel (1976) showed that a predictable outcome is that the quantity of the defining characteristics (specified in the tax statute) will decrease, and the additional characteristics, which are not subject to the tax because they are not specified in the statute, will increase on a per unit basis.

While the work by Barzel (1976) addresses quality changes resulting from taxes at the market level, his approach does not lend itself to empirical application. He defines a single demand function for a single characteristic of a commodity. This approach assumes that the single characteristic defines the quality of the commodity of interest, and that consumers are indifferent to how the characteristics are packaged into physical units (i.e., n physical units with one unit of the characteristic each are a perfect substitute for a single physical unit with n units of the characteristic). This paper allows for a more general definition of quality by adopting a multi-market framework. The commodity of interest is assumed to be available in two qualities, low and high, which are related in consumption and production. The relationship between the markets allows for some substitution (not necessarily perfect substitution) between the two qualities, both in demand and supply. Conditions under which the Alchian-Allen result exists at the market level are shown and discussed.

2 A Two-Market Model

The previous section presented several arguments for quality responses to taxes. This section develops a simple model of such quality responses. The effects of taxes are modeled by specifying an equilibrium displacement model, as used by Muth (1964), Buse (1958), Perrin (1980), Alston (1985), and others (see Piggott (1992) for a review). A two-market model is specified for the case of high- and low-quality products related in consumption and production.

A supply and demand function is specified for each market. Because the two qualities are related in consumption, the quantity demanded of each quality will depend on its own price and the price of the other quality. Similarly, the quantity supplied of each quality will depend on its own price and the price of the other quality. Other demand and supply shifters, such as income, demographic variables, production technology and input prices, are assumed fixed, and are therefore not included as arguments. These supply and demand relationships can be written in general form as:

$$Q_L^D = Q_L^D(P_L^D, P_H^D) \tag{1}$$

$$Q_L^S = Q_L^S(P_L^S, P_H^S) \tag{2}$$

$$Q_H^D = Q_H^D(P_L^D, P_H^D) \tag{3}$$

$$Q_H^S = Q_H^S(P_L^S, P_H^S) \tag{4}$$

where Q and P denote quantities and prices, subscripts L and H denote quantities and prices in the low- and high-quality markets, and superscripts D and S denote quantities and prices along the demand and supply curves, respectively. The market-clearing conditions are:

$$Q_L^D = Q_L^S \tag{5}$$

$$Q_H^D = Q_H^S \tag{6}$$

$$P_L^D = P_L^S(1+t_L) \tag{7}$$

$$P_H^D = P_H^S(1+t_H) \tag{8}$$

where t_L and t_H are proportional taxes in the low- and high-quality markets, and are initially equal to zero. Increasing either t_i term creates a wedge between the consumer price P_i^D and the producer price P_i^S in that market. Positive values for t_i correspond to taxes, while negative values represent subsidies.

Totally differentiating equations (1) through (8) yields:

$$dlnQ_L^D = \eta_{LL} dlnP_L^D + \eta_{LH} dlnP_H^D \tag{9}$$

$$dlnQ_L^S = \epsilon_{LL} dlnP_L^S + \epsilon_{LH} dlnP_H^S \tag{10}$$

$$dlnQ_H^D = \eta_{HL} dlnP_L^D + \eta_{HH} dlnP_H^D \tag{11}$$

$$dlnQ_H^S = \epsilon_{HL} dlnP_L^S + \epsilon_{HH} dlnP_H^S \tag{12}$$

$$dlnQ_L^D = dlnQ_L^S \tag{13}$$

$$dlnQ_H^D = dlnQ_H^S \tag{14}$$

$$dlnP_L^D = dlnP_L^S + t_L \tag{15}$$

$$dlnP_H^D = dlnP_H^S + t_H \tag{16}$$

where $dlnX = dX/X$, denoting a proportional change in the variable X. For instance, $dlnQ_L = dQ_L/Q_L$ is the proportional change in the quantity sold in the low-quality market. Coefficients on the $dlnP_i$ terms are elasticities: η_{ij} is the elasticity of demand for quality i with respect to the price of quality j, and ϵ_{ij} is the elasticity of supply of quality i with respect to the price of quality j. Equations (9) through (16) implicitly define the eight endogenous variables, the proportional changes in quantities demanded and supplied and the proportional changes in consumer and producer prices in each of the two markets, as functions of the two exogenous tax rates, t_L and t_H.

Imposing the market-clearing conditions in equations (13) and (14), the superscripts on the proportional quantity changes may be dropped, and the remaining six equations may be specified in matrix notation as:

$$\begin{bmatrix} 1 & 0 & -\eta_{LL} & -\eta_{LH} & 0 & 0 \\ 0 & 1 & -\eta_{HL} & -\eta_{HH} & 0 & 0 \\ 1 & 0 & 0 & 0 & -\epsilon_{LL} & -\epsilon_{LH} \\ 0 & 1 & 0 & 0 & -\epsilon_{HL} & -\epsilon_{HH} \\ 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} dlnQ_L \\ dlnQ_H \\ dlnP_L^D \\ dlnP_H^D \\ dlnP_L^S \\ dlnP_H^S \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ t_L \\ t_H \end{bmatrix},$$

or $AY = X$. Inverting the coefficient matrix A and pre-multiplying both sides of the equation with the inverse, A^{-1}, yields an expression of the endogenous variables as functions of the exogenous tax rates and elasticities, i.e., $Y = A^{-1}X$. Because each element of the first four columns of the inverted matrix will be multiplied by the zero terms in the right-hand side vector X, they may be eliminated, along with the first four rows of X.

Thus, the solution for the endogenous variables is:

$$\begin{bmatrix} dlnQ_L \\ dlnQ_H \\ dlnP_L^D \\ dlnP_H^D \\ dlnP_L^S \\ dlnP_H^S \end{bmatrix} = \frac{1}{D} \begin{bmatrix} \eta_{LL}(\epsilon_{LL}\epsilon_{HH} - \epsilon_{LH}\epsilon_{HL}) - \epsilon_{LL}(\eta_{LL}\eta_{HH} - \eta_{LH}\eta_{HL}) \\ \eta_{HL}(\epsilon_{LL}\epsilon_{HH} - \epsilon_{LH}\epsilon_{HL}) - \epsilon_{HL}(\eta_{LL}\eta_{HH} - \eta_{LH}\eta_{HL}) \\ \epsilon_{LL}(\epsilon_{HH} - \eta_{HH}) + \epsilon_{HL}(\eta_{LH} - \epsilon_{LH}) \\ \eta_{HL}\epsilon_{LL} - \eta_{LL}\epsilon_{HL} \\ \eta_{LL}(\epsilon_{HH} - \eta_{HH}) + \eta_{HL}(\eta_{LH} - \epsilon_{LH}) \\ \eta_{HL}\epsilon_{LL} - \eta_{LL}\epsilon_{HL} \end{bmatrix} t_L$$

$$+\frac{1}{D} \begin{bmatrix} \eta_{LH}(\epsilon_{LL}\epsilon_{HH} - \epsilon_{LH}\epsilon_{HL}) - \epsilon_{LH}(\eta_{LL}\eta_{HH} - \eta_{LH}\eta_{HL}) \\ \eta_{HH}(\epsilon_{LL}\epsilon_{HH} - \epsilon_{LH}\epsilon_{HL}) - \epsilon_{HH}(\eta_{LL}\eta_{HH} - \eta_{LH}\eta_{HL}) \\ \eta_{LH}\epsilon_{HH} - \eta_{HH}\epsilon_{LH} \\ \epsilon_{HH}(\epsilon_{LL} - \eta_{LL}) + \epsilon_{LH}(\eta_{HL} - \epsilon_{HL}) \\ \eta_{LH}\epsilon_{HH} - \eta_{HH}\epsilon_{LH} \\ \eta_{HH}(\epsilon_{LL} - \eta_{LL}) + \eta_{LH}(\eta_{HL} - \epsilon_{HL}) \end{bmatrix} t_H$$

where:

$$D = (\epsilon_{LL} - \eta_{LL})(\epsilon_{HH} - \eta_{HH}) - (\epsilon_{LH} - \eta_{LH})(\epsilon_{HL} - \eta_{HL})$$

In order to determine the direction of change for each endogenous variable, the signs, and in some cases, the magnitudes, of the supply and demand elasticities must be determined. Determining the sign of the own-price elasticities is straightforward. The cross-price elasticities may be estimated empirically, but a strictly theoretical approach would leave their signs unknown (since they are Marshallian, and include income effects). More importantly, the link between the results from this two-market specification and a single-market representation that assumes a homogeneous good is unclear. The next section develops a means for simplifying the terms in the above matrix and for linking the two-market results to those from a single-market representation.

3 An Armington Approach

Many of the problems with analyzing the results from the model presented above can be alleviated by interpreting the results in the context of an Armington model. This model was originally designed to represent the demand for internationally traded commodities. Armington (1969) noted that a single commodity may be produced in many different countries or geographic regions, but the nature of the commodity would vary, depending on the country of origin. If each of m countries produces and consumes n commodities, and each commodity-origin combination were modeled as a distinct product, then m^2n demand functions would have to be estimated. By imposing some restrictions on the relationships among commodities of the same type but different origins, Armington (1969) reduced the number of demand parameters to be estimated.

The Armington model, while less restrictive than a homogeneous-goods model, imposes two important restrictions. First, the marginal rate of substitution between any two commodities of the same type and different origins is independent of the consumption of other commodity types. This restriction implies that commodities of the same type comprise a weakly separable group of products. The second important restriction is that the aggregation functions are homogeneous of degree one–Armington specified them as constant elasticity of substitution (CES) functions.[1] These assumptions imply that the budgeting process may be represented in two stages. In the first stage, total utility is expressed as a

[1]The function used to aggregate quantities can also be thought of as a sub-utility function that is maximized in the second stage of the budgeting process, subject to the expenditure allocation from the first stage.

function of aggregated quantities of commodity types, and the budget is allocated among the commodity types. In the second stage, the expenditures for each particular commodity type are allocated among the quantities of that commodity from different origins.

The relationship of the Armington model to the problem at hand is relatively straightforward. Rather than differing by country of origin, the commodities considered here are of the same general type, but vary in quality. If the high- and low-quality varieties of the commodity under consideration meet the conditions specified by Armington(1969), then in the first stage of the budgeting process, the consumer maximizes utility derived from consumption of the aggregate commodity and all other goods, subject to the budget constraint. This determines the quantity aggregate, Q (the absence of a subscript denotes aggregated quantity or price). In the second stage, total expenditure on the commodity is allocated between the high- and low-quality varieties. Thus, demand functions for each quality can be expressed as functions of the prices of the low- and high-quality goods and expenditure on the commodity group (a function of the aggregate price).

Imposing the assumptions described by Armington (1969), the elasticities of demand for the individual qualities with respect to individual prices can be expressed as:

$$\eta_{LL} = s_L\eta - s_H\sigma \tag{17}$$

$$\eta_{LH} = s_H(\eta + \sigma) \tag{18}$$

$$\eta_{HL} = s_L(\eta + \sigma) \tag{19}$$

$$\eta_{HH} = s_H\eta - s_L\sigma \tag{20}$$

where $s_i = \frac{P_i Q_i}{PQ}$ is the value-share of quality i. Demand responses to a given price change are clearly comprised of two effects. The scale effect is given by the η term, where $\eta \leq 0$ is the overall elasticity of demand, defined as the elasticity of the aggregate quantity with respect to the aggregate price. The substitution effect is given by the σ term, where $\sigma \geq 0$ is the elasticity of substitution between the two qualities. Thus, for example, when the price of the low-quality good increases, the quantity demanded of the high-quality good increases through the substitution effect, since $\sigma \geq 0$, and decreases through the scale effect, since the increase in P_L increases the aggregate price.

A similar representation of the individual firm's profit maximization problem can be specified in order to derive expressions for the supply elasticities:

$$\epsilon_{LL} = s_L \epsilon - s_H \tau \quad (21)$$

$$\epsilon_{LH} = s_H(\epsilon + \tau) \quad (22)$$

$$\epsilon_{HL} = s_L(\epsilon + \tau) \quad (23)$$

$$\epsilon_{HH} = s_H \epsilon - s_L \tau \quad (24)$$

These elasticities can be interpreted similarly to the elasticities of demand. The first term in each equation gives the scale effect of supply responses to price changes, where ϵ is the overall supply elasticity, or the elasticity of aggregate quantity with respect to aggregate price ($\epsilon \geq 0$). The second term gives the substitution effect of supply responses to price changes, where $\tau \leq 0$ is the elasticity of transformation in the production process. When the price of the low-quality product increases, the quantity supplied of the high-quality product will increase through the scale effect and decrease through the substitution effect.

The differentiated goods model laid out above provides a means of relaxing the assumption of product homogeneity while limiting the number of parameters to be estimated. In addition, the explicit modeling of the two-stage budgeting process clarifies the consequences of aggregating commodities. As described above, a change in the price of one quality will change the aggregate price of the commodity group. In the first stage of the budgeting process, the change in the aggregate price will alter the consumer expenditure allocated to the group of commodities (which is also the producer revenue from that commodity group). These first-stage effects are the scale effects represented by the overall demand and supply elasticities. In the second stage of the budgeting process, the individual price change alters the relative prices of the commodities within the group. As a result, consumers will change the mix of commodities consumed, and producers will alter the mix of commodities produced (providing they are not produced and consumed in fixed proportions). These second-stage effects are the substitution effects represented by the σ and τ terms discussed above.

An aggregate analysis of a group of commodities would only account for the first stage of the budgeting process, ignoring the substitution effects in the second stage. Thus, treating a group of commodities as if it were a single commodity is equivalent to setting those substitution effects equal to zero, i.e., $\sigma = 0$ and $\tau = 0$. Setting either one of these parameters equal to zero imposes that the two qualities are produced and consumed in fixed proportions.

Substituting the expressions for the elasticities of demand and supply into the solution of the two-market equilibrium displacement model yields the following solution:

$$\begin{bmatrix} dlnQ_L \\ dlnQ_H \\ dlnP_L^D \\ dlnP_H^D \\ dlnP_L^S \\ dlnP_H^S \end{bmatrix} = \frac{1}{D'} \begin{bmatrix} s_L\epsilon\eta(\sigma-\tau) + s_H\sigma\tau(\epsilon-\eta) & s_H\epsilon\eta(\sigma-\tau) - s_H\sigma\tau(\epsilon-\eta) \\ s_L\epsilon\eta(\sigma-\tau) - s_L\sigma\tau(\epsilon-\eta) & s_H\epsilon\eta(\sigma-\tau) + s_L\sigma\tau(\epsilon-\eta) \\ s_L\epsilon(\sigma-\tau) - s_H\tau(\epsilon-\eta) & s_H\epsilon(\sigma-\tau) + s_H\tau(\epsilon-\eta) \\ s_L\epsilon(\sigma-\tau) + s_L\tau(\epsilon-\eta) & s_H\epsilon(\sigma-\tau) - s_L\tau(\epsilon-\eta) \\ s_L\eta(\sigma-\tau) - s_H\sigma(\epsilon-\eta) & s_H\eta(\sigma-\tau) + s_H\sigma(\epsilon-\eta) \\ s_L\eta(\sigma-\tau) + s_L\sigma(\epsilon-\eta) & s_H\eta(\sigma-\tau) - s_L\sigma(\epsilon-\eta) \end{bmatrix} \begin{bmatrix} t_L \\ t_H \end{bmatrix}$$

where:

$$D' = (\epsilon-\eta)(\sigma-\tau)$$

This substitution achieves three goals. First, it reduces the number of parameters appearing in the solution matrix. The eight elasticities of supply and demand are replaced with five parameters: a value share (s_H, noting that $s_L = 1 - s_H$), the overall elasticity of demand (η), the elasticity of substitution in consumption (σ), the overall elasticity of supply (ϵ), and the elasticity of transformation in production (τ). Second, all of the parameters are of known sign. Third, evaluating the expressions as either σ or τ approaches zero yields the effects on prices and quantities that would result from a single-market approach.

4 Price and Quantity Effects of a Tax

Using the solution to the equilibrium displacement model specified above, the proportional changes in the quantities and prices resulting from proportional taxes of t_L in the low-quality market, and t_H in the high-quality market are:

$$dlnQ_L = \frac{\epsilon\eta}{\epsilon-\eta}(s_Lt_L + s_Ht_H) - \frac{s_H\sigma\tau}{\sigma-\tau}(t_H - t_L) \qquad (25)$$

$$dlnQ_H = \frac{\epsilon\eta}{\epsilon-\eta}(s_Lt_L + s_Ht_H) + \frac{s_L\sigma\tau}{\sigma-\tau}(t_H - t_L) \qquad (26)$$

$$dlnP_L^D = \frac{\epsilon}{\epsilon-\eta}(s_L t_L + s_H t_H) + \frac{s_H\tau}{\sigma-\tau}(t_H - t_L) \quad (27)$$

$$dlnP_H^D = \frac{\epsilon}{\epsilon-\eta}(s_L t_L + s_H t_H) - \frac{s_L\tau}{\sigma-\tau}(t_H - t_L) \quad (28)$$

$$dlnP_L^S = \frac{\eta}{\epsilon-\eta}(s_L t_L + s_H t_H) + \frac{s_H\sigma}{\sigma-\tau}(t_H - t_L) \quad (29)$$

$$dlnP_H^S = \frac{\eta}{\epsilon-\eta}(s_L t_L + s_H t_H) - \frac{s_L\sigma}{\sigma-\tau}(t_H - t_L) \quad (30)$$

In examining each of these effects, it is useful to note that in a single-market model (as can be seen by letting one of the shares go to zero), the effects of a 100t percent tax are:

$$dln\tilde{Q} = \frac{\epsilon\eta}{\epsilon-\eta}t \quad (31)$$

$$dln\tilde{P}^D = \frac{\epsilon}{\epsilon-\eta}t \quad (32)$$

$$dln\tilde{P}^S = \frac{\eta}{\epsilon-\eta}t \quad (33)$$

where tildes (˜ 's) denote that the result is derived from a single-market representation.

Given equations (31) through (33), it is clear that the first term in each of equations (25) through (30) is analogous to the single-market effect, where the single-market tax rate is a value-share weighted sum of the individual tax rates. Thus, when the second term in each equation is equal to zero in each of equations (25) through (30), the quantity and price effects in the market for each quality will be the same as would be predicted by a single-market model using $t = s_L t_L + s_H t_H$. All of these second terms will equal zero under either of two conditions. The first condition is that the tax rates in the two markets are equal (i.e., if $t_L = t_H$), as would be the case if a single ad valorem tax were imposed in both markets. The second condition is that the two qualities are consumed and produced in fixed proportions (i.e., if $\sigma = \tau = 0$). When the second terms are not equal to zero, they adjust the single-market result (where $t = s_L t_L + s_H t_H$) for the differential tax rates and for substitution in consumption and production.

Using a single-market model to represent an aggregate of products of multiple qualities implicitly assumes that the quality of the aggregate is constant. Suppose the quantity of high-quality product divided by the quantity of low-quality product, Q_H/Q_L, is used as a measure of average quality. The difference between the proportional quantity changes, $dlnQ_H - dlnQ_L$, measures the change in the average quality resulting from the tax policy. In order for average quality to remain constant, the quantity of each quality must change by the same proportion, i.e., $dlnQ_L = dlnQ_H = dln\tilde{Q}$ (which will be true if either of the two conditions described above is met—when the tax is specified on an ad valorem basis or when the two qualities are produced and consumed in fixed proportions). If $dlnQ_H - dlnQ_L > 0$, then the quantity would be reduced by a larger proportion in the low-quality market than in the high-quality market, and the average quality would increase as a result of the tax. If the inequality were reversed, then average quality would have decreased. Similarly, the price premium for high quality can be expressed as the ratio of the price of high-quality product to the price of the low-quality product (i.e., P_H/P_L). Thus, the proportional change in the price premium will equal the difference between the proportional price changes, $dlnP_H - dlnP_L$. If this difference is positive (negative), then the price premium for high quality would increase (decrease) as a result of the tax.

The differences between the proportional quantity and price changes are:

$$dlnQ_H - dlnQ_L \quad = \quad \frac{\sigma\tau}{\sigma - \tau}(t_H - t_L) \qquad > 0 \qquad if \qquad t_H < t_L \tag{34}$$

$$dlnP_H^D - dlnP_L^D = -\frac{\tau}{\sigma - \tau}(t_H - t_L) \qquad < 0 \qquad if \qquad t_H < t_L \tag{35}$$

$$dlnP_H^S - dlnP_L^S = -\frac{\sigma}{\sigma - \tau}(t_H - t_L) \qquad > 0 \qquad if \qquad t_H < t_L \tag{36}$$

The changes in average quality and the quality premiums hinge on the relationship between the two tax rates and the substitution parameters. In the case where the tax rates are equal for the two qualities (i.e., a uniform ad valorem tax), there are no changes in average quality or in the quality premiums. Similarly, if σ and τ both equal zero, there will be no changes in quality or the quality premia. If the tax rate in the high-quality market is smaller than that in the low-quality market (i.e., $t_H < t_L$), average quality would increase, the consumer's quality premium would decrease, and the producer's quality premium would increase, relative to the no-intervention case. The directions of the quality changes are reversed when the tax rate in the low-quality market is smaller than that in the high-quality market. These changes in the distribution of quality and the quality premium would not be taken into account in a single-market analysis.

5 Per Unit Taxes

The results in the previous section show that an ad valorem tax leaves average quality and the quality premia unchanged. Because there is no change in quality, there are no errors caused by assuming product homogeneity. This is not the case, however, when taxes are specified on a per unit basis. When a tax of T per unit is imposed on products of both high and low quality, the quality-specific prices must be used to convert the tax to proportional terms. Thus, the two tax rates are specified as $t_L = \frac{T}{P_L}$ and $t_H = \frac{T}{P_H}$, where P_L and P_H

are the initial prices, and $P_i^D = P_i^S$, so the superscripts may be dropped. In this case, the proportional taxes differ, so that the second terms in equations (25) through (30) no longer vanish. The proportional changes in the quantity and price in each market for this type of policy are:

$$\begin{aligned}
dlnQ_L &= \frac{\epsilon\eta}{\epsilon-\eta}\frac{T}{P} - \frac{s_H\sigma\tau}{\sigma-\tau}\frac{T}{P_LP_H}(P_H-P_L) && (37)\\
dlnQ_H &= \frac{\epsilon\eta}{\epsilon-\eta}\frac{T}{P} + \frac{s_L\sigma\tau}{\sigma-\tau}\frac{T}{P_LP_H}(P_H-P_L) && (38)\\
dlnP_L^D &= \frac{\epsilon}{\epsilon-\eta}\frac{T}{P} + \frac{s_H\tau}{\sigma-\tau}\frac{T}{P_LP_H}(P_H-P_L) && (39)\\
dlnP_H^D &= \frac{\epsilon}{\epsilon-\eta}\frac{T}{P} - \frac{s_L\tau}{\sigma-\tau}\frac{T}{P_LP_H}(P_H-P_L) && (40)\\
dlnP_L^S &= \frac{\eta}{\epsilon-\eta}\frac{T}{P} + \frac{s_H\sigma}{\sigma-\tau}\frac{T}{P_LP_H}(P_H-P_L) && (41)\\
dlnP_H^S &= \frac{\eta}{\epsilon-\eta}\frac{T}{P} + \frac{s_H\sigma}{\sigma-\tau}\frac{T}{P_LP_H}(P_H-P_L) && (42)
\end{aligned}$$

using:

$$\begin{aligned}
s_Lt_L + s_Ht_H &= \frac{T}{P}\\
t_H - t_L &= -\frac{T}{P_LP_H}(P_H-P_L)\\
P &= \frac{P_LQ_L + P_HQ_H}{Q_L+Q_H}
\end{aligned}$$

Here, P is the average unit value of the two qualities at the initial equilibrium, and the aggregate quantity Q is defined as a simple sum of the quantities of each quality.

As in equations (25) through (30), the first term in each of equations (37) through (42) is equivalent to the effects found in a single-market model for a per unit tax of T and an initial price of P. In contrast to the ad valorem tax, though, when the same per unit tax is imposed in the two markets, the adjustment terms (the second terms in each equation) are no longer equal to zero. By definition, the price of the high-quality good is larger than that of the low-quality product, so the price difference in parentheses is positive. All other elements in the adjustment terms are of known sign, and it is clear what adjustments must be made to a single-market result to allow for the different tax rates and the substitution possibilities in production and consumption. The quantity reduction in the low-quality market is greater and the quantity reduction in the high-quality market is smaller than those predicted in a single-market model of a homogeneous good. Similarly, the consumer price effect in the low-quality market is larger and that in the high-quality market is smaller as a result of the adjustment. A similar pattern holds for producer prices: the producer price of the low-quality good decreases by a larger proportion and the producer price of the high-quality good decreases by a smaller proportion than the single-market model would indicate.

What happens to the average quality sold in the two markets when a per unit tax is imposed? The differences between the proportional quantity and price changes are:

$$dlnQ_H - dlnQ_L = -\frac{\sigma\tau}{(\sigma-\tau)}\frac{T}{P_L P_H}(P_H - P_L) \quad > 0 \tag{43}$$

$$dlnP_H^D - dlnP_L^D = \frac{\tau}{(\sigma-\tau)}\frac{T}{P_L P_H}(P_H - P_L) \quad < 0 \tag{44}$$

$$dlnP_H^S - dlnP_L^S = \frac{\sigma}{(\sigma-\tau)}\frac{T}{P_L P_H}(P_H - P_L) \quad > 0 \tag{45}$$

The proportional quantity reduction in the high quality market is smaller (in absolute terms) than that in the low-quality market, indicating that average quality increases as a result of the tax. The proportional increase in the consumer price in the high-quality market is smaller than that in the low-quality market, indicating that the consumer's quality premium decreases. Finally, the producer price decreases by a smaller proportion in the high-quality market than in the low quality market, so that the producer's quality premium increases. The effects on the consumer and producer quality premiums are intuitive. If average quality increases as a result of the tax, consumers require an incentive to consume higher quality: a lower quality premium. Similarly, producers require an incentive to producer higher quality: a higher quality premium.

These results prove the Alchian-Allen effect at the market level, under the specified demand and supply conditions. In addition, they confirm the quality change predicted by Barzel (1976) while allowing for a more general definition of quality. In order to determine how robust these results are, the solution is derived under more general elasticity decompositions next.

6 Results with a More General Separability Assumption

While the Armington decompositions used above are simple and manageable, they are fairly restrictive. Namely, they impose homothetic separability, a special case of weak separability in which the elasticity of demand for each quality with respect to expenditure on the commodity group is equal to one. Intuition suggests that this assumption may not be appropriate: the demand for a high-quality is likely to be more responsive to changes in expenditure than its low-quality counterpart. Thus, a less restrictive representation of demand and supply conditions seems necessary. The assumption of weak separability is sufficient for the second stage of budgeting to exist. In addition, if the price indexes used for the commodity groups are invariant to income, then the budgeting process may be represented in two stages. These conditions are clearly met for systems of demand functions that are not homothetic. Here, the group price index is a weighted average of quality-specific prices, where the weights are the original (undistorted quantity shares). This price index is invariant to income, as required, along with weak separability, so that the two-stage budgeting representation is appropriate. The demand elasticities can therefore be decomposed as:

$$\eta_{LL} = s_L \gamma_L \eta - s_H \sigma \tag{46}$$

$$\eta_{LH} = s_H(\gamma_L \eta + \sigma) \tag{47}$$

$$\eta_{HL} = s_L(\gamma_H \eta + \sigma) \tag{48}$$

$$\eta_{HH} = s_H \gamma_H \eta - s_L \sigma \tag{49}$$

Similarly, the elasticities of supply can be decomposed as:

$$\epsilon_{LL} = s_L \rho_L \epsilon - s_H \tau \tag{50}$$

$$\epsilon_{LH} = s_H(\rho_L \epsilon + \tau) \tag{51}$$

$$\epsilon_{HL} = s_L(\rho_H \epsilon + \tau) \tag{52}$$

$$\epsilon_{HH} = s_H \rho_H \epsilon - s_L \tau \tag{53}$$

where γ_i is the elasticity of demand for quality i with respect to group expenditure, and ρ_j is the elasticity of supply of quality j with respect to group revenue. The link between these decompositions and the Armington decompositions above is clear. When $\gamma_L = \gamma_H =$

1, equations (46) through (53) reduce to the Armington decompositions in equations (17) through (24). Recall that the Armington decompositions were functions of five parameters: η, ϵ, σ, τ, and s_H. These less restrictive decompositions only add two more variables, γ_H and ρ_H, since by Engel aggregation of the second-stage elasticities, $s_L\gamma_L + s_H\gamma_H = 1$, and $s_L\rho_L + s_H\rho_H = 1$. Note that the elasticity decompositions can be re-written fairly simply as:

$$
\begin{array}{rclcrcl}
\eta_{LL} & = & s_L\eta_L - s_H\sigma & \qquad & \epsilon_{LL} & = & s_L\epsilon_L - s_H\tau \\
\eta_{LH} & = & s_H(\eta_L + \sigma) & & \epsilon_{LH} & = & s_H(\epsilon_L + \tau) \\
\eta_{HL} & = & s_L(\eta_H + \sigma) & & \epsilon_{HL} & = & s_L(\epsilon_H + \tau) \\
\eta_{HH} & = & s_H\eta_H - s_L\sigma & & \epsilon_{HH} & = & s_H\epsilon_H - s_L\tau
\end{array}
$$

where

$$
\begin{array}{rclcrcl}
\eta_L & = & \gamma_L\eta & \qquad & \epsilon_L & = & \rho_L\epsilon \\
\eta_H & = & \gamma_H\eta & & \epsilon_H & = & \rho_H\epsilon
\end{array}
$$

Under these more general separability assumptions, the new solutions for the eight endogenous variables are equal to the solutions laid out in equations (25) through (30) plus some adjustment terms. Using hats (ˆ 's) to denote the new solutions for the endogenous variables, they can be written:

$$
\begin{bmatrix} dln\hat{Q}_L \\ dln\hat{Q}_H \\ dln\hat{P}_L^D \\ dln\hat{P}_H^D \\ dln\hat{P}_L^S \\ dln\hat{P}_H^S \end{bmatrix}
=
\begin{bmatrix} dlnQ_L \\ dlnQ_H \\ dlnP_L^D \\ dlnP_H^D \\ dlnP_L^S \\ dlnP_H^S \end{bmatrix}
+ \frac{\eta\epsilon}{D'}
\begin{bmatrix}
-s_H\sigma(\rho_H - 1) + s_H\tau(\gamma_H - 1) & -s_H\sigma(1-\rho_L) + s_H\tau(1-\gamma_L) \\
s_L\sigma(\rho_H - 1) - s_L\tau(\gamma_H - 1) & s_L\sigma(1-\rho_L) - s_L\tau(1-\gamma_L) \\
s_H(\rho_H - \gamma_H) & -s_H(\rho_L - \gamma_L) \\
-s_L(\rho_H - \gamma_H) & s_L(\rho_L - \gamma_L) \\
s_H(\rho_H - \gamma_H) & -s_H(\rho_L - \gamma_L) \\
-s_L(\rho_H - \gamma_H) & s_L(\rho_L - \gamma_L)
\end{bmatrix}
\begin{bmatrix} t_L \\ t_H \end{bmatrix}
$$

Not surprisingly, the expenditure elasticities on the demand and supply sides are the primary determinants of the adjustments made to the results presented in the previous section. The proportional changes in each endogenous variable are omitted to avoid clutter. The changes in average quality and the quality premia are:

$$dlnQ_H - dlnQ_L = \left[\frac{\sigma\tau}{(\sigma-\tau)} + \frac{\eta\epsilon}{(\epsilon-\eta)}\right](t_H - t_L) \tag{54}$$

$$+\frac{\eta\epsilon}{D'}\left[\sigma(\rho_H t_L - \rho_L t_H) - \tau(\gamma_H t_L - \gamma_L t_H)\right] \tag{55}$$

$$dlnP_H^D - dlnP_L^D = -\frac{\tau}{(\sigma-\tau)}(t_H - t_L) - \frac{\eta\epsilon}{s_L D'}(\rho_H - \gamma_H)(s_L t_L + s_H t_H) \tag{56}$$

$$dlnP_H^S - dlnP_L^S = -\frac{\sigma}{(\sigma-\tau)}(t_H - t_L) - \frac{\eta\epsilon}{s_L D'}(\rho_H - \gamma_H)(s_L t_L + s_H t_H) \tag{57}$$

In this case, even for a uniform ad valorem tax, there will be induced changes in the average quality produced and consumed, as long as the expansion effects of a given quality differ in supply and demand (i.e., as long as $\rho_H \neq \gamma_H$, which in turn implies that $\rho_L \neq \gamma_L$). This is a somewhat unexpected result: even an ad valorem tax can distort incentives to produce and consume quality when more general (and realistic) supply and demand conditions are incorporated in the analysis.

7 Concluding Remarks

The differences between the relative changes in quantities and prices in the two markets, summarized in equations (34) through (45) and (55) through (57) are implicitly assumed to be equal to zero in a single-market model for a homogeneous good of constant quality. If these quality effects are relatively small, then the policy effects from a single-market model for an aggregate good may reasonably approximate the actual policy effects in markets for heterogeneous products. However, the larger are these quality effects, the less accurate will be the results from a model of a homogeneous good. The magnitudes of the effects on average quality and the quality premiums increase as the amount of the per unit tax increases, as the difference in the prices of the high- and low-quality products increases, as the degree of substitutability in consumption or production increases, and as the supply and demand expansion effects become increasingly disparate. In other words, as T, $(P_H - P_L)$, σ, $\mid \tau \mid$ and $\mid \rho_H - \gamma_H \mid$ increase, the magnitudes of the changes in average quality and the consumer and producer quality premiums increase.

While the assumption of product homogeneity is convenient, it is important to recognize that it may not always be appropriate. In particular, the implementation of some policies will induce changes in the relative prices of different qualities, and thus in the quality mix of units produced and consumed. The simple model presented here contrasts ad valorem and per unit taxes when demand and supply conditions are characterized by homothetic separability in order to demonstrate such policy-induced changes in quality. In this case, an ad valorem tax leaves the relative prices of different qualities unchanged and reduces the quantity of each quality by the same proportion, so that average quality does not change. In contrast, the same per unit tax imposed in the two markets amounts to different proportional tax rates, so that the relative prices of the different qualities and the mix of qualities sold change. These Alchian-Allen types of effects have been discussed in the literature at length in the context of the individual consumer or producer. However, one shortcoming in this literature is the ambiguity of the change in quality when income effects or other goods are introduced. Under these Armington-type assumptions, which may be empirically very reasonable for some agricultural commodities, the quality response to per unit taxes (or other like policies) is unambiguous at the market level, where scale effects are incorporated and the different qualities of the same good comprise a weakly separable group.

The unambiguous nature of this result is contrasted with the result from the more general specification, where the assumption of homotheticity is relaxed. In this case, quality effects result from either tax policy, ad valorem or per unit.

As the degree of quality responses increases, the accuracy of policy effects derived from a single-market model as an approximation of true policy effects diminishes. Similar quality responses can be expected from other policies, such as quotas and target prices. In cases where quality responses are important, they should be incorporated in estimates of quantity, price, and welfare effects of introducing or changing a policy. This paper provides a first step in the means to that end.

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