The Precautionary Principle in Practice: How to Write a Call Option on the Environment

Greg Hertzler[1]
Agricultural and Resource Economics
University of Western Australia.

Over the years, there have been three major approaches to making decisions under risk. The first, static expected utility theory, is perhaps the most popular, used by many applied economists. The second is dynamic investment theory, used by financial economists. The third I have no name for. It is the approach we as natural resource and environmental economists use. It seems to be a mixture of philosophy, common sense and confusion. We still haven't sorted out what we mean by the term "option value." To avoid confusion in this paper, I will try to be precise in defining how risk affects decisions. This will, hopefully, lead to more precise results about how to value our options for preserving the environment.

We often talk about the "risk preferences" of decision makers. This leads to confusion and misinterpretation because, strictly speaking, preferences about risk aren't in our models. We include preferences about consumption, wealth and time, but not about risk. Risk affects decisions whenever the world is non linear. One source of non linearity is the preference or utility functions in our models. There are many other sources, however. Behaviour under risk can be explained by the non linearity which caused it. For the purposes of this discussion, non linearities will fall into the following categories:

1) Functions
    a. Consumption preferences
    b. Wealth preferences
    c. Time preferences
    d. Production and cost functions
    e. Endogenous prices
2) Probability distributions
    a. Non normal distributions
    b. Co variances
3) Asymmetries
    a. Infeasibilities
    b. Options
    c. Irreversibilities

Usually, preferences are singled out as the source "risk aversion". However, any non linear function will alter behaviour under risk. Production functions are almost always non linear and stochastic. Endogenous prices are multiplied by quantities and introduce non linearity.

Only normal probability distributions are defined by linear differential equations. Any other distribution is non linear. In finance, distributions are usually assumed to be log normal. As a consequence, risk exposure increases with the size of an investment. Co variances are a source of non linearity because they alter means and variances in a way similar to endogenous prices.

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Asymmetries are non linearities which alter the probabilities of events. Sometimes events simply can't happen and we use inequality constraints in our models as an impermeable barrier. All decisions must be on the feasible side of the barrier. In many cases, other approaches may be more realistic. Options are more like drawing a line in the sand. Events can push us to either side of the line and we invest in an option to compensate us if we end up on the on the wrong side. Perhaps most of the big environmental questions involve irreversibilities. An irreversibility is like a turnstile at the train station. Being on either side is feasible, but if you are not careful, events in an unpredictable crowd will push you through and you can't get back. If going through is undesirable, you must alter your decisions well before an unpredictable event pushes you through.

This study examines options and irreversibilities in environmental decision making. The aim is to develop option pricing formulas and determine how much society will invest to avoid undesirable outcomes. As examples, I will use stocks of exhaustible resources and water rights. With a bit of imagination, the same results might apply to your favourite resource and environmental problem.

The approach will be to adapt dynamic investment theory from finance. There are many assumptions made in finance which do not apply to natural resources and the environment, including:

1) complete markets of many investors;
2) continuous trading;
3) no profitable arbitrage.

For natural resources and the environment, there often are no markets. Many decisions are taken only once by a single agent, let alone continuously by many investors. Environmental variables probably don't have prices and profit is rarely the objective of environmental management.

So studying option pricing for the environment is more like a career than a conference paper. Never-the-less, this paper will show that the assumptions of finance aren't needed to define options. However, the formulas for pricing options are different and give much different answers. The examples in the paper are small ones and there are bigger problems yet to study. The paper concludes with suggestions about how this might be done.

Stochastic Dynamic Programming

Before presenting the results, there are a few mathematical details to attend. Option pricing is an application of Ito stochastic calculus. Stochastic calculus is a quick way to calculate expected values and variances (Hertzler, Harman and Lindner). Extending the theory of option pricing to environmental decisions requires stochastic dynamic programming (Dixit and Pindyck, Hertzler).

In a general formulation of a stochastic dynamic program, decision-makers in society are assumed to behave as if they maximise their expected utility subject to a budget constraint for the change in wealth.

$$J(0,W_0) = \max E\left\{\int_0^T e^{-\rho t}U(Q)dt + J(T,W_T)\right\}$$

$$\text{subject to:}$$

$$dW = \delta(W,Q,D)dt + \sigma(Q,D)dZ; \qquad W_0 \text{ is given.}$$

A society's satisfaction is summarized by an expected social welfare function, J. Satisfaction is derived from the utility of consumption, $U(Q)$, integrated over all years, t, and discounted at the rate of time preference, ρ. Satisfaction also includes expected utility of wealth at the end of the planning horizon, T. Starting from time zero, initial wealth, W_0, increases with changes in wealth, dW. A change in wealth has an expected change δdt, where δ is the instantaneous mean, and an error term σdZ, where σ is a vector of instantaneous standard deviations and dZ is a vector of Weiner increments. The mean and standard deviations are functions of wealth, W, consumption, Q, and decision variables, D, chosen at time t to apply over a decision interval of length dt.

This model assumes that wealth is a continuous-time Markov process with rapid uncertainty. The Markov property says that, conditional upon current wealth, future wealth doesn't depend upon the past. It allows the stochastic change in wealth to be decomposed into its expected change and its error (Grimmett and Stirzaker, p. 447). Rapid uncertainty has many random events occurring within each decision interval. These random events may be drawn from different probability distributions but they must have the same mean and variance. According to the central limit theorem, their sum converges to a normally distributed process. In continuous time, this process is white noise, ε, which has mean $E\{\varepsilon\} = 0$ and covariance $E\{\varepsilon\varepsilon'\} = \Omega/dt$, where Ω is a correlation matrix. Weiner increments are normally distributed white noise over time, $dZ = \varepsilon dt$, and have mean $E\{dZ\} = 0$ and covariance $E\{dZdZ'\} = \Omega dt$. The change in wealth, $dW = \delta dt + \sigma dZ$, is a transformation of Weiner increments and has mean $E\{dW\} = \delta dt$ and covariance $E\{(dW - \delta dt)(dW - \delta dt)'\} = \sigma\Omega\sigma' dt$.

A continuous-time Markov process with rapid uncertainty has a probability density that is the solution of two partial differential equations called the forward and backward equations (Grimmett and Stirzaker, p. 494). These equations depend upon the functions δ and σ. If the functions are constant, the forward and backward equations are linear and integrate to become the normal probability density. Over a short decision interval of length dt, functions δ and σ are approximately constant and the change in wealth is normally distributed. Over longer intervals, however, functions δ and σ are not constant. The forward and backward equations are non linear and do not integrate to become the normal density. Although the probability density of wealth exists, its functional form may be unknown. This is an advantage, however, because it allows stochastic dynamic programming to model probability distributions in a general way, simply by specifying the functions δ and σ.

Maximising expected utility over society's time horizon is equivalent to maximising the Hamilton-Jacobi-Bellman equation in each decision interval. The Hamilton-Jacobi-Bellman equation is a partial differential equation in time and wealth, subject to a boundary condition at time T.

$$(1)\qquad \frac{\partial J}{\partial t}+\max\left\{U+\frac{\partial J}{\partial W}\delta+\tfrac{1}{2}\frac{\partial^2 J}{\partial W^2}\sigma\Omega\sigma'\right\}-\rho J=0;$$

$$J(T,W_T)=\begin{cases} e^{-\rho(T-t)}V\left(W_T-\hat{W}\right); & W_T>\hat{W} \\ 0; & W_T\le\hat{W}. \end{cases}$$

In the boundary condition, expected utility at the terminal time equals the discounted utility of wealth, $e^{\rho(T-t)}V(W_T-\hat{W})$, if wealth is above a subsistence level, $\hat{W}$, and equals zero if wealth is below the subsistence level. This is the overall constraint at the lowest sustainable level of society. It is the only inequality in the model but it is not a hard constraint. Extinction is feasible, just not pleasant.

The expression to maximise in brackets is discounted current utility of consumption *U*, plus the marginal utility of wealth, $\partial J/\partial W$, multiplied by the instantaneous mean, δ, plus one-half the derivative of the marginal utility of wealth, $\tfrac{1}{2}\partial^2 J/\partial W^2$, multiplied by the instantaneous covariance, $\sigma\Omega\sigma'$, which equals the standard deviation squared. Optimality conditions are the derivatives set equal to zero.

$$(2)\qquad \frac{\partial U/\partial Q}{\lambda}+\frac{\partial\delta}{\partial Q}-\tfrac{1}{2}R\frac{\partial(\sigma\Omega\sigma')}{\partial Q}=0;$$

$$\frac{\partial\delta}{\partial D}-\tfrac{1}{2}R\frac{\partial(\sigma\Omega\sigma')}{\partial D}=0;$$

$$\text{where:}\qquad \lambda(W)=\frac{\partial J}{\partial W};$$

$$R(W)=-\frac{\partial^2 J/\partial W^2}{\partial J/\partial W}.$$

The first optimality condition is for consumption and the second is for production and investment decisions. The marginal utility of consumption is normalised by the expected marginal utility of wealth, λ. The terms containing *R* are marginal risk premiums. To simplify notation, *R* is defined as the coefficient of absolute risk aversion. It measures the curvature of expected social welfare with respect to current wealth. It is distinguished from an Arrow-Pratt coefficient of risk aversion that would measure the curvature with respect to terminal wealth. Marginal utility of wealth and the risk aversion coefficient encapsulate all of the information about the future. If their current values can be measured, optimal decisions in a single period are also dynamically optimal.

A Model of Options Including the Environment

The instantaneous mean and covariance required in equation (2) are determined by the stochastic differential equation for wealth. This differential equation is derived in the Appendix. From Appendix equation (A1), the instantaneous mean is:

$$(3)\qquad \begin{aligned}\delta(W,Q,D)=&\ \delta_W W+m(\delta_m-\delta_W)M+n(\delta_n-\delta_W)N+e(\delta_e-\delta_W)E\\ &-qQ+\left(y(1+\delta_y)+\sigma_y\sigma_Y\omega_{yY}\right)Y-c(Y)+\left(y(\delta_y-\delta_W-\delta_S)-s\right)S\\ &+\left(\overline{X}-X\right)K+\left(\overline{h}-y(1+\delta_y)\right)H-f\delta_f F.\end{aligned}$$

where:
$$m\delta_m = \frac{\partial m}{\partial t} + \frac{\partial m}{\partial y} y\delta_y + \tfrac{1}{2}\frac{\partial^2 m}{\partial y^2} y^2\sigma_y^2;$$
$$n\delta_n = \frac{\partial n}{\partial t} + \frac{\partial n}{\partial f} f\delta_f + \tfrac{1}{2}\frac{\partial^2 n}{\partial f^2} f^2\sigma_f^2;$$
$$e\delta_e = \frac{\partial e}{\partial t} + \tfrac{1}{2}\frac{\partial^2 e}{\partial X^2} X^2\sigma_X^2.$$

On the left hand side, the instantaneous mean is a function of wealth, W, consumption, Q, and decision variables, D. On the right-hand side, the first line includes investment in options; the second line includes expenditures on consumption, returns from production and returns from resource stocks; and the third line includes returns from production, forward and futures contracts.

Upper case letters denote quantities. The expected change in wealth depends upon wealth itself, W, consumption, Q, and the vector of decision variables, D. This vector includes investments in minimum price contracts, M, options on futures, N, and options on the environment, E. It includes production, Y, and resource stocks, S. It also includes hedging with production contracts, K, forward contracts, H, and futures contracts, F. An environmental variable, X, is also included, but it is not a decision variable and cannot be controlled.

Lower case letters denote prices. Prices include the value of minimum price contracts, m, the price of options on futures, n, the value of options on the environment, e, the consumption price, q, the commodity price, y, and the futures price, f. Prices expectations are modelled as log normal distributions with percentage changes denoted by δ, standard deviations by σ and correlation coefficients by ω.

In equation (3), investments include minimum price contracts, options on futures and options on the environment. Wealth, W, could be invested at a risk-free rate of δ_W, even if the risk-free rate equals zero. Minimum price contracts, M, options on futures, N, and options on the environment, E, are valued at prices m, n and e and attract returns above the risk free rate of $m(\delta_m - \delta_W)$, $n(\delta_n - \delta_W)$ and $e(\delta_e - \delta_W)$. The expected returns on investments, $m\delta_m$, $n\delta_n$ and $e\delta_e$, are defined by the partial differential equations shown in equation (3) and are functions of the commodity price, the futures price and the environmental variable. The environmental variable could be almost anything about which data is available and a prediction can be made. Examples include the Southern Oscillation Index, the size of the ozone hole, changes in world temperature, stream flow in a catchment, rainfall on a farm, kangaroos in the back paddock and sunspots.

Decisions are made about physical quantities, including consumption, production and resource stocks. Consumption, Q, is purchased at price q. Production, Y, has expected returns equal to the expected price at the end of the season, $y(1 + \delta_y)$, as modified by the covariance between the commodity price and production, $\sigma_y\sigma_y\omega_{yY}$. A negative covariance will reduce expected returns. Production costs $c(Y)$. In this simple model, resource stocks, S, are valued at commodity price for the economy with a return on investment above the risk-free rate of $y(\delta_y - \delta_W)$. Stocks will depreciate at the rate δ_S, with a depreciation cost of $y\delta_S$. Maintaining the resource stocks will cost s dollars per unit.

Some risk management decisions are made once in a year. These include production contracts and forward contracts. Production contracts, *K*, are not for production directly, but for the environmental variable, *X*. If the environmental variable is less than a contract level, $\overline{X}$, production contracts will be profitable. Production contracts are an artificial construct that may, but probably won't exist. They are included in the model as a way to design options on the environment. Forward contracts, *H*, are profitable if the expected price at the end of the decision interval, $y(1 + \delta_y)$, is less than the contract price, $\overline{h}$, at which commodities must be sold. Commodity futures are traded continuously and there is no fixed contract price that remains in force throughout the life of the contract. Trading in commodity futures will be profitable if the change in the futures price, δ_f, is negative.

Also from equation (A1) in the Appendix, the error terms for the change in wealth are:

$$\sigma(D)dZ = y(Y + S - H + (\partial m/\partial y)M)\sigma_y dz_y - f(F - (\partial n/\partial f)N)\sigma_f dz_f$$
$$+ yY\sigma_Y dZ_Y - (K - (\partial e/\partial X)E)X\sigma_X dZ_X .$$

The first line of this equation includes price risks and the second line includes quantity risks. Variances and co variances are found by squaring the errors.

$$\sigma(D)\Omega\sigma'(D) = y^2(Y + S - H + (\partial m/\partial y)M)^2\sigma_y^2 + f^2(F - (\partial n/\partial f)N)^2\sigma_f^2$$
$$+ y^2Y^2\sigma_Y^2 + (K - (\partial e/\partial X)E)^2X^2\sigma_X^2$$
$$- 2y(Y + S - H + (\partial m/\partial y)M)f(F - (\partial n/\partial f)N)\sigma_y\sigma_f\omega_{yf}$$
$$+ 2y(Y + S - H + (\partial m/\partial y)M)yY\sigma_y\sigma_Y\omega_{yY} - 2yY(K - (\partial e/\partial X)E)X\sigma_Y\sigma_X\omega_{YX} . \quad (4)$$

The first line includes variances for price risks; the second line includes variances for quantity risks; and the third and fourth lines contain co variances.

Optimal Decisions

More specific versions of the optimality conditions in equation (2) are derived using the mean in equation (3) and the variances in equation (4). Decisions about consumption and production will not be analysed. Decisions about resource stocks, production contracts, forward contracts, futures contracts, minimum price contracts, options on futures and options on the environment are:

Resource Stocks

$$y(\delta_y - \delta_W - \delta_S) - s - R\big[y^2(Y + S - H + (\partial m/\partial y)M)\sigma_y^2$$
$$- yf(F - (\partial n/\partial f)N)\sigma_y\sigma_f\omega_{yf} + y^2Y\sigma_y\sigma_Y\omega_{yY}\big] = 0. \quad (5)$$

Production Contracts

$$\overline{X} - X - R\big[(K - (\partial e/\partial X)E)X^2\sigma_X^2 - yYX\sigma_Y\sigma_X\omega_{YX}\big] = 0. \quad (6)$$

Forward Contracts

$$y(1 + \delta_y) - \overline{h} - R\big[y^2(Y + S - H + (\partial m/\partial y)M)\sigma_y^2$$
$$- yf(F - (\partial n/\partial f)N)\sigma_y\sigma_f\omega_{yf} + y^2Y\sigma_y\sigma_Y\omega_{yY}\big] = 0. \quad (7)$$

Futures Contracts

$$(8) \qquad f\delta_f - R\left[yf\left(Y + S - H + (\partial m/\partial y)M\right)\sigma_y\sigma_f\omega_{yf} - f^2\left(F - (\partial n/\partial f)N\right)\sigma_f^2\right] = 0.$$

Minimum Price Contracts

$$(9) \qquad m(\delta_m - \delta_W) - R\left[y^2\left(Y + S - H + (\partial m/\partial y)M\right)\sigma_y^2 - yf\left(F - (\partial n/\partial f)N\right)\sigma_y\sigma_f\omega_{yf} + y^2Y\sigma_y\sigma_Y\omega_{yY}\right](\partial m/\partial y) = 0.$$

Options on Futures

$$(10) \qquad n(\delta_n - \delta_W) - R\left[yf\left(Y + S - H + (\partial m/\partial y)M\right)\sigma_y\sigma_f\omega_{yf} - f^2\left(F - (\partial n/\partial f)N\right)\sigma_f^2\right](\partial n/\partial f) = 0.$$

Options on the Environment

$$(11) \qquad e(\delta_e - \delta_W) + R\left[\left(K - (\partial e/\partial X)E\right)X^2\sigma_X^2 - yYX\sigma_Y\sigma_X\omega_{YX}\right](\partial e/\partial X) = 0.$$

Production

$$(11) \qquad e(\delta_e - \delta_W) + R\left[\left(K - (\partial e/\partial X)E\right)X^2\sigma_X^2 - yYX\sigma_Y\sigma_X\omega_{YX}\right](\partial e/\partial X) = 0.$$

Although the optimality conditions seem complex, they are easy to interpret. Terms beginning with *R* are marginal risk premiums. If agents are risk neutral, the marginal risk premiums are zero and the optimality conditions collapse to simple profit maximising behaviour. If agents are averse to risk they will choose an optimal portfolio that balances profit and risk.

In the finance literature, lack of profitable arbitrage and continuous trading are assumed. The portfolio becomes riskless and the prices of options are independent of risk preferences (Merton, p. 281, Hull, p. 539). For a society making decisions about resources and the environment there is no arbitrage, there may be no trading at all, the portfolio is risky and agents may be risk-averse. Never-the-less, option pricing formulas can be derived from optimal decisions. These formulas will be illustrated for the parameters in Table 1.

Table 1: Parameter Values.

y	160	δ_W	0.05	σ_y	0.20
f	155	δ_y	0.10	σ_f	0.30
s	10	δ_f	0.11	σ_Y	0.30
X	1	δ_S	0.03	σ_S	0.15
$\bar{X}$	0.90			σ_X	0.30
$\bar{h}$	156				

Options on Futures

Options can be priced without assumptions about how markets work. To demonstrate this, consider how a single agent would price options on futures. If the agent takes out a futures contract there will be a loss if the futures price rises. An option on futures lets the agent avoid losses from a rising futures price, but gain from a falling futures price. A call option on futures gives the agent the right but not the obligation to buy a futures contract at an agreed exercise price.

Combining the optimality conditions for futures contracts and options on futures, equations (8) and (10), shows that futures and options on futures are equivalent hedges against price risk.

$$f\delta_f = \frac{n(\delta_n - \delta_W)}{\partial n / \partial f}.$$

The marginal risk premiums are eliminated and the expected loss or gain from holding a futures contract equals the expected capital gains from owning options on futures. Options incorporate the same information as futures prices and hedge against the same risks.

Substituting for the expected change in the price of an option on futures, $n\delta_n$, from equation (3) gives a partial differential equation defining how the price of an option evolves beginning from the current time. This is combined with boundary conditions that an option must satisfy at the time of maturity.

$$\frac{\partial n}{\partial t} + \tfrac{1}{2}\frac{\partial^2 n}{\partial f^2} f^2 \sigma_f^2 - n\delta_W = 0;$$

$$n(\tau, f) = \begin{cases} f - \hat{f}; & f > \hat{f} \\ 0; & f \le \hat{f}. \end{cases}$$

The boundary conditions are for a European call option that can be exercised only at the time of maturity, τ, but not before. If the futures price, f, exceeds exercise price, $\hat{f}$, the option is "in the money" and will be exercised; otherwise it is worthless. With the boundary conditions, the solution becomes a cumulative probability, in this case the Black-Scholes formula for pricing options on futures (Merton, p. 347, Hull, p. 277).

Figure 1 shows the prices of a call option on futures as a function of the futures price and time. The exercise price is set to equal the agent's expected futures price, $f(1 + \delta_f)$, or \$172 / tonne and the options prices are the amounts the agent would pay to avoid the upside risk of a higher than expected futures price. In Table 1, the current futures price is \$155 / tonne. Suppose the option is 52 weeks from maturity at time zero. In this case, the agent would pay \$11.55 / tonne for the option. The price changes as the option matures. If the futures price at maturity of the option is less than \$172 / tonne, the option will be worthless. The agent will throw away the option and buy futures contracts for less than the exercise price. Otherwise, the option at maturity is valued as the difference between the futures price and the exercise price. The agent will exercise the option to buy futures contracts at the exercise price.

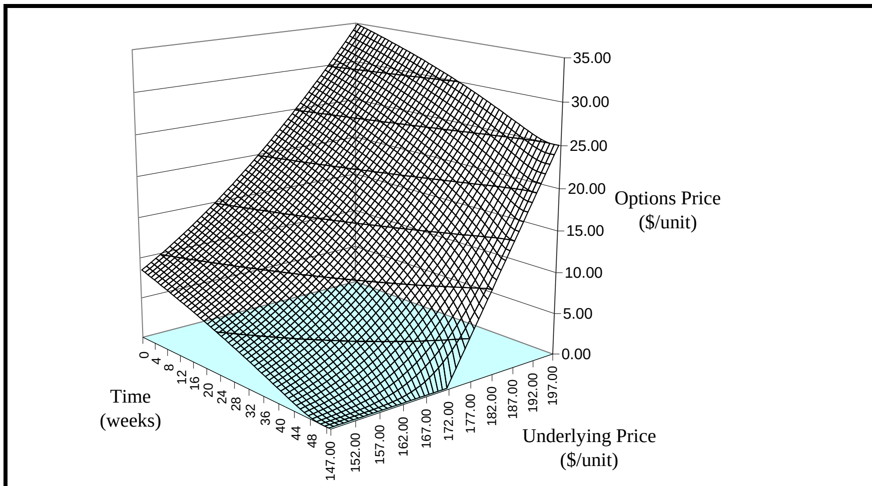

Figure 1: Prices of a European Call Option on Futures.

Options on Forward Contracts

In much of the literature on hedging, forward contracts and futures contracts are treated as equivalent. However, there is a distinction. Forward contracts have a fixed contract price and the agent gains or loses depending upon whether the commodity price is below or above the contract price at the time of maturity. Although futures contracts "locks in" a price, there is no fixed contract price and the agent gains or loses depending upon whether the futures price falls or rises. This distinction may not be important for choosing between forward contracts or futures contracts, but options on forward contracts are less valuable than options on futures because forward contracts are less flexible.

To show this, the optimality conditions for forward contracts and minimum price contracts in equations (7) and (9) are combined.

$$\bar{h} - y\left(1 + \delta_y\right) = \frac{m\left(\delta_m - \delta_W\right)}{-\partial m / \partial y}.$$

The expected return from holding a forward contract equals the expected capital gains from owning a minimum price contract. Substituting for the expected change in the value of a minimum price contract, $m\delta_m$, from equation (3) gives a partial differential equation which is solved subject to boundary conditions.

$$\frac{\partial m}{\partial t} + \frac{\partial m}{\partial y}\left(\bar{h} - y\right) + \tfrac{1}{2}\frac{\partial^2 m}{\partial y^2} y^2 \sigma_y^2 - m\delta_W = 0$$

$$m(\tau, y) = \begin{cases} \hat{y} - y; & \hat{y} > y \\ 0; & \hat{y} \le y. \end{cases}$$

Minimum price contracts have an additional term, $\left(\partial m/\partial y\right)\left(\bar{h} - y\right)$, in the option pricing formula. This term compares the fixed contract price to the changing commodity price. This differs from options on futures for which the "contract price" is simply the current futures price. The equivalent term would be $\left(\partial n/\partial f\right)\left(f - f\right)$ which is always zero.

The boundary conditions are for a European put option. At the time of maturity, if the exercise price, $\hat{y}$, exceeds the commodity price, y, the minimum price contract is "in the money" and will be exercised. The option pricing formula is a non linear differential equation and no analytical solution is known. Therefore, the solution was calculated numerically using the Crank-Nicholson method for finite differences (Hill, p. 378, Burden and Faires, p. 692). The software is available upon request.

Figure 2 shows the values of a minimum price contract as a function of the commodity price and time. The exercise price is set to \$156 / tonne which equals to the contract price for a forward contract in Table 1. Therefore, the minimum price contract avoids the downside of a commodity price that is lower than the contract price but retains the upside of a commodity price that is higher. The value of the minimum price contract is the amount the agent would pay to avoid the downside and retain the upside. From Table 1, the current commodity price is \$160 / tonne. For this commodity price a year from maturity, the minimum price contract has a value of \$7.15 / tonne. The agent's expected commodity price, $y(1 + \delta_y)$, is \$176 / tonne. Even though this is \$20 / tonne above the exercise price, the agent is still willing to pay to avoid the downside.

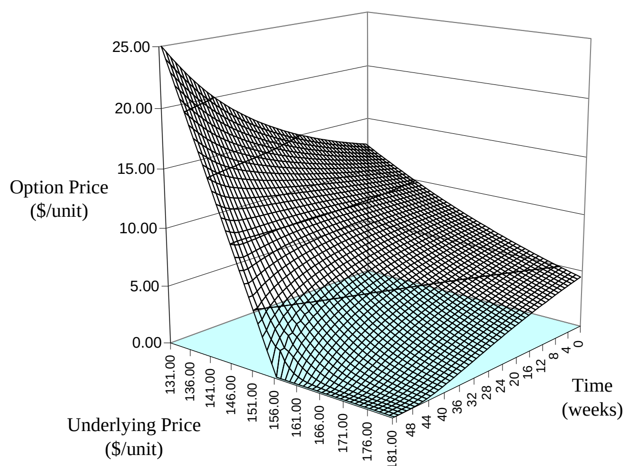

Figure 2: Values of a Minimum Price Contract.

Petzel first proposed and Bardsley and Cashin first applied the Black-Scholes formula for options on futures to the evaluation of the benefits from government programs. The Black-Scholes formula or its equivalent is now used to value many things from crop insurance (Just, Calvin and Quiggen; Mahul; Stokes, Nayda and English) to old growth forests (Conrad). Valuing a government program as if it were an option of futures is equivalent to assuming that the support guaranteed by the government varies continuously. Valuing an old growth forest as if it were an option on futures is equivalent to assuming that the government continuously changes its mind about how much forest to save. Figure 3 shows the prices of a put option on futures for the same exercise price of \$156 / tonne. At a futures price of \$160 / tonne one year from maturity, the price of a put option on futures is \$10.16 / tonne. This overvalues the benefits to the agent by about \$3 / tonne. Comparing Figure 3 with Figure 2 shows that options on futures are priced higher than minimum price contracts.

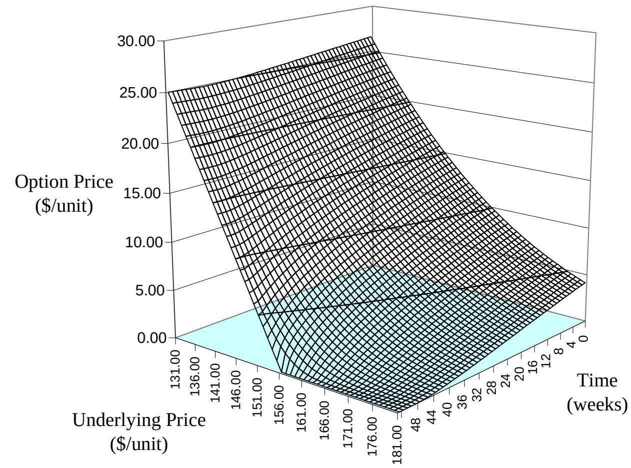

Figure 3: Prices of a Put Option on Futures.

Options on Resource Stocks

Originally, the Black-Scholes formula was derived for options on financial stocks. This can be compared with an option on resource stocks by combining equations (5) and (9).

$$y(\delta_y - \delta_W - \delta_S) - s = \frac{m(\delta_m - \delta_W)}{\partial m / \partial y}.$$

The return above costs for holding resource stocks equals the capital gains from a minimum price contract. Substituting for the expected change in the minimum price contract from equation (3) gives the option pricing formula.

$$\frac{\partial m}{\partial t} + \frac{\partial m}{\partial y}\left(y(\delta_W + \delta_S) + s\right) + \tfrac{1}{2}\frac{\partial^2 m}{\partial y^2} y^2 \sigma_y^2 - m\delta_W = 0$$

$$m(\tau, y) = \begin{cases} y - \hat{y}; & y > \hat{y} \\ 0; & y \le \hat{y}. \end{cases}$$

The Black-Scholes formula for options on stocks also has a term such as $(\partial m/\partial y) y(\delta_W + \delta_S)$ for the opportunity cost of holding stocks. Resource stocks must also be stored and maintained with an additional term $(\partial m/\partial y)s$. The prices of a European call option on resource stocks for an exercise price of $156 / tonne are shown in Figure 4.

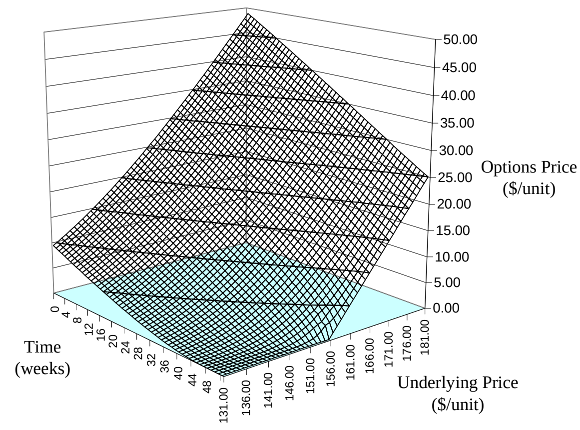

Figure 4: Prices of a Call Option on Resource Stocks.

The prices of a European call option on financial stocks are shown in Figure 5.

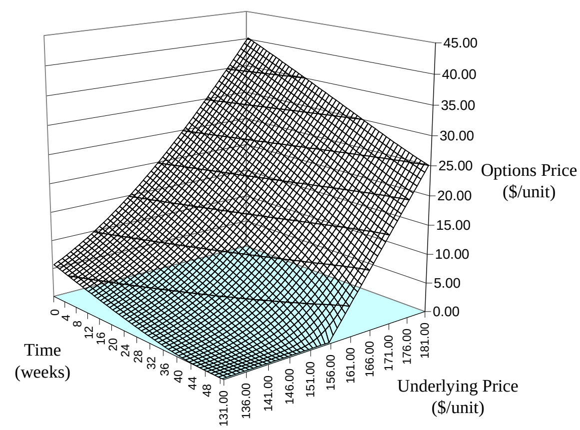

Figure 5: Prices of a Call Option on Financial Stocks.

In this case, call options on financial stocks are less valuable than call options on resource stocks. Although not shown in a figure, put options on financial stocks are more valuable than put options on resource stocks. This can be explained by the cost of maintaining the stocks. With a call option, the agent promises to buy resource stocks and doesn't have to pay the maintenance costs. With a put option, the agent promises to sell resource stocks but does have to pay the maintenance costs.

Options on the Environment

Options on the environment are not a new idea. Rainfall insurance and hail insurance for crops are examples (Bardsley, Abbey and Davenport; Quiggin). Combining equations (6) and (11) shows that options can be written on almost any environmental variable.

$$\left(\overline{X} - X\right) = \frac{e\left(\delta_e - \delta_W\right)}{-\partial e/\partial X}.$$

The expected return from a production contract equals the expected capital gains from investing in an option. Whether or not production contracts actually exist does not matter for designing an option pricing formula. Production contracts and options on the environment are equivalent methods of managing environmental risk and so long as one or the other exists, the risk can be hedged. Substituting for the expected change, $e\delta_e$, from equation (3) gives a partial differential equation for the price of options on the environment.

$$\frac{\partial e}{\partial t}+\frac{\partial e}{\partial X}\left(\overline{X}-X\right)+\tfrac{1}{2}\frac{\partial^2 e}{\partial X^2}X^2\sigma_X^2-e\delta_W=0$$

$$e(\tau,y)=\begin{cases}X-\hat{X}; & X>\hat{X}\\ 0; & X\le\hat{X}.\end{cases}$$

Like minimum price contracts, options on the environment are defined by a non linear differential equation which must be solved numerically. The boundary conditions are for a European call option. At the time of maturity, if the environmental variable, X, exceeds the exercise price, $\hat{X}$, the option on the environment is "in the money" and will be exercised. Solving the differential equation subject to the boundary conditions gives the cumulative probability that the environmental variable will exceed the exercise price. This cumulative probability is the actuarially fair price of an option on the environment.

Suppose the option is a water right to divert stream-flow from a river with multiple users. In a wet year, the option is "out of the money" because stream flow is high. In a dry year, the option can be exercised to maintain the agent's diversions while others must cut back. In Table 1, the environmental variable is scaled until it has a mean of 1 and the agent is expecting an average year. The contract amount of 0.9 can be interpreted as 90% of average. The environmental variable is also multiplied by $1 to give units of dollars. Figure 6, gives the prices of rights to divert water in a year with less than 90% of average stream flow. If, at the beginning of the year, an average year is expected, the price of the water right is $0.09.

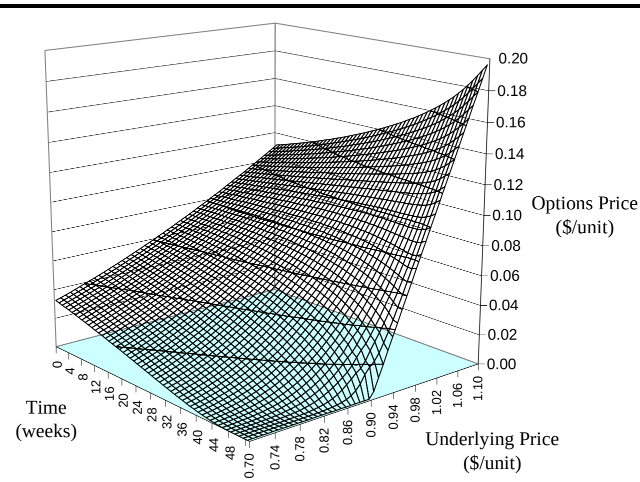

Figure 6: Prices of a Call Option on the Environment.

Figure 7 shows the prices that would be predicted if water rights were treated as if they were call options on futures.

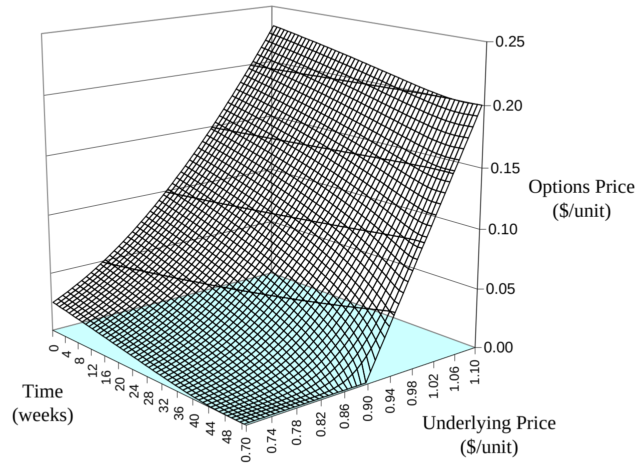

Figure 7: Prices of a Call Option on Futures.

If an average year is expected, the price of an options of futures would predict the value of the water right to be $0.17.

As with all options, prices of water rights are independent of the agent's risk aversion. In addition prices are independent of the correlation between stream flow and production. However, risk aversion and correlations affect the quantity of water rights an agent will buy. This quantity can be found by solving the optimality conditions in equations (5) through (11) as a system. Solving the system several times for all users of the river will give the demands and supplies of water rights. These could be aggregated into a model of a market for water rights. A market for water rights is a market for trading in options.

Concluding Remarks

Options can be written on any environmental stock or flow that is continuously measured and has enough historical data to estimate a stochastic differential equation. However, the Black-Scholes option pricing formula does not apply. A new option pricing formula is derived which considers the provisions of the contract that underlies the option. The price of an option is a non linear function of the environmental variable. The contract provisions will alter the degree of non linearity. A simple example is a forward contract with a fixed contract price over the life of the contract. This inflexibility lowers the value of options on forward contracts compared with options on futures. Another example is the value of a water right. The contract level must be agreed when a water right is purchased. Because of this inflexibility, the value of a water right is much less than predicted by the Black-Scholes formula or any of the usual methods of calculating cumulative probabilities.

However, the new option pricing formula is far from a complete model of options on the environment. For example, exploration for minerals is the creation of an option that may or may not be exercised in the future as the prices of minerals change. Creating the option is a complex production problem and not a simple investment as modelled here. Further, the price of minerals is an endogenous variable in the system. Perhaps the major area for future work is options on environmental variables that are endogenous to the system. Society may want to invest in options on old growth forests, but the value of those same forests depends upon how much forest is preserved. Most of the large environmental questions, such as greenhouse gas emissions, overpopulation, and available farmland, are in this category.

Pricing these options is not an impossible task. Notice that equation (1) for the formulation of a stochastic dynamic program is an option pricing formula complete with boundary conditions. Expected social welfare is the value of a call option on the wealth as affected by decisions over the planning horizon. As one example, the precautionary principle for greenhouse gases can be put into practice by formulating a suitable model of society's wealth during global warming and solving equation (1). The result will be how much society is willing to invest to avoid the risks of global warming.

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Appendix: Stochastic Wealth

To derive the stochastic differential equation for wealth, begin with society's wealth as the sum of all assets and liabilities.

$$W = bB + aA.$$

Wealth, W, consists of risk-free bonds, B, with price b, and a vector of risky assets, A, with prices a. Negative quantities of B and A are liabilities. The change in wealth is found by Ito stochastic differentiation.

$$dW = dbB + daA + (b + db)dB + (a + da)dA.$$

The first and second terms on the right hand side are capital gains or losses on beginning inventories of bonds and risky assets. The third and fourth terms are acquisitions and depreciation valued at ending prices. Assume that the quantity of bonds can change over time by acquisitions and the quantities of risky assets can change by acquisitions and physical degradation.

$$dB = \overline{B}dt;$$

$$dA = (\overline{A} - A\delta_A)dt - A\sigma_A dZ_A.$$

Acquisition of bonds is $\overline{B}$ and acquisition of risky assets is $\overline{A}$. Risky assets are expected degrade by $A\delta_A$, with error $A\sigma_A dZ_A$. Substituting these changes in bonds and risky assets gives another expression for the change in wealth.

$$dW = dbB + daA + (b + db)\overline{B}dt + (a + da)\left((\overline{A} - \delta_A A)dt - A\sigma_A dZ_A\right).$$

By definition, the acquisition of bonds and risky assets must be financed by profits generated by the economy.

$$\pi dt + d\pi = (b + db)\overline{B}dt + (a + da)\overline{A}dt.$$

The left hand side of this equation is the stochastic profit at the end of each decision interval. Profit substitutes for acquisitions in the change in wealth.

$$dW = db[W - aA]/b + daA - a\delta_A Adt - aA\sigma_A dZ_A + \pi dt + d\pi.$$

Bonds have been eliminated by rearranging the equation for wealth to solve for B and substituting into the change in wealth. The terms $da\delta_A Adt$ and $daA\sigma_A dZ_A$ have been eliminated as well because $dadt$ equals zero by the rules of stochastic differentiation and

$dadZ_A$ equals zero because the covariance between the prices of assets and their physical degradation is assumed to be zero.

The vector of risky assets can include both natural resource and financial assets. Assume that risky assets include stocks of exhaustible resources, minimum price contracts, options on futures and options on an environmental variable. Of these, only resource stocks will depreciate. Assume this depreciation is not stochastic.

$$dW = db[W - yS - mM - nN - eE]/b + dyS - y\delta_S Sdt + dmM + dnN + deE + \pi dt + d\pi.$$

Resource stocks, S, are valued at the price for all commodities in the economy, y. Minimum price contracts, M, options on futures, N, and options on the environment, E, have market prices m, n and e.

Society benefits from production, plus any gains from forward and futures contracts. Subtracting consumption expenditures, costs of production and costs of maintaining resource stocks gives profit.

$$\pi dt = \left[-qQ + yY - c(Y) - sS + \left(\overline{X} - X\right)K + \left(\overline{h} - y\right)H + \left(\overline{f} - f\right)F\right]dt.$$

Consumption goods, Q, are purchased at price q. Production, Y, will sell for price y, and will cost $c(Y)$ to produce. Resource stocks, S, will cost s per unit to maintain. K is a forward contract written on an environmental variable, X, and will add to profit if the environmental variable is less than a contract level, $\overline{X}$. Forward contracts, H, add to profit if the contract price, $\overline{h}$, exceeds the price society would receive for selling the commodity. Commodity futures contracts, F, add to profit if the contract prices, $\overline{f}$ exceeds the futures prices, f.

Profit is stochastic because the commodity price, production, the environmental variable and the commodity futures price are stochastic.

$$d\pi = dyY + (y + dy)dY - dX\,K - dyH - dfF.$$

Substituting in profit and its stochastic derivative gives another expression for the change in wealth.

$$\begin{aligned} dW = {} & db[W - yS - mM - nN - eE]/b + dyS - y\delta_S Sdt + dmM + dnN + deE \\ & + \left[-qQ + yY - c(Y) - sS + \left(\overline{X} - X\right)K + \left(\overline{h} - y\right)H\right]dt \\ & + dyY + (y + dy)dY - dX\,K - dyH - dfF. \end{aligned}$$

For commodity futures, the contract price, $\overline{f}$, is simply the futures price, f, at the time the contract was taken out. Hence, the contract price and the beginning futures prices cancel from the change in wealth.

Decision-makers in society must form expectations about prices, production and the environmental variable. Assume log-normal distributions.

$$db = b\delta_W dt;$$

$$dy = y\delta_y dt + y\sigma_y dz_y;$$

$$df = f\delta_f dt + f\sigma_f dz_f.$$

$$dY = Y\sigma_Y dZ_Y;$$

$$dX = X\sigma_X dZ_X.$$

Substituting in the differential equations for expectations and rearranging gives yet another expression for the change in wealth.

$$
\begin{aligned}
dW = {} & \delta_W\left[W - mM - nN - eE\right]dt + dmM + dnN + deE \\
& + [-qQ + \left(y\left(1+\delta_y\right) + \sigma_y\sigma_Y\omega_{yY}\right)Y - c(Y) + \left(y\left(\delta_y - \delta_W - \delta_S\right) - s\right)S \\
& + \left(\overline{X} - X\right)K + \left(\overline{h} - y\left(1+\delta_y\right)\right)H - f\delta_f F\,]dt \\
& + y\left(Y + S - H\right)\sigma_y dz_y - fF\sigma_f dz_f + yY\sigma_Y dZ_Y - KX\sigma_X dZ_X .
\end{aligned}
$$

For society, the commodity price and production will be negatively correlated with covariance *dydY* equal to $\sigma_y\sigma_Y\omega_{yY}$, where ω_{yY} is the correlation coefficient.

Minimum price contracts, options on futures and options on the environment are assets which must be purchased at prices *m*, *n* and *e*. The value of a minimum price contract is a function of the commodity price, *y*, and the time to maturity, τ - *t*, where τ is a maturity date in the future. In other words, the value of a minimum price contract is the function, $m(t, y)$. Its differential equation is found by stochastic differentiation.

$$
dm = \frac{\partial m}{\partial t} + \frac{\partial m}{\partial y}dy + \tfrac{1}{2}\frac{\partial^2 m}{\partial p^2}dy^2 .
$$

Substituting in the differential equation for the commodity price gives the final result.

$$
dm = \left[\frac{\partial m}{\partial t} + \frac{\partial m}{\partial y}y\delta_y + \tfrac{1}{2}\frac{\partial^2 m}{\partial y^2}y^2\sigma_y^2\right]dt + \frac{\partial m}{\partial y}y\sigma_y dz_y .
$$

Because *m* is nonlinear, its expected change, in brackets, depends upon the expected change in the price, $y\delta_y$, as well as the variance of the price, $y^2\sigma_y^2$. Its error term is a transformation of the error term for the price.

Similarly, options on futures are defined by a function of the futures price, $n(t, f)$, which evolves according to a stochastic differential equation.

$$
dn = \left[\frac{\partial n}{\partial t} + \frac{\partial n}{\partial f}f\delta_f + \tfrac{1}{2}\frac{\partial^2 n}{\partial f^2}f^2\sigma_f^2\right]dt + \frac{\partial n}{\partial f}f\sigma_f dz_f .
$$

Options on the environment are an asset with a value, $e(t, X)$, which depends upon the environmental variable and also has a differential equation.

$$
de = \left[\frac{\partial e}{\partial t} + \tfrac{1}{2}\frac{\partial^2 e}{\partial X^2}X^2\sigma_X^2\right]dt + \frac{\partial e}{\partial X}X\sigma_X dZ_X .
$$

Finally, substituting in these expectations for minimum price contracts, options on futures and options on the environment gives the stochastic differential equation for wealth.

$$
\begin{aligned}
(A1) \qquad dW = {} & [\delta_W W + m\left(\delta_m - \delta_W\right)M + n\left(\delta_n - \delta_W\right)N + e\left(\delta_e - \delta_W\right)E \\
& - qQ + \left(y\left(1+\delta_y\right) + \sigma_y\sigma_Y\omega_{yY}\right)Y - c(Y) + \left(y\left(\delta_y - \delta_W - \delta_S\right) - s\right)S \\
& + \left(\overline{X} - X\right)K + \left(\overline{h} - y\left(1+\delta_y\right)\right)H - f\delta_f F\,]dt \\
& + y\left(Y + S - H + \left(\partial m/\partial y\right)M\right)\sigma_y dz_y \\
& - f\left(F - \left(\partial n/\partial f\right)N\right)\sigma_f dz_f + yY\sigma_Y dZ_Y - \left(K - \left(\partial e/\partial X\right)E\right)X\sigma_X dZ_X .
\end{aligned}
$$

$$
\begin{aligned}
\text{where:} \qquad & m\delta_m = \frac{\partial m}{\partial t} + \frac{\partial m}{\partial y}y\delta_y + \tfrac{1}{2}\frac{\partial^2 m}{\partial y^2}y^2\sigma_y^2; \\
& n\delta_n = \frac{\partial n}{\partial t} + \frac{\partial n}{\partial f}f\delta_f + \tfrac{1}{2}\frac{\partial^2 n}{\partial f^2}f^2\sigma_f^2; \\
& e\delta_e = \frac{\partial e}{\partial t} + \tfrac{1}{2}\frac{\partial^2 e}{\partial X^2}X^2\sigma_X^2 .
\end{aligned}
$$

To make the presentation clearer, expected changes in the values of minimum price contracts, options on futures and options on the environment have been abbreviated as $m\delta_m$, $n\delta_n$ and $e\delta_e$.