INPUT CONTROL AND INFORMATION ASYMMETRY.

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ABSTRACT. In a production process with a labor and a non-labor input, we show that the principal's profits increase when she controls the non-labor input. Furthermore, output increases since the principal can allocate capital to help mitigate her information costs. However, this mitigation of information costs distorts the capital-labor ratio away from its production-efficient level. This distortion is socially costly. Overall, the distortion may dominate the gains realized through the increase in output and the reduction of information rents. Our result differs from the classic finding of Averch and Johnson, who found that cost-plus pricing induces overinvestment in capital equipment that is socially costly. Unlike their analysis, the problem we consider contains two offsetting social costs: asymmetric information regarding agent ability, and the production distortion in the input ratio. Neither one of the costs necessarily dominates the other, so that the social implication of input control by the principal is not a priori determinate.

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1. Introduction

Non-labor inputs often play an important role in principal-agent relationships. A principal may supply an agent with necessary non-labor inputs, or may specify contractually the inputs that the agent must use. For example, construction contracts may specify building materials. Military procurement contracts usually specify component materials. In agriculture, production contracts between farmers and processors often specify allowable fertilizers, seedstock, and other production inputs. There are a number of reasons a principal may seek to control inputs. Input quality may affect output quality, and be cheaper or easier to measure. Agents' input choices may be subject to a moral hazard problem; by specifying the input the principal may entirely avoid associated costs. (Of course, there are alternative ways of addressing this problem.) We focus on another information-driven motivation for input control by the principal: by controlling non-labor inputs, the principal can reduce the information rents she incurs due to adverse selection.

When agents' effectiveness in production (their ability) differs and is unknown to the principal, she must design an incentive-compatible contract that will induce agents to reveal their true types. When there are two possible agent types, the standard principal-agent solution involves offering a low ability agent a contract that pays him his reservation utility and distorts his production below his full information production level (due to the need to induce truthful revelation by high ability agents), and offering a high ability agent a contract that pays him his costs of production plus the returns he would obtain from choosing the low ability agent's contract (his information rents) and requires him to produce his full information output level. We show that by specifying non-labor inputs the principal can always lower the information rents for a given pair of ability-specific output levels, and that the principal's optimal contract menu will always result in higher profits when she controls inputs relative to when she does not control inputs.

Our conceptualization of input specification by the principal can be viewed as encompassing two cases: one, where the principal simply provides the input(s) in question to the agent, and two, where the principal specifies inputs in the contract with the agent and the (non-labor) inputs actually used by the agent are verifiable by a third party. There are other considerations regarding non-labor inputs in a principal-agent relationship. For example, the principal may be less informed regarding the precise nature of the production function than the agent is. This asymmetric information will impose a cost of input specification on the principal, since she may incorrectly choose the input. Similarly, an agent's choice of inputs may provide information regarding his ability. Here, we maintain that the principal and agent are equally informed about the production function, so that the only information asymmetry is that the principal does not know the agent's type.

While input control always results in larger profits for the principal under her optimal contract menu relative to her optimal contract menu without input control, the consequences for society as a whole are less clear. We develop an experiment in which an arbitrarily small amount of asymmetric information is introduced into the principal's maximization problem. We assume that with probability $1-\gamma$ a high ability agent's type is revealed to the principal, while the low ability agent's type is revealed with probability 1. In Proposition Two, we demonstrate that if γ is small enough and the "ability gap" between the two types in production is not too large, then the optimal contract for the low ability agent includes a higher level of output when non-labor inputs are specified than when inputs aren't specified. This gain in output clearly contributes to the principal's revenues. However, there is an offsetting distortion in the labor-input ratio which may reduce the total social surplus generated. Proposition 3 formalizes this observation. When the elasticity of substitution between labor and the non-labor input is too low, then the principal's ability to specify inputs will be more socially costly, and total surplus will be higher when the principal can not specify inputs.

Proposition 1, which states that input control always increases profits for the principal, can be viewed as a relatively straightforward application of the LeChatelier Principle. That is, the principal is better off when she can choose the input-labor mix for each contract output level than when the agent chooses. Here, the strict inequality is due to the fact that the agent considers only neoclassical production costs, while the principal considers information costs. Similarly, the principal's control of inputs under the optimal restricted contract is an additional constraint facing the *agent* relative to his maximization problem under the basic contract. Given the differing interests of the principal and the agent, the LeChatelier Principle can not be used to rank total social surplus under the two contracts.

Literature Review

Our analysis is related to the literature initiated by Averch and Johnson (1962) on the effect of cost-plus pricing regulation on firm behavior. In their seminal paper, public utilities regulated under cost-plus pricing have an incentive to overinvest in capital, since it will increase the base for their rate of return. This distorts the capital-labor ratio from its first-best level. In their case, the distortion is not a response to a market failure, such as the asymmetric information case we examine. Since the distortion moves the utility away from the most efficient solution, it always reduces social welfare. In contrast, we find that a distorted capital-labor ratio may be associated with a higher level of social surplus than would a non-distorted ratio, in the presence of asymmetric information. While formal differences exist between our analyses (most importantly their assumption of a natural monopoly while we assume decreasing returns to scale and a constant price), our findings suggest that in some cases an inequality between the ratio of marginal revenue products and the wage ratio may be associated with efficient, rather than inefficient, regulation.

The effect of the principal's control of non-labor inputs on information rents under adverse selection has largely been ignored in the agency theory literature. Implicitly, the literature has assumed that there is no substitutability between labor and inputs that may be controlled by the principal. Perhaps the closest line of research focuses on the principal's choice between monitoring output and monitoring agent effort when both are feasible but costly. Maskin and Riley (1985) find that the principal prefers to monitor output when the agent is the residual claimant, since high ability agents exert more effort when their marginal incentives are not distorted. Khalil and Lawarree (1995) find that the principal will prefer to monitor labor when she is the residual claimant and output when the agent is the residual claimant, provided that input and output monitoring are feasible and equally costly to the principal.

2. The model

We begin with a standard principal-agent model. The agent may be one of two types, where types affect the productivity of capital and labor in the production process. While the effect of the two types on production and the probability that the agent is of a given type are common information, the agent's actual ability is unknown to the principal. The principal wishes to maximize her profits from production, which depend on the agent's ability. In order to get the agent to reveal his true type, she must provide him with the correct incentives to do so in the menu of contracts she offers the agent. We assume, as is the convention in models of this type, that the principal can not observe labor supplied by the agent. Further, we assume that the principal can not observe *capital* when it is supplied by the agent, nor can capital supplied by the agent be verified by a third party. She can observe capital if she chooses to control capital by supplying it herself. We assume that there is a single quality of capital, so that only the quantity used is relevant to production. We further assume that both parties are perfectly informed about the production function; the only asymmetric information is that the principal does not know the agent's type. In this section we formally develop the components of our analysis, and examine the principal's problem when she can and can not specify capital.

<u>The Production Function</u>: Production depends on capital, labor and the labor supplier's ability level, or type. There are two types, "worse" and "better". $\theta \in \{\theta^w, \theta^b\}$ is the agent's true type. $\Pr(\theta), \theta \in \{\theta^w, \theta^b\}$ is the probability that an agent's type is θ. $\theta' \in \{\theta^w, \theta^b\}$ is the agent's announced type.

We make a number of assumptions regarding the production function f. For each θ, f is strictly quasi-concave in (ℓ, k), i.e., $f_{\ell\ell}, f_{kk} < 0$ and $f_{\ell\ell} f_{kk} > f_{\ell k}^2$. For $f(\ell, k, \theta)$ that the marginal products of labor, capital and ability are all positive ($f_\ell, f_k, f_\theta > 0$), and that an increase in ability positively affects the marginal products of labor and capital ($f_{\ell\theta} > 0$, and $f_{k\theta} > 0$). The following conditions on f are satisfied: 1.) for each θ, f is homogeneous of degree less than unity in ℓ and k, and 2.) θ is "technologically neutral" in the sense that for each θ, θ', $\frac{f_\ell(\ell,k,\theta)}{f_k(\ell,k,\theta)} = \frac{f_\ell(\ell,k,\theta')}{f_k(\ell,k,\theta')}$ for fixed ℓ and k. These assumptions ensure that isoquants for different ability levels are parallel. If the isoquants were allowed to cross, the analysis would become much more complex, with little insight added.

This restriction is functionally similar to the single-crossing property that is often imposed in single input principal-agent problems.

Agent's Utility Function: The agent will receive a lump-sum transfer payment from the principal and in return will deliver a specified level of output, contributing labor and capital. The agent's outside alternative is to provide his labor at the given wage-rate w per unit labor supplied. The wage rate, w, exactly compensates for the agent's constant marginal disutility of labor. His reservation utility when he does not supply labor is zero. In order to induce the agent to participate at a labor level ℓ and capital level k, the principal's transfer payment must at least cover the agent's cost, $w\ell + rk$.

Input levels: Whether the principal or the agent chooses the level of capital, we will write labor as a function of output and capital, i.e., $\ell(q, k, \theta)$ is defined implicitly by the condition:

$$q \quad = \quad f(\ell(q, k, \theta), k, \theta). \tag{1}$$

If the agent accepts a basic contract (defined below as a contract where the agent chooses k), he will solve the following one-variable optimization problem in k: $\min_k w\ell(q, k, \theta) + rk$. The first order condition for this problem is

$$0 \quad = \quad w\ell_k(q, k, \theta) \quad + \quad r, \tag{2}$$

where $\ell_k(q, k, \theta) \quad = \quad -\frac{f_k(\ell(q,k,\theta),k,\theta)}{f_\ell(\ell(q,k,\theta),k,\theta)}$. Let $\tilde{k}(q, \theta)$ denote the solution to (2) and let $\tilde{\ell}(q, \theta) = \ell\Big(q(\theta), \tilde{k}(q(\theta), \theta), \theta\Big)$. We will refer to the input vector $\Big(\tilde{k}(q, \theta), \tilde{\ell}(q, \theta)\Big)$ as the *neoclassical input mix for* q. Note that under our assumptions on f (strict concavity), this vector is uniquely defined. Let $\tilde{C}(q, \theta)$ denote the type θ agent's *production cost* of delivering the output level q with the neoclassical input mix:

$$\tilde{C}(q, \theta) \quad = \quad w\tilde{\ell}(q, k(q, \theta), \theta) \quad + \quad r\tilde{k}(q, \theta) \tag{3}$$

Similarly, let $\bar{C}(q, k, \theta)$ denote the type θ agent's production cost of delivering the output level q with capital level k:

$$\bar{C}(q, k, \theta) \quad = \quad w\ell(q, k, \theta) \quad + \quad rk \tag{4}$$

For future reference, note that by definition of $\tilde{k}(q,\theta)$

$$\frac{\partial \bar{C}(q,\tilde{k}(q,\theta),\theta)}{\partial k} = 0, \quad \text{for all } q \text{ and all } \theta. \tag{5}$$

<u>Contracts</u>: A *basic contract* is a mapping from types to output levels and transfers, $\theta \rightarrowtail (\tilde{\boldsymbol{q}},\tilde{\boldsymbol{t}}) = (\tilde{q}(\theta),\tilde{t}(\theta))$. We will sometimes write $(\tilde{\boldsymbol{q}},\tilde{\boldsymbol{t}})$ as $\Big((\tilde{q}^w,\tilde{t}^w),(\tilde{q}^b,\tilde{t}^b)\Big)$. A *restricted contract* is a mapping from types to output levels, capital levels and transfers. We will write $(\bar{\boldsymbol{q}},\bar{\boldsymbol{k}},\bar{\boldsymbol{t}})$ either as $(\bar{q}(\theta),\bar{k}(\theta),\bar{t}(\theta))$ or as $\Big((\bar{q}^w,\bar{k}^w,\bar{t}^w),(\bar{q}^b,\bar{k}^b,\bar{t}^b)\Big)$.

Our model has the standard property that in any optimal contract, the difference between the transfer offered to the lower ability agent and the agent's production cost of delivering the designated output level must just equal the agent's reservation utility, which in our case is zero. That is, for a basic contract $(\tilde{\boldsymbol{q}},\tilde{\boldsymbol{t}})$,

$$\tilde{t}^w = \tilde{C}(\tilde{q}^w,\theta^w) \tag{6-$\tilde{t}^w$}$$

while for a restricted contract:

$$\bar{t}^w = \bar{C}(\bar{q}^w,\bar{k}^w,\theta^w). \tag{7-$\bar{t}^w$}$$

On the other hand, the transfer offered to the more efficient agent includes a premium, referred to as his *information rent*. In an optimal basic contract $(\tilde{\boldsymbol{q}},\tilde{\boldsymbol{t}})$, this premium $\big(\tilde{t}^b - \tilde{C}(\tilde{q}^b,\theta^b)\big)$ must be just sufficient to offset the utility, $\big(\tilde{t}^w - \tilde{C}(\tilde{q}^w,\theta^b)\big)$, that the more efficient agent would derive by adopting the low agent's contract. It follows from (6-$\tilde{t}^w$) that:

$$\tilde{t}^b = \tilde{C}(\tilde{q}^b,\theta^b) + \big(\tilde{C}(\tilde{q}^w,\theta^w) - \tilde{C}(\tilde{q}^w,\theta^b)\big) \tag{6-$\tilde{t}^b$}$$

while for a restricted contract:

$$\bar{t}^b = \bar{C}(\bar{q}^b,\bar{k}^b,\theta^b) + \big(\bar{C}(\bar{q}^w,\bar{k}^w,\theta^w) - \bar{C}(\bar{q}^w,\bar{k}^w,\theta^b)\big) \tag{7-$\bar{t}^b$}$$

<u>The principal's problem: basic contracts</u>. For all contracts, we assume that output is sold on a perfectly competitive market at a price of p. Given a basic contract $(\tilde{\boldsymbol{q}},\tilde{\boldsymbol{t}})$, the principal's profit

from an agent who declares a type of θ' is $p\tilde{q}(\theta') - \tilde{t}(\theta')$. [1] Thus, the principal's problem is to choose the contract $(\tilde{\boldsymbol{q}}, \tilde{\boldsymbol{t}})$ that maximizes $\sum_{\theta \in \{\theta^w, \theta^b\}} \left\{ \Pr(\theta)\left(p\tilde{q}(\theta) - \tilde{t}(\theta)\right)\right\}$ subject to incentive and participation constraints. By invoking the necessary conditions (6), we can reduce the principal's program to the problem of finding an (unconstrained) maximum over $\boldsymbol{q}$ of the following expression:

$$\max_{\boldsymbol{q}} \quad \left\{ \sum_{\theta \in \{\theta^w, \theta^b\}} \Pr(\theta)\left(pq(\theta) - \tilde{C}(q,\theta)\right) \right\} \quad + \quad \Pr(\theta^b)\tilde{I}(q) \tag{8}$$

where $\tilde{I}(q) = \left(\tilde{C}(\tilde{q}^w, \theta^w) - \tilde{C}(\tilde{q}^w, \theta^b)\right)$ denotes the *information cost* of having the lower ability agent produce q under a basic contract. Together with (6-$\tilde{t}^w$) and (6-$\tilde{t}^b$), the necessary conditions for $(\tilde{\boldsymbol{q}}, \tilde{\boldsymbol{t}}) = \left((\tilde{q}^w, \tilde{t}^w), (\tilde{q}^b, \tilde{t}^b)\right)$ to maximize (8) are:

$$p = \frac{\partial \tilde{C}(\tilde{q}^b, \theta^b)}{\partial q} \tag{6-$\tilde{q}^b$}$$

$$p = \Pr(\theta^w)\frac{\partial \tilde{C}(\tilde{q}^w, \theta^w)}{\partial q} \quad + \quad \Pr(\theta^b)\frac{\partial \tilde{I}(\tilde{q}^w, \theta^w)}{\partial q} \tag{6-$\tilde{q}^w$}$$

The standard results follow immediately: while the more efficient agent will produce the neoclassical level of output for his type, the less efficient agent will produce less than the neoclassical level of output for his type, provided that $\frac{\partial \tilde{I}(\cdot,\theta^w)}{\partial q}$ is positive. To see that $\frac{\partial \tilde{I}(\cdot,\theta^w)}{\partial q}$ is positive, observe that since production technology exhibits decreasing returns to scale, $\frac{\partial \tilde{C}(\cdot,\theta^w)}{\partial q}$ is increasing in q. Hence if $\frac{\partial \tilde{I}(\cdot,\theta^w)}{\partial q} > 0$, (6-$\tilde{q}^w$) can hold only if $\tilde{q}^w$ is lower than the q-value at which marginal cost equals price.

The principal's problem: restricted contracts. Now consider a restricted contract $(\bar{\boldsymbol{q}}, \bar{\boldsymbol{k}}, \bar{\boldsymbol{t}})$. As before, by invoking the necessary conditions (7), we can reduce the principal's program to the problem of finding an (unconstrained) maximum over $(\boldsymbol{q}, \boldsymbol{k})$ of the following expression:

$$\sum_{\theta \in \{\theta^w, \theta^b\}} \Pr(\theta)\left(pq(\theta) - \bar{C}(q,k,\theta)\right) \quad + \quad \Pr(\theta^b)\bar{I}(q,k) \tag{9}$$

where $\bar{I}(q,k) = \left(\bar{C}(q,k,\theta^w) - \bar{C}(q,k,\theta^b)\right)$ denotes the *information cost* of having the lower ability agent produce q with capital level k under a restricted contract. Together with (7-$\bar{t}^w$) and (7-$\bar{t}^b$),

[1] Notice that the principal's profit depends only on agents' *announced* types. The reason is that the contract is written in terms of the agent's deliverable, q. This would not be the case if the principal specified a piecerate and the agent's deliverable were not verifiable.

the necessary conditions for $(\bar{\boldsymbol{q}}, \bar{\boldsymbol{t}}) = \left((\bar{q}^w, \bar{k}^w, \bar{t}^w), (\bar{q}^b, \bar{k}^b, \bar{t}^b)\right)$ to maximize (9) are:

$$p = \frac{\partial \bar{C}(\bar{q}^b, \bar{k}^b, \theta^b)}{\partial q} \tag{7-$\bar{q}^b$}$$

$$0 = \frac{\partial \bar{C}(\bar{q}^b, \bar{k}^b, \theta^b)}{\partial k} \tag{7-$\bar{k}^b$}$$

$$p = \Pr(\theta^w)\frac{\partial \bar{C}(\bar{q}^w, \bar{k}^w, \theta^w)}{\partial q} + \Pr(\theta^b)\frac{\partial \bar{I}(\bar{q}^w, \bar{k}^w, \theta^w)}{\partial q} \tag{7-$\bar{q}^w$}$$

$$0 = \Pr(\theta^w)\frac{\partial \bar{C}(\bar{q}^w, \bar{k}^w, \theta^w)}{\partial k} + \Pr(\theta^b)\frac{\partial \bar{I}(\bar{q}^w, \bar{k}^w, \theta^w)}{\partial k} \tag{7-$\bar{k}^w$}$$

As with a basic contract, the more efficient agent will produce the neoclassical level of output for his type, while the less efficient agent will produce less than the neoclassical level for his type, provided that $\frac{\partial \tilde{I}(\cdot,\cdot,\theta^w)}{\partial q}$ is positive. Analogous to output, the more efficient agent will use the neoclassical input mix, while the input mix for the low ability agent will be affected by the information problem. Since $\frac{\partial \bar{C}(\bar{q}^w, k(\bar{q}^w,\theta^w),\theta^w)}{\partial k}$ is zero (see (5)),the neoclassical capital choice $k(\bar{q}^w, \theta^w)$ will satisfy (7-$\bar{k}^w$) only if $\frac{\partial \bar{I}(\bar{q}^w, k(\bar{q}^w,\theta^w),\theta^w)}{\partial k}$ is zero also. We will establish below that this will *not* be the case given the assumptions we have imposed, so that the less efficient agent's prescribed input mix under an optimal restricted contract will differ from the neoclassical mix.

The following result follows immediately from expressions (6), (7), (8) and (9).

Proposition 1. *The principal's profits under the optimal restricted contract are always strictly higher than his profits under the optimal basic contract.*

The proofs of this and following propositions are in the appendix. Intuition for the proof is provided by Fig. 1. The parallel *curves* represent isoquants for the output level designated for the low-ability agent when $\gamma = 0$: the higher isoquant indicates the inputs that the low ability agent needs to produce the specified output level and the lower isoquant indicates the inputs that the high ability agent needs to imitate the low one. The parallel *lines* represent isocost curves. The graph shows the effect on information rents of constructing a restricted contract that requires the same inputs that agents of each type would choose to fulfill their type-appropriate contract. Simply by requiring an agent choosing a θ^w contract to use $\hat{k}^w = \tilde{k}(\hat{q}^w, \theta^w)$, the principal reduces information rents from $\tilde{I}(\hat{q}^w)$ to $\bar{I}(\hat{q}^w, \hat{k}^w)$. The principal's revenues are the same under the constructed restricted contract and the basic contract, since the outputs for agents are the same. Production costs are the same

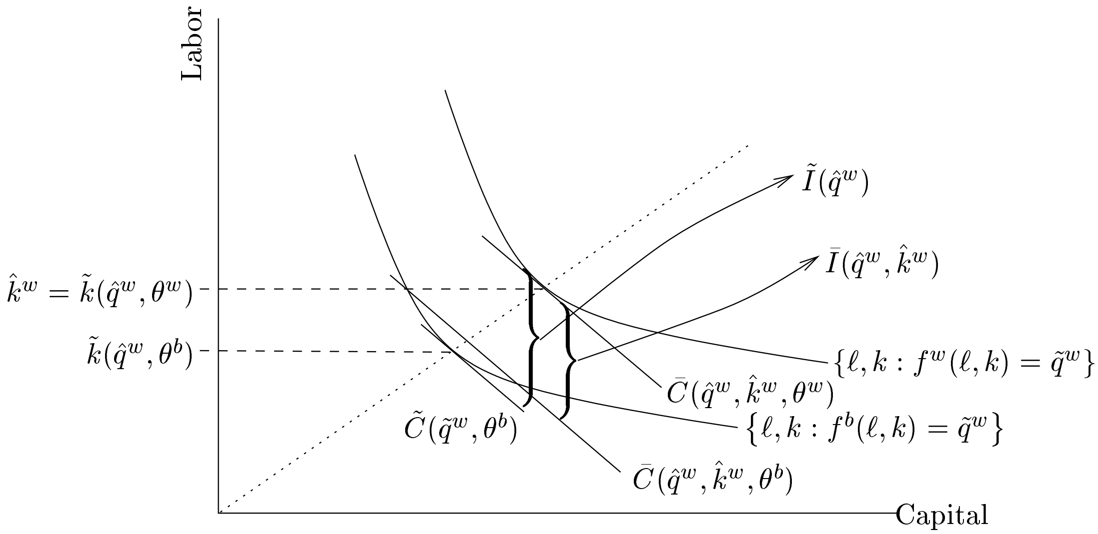


FIGURE 1. Information Cost of Producing $\tilde{q}^w$

for the two contracts, but information rents are lower under the constructed contract, so profits are higher under the constructed contract. Since the principal can choose this contract, it follows that profits must be higher under the optimal restricted contract than under the basic contract.

This result can be viewed as an application of the LeChatelier Principle. Under the restricted contract, the principal is free to specify capital levels as well as output levels, which implictly specify labor levels for agents of each ability. Under a basic contract, the principal faces the additional constraint that the agent will combine capital and labor in the neoclassical production cost-minimizing ratio.

3. The social cost of information asymmetry.

As we've seen, the principal's profits are higher under a restricted contract than under a basic contract. This does not necessarily imply, however, that restricted contracts are preferable to basic contracts from a *social* perspective. Briefly, the principal wishes to minimize the sum of production and information costs, but only production costs matter for social surplus. Information costs are simply a transfer from the principal to the high ability agent. In terms of the LeChatelier principle, while the principal is made better off, the agent is made worse off, so that the net effect is not pre-determined. In this section we compare the two kinds of contracts from a social perspective, focusing on the effect of introducing a "small" amount of informational asymmetry, in a sense to be defined below.

In the present model with perfectly elastic demand, social surplus is the sum of the principal's profit and the information rent received by the high ability agent. Since the information rent is a pure transfer, social surplus is equal to the principal's total revenue minus *production* cost. Although information rents are lower under the optimal restricted contract, average production costs are higher, because the input mix is sub-optimal from a pure production standpoint. A second factor which affects social surplus is the *level* of production. It can be shown that under weak conditions production is always greater under the optimal restricted contract. It turns out that either of these effects can dominate, so that the two kinds of contracts cannot be unambiguously ranked from a social perspective.

We wish to isolate the effect of a small increase in the degree of information asymmetry on the social cost of information. The simplest way to address this question is to vary the probability of realizing each type of agent. Specifically, in a two-type model, let λ denote the probability that the agent's ability is high. Obviously if λ is either zero or one then information is perfectly symmetric. The degree of asymmetry increases as λ moves towards one-half, and is maximized at this point. From our perspective, this kind of variation in information is not fully satisfactory, because it necessarily involves changing the principal's (stochastic) *production possibilities* along with her information. In other words, the first-best, symmetric information benchmark changes along with λ. For this reason we propose a test that holds *everything* constant except information asymmetry.

Our proposed test involves the following (somewhat contrived) scenario.[2] Agents differ according to (a) their ability and (b) whether the principal will ultimately know their type. There are, as before, two types of agent, θ^w and θ^b. For analytical convenience, we impose the further assumption that $\theta^b = 1 - \theta^w$, so that $\theta_i = Pr(\theta_i)$. With probability $1 - \gamma$, $\gamma \in [0, 1]$ the high ability agent's type will be revealed to the principal. Revelation occurs *after* production has occurred but *before* payment is made. The agent, on the other hand, know from the outset his type and whether or not it will be revealed in the future. (To fix ideas, suppose for example that with probability $1 - \gamma$, high-ability agents will need to wear spectacles on the job in the near future. The short-sighted agent knows the state of his vision well in advance, but the principal learns about it only when the be-spectacled agent reports for work.) With this timing specification, the principal can condition her *payment* to the agent on whether or not his type has been revealed but cannot condition the *production* requirements of the contract on whether an agent's type is ultimately knowable or not. The advantage of this construction is that as we vary γ holding (θ^w, θ^b) constant, the first-best, symmetric information benchmark remains constant. When $\gamma > 0$, the extent to which the principal's profits fall short of their level when $\gamma = 0$ is thus a pure measure of the cost of asymmetric information, and we can compare this cost under alternative specifications of the production contract.

Clearly, to minimize information rent payments, the principal should offer a conventional contract designed for the low-ability agent and a *contingent* contract designed for the high-ability agent. The latter should specify a common production level across contingencies and, if it is a restricted contract, a common input level across contingencies, together with a payment scheme contingent on whether the agent's type has been revealed by the time payment is scheduled. Naturally, the payment to an agent whose high ability is commonly known will just cover the agent's costs and reservation utility; this kind of agent will receive zero information rents. A high ability agent whose type has not been revealed will receive an information rent in the usual way.

Formally, the principal's task is to maximize the following objective function:

$$\max_{\tilde{q}} \quad \left\{ \sum_{\lambda \in \left\{\theta^w, \theta^b\right\}} \left\{ \lambda \Big(pq(\lambda) - C(q, \lambda) \Big) \right\} \quad + \quad \gamma \theta^b I(q) \right\} \tag{10}$$

[2] The test should be viewed as a pure thought experiment. We are not suggesting that the scenario we propose corresponds to any actual institutional arrangement.

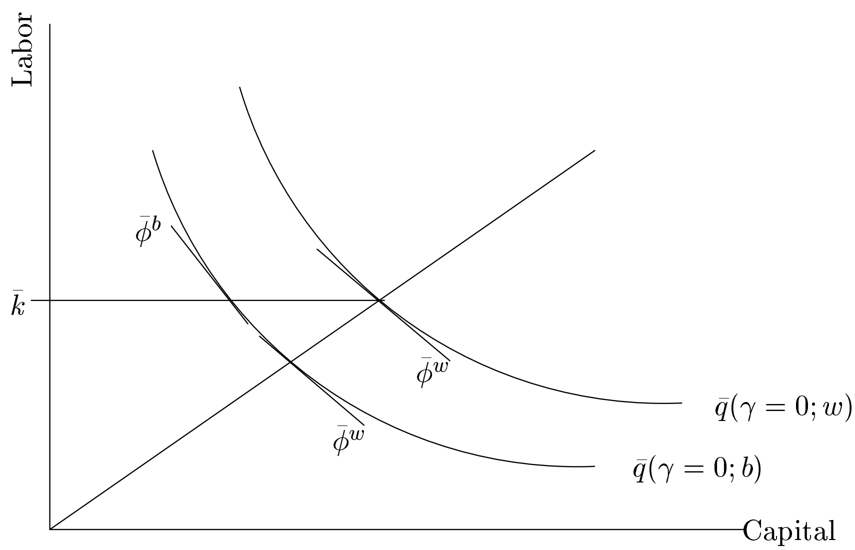
That is, the principal is required to pay all *production* costs with probability one, but the need to pay an information rent arises only with the probability, $\gamma\theta^b$ of realizing a high ability agent whose type will not be revealed to the principal. By standard arguments, the contractual requirements imposed on the high-ability type will coincide with first-best inputs and outputs, regardless of the nature of the contract and the value of γ. Therefore, we can focus on the solution to the assignment problem for the low-ability agent.

Analyzing the principal's problem in the case of the restricted contract results in the following proposition:

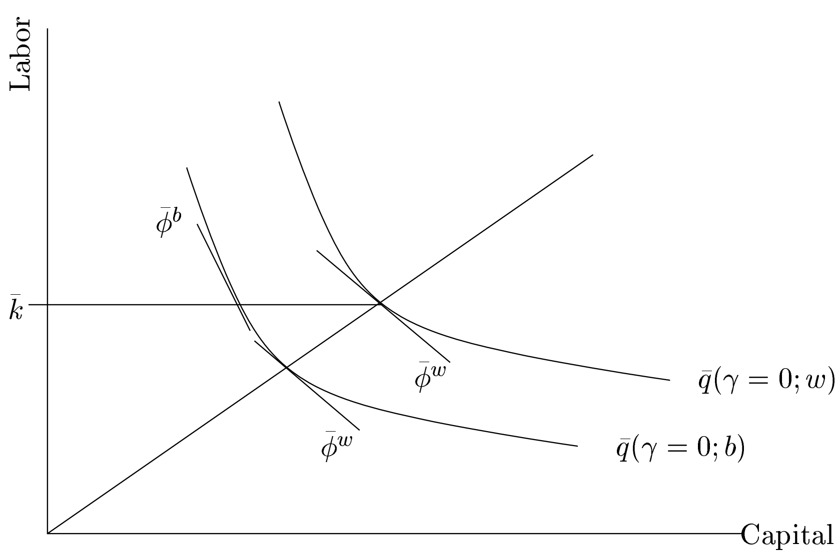
Proposition 2. *As γ increases from zero, in the contract designed for the low-ability agent the capital-labor ratio exceeds the neoclassical ratio and the output level decreases. Further, the social inefficiency due to the distortion in the capital-labor ratio is larger the lower the elasticity of substitution.*

As we introduce an arbitrarily small amount of asymmetric information into the principal's maximization problem, she reduces the amount of output specified in the low-ability agent's contract. This is the standard downward distortion observed in the standard adverse selection problem. Here, the principal also increases the level of capital relative to the neoclassical level for that amount of output, which increases the capital-labor ratio. This is a source of social inefficiency: the low-ability agent's *average* production cost is higher than it would be if the agent chose his own input levels. The significance of this inefficiency is determined by the elasticity of substitution of the production function: less substitutability implies greater inefficiency. This is illustrated in Fig. 2. In each panel of the figure, the parallel curves represent isoquants for the two agent types for producing the common output level specified by the principal when $\gamma = 0$: the outer isoquant applies to the lower ability agent. Our two homogeneity assumptions imply that the isoquants are parallel. Under the restricted contract, the higher ability agent would be required to use input level $\bar{k}$ if he chose the other type agent's contract. Note that as the isoquant becomes more convex, going from panel (a) to panel (b), the difference between $\bar{\phi}^b$ and $\bar{\phi}^w$ increases so that an increase in γ has greater inefficiency implications through its effect on average cost.

In a basic contract, the agent controls the non-labor input. The principal can not condition her contract for each agent type on input use. This restriction on the principal increases the information



(a) Inputs highly substitutable



(b) Inputs less substitutable

FIGURE 2. Elasticity of Substitution and Input Mix Distortion.

rents available to a high ability agent, since the agent's ability to substitute between inputs can not be limited by the producer. We summarize our analysis of the basic contract in the following proposition:

Proposition 3. *As γ increases from zero, in the contract designed for the low-ability agent the output level decreases. Since the agent controls capital, the capital-labor ratio remains at its neo-classical level for the specified output level.*

Now we compare the output produced by the low ability agent under the two optimal contracts.

Proposition 4. *If γ is sufficiently close to zero and if θ^b is sufficiently close to θ^w, then output produced by the low ability agent is higher under the optimal restricted contract than under the optimal basic contract.*

An interpetation of the above result is that the marginal cost curve (including production and information rent costs), *at the point at which price equals marginal cost*, increases less rapidly with gamma under the restricted contract than under the basic contract. Intuitively, this is because with the restricted contract the principal can limit the extent to which the high ability agent can substitute between labor and capital when he defects.

Proposition 4 implies that in equilibrium *revenue* is more sensitive to information asymmetry *at the margin* under the basic contract than under the restricted contract. We can't conclude from this, however, that the social cost of information asymmetry is lower under the restricted contract. At intra-marginal units of output, the marginal cost curve may be *higher* under the restricted contract than under the basic contract. The reason is that under the restricted contract, the input mix is distorted relative to the neoclassical ratio, i.e., too much capital is used and not enough labor. Hence even though output falls by less under the restricted contract, total producer surplus (divided between the principal and the agent) may be lower. As noted above the input mix distortion will be greater, the less substitutable are the inputs in production. Thus we have:

Proposition 5. *The social cost of information asymmetry may be greater or lower under the restricted contract relative to the basic contract. Holding all else constant, there exists $\underline{\xi}$ such that if the elasticity of substitution between capital and labor is less than $\underline{\xi}$, the information asymmetry will be more costly under the restricted contract.*

While the principal's profits are always greater under the optimal restricted contract (proposition 1), proposition 5 highlights that these additional profits may be socially costly. The principal minimizes the sum of production and information costs. Unlike production costs, information costs are a transfer, and so do not affect overall social surplus. When the principal exercises her power under the restricted contract to specify capital levels (and increase production costs) in order to reduce information rents, she introduces a downward distortion in social surplus. When there is limited substitutability between the two inputs, the increase in production costs at intra-marginal units of output is higher. The less substitutability between the inputs, the greater the downward distortion in social surplus due to the role of information rents in the principal's objective function. The greater the distortion due to information rents, the more likely this distortion is to outweight the increase in social surplus due to increased production when the principal controls capital.

4. Conclusion

We have shown that the principal's profits increase when she controls the non-labor input. Further, output increases, since the principal can allocate capital to help mitigate her information costs. However, this mitigation of information costs distorts the capital-labor ratio away from its production-efficient level. This distortion is socially costly. Overall, the distortion may dominate the gains realized through the increase in output and the reduction of information rents.

Our result differs from the classic finding of Averch and Johnson, who found that cost-plus pricing induces overinvestment in capital equipment that is socially costly. Essentially, the asymmetric information problem we consider contains an off-setting social cost. Neither one of the costs necessarily dominates the other, so that the social implication of input control by the principal is not a priori determinate. In terms of the LeChatelier principle, this can be explained by observing that input control for the principal is the same as removing a constraint for her, but it simultaneously imposes an additional constraint on the agent. The net effect of these adjustments is not predetermined.

Appendix

Proof of Proposition 1.

Let $(\tilde{\boldsymbol{q}}, \tilde{\boldsymbol{t}}) = \Big((\tilde{q}^w, \tilde{t}^w), (\tilde{q}^b, \tilde{t}^b)\Big)$ denote the optimal basic contract. Construct the restricted contract $(\hat{\boldsymbol{q}}, \hat{\boldsymbol{k}}, \hat{\boldsymbol{t}}) = \Big((\hat{q}^w, \hat{k}^w, \hat{t}^w), (\hat{q}^b, \hat{k}^b, \hat{t}^b)\Big)$, where $\hat{\boldsymbol{q}} = \tilde{\boldsymbol{q}}$ and for $\theta \in \{\theta^w, \theta^b\}$, $\hat{k}(\theta) = \tilde{k}(\tilde{q}(\theta), \theta)$. That is, under this constructed restricted contract, the outputs that were produced under the original basic contract are once again produced using the (neoclassical) input mix that was endogenously selected under the original basic contract. Thus for each θ, the production cost of $\hat{q}(\theta)$ is identical under both contracts. On the other hand, $\hat{k}^w = \tilde{k}(\hat{q}^w, \theta^w)$ is distinct from $\tilde{k}(\hat{q}^w, \theta^b)$ the unique solution to the unique k that solves the first order condition (2), $0 = w\ell_k(\hat{q}^w, k, \theta^b) + r$. Hence, we have $\bar{C}(\hat{q}^w, \hat{k}^w, \theta^b) > \bar{C}(\hat{q}^w, \tilde{k}(\hat{q}^w, \theta^b), \theta^b) = \tilde{C}(\tilde{q}^w, \theta^b)$. Hence

$$
\begin{aligned}
\tilde{I}(\hat{q}^w) &= \tilde{C}(\tilde{q}^w, \theta^w) - \tilde{C}(\tilde{q}^w, \theta^b) \\
&> \tilde{C}(\tilde{q}^w, \theta^w) - \bar{C}(\hat{q}^w, \hat{k}^w, \theta^b) && (11) \\
&= \bar{C}(\hat{q}^w, \hat{k}^w, \theta^w) - \bar{C}(\hat{q}^w, \hat{k}^w, \theta^b) = \bar{I}(\hat{q}^w, \hat{k}^w) && (12)
\end{aligned}
$$

The restricted contract we have constructed thus delivers the same output at a strictly lower cost to the principal. Hence the *optimal* restricted contract for the principal must also yield the principal strictly higher profits than the original basic contract. ■

Proof of Proposition 2. Assuming a restricted contract, our producer faces the following (unconstrained) maximization problem for choosing the contractually specified level of output and capital for the lower ability agent (Recall that the higher ability agent will produce the first best level of output using the first best input ratio):

$$\max_{k,q} \quad pq \quad - \quad \Big((1+\gamma)w\ell^w(k,q) \; - \; w\gamma\ell^b(k,q) \; + \; rk\Big) \tag{13}$$

where for $i = w, b$, $\ell^i(k,q)$ is defined implicitly by $q - f^i(\ell^i(k,q),k) = 0$. Hence $\frac{\partial \ell^i}{\partial q} = \frac{1}{f^i_\ell}$ and $\frac{\partial \ell^i}{\partial k} = -\frac{f^i_k}{f^i_\ell}$. The first order conditions for a solution to (13) are:

$$\begin{aligned} 0 &= p - w(1+\gamma)\frac{\partial \ell^w}{\partial q} + w\gamma \frac{\partial \ell^b}{\partial q} \\ &= \frac{p}{w} - (1+\gamma)\frac{1}{f^w_\ell} + \gamma\frac{1}{f^b_\ell} \qquad (13\text{-}\partial q) \\ 0 &= r - w\gamma\frac{\partial \ell^b}{\partial k} + w(1+\gamma)\frac{\partial \ell^w}{\partial k} \\ &= r - w(1+\gamma)\frac{f^w_k}{f^w_\ell} + w\gamma\frac{f^b_k}{f^b_\ell} \\ &= \frac{r}{w} - \Big((1+\gamma)\phi^w - \gamma\phi^b\Big) \qquad (13\text{-}\partial k) \end{aligned}$$

For $i = w, b$, let $\bar{f}^i_\ell = \frac{\partial}{\partial \ell} f^i(\ell^w(\bar{q}(0), \bar{k}(0)), \bar{k}(0))$, i.e., the bar indicates that the function is evaluated at the solution to (13) with $\gamma = 0$. That is, we evaluate the principal's problem at the full information solution, and analyze the effect of introducing an arbitrarily small amount of imperfect information. Define $\bar{f}^i_k$ similarly and let $\bar{\phi}^i = \frac{\bar{f}^i_k}{\bar{f}^i_\ell}$. For future reference, note that since, obviously, $\ell^w(\bar{q}(0), \bar{k}(0)) > \ell^b(\bar{q}(0), \bar{k}(0))$, we have:

$$\bar{f}^w_\ell < \frac{d}{d\ell} f^b(\ell^w(\bar{q}(0), \bar{k}(0)), \bar{k}(0)) < \frac{d}{d\ell} f^b(\ell^b(\bar{q}(0), \bar{k}(0)), \bar{k}(0)) = \bar{f}^b_\ell \qquad (14)$$

Also note that since $\frac{\bar{k}^b}{\bar{\ell}^b} > \frac{\bar{k}^w}{\bar{\ell}^w}$ and θ is technologically neutral[3],

$$\bar{\phi}^w > \bar{\phi}^b \qquad (15)$$

Totally differentiating this system w.r.t. γ we obtain:

$$\begin{bmatrix} \frac{1}{\bar{f}^w_\ell} - \frac{1}{\bar{f}^b_\ell} \\ \bar{\phi}^w - \bar{\phi}^b \end{bmatrix} = -\begin{bmatrix} (1+\gamma)\frac{\partial(1/\bar{f}^w_\ell)}{\partial q} - \gamma\frac{\partial(1/\bar{f}^b_\ell)}{\partial q} & (1+\gamma)\frac{\partial(1/\bar{f}^w_\ell)}{\partial k} - \gamma\frac{\partial(1/\bar{f}^b_\ell)}{\partial k} \\ (1+\gamma)\frac{\partial\bar{\phi}^w}{\partial q} - \gamma\frac{\partial\bar{\phi}^b}{\partial q} & (1+\gamma)\frac{\partial\bar{\phi}^w}{\partial k} - \gamma\frac{\partial\bar{\phi}^b}{\partial k} \end{bmatrix} \begin{bmatrix} \frac{d\bar{q}}{d\gamma} \\ \frac{d\bar{k}}{d\gamma} \end{bmatrix}$$

which, evaluated at $\gamma = 0$

$$= -\begin{bmatrix} \frac{\partial(1/\bar{f}^w_\ell)}{\partial q} & \frac{\partial(1/\bar{f}^w_\ell)}{\partial k} \\ \frac{\partial\bar{\phi}^w}{\partial q} & \frac{\partial\bar{\phi}^w}{\partial k} \end{bmatrix} \begin{bmatrix} \frac{d\bar{q}}{d\gamma} \\ \frac{d\bar{k}}{d\gamma} \end{bmatrix} \qquad (16)$$

where

$$\begin{aligned}
\frac{\partial(1/\bar{f}^w_\ell)}{\partial q} &= -\frac{\bar{f}^w_{\ell\ell}}{(\bar{f}^w_\ell)^3} > 0 && \text{(16-1,1)}\\
\frac{\partial(1/\bar{f}^w_\ell)}{\partial k} &= (\bar{f}^w_\ell)^{-3}\left(\bar{f}^w_k\bar{f}^w_{\ell\ell} - \bar{f}^w_\ell\bar{f}^w_{\ell k}\right) < 0 && \text{(16-1,2)}\\
\frac{\partial\bar{\phi}^w}{\partial q} &= (\bar{f}^w_\ell)^{-3}\left(\bar{f}^w_\ell\bar{f}^w_{\ell k} - \bar{f}^w_k\bar{f}^w_{\ell\ell}\right) = -\frac{\partial(1/\bar{f}^w_\ell)}{\partial k} > 0 && \text{(16-2,1)}\\
\frac{\partial\bar{\phi}^w}{\partial k} &= (\bar{f}^w_\ell)^{-3}\left(\bar{f}^w_{kk}(\bar{f}^w_\ell)^2 - 2\bar{f}^w_k\bar{f}^w_\ell\bar{f}^w_{\ell k} + (\bar{f}^w_k)^2\bar{f}^w_{\ell\ell}\right) < 0 && \text{(16-2,2)}
\end{aligned}$$

$\frac{\partial(1/\bar{f}^w_\ell)}{\partial q}$ is positive since $\bar{f}^w_{\ell\ell} < 0$; $\frac{\partial(1/\bar{f}^w_\ell)}{\partial k} = -\frac{\partial(1/\bar{\phi}^w)}{\partial q}$ is negative because f is homogeneous.[4] $\frac{\partial(1/\bar{\phi}^w)}{\partial k}$ is negative since it is the second principal minor of the bordered hessian of f, which is quasi-concave.

Note that as k varies holding q constant, $\ell^w(k,q)$ adjusts so that $(\ell^w(k,q),k)$ remains on the same isoquant, i.e., the one corresponding to q. That is, in (ℓ,k) space, the notation "∂k" in the present context indicates a shift *northwest* rather than a shift *due north*.

An immediate implication of (16) is that

$$\frac{d\bar{\phi}}{d\gamma} = \left(\frac{\partial\bar{\phi}}{\partial q}\frac{d\bar{q}}{d\gamma} + \frac{\partial\bar{\phi}}{\partial k}\frac{d\bar{k}}{d\gamma}\right) = \left(\bar{\phi}^b - \bar{\phi}^w\right) < 0 \tag{18}$$

That is, as γ increases, the capital-labor ratio assigned to the low-ability agent (i.e., a capital level is assigned and a labor level is implied by the assigned quantity level) exceeds the neoclassical

4 $\frac{\partial(1/\bar{f}^w_\ell)}{\partial k} = (\bar{f}^w_\ell)^{-3}\left(\bar{f}^w_k\bar{f}^w_{\ell\ell} - \bar{f}^w_\ell\bar{f}^w_{\ell k}\right)$ is obviously negative if $\bar{f}^w_{\ell k}$ is positive. Assume therefore that $\bar{f}^w_{\ell k}$ is negative. Since f is homogeneous of degree $k < 1$, $\bar{f}^w_\ell$ and $\bar{f}^w_k$ are both homogeneous of degree $k - 1 < 0$. Hence by Euler's theorem,

$$\left(\bar{f}^w_{\ell\ell}\bar{\ell} + \bar{f}^w_{\ell k}\bar{k}\right)/\bar{f}^w_\ell = \left(\bar{f}^w_{kk}\bar{k} + \bar{f}^w_{\ell k}\bar{\ell}\right)/\bar{f}^w_k = k - 1$$

Rearranging,

$$\frac{\bar{\ell}\bar{f}^w_{\ell k}}{\bar{f}^w_\ell}\left(\frac{\bar{f}^w_{\ell\ell}}{\bar{f}^w_{\ell k}} - \frac{\bar{f}^w_\ell}{\bar{f}^w_k}\right) = \frac{\bar{k}\bar{f}^w_{\ell k}}{\bar{f}^w_k}\left(\frac{\bar{f}^w_{kk}}{\bar{f}^w_{\ell k}} - \frac{\bar{f}^w_k}{\bar{f}^w_\ell}\right) \tag{17}$$

Hence the expressions in parentheses on either side of (17) have the same sign. We need to prove that both are positive. Assume therefore that both are negative, i.e., that $\frac{\bar{f}^w_{\ell\ell}}{\bar{f}^w_{\ell k}} < \frac{\bar{f}^w_\ell}{\bar{f}^w_k}$ and $\frac{\bar{f}^w_{kk}}{\bar{f}^w_{\ell k}} < \frac{\bar{f}^w_k}{\bar{f}^w_\ell}$. But in this case $\frac{\bar{f}^w_{\ell\ell}}{\bar{f}^w_{\ell k}}\frac{\bar{f}^w_{kk}}{\bar{f}^w_{\ell k}} < \frac{\bar{f}^w_k}{\bar{f}^w_\ell}\frac{\bar{f}^w_\ell}{\bar{f}^w_k} = 1$, i.e., $\bar{f}^w_{\ell\ell}\bar{f}^w_{kk} < (\bar{f}^w_{\ell k})^2$, which cannot be true (cf (19-$\bar{\Delta}$)). Rewriting each side of (17), then, we have established that both $\bar{\ell}\left(\frac{\bar{f}^w_{\ell\ell}}{\bar{f}^w_\ell} - \frac{\bar{f}^w_{\ell k}}{\bar{f}^w_k}\right)$ and $\bar{k}\left(\frac{\bar{f}^w_{kk}}{\bar{f}^w_k} - \frac{\bar{f}^w_{\ell k}}{\bar{f}^w_\ell}\right)$ are negative. Multiplying both of these expressions by $\bar{f}^w_\ell\bar{f}^w_k > 0$, we find that $\left(\bar{f}^w_k\bar{f}^w_{\ell\ell} - \bar{f}^w_\ell\bar{f}^w_{\ell k}\right)$ is negative. Moreover, for future reference, $\left(\bar{f}^w_\ell\bar{f}^w_{kk} - \bar{f}^w_k\bar{f}^w_{\ell k}\right)$ is negative also.

ratio. Note also that extent of this inefficiency is determined by the elasticity of substitution of the production function: less substitutability implies greater inefficiency.

We now analyze the effect of γ on output. Inverting the matrix on the right hand side of (16) above, we obtain

$$\begin{aligned}
\begin{bmatrix} \frac{d\bar{q}}{d\gamma} \\ \frac{d\bar{k}}{d\gamma} \end{bmatrix} &= -\begin{bmatrix} \frac{\partial(1/\bar{f}_\ell^w)}{\partial q} & \frac{\partial(1/\bar{f}_\ell^w)}{\partial k} \\ \frac{\partial\bar{\phi}^w}{\partial q} & \frac{\partial\bar{\phi}^w}{\partial k} \end{bmatrix}^{-1} \begin{bmatrix} \frac{1}{\bar{f}_\ell^w} - \frac{1}{\bar{f}_\ell^b} \\ \bar{\phi}^w - \bar{\phi}^b \end{bmatrix} \qquad (19) \\
&= -\bar{\Delta}^{-1} \begin{bmatrix} \frac{\partial\bar{\phi}^w}{\partial k} & -\frac{\partial(1/\bar{f}_\ell^w)}{\partial k} \\ -\frac{\partial\bar{\phi}^w}{\partial q} & \frac{\partial(1/\bar{f}_\ell^w)}{\partial q} \end{bmatrix} \begin{bmatrix} \frac{1}{\bar{f}_\ell^w} - \frac{1}{\bar{f}_\ell^b} \\ \bar{\phi}^w - \bar{\phi}^b \end{bmatrix}
\end{aligned}$$

where

$$\begin{aligned}
\bar{\Delta} &= \left(\frac{\partial(1/\bar{f}_\ell^w)}{\partial q}\right)\left(\frac{\partial\bar{\phi}^w}{\partial k}\right) - \left(\frac{\partial(1/\bar{f}_\ell^w)}{\partial k}\right)\left(\frac{\partial\bar{\phi}^w}{\partial q}\right) \qquad (19\text{-}\bar{\Delta}) \\
&= (\bar{f}_\ell^w)^{-6}\left\{\left[\bar{f}_{\ell\ell}^w \bar{f}_k^w - \bar{f}_{\ell k}^w \bar{f}_\ell^w\right]^2 - \bar{f}_{\ell\ell}^w\left[\bar{f}_{kk}^w (\bar{f}_\ell^w)^2 - 2\bar{f}_k^w \bar{f}_\ell^w \bar{f}_{\ell k}^w + (\bar{f}_k^w)^2 \bar{f}_{\ell\ell}^w\right]\right\} \\
&= (\bar{f}_\ell^w)^{-4}\left\{(\bar{f}_{\ell k}^w)^2 - \bar{f}_{\ell\ell}^w \bar{f}_{kk}^w\right\} < 0
\end{aligned}$$

The determinant is negative since $\left\{\bar{f}_{\ell\ell}^w \bar{f}_{kk}^w - (\bar{f}_{\ell k}^w)^2\right\}$ is the determinant of the Jacobian of f, which is assumed to be negative definite. Note also from (14) and (15) that $\begin{bmatrix} \frac{1}{\bar{f}_\ell^w} - \frac{1}{\bar{f}_\ell^b} \\ \bar{\phi}^w - \bar{\phi}^b \end{bmatrix} > 0.$

Hence at $\gamma = 0$

$$\begin{bmatrix} \frac{d\bar{q}}{d\gamma} \\ \frac{d\bar{k}}{d\gamma} \end{bmatrix} = \frac{-(\bar{f}_\ell^w)^4}{(\bar{f}_{\ell k}^w)^2 - \bar{f}_{\ell\ell}^w \bar{f}_{kk}^w} \begin{bmatrix} \frac{\partial\bar{\phi}^w}{\partial k} & -\frac{\partial(1/\bar{f}_\ell^w)}{\partial k} \\ -\frac{\partial\bar{\phi}^w}{\partial q} & \frac{\partial(1/\bar{f}_\ell^w)}{\partial q} \end{bmatrix} \begin{bmatrix} \frac{1}{\bar{f}_\ell^w} - \frac{1}{\bar{f}_\ell^b} \\ \bar{\phi}^w - \bar{\phi}^b \end{bmatrix}$$

In particular

$$\frac{d\bar{q}}{d\gamma} = \frac{-(\bar{f}^w_\ell)^4}{(\bar{f}^w_{\ell k})^2 - \bar{f}^w_{\ell\ell}\bar{f}^w_{kk}}\left\{\frac{\partial\bar{\phi}^w}{\partial k}\left(\frac{1}{\bar{f}^w_\ell} - \frac{1}{\bar{f}^b_\ell}\right) - \frac{\partial(1/\bar{f}^w_\ell)}{\partial k}\left(\bar{\phi}^w - \bar{\phi}^b\right)\right\} \tag{20}$$

$$= \frac{-\bar{f}^w_\ell}{(\bar{f}^w_{\ell k})^2 - \bar{f}^w_{\ell\ell}\bar{f}^w_{kk}}\left\{\left(\bar{f}^w_{\ell\ell}(\bar{f}^w_k)^2 - \bar{f}^w_k\bar{f}^w_\ell\bar{f}^w_{\ell k}\right)\frac{1}{\bar{f}^b_\ell}\left(\frac{\bar{f}^b_k}{\bar{f}^w_k} - 1\right) + \left(\bar{f}^w_{kk}(\bar{f}^w_\ell)^2 - \bar{f}^w_\ell\bar{f}^w_k\bar{f}^w_{\ell k}\right)\left(\frac{1}{\bar{f}^w_\ell} - \frac{1}{\bar{f}^b_\ell}\right)\right\}$$

which, after rearranging terms

$$= \underbrace{\frac{\bar{f}^w_\ell(\bar{f}^w_\ell - \bar{f}^b_\ell)}{\bar{f}^b_\ell\left((\bar{f}^w_{\ell k})^2 - \bar{f}^w_{\ell\ell}\bar{f}^w_{kk}\right)}}_{\oplus}\left\{\underbrace{\frac{\bar{f}^w_k - \bar{f}^b_k}{\bar{f}^w_\ell - \bar{f}^b_\ell}}_{?}\underbrace{\left(\bar{f}^w_{\ell\ell}\bar{f}^w_k - \bar{f}^w_\ell\bar{f}^w_{\ell k}\right)}_{\ominus} + \underbrace{\left(\bar{f}^w_{kk}\bar{f}^w_\ell - \bar{f}^w_k\bar{f}^w_{\ell k}\right)}_{\ominus}\right\}$$

Positivity of $\frac{\bar{f}^w_\ell(\bar{f}^w_\ell - \bar{f}^b_\ell)}{\bar{f}^b_\ell\left((\bar{f}^w_{\ell k})^2 - \bar{f}^w_{\ell\ell}\bar{f}^w_{kk}\right)}$ follows from the negativity of $(\bar{f}^w_\ell - \bar{f}^b_\ell)$, which was established in (14). Negativity of the two expressions inside the curly brackets was established in footnote 4. We will establish that $\frac{d\bar{q}}{d\gamma}$ is negative. This is obviously true if $\frac{\bar{f}^w_k - \bar{f}^b_k}{\bar{f}^w_\ell - \bar{f}^b_\ell}$ is positive. Assume therefore, that it is negative, i.e., that $\frac{\bar{f}^b_k - \bar{f}^w_k}{\bar{f}^w_\ell - \bar{f}^b_\ell}$ is positive. Since f is homogeneous of degree r for each value of λ, it follows from Euler's theorem that

$$\bar{f}^b_k\bar{k} + \bar{f}^b_\ell\bar{\ell}^b = \bar{f}^w_k\bar{k} + \bar{f}^w_\ell\bar{\ell}^w = r\bar{q}.$$

Rearranging, we obtain

$$\bar{f}^b_\ell\left(\bar{\ell}^b - \bar{\ell}^w\right) + \bar{\ell}^w\left(\bar{f}^b_\ell - \bar{f}^w_\ell\right) = \bar{k}\left(\bar{f}^w_k - \bar{f}^b_k\right).$$

But, since $\bar{f}^b_\ell\left(\bar{\ell}^b - \bar{\ell}^w\right)$ is negative, this implies that

$$\bar{\ell}^w\left(\bar{f}^b_\ell - \bar{f}^w_\ell\right) > \bar{k}\left(\bar{f}^w_k - \bar{f}^b_k\right).$$

i.e., that

$$\frac{\bar{\ell}^w}{\bar{k}} > \frac{\bar{f}^w_k - \bar{f}^b_k}{\bar{f}^w_\ell - \bar{f}^b_\ell}.$$

Now, rewriting (17), we obtain

$$\frac{\bar{\ell}}{\bar{k}} \quad = \quad \frac{\left(\frac{\bar{f}^w_{kk}}{\bar{f}^w_k} \; - \; \frac{\bar{f}^w_{\ell k}}{\bar{f}^w_\ell}\right)}{\left(\frac{\bar{f}^w_{\ell\ell}}{\bar{f}^w_\ell} \; - \; \frac{\bar{f}^w_{\ell k}}{\bar{f}^w_k}\right)}.$$

It follows, therefore that

$$\left|\frac{\bar{f}^w_k - \bar{f}^b_k}{\bar{f}^w_\ell - \bar{f}^b_\ell}\left(\bar{f}^w_{\ell\ell}\bar{f}^w_k \; - \; \bar{f}^w_\ell \bar{f}^w_{\ell k}\right)\right| \quad < \quad \left|\left(\bar{f}^w_{kk}\bar{f}^w_\ell \; - \; \bar{f}^w_k \bar{f}^w_{\ell k}\right)\right|$$

which concludes the argument that $\frac{d\bar{q}}{d\gamma}$ is negative when $\frac{\bar{f}^b_k - \bar{f}^w_k}{\bar{f}^w_\ell - \bar{f}^b_\ell}$ is positive. Hence, $\frac{d\bar{q}}{d\gamma}$ is always negative. ∎

Proof of Proposition 3. Under a basic contract the producer faces the following maximization problem:

$$\max_{k,q} \quad pq \quad - \quad \Big((1+\gamma)(w\ell^w(k,q) + rk^w) \; - \; \gamma(w\ell^b(k,q) \; + \; rk^b)\Big) \tag{21}$$

The first order conditions for a solution to (21) are:

$$\begin{aligned}
0 \quad &= \quad p \quad - \quad w(1+\gamma)\frac{\partial \ell^w}{\partial q} \quad + \quad w\gamma\frac{\partial \ell^b}{\partial q} \\
&= \quad \frac{p}{w} \quad - \quad (1+\gamma)\frac{1}{f^w_\ell} \quad + \quad \gamma\frac{1}{f^b_\ell} \\
0 \quad &= \quad -(1+\gamma)(r \quad - \quad w(1+\gamma)\frac{\partial \ell^w}{\partial k}) \\
&= \quad \frac{r}{w} \quad - \quad \phi^w \\
0 \quad &= \quad -\gamma(r \quad - \quad w\gamma\frac{\partial \ell^b}{\partial k}) \\
&= \quad \frac{r}{w} \quad - \quad \phi^b
\end{aligned}$$

Note that in contrast to our analysis of the restricted contract, the marginal rate of substitution for the low ability agent is independent of γ. Applying the implicit function theorem to the above system, evaluated at $\gamma = 0$ and letting tilde's over a function indicate that the function is evaluated at the solution for the basic contract with $\gamma = 0$, we obtain

$$\begin{bmatrix} \frac{d\tilde{q}}{d\gamma} \\ \frac{d\tilde{k}^w}{d\gamma} \\ \frac{d\tilde{k}^b}{d\gamma} \end{bmatrix} = - \begin{bmatrix} \frac{\partial(1/\tilde{f}_\ell^w)}{\partial q} & \frac{\partial(1/\tilde{f}_\ell^w)}{\partial k^w} & 0 \\ \frac{\partial\tilde{\phi}^w}{\partial q} & \frac{\partial\tilde{\phi}^w}{\partial k^w} & 0 \\ \frac{\partial\tilde{\phi}^b}{\partial q} & 0 & \frac{\partial\tilde{\phi}^b}{\partial k^b} \end{bmatrix}^{-1} \begin{bmatrix} \frac{1}{\tilde{f}_\ell^w} - \frac{1}{\tilde{f}_\ell^b} \\ 0 \\ 0 \end{bmatrix}$$

Computing the top-left entry of the inverse matrix, we obtain:

$$\frac{d\tilde{q}}{d\gamma} = \underbrace{\frac{-(\bar{f}_\ell^w)^4}{(\bar{f}_{\ell k}^w)^2 - \bar{f}_{\ell\ell}^w \bar{f}_{kk}^w}}_{\oplus} \underbrace{\frac{\partial\tilde{\phi}^w}{\partial k}}_{\ominus} \underbrace{\left(\frac{1}{\tilde{f}_\ell^w} - \frac{1}{\tilde{f}_\ell^b}\right)}_{\oplus} < 0 \tag{22}$$

■

Proof of Proposition 4.

We need to show that the right-hand side of (20) is smaller in absolute magnitude than the right-hand side of (22), i.e., that $\left(\frac{d\bar{q}}{d\gamma} - \frac{d\tilde{q}}{d\gamma}\right)$ is positive. Since at $\gamma = 0$, the solution for agent "w" to problems (13) and (21) are identical, we have $\tilde{f}_\ell^w = \bar{f}_\ell^w$ and $\tilde{\phi}^w = \bar{\phi}^w$, $\frac{\partial\tilde{\phi}^w}{\partial k} = \frac{\partial\bar{\phi}^w}{\partial k}$, etc. Also, because f is homothetic, $\tilde{f}_\ell^b = \tilde{f}_\ell^w$ and $\tilde{\phi}^b = \tilde{\phi}^w$. Hence

$$\frac{d\bar{q}}{d\gamma} - \frac{d\tilde{q}}{d\gamma} = \bar{\Delta}^{-1}\left\{\frac{\partial\tilde{\phi}^w}{\partial k}\left(\frac{1}{\bar{f}_\ell^b} - \frac{1}{\tilde{f}_\ell^b}\right) - \frac{\partial(1/\tilde{f}_\ell^w)}{\partial k}\left(\bar{\phi}^b - \tilde{\phi}^b\right)\right\}$$

Observe however that if the ability difference between the two types of agent (i.e., $\theta^b - \theta^w$) is sufficiently small, $\bar{k}^b \approx \tilde{k}^b$. Now observe that $\left(\ell^b(\bar{k}^b, q), \bar{k}^b\right)$ lies to the northwest of, but on the same isoquant as $\left(\ell^b(\tilde{k}^b, q), \tilde{k}^b\right)$. In particular, $\bar{k}^b = \bar{k}^w > \tilde{k}^b$. Moreover, since as we have observed, $\frac{\partial(1/\tilde{f}^b_\ell)}{\partial k}$ (resp. $\frac{\partial\tilde{\phi}^b}{\partial k}$) measures the change in $(1/\tilde{f}^b_\ell)$ (resp. $\tilde{\phi}^b$) as (ℓ, k) moves northwest along an isoquant, we have, for $(\theta^b - \theta^w)$ and hence $(\bar{k}^b - \tilde{k}^b)$ sufficiently small, sign $\left(\frac{1}{\bar{f}^b_\ell} - \frac{1}{\tilde{f}^b_\ell}\right) = \text{sign}\left(\frac{\partial(1/\tilde{f}^b_\ell)}{\partial k}(\bar{k}^b - \tilde{k}^b)\right)$ while sign $\left(\bar{\phi}^b - \tilde{\phi}^b\right) = \text{sign}\left(\frac{\partial\tilde{\phi}^b}{\partial k}(\bar{k}^b - \tilde{k}^b)\right)$. Therefore

$$\frac{d\bar{q}}{d\gamma} - \frac{d\tilde{q}}{d\gamma} \approx \bar{\Delta}^{-1}(\bar{k}^b - \tilde{k}^b)\left\{\frac{\partial\tilde{\phi}^w}{\partial k}\frac{\partial(1/\tilde{f}^b_\ell)}{\partial k} - \frac{\partial(1/\tilde{f}^w_\ell)}{\partial k}\frac{\partial\tilde{\phi}^b}{\partial k}\right\} \tag{23}$$

Next note that for $i = w, b$ (cf., (16-1,2) and (16-2,1)):

$$\begin{aligned}
\frac{\partial\tilde{\phi}^i}{\partial k} &= (\tilde{f}^i_\ell)^{-3}\left(\tilde{f}^i_{kk}(\tilde{f}^i_\ell)^2 - 2\tilde{f}^i_k\tilde{f}^i_\ell\tilde{f}^i_{\ell k} + (\tilde{f}^i_k)^2\tilde{f}^i_{\ell\ell}\right) \\
&= (\tilde{f}^i_\ell)^{-1}\left(\tilde{f}^i_{kk} - 2\tilde{\phi}^i\tilde{f}^i_{\ell k} + (\tilde{\phi}^i)^2\tilde{f}^i_{\ell\ell}\right) \\
&= (\tilde{f}^i_\ell)^{-1}\tilde{f}^i_{\ell k}\left(\frac{\tilde{f}^i_{kk}}{\tilde{f}^i_{\ell k}} - 2\tilde{\phi}^i + (\tilde{\phi}^i)^2\frac{\tilde{f}^i_{\ell\ell}}{\tilde{f}^i_{\ell k}}\right)
\end{aligned}$$

while

$$\begin{aligned}
\frac{\partial(1/\tilde{f}^i_\ell)}{\partial k} &= (\tilde{f}^i_\ell)^{-4}\left((\tilde{f}^i_k)^2\tilde{f}^i_{\ell\ell} - \tilde{f}^i_k\tilde{f}^i_\ell\tilde{f}^i_{\ell k}\right) \\
&= (\tilde{f}^i_\ell)^{-2}\left((\tilde{\phi}^i)^2\tilde{f}^i_{\ell\ell} - \tilde{\phi}^i\tilde{f}^i_{\ell k}\right) \\
&= (\tilde{f}^i_\ell)^{-2}\tilde{f}^i_{\ell k}\left((\tilde{\phi}^i)^2\frac{\tilde{f}^i_{\ell\ell}}{\tilde{f}^i_{\ell k}} - \tilde{\phi}^i\right)
\end{aligned}$$

Now $\tilde{\phi}^w = \tilde{\phi}^b = \frac{r}{w}$. Also, since f is homogeneous and neutral with respect to λ, $\frac{\tilde{k}^w}{\tilde{\ell}^w} = \frac{\tilde{k}^b}{\tilde{\ell}^b}$. Moreover, since f_ℓ and f_k are homogeneous also, $\frac{\tilde{f}^w_{\ell\ell}}{\tilde{f}^w_{\ell k}} = \frac{\tilde{f}^b_{\ell\ell}}{\tilde{f}^b_{\ell k}}$ and $\frac{\tilde{f}^w_{kk}}{\tilde{f}^w_{\ell k}} = \frac{\tilde{f}^b_{kk}}{\tilde{f}^b_{\ell k}}$. This in turn implies that $\frac{\tilde{f}^w_{\ell k}}{\tilde{f}^b_{\ell k}} > 0$. Finally, due to linear disutility of labor, $\frac{\tilde{f}^w_\ell}{\tilde{f}^w_k} = \frac{\tilde{f}^b_\ell}{\tilde{f}^b_k}$. Appealing to homotheticity, this implies that $\frac{k_w}{\ell_w} = \frac{k_b}{\ell_b}$. Now, under homogeneity of degree α, $k_w \tilde{f}^w_k + \ell_w \tilde{f}^w_\ell = k_b \tilde{f}^b_k + \ell_b \tilde{f}^b_\ell = \alpha f$. Observe that both the levels and the derivatives of f are proportional to each other. Given this, we consider a one-variable problem for the two ability types: $\omega_w f'(\omega_w) = \omega_b f'(\omega_b) = \alpha f$. Under this equivalent formulation, note that diminishing marginal returns to θ imply that $\frac{\omega_b}{\omega_w} < \frac{\theta_b}{\theta_w}$. Accordingly, $f'(\omega_w) < f'(\omega_b)$. Since the one- and two-variable formulations are equivalent, $\tilde{f}^w_{\ell\ell} < \tilde{f}^b_{\ell\ell}$. Therefore from (23), we have

$$
\begin{aligned}
\operatorname{sign}\left\{\frac{d\bar{q}}{d\gamma} - \frac{d\tilde{q}}{d\gamma}\right\} &= \operatorname{sign}\Bigg\{\bar{\Delta}^{-1}(\bar{k}^b - \tilde{k}^b)\frac{\tilde{f}^b_{\ell k}}{\tilde{f}^b_\ell}\Bigg[\frac{\partial\tilde{\phi}^w}{\partial k}\left(\frac{\tilde{f}^b_{kk}}{\tilde{f}^b_{\ell k}} - 2\tilde{\phi}^b + (\tilde{\phi}^b)^2\frac{\tilde{f}^b_{\ell\ell}}{\tilde{f}^b_{\ell k}}\right) \\
&\qquad - \frac{\partial(1/\tilde{f}^w_\ell)}{\partial k}(\tilde{f}^b_\ell)^{-1}\left((\tilde{\phi}^b)^2\frac{\tilde{f}^b_{\ell\ell}}{\tilde{f}^b_{\ell k}} - \tilde{\phi}^b\right)\Bigg]\Bigg\} \\
&= \operatorname{sign}\Bigg\{\bar{\Delta}^{-1}(\bar{k}^b - \tilde{k}^b)\frac{\tilde{f}^b_{\ell k}}{\tilde{f}^b_\ell}\Bigg[\frac{\partial\tilde{\phi}^w}{\partial k}\left(\frac{\tilde{f}^w_{kk}}{\tilde{f}^w_{\ell k}} - 2\tilde{\phi}^w + (\tilde{\phi}^w)^2\frac{\tilde{f}^w_{\ell\ell}}{\tilde{f}^w_{\ell k}}\right) \\
&\qquad - \operatorname{sign}\frac{\partial(1/\tilde{f}^w_\ell)}{\partial k}(\tilde{f}^b_\ell)^{-1}\left((\tilde{\phi}^w)^2\frac{\tilde{f}^w_{\ell\ell}}{\tilde{f}^w_{\ell k}} - \tilde{\phi}^w\right)\Bigg]\Bigg\} \\
&= \operatorname{sign}\left\{\underbrace{\bar{\Delta}^{-1}}_{\ominus}\underbrace{(\bar{k}^b - \tilde{k}^b)}_{\oplus}\underbrace{\frac{\tilde{f}^b_{\ell k}}{\tilde{f}^b_\ell}\frac{\tilde{f}^w_\ell}{\tilde{f}^w_{\ell k}}}_{\oplus}\underbrace{\frac{\partial\tilde{\phi}^w}{\partial k}}_{\ominus}\underbrace{\frac{\partial(1/\tilde{f}^w_\ell)}{\partial k}}_{\ominus}\underbrace{\left[1 - \frac{\tilde{f}^b_\ell}{\tilde{f}^w_\ell}\right]}_{\ominus}\right\} > 0
\end{aligned}
$$

■

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