Spatial Replicator Dynamics as a Model of Agricultural Technology Adoption*

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Abstract

Many explanations of agricultural technology adoption have either been based loosely on diffusion concepts and have ignored economic factors or have been static economic models that ignore dynamic and spatial aspects of adoption. In this paper we propose a model of adoption which incorporates economic factors in a spatial and dynamic framework. the approach used is based on extending replicator dynamics models in evolutionary game theory to a spatial setting. The replicator dynamics of adopters versus non-adopters are characterised by a spatial diffusion model the solution of which illustrates how local institutions drive spatial technology adoption processes via the rules of the game. The model is applied to forage technology adoption in the Philippines and a method of empirically testing the model is presented.

1 Introduction

Theories and models of agricultural technology adoption have been plagued by an apparent incompatibility between the diffusion models of agricultural extension that have been largely devoid of economic content and the view of many economists that adoption is based

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on economic incentives. If the factors which lead to adopion of some agricultural technology and the failure to adopt others are to be properly understood, a theoretical framework that overcomes the limitations of both the diffusion approach and the incentive approach needs to be developed.

Traditionally model of technology adoption have been based upon logistic diffusion curves whereby the percentage of the population that have adopted an innovation grows according to a logistic differential equation. In the agricultural extension literature the differential equation is typically ignored and the logistic "adoption curve", which is actually the solution of the logistic differential equation, is used to explain the adoption process in terms of stages of "early" and "late" adoption of technology.

The logistic diffusion model as used in agricultural extension avoids a discussion of the spatial aspect of adoption and concentrates solely on temporal aspects. The spatial dimension of agricultural technology diffusion has been emphasized by Haägerstrand (1967). Cavalli-Sforza and Feldman (1981) suggested ways of mathematically modelling the spatial diffusion of innovation using a spatial version of the logistic equation - The Fisher equation:

$$\frac{\partial x}{\partial t} == nx(t,s)(1-x(t,s)/K) + \nabla^2 x$$

where $x(t,s)$ represents for example the proportion of the population that have adopted an innovation and $\nabla^2 x$ represents a spatial interaction term.

The Fisher equation is a non-linear partial differential equation of diffusion type. At any given location it assumes that adoption follows a logistic pattern at that location and that the technology then spreads out spatially according to the diffusion term $\nabla^2 x$. Space may be modelled as a line, a plain, a volume or even in higher dimensions if need be. note that this model completely lacks any economic basis and is purely mechanistic in nature.

An alternative approach suggested by (1981) is based on the analogy of the diffusion of technology with an epidemic.

This approach has also been adopted in the literature on rumour modelling(Bartholomew 1967). Rumour models treat the spreaders of news and rumours as "infectives" and the "hearers" of news and rumours as "susceptibles" analogous with mathematical models of epidemics.

Within economics, evolutionary economists have also shown an interest in technology diffusion and adoption. Dalle (1997) for example proposes a model of technology adoption based on random fields. A random field is a generalisation of a stochastic process which depends on both time as a parameter and a spatial parameter. the use of random field models by evolutionary economists has been restricted to discrete-time and discrete-space Markov random fields, such as Gibbs random fields and Ising type spin-glass models.

The Ising spin glass model of a socioeconomic system is represented on a grid as follows:

+	+	−	+	+	−	+
−	+	+	+	+	−	+
+	−	+	+	+	−	+
−	+	−	+	−	−	+
+	−	+	+	−	+	+

where + represents an individual that has adopted and − an individual with the old technology.

the analogoy here is that technology adoption can be viewed as analogous to a "‘magnetization" process.

Each of the approaches reviewed here to modelling the diffusion of technology suffers from the drawback that economic incentives are not incorporated explicitly into the modelling framework.

In this paper we suggest an alternative approach based on evolutionary game theory that combines features of the models discussed above with game theoretic ideas concerning economic incentives.

2 Learning versus Rumours

It is important to distinguish firstly between the diffusion of technology as a learning process and diffusion of technology as a process involving the spread of ideas. Models based on learning are not necessarily spatial in nature. Game theoretic model of learning (see Fudenberg and Levine) fall into this class.

The literature on rumour modelling has much to offer if one is interested in modelling the adoption of technology(1967, pp. 204-260). Rumour models have typically modelled the spread of rumours and news in analogy with epidemics. Mathematically epidemiological and rumour models are virtually identical[1].

Such models have as their basis sociologically concepts of communication between memebers of a society.

The population is divided up into three groups(1967, p.224)

$m(t)$ those who have not heard the rumour/news

$n(t)$ those who have heard and are spreading the rumour

$l(t) = N - m(t) - n(t)$ persons who have heard the news but have ceased to spread it

The dynamics of the rumour can be described by the following system of differential equations

$$\dot{n} = \beta nm - \mu n$$

$$\dot{m} = -\beta mn$$

$$\dot{l} = \mu n$$

where β is the rate of contact between rumourmongers and those yet to hear the rumour. μ is the rate at which rumourmongers lose interest in the rumour.

[1]Dare one suggest that extension agents may be viewed as analogous to vectors in epidemiologial models?

Note that in this formulation spatial aspects of the problem are completely ignored. In the agricultural setting, especially, in deeloping countries information spreads within the context of a community within a particular geographical context.

This spatial context of the spread of technology through rural communities is clearly demonstrated by (1967). To account for this the rumour model presented above would have to be modified to incorproate a spatial dimension. this could be done by allowing spreaders to become "mobile". The population dynamics of the spreaders would then be represented by a partial differential equation.

The problem with this approach is that it is in some sense too sociological, it fails to take into account economic aspects of the problem. this is the essence of the distinction between learning and simple acquisition of information.

The rumour models used above could be coupled to a simple profit function. Thenif it is assumed that producers adopt better technologies on hearing of the rumour changing adoption patterns over time and space could be studied. But this does not explain why producers fail to adopt even after being informed of the supposed benefits of the technology - this model still assumes rational decision-making. If one introduces learning into the model, producers will not necessarly adopt even after hearing of the supposed benefits of the technology. Instead they will observe the actual benefits within the community and adopt if it appears that they have a chance of gaining from doing so. This approach leads to one viewing technology adoption from the perspective of evolutionary game theory.

3 Spatial Replicator Dynamics

There are a number of different approaches to incorporating learning in evolutionary games. The approach taken here makes use of replicator dynamics but other approaches such as those based on "imitation" dynamics are also possible.

We distinguish to types of agents those who have adopted a new technolgy and those who have not. The benefits of the technology are assumed to depend also on whether or not others in the community have also adopted the technology. This is primarily a technical assumption as it may well be that the benefits to a particular producer are the same whether or not others have adopted or not.

In developing countries in particular there are many uncertainties in the adoption of technology, concerning how widespread a particuar technology is at a particular time and a particular location. In addition, agents may adopt technology briefly only to give it up a short while later. To capture the uncertain nature of the adoption process the population dynamics of the adoption process are modelled as a system of stochastic partial differential equations. To analyse trend effects we can recover a deterministic system by setting the noise parameter σ to zero where appropriate.

The population dynamics for agents using old and new technology are given by

$$\partial p_{old} = \left[p_{old}(\beta_{old} + \Pi_{old} - \delta_{old}) + \nabla^2 p_{old}\right] dt + \sigma^2 p_{old} \frac{\partial^2 B}{\partial s \partial t}$$

$$\partial p_{new} = \left[p_{new}(\beta_{new} + \Pi_{new} - \delta_{new}) + \nabla^2 p_{new} \right] dt + \sigma^2 p_{new} \frac{\partial^2 B}{\partial s \partial t}$$

where

p_i $i = old, new$ represents the poluation of i-th player strategy.

Π_i is the expected payoff to the i-th player strategy

β_i is the brith rate of the i-th player strategy

δ_i is the death rate of the i-th player strategy

σ

s is a spatial parameter.

the partial "derivative" $\frac{\partial^2 B}{\partial s \partial t}$ is the derivative[2] of the Brownian sheet.

This system of equations can be viewed as an extension of the stochastic evolutionary game model of Fudenberg and Harris (1992) to a spatial setting. In (1992) model a stochastic evolutionary game using Ito stochastic differential equations. In the spatial setting the Wiener increments of the ito equations need to be replaced by the derivative of the Brownian sheet. Note that the derivative of the Brownian sheet and the Brownian sheet $W(s,t)$ possess the same distribution(Walsh 1984, pp. 284-285). Thus $\frac{\partial^2 B}{\partial s \partial t}$ is a normally distributed random variable with mean 0 and standard deviation of finite measure. The solution of tese equations will be a random field, i.e. $p_(s,t,\omega)$ is a random filed.

The Brownian sheet was first introduced by Kitagawa (1951) as a means carrying out analysis of variance in continuous-time(1984, p. 270).

In order to derive the spatial stochastic replicator dynamics, i.e. the dynamics of the population proportions. the following identity is used:

$$x_i(s,t) = \frac{p_i(t,s)}{\sum_i^N p_i(s,t)} \forall s,t$$

$$x_i(s,t) = \frac{p_i(s,t)}{p_1(s,t) + p_2(s,t)}$$

Thus x_i is the population proportion of the i-th player strategy.

Rearranging one obtains

$$p_i(s,t) = x_i(s,t)(p_1(s,t) + p_2(s,t))$$

[2]the derivative exists only in the more general sense of a Schwartz distribution. Not in the usual sense of ordinary calculus.

The total change in this is then given by

$$\partial p_i(s,t) = \partial x_i(s,t)(p_1(s,t) + p_2(s,t)) + x_i(s,t)(\partial p_1(s,t) + \partial p_2(s,t)$$

Which on rearranging gives

$$\partial x_i(s,t) = \frac{\partial p_i(s,t)}{p_1(s,t) + p_2(s,t)} - x_i(s,t)(\frac{\partial p_1(s,t)}{p_1(s,t) + p_2(s,t)} + \frac{\partial p_2(s,t)}{p_1(s,t) + p_2(s,t)})$$

Substituting in gives the stochastic spatial replicator dynamics model:

$$\partial x_i(s,t) = x_i \left[\Pi_i - \bar{P}i\right] + \nabla^2 x)dt + \sigma^2 x^2 \frac{\partial^2 B}{\partial s \partial t}$$

where $i = old, new$.

4 Numerical Solution

The stochastic spatial replicator dynamics model presented in the previous section may be solved numerically using a method known as method of lines[3]. The idea behind method of lines is to first discretize the SPDEs' in the spatial direction. This is done using a finite difference procedure wherby partial derivatives are represented using finite difference approximations. The result is that one obtains a systems of ordninary stochastic differential equations that must then be solved by an implicit scheme for solving stochastic differential equations. The reason for this is that such systems are generally stiff requiring very small step sizes and hence large amounts of compoutational power to solve.

Due to the nonlinear nature of the model the usual method of simply rearrangingthe right-hanfd side of the equation to obtain the current value of the differential equation fails. However backward solution appears a possibility.

In general a backward SDE will not have the same solution a s a forward SDE, however, one should bear in mind two things:

> Firstly, we are really only going to solve the a system of finite difference equations backwards not the SDE itself.
>
> Secondly, the point is rather technical as the backward solution could be defined if the stochastic integral were anticipative, e.g. Skorokhod rather than Ito integral[4].

Thus for each location s on a discretized grid. One solves a system of SDE's backwards using the Euler-Maruyama method for stochastic differential equations.

For small to moderate spatial grids this method may be implemented in a Spreadsheet. The following graphs present numerical results of solving the stochastic spatial replicator dynamics in this way.

[3]See (Ames 1992, pp. 33-34) for a discussion of method of lines.

[4]For further discussion of backward stochastic differential equations see (Ma and Yong 1999)

Setting $\sigma = 0$ gives us the numerical solution of the deterministic spatial replicator dynmamics model thus allowing us to analyse trends. Whilst this is more suited for model validation and prediction. Solution of the stochastic model allows one to “bootstrap” confidence intervals that are more appropriate in the small sample setting typical of developing countries.

5 Case Study: Forage Crop Adoption in the Philippines

The case study involves the adoption of forage crops in Mindanao in the Philippines. These crops were introduced into the Malitbog region of northern Mindanao. Malitboglies in the province of Bukidnon at latitude 8^o N and longitude 124^o E. the area is a mountainous area ranging in altitude from 250-1000 m above sea-level. The Malitbog muncicpality is spread over 58000 hectares.

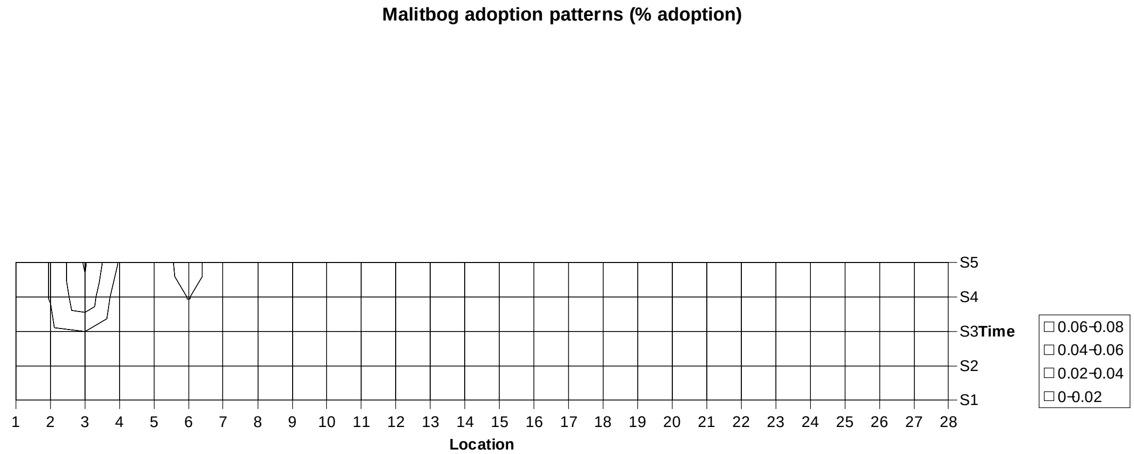
Farming is the main source of livelihood for 90% of the people. Most farmers possess one or two hectares of land. The major crops are corn and bananas and to a lesser extent coconut, rice, coffee, and vegetables. Livestock include cattle, carabao, goats, horses, swine and poultry.

The data were collected as part of the Forages for Smallholders Project (FSP)in which one of the authors (Purcell) is involved. The FSP is funded by AUSAID and coordinated by CIAT and CSIRO. A precursor to this project the Pilot provincial Agricultural Extension Project introduced forages to the region in 1995 as an attempt to supplement existing natural feeds available on roadsideds and in fields. Since then the FSP has been asked in response to farmer interest to help farmers integrate forage technologies into their farm activities.

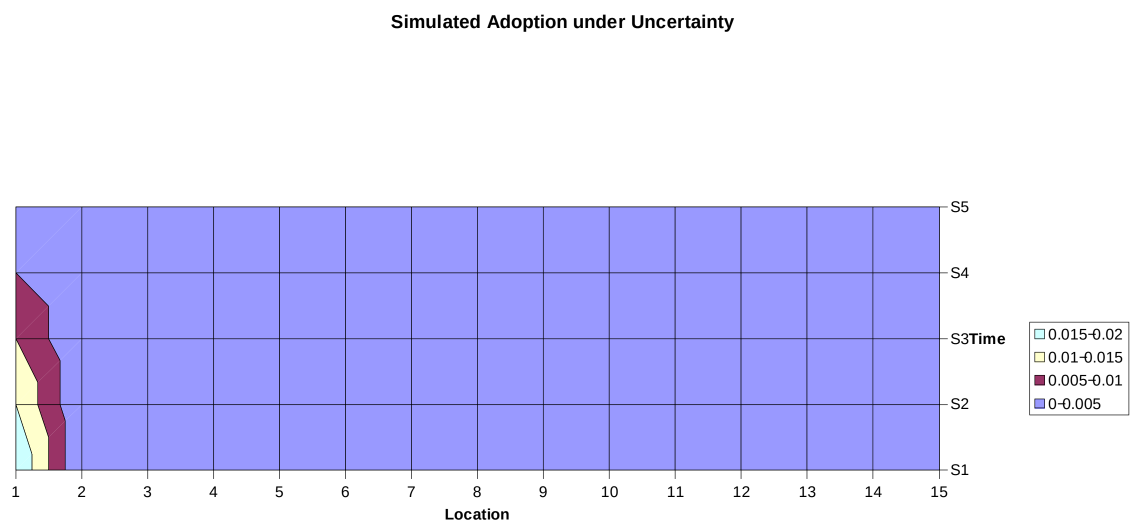
The approach of the FSP has been participatory in nature. Of interest is whether the extension work of FSP has adided adoption or whether the adoption process can be explained by purely economic motives. To this end it is of interest to determine whether the proposed economic model of adoption using spatial replicator dynamic is consistent with the observed pattern of adoption or whether the extension program explains adoption more plausibel.

Clearly without an extension program forages would not have been introduced into the area at the time they were introduced. However, the dynamics of adoption may be more plausibly expalined by economic arguments than by diffusion of information.

The following graph shows percentage adoption of forages by Malitbog farmers over time.



This may be contrasted with the simulated graph showing adoption patterns over tiem based on replicator dynamics.



One notices that with the empirical data a number of jumps occur due to visits of extension officers to villages. thus information transmission in reality does not appear to follow the diffusion model that is illustrated in the second graph.

In order to account for these jumps the model needs to be modified we have not yet extended the model to account for this.

One possible way of doing this is by using a jump-diffusion approach with a Poisson distributed random variable appended to the equations.

In the following we outline the proposed means of empriical testing the final model.

6 A Statistical Test Procedure

A popular procedure for model validation in population biology is to compare predicted versus actual using a χ^2 test for goodness of fit. In the partial differential equation setting a similar procedure can be used for model validation but instead of interpreting χ^2 in terms of goodness of fit, it is now interpreted in terms of homogeneity.

The patterns of adoption clearly differ from the model as it now stands so we have not bothered to implement this test. On extending the model to account for jumps and leaps in the adoption pattern the χ^2 method seems appropriate.

7 Conclusion

Adoption of forages by farmers in the Malitbog region of the Philippines does not appear to follow a pure replicator based diffusion process but involves jumps in the adoption process. Nevertheless the basic methodology for modfelling adoption appears promising and we intend to continue refining the methodology.

Lack of detqailed dat on economic benefits of technology and how they are driven by community interaction has to some extent hampered our analysis.

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