CLOSED-LOOP SOLUTION FOR OPTIMAL SEQUENTIAL HEDGING AND FORWARD CONTRACTING IN U.S. HOG PRODUCTION

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Abstract

This paper develops a multiperiod model in which hedge adjustments are allowed. The two major marketing alternatives specified in the model are to sell in the spot market or to forward contract using formula pricing. To proxy the underlying forward contract value, the American put-call parity (APCP) technique is used. The conceptual framework considers a mean-variance utility function that is maximized sequentially to obtain optimal forward contract and hedge ratios. The closed loop solution guides the dynamic flow of information between decision stages via three essential features: sequential dependence, feedback, and anticipated revision. The empirical model considers a multivariate ARMA-GARCH framework that estimates the time series of APCP values, live hog prices, and futures gains/losses simultaneously with the conditional (time-varying) variance-covariance structure. This provides a superior forecasting tool to capture the second-moment dynamics for computing optimal forward contract and hedge ratios. The simulation results recommend significant upward hedge adjustments on average. This reflects the time-varying pattern of optimal hedge ratios. The optimal forward contract ratios indicate in almost all instances that hogs should be marketed entirely in advance. The effectiveness of CME live/lean hog futures hedging is assessed for its ability to reduce both spot price and contract value risks under the closed loop solution. The percentage improvement in utility is compared with two other alternative portfolios of no hedging and non-adjustable MGARCH hedging. The findings indicate that the closed loop solution is able to achieve the best utility outcome especially in volatile market situations.

Key Words: closed loop solution, American put-call parity, MGARCH, multiperiod hedging

1. Introduction

U.S. hog production has undergone a significant change in its market structure. The practice of vertical coordination via contractual arrangements has become prevalent in recent years, thereby reducing the conventional role of spot market transactions. The University of Missouri and National Pork Producers Council (2000) reported that 74.3% of all U.S. hogs slaughtered were sold under non-spot market transactions in January 2000. In contrast, this number was 64.2% in the same month of 1999 and 56.6% in the year of 1997.

The most common marketing practice at this time is forward contracts under the formula pricing method. This accounted for 47.2% of all U.S. hogs sold in January 2000[1]. Formula pricing is generally tied to a reported price plus some fixed premium amount about US$1-US$3 per cwt. The premium acts as a monetary incentive for producers to provide a consistent level of meat quality, whereas the reported price is based on a quoted spot market price when hogs are delivered. In this respect, hog producers are not shielded from price risk as the formula moves dollar for dollar with the cash market. This is contrary to the belief that producers enter into contractual arrangements primarily for the purpose of minimizing price risk in the market place[2]. Adding the spot market transaction figure of 25.7% [3] to the formula-pricing one, 72.9% of all hogs sold were susceptible to spot price risk (directly or indirectly) and together they represented a substantial marketing share.

While many studies have examined futures hedging models to manage spot market risk, the rising importance of forward contracting suggests that such models must be extended to fully address forward contracting. Consider a hog producer who enters into a forward contract under the formula pricing method six months before delivery. The producer makes both optimal hedge and forward contract decisions at the initial stage. The forward contract ratio determines the portion of hogs to be marketed in advance while the remaining portion is to be sold on the spot market. At the subsequent stages

[1] 2000 Hog Marketing Contract Study by University of Missouri and National Pork Producers Council, March 2000.
[2] Producers enter into formula pricing contracts because they can be guaranteed future market access or shackle space when the available hog supply is above slaughter capacity (USITC 1999). This issue will not be discussed in this study.
[3] See [1].

(two and four months later), the forward contract position cannot be adjusted as it becomes a legal and binding agreement. Further complications introduced by forward contracting, beyond the sequencing of hedge decisions, include volatility, time, and time value for example. The forward contract value accounts for these important factors and this risky variable serves as a forward market signal in the decision-making process.

The purpose of this paper is to develop a mean-variance hedge model of optimal sequential forward contracting and hedging decisions when output price and contract value risks are both present for a representative producer. The model builds upon previous research by Haigh and Holt, Mathews and Holthausen, and Antle that takes into consideration all three essential features of a closed loop solution. In addition, this paper introduces a new technique based upon the American put-call parity (APCP) as described in Black (1973) to proxy the effects of forward contract value on optimal sequential decisions. Assuming hedging begins six months before delivery, this requires that the multi-step-ahead forecasts of conditional (time-varying) variances and covariances of risky variables be estimated from a MGARCH model. With changing market conditions (e.g., 1998 hog price crisis), it is important that the producer is able to incorporate dynamic flow of information and to periodically update the "portfolio optimality" over time. The MGARCH methodology combined with the closed loop solution equip producers with a tool to efficiently utilize information of time-varying risks (conditional variances and covariances) in developing sequential optimal hedging strategy.

The paper proceeds as follows. The next sections will detail the conceptual framework of American put-call parity as a proxy for forward contract value, the closed loop solution, sequential forward contracting and hedging, as well as the complete strategies for optimal adjustment of hedging decisions with information revision. This is followed by a description of recent hog market data used in the time-series ARMA-MGARCH estimation. The in-sample and out-of-sample simulated results are discussed. The final section concludes with a summary of the findings.

2. American Put-Call Parity as a Proxy for Forward Contract Value

Hull defines a forward contract as a contract that obligates the holder to buy or sell an asset at a predetermined delivery price at a predetermined future time. In fact, a forward contract can be considered as a risk swap between the producers and processors. The formula for a forward contract value on an investment asset that provides no income is

(1) $f = S - Ke^{-rT}$

where f is the forward contract value of live hogs in a long position today; S is the price of the asset (live hogs) underlying the forward contract today; K is the delivery price of the forward contract; r is the risk-free interest rate per annum today; and T is the time until delivery date (in years). The formula of put-call parity for American option prices is defined as

(2) $C - P \prec S - Xe^{-rT}$

where C is the American call option value to long one call option contract; P is the American put option value to long one put option contract; S is the current asset (live hog) price; X is the exercise price of option; r is the risk-free interest rate; and T is the time to expiration of the option. Hence, the relationship between forward contract value and put-call option can be expressed as the following

(3) $C - P \prec f = S - Ke^{-rT}$.

The difference between the call and put option values provides the lower bound for the forward contract value[4].

Figure 1 illustrates the relationship between the forward contract value and put-call parity. Suppose a call option, a put option, and a forward contract (all in a long position) have the same expiration date T and strike price X. There are no transaction costs involved. The forward contract value can be considered as a swap agreement between two individuals for the payoffs that may otherwise be obtained in the option market. In fact, areas underneath the call and put option lines and above zero represent risky

[4] If the options are European, the difference between call and put options will exactly equal the forward contract value.

outcomes. They describe the expected profits of producers and processors respectively by integrating all the possible profits with respect to the probability density function. The exact value of the contract will be the same as respective put and call values at the expiration date. An individual (processor) may agree to compensate the other party the equivalent payoff of a put option, e.g. at point A, should the price fall below the strike price (delivery price) at the expiration date T (delivery date). Similarly the same individual will be compensated in exchange by the other party by the equivalent payoff of a call option, e.g. at point B, should the price rise above the strike price. This individual effectively transforms his or her financial situation into a long forward contract position by exchanging the cash flows with the other party. Put-call parity actually can be used to describe the dynamic gains and losses relative to producer or processor.

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Formula pricing contracts may involve a system whereby a selling price is normally based on the spot price (or a reported price) plus some premium. In this case, the expected delivery price can be thought of as the expected cash price prevailing on the delivery date T. According to Black, the value of a forward contract when it is initiated is always zero as the delivery price is always set equal to the current futures price for a transaction that will occur at T. Therefore, the expected delivery price can be approximated by the futures price, using the closest futures contract month to the delivery date, at the beginning of production process (when the contract is initiated). As the futures contracts are settled and prices are rewritten on a daily basis[5], producers or processors compensate the other party implicitly should the daily futures prices deviate from the initial expected delivery price. Only the actual gains and losses on the delivery date will be transferred and realized. If the spot price falls below the initial expected cash price at T, producers yield a positive gain. With the formula pricing scheme, this will be transferred back to processors by bringing the initial expected delivery price (strike price) to the realized cash level. This makes the formula price move dollar-for-dollar with the spot market.

For example, Figure 2 illustrates an example of February forward contract values with different strike prices generated using the put-call parity formula. The graphs

[5] Black (1976) describes a futures contract as a series of forward contracts. Each day, yesterday's contract is settled, and today's contract is written with a contract price equal to the futures price with the same maturity as the futures contract.

fluctuate around the mean-zero level and these patterns constitute the forward contract value cycles. Positive forward contract values provide gains (losses) for processors but losses (gains) for producers. Positive values are mutually exclusive so that producers and processors cannot benefit simultaneously. It is a zero-sum game in which the participants swap cash flows of gains and losses. Specifically, the amplitude of the February contract values was the greatest below the mean-zero line during the "hog price crisis", indicating the high volatility (risk) in that period relative to others. It also represents the processors' potential financial losses if the contracts were fixed price and they were to be sold back to producers before the delivery date. Depending on the current and expected future market conditions, gains and losses are uncertain and they may shift between producers and processors within the life of a particular forward contract. Forward contract values evolve dynamically with uncertainty.

3. Features of the Closed Loop Solution to Optimal Contracting and Hedging

Previous studies (e.g., Mathews and Holthausen 1991; and Holt and Brandt 1985) have primarily focused on sequential hedging as a risk management tool for reducing the spot price risk. In Mathews and Holthausen's multiperiod hedge model, a producer is permitted to update the futures position over the production process so that the spot price risk can be managed periodically. With four trading opportunities, an initial hedge is placed six months before delivery and adjustments are made two months and four months thereafter. At the final stage (delivery), the hedge is lifted and all hogs are sold in the spot market. The model is set up with the objective to minimize the portfolio risk by choosing the optimal hedge ratios[6]. These results exhibit three essential features of the closed loop solution presented by Antle (1983) and they relate to how the information is utilized by the decision-maker. Firstly, the portion of hogs to be hedged at each stage is sequentially dependent. For example, the initial hedge ratio is a function of the adjustment ratios such that the risk expectations in subsequent periods are linked to the initial hedging decision. Secondly, earlier information is always learned and incorporated into the subsequent

[6] The hedge ratio is expressed in terms of the proportion of futures positions to cash positions (risk exposure) for an asset.

decisions through the feedback mechanism. Hence, the adjustment ratios are optimized under a lesser risky environment. Thirdly, hedging decisions depend on the conditional second moments in the spot and futures prices. These conditional expectations are formed based on the information set that is revisable. This ensures that the initial hedging optimality can be adjusted when new information becomes available. Holt and Brandt's selective model entails a simple binary strategy: to hedge fully or not to hedge at all depending on the expected price signals at each adjustment stage, and it fails to implement these features. Consequently, their sequential hedging outcome may be limited by the inferior information set.

The core function of Antle's closed loop solution consists of three essential features outlining how information should be utilized:

(a) *Sequential dependence of decisions (open loop solution)*
- *decisions made earlier may affect those made later;*

(b) *Information feedback (sequential updating solution)*
- *information that becomes available during earlier stages may be utilized in subsequent decisions; and*

(c) *Anticipated revision (conditional expectation)*
- *decisions made earlier may be revised later as new information becomes available.*

Suppose initial forward contracting and futures hedging decisions are made at time t and hedge adjustment decisions are followed at later times $t+j$ $(j = 1, 2,...,n)$. Feature (a) is known as the open loop solution. For example, it affects how the initial forward contract ratio is determined at t. As the forward contract position remains fixed, its optimal choice made at t is linked to the optimal hedge adjustment decisions made at $t+j$. In other words, the flexibility in sequential hedge adjustments at $t+j$ $(j = 1, 2,...,n)$ may be used to complement the forward contract rigidity and its inability to respond to risk. Consequently, optimal hedge ratios are dependent on initial forward contract ratio under feature (a). The main limitation of (a) is that information cannot be updated and all unconditional expectations in hedge adjustment decisions are based on the minimal information set prescribed by feature (a).

The information feedback feature (b) is known as the sequential updating solution. It specifies that information that becomes available in the earlier stages will always be learned and incorporated into subsequent optimal decision process. For example, a hedge adjustment replaces the previous futures position at t with a new futures position at $t+2$. The actual profit/loss from the previous futures position is the difference between the corresponding futures prices at t and at $t+2$. It will be observed and known (non-risky) when determining the hedge adjustment ratios at subsequent stages. Although feature (b) does not allow for sequential dependence of decisions as in the open loop solution (a), all unconditional expectations in the hedge adjustment decisions are based on a richer information set than the one of (a). Uncertainty costs may also be reduced due to observability.

The closed loop solution combines both open loop and sequential updating solutions (a) and (b), plus the anticipated revision feature (c). This is considered to be a superior solution to (a), (b), or both. The anticipated revision feature (c) modifies features (a) and (b) with conditional expectations in each sequential stage. Hedging decisions determined in the earlier stages can be revised as new information becomes available. This means that optimal hedge ratios may become time varying and thus more responsive to changing market conditions.

Mathews and Holthausen estimated the sequential hedge ratios for hogs showing an increasing trend through time from 0.912 to 0.942 (from the initial stage to the final stage). This upward adjustment pattern suggests that the live-hog futures contract becomes slightly more effective in reducing spot price risk as delivery nears. One notable observation is the recommendation of 91.2% of hogs to be hedged at the initial stage. This necessarily reflects the high correlation between the spot and futures prices predicted six months ahead and it also means greater ability for futures contracts to mitigate risk. Despite the promising results, Mathews and Holthausen only partially utilize the property of revisable information set. The risk expectations are formed conditional on the information set available at each initial stage and new information revealed through the subsequent stages is not incorporated into adjustment hedge ratio estimations (e.g., information is only revisable at each initial stage). This simplifying assumption is made for the tractability purpose of variance and covariance computations using the method of mean products of forecast errors (Peck 1975), or else it would require too many of them to be estimated. Clearly, the method used to generate the conditional variances and covariances of risky variables could have implications on the hedge ratio performance. This raises the question as to whether the high initial hedge ratio and insignificant upward hedge adjustments obtained by Mathews and Holthausen are in fact "optimal".

To remedy this shortcoming, Haigh and Holt's (2000) mean-variance hedge model incorporates the Multivariate Generalized Autoregressive Conditional Heteroscedasticity (MGARCH) approach to estimate conditional variances and covariances between spot and futures prices simultaneously within a system of equations. These one-step-ahead estimates of variances and covariances are used to calculate the time-varying hedge ratios. Several previous articles have also developed similar GARCH-type models to analyze the impact of time-varying risk on production levels (Holt and Aradhyula 1998, Holt 1993, Holt and Moschini 1992, Holt and Aradhyula 1990). The GARCH-type framework is appropriate because short-term volatility is reflected directly in the optimal hedge ratios conditional on the information set available at the time of decision-making. This is achieved by the expected variance-covariance structure of risky variables being a function of their own lagged values and also lagged values of squared innovations. Hence, a temporary change in volatility would be carried over to subsequent risk expectations and to the optimal hedge ratios in a dynamic way. This is important for modeling risk empirically in U.S. hog production given that there exist periods of high price volatility and also periods of price tranquility[7].

The MGARCH methodology satisfies feature (c) and it is the choice in this study to generate the conditional risk expectations used in optimal hedging decisions. It is also the only feature utilized in Haigh and Holt's non-sequential framework. In contrast, Mathews and Holthausen's sequential framework used all three features but only partially implemented feature (c), constrained by the computational complexity of Peck's method compared to MGARCH.

[7] This also translates to periods of high contract value volatility and tranquility.

4. Sequential Forward Contracting and Hedging Model

Following Haigh and Holt, the optimal hedge framework considers a mean-variance utility function. It is expressed as a linear certainty equivalent profit function, increasing in expected return and decreasing in return variance. The extension to Haigh and Holt's MGARCH framework is that hedging decisions can be sequentially optimized under the closed loop solution presented by Antle and partially implemented by Mathews and Holthausen.

It takes approximately six months for hogs to reach maturity from the birth stage to the slaughter stage. Suppose the birth stage is denoted by t and the marketing stage is denoted by $t+6$. Two-month intervals divide the intermediate adjustment stages at $t+2$ and at $t+4$. A representative hog producer is limited to only the two major marketing alternatives: (i) to sell in the spot market at $t+6$; and/or (ii) to forward contract in advance with formula-pricing method at t for delivery at $t+6$. It is assumed that all hedging, forward contracting, as well as marketing decisions are made in the first week of the month.

For simplicity, this paper only considers hogs that are born and marketed in the months of June and December. Hence, futures hedges and forward contracts are initiated in the first week of June (December) and they are lifted or expired in the first week of December (June). Hedge adjustments are made every two months in the first weeks of February and April for the June futures contracts (August and October for the December futures contracts). A commission cost of US$0.15/cwt. is levied when a futures hedge is placed.

At stage t, the producer chooses the optimal forward contract and futures hedge (by going short in the futures market) positions concurrently. The forward contract position cannot be adjusted once it is locked in, since it becomes a legal binding agreement. However, the hedge position can be adjusted at intermediate dates $t+2$ and $t+4$ (every two months) by closing the previous futures position and opening a new one. This allows the producer to respond to any changes in market conditions when new information arrives and to update the "optimality" of the initial hedge position accordingly. At stage $t+6$, the producer lifts his or her hedge and completes the transactions in both spot and forward contract markets.

Initial Hedging and Forward Contracting Decisions with Closed Loop Features

The first-period forward contract and hedge ratios at the initial stage t are determined using a three-period optimization process beginning at stage $t+4$[8]. The producer maximizes a mean-variance utility function, with respect to the expected third-period ($t+4$) hedge ratio h_3, conditional on the information set Ω_t available at t:

$$(4) \qquad \underset{h_3}{Max}\, E_t[U_{t+6}] = \underset{h_3}{Max}\, E_t[(\pi_{t+6} \mid \Omega_t) - \frac{\lambda}{2}\sigma^2(\pi_{t+6} \mid \Omega_t)].$$

E_t is the conditional expectation operator (feature (c)) based on the information set Ω_t available at birth stage t, λ is the risk aversion coefficient set to equal two, as also assumed by Haigh and Holt in their work. The fourth-period $(t+6)$ profit Π_{t+6} (per cwt.) can be expressed as

$$(5) \qquad \begin{aligned} \pi_{t+6} &= P_{t+6}(1-\alpha_1) + (\overline{F_{t|t+6}} + APCP_{t+6} + K)\alpha_1 + (F_{t+6|t+6} - F_{t+4|t+6} - M)h_3 \\ &+ (F_{t+4|t+6} - F_{t+2|t+6} - M)h_2 + (F_{t+2|t+6} - F_{t|t+6} - M)h_1 - c \end{aligned}$$

where P_{t+6} is the spot price at $t+6$. $\overline{F_{t|t+6}}$ is the futures price at t expiring at $t+6$ and it is also taken as the expected delivery price at t (strike price). $F_{t+2|t+6}$ and $F_{t+4|t+6}$ are the futures prices at $t+2$ and at $t+4$ expiring at $t+6$. $APCP_{t+6}$ is the American put-call parity value at $t+6$ having a strike price of (or close to) $\overline{F_{t|t+6}}$. In fact, $APCP_{t+6}$ corrects the expected delivery price back to the spot level and it can also be viewed as the deviation above or below $\overline{F_{t|t+6}}$. K is the fixed formula-pricing premium and it is set at US\$2/cwt (taken at the midpoint of \$1-\$3 per cwt). c is constant marginal cost assuming the production exhibits constant-returns-to-scale technology. α_1 is the forward contract ratio selected at t ($0 \le \alpha_1 \le 1$). h_1, h_2, and h_3 are the sequential hedge ratios selected at t, $t+2$, and $t+4$ successively and these are negative if the producer goes short on the positions and positive otherwise. The difference terms $(F_{t+6|t+6} - F_{t+4|t+6})$, $(F_{t+4|t+6} - F_{t+2|t+6})$, and $(F_{t+2|t+6} - F_{t|t+6})$ represent the profits/losses from hedging at different sequential stages at t, $t+2$, and $t+4$. M is the commission cost of US\$0.15/cwt. for futures contract trade and it

[8] This follows from Mathews and Holthausen's optimization method, moving backward from $t+4$ to t. However, the results would be the same if optimization is taken forward from t to $t+4$ under the closed loop features.

is subtracted from these profits/losses. To simplify the notations, $(F_{t+6|t+6}$ $-F_{t+4|t+6}$ $-M)$, $(F_{t+4|t+6}$ $-F_{t+2|t+6}$ $-M)$, and $(F_{t+2|t+6}$ $-F_{t|t+6}$ $-M)$ are replaced by dF_{6t}, dF_{4t}, and dF_{2t} respectively.

By implementing the closed loop solution feature (b), the only risky variables in equation (6) are P_{t+6}, dF_{6t} and $APCP_{t+6}$ at $t+4$. The information feedback feature (b) allows the previous hedge ratios h_2 and h_1, hedge adjustment profits/losses dF_{4t} and dF_{2t}, initial forward contract ratio α_1, fixed premium K, and constant marginal cost c all be learned and incorporated at $t+4$. There will be no uncertainty associated with them and the variance of the profit $\sigma^2(\Pi_{t+6})$ at $t+4$ is

$$\text{(6)}\quad \begin{aligned}\sigma^2(\pi_{t+6}): at\, t+4 := {} & (1-\alpha_1)^2\sigma^2_{Pt+6} + \alpha_1{}^2\sigma^2_{APCPt+6} + h_3^2\sigma^2_{dF6t} + 2(1-\alpha_1)\alpha_1\sigma_{Pt+6,PCt+6} \\ & + 2(1-\alpha_1)h_3\sigma_{Pt+6,dF6t} + 2\alpha_1 h_3\sigma_{APCPCt+6,dF6t},\end{aligned}$$

where σ^2_x represents the variance of risky variable x and $\sigma_{x,y}$ represents the covariance between risky variables x and y. Substituting equations (5) and (6) into (4) and maximizing with respect to the third-period hedge ratio h_3 conditional on the information set available at t, the first order condition is

$$\text{(7)}\quad \frac{\partial E_t[U_{t+6}]}{\partial h_3} = E_t[dF_{6t} - \lambda(h_3\sigma^2_{dF6t} + (1-(\alpha_1^*|_t))\sigma_{Pt+6,dF6t} + (\alpha_1^*|_t)\sigma_{APCPt+6,dF6t})] = 0.$$

Solving for the optimal hedge ratio $h_3^*|_t$ (conditional on the information set t) from (7),

$$\text{(8)}\quad h_3^*|_t = \frac{E_t(dF_{6t}) - \lambda[(1-(\alpha_1^*|_t))E_t(\sigma_{Pt+6,dF6t}) + (\alpha_1^*|_t)E_t(\sigma_{APCPt+6,dF6t})]}{\lambda E_t(\sigma^2_{dF6t})}.$$

Similarly moving from $t+4$ to $t+2$ and using feature (b), the risky variables in equation (9) this time are P_{t+6}, dF_{6t}, dF_{4t}, and $APCP_{t+6}$. The uncertainty cost becomes greater due to the smaller information set when the decision stage gets closer to t. The variance of the profit $\sigma^2(\Pi_{t+6})$ at $t+2$ is

$$\text{(9)}\quad \begin{aligned}\sigma^2(\pi_{t+6}): at\, t+2 := {} & (1-\alpha_1)^2\sigma^2_{Pt+6} + \alpha_1{}^2\sigma^2_{APCPCt+6} + h_3^2\sigma^2_{dF6t} + h_2^2\sigma^2_{dF4t} \\ & + 2(1-\alpha_1)\alpha_1\sigma_{Pt+6,APCPt+6} + 2(1-\alpha_1)h_3\sigma_{Pt+6,dF6t} + 2(1-\alpha_1)h_2\sigma_{Pt+6,dF4t} \\ & + 2\alpha_1 h_3\sigma_{APCPt+6,dF6t} + 2\alpha_1 h_2\sigma_{APCPt+6,dF4t} + 2h_2h_3\sigma_{dF6t,dF4t}.\end{aligned}$$

Substituting equations (5), (8) and (9) into (4) and maximizing with respect to the expected second-period hedge ratio h_2 conditional on the information set available at t, the first order condition is

$$(10)\quad \frac{\partial E_t[U_{t+6}]}{\partial h_2} = E_t[dF_{4t} - \lambda(h_2\sigma^2_{dF4t} + (1-(\alpha_1|_t^*))\sigma_{Pt+6,dF4t} + (\alpha_1|_t^*)_1\sigma_{APCPt+6,dF4t} + (h_3^*|_t)\sigma_{dF6t,dF4t})] = 0.$$

Solving for the optimal hedge ratio $h_2^*|_t$ (conditional on the information set t) from (10),

$$(11)\quad h_2^*|_t = \frac{E_t(dF_{4t}) - \lambda[(1-(\alpha_1^*|_t))E_t(\sigma_{Pt+6,dF4t}) + (\alpha_1^*|_t)E_t(\sigma_{APCPt+6,dF4t}) + (h_3^*|_t)E_t(\sigma_{dF6t,dF4t})]}{\lambda E_t(\sigma^2_{dF4t})}.$$

Lastly moving from $t+2$ to the initial stage t and again using feature (b), the risky variables are P_{t+6}, dF_{6t}, dF_{4t}, dF_{2t}, and $APCP_{t+6}$. The variance of the profit $\sigma^2(\Pi_{t+6})$ at t is

$$(12)\quad \begin{aligned}\sigma^2(\pi_{t+6}): at\ t := {} & (1-\alpha_1)^2\sigma^2_{Pt+6} + \alpha_1^2\sigma^2_{APCPt+6} + h_3^2\sigma^2_{dF6t} + h_2^2\sigma^2_{dF4t} + h_1^2\sigma^2_{dF2t} \\ & + 2(1-\alpha_1)\alpha_1\sigma_{Pt+6,APCPt+6} + 2(1-\alpha_1)h_3\sigma_{Pt+6,dF6t} + 2(1-\alpha_1)h_2\sigma_{Pt+6,dF4t} \\ & + 2(1-\alpha_1)h_1\sigma_{Pt+6,dF2t} + 2\alpha_1 h_3\sigma_{APCPt+6,dF6t} + 2\alpha_1 h_2\sigma_{APCPt+6,dF4t} \\ & + 2\alpha_1 h_1\sigma_{APCPt+6,dF2t} + 2h_2h_3\sigma_{dF6t,dF4t} + 2h_1h_3\sigma_{dF6t,dF2t} + 2h_1h_2\sigma_{dF4t,dF2t}.\end{aligned}$$

Substituting (5), (8), (11), and (12) into (4) and maximizing with respect to the first-period hedge ratio h_1 and the first-period forward contract ratio α_1 conditional on the information set available at t, the first order conditions are

$$(13)\quad \frac{\partial E_t[U_{t+6}]}{\partial h_1} = E_t[dF_{2t} - \lambda(h_1\sigma^2_{dF2t} + (1-(\alpha_1|_t^*))\sigma_{Pt+6,dF2t} + (\alpha_1|_t^*)_1\sigma_{APCPt+6,dF2t} + (h_3^*|_t)\sigma_{dF6t,dF2t}) + (h_2^*|_t)\sigma_{dF4t,dF2t})] = 0.$$

$$(14)\quad \frac{\partial E_t[U_{t+6}]}{\partial \alpha_1} = E_t[G_1 + (h_3^*|_t)G_2 + (h_2^*|_t)G_3 + (h_1^*|_t)G_4 + (\alpha_1)G_5] = 0,$$

where

$$G_1 = (\overline{F_{t|t+6}} + APCP_{t+6} + K) - P_{t+6} + (dF_{6t} - \lambda\sigma_{Pt+3,dF3t})\frac{\partial h_3^*|_t}{\partial \alpha_1}$$

$$+ (dF_{4t} - \lambda\sigma_{Pt+6,dF4t})\frac{\partial h_2^*|_t}{\partial \alpha_1} + (dF_{2t} - \lambda\sigma_{Pt+6,dF2t})\frac{\partial h_1^*|_t}{\partial \alpha_1} + \lambda\sigma^2{}_{Pt+6}$$

$$- \lambda\sigma_{Pt+6,APCPt+6};$$

$$G_2 = -\lambda[(\sigma^2{}_{dF6t})\frac{\partial h_3^*|_t}{\partial \alpha_1} - \sigma_{Pt+6,dF6t} + \sigma_{APCPt+6,dF6t} + (\sigma_{dF6t,dF4t})\frac{\partial h_2^*|_t}{\partial \alpha_1}$$

$$+ (\sigma_{dF6t,dF2t})\frac{\partial h_1^*|_t}{\partial \alpha_1}];$$

$$G_3 = -\lambda[(\sigma^2{}_{dF4t})\frac{\partial h_2^*|_t}{\partial \alpha_1} - \sigma_{Pt+6,dF4t} + \sigma_{APCPt+6,dF4t} + (\sigma_{dF6t,dF4t})\frac{\partial h_3^*|_t}{\partial \alpha_1}$$

$$+ (\sigma_{dF4t,dF2t})\frac{\partial h_1^*|_t}{\partial \alpha_1}];$$

$$G_4 = -\lambda[(\sigma^2{}_{dF2t})\frac{\partial h_1^*|_t}{\partial \alpha_1} - \sigma_{Pt+6,dF2t} + \sigma_{APCPt+6,dF2t} + (\sigma_{dF6t,dF2t})\frac{\partial h_3^*|_t}{\partial \alpha_1}$$

$$+ (\sigma_{dF4t,dF2t})\frac{\partial h_2^*|_t}{\partial \alpha_1}];$$

and

$$G_5 = -\lambda[\sigma^2{}_{Pt+6} + \sigma^2{}_{APCPt+6} - 2\sigma_{Pt+6,APCPt+6} - (\sigma_{Pt+6,dF6t})\frac{\partial h_3^*|_t}{\partial \alpha_1} - (\sigma_{Pt+6,dF4t})\frac{\partial h_2^*|_t}{\partial \alpha_1}$$

$$+ (\sigma_{Pt+6,dF2t})\frac{\partial h_1^*|_t}{\partial \alpha_1} + (\sigma_{APCPt+6,dF6t})\frac{\partial h_3^*|_t}{\partial \alpha_1} + (\sigma_{APCPt+6,dF4t})\frac{\partial h_2^*|_t}{\partial \alpha_1}$$

$$+ (\sigma_{APCPt+6,dF2t})\frac{\partial h_1^*|_t}{\partial \alpha_1}].$$

Solving for the optimal hedge ratio $h_1^*|_t$ and forward contract ratio $\alpha_1^*|_t$ (conditional on the information set t) from (13) and (14),

$$(15) \qquad h_1^*|_t = \frac{\begin{array}{c} E_t(dF_{2t}) - \lambda[(1-(\alpha_1^*|_t))E_t(\sigma_{Pt+6,dF2t}) + (\alpha_1^*|_t)E_t(\sigma_{APCPt+6,dF2t}) \\ + (h_3^*|_t)E_t(\sigma_{dF6t,dF2t}) + (h_2^*|_t)E_t(\sigma_{dF4t,dF2t})] \end{array}}{\lambda E_t(\sigma^2_{dF2t})},$$

$$(16)\qquad \alpha_1^*|_t = \frac{-E_t(G_1 + (h_3^*|_t)G_2 + (h_2^*|_t)G_3 + (h_1^*|_t)G_4)}{E_t(G_5)}.$$

All optimal hedge ratios are negative for short positions but positive for the optimal forward contract ratio. Equation (15) provides the optimal decision rule for the first-period hedging conditional on the information set available at t. The first term $E_t(dF_{2t})$ in the equation represents the speculative part of the decision. For example, if the next period futures price (F_{t+2}) is expected to increase, the speculative part is positive and the producer reduces his or her first-period hedge ratio in the short position in response to the futures loss. The conditional covariance terms associated with $h_2^*|_t$ and $h_3^*|_t$ ($\sigma_{dF4t,dF2t}$ and $\sigma_{dF6t,dF2t}$) represent the risk management part of the decision. If these covariances are positive, the futures gain/loss in the first period (dF_{2t}) is expected to move in the same direction of those in the second and third periods (dF_{4t} and dF_{6t}). With rising futures prices, each sequential stage could expose the producer to the downside risk of futures losses. This has a negative effect on the first-period hedge ratio ($h_1^*|_t$) given the expectation that hedge adjustments may not be effective for subsequent risk reduction. Similarly, the conditional covariance terms with $\alpha_1^*|_t$ ($\sigma_{Pt+6,dF2t}$ *and* $\sigma_{APCPt+6,dF2t}$) also represent the risk management part of the decision but they are related to how hedging can reduce spot price and contract value risks. If $\sigma_{Pt+6,dF2t}$ and $\sigma_{APCPt+6,dF2t}$ are positive and $\sigma_{Pt+6,dF2t} > \sigma_{APCPt+6,dF2t}$, it means that the expected spot price is more correlated with the first-period futures gain/loss than is the expected contract value at the delivery date $t+6$. Therefore, hedging is expected to be relatively effective in reducing the contract value risk. The producer substitutes away from the spot market transaction towards the forward contract one at t, and this has a negative effect on the first-period hedge ratio ($h_1^*|_t$)[9].

Equation (16) provides the optimal decision rule for the first-period forward contracting conditional on the information set available at t. This equation contains some partial derivative terms of hedge ratios ($h_1^*|_t$, $h_2^*|_t$, and $h_3^*|_t$) taken with respect to the forward contract ratio ($\alpha_1^*|_t$). This results from the implementation of feature (a), sequential dependence of decisions, which explicitly includes the indirect effect of the current forward contracting decision (at t) on current and subsequent hedging stages (*at t, $t+2$,* and *$t+4$*). However, feature (a) could become irrelevant if first-order conditions (8) and (11) are independent of the optimal forward contract choice ($\alpha_1^*|_t$) made at t. This requires that the conditional covariances associated with α_1 within (8) and (11) are either zero or identical.

Hedging Adjustment Decisions with Information Revision

The optimal forward contract and hedge ratios $\alpha_1^*|_t$ and $h_1^*|_t$ are determined simultaneously by (8), (11), (15), and (16) with conditional expectations based on the initial information set available at t, with the knowledge that more information will become available in the future under anticipated revision feature (c). Feature (c) allows the producer to revise these conditional expectations in later stages. By carrying over $\alpha_1^*|_t$ from t, the revised optimal hedge ratio $h_3^*|_{t+1}$ can be found by using first-order conditions (7) based on the new information set available at $t+2$

$$(17)\quad \frac{\partial E_{t+2}[U_{t+6}]}{\partial h_3} = E_{t+2}[dF_{6t} - \lambda(h_3\sigma^2_{dF6t} + (1-(\alpha_1^*|_t))\sigma_{Pt+6,dF6t} + (\alpha_1^*|_t)\sigma_{APCPt+6,dF6t})] = 0.$$

Solving for $h_3^*|_{t+2}$ (conditional on the information set $t+2$) from (17),

$$(18)\quad h_3^*|_{t+2} = \frac{E_{t+2}(dF_{6t}) - \lambda[(1-(\alpha_1^*|_t))E_{t+2}(\sigma_{Pt+6,dF6t}) + (\alpha_1^*|_t)E_{t+2}(\sigma_{APCPt+6,dF6t})]}{\lambda E_{t+1}(\sigma^2_{dF6t})}.$$

The revised optimal hedge ratio $h_2^*|_{t+2}$ can be found by using first-order conditions (10) based on the new information set available at $t+2$

$$(19)\quad \frac{\partial E_{t+2}[U_{t+6}]}{\partial h_2} = E_{t+2}[dF_{4t} - \lambda(h_2\sigma^2_{dF4t} + (1-(\alpha_1|_t^*))\sigma_{Pt+6,dF4t} + (\alpha_1|_t^*)_1\sigma_{APCPt+6,dF4t} + (h_3^*|_{t+2})\sigma_{dF6t,dF4t})] = 0.$$

Solving for $h_2^*|_{t+2}$ (conditional on the information set $t+2$) from (19),

[9] The speculative and risk management parts in (12) and (15) can be explained similarly.

$$(20)\quad h_2^*|_{t+2} = \frac{E_{t+2}(dF_{4t}) - \lambda[(1-(\alpha_1^*|_t))E_{t+2}(\sigma_{Pt+6,dF4t}) + (\alpha_1^*|_t)E_{t+2}(\sigma_{APCPt+6,dF4t}) + (h_3^*|_{t+2})E_{t+2}(\sigma_{dF6t,dF4t})]}{\lambda E_{t+1}(\sigma^2_{dF4t})}.$$

The revised optimal hedge ratio $h_2^*|_{t+2}$ is determined simultaneously by (18) and (20) based on the new information set available at $t+2$.

At the last hedge adjustment stage $t+4$, the information set used to determine the optimal hedge ratio is revised again with the new information set. Carrying over $\alpha_1^*|_t$ from t, first-order condition (7) becomes

$$(21)\quad \frac{\partial E_{t+4}[U_{t+6}]}{\partial h_3} = E_{t+4}[dF_{6t} - \lambda(h_3\sigma^2_{dF6t} + (1-(\alpha_1^*|_t))\sigma_{Pt+6,dF6t} + (\alpha_1^*|_t)\sigma_{APCPt+6,dF6t})] = 0.$$

Solving for $h_3^*|_{t+4}$ (conditional on the information set $t+4$) from (21),

$$(22)\quad h_3^*|_{t+4} = \frac{E_{t+4}(dF_{6t}) - \lambda[(1-(\alpha_1^*|_t))E_{t+4}(\sigma_{Pt+6,dF6t}) + (\alpha_1^*|_t)E_{t+4}(\sigma_{APCPt+6,dF6t})]}{\lambda E_{t+4}(\sigma^2_{dF6t})}.$$

The revised optimal hedge ratio $h_3^*|_{t+4}$ is determined by (22) based on the information set available at $t+4$.

Complete Strategies for Optimal Sequential Hedging and Forward Contracting

The complete strategies for optimal sequential hedging and forward contracting under the closed-loop solution are as follows:

(i) At stage t, initial forward contract and hedge ratios $\alpha_1^*|_t$ and $h_1^*|_t$ are obtained by solving

$$\begin{bmatrix} 1 & \frac{E_t(\sigma_{dF4t,dF2t})}{E_t(\sigma^2_{dF2t})} & \frac{E_t(\sigma_{dF6t,dF2t})}{E_t(\sigma^2_{dF2t})} & \frac{E_t(\sigma_{Pt+6,dF2t} - \sigma_{APCPt+6,dF2t})}{E_t(\sigma^2_{dF2t})} \\ 0 & 1 & \frac{E_t(\sigma_{dF6t,dF4t})}{E_t(\sigma^2_{dF4t})} & \frac{E_t(\sigma_{Pt+6,dF4t} - \sigma_{APCPt+6,dF4t})}{E_t(\sigma^2_{dF4t})} \\ 0 & 0 & 1 & \frac{E_t(\sigma_{Pt+6,dF6t} - \sigma_{APCPt+6,dF6t})}{E_t(\sigma^2_{dF6t})} \\ \frac{E_t(G_4)}{E_t(G_5)} & \frac{E_t(G_3)}{E_t(G_5)} & \frac{E_t(G_2)}{E_t(G_5)} & 1 \end{bmatrix} \begin{bmatrix} h_1^*|_t \\ h_2^*|_t \\ h_3^*|_t \\ \alpha_1^*|_t \end{bmatrix} = \begin{bmatrix} \frac{E_t(dF_{2t} - \lambda\sigma_{Pt+6,dF2t})}{\lambda E_t(\sigma^2_{dF2t})} \\ \frac{E_t(dF_{4t} - \lambda\sigma_{Pt+6,dF4t})}{\lambda E_t(\sigma^2_{dF4t})} \\ \frac{E_t(dF_{6t} - \lambda\sigma_{Pt+6,dF6t})}{\lambda E_t(\sigma^2_{dF6t})} \\ -\frac{E_t(G_1)}{E_t(G_5)} \end{bmatrix};$$

(ii) At stage $t+2$, initial forward contract ratio $\alpha_1^*|_t$ is carried over. The information set is revised to $t+2$ and hedge adjustment ratio $h_2^*|_{t+2}$ is obtained by solving

$$\begin{bmatrix} 1 & \frac{E_{t+2}(\sigma_{dF6t,dF4t})}{E_{t+2}(\sigma^2{}_{dF4t})} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} h_2^*|_{t+2} \\ h_3^*|_{t+2} \end{bmatrix} = \begin{bmatrix} \frac{E_{t+2}(dF_{4t}) - \lambda[(1-\alpha_1^*|_t)E_{t+2}(\sigma_{Pt+6,dF4t}) + (\alpha_1^*|_t)E_{t+2}(\sigma_{APCPt+6,dF4t})]}{\lambda E_{t+2}(\sigma^2{}_{dF4t})} \\ \frac{E_{t+2}(dF_{6t}) - \lambda[(1-\alpha_1^*|_t)E_{t+2}(\sigma_{Pt+6,dF6t}) + (\alpha_1^*|_t)E_{t+2}(\sigma_{APCPt+6,dF6t})]}{\lambda E_{t+2}(\sigma^2{}_{dF6t})} \end{bmatrix}; \quad \textit{and}$$

(iii) At stage $t+4$, initial forward contract ratio $\alpha_1^*|_t$ is carried over. The information set is revised to $t+4$ and hedge adjustment ratio $h_3^*|_{t+4}$ is obtained by solving

$$h_3^*|_{t+4} = \frac{E_{t+4}(dF_{6t}) - \lambda[(1-(\alpha_1^*|_t))E_{t+4}(\sigma_{Pt+6,dF6t}) + (\alpha_1^*|_t)E_{t+4}(\sigma_{APCPt+6,dF6t})]}{\lambda E_{t+4}(\sigma^2_{dF6t})}.$$

5. Data and Time-Series ARMA-MGARCH Estimation Results

Data

All of the monthly data series are collected for the period from January 1988 to May 1999 for a total of 137 observations. The in-sample analysis covers March 1988 to December 1995. The remaining data from January 1996 to May 1999 will be reserved for assessing the out-of-sample effectiveness of the closed loop hedging strategy vis-à-vis other alternatives (no hedging and non-adjustable MGARCH hedging).

The weekly IA-MN live hog prices (US$/cwt., barrow and gilts, 230-240 lbs) were obtained from the Livestock Marketing Information Center (http://lmic1.co.nrcs.usda.gov/). The average live hog price for the first week of the relevant month is taken to represent the monthly price P_t. If the first working day of the month begins on a Friday, the average price of the second week is used.

The daily CME live/lean hog futures and options prices were purchased from the Futures Industry Institute. A simple average of the daily June futures prices during the first week of each month are used to represent the monthly futures price series for the June contracts. These weekly averages contain the first working day of the month. If the first working day begins on Friday, the following week is used. The monthly futures price series for the December contracts are also constructed following the same process.

To obtain futures contract gains/losses from hedging (US$/cwt.), these futures prices are differenced between two-month intervals to reflect the hedge adjustment frequency. The June and December futures contracts are combined to form the monthly data series of futures gains/losses dF_t. This allows for consecutive hedging at two marketing stages in June and December (January to June- June futures contracts; July to December- December futures contracts). For example, the futures gain/loss in the month of January is the difference between the June futures price in January and the June futures price in November of the previous year. The July futures gain/loss is the difference between the December futures price in July and the December futures price in May.

To obtain the APCP values (US$/cwt.), the strike price for the June (December) call and put options is taken to be the one closest to the average June (December) futures price in December of the previous year (June of the following year) when forward contracts are entered. The strike price is fixed thereafter until the delivery month of June (December). The APCP values are the difference between the call and put settle prices, with respect to the chosen strike price. Similar to the futures price series, a simple average of the daily June APCP values during the first week in each month are calculated to represent the monthly APCP value series for the June contracts. These weekly averages contain the first working day of the month. If the first working day begins on Friday, the following week is used. The monthly APCP value series for the December contracts are also constructed using the same process. The June and December options contracts are combined to represent the monthly series of $APCP_t$. This allows for consecutive forward contracting at two birth stages in June and December (January to June- June call and put options contracts; July to December- December put and call options contracts).

With the exception of delivery months in June and December, the respective APCP values are calculated using equation (3) due to unavailability of the options data. This is supported by the fact that T (time to expiration of the option) is close to zero in the months that the options contracts expire. As a result, the discount factor can be disregarded and equation (3) reduces to be only the difference between the current price and the initial strike price. For example, the June APCP value (June options contracts) is the difference between the live hog price P_t in June and the initial strike price (June futures price) chosen in December of the previous year.

As for the missing options data from July 1993 to November 1993, this data is also approximated using equation (3) but the discount factor is taken into consideration. The risk-free interest rates used for discounting are the monthly one-year treasury bill rates, auction average, from the Federal Reserve Bank of St. Louis (www.stls.frb.org/fred/data/irates/tb1ya).

The live hog futures and options contracts were replaced by the lean hog contracts in January 1997. The lean hog futures gains/losses and APCP values from January 1997 to May 1999 in the out-of-sample analysis are converted to their corresponding live hog values by a multiplication factor of 0.74 (Wellman 1996).

Each data series is tested for stationarity and this is conducted by the augmented Dickey-Fuller test (1981) (ADF) over the in-sample period. The presence of a unit root is rejected in all series (they are concluded to be stationary).

ARMA-MGARCH Specification

Bollerslev's GARCH model (1986) is an extension to the original ARCH model developed by Engle (1982). The main shortcoming of ARCH is that it can be non-parsimonious when the lag length becomes undesirably large. GARCH can be used to circumvent this possibility: the conditional variance is not only dependent on the past realized values of errors but also on the past conditional variances. As GARCH is able to track/forecast the second-moment dynamics, this facilitates the conditional expectations to be revised periodically and it is essential for implementing feature (c) of the closed loop solution.

In the multivariate GARCH setting, the parsimonious MGARCH(1,1) specification is chosen to model the time-varying conditional variances and covariances. According to Engle and Kroner, the BEKK parameterization of W_t is specified as

$$W_t = C'C + A'\varepsilon_{t-1}\varepsilon_{t-1}'A + B'W_{t-1}B$$

$$\begin{bmatrix} w_{11t} & w_{12t} & w_{13t} \\ w_{12t} & w_{22t} & w_{23t} \\ w_{13t} & w_{23t} & w_{33t} \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ 0 & c_{22} & c_{23} \\ 0 & 0 & c_{33} \end{bmatrix}' \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ 0 & c_{22} & c_{23} \\ 0 & 0 & c_{33} \end{bmatrix}$$

$$(23) \quad + \begin{bmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} \\ \alpha_{21} & \alpha_{22} & \alpha_{23} \\ \alpha_{31} & \alpha_{32} & \alpha_{33} \end{bmatrix}' \begin{bmatrix} \varepsilon_{1t-1}^2 & \varepsilon_{1t-1}\varepsilon_{2t-1} & \varepsilon_{1t-1}\varepsilon_{3t-1} \\ \varepsilon_{1t-1}\varepsilon_{2t-1} & \varepsilon_{2t-1}^2 & \varepsilon_{2t-1}\varepsilon_{3t-1} \\ \varepsilon_{1t-1}\varepsilon_{3t-1} & \varepsilon_{2t-1}\varepsilon_{3t-1} & \varepsilon_{3t-1}^2 \end{bmatrix} \begin{bmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} \\ \alpha_{21} & \alpha_{22} & \alpha_{23} \\ \alpha_{31} & \alpha_{32} & \alpha_{33} \end{bmatrix},$$

$$+ \begin{bmatrix} \beta_{11} & \beta_{12} & \beta_{13} \\ \beta_{21} & \beta_{22} & \beta_{23} \\ \beta_{31} & \beta_{32} & \beta_{33} \end{bmatrix}' \begin{bmatrix} w_{11t-1} & w_{12t-1} & w_{13t-1} \\ w_{12t-1} & w_{22t-1} & w_{23t-1} \\ w_{13t-1} & w_{23t-1} & w_{33t-1} \end{bmatrix} \begin{bmatrix} \beta_{11} & \beta_{12} & \beta_{13} \\ \beta_{21} & \beta_{22} & \beta_{23} \\ \beta_{31} & \beta_{32} & \beta_{33} \end{bmatrix}$$

where 1 = futures gains/losses dFP_t; 2 = American put-call parity values $APCP_t$; 3 = live hog price P_t; W_t is a (3x3) symmetric conditional variance-covariance matrix; and C, A, and B are (3x3) parameter matrices with C being triangular. The BEKK[10] representation of (23) ensures the conditional variance-covariance matrix is positive semidefinite. The Vector Autoregressive Moving Average (VARMA) structure also allows for parametric interactions in the MGARCH process to be present across all second-moment equations.

The econometric specification of the multivariate model is

$$(24) \quad \boxed{\begin{aligned} & dFP_t = \delta_1 dFP_{t-1} + \delta_2 dFP_{t-2} + \varepsilon_{1t}; \\ & APCP_t = \lambda_1 APCP_{t-1} + \lambda_2 APCP_{t-2} + \varepsilon_{2t}; \\ & P_t = \phi_0 + \phi_1 P_{t-1} + \varepsilon_{3t} + \gamma_{12}\varepsilon_{3t-12}; \\ & \varepsilon_t' = [\varepsilon_{1t} \quad \varepsilon_{2t} \quad \varepsilon_{3t}] \mid \Omega_{t-1} \sim N(0, W_t); \text{ and} \\ & W_t = C'C + A'\varepsilon_{t-1}\varepsilon_{t-1}'A + B'W_{t-1}B_t. \end{aligned}}$$

The appropriate p-q-order of ARMA(p, q) mean equations is selected by using the sample autocorrelation function and partial autocorrelation function of each series in conjunction with the Schwartz Bayesian Criterion (SBC). The system of equations contains 31 parameters in (24). Under the assumption of conditional normality, they are estimated simultaneously using the nonlinear FIML procedure. The Broyden-Fletcher-Goldfarb-Shanno (BFGS) algorithm from the software package RATS Version 4 is used to obtain the estimation results.

[10] The acronym BEKK comes from the earlier paper by Baba, Engle, Kraft, and Kroner (1989).

Multivariate ARMA-GARCH Estimation Results

Table 1 summarizes the estimation results of (24). All coefficients in the ARMA mean equations are significantly different from zero at the 5% level, with the exception of the second autoregressive coefficient λ_2 in $APCP_t$. In addition, the characteristic roots of the mean equations lie within the unit circle and this confirms that all series dFP_t, $APCP_t$, and P_t are convergent.

Analysis of the estimated BEKK parameters of the MGARCH matrices A and B reveals that the majority of them are highly significant at the 5% level. Particularly, the off-diagonal elements in both matrices show that there are notable parametric linkages between the conditional variance and covariance equations. In other words, the live hog price, futures gains/losses, and APCP values all have influences on each other's innovations and variances. This is not surprising as these series react to current and expected market conditions. Any perturbations would result in movements in the live hog prices, futures gains/losses, and APCP values. The results indicate that the BEKK parameterization of MGARCH(1,1) is able to capture such cross-effects in volatility reasonably well.

In all diagnostic tests of the standardized and squared standardized residuals (Ng 1991), Q(12) and $Q^2(12)$ of the Ljung-Box statistics show strong evidence of no serial correlation in both conditional first and second moments at the 5% level. Hence, the multivariate model provides adequate descriptions of conditional mean and variance-covariance dynamics.

6. The Closed Loop Solution for Optimal Sequential Hedging and Forward Contracting

The Complete Strategies: In-Sample Simulations

The conditional variances and covariances at different sequential stages are computed from the MGARCH parameter estimates so that the complete strategies can be implemented[11]. The in-sample simulations are performed using the data from December 1988 to October 1995, consisting of 14 production cycles from birth to marketing. The optimal sequential hedge and forward contract ratios under the closed loop solution are presented in Table 2. The optimal forward contract ratios indicate that, in almost all instances, the producer should market his/her hogs entirely in advance at the initial stage t under the formula-pricing forward contracts. The negative sign associated with all of the hedge ratios implies that the producer should go short with the CME hog futures contracts, which also agrees with the appropriate hedge position if the spot price and contract value risks are to be neutralized.

The plots of the optimal hedge ratios at t, $t+2$, and $t+4$ are illustrated separately in Figure 3, Figure 4, and Figure 5. The time-varying pattern suggests that the hedge ratios tend to increase over the adjustment stages and the negative signs also reflect that short positions are taken in order to reduce risks. In fact, the optimal results recommend significant upward adjustments by twofold from –0.3006 at t to –0.6350 at $t+4$ at the mean level. This trend reflects the greater positive correlation between the expected spot price and futures gain/loss when the delivery month approaches. Hence, the risk management part of the decision calls for additional hedging in the CME futures market.

In order to assess the effectiveness of the closed loop solution for reducing both spot price and contract value risks, two different strategies are also considered for comparative purposes: (i) no hedging and (ii) non-adjustable MGARCH hedging. This provides the representative hog producer with the options to do nothing in the CME futures market (no hedging), or to hedge optimally in the initial period t but with no adjustments and information revisions taking place at $t+2$ and $t+4$ (non-adjustable MGARCH hedging). The expected utility improvement of the complete strategy is measured by the percentage change of the certainty equivalent profit level (expected income utility) over these two alternative portfolios.

In contrast with the closed loop solution, the no hedging alternative sets the hedge ratios constant so that $h_1^{NH}\,|_t = h_2^{NH}\,|_{t+2} = h_3^{NH}\,|_{t+4} = 0$. It does not change across and within each hedge horizon as the cash position is always chosen. The non-adjustable MGARCH

[11] To derive the conditional covariance between the same risky variable forecasted for different periods (e.g. $\sigma_{dF4t,dF2t}$), a simple example is illustrated in Appendix 1.

hedging alternative is only time variant at t. All conditional expectations are solely based on the information set at t when the hedge decision is made. Hence, the initial hedge ratio is fixed at all subsequent adjustment stages so that $h_1^{NA}|_t = h_2^{NA}|_{t+2} = h_3^{NA}|_{t+4}$. It only changes across each hedge horizon but not within, and both features (a) and (b) of the closed loop solution do not apply in this case. No hedging and non-adjustable MGARCH hedging strategies obviously lack the flexibility of the closed loop solution for utilizing new information: its hedge ratios are allowed to be time varying both across and within hedge horizons.

The optimal hedge and forward contract ratios under the alternative strategies are presented in Table 3. The conditional mean, variances, and covariances used in deriving these ratios all utilize the MGARCH parameter estimates and equations from (24)[12]. Similar to the closed loop solution, the forward contract ratios of both strategies indicate that the producer should market his/her hogs entirely at t in almost all instances. The time-series plot of the non-adjustable MGARCH hedge ratios, presented in Figure 6, shows some variations across hedge horizons. Compared with the closed loop hedge ratios at the mean level, the non-adjustable MGARCH alternative requires more hedging in short positions at t and $t+2$ (0.5384 > 0.3006 and 0.5384 > 0.4767 respectively) but less at $t+4$ (0.5384 < 0.6350).

Table 4 reports the descriptive statistics of expected income utility for each portfolio, along with the percentage improvement from using the closed loop solution. The results indicate that the closed loop solution clearly outperforms both alternative choices with the greatest utility value (48.2849). On average, the closed loop solution is able to enhance the producer's utility by 4.89% over no hedging and by 1.92% over non-adjustable MGARCH strategies. Figure 7 and Figure 8 further depict the time-varying percentage improvement. There are a few occasions in which the no hedging and non-adjustable MGARCH hedging portfolios outperform the closed loop solution. These negative percentages are shown to be modest, except for one instance when the utility loss reaches as high as –20.41% relative to the no hedging case. Nonetheless, in the long

[12] The hedge ratios for no hedging are constrained to be zero in all periods.

run, the gains to the producer from using the closed loop solution still outweigh these sporadic losses.

The Complete Strategies: Out-of-Sample Simulations

The multivariate model in the previous section uses the in-sample data to obtain the parameter estimates and hence the in-sample simulations. The results point to the closed loop solution for providing the best hedging performance relative to both no hedging and non-adjustable MGARCH hedging alternatives. However, it is also important for the representative producer to learn the predictive power of the closed loop solution outside this data range. The out-of-sample simulations provide this assessment.

The out-of-sample forecasts for the conditional mean values, variances, and covariances cover the period from January 1996 to May 1999, consisting of 7 production cycles. As hog price crisis occurred in the 1998, the out-of-sample data set also provides an opportunity to examine the hedging performance of the closed loop solution in a highly volatile situation.

Table 5 presents the optimal forward contract and hedge ratios for the three portfolios. In fact, the out-of-sample results do not differ very much from those generated by the in-sample data at the mean level. In all periods, the unity ratios suggest that hogs should be forward contracted entirely. Approximately half of the hogs should be hedged under the non-adjustable MGARCH strategy (-0.5075), while significant upward adjustments are recommended once again under the closed loop solution (from –0.2443 to –0.7042). However, these upward adjustments are in contrast with modest results shown in Mathews and Holthausen's study (from –0.9120 to –0.9420)[13]. Mathews and Holthausen may have "over-estimated" the initial hedge ratio without taking time-varying conditional variances and covariances into consideration.

More interestingly, the low initial hedge ratio of the closed loop solution may well signify that a short futures position can be initiated reasonably four months prior to delivery. Except during the volatile period of hog price crisis, the initial hedge ratio for the June 1999 contract taken in December 1998 shows a rise in value (-0.4637). This is

[13] The conditional variances and covariances in Matthews and Holthausen's multi-period hedge model are calculated using the Peck's method of mean products of forecast errors.

mainly due to the temporary shock in the spot price market being immediately translated into the futures and options markets. Any corresponding changes in risk expectations are expressed by the conditional covariance terms in (15): $E_t(\sigma_{APCPt+6,dF2t})$, $E_t(\sigma_{dF4t,dF2t})$, and $E_t(\sigma_{dF6t,dF2t})$[14].

Comparing with the June 1996 futures contract taken in December 1995, the conditional covariance $E_t(\sigma_{APCPt+6,dF2t})$ shows an increase from 0.33 to 2.07. This necessarily means that co-movements of the APCP value and futures gain/loss are strengthened considerably due to the market perturbation, thereby making short hedging more attractive in reducing contract value risk at the initial stage. The conditional covariances $E_t(\sigma_{dF4t,dF2t})$ and $E_t(\sigma_{dF6t,dF2t})$ also show an increase in absolute value from –0.14 to –0.29 and from –0.62 to –2.67 respectively. This necessarily means that the futures gain/loss in the first period (dF_{2t}) becomes even more negatively correlated with those in the second and third periods (dF_{4t} and dF_{6t}). This further reduces the possibility of downside risk exposure of futures losses in the subsequent stages and this increases the initial hedge ratio.

In addition, the shock effect appears to get carried over to the second period (at $t+2$) in February 1999 and the conditional covariances continue to give rise to even greater adjustment hedge ratio ($h_2^* |_{t+2} = -0.9308$). In the third period (at $t+4$), all market signals begin to reflect the "true" expected demand/supply conditions as the shock effect subsides. With the revision feature (c) facilitated by MGARCH, such new information can be incorporated into the adjustment decision. Accordingly, the second-period hedge ratio is corrected downward as a consequence of overshooting ($h_3^* |_{t+4} = -0.8493$).

Figure 9 illustrates the time-varying pattern of percentage improvement from using the closed loop solution over no hedging and non-adjustable MGARCH hedging (with optimal forward contract ratios). By examining the plot, the closed loop solution yields significant gains during the hog price crisis. From June 1998 to June 1999 when the high market volatility occurred, the closed loop solution improves over no hedging by 25.76% and over non-adjustable MGARCH hedging by 2.21% on average. These are

[14] In equation (15), the optimal forward contract ratio is set to unity; hence, the conditional covariance between the spot price and futures gain/loss is irrelevant. For simplicity, effects of sequential dependence of decisions on the initial hedge ratio are also suppressed.

indeed higher than the out-of-sample averages 12.12% and 1.47% respectively. The results confirm the effectiveness of the closed loop solution for reducing both the spot price and contract value risks via the CME live/lean hog futures contracts. More importantly, the three essential features of the closed loop utilize dynamic changes of information and make necessary sequential adjustments along the hedge optimality path. Its superior performance has also been demonstrated in the presence of volatile market situations.

7. Summary

Formula-pricing arrangements have become the most common marketing practice in the U.S. hog industry and this form of forward contracting is a source of risk to income stability. Previous research has primarily focused on futures hedging and its ability to reduce spot price risk. No attempts have been made in developing hedge strategies that reflect the current marketing changes. Moreover, hedge recommendations have been based on variance-covariance forecasts other than those from time-series econometric models. This could limit the predictive accuracy of volatility movements and consequently the performance of hedging.

Adding to the existing literature, this research is the first analysis to consider both spot price and contract value risks in a sequential hedging and forward contracting framework for U.S. hog production. The three main contributions are the theoretical and empirical applications of APCP for forward contract valuations, the closed loop solution for dynamic decision process, and application of the MGARCH methodology for time-varying conditional variance-covariance forecasts.

The APCP technique is a useful proxy measure for quantifying the forward contract gains or losses between two parties and incorporating this risky variable into hedging decisions to account for the contract effects. APCP is not only applicable to forward contract valuations for output marketing but also to those for purchasing input factors (e.g., corn or barley feed), or both under different pricing or risk-sharing contract schemes. However, an APCP limitation arises when futures and options contracts do not exist for a forward-contracted commodity.

The conceptual framework for sequential hedging and forward contracting decisions is presented. The representative producer maximizes a mean-variance utility function conditional on the information set available at t, $t+2$, and $t+4$. The optimal forward contract and hedge ratios derived from the first-order conditions embody all three essential features of the closed loop solution. These features outline how the information flows in the dynamic optimization process. Feature (a) specifies dependence between sequential forward contracting and hedging decisions. Feature (b) feeds back information from earlier stages so that uncertainty cost can be reduced in subsequent hedging decisions. Feature (c) provides anticipated revision and this "closes" the information loop. As the sequential hedge ratios contain conditional variance and covariance terms, this feature allows the producer to revise risk expectations sequentially.

To model the conditional variances and covariances of multiple risky variables, the MGARCH methodology is used to implement anticipated revision feature (c). MGARCH is a time-series process that specifies the conditional variance (covariance) as a function of the past realized errors as well as conditional variances and covariances. Therefore, periods of tranquility or volatility may lead to similar forecasts being incorporated into hedge ratio calculations. Previous research of hedging in U.S. hog production has overlooked MGARCH as a superior forecasting tool to capture both variance and covariance dynamics in sequential decisions.

The multivariate ARMA-GARCH model is estimated and the empirical results indicate evidence of GARCH behavior in the equation system. Peck's method of mean products of forecast errors has undesirable complexity in order to facilitate information revision. It lacks the econometric time-series specification to closely track changing risk conditions and this limitation similarly extends to the constant variance assumption of OLS or SUR. As a result, such methods would not be able to render full risk management capability especially in volatile situations. This further confirms the importance of GARCH and its potential to improve hedge performance.

The multivariate ARMA-GARCH parameter estimates are used to form the conditional expectations necessary for the computations of optimal forward contract and hedge ratios under the closed loop solution. Both in-sample and out-of-sample simulation results recommend considerable upward hedge adjustments. This gives rise to the time-varying pattern of optimal hedge ratios in response to a changing risk environment. The low initial hedge ratio—in contrast with results of Mathews and Holthausen—may well signify that hedging with futures contracts could be delayed, reasonably four months prior to delivery. This suggests the greater positive correlation between the expected spot price (APCP value) and futures gain/loss as the delivery month approaches. The optimal forward contract ratios indicate in almost all instances that hogs should be marketed entirely in advance under the formula-pricing contracts. This could explain the reason why an increasing trend toward contract production has been observed over the years.

To evaluate the futures hedge performance under the closed loop solution, the percentage improvement in utility is compared with two other portfolios of no hedging and non-adjustable MGARCH hedging. The in-sample and out-of-sample findings indicate that closed loop hedging provides the best hedge performance of all portfolios. The improvement over non-adjustable MGARCH hedging is modest possibly due to its ability to revise information across the hedge horizons. On the other hand, the improvement over no hedging is quite significant. In volatile market situations such as the hog price crisis in 1998, the closed loop hedging is proven to achieve notable utility gains over both alternative portfolios. This research combines the APCP technique, closed loop solution, as well as MGARCH methodology to help assess the effectiveness of the CME live/lean hog futures contracts for reducing both spot price and contract value risks. The results confirm this fact and provide valuable information to hog producers.

Future research may expand the same modeling framework to other commodities and different hedging horizons, explore cross-hedging opportunities and other types of contract value risks (e.g. fixed price tied to a feed price with ledger maintained), or consider the effects of government-sponsored price stabilization programs on optimal hedging choices.

Table 1. Multivariate ARMA-GARCH Estimation Results:

ARMA(2,0) Coefficients for Series dFP_t

δ_1	0.4276	δ_2	-0.2558
	(7.3119)		(-4.1728)
Q(12)	20.0026		
	(0.0670)		
$Q^2(12)$	5.3161		
	(0.9466)		

ARMA(2,0) Coefficients for Series $APCP_t$

λ_1	0.6051	λ_2	-0.0390
	(7.4945)		(-0.5949)
Q(12)	5.4231		
	(0.9423)		
$Q^2(12)$	0.0450		
	(1.0000)		

ARMA(1,12) Coefficients for Series P_t

ϕ_0	8.8442	ϕ_1	0.8023	γ_{12}	0.3306
	(11.8686)		(35.7501)		(3.9295)
Q(12)	9.7027				
	(0.5660)				
$Q^2(12)$	6.7353				
	(0.8746)				

MGARCH(1,1)-BEKK Coefficients

$$C = \begin{bmatrix} C_{11} = 0.6464 & C_{12} = -0.1089 & C_{13} = 0.6627 \\ (2.0647) & (-0.4781) & (1.4215) \\ 0 & C_{22} = -0.1605 & C_{23} = 1.6351 \\ & (-0.7641) & (6.4328) \\ 0 & 0 & C_{33} = 0.4571 \\ & & (4.6373) \end{bmatrix}$$

$$A = \begin{bmatrix} \alpha_{11} = 0.9252 & \alpha_{12} = 1.0474 & \alpha_{13} = 0.3421 \\ (6.2021) & (8.6773) & (1.8035) \\ \alpha_{21} = -0.3031 & \alpha_{22} = 0.2328 & \alpha_{23} = 0.0417 \\ (-2.9252) & (2.4943) & (0.2784) \\ \alpha_{31} = -0.1435 & \alpha_{32} = -0.2920 & \alpha_{33} = -0.0773 \\ (-2.1560) & (-4.9067) & (-0.8718) \end{bmatrix}$$

$$B = \begin{bmatrix} \beta_{11} = 0.4840 & \beta_{12} = 0.7855 & \beta_{13} = 0.2458 \\ (2.9156) & (10.1969) & (1.9066) \\ \beta_{21} = -0.1371 & \beta_{22} = -0.2584 & \beta_{23} = 0.5806 \\ (-0.96221) & (-3.0136) & (3.7611) \\ \beta_{31} = 0.4892 & \beta_{32} = 0.1122 & \beta_{33} = -0.0525 \\ (6.9522) & (2.2677) & (-0.4140) \end{bmatrix}$$

Log-likelihood

$L = -319.0902$

Note: All parameter estimates: t-statistics are in parenthesis.
Ljung-Box Q-statistics: P-values are in parenthesis

Q(12): Ljung-Box Q-Statistic for the 12th Order Autocorrelation in Standardized Residuals
$Q^2(12)$: Ljung-Box Q-Statistic for the 12th Order Autocorrelation in Squared Standardized Residuals

Table 2. Optimal Hedge and Forward Contract Ratios under the Closed Loop Solution: In-Sample Simulations

	at t		**at $t+2$**	**at $t+4$**
	$h_1^* \|_t$	$\alpha_1^* \|_t$	$h_2^* \|_{t+2}$	$h_3^* \|_{t+4}$
1988.12	-0.1541	1.0000		
1989.02			-0.4254	
1989.04				-0.5575
1989.06	-0.1551	1.0000		
1989.08			-0.2893	
1989.10				-0.6835
1989.12	-0.2038	1.0000		
1990.02			-0.6388	
1990.04				-0.7175
1990.06	-0.1762	1.0000		
1990.08			-0.5000	
1990.10				-0.6741
1990.12	-1.0003	0.0000		
1991.02			-0.7951	
1991.04				-0.3123
1991.06	-0.2343	1.0000		
1991.08			-0.2474	
1991.10				-0.5980
1991.12	-0.2339	1.0000		
1992.02			-0.3872	
1992.04				-0.6762
1992.06	-0.2239	0.8674		
1992.08			-0.3425	
1992.10				-0.6671
1992.12	-0.2109	1.0000		
1993.02			-0.4426	
1993.04				-0.6577
1993.06	-0.6210	0.3931		
1993.08			-0.7901	
1993.10				-0.5751
1993.12	-0.3572	1.0000		
1994.02			-0.3907	
1994.04				-0.7083
1994.06	-0.2470	1.0000		
1994.08			-0.5121	
1994.10				-0.6894
1994.12	-0.23336	1.0000		
1995.02			- 0.5112	
1995.04				-0.6889

1995.06	-0.1564	1.0000		
1995.08			-0.4021	
1995.10				-0.6847
Mean	-0.3006	0.8757	-0.4767	-0.6350

Table 3. Optimal Hedge and Forward Contract Ratios under No Hedging and Non-Adjustable MGARCH Hedging Strategies: In-Sample Simulations

All decisions are made at t

	No Hedging		**Non-Adjustable MGARCH Hedging**	
	$h_1^{NH}\vert_t = h_2^{NH}\vert_{t+2} = h_3^{NH}\vert_{t+4} = 0$	$\alpha_1^{NH}\vert_t$	$h_1^{NA}\vert_t = h_2^{NA}\vert_{t+2} = h_3^{NA}\vert_{t+4}$	$\alpha_1^{NA}\vert_t$
1988.12	0.0000	1.0000	-0.4766	1.0000
1989.06	0.0000	1.0000	-0.4541	1.0000
1989.12	0.0000	1.0000	-0.4638	1.0000
1990.06	0.0000	1.0000	-0.3773	1.0000
1990.12	0.0000	0.7813	-1.0085	0.0000
1991.06	0.0000	1.0000	-0.4675	1.0000
1991.12	0.0000	1.0000	-0.4977	1.0000
1992.06	0.0000	0.8848	-0.4418	0.9292
1992.12	0.0000	1.0000	-0.4749	1.0000
1993.06	0.0000	0.8635	-0.8872	0.0000
1993.12	0.0000	1.0000	-0.5957	1.0000
1994.06	0.0000	1.0000	-0.5112	1.0000
1994.12	0.0000	1.0000	-0.4759	1.0000
1995.06	0.0000	1.0000	-0.4056	1.0000
Mean	0.0000	0.9664	-0.5384	0.8521

Table 4. Descriptive Statistics for No Hedging, Non-Adjustable MGARCH Hedging, and Closed Loop Solution:

Expected Income Utility

	No Hedging	**Non-Adjustable MGARCH Hedging**	***Closed Loop Solution***
Mean	46.0523	47.4435	48.2849
Standard Error	0.5916	0.7708	0.7207
Minimum	39.1560	34.3835	39.3398
Maximum	53.8330	57.5750	58.8657
Sum (in sample)	1934.1971	1992.6269	2027.9650
In-sample size	42.0000	42.0000	42.0000

% Improvement from Using the Closed Loop Solution Relative to

No Hedging

Mean	4.89%
Standard Error	0.8824
Minimum	-20.41%
Maximum	17.21%

Non-Adjustable MGARCH Hedging

Mean	1.92%
Standard Error	0.4670
Minimum	-0.88%
Maximum	18.51%

Table 5. Optimal Hedge and Forward Contract Ratios: Out-of-Sample Simulations

Closed Loop Solution

	at t		at $t+2$	at $t+4$
	$h_1^* \vert_t$	$\alpha_1^* \vert_t$	$h_2^* \vert_{t+2}$	$h_3^* \vert_{t+4}$
1995.12	-0.1476	1.0000		
1996.02			-0.7122	
1996.04				-0.7954
1996.06	-0.2285	1.0000		
1996.08			-0.5170	
1996.10				-0.7058
1996.12	-0.1953	1.0000		
1997.02			-0.5375	
1997.04				-0.7220
1997.06	-0.2956	1.0000		
1997.08			-0.4471	
1997.10				-0.6502
1997.12	-0.1771	1.0000		
1998.02			-0.4408	
1998.04				-0.5828
1998.06	-0.2025	1.0000		
1998.08			-0.5174	
1998.10				-0.6241
1998.12	-0.4637	1.0000		
1999.02			-0.9308	
1999.04				-0.8493
Mean	-0.2443	1.0000	-0.5861	-0.7042

	No Hedging		**Non-Adjustable MGARCH Hedging**	
	$h_1^{NH} \vert_t = h_2^{NH} \vert_{t+2} = h_3^{NH} \vert_{t+4} = 0$	$\alpha_1^{NH} \vert_t$	$h_1^{NA} \vert_t = h_2^{NA} \vert_{t+2} = h_3^{NA} \vert_{t+4}$	$\alpha_1^{NA} \vert_t$
1995.12	0.0000	1.0000	-0.4488	1.0000
1996.06	0.0000	1.0000	-0.4576	1.0000
1996.12	0.0000	1.0000	-0.4455	1.0000
1997.06	0.0000	1.0000	-0.5322	1.0000
1997.12	0.0000	1.0000	-0.3737	1.0000
1998.06	0.0000	1.0000	-0.4278	1.0000
1998.12	0.0000	1.0000	-0.8672	1.0000
Mean	0.0000	1.0000	-0.5075	1.0000

Figure 1. Forward Contract Value and Put-Call Parity.

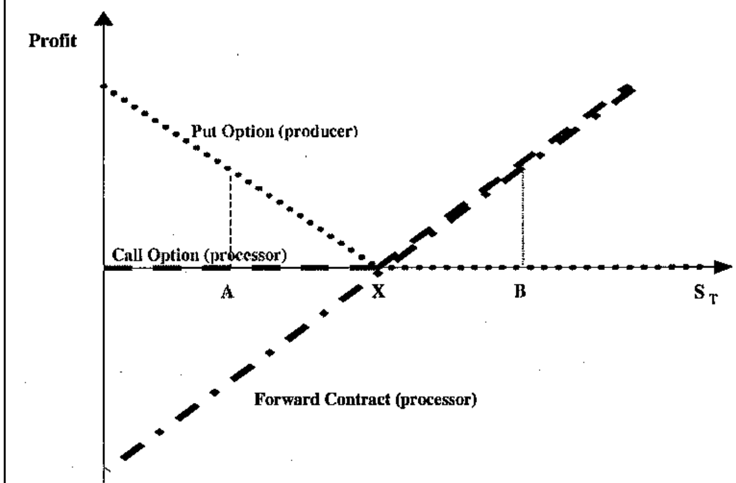


Figure 2. Average of F[illegible] Strike Price (February Contracts)

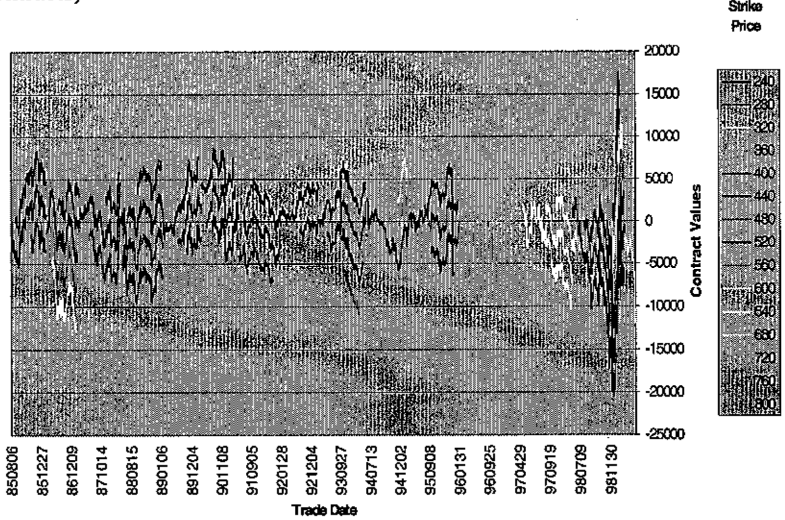


Figure 3. Optimal Hedge Ratio at t (h1|t)

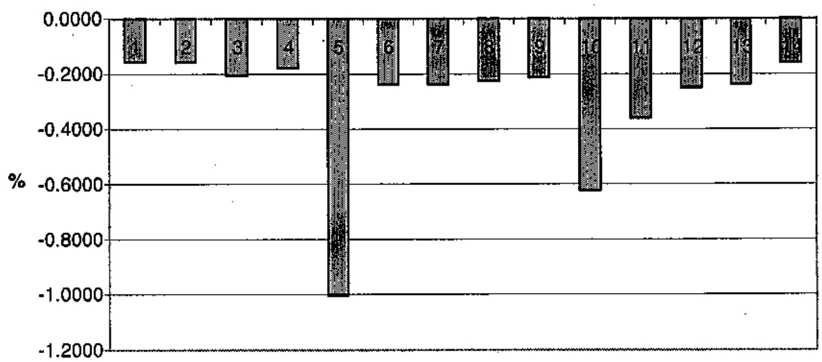


Figure 4. Optimal Hedge Ratio at t+2 (h2|t+2)

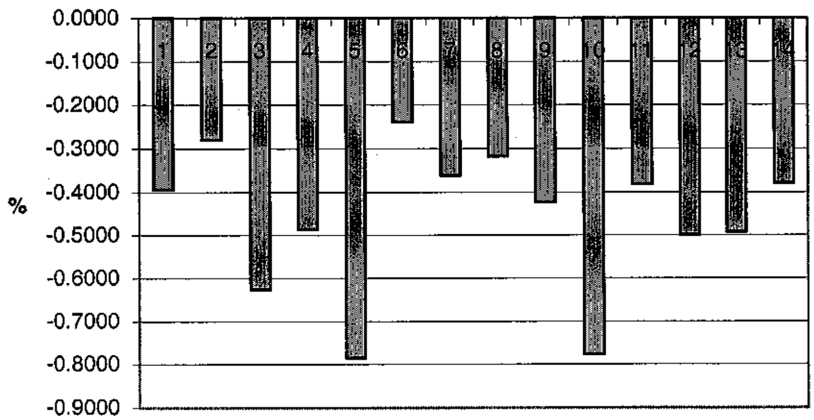


Figure 5. Optimal Hedge Ratio at t+4 (h3|t+4)

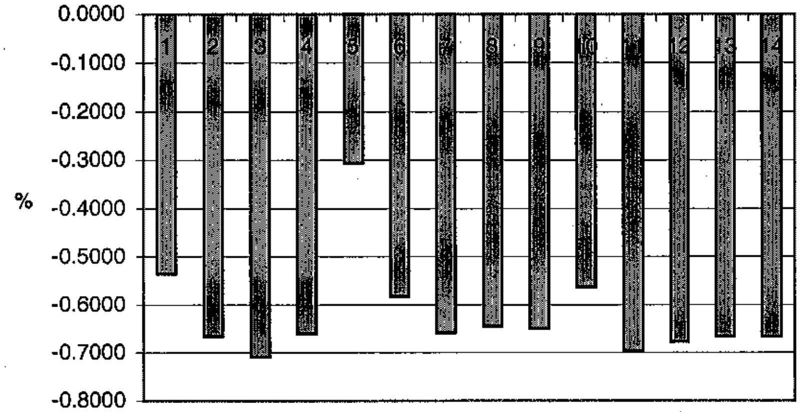


Figure 6. Optimal Hedge Ratio at t (h|t=h|t+2=h|t+4) – Non-Adjustable Hedging

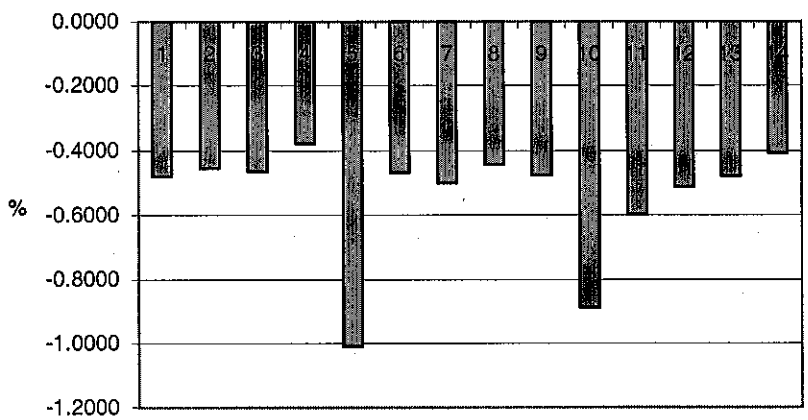


Figure 7. % Improvement of the Closed Loop Solution over No Hedging

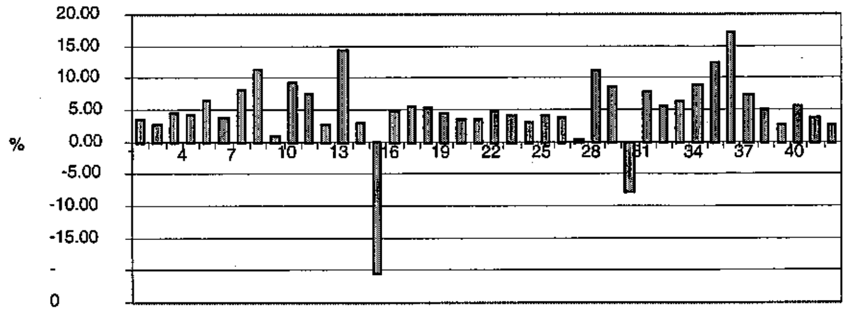


Figure 8. % Improvement of the Closed Loop Solution over Non Adjustable Hedging.

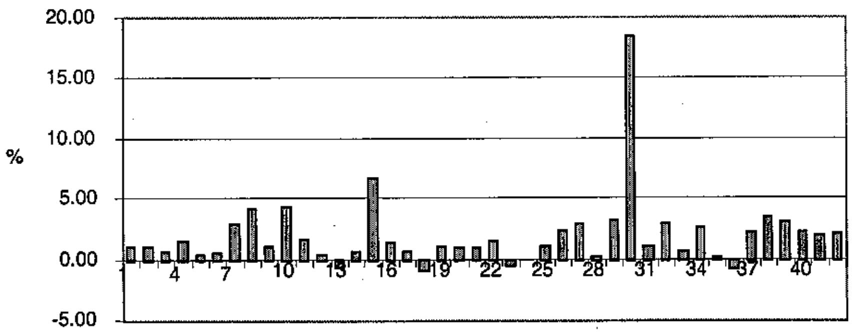
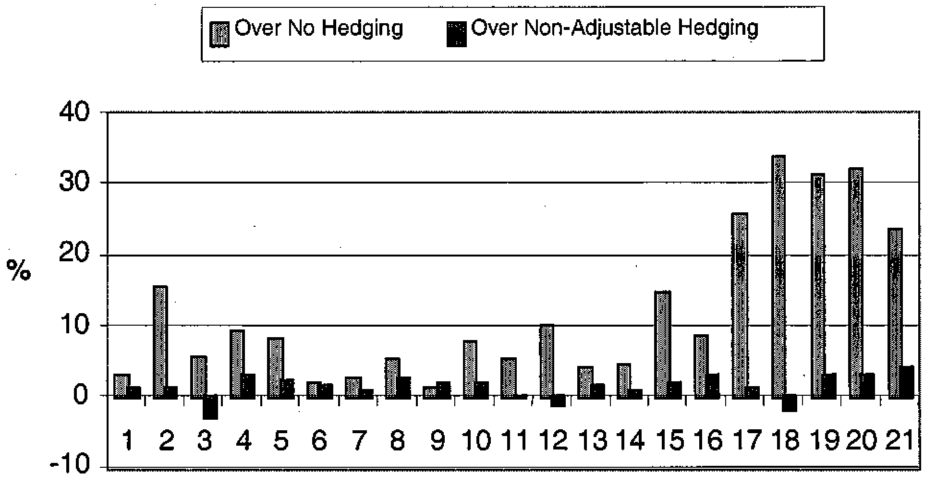


Figure 9. % Improvement of the Closed Loop Solution (w/ Optimal Forward Contract Ratios



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Appendix 1

Suppose a time series X_t is represented by a simple AR(1) process: $X_t = a_0 + a_1 X_{t-1} + \varepsilon_t$. At time $t+1$, X_{t+1} can be expressed as $X_{t+1} = a_0 + a_1 X_t + \varepsilon_{t+1}$ and similarly at time $t+2$ $X_{t+2} = a_0 + a_1 X_{t+1} + \varepsilon_{t+2}$. Based on the information set available at time t, X_{t+1} and X_{t+2} are iterated back so that $X_{t+1} = (a_0 + a_0 a_1) + a_1^2 X_{t-1} + a_1 \varepsilon_t + \varepsilon_{t+1}$ and $X_{t+2} = (a_0 + a_0 a_1 + a_0 a_1^2) + a_1^3 X_{t-1} + a_1^2 \varepsilon_t + a_1 \varepsilon_{t+1} + \varepsilon_{t+2}$. The conditional covariance $\sigma_{X_{t+1}, X_{t+2}}|_t$ therefore becomes $E_t[(X_{t+1} - E_t(X_{t+1}))(X_{t+2} - E_t(X_{t+2}))] = a_1^3 \sigma_{X_t}^2$. The result is equivalent to the conditional variance of X_t multiplied by the AR(1) coefficient a_1^3.