The Theoretical Structure of

Producer Willingness to Pay Estimates

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Abstract

This paper analyzes the theoretical underpinnings of producers' willingness to pay (WTP) for novel inputs. In addition to conceptualizing the WTP function for producers, we derive its comparative statics and demonstrate the use of these properties to estimate input quantities demanded, outputs supplied, and price elasticities. We also discuss implications of the comparative statics results for the specification of empirical producer WTP models and survey design.

Key words: Cobb-Douglas production function, contingent valuation, price elasticities, new technologies, survey design.

1. Introduction

Producers and agribusinesses are constantly seeking new technologies or inputs with novel attributes that can help them reduce production costs and increase revenues. However, the novel nature of these products also means that prospective suppliers do not have data from actual markets to estimate the potential demand for these new technologies or inputs. To estimate producers' demands, suppliers of these novel factors can rely on stated preference methods such as contingent valuation.

Contingent valuation, a survey-based methodology, was initially developed to elicit the value (i.e., willingness to pay, WTP) that people place on non-market goods and services. This elicitation methodology has been used primarily in the assessment of individuals' WTP for environmental services (e.g., Carson et *al*., 1995; Boyle, 2003; Carson and Hanemann, 2005; Zapata et *al*., 2012). More recent applications of contingent valuation methodologies are found in other areas such as health economics (e.g., Diener et *al*., 1998; Krupnick et *al*., 2002), real estate appraising (e.g., Breffle et *al*., 1998; Banfi et *al*., 2008; Lipscomb, 2011), art valuation (e.g., Thompson et *al*., 2002), and agribusiness (e.g., Lusk and Hudson, 2004).

The majority of empirical and theoretical contingent valuation literature has focused on the consumer side, rather than on the producer side. For example, applications of contingent valuation on agribusiness are mainly related to consumers' WTP for neoteric products, food quality enhancements, or specific attributes (e.g., Lusk, 2003; Carpio and Isengildina-Massa, 2009). However, little conceptual or empirical work has been conducted to understand the monetary value that producers and agribusinesses place on new production factors.

The purpose of this paper is to extend the literature regarding producers' WTP for new technologies or inputs. More specifically, we derive the producers' WTP function (also called variation function) and its corresponding comparative statics, which have implications for the specification of empirical WTP models and survey design. In fact, we demonstrate the use of these properties to estimate the quantity demanded of novel inputs, the quantity supplied of the output and price elasticities. Hence, another contribution of this paper is the establishment of a link between traditional demand analyses (with emphasis on the estimation of price and income elasticities) and contingent valuation studies (with a focus on estimating a mean WTP value). This is important because agribusinesses are mainly interested in estimating market demand for novel products and market reaction measures such as price elasticities.

This paper is laid out as follows: Section 2 presents a brief literature review of contingent valuation and its uses in agribusiness. Section 3 discusses the theoretical structure of the WTP or variation function and comparative statics. Empirical implications of the theoretical results are presented in Section 4. Finally, Section 5 provides some brief conclusions. All proofs are presented in the Appendix.

2. Literature Review

The contingent valuation methodology was first proposed and implemented by Davis (1963) who designed a hypothetical scenario to assess the economic value of recreational possibilities of Maine’s forests. Since then, great progress has been achieved in empirical procedures and theoretical foundations of the contingent valuation method (Hanemann, 1984; Cameron, 1988). Contingent valuation methods are now widely used by researchers and government agencies as crucial tools in the assessment of environmental benefits (Carson et al., 1995; Boyle, 2003; Carson and Hanemann, 2005).

The theoretical foundations of discrete choice models for contingent valuation were developed by Hanemann (1984) and Cameron (1988). Both authors assumed that individual responses arose from discrete instances of utility maximization, which would imply a consumer WTP function with properties derived from neoclassical utility functions. However, the Cameron approach facilitates the derivation of comparative statics of the WTP function and is consistent with discrete choice and continuous valuation function models (Whitehead, 1995). Consider Whitehead's (1995) definition of consumers WTP for a policy with a goal to change the quality of goods consumed from $\boldsymbol{q}^0$ to $\boldsymbol{q}^1$:

$$WTP = m[\boldsymbol{p}, \boldsymbol{q}^0, v(\boldsymbol{p}, \boldsymbol{q}^1, y)] - m[\boldsymbol{p}, \boldsymbol{q}^0, v(\boldsymbol{p}, \boldsymbol{q}^0, y)] \tag{1}$$
$$= m[\boldsymbol{p}, \boldsymbol{q}^0, v(\boldsymbol{p}, \boldsymbol{q}^1, y)] - y,$$

where $m[\cdot]$ and $v(\cdot)$ are the individual's expenditures and indirect utility functions, respectively; $\boldsymbol{p}$ is the vector of good prices; $\boldsymbol{q}$ is a vector of quality of goods consumed; and y is income. Comparative statics of the WTP function can be derived by taking derivatives of equation (1) with respect to the variables of interest. For example, Whitehead (1995) shows that the effect of the ith input price on consumer WTP is

$$\frac{\partial WTP}{\partial p_i} = x_i^m(\cdot, \boldsymbol{q}^0) - \frac{m_v^0}{m_v^1} x_i^m(\cdot, \boldsymbol{q}^1), \tag{2}$$

where $x_i^m(\cdot, \boldsymbol{q}^0)$ and $x_i^m(\cdot, \boldsymbol{q}^1)$ are *Marshallian* demand functions before and after the quality change, respectively, and m_v^ι, $\iota = 0{,}1$, is the partial derivative of the expenditure function with respect to indirect utility ($m_v^\iota = \frac{\partial m}{\partial v}[\cdot, \boldsymbol{q}^\iota]$); $\iota = 0{,}1$, and all arguments other than environment quality level are suppressed for simplicity (Whitehead, 1995).

Comparative statics results, such as those presented in (2), can be used to theoretically interpret the results of contingent valuation empirical models or predict the change in demand for goods after quality improvement (McConnell, 1990; and Whitehead, 1995).

One limitation of this theoretical work is its focus on the WTP of consumers.[1] Moreover, to the best of our knowledge, the implications of these properties for empirical work have been largely ignored. Similarly, on the empirical side, the vast majority of contingent valuation literature has focused on the consumer side rather than on producers.

Few empirical studies are found on the literature regarding the use of contingent valuation methods for producers. For example, the only studies found in the agribusiness literature related with this subject include the estimation of producers' WTP for information under risk (Roe and Antonovitz, 1985), crop insurance (Patrick, 1988), agricultural extension services (Whitehead et al., 2001, Budak et al., 2010), and novel technologies or inputs (Kenkel and Norris, 1995; Hudson and Hite, 2003). Overall, as the literature review shows, little conceptual and empirical work has been conducted to understand the monetary value that producers place on new production factors.

3. Theoretical Model and Comparative Statics

Theoretical Framework

The derivation of the producer WTP function for novel factors of production is based on the model used by McConnell and Bockstael (2005) to explain the effects of environmental changes in the firm production process. The theoretical model proposed in this paper allows the analysis of producers' WTP for a change in quality of any factor of production and not only a change in the environmental goods or services as in McConnell and Bockstael's model (2005). Suppose

[1] McConnell and Bockstael (2005) developed several theoretical models with the aim to conceptualize and measure the economic value that firms place on environmental services. However, the main emphasis of this work has been on elucidating the economic costs and benefits of environmental changes that influence production rather than explaining the economic value producers place on novel factors of production.

that an individual's utility is given by $U(\boldsymbol{Z})$, where $\boldsymbol{Z}$ is a vector of goods consumed by that individual. The problem faced by the individual consumer can be written as:

(3) $$max_{\boldsymbol{z}} U(\boldsymbol{Z}) \text{ subject to } \overline{m} + L = \boldsymbol{P}_{\boldsymbol{z}}^{`}\boldsymbol{Z},$$

where $\overline{m}$ and L are the individual's non-labor and labor income, respectively, and $\boldsymbol{P}_{\boldsymbol{z}}$ is a vector of prices. It is assumed that non-labor income $\overline{m}$ comes from a decision process independent of individual preferences. The indirect utility function is obtained by replacing the optimal quantity demanded of $\boldsymbol{Z} = \boldsymbol{Z}(\overline{m}, L, \boldsymbol{P}_{\boldsymbol{z}})$ into the utility function. Consequently, the indirect utility function is expressed in terms of variables that are assumed exogenous to the individual:

(4) $$V(\overline{m}, L, \boldsymbol{P}_{\boldsymbol{z}}) = V_0.$$

It is also assumed that the individual produces a product, Y, to sell it in the market. As a producer, she faces the following problem:

(5) $$max_Y \, \Pi = p_y Y - C(Y, \boldsymbol{r}, \boldsymbol{q}),$$

where Π is the profit function, p_y is the price of produced output, and $C(Y, \boldsymbol{r}, \boldsymbol{q})$ is the cost function of the individual's firm. The cost function can be defined as the solution of the following problem:

(6) $$min_{\boldsymbol{X}} \, C = \boldsymbol{r}^{`}\boldsymbol{X} \text{ subject to } Y = f(\boldsymbol{X}, \boldsymbol{q}),$$

where $\boldsymbol{r}$ is a vector of input prices, $\boldsymbol{X}$ is a vector of input quantities, $f(\boldsymbol{X}, \boldsymbol{q})$ is the production function of Y, and $\boldsymbol{q}$ is a vector of input quality levels. The level of $\boldsymbol{q}$ is fixed exogenously, thus the profit and cost functions are conditional on $\boldsymbol{q}$. Given p_y, $\boldsymbol{r}$, and $\boldsymbol{q}$, the producer chooses the optimal level of output, $Y(p_y, \boldsymbol{r}, \boldsymbol{q})$, and input, $\boldsymbol{X}(Y, \boldsymbol{r}, \boldsymbol{q})$, which generate the indirect profit function, $\Pi(p_y, \boldsymbol{r}, \boldsymbol{q})$, and cost function $C(Y, \boldsymbol{r}, \boldsymbol{q})$ (see Appendix A).

The link between the consumer and producer problem is given by non-labor income, $\overline{\mathrm{m}}$, which can be assumed to be a function of profits such that $\frac{\partial \overline{\mathrm{m}}}{\partial \Pi} > 0$. Thus, $\overline{m} =$

$\overline{m}(\Pi(p_y, \boldsymbol{r}, \boldsymbol{q}), k)$, where k represents other factors that affect non-labor income; therefore, (4) can be rewritten as:

(7) $$V[\overline{m}(\Pi(p_y, \boldsymbol{r}, \boldsymbol{q}), k), L, \boldsymbol{P_z}] = V_0.$$

Then, the compensated variation (CV) and equivalent variation (EV) of a change in the vector of input quality level, $\boldsymbol{q}$, from $\boldsymbol{q}^0$ to $\boldsymbol{q}^1$ are the amounts of money that make the following conditions to hold:

(8) $$V[\overline{m}(\Pi(p_y, \boldsymbol{r}, \boldsymbol{q}^0), k), L, \boldsymbol{P_z}] = V[\overline{m}(\Pi(p_y, \boldsymbol{r}, \boldsymbol{q}^1), k) - CV, L, \boldsymbol{P_z}]$$

(9) $$V[\overline{m}(\Pi(p_y, \boldsymbol{r}, \boldsymbol{q}^0), k) + EV, L, \boldsymbol{P_z}] = V[\overline{m}(\Pi(p_y, \boldsymbol{r}, \boldsymbol{q}^1), k), L, \boldsymbol{P_z}].$$

In this context, CV and EV measures represent the economic value that the producer places on upgrades in input quality levels. Positive CV and EV measures imply a welfare improvement and vice versa. In general, CV and EV measures are not equal except when the variation in welfare comes from a change in exogenous income (e.g., change in the level of non-labor income). Consequently, the CV and EV measures in expressions (8) and (9) are identical and are given by the variation function (i.e., producer WTP function) d, which can be defined as:

(10) $$d = \overline{m}(\Pi(p_y, \boldsymbol{r}, \boldsymbol{q}^1), k) - \overline{m}(\Pi(p_y, \boldsymbol{r}, \boldsymbol{q}^0), k).$$

This is a variation function because it represents the CV or EV of the individual, depending on the initial and final levels of non-labor income (McConnell, 1990). If the improvement on a particular input quality level, q_i, results in an increase in profits, such that $d > 0$, then expression (10) represents the maximum (minimum) amount of profit that a producer would be willing to forgo (accept) to obtain (give up) the benefits of the new input quality level, q_i^1.

Under the assumption that non-labor income ($\overline{m}$) is a linear function of profit (Π) and k, then the variation on welfare due to a change in $\boldsymbol{q}$ from $\boldsymbol{q}^0$ to $\boldsymbol{q}^1$ is also a linear function of the difference in profits and can be simplified to [2]:

$$d = \Pi(p_y, \boldsymbol{r}, \boldsymbol{q}^1) - \Pi(p_y, \boldsymbol{r}, \boldsymbol{q}^0). \quad (11)$$

Consequently, the maximum amount of money a producer is WTP for improvements in input quality levels reduces to the difference between the *ex post* (after adopting the new input) and *ex ante* (before adopting the new input) firm's profit levels.

Comparative Statics of the Variation Function

To derive comparative statics, equation (5) can be used to rewrite the variation function (11) as[3]:

$$\begin{aligned} d = {} & \left[p_y Y(p_y, \boldsymbol{r}, \boldsymbol{q}^1) - C\left(Y(p_y, \boldsymbol{r}, \boldsymbol{q}^1), \boldsymbol{r}, \boldsymbol{q}^1\right)\right] \\ & - \left[p_y Y(p_y, \boldsymbol{r}, \boldsymbol{q}^0) - C\left(Y(p_y, \boldsymbol{r}, \boldsymbol{q}^0), \boldsymbol{r}, \boldsymbol{q}^0\right)\right]. \end{aligned} \quad (12)$$

Without loss of generality, it is assumed that only the quality of one input (i^{th} input) changes, such that $\boldsymbol{q}^1$ contains the same elements as $\boldsymbol{q}^0$ except for the i^{th} element, which is replaced by q_i^1 and the upgraded quality level of the i^{th} input is greater than its previous level ($q_i^1 > q_i^0$). It is also assumed that the firm operates in a competitive market; thus, the change in quantity demanded of the novel input by the firm does not affect market prices.

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[2] A general form of a variation function linear in profits is given by $d = b[\Pi(p_y, \boldsymbol{r}, \boldsymbol{q}^1) - \Pi(p_y, \boldsymbol{r}, \boldsymbol{q}^0)]$, where b is a constant and can be thought of as the individual's discount factor of a firm's profits. If $b \neq 1$, then the stated individual producer WTP for novel inputs or technologies is not the value that the firm, as a whole place, on these new factors of production. Therefore, the model presented here only applies to a firm with only one owner. For a firm with multiple owners, the WTP question should be asked in terms of how much the firm is willing to pay for these inputs rather than in terms of the individual WTP value.

[3] The change in profits, due to a change in the vector of input quality levels, can also be derived by adapting the approach proposed by McConnell and Bockstael (2005) to analyze the change in producers' welfare measures of a change in the environmental quality input. Their approach involves the estimation of an essential output supply or input demand function which is later used to recover the change in profits.

To illustrate the theoretical results of the analysis, a Cobb-Douglas production function is used throughout this paper. Specifically, we consider the two input case where quality level of input 1 is upgraded and quality level of input 2 remains at its original level. The firm production process is represented by

$$(13) \qquad Y = (q_1 x_1)^{\alpha}(q_2 x_2)^{\beta},$$

where q_i and x_i, $i = 1{,}2$, are the levels of quality and quantity of input i, respectively. The product $q_i x_i$ can be seen as the total, or "true," measurement of input i (Griliches, 1957). It is also assumed that the firm has diminishing returns to scale, such that $\alpha + \beta < 1$, and the marginal products of both inputs are positive, therefore $\alpha > 0$ and $\beta > 0$. Furthermore, nput quality levels, q_1 and q_2, are positive. Therefore, the variation function (11), which corresponds to the two inputs Cobb-Douglas production function in (13) is (see Appendix B):

$$(14) \qquad d = \Pi\left(p_y, r_1, r_2, q_1^1, q_2^0\right) - \Pi\left(p_y, r_1, r_2, q_1^0, q_2^0\right)$$

$$= [1 - (\alpha + \beta)]\left[q_1^{1\frac{\alpha}{1-(\alpha+\beta)}} - q_1^{0\frac{\alpha}{1-(\alpha+\beta)}}\right]\left[\frac{p_y \beta^{\beta} q_2^{0^{\beta}} \alpha^{\alpha}}{r_1^{\alpha} r_2^{\beta}}\right]^{\frac{1}{1-(\alpha+\beta)}}.$$

Equation (14) clearly illustrates the theoretical structure of the variation or producer WTP function and reveals that WTP is not merely a quantity (i.e., the difference in *ex post* and *ex ante* profits), but is also a function of endogenous variables similar to cost, profit, or demand functions. Moreover, this theoretical structure can be used to derive comparative statics or marginal effects of a change in input and output prices and input quality levels on the variation function using known properties of the profit and cost functions.

Input Price Effects

The change in the variation function from a change in the input j price is

$$(15) \qquad \frac{\partial d}{\partial r_j} = \left.\frac{\partial C(Y,\boldsymbol{r},\boldsymbol{q}^0)}{\partial r_j}\right|_{Y=Y(p_y,\boldsymbol{r},\boldsymbol{q}^0)} - \left.\frac{\partial C(Y,\boldsymbol{r},\boldsymbol{q}^1)}{\partial r_j}\right|_{Y=Y(p_y,\boldsymbol{r},\boldsymbol{q}^1)},$$

where $\left.\frac{\partial C(Y,\boldsymbol{r},\boldsymbol{q}^{\iota})}{\partial r_j}\right|_{Y=Y(p_y,\boldsymbol{r},\boldsymbol{q}^{\iota})}$, ι=0,1, represents the change in production cost due to a change in the input j price. Because $\left.\frac{\partial C(Y,\boldsymbol{r},\boldsymbol{q})}{\partial r_j}\right|_{Y=Y(p_y,\boldsymbol{r},\boldsymbol{q})} = x_j\big(Y(p_y,\boldsymbol{r},\boldsymbol{q}),\boldsymbol{r},\boldsymbol{q}\big)$, equation (15) can be written as (see Appendix C):

$$\frac{\partial d}{\partial r_j} = x_j\big(Y(p_y,\boldsymbol{r},\boldsymbol{q}^0),\boldsymbol{r},\boldsymbol{q}^0\big) - x_j\big(Y(p_y,\boldsymbol{r},\boldsymbol{q}^1),\boldsymbol{r},\boldsymbol{q}^1\big) = x_j^0 - x_j^1. \tag{16}$$

Note that the effect of a change in input j price on the variation function is given by the difference between the quantities of the input demanded before and after the change in input i quality level. The variation function "own price effect" $\left(\frac{\partial d}{\partial r_i}\right)$ will be negative if an improvement in the quality level of input i increases the quantity of input i that is demanded, so that $\frac{\partial x_i(Y(p_y,\boldsymbol{r},\boldsymbol{q}),\boldsymbol{r},\boldsymbol{q})}{\partial q_i} > 0$[4]. Similarly, the variation function "cross price effect" $\left(\frac{\partial d}{\partial r_j}\right)$ (for all $j \neq i$) will be negative (positive) if an upgrade in the quality level of input i results in an increase (decrease) in the quantity of input j that is demanded.

In the Cobb-Douglas case, the variation function own price and cross price effects are

$$\frac{\partial d}{\partial r_1} = -\frac{\alpha}{1-(\alpha+\beta)}\frac{d}{r_1} < 0 \tag{17}$$

and

$$\frac{\partial d}{\partial r_2} = -\frac{\beta}{1-(\alpha+\beta)}\frac{d}{r_2} < 0, \tag{18}$$

respectively. For a producer willing to pay for an upgrade in the quality level of input 1 $(d > 0)$, both the variation function own price and cross price effects will be negative. Note from

[4] More precisely, the own price effect will be negative if $\frac{\partial x_i(Y(p_y,\boldsymbol{r},\boldsymbol{q}),\boldsymbol{r},\boldsymbol{q})}{\partial Y}\frac{\partial Y(p_y,\boldsymbol{r},\boldsymbol{q})}{\partial q_i} + \left.\frac{\partial x_i(Y,\boldsymbol{r},\boldsymbol{q})}{\partial q_i}\right|_{Y=Y(p_y,\boldsymbol{r},\boldsymbol{q})} > 0$, where the first term on the left-hand side is expected to be positive and the second term $\left.\frac{\partial x_i(Y,\boldsymbol{r},\boldsymbol{q})}{\partial q_i}\right|_{Y=Y(p_y,\boldsymbol{r},\boldsymbol{q})}$ is expected to be negative.

expression (14), d will be positive as long as the new quality level of input 1 is higher than its previous level (i.e., $q_1^1 > q_1^0$). Moreover, the general condition to have negative own price and cross price effects, $\frac{\partial x_i(Y(p_y,\boldsymbol{r},\boldsymbol{q}),\boldsymbol{r},\boldsymbol{q})}{\partial q_j} > 0$, $j = 1{,}2$, is met in the Cobb-Douglas case (i.e., the quantity of x_1 and x_2 demand increase with improvements in the quality level of input 1, where the specific increases are given by $\frac{\partial x_1(Y(p_y,\boldsymbol{r},\boldsymbol{q}),\boldsymbol{r},\boldsymbol{q})}{\partial q_1} = \frac{\alpha}{1-(\alpha+\beta)}\frac{x_1}{q_1} > 0$ and $\frac{\partial x_2(Y(p_y,\boldsymbol{r},\boldsymbol{q}),\boldsymbol{r},\boldsymbol{q})}{\partial q_1} = \frac{\beta}{1-(\alpha+\beta)}\frac{x_2}{q_1} > 0$).

Output Price Effect

The effect of a change in the output price on the variation function is (see Appendix C):

$$(19) \qquad \frac{\partial d}{\partial p_y} = Y\left(p_y,\boldsymbol{r},\boldsymbol{q}^1\right) - Y\left(p_y,\boldsymbol{r},\boldsymbol{q}^0\right) = Y^1 - Y^0.$$

Hence, the change in d, due to a change in the output price, is given by the difference between the *ex post* and *ex ante* level of output produced. To sign this effect, additional comparative statics of the firm's profit maximization problem, described in (5), need to be derived. At the optimal level of $Y(P_y,\boldsymbol{r},\boldsymbol{q})$, the following condition holds:

$$(20) \qquad \frac{\partial Y(p_y,\boldsymbol{r},\boldsymbol{q})}{\partial q_i} = -\frac{C_{Yq_i}}{C_{YY}} = -\left(\frac{\partial \lambda(Y(P_y,\boldsymbol{r},\boldsymbol{q}),\boldsymbol{r},\boldsymbol{q})}{\partial Y}\right)^{-1} \frac{\partial \lambda(Y,\boldsymbol{r},\boldsymbol{q})}{\partial q_i}\bigg|_{Y=Y(p_y,\boldsymbol{r},\boldsymbol{q})},$$

where $C_{YY} = \frac{\partial^2 C(Y,\boldsymbol{r},\boldsymbol{q})}{\partial Y^2}$, $C_{Yq_i} = \frac{\partial^2 C(Y,\boldsymbol{r},\boldsymbol{q})}{\partial Y \partial q_i}\bigg|_{Y=Y(P_y,\boldsymbol{r},\boldsymbol{q})}$ and λ is the Lagrangian multiplier, which represents the firm's marginal cost of production (see Appendix C). Hence, the output price effect is positive if the firm operates where the marginal costs of production increase and an increase in the quality level of input i reduces the marginal cost of production. The two conditions requiring a positive output price effect are likely to occur in practice. First, firms are expected to operate in the "second stage of production" where the marginal product of inputs decreases with each extra unit of input; therefore, the marginal cost to produce each additional unit of output increases. Second, at given input prices and output levels, the use of more efficient inputs (e.g., inputs with higher quality levels) is expected to reduce costs that are incurred in producing each additional unit of output.

In the Cobb-Douglas case, the output price effect is positive and is given by

(21) $$\frac{\partial d}{\partial p_y} = \frac{1}{1-(\alpha+\beta)}\frac{d}{p_y} > 0.$$

Once again, the output price effect will be positive if $d > 0$. Additionally, the general properties, identified in expression (20), are $\frac{\partial \lambda(Y(P_y,\boldsymbol{r},\boldsymbol{q}),\boldsymbol{r},\boldsymbol{q})}{\partial Y} = \frac{1-(\alpha+\beta)}{(\alpha+\beta)}\frac{\lambda}{Y} > 0$ and $\frac{\partial \lambda(Y,\boldsymbol{r},\boldsymbol{q})}{\partial q_1}\Big|_{Y=Y(p_y,\boldsymbol{r},\boldsymbol{q})} = -\frac{\alpha}{(\alpha+\beta)}\frac{\lambda}{q_1} < 0$, where λ is positive because the cost function is non-decreasing in output (see Appendix B).

Input Quality Effects

The effect of a change in the initial quality level of input i on the variation function is

(22) $$\frac{\partial d}{\partial q_i^0} = \frac{\partial C(Y,\boldsymbol{r},\boldsymbol{q}^0)}{\partial q_i^0}\Big|_{Y=Y(P_y,\boldsymbol{r},\boldsymbol{q}^0)} .$$

Note that expression (22) represents the change in the firm's original production cost because of a change in the initial quality level of input i. The firm's cost minimization problem described in (6) allows us to rewrite (22) as

(23) $$\frac{\partial d}{\partial q_i^0} = -\lambda(Y(P_y,\boldsymbol{r},\boldsymbol{q}^0),\boldsymbol{r},\boldsymbol{q}^0)f_{q_i^0}^0 ,$$

where $f_{q_i^0}^0 = \frac{\partial f(\boldsymbol{X},\boldsymbol{q}^0)}{\partial q_i^0}\Big|_{\boldsymbol{X}=\boldsymbol{X}(Y(P_y,\boldsymbol{r},\boldsymbol{q}^0),\boldsymbol{r},\boldsymbol{q}^0)}$. $f_{q_i^0}^0$ can also be seen as the marginal product of q_i evaluated at the original input quality levels (see Appendix C). Note that the initial input quality effect will be negative if the firm operates where both the marginal costs of production and the marginal product of q_i^o are positive. In general, a firm's marginal cost (λ) is nonnegative because the cost function is non-decreasing in output and improvements in the quality level of inputs are expected to expand the amount of output produced.

Similarly, the final input quality effect can be written as

$$(24) \qquad \frac{\partial d}{\partial q_i^1} = -\left.\frac{\partial C(Y,\boldsymbol{r},\boldsymbol{q}^1)}{\partial q_i^1}\right|_{Y=Y(P_y,\boldsymbol{r},\boldsymbol{q}^1)} = \lambda(Y(P_y,\boldsymbol{r},\boldsymbol{q}^1),\boldsymbol{r},\boldsymbol{q}^1)f_{q_i^1}^1 ,$$

where $f_{q_i^1}^1 = \left.\frac{\partial f(\boldsymbol{X},\boldsymbol{q}^1)}{\partial q_i^1}\right|_{\boldsymbol{X}=\boldsymbol{X}(Y(P_y,\boldsymbol{r},\boldsymbol{q}^1),\boldsymbol{r},\boldsymbol{q}^1)}$. As in the case of $\frac{\partial d}{\partial q_i^0}$, the final input quality effect is positive if the *ex post* marginal costs of production and marginal product of q_i^1 are both positive.

Finally, the effect on the variation function of a change in the quality level of input *j* (for all $j \neq i$), whose *ex post* and *ex ante* quality level is assumed to be the same, is

$$(25) \qquad \frac{\partial d}{\partial q_j^0} = \left.\frac{\partial C(Y,\boldsymbol{r},\boldsymbol{q}^0)}{\partial q_j^0}\right|_{Y=Y(P_y,\boldsymbol{r},\boldsymbol{q}^0)} - \left.\frac{\partial C(Y,\boldsymbol{r},\boldsymbol{q}^1)}{\partial q_j^0}\right|_{Y=Y(P_y,\boldsymbol{r},\boldsymbol{q}^1)}$$

$$= \lambda\big(Y\big(P_y,\boldsymbol{r},\boldsymbol{q}^1\big),\boldsymbol{r},\boldsymbol{q}^1\big)f_{q_j^0}^1 - \lambda\big(Y\big(P_y,\boldsymbol{r},\boldsymbol{q}^0\big),\boldsymbol{r},\boldsymbol{q}^0\big)f_{q_j^0}^0.$$

Note that the two right-hand side terms in (25) differ only in the quality level of input i; therefore, this derivative can be signed by taking the first partial derivate of $\lambda\big(Y\big(P_y,\boldsymbol{r},\boldsymbol{q}\big),\boldsymbol{r},\boldsymbol{q}\big)f_{q_j}$ w.r.t. q_i, where $f_{q_j} = \left.\frac{\partial f(\boldsymbol{X},\boldsymbol{q})}{\partial q_j}\right|_{\boldsymbol{X}=\boldsymbol{X}(Y(P_y,\boldsymbol{r},\boldsymbol{q}),\boldsymbol{r},\boldsymbol{q})}$. Let $A = \lambda\big(Y\big(P_y,\boldsymbol{r},\boldsymbol{q}\big),\boldsymbol{r},\boldsymbol{q}\big)f_{q_j}$ then it is easily verified that $\frac{\partial A}{\partial q_i} = \lambda\big(Y\big(P_y,\boldsymbol{r},\boldsymbol{q}\big),\boldsymbol{r},\boldsymbol{q}\big)f_{q_jq_i}$, where $f_{q_jq_i} = \left.\frac{\partial^2 f(\boldsymbol{X},\boldsymbol{q})}{\partial q_j \partial q_i}\right|_{\boldsymbol{X}=\boldsymbol{X}(Y(P_y,\boldsymbol{r},\boldsymbol{q}),\boldsymbol{r},\boldsymbol{q})}$ (see Appendix C).

Thus, if the marginal costs of production and $f_{q_jq_i}$ are both positive, then the input *j* quality effect is also positive. The term $f_{q_jq_i}$ is expected to be positive because an improvement in the quality of one input is likely to make other quality upgraded inputs even more productive.

The corresponding input quality effects for the Cobb-Douglas case are:

(26) $$\frac{\partial d}{\partial q_1^0} = -\frac{\alpha}{1-(\alpha+\beta)}\frac{\Pi^0}{q_1^0} < 0,$$

(27) $$\frac{\partial d}{\partial q_1^1} = \frac{\alpha}{1-(\alpha+\beta)}\frac{\Pi^1}{q_1^1} > 0$$

and

(28) $$\frac{\partial d}{\partial q_2^0} = \frac{\beta}{1-(\alpha+\beta)}\frac{d}{q_2^0} > 0,$$

where $\Pi^0 = \Pi(p_y, r_1, r_2, q_1^0, q_2^0) > 0$ and $\Pi^1 = \Pi(p_y, r_1, r_2, q_1^1, q_2^0) > 0$. Note that, in the Cobb-Douglas case, the variation function is decreasing in q_1^0 and increasing in q_1^1 and q_2^0. Moreover, the general properties needed to sign the direction of the different quality effects are given by $f_{q_1^\iota}^\iota = \alpha\frac{Y^\iota}{q_1^\iota} > 0$, $\iota = 1{,}0$, and $f_{q_2q_1} = \alpha\beta\frac{Y}{q_1q_2} > 0$.

4. Implications for Current Practice

The derived comparative statics of the variation, or WTP, function have significant implications for current practice. The first concerns the specification of empirical models and the design of surveys. The second implication relates to testing theoretical restrictions.

To clarify the role of the comparative statics results in the specification of empirical models and survey design, consider the simple case that includes only two inputs; the quality level on input 1 is upgraded while the quality level of input 2 remains constant. A linear variation function model including all the explanatory variables identified in the theoretical model (i.e., input prices, output price, and input quality levels) is[5]

(29) $$d = \beta_0 + \beta_1 r_1 + \beta_2 r_2 + \beta_3 p_y + \beta_4 q_1^0 + \beta_5 q_1^1 + \beta_6 q_2^0 + \varepsilon,$$

[5] The model could also include characteristics of the firm or firm's owner but we exclude these to simplify the analysis.

where the $\beta_i's$ are coefficients to be estimated and ε is a zero mean error term. Note that coefficients corresponding to prices or quality levels (β_1 to β_6) can only be estimated if there is variability in the levels of these variables across producers. The variability in the exogenous variables can occur if producers face different prices or use products of different quality levels and can be collected as part of the survey. Alternatively, variability in the explanatory variables can be generated as part of the contingent valuation survey design (i.e., producers are given different hypothetical price and quality levels). After estimation, the marginal effects of the variation function can be recovered using the coefficients in (29), so that $\beta_1 = \frac{\partial d}{\partial r_1}$, $\beta_2 = \frac{\partial d}{\partial r_2}$, $\beta_3 = \frac{\partial d}{\partial p_y}$, $\beta_4 = \frac{\partial d}{\partial q_1^0}$, $\beta_5 = \frac{\partial d}{\partial q_1^1}$, and $\beta_6 = \frac{\partial d}{\partial q_2^0}$ and the signs of the coefficients compared to those derived in the theoretical section.

The estimated derived marginal effects from equation (29) can also be used to estimate *ex post* input and output quantities. For example, because

(30) $$x_1^1 = x_1^0 - \frac{\partial d}{\partial r_1}$$

(from equation (16)) and

(31) $$Y^1 = Y^0 + \frac{\partial d}{\partial p_y}$$

(from equation (19)), estimates of the *ex post* quantity demanded of input 1 (x_1^1) and *ex post* output supply (Y^1) can be calculated combining the estimates of $\frac{\partial d}{\partial r_1}$ and $\frac{\partial d}{\partial p_y}$ from (29) (i.e., β_1 and β_3) with the current amounts of input demanded (x_1^0) and output supplied (Y^0); these values can also be collected during the survey stage.

One limitation of the linear variation functional form in equation (29) is that it does not allow the estimation of marginal effects or elasticities of the new demand and supply functions. Specifically, estimation of marginal effects or elasticities requires the specification of a variation function that allows, at least, second order derivative calculations (e.g., by adding quadratic terms to equation (29)). Moreover, as in the case of the *ex post* input and output quantities estimation, the calculation of *ex post* elasticities requires knowledge of *ex ante* elasticity values or marginal effects. For example, the *ex post* input 1 own price marginal effect can be obtained by taking the partial derivate of x_1^1 with respect to r_1 in equation (30), which results in

(32) $$\frac{\partial x_1^1}{\partial r_1} = \frac{\partial x_1^0}{\partial r_1} - \frac{\partial^2 d}{\partial {r_1}^2},$$

and the corresponding *ex post* input 1 own price elasticity is given by

(33) $$\varepsilon_{x_1 r_1}^1 = \frac{x_1^0}{x_1^1} \varepsilon_{x_1 r_1}^0 - \frac{\partial^2 d}{\partial {r_1}^2} \frac{r_1}{x_1^1},$$

where $\varepsilon_{x_1 r_1}^0$ is the *ex ante* input 1 own price elasticity. Likewise, the *ex post* output price marginal effect can be estimated by taking the partial derivative of Y^1 with respect to p_y in expression (31). Specifically, the *ex post* output price marginal effect and price elasticity of supply are given by

(34) $$\frac{\partial Y^1}{\partial p_y} = \frac{\partial Y^0}{\partial p_y} + \frac{\partial^2 d}{\partial {p_y}^2}$$

and

(35) $$\varepsilon_{Y p_y}^1 = \frac{Y^0}{Y^1} \varepsilon_{Y p_y}^0 + \frac{\partial^2 d}{\partial {p_y}^2} \frac{p_y}{Y^1},$$

respectively, where $\varepsilon_{Y p_y}^0$ is the *ex ante* price elasticity of supply.

It is also possible to envision an alternative use of the results obtained by estimating a variation function of the type shown in equation (29); specifically, in a case where all parameters of the production function of a firm or industry are known in advance. For example, for the 2 inputs Cobb-Douglas production function introduced earlier, the new input demand and output supply of the firm, derived from a change in the quality level of input 1 from q_1^0 to q_1^1, are given by $x_1^1 = x_1^0 + \frac{\alpha}{1-(\alpha+\beta)}\frac{d}{r_1}$, $x_2^1 = x_2^0 + \frac{\beta}{1-(\alpha+\beta)}\frac{d}{r_2}$, and $Y^1 = Y^0 + \frac{1}{1-(\alpha+\beta)}\frac{d}{p_y}$, respectively. Hence, the new inputs demand and supply values can be calculated using information from the original quantity demanded of inputs and quantity of output supplied, the WTP value, and the parameters of the production function.

If the parameters of the production were known, the relevant derivatives of the new demand for input 1 are $\frac{\partial x_1^1}{\partial r_1} = -\frac{1-\beta}{(1-(\alpha+\beta))r_1}\left(x_1^0 - \frac{\partial d}{\partial r_1}\right)$, $\frac{\partial x_1^1}{\partial r_2} = -\frac{\beta}{(1-(\alpha+\beta))r_2}\left(x_1^0 - \frac{\partial d}{\partial r_1}\right)$, $\frac{\partial x_1^1}{\partial p_y} = \frac{1}{(1-(\alpha+\beta))p_y}\left(x_1^0 - \frac{\partial d}{\partial r_1}\right)$, and $\frac{\partial x_1^1}{\partial q_1^1} = \frac{\alpha}{(1-(\alpha+\beta))r_1}\frac{\partial d}{\partial q_1^1}$. Thus, in this case, the calculation of the marginal effects of the new demands only require information on the parameters of the production function, input or output levels, prices, and the marginal effects obtained from (29). Similarly, the derivatives of the new output supply, with respect to input prices, output price, and input 1 final quality level are $\frac{\partial Y^1}{\partial r_1} = -\frac{\alpha}{(1-(\alpha+\beta))r_1}\left(Y^0 + \frac{\partial d}{\partial p_y}\right)$, $\frac{\partial Y^1}{\partial r_2} = -\frac{\beta}{(1-(\alpha+\beta))r_2}\left(Y^0 + \frac{\partial d}{\partial p_y}\right)$, $\frac{\partial Y^1}{\partial p_y} = \frac{\alpha+\beta}{(1-(\alpha+\beta))p_y}\left(Y^0 + \frac{\partial d}{\partial p_y}\right)$, and $\frac{\partial Y^1}{\partial q_1^1} = \frac{1}{(1-(\alpha+\beta))p_y}\frac{\partial d}{\partial q_1^1}$, respectively[6]. Moreover, it is easily shown that the *ex post* own input price elasticity of input 1 and price elasticity of supply for the Cobb-Douglas case are given by $\varepsilon_{x_1 r_1}^1 = -\frac{1-\beta}{(1-(\alpha+\beta))}$ and $\varepsilon_{Y p_y}^1 = \frac{\alpha+\beta}{(1-(\alpha+\beta))}$, respectively.

5. Summary and Conclusions

The main objective of this study was to analyze the theoretical underpinnings of producer WTP for new inputs. In addition to conceptualizing the producer WTP function, we derived its

[6] These marginal effects are derived using the fact that $\boldsymbol{X}(Y, \boldsymbol{r}, \boldsymbol{q})$ and $Y(p_y, \boldsymbol{r}, \boldsymbol{q})$ come from cost minimization and profit maximization (see Appendix B for specific forms), respectively. Moreover, these derivatives can be signed using the comparative statics results presented in section 3. For example, the quantity demanded of the quality upgraded input (input 1) can be shown to decrease with its own and other input prices and increase with output price and its own final quality level.

comparative statics and showed how these properties can be used to recovery quantity demanded or supplied and, in some cases, price elasticities. We also discussed implications of this relationship to specify empirical WTP models and survey design.

The WTP model presented was developed within the context of neoclassical theories of utility and profit maximization. More specifically, the variation function, or producers' WTP, for novel inputs or technologies is derived using an individual indirect utility function in combination with the firm's profit function. This theoretical model is developed in a context where the production function $f(\cdot)$ has, as arguments, a vector of input quantities $\boldsymbol{X}$ and a vector of input quality levels $\boldsymbol{q}$. The level of $\boldsymbol{q}$ is fixed exogenously, thus the profit and cost functions are also conditional on $\boldsymbol{q}$. The analysis considers an improvement on a particular input quality level, q_i.

The theoretical results imply that the maximum amount of money that a producer is WTP for a new production factor is equal to the difference between the *ex post* and *ex ante* firm's profit levels. Moreover, the results suggest that the producers' WTP is a function of output and input prices and input *ex ante* and *ex post* quality levels. Comparative statics results show that producers' WTP is a decreasing function of upgraded input price, its initial quality level, and an increasing function of output price and final quality level.

Use of the structure required by profit and utility maximization is also helpful in empirical practice. Here, we demonstrated the use of comparative statics results to estimate input demanded, output supplied, and price elasticities after the change in the input quality. However, estimation of these values is dependent upon the empirical model used and data availability. Thus, the results of this study should be of considerable use in specifying empirical WTP models and survey design.

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7. Appendices

Appendix A. Cost Minimization and Profit Maximization Problems.

Cost minimization problem

The Lagrangian function of the cost minimization problem is given by

(A1) $$\mathcal{L}(\boldsymbol{X}) = \boldsymbol{r}`\boldsymbol{X} + \lambda(Y - f(\boldsymbol{X},\boldsymbol{q}))$$

and the FOC can be represented by

(A2) $$r_i - \lambda f_{x_i} = 0 \quad \forall\, i$$

and

(A3) $$Y - f(\boldsymbol{X},\boldsymbol{q}) = 0,$$

where $f_{x_i} = \frac{\partial f(\boldsymbol{X},\boldsymbol{q})}{\partial x_i}$. The FOC imply that

(A4) $$\boldsymbol{X} = \boldsymbol{X}(Y,\boldsymbol{r},\boldsymbol{q})$$

and

(A5) $$\lambda = \lambda(Y,\boldsymbol{r},\boldsymbol{q}).$$

The cost function is obtained by replacing (A4) and (A5) into (A1)

(A6) $$C(Y,\boldsymbol{r},\boldsymbol{q}) = \boldsymbol{r}`\boldsymbol{X}(Y,\boldsymbol{r},\boldsymbol{q}) + \lambda(Y,\boldsymbol{r},\boldsymbol{q})[Y - f(\boldsymbol{X}(Y,\boldsymbol{r},\boldsymbol{q}),\boldsymbol{q})\,].$$

At the optimum, partial derivatives of (A6) w.r.t. Y, r_i and q_i, respectively, are given by

(A7) $$\frac{\partial C(Y,\boldsymbol{r},\boldsymbol{q})}{\partial Y} = \lambda(Y,\boldsymbol{r},\boldsymbol{q}) + \sum_i \frac{\partial x_i}{\partial Y}\left(r_i - \lambda f_{x_i}\right) + \frac{\partial \lambda}{\partial Y}(Y - f(\boldsymbol{X},\boldsymbol{q})\,)$$

$$= \lambda(Y,\boldsymbol{r},\boldsymbol{q}),$$

(A8) $$\frac{\partial C(Y,\boldsymbol{r},\boldsymbol{q})}{\partial r_i} = x_i(Y,\boldsymbol{r},\boldsymbol{q}) + \sum_i \frac{\partial x_i}{\partial r_i}\left(r_i - \lambda f_{x_i}\right) + \frac{\partial \lambda}{\partial r_i}(Y - f(\boldsymbol{X},\boldsymbol{q})\,)$$

$$= x_i(Y,\boldsymbol{r},\boldsymbol{q})$$

and

(A9) $$\frac{\partial C(Y,\boldsymbol{r},\boldsymbol{q})}{\partial q_i} = -\lambda(Y,\boldsymbol{r},\boldsymbol{q})f_{q_i} + \sum_i \frac{\partial x_i}{\partial q_i}\left(r_i - \lambda f_{x_i}\right) + \frac{\partial \lambda}{\partial q_i}(Y - f(\boldsymbol{X},\boldsymbol{q})\,)$$

$$= -\lambda(Y, \boldsymbol{r}, \boldsymbol{q}) f_{q_i} ,$$

where $f_{q_i} = \frac{\partial f(\boldsymbol{X},\boldsymbol{q})}{\partial q_i}\Big|_{\boldsymbol{X}=\boldsymbol{X}(Y,\boldsymbol{r},\boldsymbol{q})}$.

Profit maximization problem

The producer's cost maximization problem is given by

(A10) $$max_Y \Pi = p_y Y - \; C(Y, \boldsymbol{r}, \boldsymbol{q})$$

and the FOC from (A10) is

(A11) $$p_y - \frac{\partial C(Y,\boldsymbol{r},\boldsymbol{q})}{\partial Y} = 0.$$

From (A11) we obtain that

(A12) $$Y = Y\big(p_y, \boldsymbol{r}, \boldsymbol{q}\big).$$

The firm's profit function is obtained by replacing (A12) into (A10)

(A13) $$\Pi\big(p_y, \boldsymbol{r}, \boldsymbol{q}\big) = p_y Y\big(p_y, \boldsymbol{r}, \boldsymbol{q}\big) - C\Big(Y\big(p_y, \boldsymbol{r}, \boldsymbol{q}\big), \boldsymbol{r}, \boldsymbol{q}\Big).$$

The partial derivatives of the profit function w.r.t. p_y, r_i and q_i, respectively, are given by

(A14) $$\frac{\partial \Pi(p_y,\boldsymbol{r},\boldsymbol{q})}{\partial p_y} = Y\big(p_y, \boldsymbol{r}, \boldsymbol{q}\big) + \frac{\partial Y}{\partial p_y}\left(p_y - \frac{\partial C(Y,\boldsymbol{r},\boldsymbol{q})}{\partial Y}\right)$$

$$= Y\big(p_y, \boldsymbol{r}, \boldsymbol{q}\big),$$

(A15) $$\frac{\partial \Pi(p_y,\boldsymbol{r},\boldsymbol{q})}{\partial r_i} = -\frac{\partial C(Y,\boldsymbol{r},\boldsymbol{q})}{\partial r_i}\Big|_{Y=Y(p_y,\boldsymbol{r},\boldsymbol{q})} + \frac{\partial Y}{\partial r_i}\left(p_y - \frac{\partial C(Y,\boldsymbol{r},\boldsymbol{q})}{\partial Y}\right)$$

$$= -\frac{\partial C(Y,\boldsymbol{r},\boldsymbol{q})}{\partial r_i}\Big|_{Y=Y(p_y,\boldsymbol{r},\boldsymbol{q})}$$

and

(A16) $$\frac{\partial \Pi(p_y,\boldsymbol{r},\boldsymbol{q})}{\partial q_i} = -\frac{\partial C(Y,\boldsymbol{r},\boldsymbol{q})}{\partial q_i}\Big|_{Y=Y(p_y,\boldsymbol{r},\boldsymbol{q})} + \frac{\partial Y}{\partial q_i}\left(p_y - \frac{\partial C(Y,\boldsymbol{r},\boldsymbol{q})}{\partial Y}\right)$$

$$= -\frac{\partial C(Y,\boldsymbol{r},\boldsymbol{q})}{\partial q_i}\Big|_{Y=Y(p_y,\boldsymbol{r},\boldsymbol{q})}.$$

Appendix B. Derivation of the Variation Function under a Cobb-Douglas Two Inputs Production Function.

It is assumed that the production of the output Y is given by

(B1) $$Y = (q_1 x_1)^{\alpha}(q_2 x_2)^{\beta}.$$

Thus, the cost minimization problem of the firm is represented by

(B2) $$min_{x_1,x_2} C = r_1 x_1 + r_2 x_2 \text{ subject to } Y = (q_1 x_1)^{\alpha}(q_2 x_2)^{\beta}$$

and the first order conditions (FOC) are given by

(B3) $$r_1 = \lambda\alpha(q_1 x_1)^{\alpha-1}(q_2 x_2)^{\beta} q_1,$$

(B4) $$r_2 = \lambda\beta(q_1 x_1)^{\alpha}(q_2 x_2)^{\beta-1} q_2$$

and

(B5) $$Y = (q_1 x_1)^{\alpha}(q_2 x_2)^{\beta},$$

where λ is the Lagrangian multiplier.

From the FOC the optimal level of input 1 and 2, respectively, are

(B6) $$x_1 = \left[\frac{Y {r_2}^{\beta} \alpha^{\beta}}{r_1^{\beta} q_1^{\alpha} q_2^{\beta} \beta^{\beta}}\right]^{\frac{1}{(\alpha+\beta)}}$$

and

(B7) $$x_2 = \left[\frac{Y r_1^{\alpha} \beta^{\alpha}}{r_2^{\alpha} q_1^{\alpha} q_2^{\beta} \alpha^{\alpha}}\right]^{\frac{1}{(\alpha+\beta)}}$$

The cost function is obtained by replacing the optimal level of x_1 and x_2 into (B2)

(B8) $$C(Y, \boldsymbol{r}, \boldsymbol{q}) = (\alpha+\beta)\left[\frac{Y r_1^{\alpha} {r_2}^{\beta}}{q_1^{\alpha} q_2^{\beta} \alpha^{\alpha} \beta^{\beta}}\right]^{\frac{1}{(\alpha+\beta)}}.$$

Then, the producer's profit maximization problem is given by

(B9) $$max_Y \Pi = P_y Y - (\alpha+\beta)\left[\frac{Y r_1^{\alpha} {r_2}^{\beta}}{q_1^{\alpha} q_2^{\beta} \alpha^{\alpha} \beta^{\beta}}\right]^{\frac{1}{(\alpha+\beta)}}$$

and the optimal output level from [B9] is

(B10) $$Y = \left[\frac{P_y^{\alpha+\beta} q_1^{\alpha} q_2^{\beta} \alpha^{\alpha} \beta^{\beta}}{r_1^{\alpha} r_2{}^{\beta}}\right]^{\frac{1}{1-(\alpha+\beta)}}.$$

The profit function is obtained by replacing (B10) into (B9)

(B11) $$\Pi = [1-(\alpha+\beta)]\left[\frac{P_y q_1^{\alpha} q_2^{\beta} \alpha^{\alpha} \beta^{\beta}}{r_1^{\alpha} r_2{}^{\beta}}\right]^{\frac{1}{1-(\alpha+\beta)}}.$$

Finally, the change in profits or variation function from a change in the quality level of input 1 from q_1^0 to q_1^1 is given by

(B12) $$d = [1-(\alpha+\beta)]\left[q_1^{1\frac{\alpha}{1-(\alpha+\beta)}} - q_1^{0\frac{\alpha}{1-(\alpha+\beta)}}\right]\left[\frac{p_y \beta^{\beta} q_2^{0\beta} \alpha^{\alpha}}{r_1^{\alpha} r_2^{\beta}}\right]^{\frac{1}{1-(\alpha+\beta)}}.$$

Appendix C. Comparative Statics of the Variation Function.

Input price effects

The change in the variation function from a change in the price of input i is

(C1) $$\frac{\partial d}{\partial r_i} = \frac{\partial \Pi(p_y, \boldsymbol{r}, \boldsymbol{q}^1)}{\partial r_i} - \frac{\partial \Pi(p_y, \boldsymbol{r}, \boldsymbol{q}^0)}{\partial r_i}.$$

By replacing the appropriate forms of (A15) into (C1), then expression (C1) can be rewritten as

(C2) $$\frac{\partial d}{\partial r_i} = \frac{\partial C(Y, \boldsymbol{r}, \boldsymbol{q}^0)}{\partial r_i}\bigg|_{Y=Y(p_y, \boldsymbol{r}, \boldsymbol{q}^0)} - \frac{\partial C(Y, \boldsymbol{r}, \boldsymbol{q}^1)}{\partial r_i}\bigg|_{Y=Y(p_y, \boldsymbol{r}, \boldsymbol{q}^1)}$$

and by replacing expression (A8) into (C2) we can express $\frac{\partial d}{\partial r_i}$ as

(C3) $$\frac{\partial d}{\partial r_i} = x_i\left(Y(p_y, \boldsymbol{r}, \boldsymbol{q}^0), \boldsymbol{r}, \boldsymbol{q}^0\right) - x_i\left(Y(p_y, \boldsymbol{r}, \boldsymbol{q}^1), \boldsymbol{r}, \boldsymbol{q}^1\right).$$

Output price effect

The effect of a change in the output price on the variation function is given by

(C4) $$\frac{\partial d}{\partial p_y} = \frac{\partial \Pi(p_y, \boldsymbol{r}, \boldsymbol{q}^0)}{\partial p_y} - \frac{\partial \Pi(p_y, \boldsymbol{r}, \boldsymbol{q}^1)}{\partial p_y}.$$

From expression (A14) we can rewrite (C4) as

(C5) $$\frac{\partial d}{\partial p_y} = Y(p_y, \boldsymbol{r}, \boldsymbol{q}^1) - Y(p_y, \boldsymbol{r}, \boldsymbol{q}^0).$$

Moreover, replacing (A12) back into (A11) and taking the partial derivative of (A11) w.r.t. q_i yields

(C6) $$C_{YY}\frac{\partial Y(p_y, \boldsymbol{r}, \boldsymbol{q})}{\partial q_i} = -C_{Yq_i},$$

where $C_{YY} = \frac{\partial^2 C(Y, \boldsymbol{r}, \boldsymbol{q})}{\partial Y^2}$ and $C_{Yq_i} = \frac{\partial^2 C(Y, \boldsymbol{r}, \boldsymbol{q})}{\partial Y \partial q_i}\Big|_{Y=Y(p_y, \boldsymbol{r}, \boldsymbol{q})}$. By rearranging terms, at the optimum production level the change in $Y(p_y, \boldsymbol{r}, \boldsymbol{q})$ w.r.t. q_i is equal to

(C7) $$\frac{\partial Y(p_y, \boldsymbol{r}, \boldsymbol{q})}{\partial q_i} = -\frac{C_{Yq_i}}{C_{YY}}.$$

By (A7) it is easily verified that expression (C7) can be written as

(C8) $$\frac{\partial Y(p_y,\boldsymbol{r},\boldsymbol{q})}{\partial q_i} = -\left(\frac{\partial\lambda(Y(P_y,\boldsymbol{r},\boldsymbol{q}),\boldsymbol{r},\boldsymbol{q})}{\partial Y}\right)^{-1}\frac{\partial\lambda(Y,\boldsymbol{r},\boldsymbol{q})}{\partial q_i}\bigg|_{Y=Y(p_y,\boldsymbol{r},\boldsymbol{q})}.$$

Input quality effects

The effect of a change in the initial quality level of input i, q_i^0, on the variation function is

(C9) $$\frac{\partial d}{\partial q_i^0} = \frac{\partial\Pi(p_y,\boldsymbol{r},\boldsymbol{q}^0)}{\partial q_i^0}.$$

By replacing (A16) into (C9) we can rewrite expression (C9) as

(C10) $$\frac{\partial d}{\partial q_i^0} = -\frac{\partial C(Y,\boldsymbol{r},\boldsymbol{q}^0)}{\partial q_i}\bigg|_{Y=Y(p_y,\boldsymbol{r},\boldsymbol{q}^0)}.$$

Finally, replace (A9) into (C10) to obtain

(C11) $$\frac{\partial d}{\partial q_i^0} = -\lambda(Y(P_y,\boldsymbol{r},\boldsymbol{q}^0),\boldsymbol{r},\boldsymbol{q}^0)f^0_{q_i^0}.$$

The same logic can used to derive the marginal effects of a change in the final quality level of input i, $\frac{\partial d}{\partial q_i^1}$, or the marginal effect of a change in the quality level of input j, $\frac{\partial d}{\partial q_j^0}$, on the variation function.

In the case of $\frac{\partial d}{\partial q_j^0}$, let $A = \lambda\big(Y\big(P_y,\boldsymbol{r},\boldsymbol{q}\big),\boldsymbol{r},\boldsymbol{q}\big)f_{q_j}$, then the partial derivative of A w.r.t. q_i is given by

(C12) $$\frac{\partial A}{\partial q_i} = \left[\frac{\partial\lambda(Y(P_y,\boldsymbol{r},\boldsymbol{q}),\boldsymbol{r},\boldsymbol{q})}{\partial Y}\frac{\partial Y(P_y,\boldsymbol{r},\boldsymbol{q})}{\partial q_i} + \frac{\partial\lambda(Y,\boldsymbol{r},\boldsymbol{q})}{\partial q_i}\bigg|_{Y=Y(p_y,\boldsymbol{r},\boldsymbol{q})}\right]f_{q_j}$$

$$+\lambda\big(Y\big(P_y,\boldsymbol{r},\boldsymbol{q}\big),\boldsymbol{r},\boldsymbol{q}\big)f_{q_jq_i},$$

where $f_{q_jq_i} = \frac{\partial^2 f(\boldsymbol{X},\boldsymbol{q})}{\partial q_j\partial q_i}\Big|_{\boldsymbol{X}=\boldsymbol{X}(Y(P_y,\boldsymbol{r},\boldsymbol{q}),\boldsymbol{r},\boldsymbol{q})}$. Finally, by (C8) expression (C12) can be written as

(C13) $$\frac{\partial A}{\partial q_i} = \lambda\big(Y\big(P_y,\boldsymbol{r},\boldsymbol{q}\big),\boldsymbol{r},\boldsymbol{q}\big)f_{q_jq_i}.$$