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Production Flexibility and Technical Change in Australia's Wheat-Sheep Zone*

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The flexibility of production and the bias of technical change in the Wheat/Sheep Zone has been examined by estimating the system of derived outout and input share equations from a tianslog variable profit function. This analysis was undertaken for three outputs (sheep and wool, crops, and beef cattle and other farm output) and five inputs (labour, materials and services, livestock, capital, and land).

The supply of each of these three major groups of farm outputs has been inelastic. Sheep enterprise production has been complementary with cropping while crop and beef cattle outputs have not been complementary. The demand for materials and services inputs has been elastic while the elasticity of demand for labour has been approximately unity.

Wool and other sheep output has been relatively labour intensive while crops have been relatively capital intensive. Livestock activities (sheep and cattle) have been relatively land intensive. Hence, policies that have causea the price of labour to be greater (less) than it would have been otherwise have discouraged (encouraged) the production of sheep enterprise output relatively more than other farm outputs.

Introduction

The purpose of this paper is to assess both the flexibility of production and the bias of technical change in Australia's major mixed farming region, the Wheat/Sheep Zone. This includes assessing the ease with which the output mix of wool and sheep, crops, and beef cattle is changed in response to relative product prices and the relative intensities with which these outputs employ key inputs such as labour, capital, and land. It is important that economists have information on such characteristics of production in order to adequately assess the production effects of price and policy changes. For example, knowledge of the ease with which a producer can change his output mix and alter the quantity of each input he uses is necessary in fully assessing the likely effects of changes in wage rates or the prospects for live-sheep exports. Moreover, information on the relative intensities with which the production of each output uses the various inputs is necessary to assess the likely effects of relative input price changes on the composition of output of multi-output producers.

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Estimates of the elasticities reflecting these characteristics of the production technology were obtained by estimating a system of derived output and input share equations for the Wheat/Sheep Zone of Australia's sheep industry. Share equations were simultaneously estimated for three outputs (sheep and wool, crops, and beef cattle and other farm output) and five inputs (labour, materials and services, livestock, capital, and land). The time period employed for this estimation was 1952–53 to 1976–77.

In the past, econometric studies of rural production have generally concentrated on either the sector as a whole or specific outputs (Young 1971; Malecky 1975). Moreover, the relationship between production inputs has received more attention than that between outputs (Vincent 1977; McKay, Lawrence and Vlastuin 1980). In spite of the joint production of outputs in much of Australian agriculture, this aspect of the production system has commonly been overlooked due to limitations associated with traditional functional forms (Duloy 1964). The study by Powell and Gruen (1967) was an early attempt to incorporate joint production. More recently, Freebairn (1973) used an econometric model to examine the supply response of the New South Wales livestock sector and Wicks and Dillon (1978) examined the supply response of three outputs (wool, wheat, and beef) using the APMAA programming model. Vincent, Dixon and Powell (1980) recently developed a CRESH/CRETH model which is flexible enough to accommodate joint production but is unnecessarily restrictive in assuming that no input has a comparative advantage in the production of any particular output. The approach taken in this paper yields information on both the relationships between various inputs and selected farm outputs and the relationships between alternative outputs.

After briefly formulating a general variable profit function model of production, a specific econometric model is developed in the following section. Sufficient conditions for duality with a conventional production possibility set are given in Appendix I. Parameter estimates for the derived share equations along with the associated elasticities reflecting the production technology are presented and analysed in the third section. Conclusions are given in the fourth section and the data employed are presented in Appendix II.

Model Formulation

The analysis presented in this paper is based upon the duality that exists between the transformation function, the variable profit function and the production possibility set. After being introduced by Samuelson (1954) the concept of a variable profit function has been refined by Gorman (1968) and Diewert (1973, 1974) among others. Under the regularity conditions given in Appendix I the duality theorems establish that, if producers maximise profit, the variable profit function contains sufficient information to completely describe the production technology.

The advantage in specifying the variable profit function is that the output supply and input demand equations are easily derived as the partial derivatives of the variable profit function with respect to prices, while shadow price equations can be derived as the partial derivatives of the variable profit function with respect to fixed inputs (Diewert 1974). The parameters of the output supply, input demand and shadow price equations may be estimated using conventional multivariate regression techniques. Parameters, so estimated, describe the production technology and provide a measure of the ease with which producers have been able to alter the combination of outputs produced and inputs employed.

Henceforth, we will denote variable outputs or inputs by y_i, $i = 1, \ldots, I$ (y_i is positive if an output, negative otherwise), fixed inputs by x_j, $j = 1, \ldots, J$, prices of variable quantities by p_i and prices of fixed quantities by w_j. The vector of these quantities or prices will be denoted by the corresponding letter, but without subscript. Due to the lack of a suitable index for the level of technology, time, t, is employed as a proxy for such an index.

The following transcendental logarithmic form was postulated for the variable profit function:

$$
\begin{aligned}
(1)\ \ln \pi(p;x,t) = {} & A + A_T \ln t + A_{TT} (\ln t)^2 + \sum_{i=1}^{I} a_{io} \ln p_i \\
& + \tfrac{1}{2} \sum_{i=1}^{I} \sum_{h=1}^{I} a_{ih} \ln p_i \ln p_h + \sum_{i=1}^{I} a_{iT} \ln t \ln p_i \\
& + \sum_{j=1}^{J} b_{jo} \ln x_j + \tfrac{1}{2} \sum_{j=1}^{J} \sum_{k=1}^{J} b_{jk} \ln x_j \ln x_k \\
& + \sum_{j=1}^{J} b_{jT} \ln t \ln x_j + \sum_{i=1}^{I} \sum_{j=1}^{J} c_{ij} \ln p_i \ln x_j
\end{aligned}
$$

where $\pi(p;x,t)$ is variable profit (total returns less variable costs).[1] This functional form provides a second order approximation to an arbitrary variable profit function and, hence, to an arbitrary set of elasticities relating variable net outputs and shadow prices to changes in prices and fixed inputs. However, in the estimated econometric model, this translog variable profit function was not viewed as an approximation to an arbitrary variable profit function. Rather it was assumed to be the true data-generating function. This permitted additive disturbance terms to be specified for the derived output and input share equations and interpreted as deviations of the endogenous left-hand variables about their profit maximising values.

To ensure symmetry of the matrix of second order derivatives of a twice differentiable continuous function and permit the parameters of share equations to be identified the following restrictions are imposed (Diewert 1974);

$$(2)\quad a_{ih} = a_{hi},\ b_{jk} = b_{kj};\ i,h = 1, \ldots, I;\ j,k = 1, \ldots, J.$$

[1] With this functional form, having time as a proxy for the level of technology is equivalent to specifying a general form of exponential rates of output and input augmenting technical change. Hence, it is acknowledged that parameter estimates may be significantly dependent on the starting point and units of measurement for the time variable. In general, the higher the number taken for the starting point, the closer the results with this logarithmic time variable will approximate those that would be generated by specifying a linear time variable. In the absence of any *a priori* reason to depart from convention when postulating a logarithmic time variable, t has been set equal to 1, 2, . . ., 25 for the 25 annual observations used in this study.

The assumed linear homogeneity of this variable profit function in prices (p) and fixed inputs (x) (Conditions I.2 and I.4 of Appendix I) requires that the following restrictions be imposed on the parameter estimates:

$$(3) \quad \sum_{i=1}^{I} a_{io} = 1, \sum_{i=1}^{I} a_{ih} = 0 \text{ for all } h \neq 0, \sum_{i=1}^{I} c_{ij} = 0 \text{ for all } \quad \sum_{i=1}^{I} a_{iT} = 0$$

$$(4) \quad \sum_{j=1}^{J} b_{jo} = 1, \sum_{j=1}^{J} b_{jk} = 0 \text{ for all } k \neq 0, \sum_{j=1}^{J} c_{ij} = 0 \text{ for all } \quad \sum_{j=1}^{J} b_{jT} = 0$$

The output and input share equations obtained by differentiating the translog variable profit function with respect to the prices of outputs and variable inputs and the quantity of fixed inputs are linear in the unknown parameters (Diewert 1974). Assuming that the prices of fixed inputs are endogenous, the shadow price of each fixed input derived from partially differentiating the variable profit function with respect to that input is equal to the market price, so the fixed input share equations can be included in the set of equations to be estimated. Needless to say, the usefulness of these shadow price equations for econometric work depends on the reasonableness of this assumption. If the market price for any input that is assumed to be fixed is determined exogenously and does not correspond closely with the shadow price, using market prices in place of the non-observable shadow prices will introduce error.

Given a suitable specification of residuals, and data on the price and quantity of outputs and inputs, these equations can be estimated. In this study, additive disturbance terms are postulated and assumed to be contemporaneously correlated. Together with the fact that the shares of both variable inputs and outputs and fixed inputs sum to unity by definition, this leads to the singularity of the residual covariance matrix for the variable input and output share equations and the fixed input share equations. This can be overcome by omitting one of the variable net output share equations and one of the fixed input share equations. In this study the last variable net output share equation and the last fixed input share equation were dropped and the remaining equations estimated simultaneously with the restrictions on parameters expressed in equations (2), (3) and (4) imposed (Byron 1970). Thus the five variable net output, three fixed input model of mixed farm production can be expressed by the following set of six share equations, where the assumed linear homogeneity of $\pi(p;x,t)$ in p and in x is used to express the exogenous variables as ratios.

$$(5) \quad S_i = a_{io} + \sum_{h=1}^{4} a_{ih} \ln (p_h/p_5) + \sum_{j=1}^{2} c_{ij} \ln (x_j/x_3) + a_{iT} \ln t + e_i, \; i = 1, \ldots, 4,$$

$$(6) \quad R_j = b_{jo} + \sum_{k=1}^{2} b_{jk} \ln (x_k/x_3) + \sum_{i=1}^{4} c_{ij} \ln (p_i/p_5) + b_{jT} \ln t + e_{4+j}, \; j = 1,2$$

where S_i is the share of net output i in variable profit, R_j is the share of fixed input j in the total cost of fixed inputs and the e_i and e_{4+j} are residuals. Time subscripts are implicit on all variables except t. The coefficients of the excluded equations may be derived from the symmetry and linear homogeneity restrictions (2), (3) and (4).

If there is reason to doubt that the market prices of the relatively fixed inputs, such as land, correspond closely with the non-observable shadow prices, consideration could be given to estimating the variable net output share equations given in equation (5) alone. Apart from the potential loss in efficiency, from not using relevant information in the estimation procedure, the only loss in terms of information regarding production relationships would be that between the fixed inputs. Estimating the variable net output share equations alone would yield information on the relationship between outputs and between outputs and all inputs (variable and fixed) for a multiple-output producer. In the absence of firm evidence to the contrary market prices have been assumed to be a reasonable reflection of shadow prices in this study, and so the full six equation system was estimated.

The time coefficients measure the bias of technical change. A negative (positive) value for b_{jT} indicates that technical change has been relatively fixed input j saving (using). Similarly, a positive (negative) value for a_{iT} indicates that technical change has been biased in favour of (against) that output if i is an output, or input i saving (using) if i is a variable input.

The relationships between these various outputs and inputs of mixed farm production are reflected in the parameter estimates of this set of derived share equations. These relationships may be discussed in terms of such familiar concepts as elasticities of transformation and partial price elasticities. As the production technology has been specified in the form of a variable profit function, one set of elasticity concepts which may be employed to describe it are those developed by Diewert (1974). For the translog variable profit function this set of elasticities consists of:

(i) $\theta_{ih}\ (p;x,t)$, the elasticity of transformation between variable quantities y_i and y_h;

$$(7)\quad \theta_{ih}(p;x,t) = \begin{cases} 1 + a_{ih}/S_iS_h & \text{for } i, h = 1, \ldots, 5,\ i \neq h \\ 1 + a_{ii}/S_i^2 - 1/S_i & \text{for } i = 1, \ldots, 5, \end{cases}$$

(ii) $\sigma_{jk}(p;x,t)$, the elasticity of complementarity between fixed quantities x_j and x_k;

$$(8)\quad \sigma_{jk}(p;x,t) = \begin{cases} 1 + b_{jk}/R_jR_k & \text{for } j, k = 1, 2, 3,\ j \neq k, \\ 1 + b_{jj}/R_j^2 - 1/R_j & \text{for } j = 1, 2, 3, \end{cases}$$

and

(iii) $\psi_{ij}\ (p;x,t)$, the elasticity of intensity between variable quantity y_i and fixed quantity x_j;

$$(9)\quad \psi_{ij}(p;x,t) = 1 + c_{ij}/S_iR_j \qquad \text{for } i = 1, \ldots, 5,\ j = 1, 2, 3.$$

These three elasticities are scale invariant normalizations of

$\partial y_i / \partial p_h$ $(i, h = 1, \ldots, 5)$,

$\partial w_j / \partial x_k$ $(j, k = 1, 2, 3)$, and

$\partial y_i / \partial x_j$ (or $\partial w_j / \partial p_i$) $(i = 1, \ldots 5;\ j = 1, 2, 3)$, respectively.

Furthermore, symmetry ensures that $\theta_{ih} = \theta_{hi}$ and $\sigma_{jk} = \sigma_{kj}$ for all i, h, j and k.

An alternative means of describing the characteristics of the technology is in terms of the associated partial elasticities. For the translog variable profit function, they are:

(a) $\varepsilon_{ih}(p;x,t)$, the partial elasticity of output i with respect to price p_h;

$$(10) \quad \varepsilon_{ih}(p;x,t) = \begin{cases} S_h + a_{ih}/S_i & i, h = 1, \ldots, 5,\ i \neq h, \\ S_i + a_{ii}/S_i - 1 & i = 1, \ldots, 5, \end{cases}$$

(b) $\eta_{jk}(p;x,t)$, the inverse partial price elasticity of fixed inputs j and k;

$$(11) \quad \eta_{jk}(p;x,t) = \begin{cases} R_k + b_{jk}/R_j & j, k = 1, 2, 3,\ j \neq k \\ R_j + b_{jj}/R_j - 1 & j = 1, 2, 3, \end{cases}$$

(c) $\xi_{ij}(p;x,t)$, the partial elasticity of variable quantity y_i with respect to fixed quantity x_j;

$$(12) \quad \xi_{ij}(p;x,t) = R_j + c_{ij}/S_i \qquad i = 1, \ldots, 5,\ j = 1, 2, 3$$

and

(d) $\rho_{ji}(p;x,t)$, the partial elasticity of fixed quantity j's shadow price with respect to variable quantity i's price;

$$(13) \quad \rho_{ji}(p;x,t) = S_i + c_{ij}/R_j \qquad i = 1, \ldots, 5,\ j = 1, 2, 3.$$

Here ε_{ii} is the conventional elasticity of supply (demand) if i is an output (variable input) while η_{jj} is the elasticity of the shadow price of input j with respect to an increase in the endowment of that fixed input. In place of the single set of normalized elasticities of intensity there are two separate sets of partial elasticities linking variable net outputs with fixed inputs.

These partial elasticities are, however, interdependent due to the assumed homogeneity of $\pi(p;x,t)$ in p and in x. More specifically:

$$(15) \quad \sum_{h=1}^{I} \varepsilon_{ih} = 0 \qquad i = 1, \ldots, 5,$$

$$(16) \quad \sum_{k=1}^{J} \eta_{jk} = 0 \qquad j = 1, 2, 3,$$

$$(17) \quad \sum_{j=1}^{J} \xi_{ij} = 1 \qquad i = 1, \ldots, 5,$$

and

$$(18) \quad \sum_{i=1}^{I} \rho_{ji} = 1 \qquad j = 1, 2, 3.$$

The system of share equations estimated here contains three variable outputs:

sheep and wool (y_1),

crops (y_2),

cattle and other output (y_3),

two variable inputs:

labour ($-y_4$),

materials and services ($-y_5$),

and three relatively fixed inputs:

livestock (x_1),

capital (x_2),

land (x_3).

While it is acknowledged that the inputs of livestock, capital, and land are not fixed in an absolute sense they are relatively fixed. It appears more reasonable that they are fixed within a twelve months planning period than the alternative of their being perfectly adjustable.

Due to the absence of adjustment constraints on outputs and variable inputs, coupled with the assumed relative fixity of livestock, capital and land inputs, in this production model all elasticities are implicitly short or at most medium term. They correspond to a period of time (one year) that is sufficiently long for producers to adjust the composition of their outputs and their employment of labour, and materials and services, but is too short to adjust their relatively fixed endowments of livestock, capital and land.

Tornqvist price indexes and the corresponding implicit quantities were computed for the three outputs and five inputs for the average property in the Wheat/Sheep Zone for the period 1952–53 to 1976–77 using data obtained from the Bureau of Agricultural Economics' Australian Sheep Industry Survey. This survey presents results for properties with 200 or more sheep. All the data used in the study are recorded and further discussed in Appendix II. Although a simplification, perfect price expectations are implicitly assumed throughout. Alternatives such as myopic price expectations, together with a specified planning horizon, could be adopted and developed within this variable profit function framework but there seems little evidence or *a priori* reason to prefer such alternatives and an investigation of the expectations mechanism is beyond the scope of this paper.

Empirical Results

Estimates of the parameters of the input share demand functions are given in Table 1. The parameter estimates for the fifth variable net output (materials and services) and the third fixed input (land) were derived using the symmetry and homogeneity constraints. As all the estimated shares for variable inputs are negative while those for outputs and fixed inputs are positive the corresponding translog variable profit function satisfied the monotonicity requirements (equations I.6 and I.7 in Appendix I) for all observed sets of variable net output prices and fixed input quantities. Unfortunately, this translog variable profit function is not convex in p at mean exogenous prices and quantities as would be required to identify the dual production possibility set. Nor is it concave in fixed inputs x at mean exogenous prices and quantities as the matrix of second order partial derivatives with respect to the x_j is not negative semidefinite. However, all partial own elasticities were of the expected sign, with θ_{ii} being positive for $i = 1$, 2 and 3 and negative for $i = 4$ and 5, for each of the 25 years in the etimation period. Furthermore, all own-price coefficients were significantly different from zero at the 99 per cent confidence level. Hence, each output supply share equation is significantly upward sloping and the demand share equation for each variable input is significantly downward sloping. This suggests that the own-price elasticities derived from these parameters are also significantly different from zero.

While no goodness-of-fit measures for the system of equations are reported in Table 1, the R^2 statistics from ordinary least squares estimation of the six share equations separately ranged from 0.49 for the labour equation to 0.95 for the livestock equation. Moreover, no evidence of significant autocorrelation was found for these estimated equations as the Durbin Watson statistics ranged from 1.17 for the capital equation to 2.33 for the livestock share equation.

Table 1: Parameter Estimates for a Translog Variable Profit Function for the Wheat/Sheep Zone, 1952–53 to 1976–77

Dependent Variables	Coefficients of Explanatory Variables[a]									
	a_{i1}	a_{i2}	a_{i3}	a_{i4}	a^b_{i5}	c_{i1}	c_{i2}	c^b_{i3}	a_{i0}	a_{iT}
	Sheep Output	Crops Output	Cattle and Other Output	Labour	Materials	Livestock	Capital	Land	Constant	Time
Shares of Variable Net Outputs (i)—										
Sheep Output (S_1) ..	0.64 (4.21)	−0.55 (−7.73)	−0.37 (−5.91)	0.06 (0.69)	0.22	0.12 (7.38)	−0.01 (−0.96)	−0.10	1.53 (9.63)	−0.15 (−2.17)
Crop Output (S_2) ..	−0.55 (−7.73)	0.53 (4.89)	−0.39 (−7.41)	0.22 (3.37)	0.19	0.01 (0.42)	0.03 (1.14)	−0.04	0.49 (5.94)	0.06 (1.76)
Cattle and Other Output (S_3) ..	−0.37 (−5.91)	−0.39 (−7.41)	0.34 (6.99)	0.31 (6.99)	0.11	0.04 (2.45)	−0.04 (−2.92)	0.00	0.52 (7.79)	−0.05 (−1.80)
Labour Inputs (S_4) ..	0.06 (0.69)	0.22 (3.37)	0.31 (6.99)	−0.36 (−4.12)	−0.23	−0.11 (−3.60)	−0.03 (−1.07)	0.14	−0.75 (−7.92)	0.10 (2.47)
Materials and Services Input (S_5)[b]	0.22	0.19	0.11	−0.23	−0.29	−0.06	0.05	0.00	−0.79	0.04

Dependent Variables	Coefficients of Explanatory Variables									
	c_{1j}	c_{2j}	c_{3j}	c_{4j}	c^b_{5j}	b_{j1}	b_{j2}	b^b_{j3}	b_{j0}	b_{iT}
Shares of Fixed Inputs (j)—										
Livestock (R_1) ..	0.12 (7.38)	0.01 (0.42)	0.04 (2.45)	−0.11 (−3.60)	−0.06	0.13 (5.16)	−0.03 (−2.10)	−0.10	0.34 (19.76)	−0.01 (−1.48)
Capital (R_2)	−0.01 (−0.96)	0.03 (1.14)	−0.04 (−2.92)	−0.03 (−1.07)	0.05	−0.03 (−2.11)	0.03 (0.03)	0.00	0.29 (19.70)	0.03 (4.19)
Land (R_3)[b]	−0.10	−0.04	0.00	0.14	0.00	−0.10	0.00	0.10	0.37	−0.02

[a] t-statistics in parentheses.

[b] Implied estimates computed using the homogeneity constraint.

The vast majority of parameter estimates were significantly different from zero at the 95 per cent confidence level. In particular, all price coefficients in the variable net output share equations with the exception of that between sheep and labour were significantly different from zero. The shares of outputs and variable inputs in variable profit have been significantly responsive to changes in relative prices. In addition, the proportion of variable profit from the sheep enterprise has been significantly dependent upon the quantity of livestock input while cattle and other output's share has been significantly dependent on the composition of fixed inputs. Therefore, due to the symmetry of this specification, the share of livestock in fixed costs has been significantly dependent on the price of sheep and cattle output as expected. Moreover, the shares of fixed inputs in fixed cost have in general been significantly dependent on the level and composition of fixed inputs.

From the time coefficients, a_{iT} and b_{jT}, technological change has been biased in favour of crop production and against sheep and beef cattle. Moreover, it has been strongly labour saving and capital using. Hence, the optimal capital-to-labour ratio, for a given set of relative prices, has increased over time. Similarly, the negative time coefficients in the livestock and land share equations indicate that technical change has been relatively livestock and land saving. The share of capital (livestock and land) in "fixed" cost would have increased (decreased) as a result of technical progress even if relative prices remained unchanged.

Estimated elasticities of transformation, complementarity and intensity are given in Table 2 for five of the twenty-five years and for mean exogenous prices and quantities. The analogous partial elasticities are given in Tables 3 and 4. Sheep and cropping activities have clearly been complementary as the elasticity of transformation between sheep output and crop output has been positive throughout this period. An increase in the price of sheep or crop commodities has been accompanied by an increase in the output from both activities. In contrast, cropping and beef cattle production have generally not been complementary outputs as an increase in the price of one has been associated with a decrease in the quantity produced of the other. The elasticity of transformation between sheep output and beef cattle output has been close to zero.

In spite of the scope for changing the output composition, the supply of major farm outputs from this mixed farming region has been inelastic. The own-price elasticity of supply of sheep and wool, crops, and beef cattle and other output at mean explanatory variables has been 0.7, 0.5 and 0.3, respectively. In contrast, the partial elasticity of sheep and wool output with respect to the price of crop output has been 0.3 while that of beef cattle and other output with respect to the price of crop output has been -0.3. A 10 per cent increase in the relative price of crop output has typically lead to a 5 per cent expansion of crop output, a 3 per cent increase in sheep and wool output and a 3 per cent decrease in beef cattle and other output. In times of relatively high grain or sheep output prices, the complementary farm activities of sheep grazing and cropping have displaced beef cattle and other activities while cropping has been reduced to accommodate an expansion of beef production during times of relatively high beef prices.

Table 2: Elasticities of Transformation, Complementarity and Intensity for the Wheat/Sheep Zone: Selected Years 1952–53 to 1976–77

Elasticity[a]	1953–54	1958–59	1964–65	1970–71	1975–76	Mean Explanatory Variables
			Elasticity of Transformation			
θ_{11}	0.63	0.61	0.61	0.70	0.68	0.61
θ_{12}	0.24	0.37	0.34	0.30	0.57	0.40
θ_{13}	−0.14	0.19	0.07	0.38	−0.02	0.18
θ_{14}	0.95	0.92	0.91	0.89	0.92	0.92
θ_{15}	0.80	0.76	0.74	0.71	0.75	0.75
θ_{22}	1.56	0.64	0.69	0.55	0.56	0.60
θ_{23}	−3.77	−0.47	−0.83	0.37	0.31	−0.35
θ_{24}	0.23	0.47	0.40	0.60	0.80	0.52
θ_{25}	0.32	0.64	0.59	0.76	0.86	0.67
θ_{33}	4.97	0.74	1.17	0.25	0.60	0.71
θ_{34}	−1.47	−0.46	−0.81	0.24	−0.01	−0.41
θ_{35}	0.10	0.59	0.49	0.81	0.71	0.60
θ_{44}	1.68	1.64	1.60	1.66	1.68	1.65
θ_{45}	0.47	0.46	0.39	0.56	0.69	0.48
θ_{55}	1.86	1.83	1.85	1.78	1.74	1.83
			Elasticity of Complementarity			
σ_{11}	−0.80	−0.95	−0.91	−0.95	0.17	−0.95
σ_{12}	0.74	0.69	0.73	0.68	0.50	0.71
σ_{13}	0.13	−0.01	0.01	−0.06	−0.53	−0.02
σ_{22}	−1.76	−1.49	−1.46	−1.43	−1.23	−1.45
σ_{23}	0.98	0.99	0.98	0.99	0.99	0.99
σ_{33}	−1.15	−0.98	−1.13	−0.95	−0.76	−1.03
			Elasticity of Intensity			
ψ_{11}	1.19	1.36	1.31	1.54	1.88	1.36
ψ_{12}	0.98	0.97	0.97	0.96	0.97	0.97
ψ_{13}	0.81	0.77	0.74	0.68	0.74	0.75
ψ_{21}	1.07	1.06	1.05	1.05	1.05	1.05
ψ_{22}	1.18	1.09	1.10	1.07	1.04	1.09
ψ_{23}	0.74	0.87	0.84	0.90	0.94	0.87
ψ_{31}	1.62	1.43	1.44	1.25	1.72	1.41
ψ_{32}	0.39	0.72	0.68	0.85	0.77	0.73
ψ_{33}	0.93	0.97	0.96	0.98	0.98	0.97
ψ_{41}	1.48	1.74	1.68	1.75	1.99	1.70
ψ_{42}	1.13	1.13	1.13	1.12	1.08	1.12
ψ_{43}	0.34	0.35	0.21	0.39	0.59	0.33
ψ_{51}	1.25	1.29	1.27	1.27	1.41	1.28
ψ_{52}	0.75	0.81	0.80	0.84	0.87	0.81
ψ_{53}	0.98	0.99	0.98	0.99	0.99	0.99

[a] For θ_{ih}, the subscripts i and h refer to the five variable net outputs, sheep and wool, crops, cattle and other output, labour, and materials and services. For σ_{jk}, the subscripts j and k refer to fixed inputs, livestock, capital and land. For ψ_{ij}, i is a variable net output and j a fixed input.

Table 3: *Partial Elasticities of Demand, Transformation and Complementarity for the Wheat/Sheep Zone: Selected Years* 1952–53 *to* 1976–77

Partial Elasticity[a]	1953–54	1958–59	1964–65	1970–71	1975–76	Mean Explanatory Variables
ε_{11}	1.07	0.74	0.75	0.61	0.61	0.72
ε_{12}	0.11	0.27	0.23	0.28	0.81	0.31
ε_{13}	−0.03	0.07	0.02	0.26	−0.01	−0.07
ε_{14}	−0.63	−0.53	−0.49	−0.53	−0.71	−0.54
ε_{15}	−0.52	−0.55	−0.51	−0.60	−0.71	−0.56
ε_{21}	0.41	0.45	0.42	0.26	0.51	0.47
ε_{22}	0.67	0.46	0.47	0.50	0.80	0.46
ε_{23}	−0.72	−0.17	−0.26	0.25	0.12	−1.13
ε_{24}	−0.15	−0.27	−0.21	−0.36	−0.62	−0.30
ε_{25}	−0.23	−0.47	−0.40	−0.65	−0.82	−0.21
ε_{31}	−0.23	0.23	0.08	0.32	−0.02	0.21
ε_{32}	−1.63	−0.34	−0.56	0.34	0.44	−0.27
ε_{33}	0.95	0.28	0.37	0.17	0.24	0.27
ε_{34}	0.97	0.26	0.44	−0.15	0.01	0.24
ε_{35}	−0.06	−0.43	−0.33	−0.69	−0.67	−0.44
ε_{41}	1.60	1.11	1.12	0.76	0.82	1.08
ε_{42}	0.10	0.34	0.27	0.55	1.14	0.40
ε_{43}	0.28	−0.17	−0.26	−0.17	0.00	−0.15
ε_{44}	−1.11	−0.94	−0.87	−1.00	−1.30	−0.97
ε_{45}	−0.30	−0.34	−0.26	−0.48	−0.66	−0.35
ε_{51}	1.35	0.92	0.92	0.61	0.67	0.89
ε_{52}	0.14	0.46	0.40	0.70	1.23	0.52
ε_{53}	0.02	0.22	0.16	0.55	0.28	0.23
ε_{54}	−0.31	−0.27	−0.21	−0.34	−0.54	−0.29
ε_{55}	−1.20	−1.33	−1.26	−1.52	−1.64	−1.35
η_{11}	0.28	−0.25	−0.28	0.24	0.02	−0.26
η_{12}	0.24	0.25	0.27	0.26	0.21	0.26
η_{13}	0.04	0.00	0.00	−0.02	−0.23	−0.01
η_{21}	0.26	0.18	0.22	0.17	0.07	0.19
η_{22}	0.58	−0.55	−0.54	−0.54	−0.51	−0.54
η_{23}	0.31	0.36	0.32	0.37	0.44	0.35
η_{31}	0.05	0.00	0.00	−0.01	−0.08	0.00
η_{32}	0.32	0.36	0.37	0.37	0.41	0.37
η_{33}	0.37	0.36	−0.37	−0.36	−0.33	−0.36

[a] ε_{ii} is the elasticity of supply (demand) for variable net output (input) *i*. For ε_{ih} the subscripts *i* and *h* refer to the five variable net outputs sheep and wool, crops, cattle and other output, labour, and materials and services. For η_{jm}, the subscripts *j* and *m* refer to the fixed inputs, livestock, capital and land.

Table 4: Partial Elasticities of Intensity for the Wheat/Sheep Zone: Selected Years 1952–53 *to* 1976–77

Partial Elasticity[a]	1953–54	1958–59	1964–65	1970–71	1975–76	Mean Explanatory Variables
ξ_{11}	0.42	0.36	0.40	0.38	0.27	0.37
ξ_{12}	0.32	0.36	0.36	0.36	0.40	0.36
ξ_{13}	0.26	0.28	0.24	0.26	0.33	0.27
ξ_{21}	0.38	0.28	0.32	0.26	0.15	0.29
ξ_{22}	0.38	0.40	0.41	0.40	0.43	0.41
ξ_{23}	0.24	0.32	0.27	0.34	0.42	0.31
ξ_{31}	0.57	0.38	0.44	0.31	0.25	0.39
ξ_{32}	0.13	0.27	0.25	0.32	0.32	0.27
ξ_{33}	0.30	0.36	0.31	0.37	0.43	0.34
ξ_{41}	0.52	0.46	0.51	0.43	0.29	0.46
ξ_{42}	0.37	0.41	0.42	0.42	0.45	0.42
ξ_{43}	0.11	0.13	0.07	0.15	0.26	0.12
ξ_{51}	0.44	0.34	0.39	0.31	0.21	0.35
ξ_{52}	0.25	0.30	0.30	0.32	0.36	0.30
ξ_{53}	0.31	0.36	0.32	0.37	0.44	0.35
ρ_{11}	2.01	1.65	1.61	1.33	1.69	1.60
ρ_{21}	1.65	1.18	1.20	0.83	0.87	1.14
ρ_{31}	1.36	0.93	0.91	0.59	0.66	0.88
ρ_{12}	0.46	0.76	0.71	0.96	1.50	0.81
ρ_{22}	0.51	0.79	0.74	0.98	1.49	0.84
ρ_{32}	0.32	0.63	0.56	0.82	1.34	0.67
ρ_{13}	0.31	0.53	0.46	0.85	0.69	0.53
ρ_{23}	0.07	0.27	0.22	0.58	0.31	0.28
ρ_{33}	0.18	0.36	0.31	0.67	0.39	0.37
ρ_{14}	−0.98	−1.00	−0.91	−1.06	−1.54	−1.00
ρ_{24}	−0.75	−0.65	−0.61	−0.68	−0.84	−0.66
ρ_{34}	−0.23	−0.20	−0.11	−0.23	−0.46	−0.19
ρ_{15}	−0.80	−0.94	−0.87	−1.08	−1.33	−0.94
ρ_{25}	−0.49	−0.59	−0.54	−0.72	−0.82	−0.60
ρ_{35}	−0.63	−0.72	−0.67	−0.84	−0.94	−0.73

[a] ξ_{ij} is the partial elasticity of variable quantity y_i with respect to a change in fixed quantity x_j. ρ_{ji} is the partial elasticity of the shadow price of fixed input j (w_j) with respect to the price of variable net output i (p_i).

The estimates of the own-price elasticities of output supply reported in this paper differ noticeably from the findings of some previous analysts. In particular, our estimate of 0.7 for the own-price elasticity of supply for wool and sheep output is generally higher than previous studies' estimates. For instance, Malecky (1975) obtained an estimate of 0.35 for the own-price elasticity of supply for wool while Wicks and Dillon (1978) obtained an estimate of 0.17 for this elasticity in the Wheat/Sheep Zone. On the other hand, our estimates of 0.5 and 0.3 for the own-price supply elasticities of crops and cattle outputs, respectively, are substantially lower than Wicks and Dillon's estimates of 1.31 for wheat and 0.46 for beef. Wicks and Dillon's estimates were, however, derived from a synthetic aggregative programming model whereas our estimates were derived from direct econometric analysis of farm survey data. While our estimate of the own-price supply elasticity for sheep and wool output is also substantially higher than Vincent, Dixon and Powell's (1980) estimates of 0.26 and 0.23 for wool and sheep outputs, respectively, our estimates for the own-price supply elasticities for crops and cattle outputs are close to their estimates for the Wheat/Sheep Zone of 0.77, 0.50 and 0.48 for wheat, barley and cattle, respectively. Neither Wicks and Dillon, nor Vincent, Dixon and Powell found any complementarity between the major farm outputs. It should be noted, however, that the methodology adopted by the latter specifically precluded the possibility of complementarity (Vincent, Dixon and Powell 1980, p. 228).

The demand for materials and services by Australia's mixed cropping-livestock properties has generally been elastic while that for labour has been close to unity. Furthermore, labour, and materials and services have been substitutes with the elasticity of transformation between them ranging from 0.4 with 1964–65 prices and technology to 0.7 with 1975–76 prices and technology. In terms of the partial elasticities, a 10 per cent increase in the price of labour (materials and services) has been associated with a 3 (4) per cent increase in absolute terms in the quantity of materials and services (labour) demanded.

As may be expected, sheep production has been relatively intensive in its use of labour. The elasticity of transformation between labour input and sheep output has been about 0.9. In contrast, the elasticity of transformation between crops and labour has been about 0.5 while that between labour and beef cattle and other output has been quite variable ranging from -1.5 in 1953–54 to 0.2 in 1970–71. In terms of the partial elasticities a 10 per cent increase in the price of labour leads to a 5 per cent decline in sheep output, a 3 per cent decline in crop output and a 2 per cent increase in beef cattle and other output at mean historical prices. A rise in the relative price of labour has generally led to an increase in the proportion of beef cattle output in total farm output.

All three output groups, sheep, crops, and beef cattle and other output, use materials and services inputs with similar intensity. A 10 per cent increase in the price of materials and services has generally been accompanied by a 5 per cent decrease in the supply of each output so there is little change in the composition of output.

Turning to consider the elasticities of intensity of variable net outputs with respect to fixed inputs it is immediately apparent that no output is absolutely intensive in any of the fixed inputs as the elasticities of output supply with respect to the quantity of each fixed input are all positive but less than unity. However, output goods may be classified according to their relative input intensities by defining output i to be relatively input j intensive if an increase in p_i would lead to a proportionally higher increase in the return to input j than in the return to any other input. Under this definition, both sheep and wool output and cattle output are relatively livestock intensive while crops are relatively capital intensive. An increase in the price of crop output (p_2), for example, increases w_2, the shadow price of capital, proportionally more than it increases w_1 and w_3, the shadow prices of livestock and land.

The findings of this study correspond closely with the widely held view that sheep are relatively labour intensive and crops are relatively capital intensive. Changes in the prices of variable inputs or the endowment of fixed inputs, influence the composition of output by fostering the production of some commodities more than others. In general, a policy which results in the relative price of a variable input being less than it would be otherwise results in the contribution to total output of the output which uses that input relatively intensively being larger than it would be otherwise. For example, while policies that would increase the price of labour would lead to a reduction of both sheep and crop output, this reduction would be largest in the case of sheep.

The final set of relationships to be considered is that between fixed inputs. Capital inputs have clearly been complementary with both livestock and land inputs. The elasticity of complementarity between livestock and capital has varied between 0.5 and 0.7 while that between land and capital has been quite stable between 0.9 and 1.0. The elasticity of complementarity between livestock and land inputs has been close to zero throughout this time period in this mixed-farming region.

As mentioned earlier, the wage or price function for each of the three fixed inputs is downward sloping as expected. The own-quantity elasticity of input prices range from −0.5 for capital to −0.3 for livestock. Increases in the quantity of each fixed input lead to less than proportional falls in their unit value.

Conclusions

The supply of sheep and wool, crops, and beef cattle and other output by the average Wheat/Sheep Zone property has been inelastic, with the price elasticity of supply for these three groups of output being estimated as 0.7, 0.5 and 0.3, respectively, at mean prices and quantities. During the 25-year period 1952–53 to 1976–77, the production of wool and other sheep outputs has clearly been complementary with the production of crops. In contrast, crops and beef cattle have been substitutes rather than complementary in production as an increase in the price of one of these outputs has been associated with a decrease in the quantity of the other produced.

The demand for materials and services inputs by mixed farming properties over this 25-year period has been elastic while that for labour has been close to unity. Hence, increases in the price of labour have been met with a similar proportional reduction in the amount of labour employed leading to there being little change in expenditure on labour.

Wool and other sheep output has been relatively labour intensive while crops have been relatively capital intensive. Livestock activities in general have been relatively land intensive. Hence, as labour is a variable input, policies that have caused the price of labour to be greater (less) than it would have been otherwise have discouraged (encouraged) the production of wool and other sheep outputs relative to other farm outputs such as crops and beef cattle.

Appendix I

To satisfy the necessary conditions for duality with a corresponding production possibility set or transformation function the variable profit function $\pi(p;x,t)$ was assumed to be:

(I.1) a non-negative real valued function defined for all $p >> 0$[2] and any x;

(I.2) homogeneous of degree one in p;

(I.3) convex and continuous in p for every fixed x;

(I.4) homogeneous of degree one in x;

(I.5) concave and continuous in x for every fixed p;

(I.6) non-decreasing in x for every fixed p; and

(I.7) non-decreasing (non-increasing) in p_i if i is an output (variable input) for every fixed x.

[2] $p >> 0$ is used here to mean that every element of the vector p is greater than 0.

Appendix II

The data employed in this analysis were Tornqvist price indexes for the three output categories (sheep and wool, crops, and cattle and other output) and five input categories (labour, materials and services, livestock, capital, and land), together with the corresponding implicit quantity indexes. The Tornqvist (1936) index is quite flexible as it is based on a homogeneous translog production function which provides a second-order approximation to an arbitrary production function at any given point (Christensen, Jorgenson and Lau 1973). It can precisely reflect an arbitrary set of substitution possibilities at any given feasible point. The formula for the Tornqvist price index in log change form for an input group consisting of n items is as follows:

$$\log (P_t/P_{t-1}) = \sum_{i=1}^{n} v_{it} \log (p_{it}/p_{it-1})$$

where

$$v_{it} = (p_{it}\, q_{it} \;/\; \sum_{j=1}^{n} p_{jt}\, q_{jt} + p_{it-1}\, q_{it-1} \;/\; \sum_{j=1}^{n} p_{jt-1}\, q_{jt-1})/2$$

p_{it} is the price of item i at time t;

q_{it} is the quantity of item i at time t; and

P_t is the price of the input group at time t.

Clearly, a price and dollar value were required for each item in each of the three output and five input groups to calculate the eight Tornqvist price indexes. The price indexes were then divided into the corresponding group values to derive implicit quantity indexes.

The sheep and wool output category contained all wool production, sheep sales and positive changes in on-farm sheep inventories between the beginning and end of each year while the crops output category contained all wheat, barley and oats production. The cattle and other outputs category contained all cattle sales and positive changes in on-farm cattle inventories within each year along with all other outputs such as pig and horse production, crops other than cereals and dairy products.

Labour inputs included all labour used in operating the property. That is, the operator's own labour, his family's labour and all farm operation contracts were included in addition to hired labour. Similarly, all materials and services used by the property, including maintenance of plant and improvements, were contained in the materials and services category.

The prices used for the three durable input categories (livestock, capital, and land) represent the price of the service flow from those inputs as opposed to the stock price of the durable inputs. Durable inputs are not completely consumed in the year of purchase but provide a flow of services over several years. The value of this service flow consists of two components: depreciation and opportunity cost. A lack of data precluded the incorporation of capital gains. The values of both purchases and any reductions in inventories were added in the case of livestock. The quantity of service flow from capital and land was assumed to be proportional to the quantity of the stock of the input held. The capital group included water, fencing and yards, buildings, and plant inputs. Livestock input quantity was taken to be proportional to the sum of opening numbers, purchases and reductions in on-farm inventories during the year. A more detailed discussion of the treatment of durable inputs and of the components of each output and input category is presented in Lawrence and McKay (1980).

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