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A Description of the Methods and Data Employed in the U.S. Dairy Sector Simulator, Version 97.3

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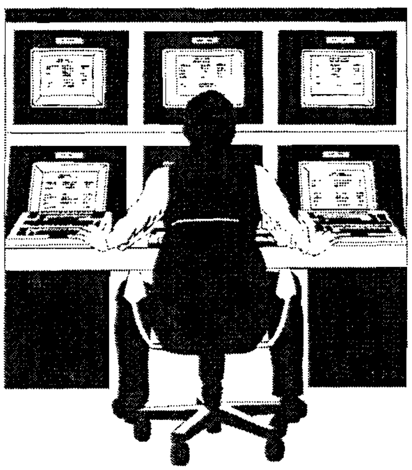

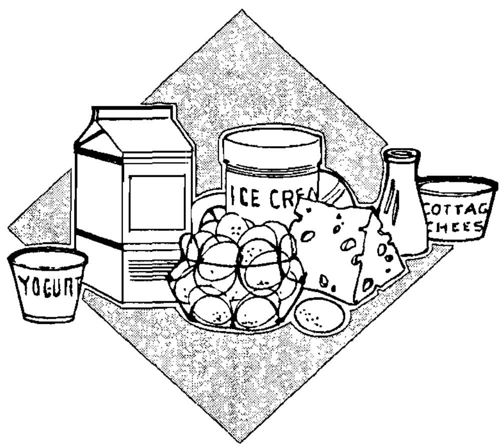

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R.B. 97–09 Errata & Addendum

Subsequent to the publication of *A Description of the Methods and Data Employed in the U.S. Dairy Sector Simulator, Version 97.3* (R.B. 97–09), some typographical errors have been brought to our attention. We would like to thank Richard Kilmer of the University of Florida for alerting us to a couple of these errors and would encourage any reader of either our printed material or our web–based publications to do the same.

We should stress that the errors noted below are purely typographical and were not incorporated into any of the analytical work we released prior to or since the publication of R.B. 97–09.

Page 33:

The U.S. average fat content for May was incorrectly stated as 3.68%, thus the last sentence of the first paragraph on this page should read "The index was constructed in such a way that the resulting monthly U.S. average fat content was the same as that reported in *Agricultural Prices* (3.61 percent in May and 3.72 percent in October)."

Page 34:

Table 3 contained incorrect data. For May, 1995, the weighted average fat percent should have read 3.605; 1,000 lbs. fat should have read 492,115; weighted average SNF percent should have read 8.709; and 1,000 lbs. SNF should have read 1,188,865. For October, 1995, 1,000 lbs. fat should have read 469,940.

Page 35:

Figure 7 incorrectly shows the weighted average = 3.68%. It should have read "3.61%".

Page 47:

The algorithm at the top of page 47 contains two terms which are incorrectly signed. The algorithm should be consistent with the assumptions listed on page 46.

Step (iii) of the algorithm should read:
Set Domestic Consumption = Production + Imports – Exports – ΔStocks – Products
Used in the Production of Other Dairy Products

Step (vii) of the algorithm should read:
Set Domestic Consumption = Production + Imports – Exports – ΔStocks – Products
Used in the Production of Other Dairy Products

Actually, in step (vii), the term 'ΔStocks' is always zero so the sign and even the term itself is irrelevant.

<u>Page 55, Tables 11 and 12</u>

The lower two panels of Tables 9 through 12 (*i.e.*, the panels immediately above and below the double line) present the result of applying the algorithm on page 47 to the data contained in the upper two panels of the tables. Due to the typographical error noted above, some of the calculations were performed incorrectly when constructing these tables. Note that the error did not effect all calculations as it is contingent upon the particular values for Exports and ΔStocks in each case. We should stress again that the errors in Tables 11 and 12 occurred at the time of writing and were not contained in the data files used to run the model.

For completeness, the lower two panels of Tables 11 and 12 are reproduced with all figures calculated according to the corrected algorithm.

Table 11
(Correction to Lower Two Panels)

	May	October
Domestic Consumption; 1,000 lbs.	100,707	93,518
Exports; 1,000 lbs.	16,687	0
Changes in Stocks; 1,000 lbs.	2,250	0
Aggregate Consumption; 1,000 lbs.	119,644	93,518

Table 12
(Correction to Lower Two Panels)

	May	October
Domestic Consumption; 1,000 lbs.	120,269	84,840
Exports; 1,000 lbs.	17,955	15,962
Changes in Stocks; 1,000 lbs.	304	0
Aggregate Consumption; 1,000 lbs.	138,528	100,802

Page 48, Table 7

Table 7 on page 48 contains no errors. However, it may have been helpful if we had included in Table 7 a presentation of the data used to construct the index which enabled us to regionally adjust consumption. While the description on pages 47 through 49 of the indexing procedure refers to daily consumption by region, the table below presents the regional consumption in pounds per month.

Regional Monthly Consumption, Pounds; May and October 1995								
	Northeast		Southern		Midwest		West	
	May	Oct.	May	Oct.	May	Oct.	May	Oct.
Soft Products	3.30	3.30	2.95	2.95	3.58	3.28	3.65	3.65
Cheese	2.38	2.39	2.20	2.21	2.75	2.76	2.38	2.39
Butter	0.46	0.45	0.30	0.28	0.46	0.45	0.38	0.38
Dry, Condensed, and Evaporated Products	0.37	0.26	0.51	0.36	0.51	0.36	0.42	0.29

PREFACE

The model which is described in this document is the result of modeling research undertaken by the Cornell Program on Dairy Markets and Policy (CPDMP). Over the years, contributions have been made by a variety of people, not all of whom are authors of this paper.

The original concept for this model was developed by Dr. James Pratt (Senior Research Associate) and Dr. Andrew Novakovic (Director of the CPDMP and the E.V. Baker Professor of Agricultural Economics) in the early 1980s. As computational capacities increased, it became possible to expand the model's size and scope. The core objective has and continues to be the representation of the dairy economy in ways that recognize its geographic (spatial), processing, market level, and regulatory complexity.

Phillip Bishop (Ph.D. candidate and Research Associate) and Eric Erba contributed much to the current phase of the model. In particular, Mr. Bishop provided major contributions in the areas of programming the model, compiling the data, and preparing the output. Dr. Erba was responsible for gathering much of the data. Dr. Mark Stephenson is a Senior Extension Associate. Excepting Erba, the authors are in the Department of Agricultural, Resource, and Managerial Economics at Cornell University. Dr. Erba has worked for the California Department of Food and Agriculture since April 1997, and attained his Ph.D. from Cornell in August 1997.

Others who have contributed in varying measures to the development of the U.S. Dairy Sector Simulator over the years include Dr. David Jensen, Mr. Will Francis, Dr. Maurice Doyon, and Mr. Geoff Green.

We would also like to thank IBM Corporation and, in particular, Chris Marcy, RISC System Product Specialist, for their assistance in making this project computationally feasible for us.

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A Description of the Methods and Data Employed in the U.S. Dairy Sector Simulator, Version 97.3

INTRODUCTION

The United States Dairy Sector Simulator (USDSS) is a spatially detailed model of the U.S. dairy industry. Its development has been ongoing for nearly two decades and over that time it has been applied to a wide array of research efforts concerning the dairy sector. The USDSS is formulated as a single–time period, network transshipment model and is solved as a linear program. The goal of the model is to obtain an efficient solution to a complex spatial markets problem. In so doing, the model minimizes the total costs associated with marketing milk and milk products. These costs include the cost of raw milk assembly, shipping intermediate products between plants, plant processing costs, and the distribution of finished products. Economic activity at three market levels is represented in the model; farm milk supply, dairy product processing, and dairy product consumption. These three market levels are inextricably linked to one another in the U.S. dairy sector. So too are the various product sectors. Recognition of these two fundamental aspects provides the framework in which the USDSS is constructed.

The purpose of this paper is to provide a detailed description of the USDSS and its data requirements. Most recently, the model has been revised and updated as part of the research supporting reform of the Federal Milk Marketing Order (FMMO) program currently being undertaken by the staff of the Cornell Program on Dairy Markets and Policy (CPDMP). To that end, this paper serves as a methodological reference for subsequent publications emanating from the CPDMP's current research efforts. A comprehensive treatment will be given to all aspects of the model, however, even those not central to the present application. The paper proceeds as follows. First, the history of dairy market modeling is briefly surveyed and the methods employed are chronicled. From this perspective, the lineage and ancestry of the USDSS can be appreciated. Second, the conceptual models upon which the USDSS is based are reviewed. The USDSS is then presented, and finally, all of the data requirements are described, their construction explained, and data summaries provided.

A BRIEF HISTORY OF DAIRY MARKET MODELING

Spatially formulated trading models were prominent among the first uses of the newly discovered linear programming methods developed by George Dantzig in 1947 (Dantzig, 1948). In fact, the first use of Dantzig's simplex method was for a logistics problem involving troop deployment across space. Economists and agricultural economists alike embraced the new programming methods and quickly began applying them to pressing problems of the day, many of which were spatially oriented. Paul Samuelson's famous 1952 paper in the *American Economic Review*, integrating spatial price equilibrium and linear programming, spawned the later works of T. Takayama and G. Judge in using nonlinear programming methods for similar problems (*e.g.* Takayama and Judge, 1971). More recently, Takayama (1994) himself acknowledges the place of linear programming in the tools of a spatial economist and credits E.O.

Heady, a pioneer of agricultural modeling, with promoting the use of linear programming by 'energetically' applying linear programming methods to economic decision–making in agriculture. The dairy industry was a fertile sector in which to use these newly developed techniques.

Even prior to the modeling revolution brought about by the simplex method, researchers were formulating spatial dairy problems for analytical examination. In 1941, Kasten Gailius wrote "The Price and Supply Interrelationships for New England Milk Markets," an M.S. thesis from the Department of Agricultural Economics at the University of Connecticut. The very next year Hammerberg, Parker, and Bressler employed a heuristic procedure to derive an optimum dairy market organization for the state of Connecticut (Hammerberg *et al.*, 1942). Ten years later, Bredo and Rojko, also studying the northeast dairy sector, published an award winning study which laid out the structure of a spatial programming problem which would be used in later studies implementing the new solution algorithms (Bredo and Rojko, 1952).

Use of the new, powerful applied methods found a natural home in applied dairy marketing. Snodgrass (1956) and Snodgrass and French (1958) used linear programming to simulate efficient spatial organization of the U.S. dairy sector. Adoption of the new techniques was not confined to U.S. researchers, however. Louwes *et al.* (1963), for instance, applied the quadratic programming technique to modeling the optimal use of milk in the Netherlands. Subsequent to these and other early works, many applications of programming methods using both linear and nonlinear techniques to analyze spatial issues in the dairy industry followed. An apex in the use of programming methods for such spatial studies was reached during the late 1970s and early 1980s when a large number of studies of the dairy industry emerged. Some examples include Beck and Goodin (1980); Riley and Blakley (1976); Boehm and Conner (1976); Kloth and Blakley (1971); McDowell (1978 and 1982); Thomas and DeHaven (1977); McClean *et al.* (1982); and Pratt *et al.* (1986). These models, as well as many others constructed during this time, were concerned primarily with issues such as market organization and the opportunity for efficiency improvements; optimal plant size, numbers, and location; transportation arrangements; and the analysis of various pricing schemes.

At the same time that the mathematical programming models of spatial organization and trade were rapidly developing, there were also new developments in the use of more statistically oriented methods (see Thompson, 1981). It is fair to characterize the statistical trade models as being much more oriented toward studies of international trade rather than toward regional or sub–regional analyses. Few statistical trade models at a smaller–than–country level have been constructed. This is mainly because of data limitations. The statistical models must rely upon observations over time. Therefore, it is necessary that actual trade flows between the units being analyzed are observed and recorded. Commodity flow data in international markets are routinely compiled because of concerns for compliance with government imposed economic and health regulations. Data on commodity flows within a specific country are much less likely to be compiled (an exception would be cases such as interprovincial trade in Canada where regulations are applied on a provincial basis). Additionally, statistical trade models rely heavily on past observations for their prescriptive results. When the analysis involves no changes in the regulatory or technological regimes, or when these changes are minor, the past may well be a robust predictor of the future. In contrast, when there are significant regulatory or technological changes, or when the specific purpose of the analysis is to study the impacts of such changes, the heavy reliance on observations generated by a system which did not include these new regulations or technology makes it much more difficult to predict the impacts of such changes. Heady (1963) had this to say on the matter:

> "However, regression models, while they may be useful in estimating *ex post* commodity supply and factor demand relationships by regions, can hardly serve as useful tools for analysis of the important structural changes (especially when these revolve around technology) which cause change in competitive or equilibrium positions among regions and, thus, cause the useful questions of interregional competition to be posed."

Programming models, while being notoriously poor predictors of actual trade flows but very good at predicting trade prices, require the analyst to explicitly or implicitly express the regulatory and technological parameters used in the analysis. Statistically based models are typically used to estimate these parameters.

The USDSS is a descendant of the early spatial models of the dairy industry and draws most directly upon work completed at Purdue University and Cornell University (*e.g.* Babb, 1967; Babb *et al.*, 1977; Novakovic *et al.*, 1980; and Pratt *et al.*, 1986). It has been applied to numerous research efforts over the past two decades and is constantly being revised, improved, and updated. In recent years it has been used to undertake analyses of the 1985, 1990 and 1995 farm bills; analyze federal milk marketing order regulation (Francis, 1992); investigate questions of class price alignment (Novakovic *et al.*, 1995); explore the regional impacts of dairy trade liberalization with Canada (Doyon, 1996); and understand the implications for milk marketing orders of trade agreements such as the General Agreement on Tariffs and Trade (GATT) and the North American Free Trade Agreement (NAFTA) (Bishop, 1996; Bishop and Novakovic, 1997).

A REVIEW OF SOME MATHEMATICAL PROGRAMMING MODELS AND CONCEPTS

Overview

The purpose of this section is to provide the mathematical basis from which the USDSS was developed. Much of the material presented here can be found in almost any mathematical programming textbook, and, moreover, in much greater detail. While the USDSS is, in a strictly technical sense, a linear programming (LP) model, much of its construction derives from the strand of optimization problems known as network models. It is therefore useful to review this class of models and examine their association with LP models. Some other model types which have a bearing on the USDSS will also be highlighted.

Mathematical programming is a collective term referring to the many techniques available for solving problems which require the coordination of interrelated activities to meet some overall goal, or objective. As a field of study, it really began to emerge in the 1950s following the developments in the area of production planning. Production planning was the focus of much attention during the war years. Key among these developments were the efforts of George B.

Dantzig and his colleagues at the Pentagon. Dantzig,[1] *op. cit.*, is credited with having developed the simplex method for solving linear programs. The key feature of mathematical programming which distinguishes it from the classical methods of optimization using calculus is its capability to handle inequality constraints. The distinction between the use of inequalities rather than equations can be likened to taking a prescriptive instead of a descriptive approach to mathematically characterizing the underlying economic system. Inequalities permit a more realistic problem–solving framework because they allow the presence of resources in amounts exceeding those actually required to perform a given activity. Such behavior is frequently observed and is, in fact, often necessary due to the many constraints on a given process. The solution procedures used in mathematical programming problems typically involve finding an optimal, or best, solution to an objective function while satisfying a set of constraints.

Linear programming problems belong to a general class of optimization problems which envelop many others as special cases. Dantzig proposed the simplex method as a means of solving linear programs although many other techniques have since been developed. Indeed, the emergence of the many sub–classes of problems has generally followed the discovery of a specialized algorithm to solve a particular type of problem. In this regard, network models are no exception. Because the USDSS embodies many aspects of the canonical network problem, the next few pages will briefly examine network flow theory. The transportation problem, the transshipment problem, the shortest path problem, and the minimum spanning tree problem can all be cast as network models and will each be described in the following discussion, as they all play a role in the USDSS.

<u>Network Concepts</u>

Many important optimization problems can best be analyzed by means of a graphical or network representation (Winston, 1995). In fact, the use of a network to describe an economic system dates back to 1838 when Cournot used the concept to describe market equilibria. Network flow theory concerns a class of problems having a very special network structure (Dantzig and Thapa, 1997). The combinatorial nature of this structure has led to the development of very efficient algorithms which can take advantage of the network structure to find solutions to very complex problems, and, perhaps more importantly, find them in a timely manner.

The basic building blocks of a network are nodes and arcs. Nodes are simply a set of points, or vertices. Loosely speaking, we use cities to define a set of nodes in the USDSS although in a stricter sense, more than one node can be present at a specific geographic location. An arc is an ordered pair of nodes and represents a possible direction of motion that may occur between nodes. Thus if $V = \{1,2,3,4\}$ is a set of nodes, then $\{(1,2),(2,3),(3,4)\}$ might be a sequence of arcs. In the USDSS we say that the shipment of raw milk or of an intermediate or final milk product between two cities represents an arc. In general, the arc (i,j) denotes the possibility for motion from node i to node j where node i is said to be the initial node and node j is called the terminal node. A chain is a sequence of arcs such that every arc has exactly one node

[1] Dantzig's classic reference is Dantzig, G. *Linear Programming and Extensions*. Princeton, N.J.: Princeton University Press, 1963.

in common with the previous node while a path is defined as a chain in which the terminal node of each arc is identical to the initial node of the next arc. Thus in the simple example above, the sequence of arcs $\{(1,2),(3,2),(3,4)\}$ is a chain but not a path, while the sequence $\{(1,2),(2,3),(3,4)\}$ is both a chain and a path. Two final concepts used to describe basic networks are circuits and cycles. A circuit is a path where the terminal node of the last arc in the sequence is identical to the initial node of the first arc in the sequence. A cycle, on the other hand, is a chain such that the terminal node of the last arc in the sequence is identical to the initial node of the first arc in the sequence. So, the sequence $\{(1,2),(2,3),(3,4),(4,1)\}$ would be both a circuit and a cycle while the sequence $\{(1,2),(3,2),(3,4),(4,1)\}$ would be a cycle. These concepts are illustrated in Figure 1.

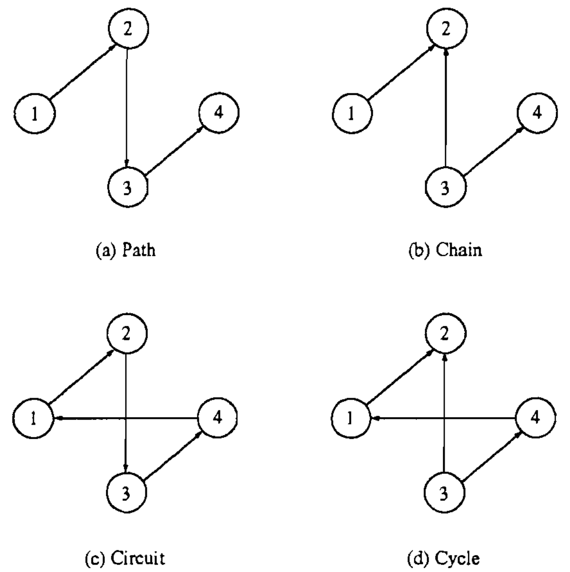

Figure 1. A Path, Chain, Circuit, and Cycle
Source: Dantzig and Thapa (1997)

The Transportation Problem

A very simple network model is the transportation problem due to Hitchcock (1941). This particular problem can be regarded as one of finding the minimum cost to move objects through a distinctive type of network. Suppose that m suppliers, denoted $S=\{s_1,s_2,...,s_m\}$, are to supply a commodity to a set of n demanders, denoted $D=\{d_1,d_2,...,d_n\}$. The problem is one of how, at minimum cost, to satisfy all the demands while not exceeding the capacity of any supplier. The basic transportation problem can be stated mathematically as:

$$\text{Minimize} \sum_{i=1}^{m} \sum_{j=1}^{n} c_{i,j} x_{i,j}$$

subject to :

$$\sum_{j=1}^{n} x_{i,j} \leq s_i \quad \text{for } i = 1, 2, \ldots, m$$

$$\sum_{i=1}^{m} x_{i,j} \geq d_j \quad \text{for } j = 1, 2, \ldots, n$$

$$x_{i,j} \geq 0$$

where:

$c_{i,j}$ = the cost of shipping one unit of commodity x from the i^{th} supply point to the j^{th} destination,

$x_{i,j}$ = the amount of commodity x shipped from the i^{th} supply point to the j^{th} destination,

s_i = the amount of commodity x available at supply point i, and

d_j = the amount of commodity x demanded at destination j.

The transportation model has been widely applied to problems involving distribution. A typical example would entail a company wishing to find the least cost method of utilizing a fleet of trucks to move goods from warehouses to retail or distribution outlets. At its most fundamental level, the U.S. dairy sector can be thought of as three separate transportation problems; a) moving milk from farms to processing facilities, b) moving intermediate products between processing plants, and c) moving finished products from processing plants to consumers. In fact, the structure of the U.S. dairy sector is more accurately characterized by the so–called capacitated transshipment model, a variation of the basic transportation problem.

The Transshipment Problem

The transshipment problem includes not only points of supply (sources) and demand (sinks), but intermediate points as well. As the good moves from the points of supply to the final demand locations, it passes through the intermediate nodes, or transshipment points. Applying the standard conservation-of-flow assumption, the basic transshipment model, originally associated with Orden (1956), can be stated mathematically as:

$$\text{Minimize} \sum_i \sum_j c_{i,j} x_{i,j}$$

subject to :

$$\sum_j x_{i,j} - \sum_j x_{j,i} = s_i \quad \text{for i = a source node}$$

$$\sum_j x_{i,j} - \sum_j x_{j,i} = 0 \quad \text{for i = a transshipment node}$$

$$\sum_j x_{i,j} - \sum_j x_{j,i} = -d_i \quad \text{for i = a sink node}$$

$$0 \le l_{i,j} \le x_{i,j} \le u_{i,j}$$

where:

- i,j = the locations of source, transshipment, or sink nodes,
- $c_{i,j}$ = the cost of shipping one unit of commodity x from the i^{th} originating point to the j^{th} destination,
- $x_{i,j}$ = the amount of commodity x shipped from the i^{th} originating point to the j^{th} destination,
- s_i = the amount of commodity x available at the i^{th} supply point,
- d_i = the amount of commodity x demanded at the i^{th} destination,
- $l_{i,j}$ = a lower bound on the capacity of the $(i,j)^{th}$ arc, and
- $u_{i,j}$ = an upper bound on the capacity of the $(i,j)^{th}$ arc.

Note that this representation is quite general in that any or all of the supply and demand nodes can also serve as transshipment points. Hence, the summation operators do not specify an exact dimension over which to sum the flows. Note too that each set of constraints are strict equalities. This implies that the commodity is not able to accumulate at transshipment nodes. Such a condition could be relaxed. Some additional structure has been added to this network in the form of capacity flow constraints along the arcs. In other words, this is a capacitated transshipment model. Both the transportation and transshipment models can be formulated and solved as LP problems, although from a computational viewpoint it rarely makes sense to do this because network solution algorithms are generally much more efficient than LP solvers.

The Shortest Path and Minimum Spanning Tree Problems

Two additional models worth noting, as they each play a role in the USDSS, are the shortest path and minimum spanning tree problems. Each of these models are very specialized network models with a particular structure that has been exploited to design extremely efficient solution algorithms. If a particular problem can be formulated as one of these types of model then it is generally advantageous to do so. The shortest path problem is that of finding the minimum accumulated distance along the paths in a network from an initial node to some final destination node. A minimum spanning tree is that collection of *(n-1)* arcs that will connect the *n* nodes of a network such that the sum of the distances along the arcs is minimized. Minimizing the sum of the distances turns out to be the same as requiring that the resulting tree contain no cycles. A key point is that the 'distance' associated with the arcs need not be confined to some notion of geographic separation, such as miles. Rather, the mathematical concept of a distance is used and thus it can be measured as a cost, a quantity, or a time. This then leads to a large array of problems that can be formulated as shortest path or minimum spanning tree problems.

The Linear Programming Problem

Thus far, the discussion has focused on single commodity models where the commodity is assumed to be homogeneous. Gass (1985) demonstrates how to reformulate these simple network models as multi–commodity models. However, such formulations are accomplished through the introduction of additional commodity–specific nodes and are really just a collection of single commodity networks all combined into one model. Multi–commodity networks do not address the issue of joint production which is crucially important in the dairy sector; one must use the more general LP formulation to handle this.

The generalized LP problem consists of a set of linear inequalities representing the technical constraints on the problem and a linear function which expresses the objective of the problem. The LP problem can be stated mathematically as:

$$\text{Minimize } \sum_{j=1}^{n} c_j x_j$$

subject to :

$$\sum_{j=1}^{n} a_{i,j} x_j \le b_i \qquad \text{for } i = 1, 2, \ldots, m$$

$$x_j \ge 0 \qquad \text{for } j = 1, 2, \ldots, n$$

where:

c_j = the objective function coefficients or, alternatively, the cost associated with the j^{th} activity. Frequently, these coefficients are referred to as the c_j's.

x_j = the activities or decision variables,

$a_{i,j}$ = the technical coefficients describing the number of units of the i^{th} resource required per unit of output from the j^{th} activity. The j^{th} activity might be the production of a particular good or the shipment of a good from one place to another,

b_i = the available quantity of the i^{th} resource, or more commonly, the right–hand side (RHS) constraint value,

m = the number of constraints, or rows, and

n = the number of activities, or columns.

This example specifies a minimization problem. It is a simple matter, however, of negating the objective function to restate the problem as a maximization problem. An optimal solution to the above minimization problem is the set of x_j's yielding the minimum value of the objective function while simultaneously satisfying the *m* constraints. Such a solution will be unique in terms of the value obtained for the objective function, but may not be unique with respect to the choice and level of the x_j*'s*. In other words, the use of different combinations of resources may yield identical, optimal values for the objective function. It is possible that there may be no set of activities which satisfy the constraints imposed on the problem, and hence, there is no solution. Such a problem is said to be infeasible. It is worth noting that the objective function coefficients need not be costs. Problems seeking to maximize gross or net revenues, for example, are very common and usually use prices as the coefficients on some or all variables.

In addition to yielding the levels of all the variables when an optimal solution obtains, the linear programming procedure also returns resource values, or shadow prices. There exists a shadow price associated with every constraint. Specifically, the shadow price, also known as the marginal value, of the i^{th} constraint is defined to be the rate of change in the objective function value accompanying an incremental change in the value of b_i, the RHS of the i^{th} constraint. In classical optimization, the marginal value represents a derivative. In linear programming, it is not, strictly speaking, a derivative but a directionally specific arc tangent. This mathematical notion has a certain intuitive appeal when viewed in an economic context. Essentially, it is saying that a particular resource has some value to the process using that resource. Moreover, the greater the relative scarcity of a resource, the greater its value will be. Conversely, an optimal solution whose processes do not require the use of all of an available resource will place no value on additional units of that resource.

Corresponding to every allocative linear program there is a counterpart valuative formulation, with the important property that the objective function value for each is identical. The original problem is called the primal and its counterpart is known as the dual, hence the term 'duality' to describe this relationship. Given all of the information available from the optimal solution to one formulation, *i.e.* variable levels and marginal values, it is possible to obtain the optimal solution to the other. In fact, when formulating the problem, the b_i*'s* of the primal are the objective function coefficients in the dual. The marginal values from the primal solution turn out to be the choice, or activity, variable levels in the dual and *vice versa*. This suggests that a choice as to formulation is available to the analyst. Frequently, that choice boils down to one of dimensions and therefore ease of solution. If the primal has *m* constraints and *n* variables, then the corresponding dual will have *n* constraints and *m* variables. Hence, the formulation with the fewer constraints is typically chosen as the mode in which the problem will be solved. Fewer constraints generally implies the problem will be easier to solve and will yield a solution more quickly.

Shadow Values as Prices

The proliferation of complex spatial trading models, or inter–industry models with a spatial context, has been quite remarkable. When considered in its simplest form as an exercise in finding the intersection of a supply and demand response function, or a set of such functions, it is of little surprise that these models have found great appeal with economists. It is somewhat surprising, however, that those interested in the spatial trading model have all but forgotten its roots in linear programming, particularly in light of the fact that Samuelson's (*op. cit.*) path–breaking article made this connection explicit. These 'fixed production and consumption' models, where quantities, both desired and available, are considered predetermined, provided Samuelson with his 'inside' problem—a transportation formulation whose dual information, along with transportation costs, could be used, in turn, to compute equilibrium market prices in the more familiar setting of price responsive supply and demand. This type of transportation problem, where we have only two market levels trading with each other, was formalized by Tjalling Koopmans in the 1940s and 1950s (for which, in part, Koopmans was awarded the Nobel prize in economics in 1975), and was one of the first problem types to be rigorously attacked with the new linear programming algorithms. Maintaining the notation presented above with the transportation problem, we have:

m = the number of supply sources,
n = the number of demand sinks,
s_i = the supply at source *i*,
d_j = the demand at sink *j*,
$c_{i,j}$ = the per unit cost of transporting the commodity from source *i* to sink *j*, and
$x_{i,j}$ = the quantity shipped from *i* to *j*.

There are three basic conditions related to the quantities shipped which must be met in order for this type of problem to have a feasible solution:

(i) $x_{i,j} \geq 0 \text{ for } i = 1,\ldots, m \text{ and } j = 1,\ldots, n$

We have nonnegative quantities shipped. This literally means that we cannot operate the process in reverse and thereby create supply out of demand. This is not to be confused with having other types of nodes, such as intermediaries, which can both receive and send shipments, that is, a transshipment problem. The USDSS is such a transshipment formulation—dairy processing plants receive raw materials *and* ship final or intermediate products.

(ii) $\sum_{j=1}^{n} x_{i,j} \leq s_i \text{ for } i = 1,\ldots, m$

Total shipments emanating from any supply source *i* must not exceed the quantity available at that source.

(iii) $\sum_{i=1}^{m} x_{i,j} \geq d_j \text{ for } j = 1,\ldots,n$

Total shipments to any demand sink *j* must meet or exceed the quantity required at that sink.

Now, we wish to find a set of $x_{i,j}$'s which yields a feasible solution to (i), (ii), and (iii) while minimizing the total transportation cost:

(iv) $\text{Minimize} \sum_{i=1}^{m} \sum_{j=1}^{n} c_{i,j} x_{i,j}$

A necessary condition for the solution of this problem is that total demand must be less than or equal to total supply. By summing (ii) over *m* and (iii) over *n*, we can derive the following relationship:

(v) $\sum_{j=1}^{n} d_j \leq \sum_{i=1}^{m} \sum_{j=1}^{n} x_{i,j} \leq \sum_{i=1}^{m} s_i$

This necessary condition states the obvious—that total demand can be no larger than total supply, provided a feasible solution exists. We can now restate this problem to yield identically the transportation problem seen above. This is the 'primal' form of the transportation problem, whereby the optimal, yet initially unknown, shipments, $x_{i,j}$*'s*, can be selected in such a way so as to minimize total transport costs, while simultaneously satisfying demand requirements, respecting supply limitations, disallowing the creation of supplies from demands, and observing the nonnegativity conditions on $x_{i,j}$. This very simple primal problem structure fits a surprising number of dissimilar applied optimization problems, and has proven itself very useful for problems of spatial organization. Modern computers with state–of–the–art software are able to solve problems of this type with millions of variables, $x_{i,j}$*'s*, and tens of thousands of constraints, conditions (i)–(iii).

Accompanying every allocation problem, *i.e.*, the primal noted above, is the previously noted mathematically defined equivalent 'dual' problem which provides the concomitant optimal valuation of the resource limits embodied in the constraints of the primal. The optimal objective values for the primal and the dual problems are identical, *i.e.*, the sum of the dual values multiplied by their respective resource levels gives the same value as the minimized total cost from the primal. There is an optimal dual value associated with each resource constraint in a mathematical programming formulation. These optimal dual values are an integral and useful part of any mathematical programming solution. Quite literally, the dual values specify the change in the objective function resulting from a one unit change in the availability of a resource. These 'imputed' values give important information about the optimal resource valuations which can be interpreted in a managerial context. The imputed values provide the change in the optimal value of the objective function associated with a change in the availability of a resource. Resources which are in excess, *i.e.*, which are not totally exhausted by the activities associated with the optimal primal solution, will have an imputed value of zero. At the margin, adding or removing another unit of a resource which is already underutilized will add nothing to one's ability to improve the given objective. In contrast, adding or removing another unit of a resource which is fully utilized *will* change one's ability to optimize the objective. In other words, a resource whose availability is completely exhausted will have a strictly positive imputed value. These types of derived relationships for dual values result from the 'complementary slackness' conditions—a set of mathematically determined primal/dual conditions which must hold for any optimal solution to a mathematical programming problem. The dual values are imputed, meaning that the value of a resource is determined solely from its utility to the optimal solution as opposed to summing the costs of its constituent elements. To wit, these values are determined entirely within the context of the mathematical program at hand. No other information or opportunities, other than those contained in the program, has any role in determining these imputed values.

Duality holds a special place in the world of spatial economics. In mathematical programming, the dual, or shadow, or imputed, values which are associated with the optimization of some objective over a set of given resources, can be interpreted as the optimal values of those resources. In the simple, previously presented, transportation context, these resources are a) supplies of the commodity available to be shipped from the various supply sources, b) demands for the commodity required to be shipped to the various consuming locations, and c) capacity limitations on the transportation activity. Within the context of the slightly more complex transshipment problem, of which the USDSS is an example, the transshipping activity can also be considered a resource and therefore has an associated value. In fact, as noted above, the transshipping nodes in the USDSS are actually the processing locations.

In keeping with the pedagogical nature of this discussion, the following will focus on the simple transportation problem. However, the concepts readily translate to the transshipment problem and thus to the USDSS. The resources in a mathematical programming problem are actual quantities of a commodity (*e.g.* milk). The objective of the problem is stated in terms of dollars per unit of the commodity and the dual values are also denominated in these units. A change in a resource, in this case, is literally a change in the quantity of milk supply or consumption. The dual value associated with such a change is then denominated in dollars per unit of supply or consumption, *i.e.*, an imputed price. (See Thompson and Thore, p. 170, 1992. "When Can a Dual Variable Be Interpreted as a Market Price?"). The set of imputed values for the supplies and demands associated with specific locations defines the set of equilibrium prices for those locations. This dual problem, or 'inside' problem to which Samuelson referred, provides us with some familiar rules governing price relationships in this simple, fixed production/fixed consumption model when there are no trade flow distorting mechanisms; a) any location at which supplies are not totally exhausted will have a local price of zero, b) the imputed price difference between two points in geographic space can not exceed the transportation cost between these two places, c) the imputed price difference between two points in geographic space which actually do trade with each other must exactly equal the cost of transportation between these points, and d) two places whose local price difference is strictly less than transportation costs will not trade with one another.

By way of complementary slackness, any supply location which has a surplus of a commodity in the optimal solution, *i.e.*, resources are underutilized, will have an imputed price of zero. What sense does such a price make? Given, as noted above, that the dual prices are imputed from the programming model only, they only embody the information present in the model. In the transportation formulation above, if the $c_{i,j}$'s, the costs of moving a unit of commodity from location i to location j, do not include the cost of producing or extracting that commodity to make it available at location i in the first place, then the imputed prices will not include the initial production or extraction costs. Even if such costs were included in the $c_{i,j}$'s, in cases where the total supply in the model is greater than total consumption, at least one supply source would have an imputed value equal to zero—the marginal unit of supply at that source can have no impact on the objective function. While such underutilized supply might have an actual reservation price or salvage value, unless this value were explicitly included in the programming formulation, say as the price on a super source point (a point from which all supplies emanate), unused supplies would be valued at zero. These imputed value interpretations provide useful managerial information which offer guidance when making decisions with respect to logistical management and control.

If two potential trading locations have a non–zero imputed price difference, then at least one of those locations must have a positive imputed price. If the transportation cost between these two locations is less than the imputed price difference, moving a unit of commodity from the low priced location to the higher priced location would result in a net gain in value for that unit, or, equivalently, a reduction in total costs. At optimality, total costs are minimized. Therefore, the imputed value differences between potential trading locations would not and could not be greater than their associated transportation costs.

If, in the optimal solution, two locations, i and j, trade, their imputed values will be linked by this primal trade flow. In other words, the value difference will exactly equal transportation costs thereby creating a spatial price equilibrium. This would not be the case if the trade flow was from i to j and the imputed value difference between i and j was less than transportation costs. Shipping from i to j would, in such a case, result in a loss of total value, because the transportation cost would outweigh the gain in location value. If the trade flow was from i to j and the imputed value difference between i and j was more than transportation costs, shipping from i to j would lead to a gain in total value because the transportation cost would be outweighed by the gain in location value. In this second condition, more commodity would be shipped from i to j, thereby increasing total value. Such a solution would clearly not be optimal, because, at optimality, total value from the dual and total costs from the primal must be equal. Non-equality of these two objectives would provide the opportunity to gain total value and the solution procedure would therefore continue adjusting the pattern of trade flows and the imputed values until the difference in imputed values between locations engaged in trade exactly equaled transportation costs. Two potential trading locations whose value differences are less than their associated transportation costs at optimality will not trade. Two potential trading locations who do trade in the optimal solution, must have imputed values differences which are equal to transportation costs.

The imputed values from transportation/transshipment problems can be interpreted as market prices. They are expressed in the correct units and they are associated with quantities supplied and consumed. They may not, however, include all of the elements which make–up an actual 'observed' price, *i.e.*, those production or extraction costs noted earlier. When these price elements are not included in the primal problem, the imputed value differences represent the location and transportation cost determined *differences* in spatial prices rather than the spatial market prices themselves. Issues involving raw material costs and/or marketing margins may best be approached as 'side analyses' (see Bressler and King, p.98, 1978) where the basic transportation problem solution is augmented with additional market information. Finally, it must be stressed that the simulated shadow values we obtain from the USDSS have concomitant primal solutions. Each value surface has associated with it a set of milk and milk product flows which are derived by minimizing dairy industry costs across the entire U.S. for all dairy products.

THE UNITED STATES DAIRY SECTOR SIMULATOR

Introduction

We now turn to presenting the USDSS itself. The USDSS is a spatially detailed model of the U.S. dairy industry. It is formulated as a capacitated transshipment model with three market levels: farm milk supply, dairy product processing, and dairy product consumption. The goal of the model is to obtain an efficient solution to a complex spatial markets problem. In so doing, the model minimizes the total costs associated with marketing milk and milk products. These costs include the cost of raw milk assembly, shipping intermediate products between plants, plant processing costs, and the distribution of finished products. While few trade models include more than two market levels, it would be difficult to argue that producers, on the whole, trade directly with consumers without the involvement of some type of intermediary. These intermediaries could simply be wholesalers and/or retailers, or they could provide substantial value-added functions and services such as a dairy processing plant would do. In any case, recent research has begun to focus on the role of intermediaries in determining market outcomes in a spatial trading context (for example, see Anania and McCalla, 1991; Bishop *et al.*, 1994; and Roy, 1994). The USDSS explicitly recognizes the role of intermediaries.

This section of the paper continues as follows. First, without any loss of generality, the model is presented algebraically. The dimensions and the sectoral detail pertaining to the present application are then presented and discussed. The structure of the model is explained and the purpose for each set of constraints is explicated. A detailed description of the data and its construction is reserved for the next section of the paper.

Algebraic Presentation of the Model

Defining all indexes, parameters, and variables as follows, the model can be stated algebraically. A few conventions regarding notation are observed—in order to easily distinguish parameters from variables, all parameter labels have a bar over them or, in a few cases, are defined as Greek symbols; the indexes are lower case; and variables beginning with the letter 'Q' denote production quantities while all flow variables begin with the letter 'X.'

i,k,j	=	1, 2,..., (I,K,J) cities. Specifically, let *i* refer to supply points, *k* to plant locations, and *j* the points of consumption.
m	=	1, 2,..., t, t+1, ..., M intermediate product types where *t* is just a place–holder in the sequence running from 1 to M.
n	=	1, 2,..., t, t+1, ..., N final product classifications where, again, *t* is nothing more than a place–holder in the sequence.
q	=	1, 2, .., Q components of milk.
r	=	1,2, .., R plant sizes.

$\overline{AC}_{i,k}$ = the per unit cost of assembling milk from the i^{th} supply point to the k^{th} plant location.

$\overline{IC}_{k,k',m}$ = the per unit cost of shipping the m^{th} intermediate product from the k^{th} plant location to the k'^{th} plant location.

$\overline{DC}_{k,j,n}$ = the per unit cost of distributing the n^{th} final product type from the k^{th} plant location to the j^{th} consumption area.

$\overline{PC}_{k,n}$ = the per unit cost of processing the n^{th} final product at the k^{th} plant location.

$\overline{FC}_{k,n,r}$ = the fixed cost of establishing a plant of size r and of the n^{th} type at the k^{th} plant location.

$\overline{VC}_{k,n,r}$ = the variable, or marginal, cost of processing the n^{th} final product in a plant of size r at the k^{th} plant location.

$\overline{QRM}_{i}$ = the supply of raw milk available at the i^{th} supply point.

$\overline{QFP}_{j,n}$ = the quantity of the n^{th} final product demanded at the j^{th} consumption area.

$\overline{CRM}_{i,q}$ = the proportion of the q^{th} component available in raw milk at the i^{th} supply point.

$\overline{CIP}_{m,q}$ = the proportion of the q^{th} component contained in the m^{th} intermediate product.

$\overline{CFP}_{j,n,q}$ = the proportion of the q^{th} component contained in the n^{th} final product demanded at the j^{th} consumption area.

$\overline{CAP}_{k,n,r}$ = the capacity of the n^{th} plant type of plant size r at the k^{th} plant location.

ϕ_i = the operational reserve proportion, *i.e.* the proportion of the raw milk available at the i^{th} supply point which is ineligible for shipment to fluid plants.

φ_n = the maximum ratio, on a volume produced basis, of intermediate products to the n^{th} final product at the n^{th} plant type.

$\delta_{n,q}$ = the proportion of the q^{th} component at the n^{th} plant type which may be shipped into that plant in the form of specified intermediate products.

$XRM_{i,k,n}$ = a shipment of raw milk from the i^{th} supply point to the n^{th} plant type at the k^{th} plant location.

$XIP_{k,n,k',n',m}$ = a shipment of the m^{th} intermediate product type from the n^{th} plant type at the k^{th} plant location to the n'^{th} plant type at the k'^{th} plant location.

$XFP_{k,j,n}$ = a shipment of the n^{th} final product type from the k^{th} plant location to the j^{th} consumption area.

$QCR_{k,n,q}$ = the quantity of the q^{th} component received in the form of raw milk at the n^{th} plant type at the k^{th} plant location.

$QCP_{k,n,q}$ = the quantity of the q^{th} component processed at the n^{th} plant type at the k^{th} plant location.

$QPROC_{k,n}$ = the quantity of product processed at the n^{th} plant type at the k^{th} plant location. (Note that this quantity may include both intermediate and final products. The set n refers to a plant and final product type. The set m, which defines intermediate products, however, is uniquely mapped into the plant type set. Thus, each plant type is only able to produce and ship specified intermediate products).

$NPLTS_{k,n,r}$ = the number of plants of size r and type n to be situated at the k^{th} plant location.

Z = the objective function value, *i.e.*, minimized sum of all costs.

$$\begin{aligned}\text{Minimize } Z = & \sum_{i=1}^{I}\sum_{k=1}^{K}\sum_{n=1}^{N} \overline{AC}_{i,k} * XRM_{i,k,n} \\ & + \sum_{k=1}^{K}\sum_{n=1}^{N}\sum_{k'=1}^{K}\sum_{n'=1}^{N}\sum_{m=1}^{M} \overline{IC}_{k,k',m} * XIP_{k,n,k',n',m} \\ & + \sum_{k=1}^{K}\sum_{n=1}^{N} \overline{PC}_{k,n} * QPROC_{k,n} \\ & + \sum_{k=1}^{K}\sum_{j=1}^{J}\sum_{n=1}^{N} \overline{DC}_{k,j,n} * XFP_{k,j,n}\end{aligned} \tag{1a}$$

subject to

$$\overline{QRM}_i \geq \sum_{k=1}^{K}\sum_{n=1}^{N} XRM_{i,k,n} \tag{2}$$

$$\sum_{i=1}^{I} \overline{CRM}_{i,q} * XRM_{i,k,n} \geq QCR_{k,n,q} \tag{3}$$

$$\begin{aligned}QCR_{k,n,q} + \sum_{k'=1}^{K}\sum_{n'=1}^{N}\sum_{m=1}^{M} \overline{CIP}_{m,q} * XIP_{k',n',k,n,m} \geq \\ \sum_{k'=1}^{K}\sum_{n'=1}^{N}\sum_{m=1}^{M} \overline{CIP}_{m,q} * XIP_{k,n,k',n',m} + QCP_{k,n,q}\end{aligned} \tag{4}$$

$$QCP_{k,n,q} \geq \sum_{j=1}^{J} \overline{CFP}_{j,n,q} * XFP_{k,j,n} \tag{5}$$

$$QPROC_{k,n} = \sum_{k'=1}^{K}\sum_{n'=1}^{N}\sum_{m=1}^{M-t} XIP_{k,n,k',n',m} + \sum_{j=1}^{J} XFP_{k,j,n} \tag{6a}$$

$$\sum_{k=1}^{K} XFP_{k,j,n} \geq \overline{QFP}_{j,n} \qquad (7)$$

Additional Constraints

Although the model as defined thus far is a well–formed and adequately specified problem, the following three sets of constraints were added in order to impose on the model an even greater level of 'real–world' structure.

$$\sum_{k=1}^{K} \sum_{n=1}^{N-t} XRM_{i,k,n} \geq \phi_i * \overline{QRM_i} \qquad (8)$$

$$\varphi_n * QPROC_{k,n} - \sum_{k'=1}^{K} \sum_{n'=1}^{N} \sum_{m=1}^{M} XIP_{k,n,k',n',m} \geq 0 \qquad (9a)$$

$$\delta_{n,q} * QCP_{k,n,q} - \sum_{k'=1}^{K} \sum_{n'=1}^{N} \sum_{m=1}^{M} \overline{CIP}_{m,q} * XIP_{k',n',k,n,m} \geq 0 \qquad (10)$$

Fixed Charge Transformation

One particular feature of the USDSS is that it can be used to model scale economies in the processing sector. The introduction of scale economies would, typically, require a problem formulation which contained nonlinear constraints and would therefore be difficult to formulate and solve within a linear programming framework. However, several methods of approximating nonlinear problems with either integer programming or linear programming techniques have been developed. One such method is the so–called fixed charge problem, which is particularly well–suited to the question of scale economies, and can be solved using integer programming techniques, a close cousin of linear programming. The fixed charge problem grew out of the well–studied 'facility location problem' and dates back to the work of Balinski (1966), Kuehn and Hamburger (1963), Manne (1964), and Stollsteimer (1961, 1963). As Stollsteimer (1963) notes, the issue is, in its simplest form, one of answering the following questions. How many plants should be built? Where should they be located? How large should each one be? Where should the raw materials be obtained and which clients, or markets, should be served by each plant? An optimal solution to the problem is one which answers all of these questions while minimizing the total associated costs.

It is relatively straightforward to transform the above LP into a fixed charge problem and formulate it as a Mixed Integer Program (MIP). First, redefine the variable QPROC such that it is indexed on *r*, plant size, in addition to *k* and *n*. Second, add a general integer variable, $NPLTS_{k,n,r}$, where $NPLTS_{k,n,r}$ denotes the number of plants of size *r* and type *n* to be situated at the k^{th} plant location. Finally, as indicated below, modify objective function (1a) to become (1b), update the two sets of constraints containing the variable QPROC, *i.e.*, (6a) and (9a) to (6b) and (9b) respectively, and add a new plant capacity constraint, (11).

$$\begin{aligned}\text{Minimize } Z = & \sum_{i=1}^{I}\sum_{k=1}^{K}\sum_{n=1}^{N} \overline{AC}_{i,k} * XRM_{i,k,n} \\ & +\sum_{k=1}^{K}\sum_{n=1}^{N}\sum_{k'=1}^{K}\sum_{n'=1}^{N}\sum_{m=1}^{M} \overline{IC}_{k,k',m} * XIP_{k,n,k',n',m} \\ & +\sum_{k=1}^{K}\sum_{n=1}^{N}\sum_{r=1}^{R} \overline{FC}_{k,n,r} * NPLTS_{k,n,r} + \overline{VC}_{k,n,r} * QPROC_{k,n,r} \\ & +\sum_{k=1}^{K}\sum_{j=1}^{J}\sum_{n=1}^{N} \overline{DC}_{k,j,n} * XFP_{k,j,n}\end{aligned} \tag{1b}$$

$$\sum_{r=1}^{R} QPROC_{k,n,r} = \sum_{k'=1}^{K}\sum_{n'=1}^{N}\sum_{m=1}^{M-t} XIP_{k,n,k',n',m} + \sum_{j=1}^{J} XFP_{k,j,n} \tag{6b}$$

$$\sum_{r=1}^{R} \varphi_n * QPROC_{k,n,r} - \sum_{k'=1}^{K}\sum_{n'=1}^{N}\sum_{m=1}^{M} XIP_{k,n,k',n',m} \geq 0 \tag{9b}$$

$$\overline{CAP}_{k,n,r} * NPLTS_{k,n,r} \geq QPROC_{k,n,r} \tag{11}$$

For much of the analysis undertaken in the present study of federal order class I prices, it was not appropriate to incorporate the scale economies feature of the USDSS. Thus, objective function (1b) and constraints (6b), (9b), and (11) were generally not used. However, the fixed charge formulation has been thoroughly validated and has been employed in analyses that address more location specific efficiency questions.

Explanation of Objective Function and Constraints

Before proceeding to explain each set of constraints, the underlying structure and dimensions of the USDSS, as applicable in the base case analysis of the federal order class I price study, are presented graphically. From a model–building perspective, it is a simple and straightforward task to change any of these dimensions. However, compiling the necessary data may in some cases be an extremely time–consuming undertaking. Figure 2 illustrates the network structure of just the domestic portion of the USDSS. U.S. milk supply is represented by 240 specific geographic locations in the USDSS, the circles in Figure 2. Each location represents the milk supply available from a contiguous multi–county aggregate set of counties selected from all 3,111 counties in the U.S. Similarly, total U.S. dairy product consumption for each of five product groups, encompassing the entire consumption of U.S. dairy products, is represented by 334 specific geographic locations, the squares in Figure 2. The five dairy product groups distinguished at the processing and consumption levels in the USDSS are fluid milk products; soft dairy products; hard cheeses; butter; and dry, condensed, and evaporated (DCE) dairy products.

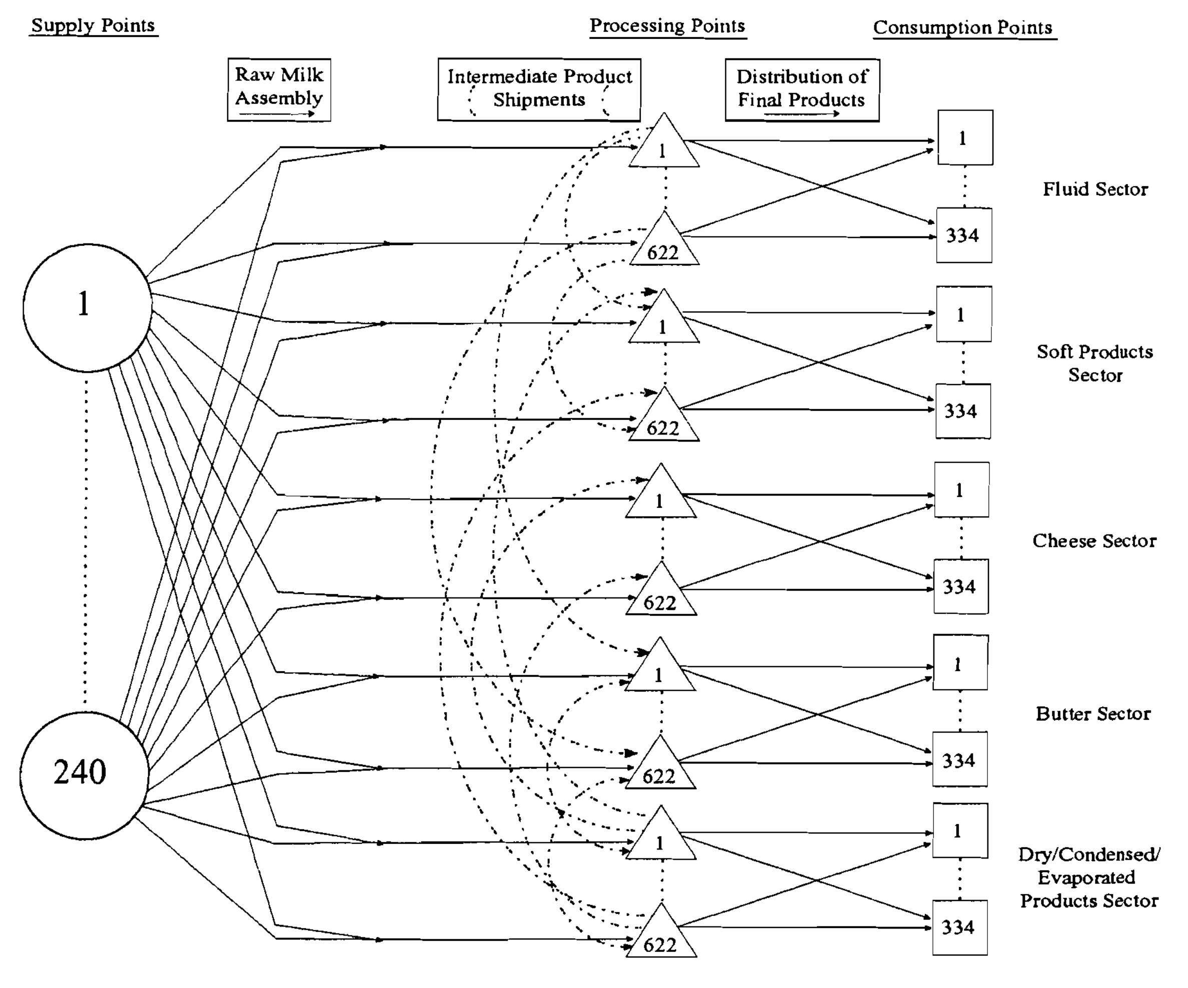

Figure 2. Network Representation of the United States Dairy Sector Simulator

As currently configured, there are 622 potential processing locations, the triangles in Figure 2, at which each group of dairy products may be processed. Raw milk, intermediate products, and all final products are represented on a multiple component basis; fat and solids–not–fat (SNF) are the two components currently being used. The integer formulation of the model requires that, in addition to type and location, plants also be specified according to size. Currently we classify plants as being either medium or large. In analyses which are configured to have the model determine optimum plant size, it has been found that small plants rarely appear in the optimal solution. Thus, we don't model them.

Assembly of raw milk from farms to plants is illustrated in Figure 2 by the lines connecting the circles to the triangles. There is, of course, a cost associated with moving milk along these arcs. Note that every supply point has the potential to ship to any plant type at any geographic location. Once the milk is at the plants, it is transformed, either into intermediate products which are shipped to other plants, the dashed lines connecting the triangles, or into final products which are transported to the consumption points. The model is currently specified with four intermediate product types which are able to move from plants of one type to plants of some other type in nine different combinations. For example, the shipment of cream from fluid plants to soft products plants and butter plants constitutes two such combinations. As with assembly, there are costs associated with making shipments of intermediate and final products. Costs may also be applied at the processing level, that is, at the triangles. Observe that there are five sets of 622 triangles, one set for each of the five product types. Quite obviously, it is not possible for a plant to ship products of a type for which it is incapable of producing. However, provided the technology constraints are observed, each processor can ship to every consumption point. Therefore, every triangle from within each set of triangles is connected to every square in the corresponding set of squares.

We now dispense with the generality of the algebraic presentation above and describe each set of constraints in the precise manner in which they are employed in the current application. Even though the model accounts for trade on both the import and the export side, the discussion to follow refers primarily to the domestic U.S. portion of the model. While the trade sector is modeled similarly, it constitutes a small part of the overall U.S. dairy sector. It will be adequately described in the data section of the paper.

Objective Function

Equations (1a) and (1b) define the two alternative objective functions. Only one of these may be used at a time. Equation (1a) corresponds to the LP formulation of the model while (1b) corresponds to the mixed integer, or MIP, formulation. The objective function simply states the goal of the problem which in both cases is to minimize the sum of all costs. Equation (1b) differs from (1a) only in the manner in which costs are applied to the processing sector. The LP formulation applies processing costs on a per unit of product processed basis. The MIP formulation, on the other hand, splits processing costs into a fixed and a variable component. Moreover, a fixed and a variable cost can be specified for a range of plant sizes. Thus, at any location and for any plant type, the fixed cost is incurred for each new plant the model brings into the solution. The variable cost is, of course, incurred on each unit of product processed. All transportation costs enter both objective functions identically.

Raw Milk Assembly

The set of constraints numbered (2) are referred to as the raw milk assembly constraints. One of these constraints is generated for every supply point, of which there are 240. Quite simply, these constraints ensure that the sum of all raw milk shipments to plants, XRM, emanating from a supply point can be no more than the amount of raw milk available at that supply point, $\overline{QRM}$. In other words, a supply point can't ship any more milk than it has available. The raw milk assembly constraints impose no restrictions on which locations or plant types receive the milk. However, there are other constraints that do impose such restrictions.

Receiving of Milk Components at Plants

The next set of constraints, (3), performs two functions. First, it determines the quantity of fat and SNF that is contained in each unit of milk being shipped from a supply point to a plant. Second, it ensures that the total quantity of each component actually received at a plant is no greater than the quantity of each component shipped into the plant. It accomplishes this regardless of how many supply points do the shipping and what the composition of raw milk is at each of them. Note that because these constraints are inequalities rather than strict equalities, it is mathematically possible for raw milk to be shipped from a supply point but not actually received at any plant. Hence, the need for the subtle distinction between the quantity shipped and the quantity received. Two of these constraints, one for fat and one for SNF, are generated for every plant type at every location. Thus, if all 622 locations were able to process all five product types, constraint set (3) would generate 6,220 individual constraints.

Interplant Transfers of Intermediate Products and Component Balancing

The fourth set of constraints deal with interplant shipments of intermediate products and component balancing. Constraint set (4) literally states that the quantity of each component arriving at a plant either in the form of raw milk, QCR, or in the form of an intermediate product, $\overline{\text{CIP}} * \text{XIP}$, must at least exceed the quantity of each component that either leaves the plant in the form of an intermediate product, $\overline{\text{CIP}} * \text{XIP}$, or is processed at the plant into a final product, QCP. As with constraint set (3) above, 6,220 individual constraints would be generated by this set of constraints if all 622 locations were able to process all five product types.

Processing; Milk Components and Products

Constraints (5), (6a), and (6b) take care of the processing activity in the USDSS. The physical constraint on components being processed is controlled by (5) while (6a), or (6b), facilitate the application of costs to the processing activity. Constraint (5) requires that the quantity of each component actually processed at a plant be greater than or equal to the quantity of each component contained in the sum of all final product shipments leaving the plant. Note that the right–hand side of (5) makes no reference to the components leaving a plant in the form of intermediate products. Such components are not, strictly speaking, processed so are therefore not included in these constraints. The separation between interplant transfers and final product processing is quite subtle but important. In a strict mathematical and physical flow sense, the interplant transfer of intermediate products takes place before processing is undertaken. In other words, only final products are processed. Once again, if all 622 locations were able to process all five product types, then constraints (5) and (6a) would together generate 9,330 individual constraints.

If the left–hand side of (3) and the right–hand side of (5) were substituted for QCR and QCP, respectively, into (4), then constraints (3), (4), and (5) can be collapsed into one expression and the variables QCR and QCP can be eliminated. However, keeping track of the algebra becomes a little more difficult.

Constraint (6a) is simply an accounting identity which calculates the quantity of product actually processed at a plant. This is done so that processing costs (in the objective function) can be applied on a product basis rather than on a component basis. It would be extremely difficult to apportion costs on a component basis. Indeed, studies of dairy processing costs tend to estimate costs on the basis of product produced, or, occasionally, on the basis of raw milk entering the plant. Constraint (6b) performs the same role as does (6a) except that it calculates the quantity processed not only at all plant types and locations, but for all plant sizes as well. Note that the right–hand sides of (6a) and (6b) include a summation over a subset of the M intermediate products yet the constraints are generated for all N final product, or plant, types. The need to 'mix' intermediate products with final products for the purpose of assigning costs arises as follows.

The four intermediate products defined in the USDSS are cream, skim milk, nonfat dry milk (NDM), and ice cream mix (ICM). Only one of these, NDM, involves a substantial and costly transformation of raw milk. Hence, it is necessary to apply processing costs to NDM in the same way as they're applied to the final products. The other three intermediate products are all transported as wet solids, in much the same way as raw milk. A small charge is therefore added to the raw milk assembly cost to derive the total ('processing' and transportation) cost for these three intermediate products. Thus, the cost of 'processing' and transporting these products is all incorporated into the transportation cost.[2] While this structure is perhaps a little confusing, it is easy to implement and is logically consistent. Note that processing costs, $\overline{PC}$ [in (1a)] and $\overline{VC}$ [in (1b)], are not defined on the index for intermediate products, *m*, yet such costs must apply to NDM, an element of this set. Processing costs are actually applied to a plant type rather than a product class *per se* and intermediate products are uniquely associated with plant types. In other words, it is not possible for all plant types to ship every type of intermediate product. So, it is a simple matter to sum over all NDM (as an intermediate product) and final product shipments leaving a powder plant and apply the powder (or more correctly, DCE) plant processing costs to this quantity.

Distribution of Final Products

The next set of constraints, (7), are responsible for controlling the distribution of final products. Five of these constraints are generated for every consumption point—one for each product type. Thus there are 1,670 individual final product distribution constraints. These constraints simply require that the sum of all shipments of final product to a consumption point must be greater than or equal to the quantity of product actually consumed there.

Operational Reserve Requirement

Constraint set (8) defines what we refer to as an 'operational reserve' requirement. The manner in which we specify this constraint leads to a restriction on raw milk assembly shipments such that no more than 85 percent of the raw milk available at any supply point can be shipped to a fluid plant. More specifically, at least 15 percent must be shipped to either a butter or a butter/powder plant. Note that ϕ_i, the operational reserve parameter, is defined on index *i*. This means that it can be allowed to vary regionally although at present we specify it to be 15 percent everywhere. In general algebraic parlance, the constraint states that the sum of a subset of the shipments leaving a supply point must be greater than or equal to some specified proportion of the raw milk available at that supply point. One such constraint is generated for each of the 240 supply points in the model. The subset of raw milk shipments in question consists of shipments to butter and butter/powder plants and the sum of these shipments must be at least 15 percent of the raw milk available at the supply point.

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[2] A detailed description of costs will be provided later.

The reason for imposing this constraint is to reflect the reality that fluid plants generally do not receive and process raw milk seven days per week. But milk is, of course, marketed by farmers on a daily basis. Hence, even in areas that are highly deficit with respect to beverage milk needs, raw milk shipments to butter/powder plants are required. This constraint should not be confused with the seasonal balancing of supply and demand. Data for the present analysis has been compiled on a monthly basis which obviates the need to directly consider seasonal balancing.

Technically, there is no reason that the operational balancing could not be accomplished at plant types other than butter/powder, for example, at cheese plants. However, such practices are not typically observed. As is the case with many of the model parameters for which a degree of subjectivity exists, we have attempted to mimic the most common management practices.

Volume Balance Requirement

While designing a dairy model that represents milk and milk products on a component basis has many advantages, it also has a few disadvantages. One of these is that the model lacks a mechanism to ensure that, on a volume basis, plants don't ship more product than they receive as inputs. This problem manifests itself at plant types such as fluid plants where the production process does not result in any significant weight reduction. In contrast, at a powder plant, for example, much of the volume entering the plant in the form of milk is evaporated away during the production of powder. Constraints (9a) and (9b) are designed to rectify this anomaly and they do so by requiring a ratio of intermediate products to final products at fluid plants of no more than 0.1, *i.e.* $\varphi_n = 0.1$ where n = fluid plants only. In other words, the volume of cream and ice cream mix that leaves a fluid plant can be no more than 10 percent of the volume of packaged milk leaving that plant. Note that constraints (9a) and (9b) are generated on the indexes k and n when it is equal to fluid plants. Thus, the constraint is only applicable at the locations at which there are fluid processing opportunities. The choice of $\varphi_n = 0.1$ was not arbitrary. It was computed such that the volume balance was maintained for the most extreme combinations of raw milk and final packaged milk compositions. The composition of intermediate products is specified to be uniform across the country while that of raw milk and packaged milk varies regionally.

Essentially the problem boils down to one of not including water, or, more correctly, carrier, as one of the components of milk. An alternative remedy would have been to add a third component, namely carrier, but the payoff would have been minuscule compared to the cost of the added dimensions. Given the fixed composition of raw milk, intermediate products, and final products, and the fact that each of these may be vastly different from one another, it turns out that without the volume balance constraint, fluid plants in the model are able to ship out a greater volume of packaged milk, cream, and ice cream mix than the volume of raw milk received at the plant. While this does not lead to a situation where more components leave the plant than are received, constraints (3), (4), and (5) protect against that, it does give the appearance that plants can produce something from nothing. An additional consequence of ignoring the carrier component of milk is that fluid plants can act as pure transfer stations, that is, they can receive raw milk and ship out intermediate products but ship no final product. Again, in an attempt to reflect common management practices, the volume balance constraint is used to eliminate this possibility. Should it be deemed desirable, the USDSS can easily be formulated to allow fluid plants to 'reconstitute' fluid milk from cream and NDM and/or concentrated skim. Such reconstitution is not permitted in our current study.

Restriction on Use of Components from Intermediate Products

The set of constraints labeled (10) is used to impose limits on the proportion of components that a plant may obtain in the form of specified intermediate products. Here it is used to restrict to no more than 50 percent, the proportion of SNF that a soft products plant can obtain in the form of ice cream mix. In other words, the constraints are specified only at locations where n is equal to soft products and for q equal to SNF. As it happens, this constraint is rarely binding and could actually be omitted without any appreciable change in the model results.

Plant Capacity Constraint

Constraint set (11) applies only to the MIP formulation of the model at this time. Capacity constraints can be introduced in a variety of ways. For example, the constraint may operate on the quantity of raw milk received at a plant. In such a case it would, perhaps, be more correctly considered a throughput constraint. Alternatively, they may enter the model in the form of a limit on the volume of finished product that can be processed, as is the case here. Quite simply, the capacity constraint, as formulated here, is saying that at each plant location, the number of plants of a given type and size that the model brings into the solution multiplied by the corresponding capacity parameter for that plant type, size, and location must be greater than or equal to the quantity of product processed there.

Finally, the model dimensions and the scalar parameter values are summarized in Table 1. Parameter values which can take on many values, such as costs, supply, and consumption, are described in the data section of the paper.

Table 1. Summary of Model Dimensions and Scalar Parameter Values

	(symbol)	
Supply areas	i	240
Potential processing locations	k	622
Consumption areas	j	334
Supply goods (*i.e.*, raw milk)		1
Intermediate product types	m	4
Final product classifications	n	5
Milk components	q	2
Interplant transfer types		9
Plant sizes (MIP formulation only)	r	2
Operational reserve proportion	ϕ_i	0.15
Maximum ratio of intermediate to final products at fluid plants	φ_n	0.1
Maximum proportion SNF at soft plants from ICM	$\delta_{n,q}$	0.5

General Discussion

Five groupings of final dairy products are distinguished at the processing and consumption levels in the USDSS: fluid milk products; soft dairy products; hard cheeses; butter; and dry, condensed, and evaporated (DCE) dairy products. Because these various processed and consumed dairy products rarely use the components of milk in the same proportion as they are available in farm milk supplies, USDSS uses a multiple–component characterization of milk and dairy products. Currently, fat and solids–not–fat are used to account for the supply and use of the valuable constituents in milk. Dairy product processing plants must 'balance' the use of milk components in the various dairy products by moving intermediate dairy products between uses, and often across space (see Bishop *et al., op. cit.*). For example, unused components from one processing operation must be moved from that operation to another, which may or may not be situated at the same geographic location, for use in a subsequent dairy process. The USDSS simultaneously analyzes the optimal location of processing facilities, farm milk assembly movements, interplant transfers of intermediate dairy products, and dairy product distribution movements. Given estimates of producer milk marketings, dairy product consumption, processing costs, and transportation costs for moving milk from farms to plants, intermediate dairy products between plants, and processed dairy products from plants to consumers, the USDSS finds the least cost organization of milk, interplant, and distribution flows as well as efficient processing locations and sizes.

Substantial effort and resources were expended on maximizing the level of spatial disaggregation in the USDSS. For the milk supply and dairy product consumption nodes, U.S. counties were used as the initial unit of analysis. These were aggregated to multiple–county units which, in turn, were represented by specific geographic points. For processing nodes, actual processing facilities were aggregated directly to specific geographic points. There exists a trade–off between the level of disaggregation, the effort which must be expended to collect and update the base data, and the benefits derived from disaggregation. Somewhere a balance has to be struck. We have been guided in these decisions by the thoughts of Earl O. Heady (Heady, *op. cit.*).

> "The intensity of the aggregation problem is, partly, a function of the purposes of the investigation. If the only purpose of the model application and empirical attempt is illustrative and to show, in fact, that one can be in the 'style of the economist' by actually estimating some quantitative supply and demand relationships, deriving therefrom some equilibrium prices and quantities, concerns in aggregation can be minimized. Perhaps not a small portion of research in agricultural economics currently falls in this realm: to 'be in style' by assembling a few data and coefficients as an illustration that one has applied the latest empirical technique. When the analysis is for these style or illustrative purposes alone, basic aggregation considerations are secondary and perhaps unimportant. However, when the analysis is expected to predict response relationships, production patterns or optima which will serve in outlook and guidance for policy, educational programs or farmer investment decisions, problems of aggregation take on a great deal of importance. It is no longer sufficient to draw an arbitrary boundary around

a number of states for which data are readily available and term the contents a meaningful region."

Dairy industry issues are intensely locational. We view the USDSS as a tool which is able to provide useful policy guidance rather than an instrument enabling us to simply act 'in the style of the economist.' In our judgment, maximum, feasible spatial disaggregation is necessary to provide useful 'guidance for policy' with respect to issues which are themselves intensely locational.

The model has been coded using the General Algebraic Modeling System (GAMS). GAMS generates the problem as specified in the code and then passes it to a solution algorithm, or solver, to be solved. We use an assortment of solvers, depending on the specification of the model, from IBM's Optimization Software Library (OSL). After the model has been solved, it returns the objective function value, all variable levels, and the marginal values associated with each constraint to the problem.

DATA

Two sets of data have been compiled—one representing May 1995 and the other October 1995. Some of the required data, distances between cities for example, is common to both months. This section of the paper simply describes, in turn, each aspect of the required data.

Cities and Distances

The first step in compiling the data required by the USDSS was to specify the basis for the node structure of the underlying economic network. A total of 622 cities were selected to represent the locations at which milk production, dairy product processing, dairy product consumption, or some combination of all three activities, could occur in the model. The Venn diagram in Figure 3 illustrates the degree of overlap among the activities assigned to cities. Actually, the astute reader will ascertain from the diagram that 28 of the 622 cities are redundant when processing locations are restricted[3] to the 434 sites of known processing activity. This minor redundancy is due to the continual updating of the master plant list. Nodes in the USDSS were thus defined at specific geographic locations. Strictly speaking, nodes and cities are not literally analogous as there may be more than one node at any particular city location. For example, if a city denotes the location of a supply point, a fluid milk plant, and a cheese plant, then that single location would be associated with three nodes. In fact, it is actually even more complicated as it is the milk components, fat and SNF, which move through plants in the USDSS. Therefore the city in question would be associated with not three, but five nodes. Fortunately, such devotion to technical detail does not preclude us, as a matter of convenience, from sometimes referring to cities as nodes. The choice of cities was not arbitrary and neither was it necessary to have the cities uniformly distributed across the forty–eight contiguous

[3] The distinction between restricted and unrestricted processing locations will be made clear shortly.

states.[4] The next few pages will make clear the reasons why the 622 particular cities were chosen.

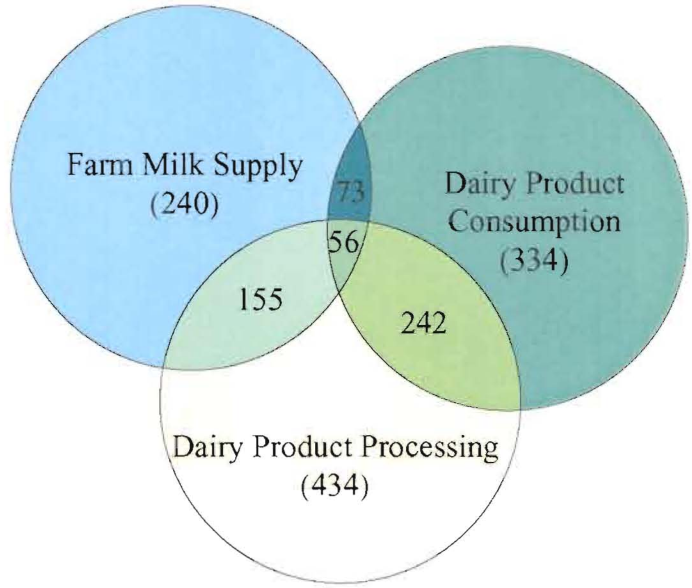

Figure 3. Venn Diagram of the Number of Cities Engaged in Various Economic Activities

Five additional 'cities' were identified to represent the trade and stockholding sub–sectors of the U.S. dairy sector. Three of these five points represent import nodes and allow the model to simulate the importation of dairy products from the rest of the world. The import nodes symbolize ports and are associated with the cities of New York, Houston, and San Francisco. It is assumed that imported dairy products reach the U.S. by ship and then move through the road network to their ultimate point of consumption. The fourth of these five nodes was designated an export transshipment point, and the last was the point at which stocks may accumulate. These last two nodes do not have specific geographic point associations.

The USDSS is based on the assumption that all transportation occurs on interstate and major state highways. Hence it is necessary to know the precise distance in miles between every pair of cities in the network. Transportation costs are directly related to distance and constitute an integral part of the USDSS. The integrity of the model's results depend crucially on the distance matrix having been constructed accurately and carefully. Not counting the trade and stocks nodes, there are 386,884 (*i.e.* 622*622) individual distances required by the model. Compiling a list of this size is clearly a task prone to errors, so computerized methods were employed to minimize the possibility of mistakes.

Given the degree of symmetry in the distance matrix, the number of distances actually required to be determined is (622*621)/2 = 193,131 (*i.e.* the distance from city i to city j is the

[4] The model does not include Alaska or Hawaii.

same as from *j* to *i*, and when $i = j$, the distance is obviously zero). While 193,131 is much less than 386,884, it is still a large number. Fortunately, it is not necessary to find each of these distances individually. A network problem known as the shortest path problem can be used to generate all required distances if only a particular subset of the distances are known in advance. Specifically, the required subset must include the distance between adjacent cities and any other pairs of cities such that a network of actual roadways connecting all of the cities is enumerated. The resulting network, comprised of just 1,665 arcs is depicted in Figure 4. Clearly, the task of assembling 1,665 individual distances is much less tedious and considerably less likely to contain errors than the task of obtaining 193,131 distances.

The 1,665 distances were obtained from a road atlas and represent actual road mileages. If Figure 4 reveals that seemingly nearby cities are not connected directly to one another, it is because there is no major road forming such a connection. Before the 1,665 distances were ascertained, an additional 335 points were identified in order to better discern the true distance between each of the 622 principal cities. In total, 957 cities, or nodes, formed a network which was spanned by 1,665 arcs, each one characterized as a distance. Note that these additional 335 points were mostly the locations of major road intersections and allowed the actual road distance between the 622 principal cities to be determined more accurately. They played no role in the USDSS other than to aid in determining the distance matrix. The 622 principal cities appear in Figure 4 as black stars, while the additional 335 cities appear as white circles.

Using the concept of a path, as described earlier, the shortest route between any two points can be described simply as the path which yields the minimum accumulated length of the arcs comprising the path. Obviously there are many paths which could be taken to get from one city to another. Note that if the path contains chains, cycles, or circuits, it would, by definition, not be the shortest. Finding the shortest path is one of the most fundamental problems emanating from network theory and there exists numerous algorithms for solving this problem efficiently. The particular algorithm used here was an all–purpose algorithm which Gilsinn and Witzgall (1973) refer to as 'S4.' After the shortest path algorithm has generated the required pair–wise distances, the three import nodes were specified to be the same distance from every other city as were their associated cities, *i.e.* New York, Houston, and San Francisco respectively. The final two nodes, the export and stocks nodes, were each specified to be zero miles from each of the 622 principal cities. In other words, exporting and stockholding activities can take place at any location.

A departure from the shortest path criteria when compiling the distance matrix was employed in the case of New York state. As will be seen later, transportation costs are specified to be not only a function of distance but of wages and gross vehicle weight limits as well. It turns out that for many trips between two cities within the state of New York, the shortest route is not always the least expensive. This arises because the state of Pennsylvania has a much lower gross vehicle weight limit than does New York and for trips from western and central New York to points in the vicinity of New York city, the shortest distance route takes a path through Pennsylvania. In such cases, the distance matrix was amended such that the longer but less expensive route was taken. This point will become clearer following the discussion of transportation costs. This was the only instance where the shortest path criteria was not used.

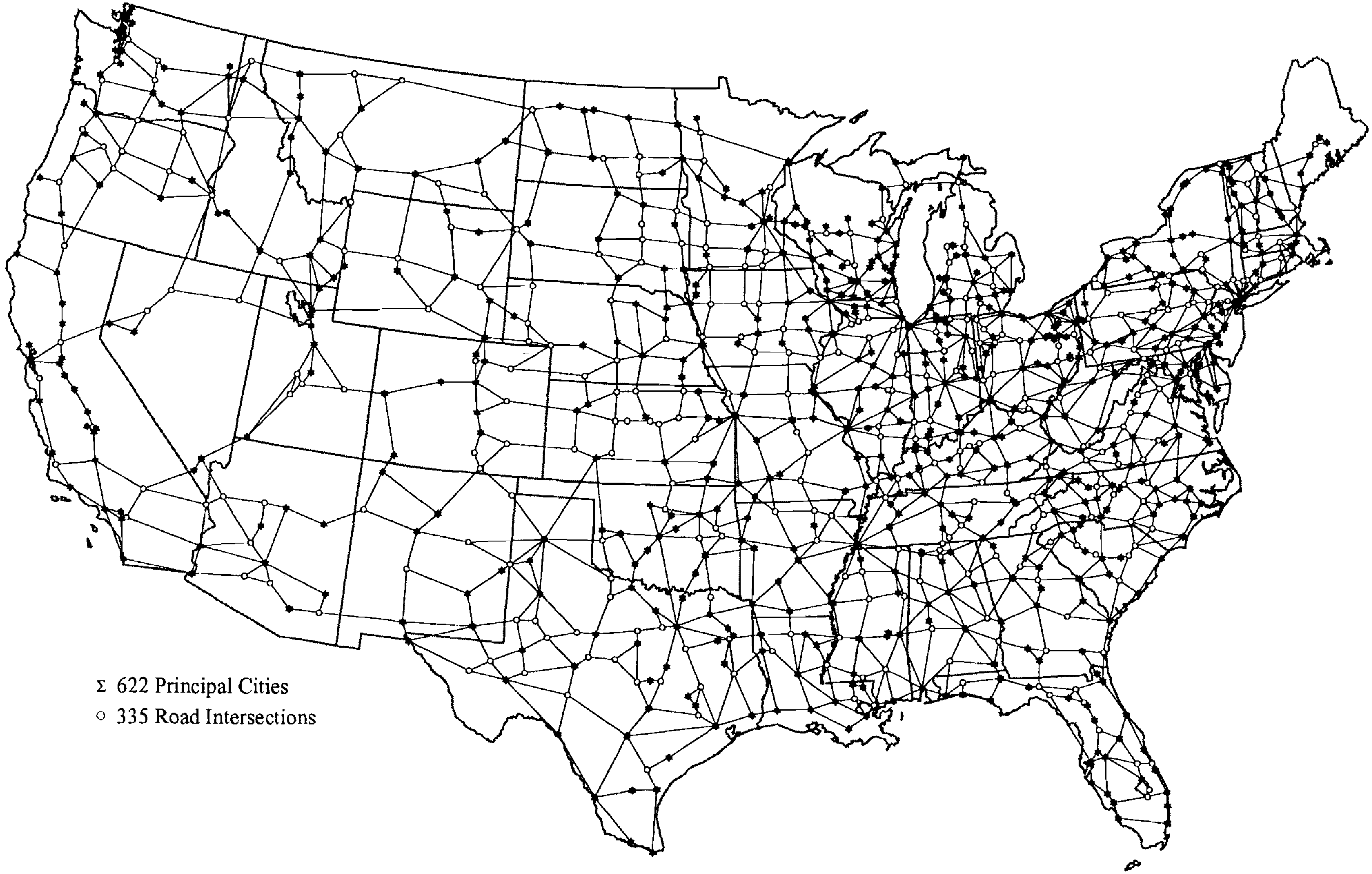

Figure 4. Road Network Used to Determine the Distance Matrix

The final step of the procedure is to check the accuracy of the matrix. For a variety of reasons, even when some of the popular GIS programs are used to generate the distance matrix, errors can, and indeed do, occur. Given the importance of distances in determining accurate costs, considerable time and effort was devoted to checking the distance matrix. Several methods were utilized. First, the distance between random pairs of cities was looked–up to see if it was correct. Second, predecessor lists were generated and checked for selected cities. A predecessor list for an individual city lists the distance from that city to every other city. More importantly, it can be used to find the path taken to get from any city back to the city for which the list was generated. A third, simple, yet revealing method, was to use mapping software to produce contour maps of the distances obtained from each predecessor list. Such maps enabled any irregularities to be quickly identified. The final checking mechanism used was to generate and plot the minimum spanning tree for the 622 principal cities. Recall from the earlier discussion that a minimum spanning tree is that set of *(n-1)* arcs that will connect *n* nodes such that the sum of the lengths of the arcs is minimized. The particular algorithm used in this case was a version of the so–called 'greedy' algorithm taken from Hillier and Lieberman (1974).

Figure 5 shows the minimum spanning tree for the 622 nodes and 1,665 road segments noted above. It also illustrates the spatial distribution of the 622 principal cities. A mistake in the distance matrix would manifest itself in the minimum spanning tree plot as either a line connecting a city to another which is clearly not the closest nearby city, or as a sequence of lines which formed a cycle. Note that Figure 5 does not purport to show the path taken to get from one city to any other. It shows the 621 arcs that connect the 622 cities, or nodes, such that the sum of the lengths of the 621 arcs is minimized. Due to the presence of ties, the set of arcs able to accomplish this may not be unique. However, the minimum sum of the lengths certainly is unique and for this particular network it is 34,612 miles.

Farm Milk Supply; Areas, Quantities, and Composition

For each of the 240 supply areas, the USDSS requires a raw milk supply quantity and an associated fat and solids–not–fat content. The first step in constructing this information was to define the multiple–county supply areas. Each of the 3,111 U.S. counties and independent cities in the 48 contiguous states was aggregated into one of the 240 areas. The number of areas and the counties to be associated with each were selected on the basis of the spatial distribution of milk cows or milk production within each state. A single city at the milk production centroid was then chosen to represent the entire supply of the aggregated area. Whence 240 of the 622 principal cities were identified. Figure 6 shows the 240 supply areas as well as a proportional representation of the milk supply associated with each area situated at the representative city.

Milk marketings, which includes direct sales to plants and dealers and direct sales to consumers but does not include milk used on farms, were used to represent milk supply. Approximately 13,651 and 12,638 million pounds of milk were marketed by producers in May and October of 1995, respectively. After estimating producer milk marketings by state, an indexing procedure was used to distribute those marketings to the counties within each state. The individual county estimates were then aggregated to their respective supply area. Note that each of the 3,111 independent cities and counties in the 48 contiguous states were uniquely assigned to one of the 240 supply areas.

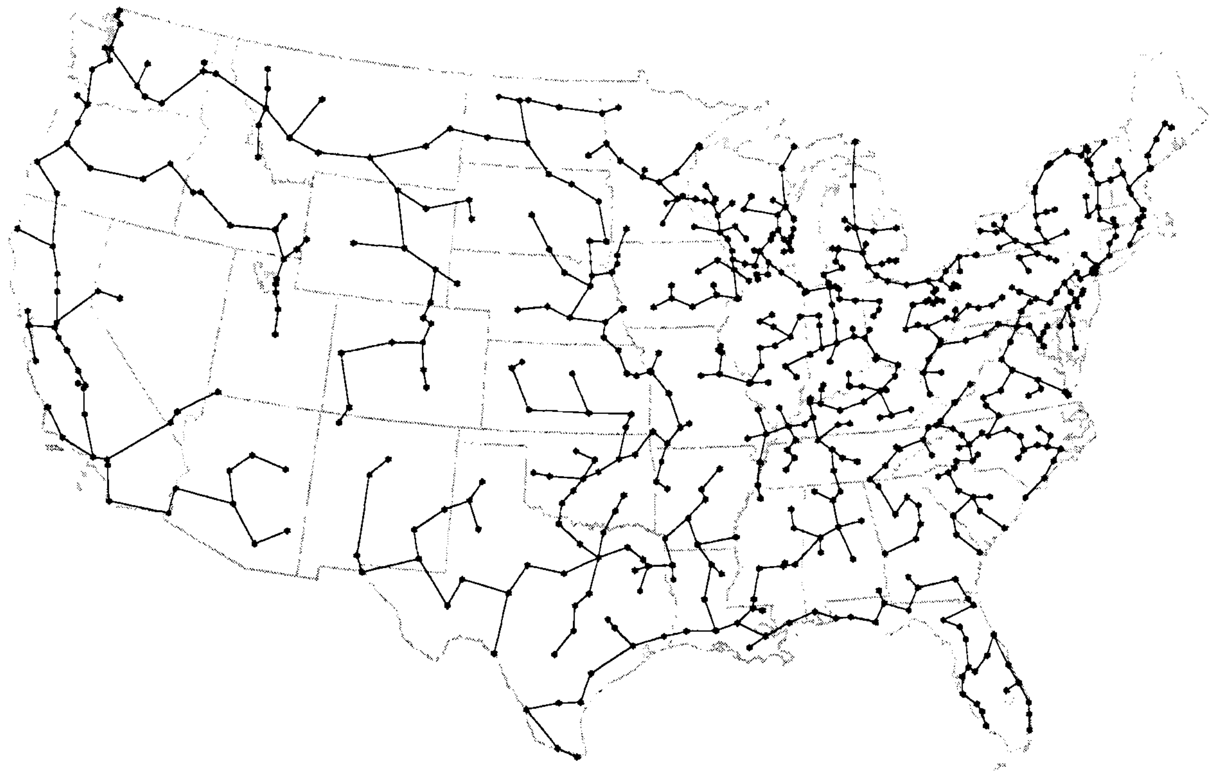

Figure 5. Minimum Spanning Tree Derived from the 622*622 Matrix of Pair–Wise Distances Between Principal Cities

Unfortunately, monthly state–level milk marketings are not reported. The following procedure was employed to estimate such marketings. For the twenty–two states which have historically had the highest milk production and in 1995 accounted for over 80 percent of all milk produced, the USDA's *Milk Production, Disposition, and Income, 1995 Annual Summary* (USDA(n), 1996) was used to estimate the proportion of production which was actually marketed on an annual basis. The particular tables used to calculate this proportion were 'Milk Cows and Production of Milk and Milkfat: By State and United States' and 'Milk Used and Marketed By Producers: By State and United States.' This proportion was then applied to the production figures reported in the USDA's *Milk Production* to yield an estimate of milk marketings for the twenty–two states (USDA(o): 6–95 for May, and 11–95 for October).

For the remaining twenty–six states, quarterly state–level milk production estimates from the 'Milk Cows and Production: By State and United States' tables in the USDA's *Milk Production* (USDA(o): 10–95 for May; and 2–96, 1996 for October) were used to derive estimates of marketings. First, the quarterly production estimates were converted to daily estimates by simply dividing by the appropriate number of days. Monthly milk production estimates were then derived from the daily figures. The proportion of production actually marketed was computed in the same way as was described above for the top twenty–two states. Again, this annual proportion was applied to the derived monthly production estimates to yield an estimate of monthly marketings.

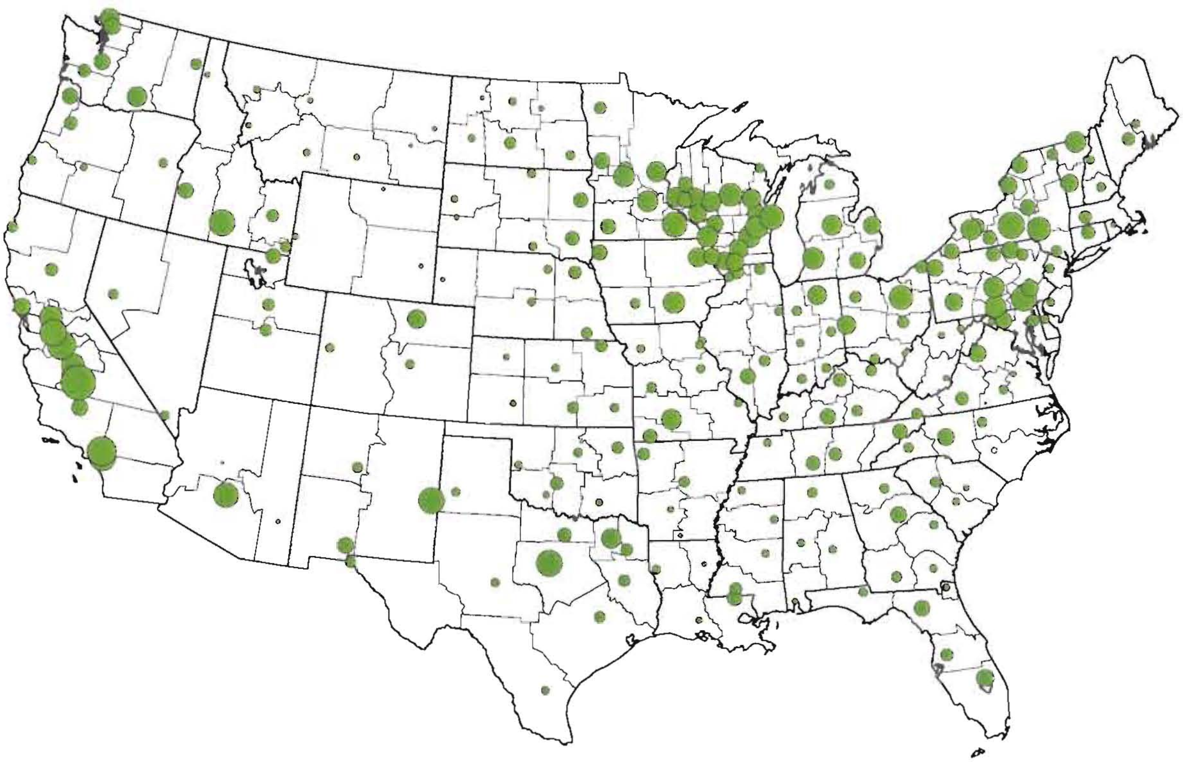

Figure 6. Proportional Representation of Milk Supply at 240 Multiple–County Supply Areas

Finally, for all 48 states, the monthly estimates for milk marketings were distributed among the counties of each state so that they could be aggregated to the 240 supply areas. The index used to do this was constructed using county–level milk production data if available, or estimates of cow numbers for the states where milk production figures were not reported by county. Both the availability and, most importantly, the reliability of data were considered when generating the indices used to distribute state–level milk marketings to individual counties. Data compiled by the National Agricultural Statistics Service (NASS) was the preferred source of county–level milk production estimates. However, not all states report county–level data, and in 1995, those not reporting milk production at the county level numbered twenty–five. For these states, an index was developed using county–level estimates of cow numbers (USDA(m), 1997). Again, NASS estimates were preferred to other sources of data. Not unexpectedly, NASS estimates were unavailable for states with only a modest dairy industry, so for these states, estimates of cow numbers were obtained from the Census of Agriculture (USDC, 1992). For Wisconsin and California, dairy industry summaries published by the states were used to complete the index (CDFA, No. 5 and No. 10, 1995; Wisconsin Agricultural Statistics Service, 1996).

Farm–level milk supplies were assumed to be homogeneous with respect to quality (*e.g.*, somatic cell count). Composition, however, varies regionally. The fat and SNF contents of farm milk were each estimated differently due to the unavailability of the necessary data. The USDA's *Agricultural Prices* (USDA(a), 1996) reports the average monthly fat content for all milk produced in 34 states as well as an annual average figure for the entire U.S. For the 34 states listed, the May and October figures were used. For the remaining states, the monthly data for marketings and the estimate of fat content by state from *Milk Production, Disposition, and Income, 1995 Annual Summary* (USDA(n), 1996) were used to construct an index that was then used to derive estimates for May and October, 1995. The index was constructed in such a way that the resulting monthly U.S. average fat content was the same as that reported in *Agricultural Prices* (3.61 percent in May and 3.72 percent in October).

Although milk composition varies regionally and seasonally in both fat and SNF content, a lack of regional SNF composition data hindered the estimation of raw milk composition parameters. To determine the monthly SNF content by state, a regression equation was developed using observed monthly data for fat and SNF content from the Ohio Valley, Eastern Ohio–Western Pennsylvania, New Mexico–West Texas, and Texas federal orders from 1994 and 1995. The model was specified with SNF as the dependent variable and fat as the independent variable and appeared to fit the data reasonably well (Table 2). The resulting regression coefficients enabled values for SNF content to be established using the equation: SNF% = 6.535 + 0.6031*Fat%.

Table 2. Regression Results for Raw Milk Composition[1]

Dependent variable: SNF, % — Independent variable: Fat, %

	d.f.	S.S	M.S.	F–value	Probability
Regression	1	0.173	0.173	84.770	<0.0001
Residual	46	0.094	0.002		
Total	47	0.267			

	Coefficients	Standard Error	t–Statistic	Probability
Constant	6.535	0.345	18.957	<0.0001
Fat	0.603	0.097	6.244	<0.0001

[1] The R^2 for the model was 0.648.

After the state–level figures for fat and SNF content were compiled, they were assigned to each of the supply areas within each state. Note that the 240 supply areas respect state boundaries. Figure 7 illustrates how the fat content of farm milk varies regionally in the month of May, 1995. While the levels are slightly different in October, the regional variation is similar. A map is not shown for SNF as it is simply a linear transformation of the fat content and, therefore, the regional pattern would be identical.

Finally, the import sector is modeled as if it were made–up of three locations engaged in the supply of milk and the processing of finished products. For the months of May and October 1995, the quantity of fat and SNF contained in all imported dairy products was calculated. This quantity was then made available at the import nodes as if it were contained in milk. In this way, the import sector was able to supply the appropriate quantity of finished product imports. The supply data, including milk components at the import sector, is summarized in Table 3.

Table 3. Summary of Supply Data; May and October, 1995

	1,000 lbs. Farm Milk	Weighted Average Fat %	1,000 lbs. Fat	Weighted Average SNF %	1,000 lbs. SNF
U.S.					
May, 1995	13,650,660	3.605	502,115	8.709	1,188,865
Oct., 1995	12,638,420	3.718	469,940	8.778	1,109,342
Import Sector					
May, 1995			7,571		9,465
Oct., 1995			8,952		9,726

Processing Locations

From the farm milk supply points, raw milk is shipped to plants for further processing. The USDSS can be directed to choose the best locations at which to process each of the product classes given the option of choosing from all 622 principal cities. Alternatively, it can be constrained such that the choice is restricted to those locations at which dairy processing is known to currently occur. Four hundred and thirty–four cities have been identified to represent locations at which dairy processing actually occurs. Specifically, there are 319 fluid processing locations, 147 for soft products, 178 for cheese, 71 for butter, and 60 for dry, condensed, and evaporated products category. Note that these five numbers don't add to 434 because, in many cases, more than one product class may be processed at a particular location. All told, when the model is specified to choose processing locations from the restricted list, there are 775 processing nodes available in the model at 434 distinct geographic locations. Figures 8 through 12 show the locations at which processing can potentially occur when restricted to actual locations.

Processing locations, whether restricted or not, are often referred to as *potential* processing locations because the model is not required to actually use them. Rather, given a set of options, the model chooses where to process and will pick locations such that the sum of all transportation and processing costs is minimized. In other words, it will make efficient choices on the basis of costs. The processing locations actually chosen by the model will represent optimal processing locations. Depending on the locations made available to the model, the resulting optimal plant locations could include places were no dairy processing currently occurs.

The procedure for selecting the 434 cities comprising the restricted plant location list is a little complicated. First of all, the 434 cities do not represent the location of every known dairy processing facility; rather, they represent points to which a number of known facilities have been aggregated. It should also be pointed–out that in the LP specification of the model, it is not possible to allow more than one plant, of a particular type, to be situated at a given location. (The MIP formulation does allow this and is one of its appealing attributes.) In other words, at the very least, aggregation of all plants of the same type known to exist within a single city is unavoidable unless some of the 622 cities were duplicated thereby adding yet more dimensions to the model. The process of aggregating plants first requires that all dairy processing facilities operating in the contiguous 48 states be identified. Unfortunately, such a master plant list does not exist. With assistance from staff at the USDA's Dairy Division and from various state regulatory agencies, a plant list was developed which included 1,392 individual dairy processing facilities, their geographic locations, and the principal products processed at each. Many, but not all producer–dealers were included as fluid processing locations. Only those locations with actual processing activity in 1996 were considered.

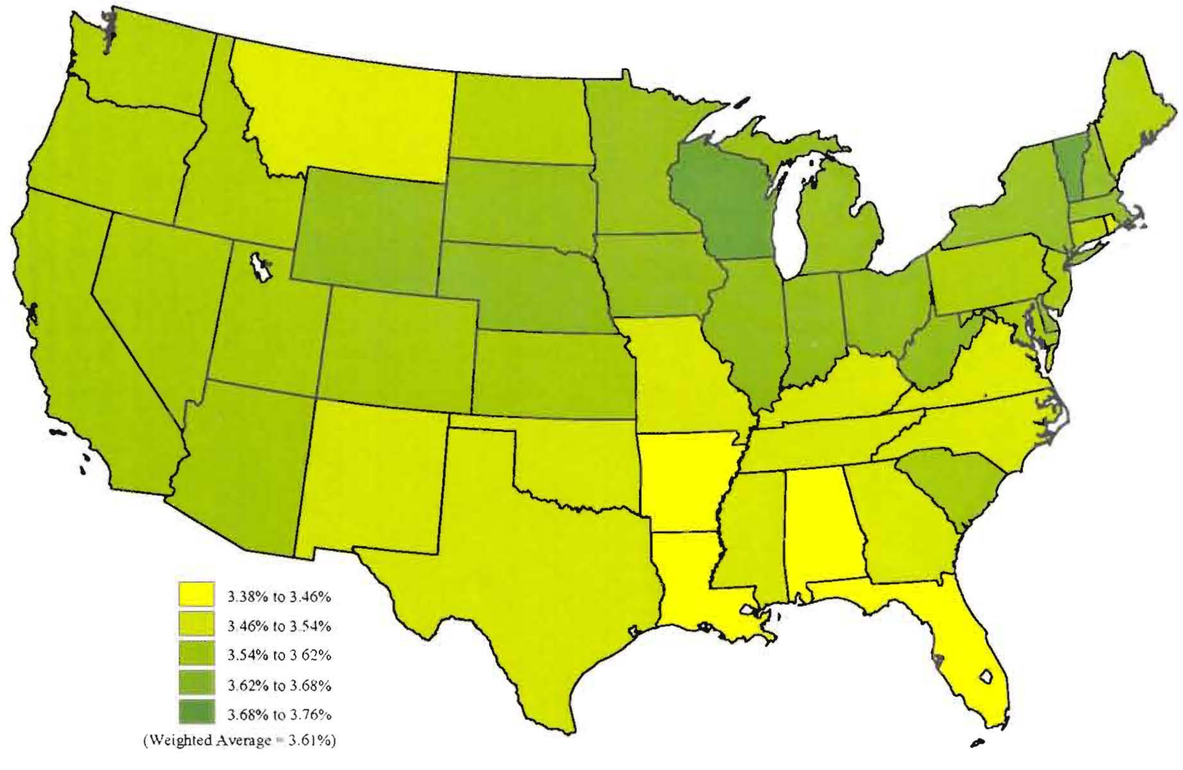

Figure 7. Percent Fat in Farm Milk by State; May 1995

We began with the 367 cities which had been identified as aggregated processing locations in the USDSS prior to the present revision of the model and its data. Added to this list were the 240 cities representing supply points (described earlier) and the 334 cities representing consumption points (to be described shortly). Additional cities known to be places where new plants have been constructed since the USDSS was last revised were also included, Roswell, NM for example. All told, a list of 622 cities was compiled (refer to Figure 5 for a map of the 622 principal city locations). Each of the 1,392 known processing facilities was then assigned to the closest city available from the list of the 622 principal cities included in the USDSS, yielding the final tally of 434 cities in the model representing processing locations.

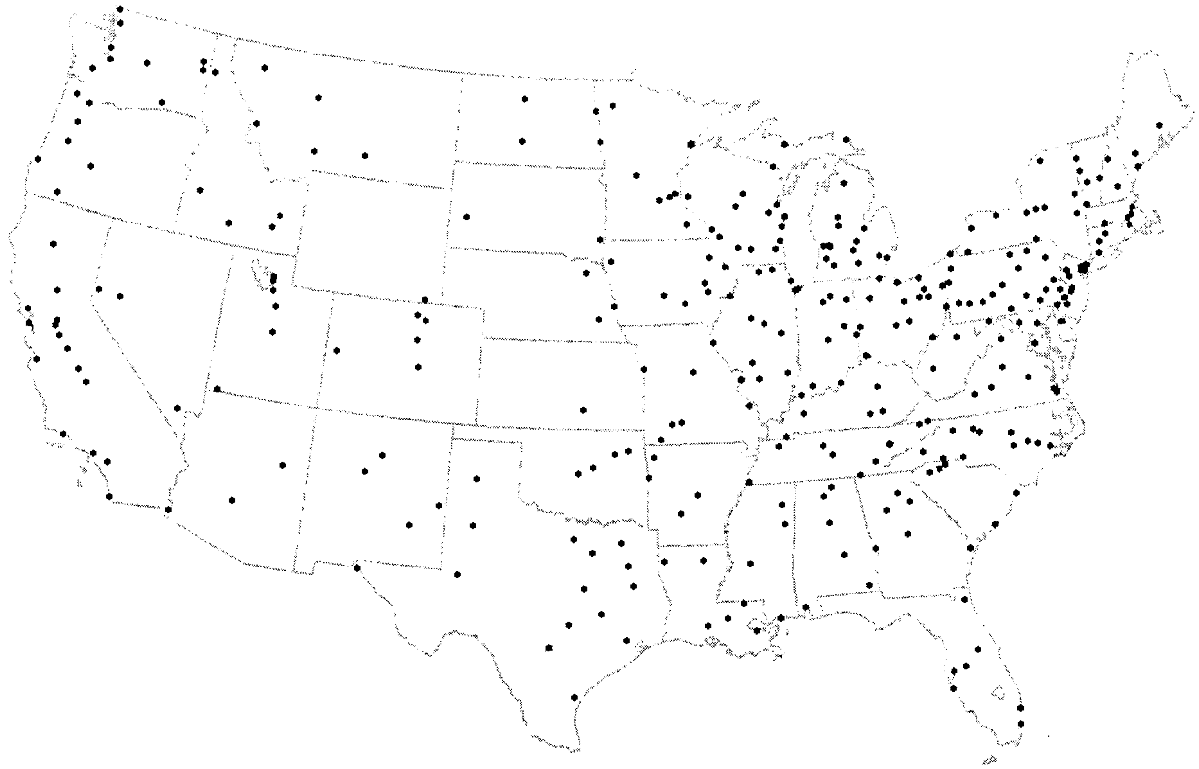

Figure 8. 319 Potential Fluid Milk Processing Sites

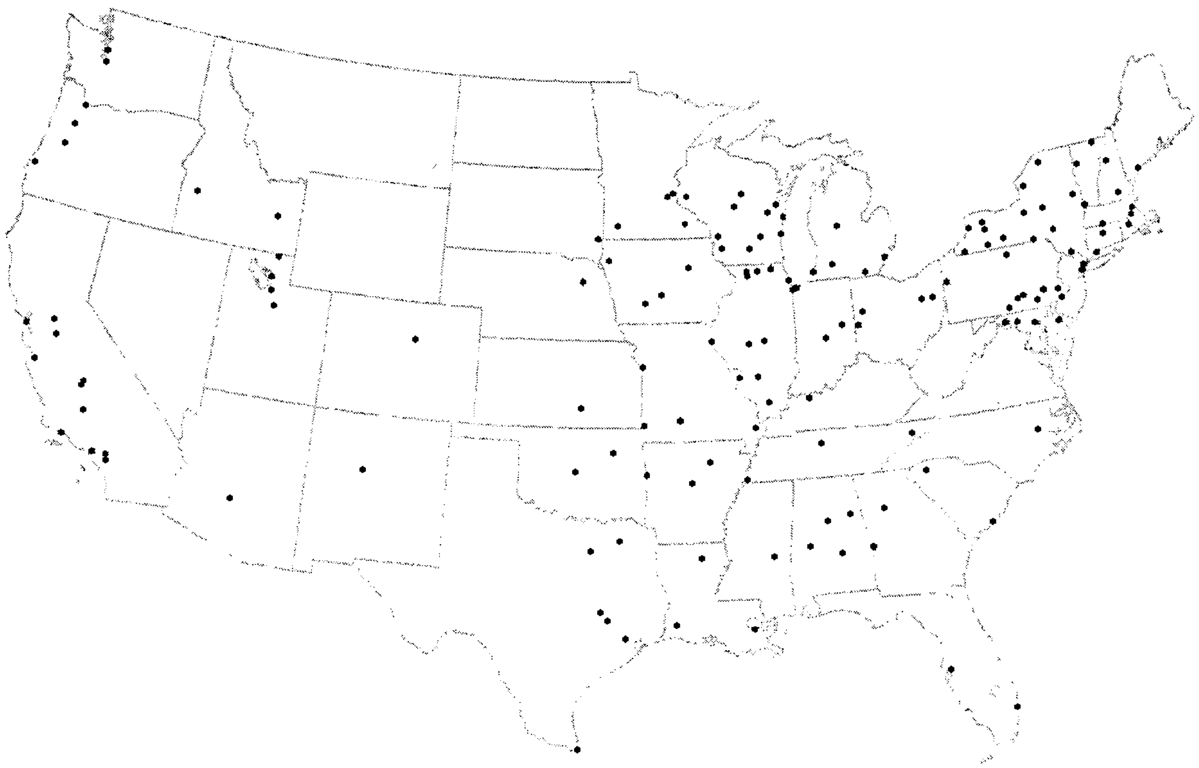

Figure 9. 147 Potential Soft Products Processing Sites

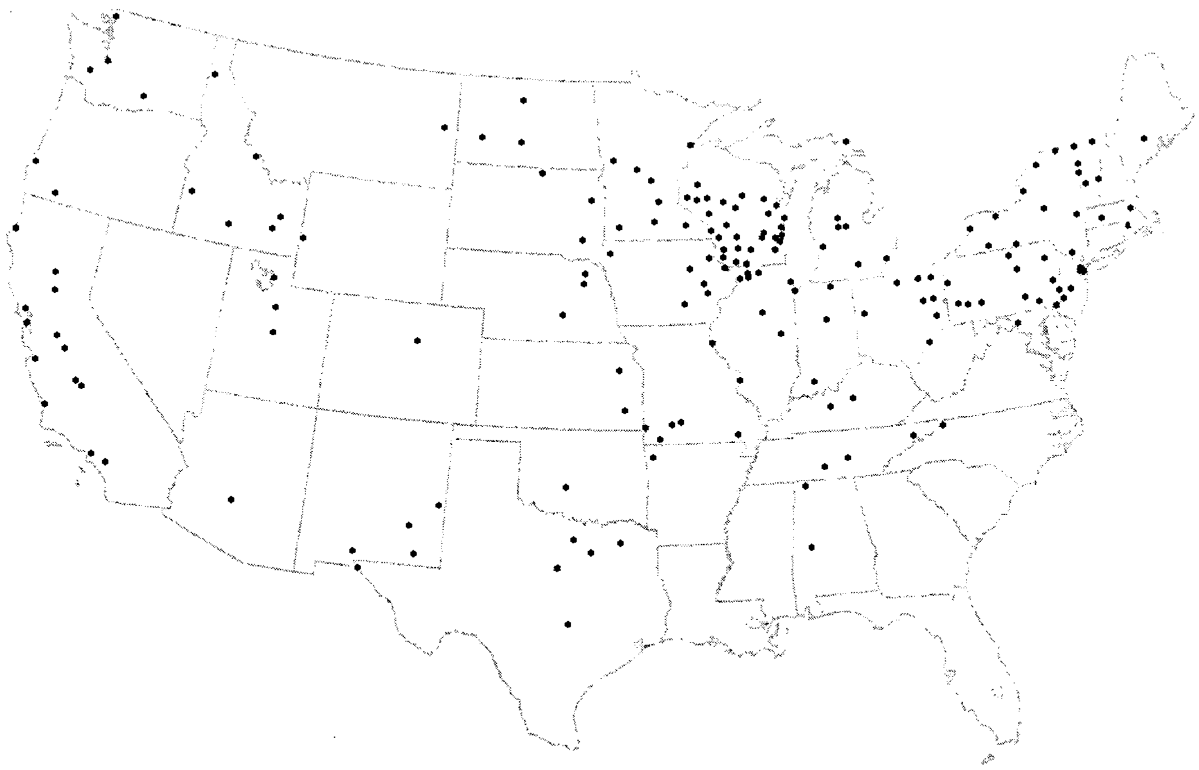

Figure 10. 178 Potential Cheese Processing Sites

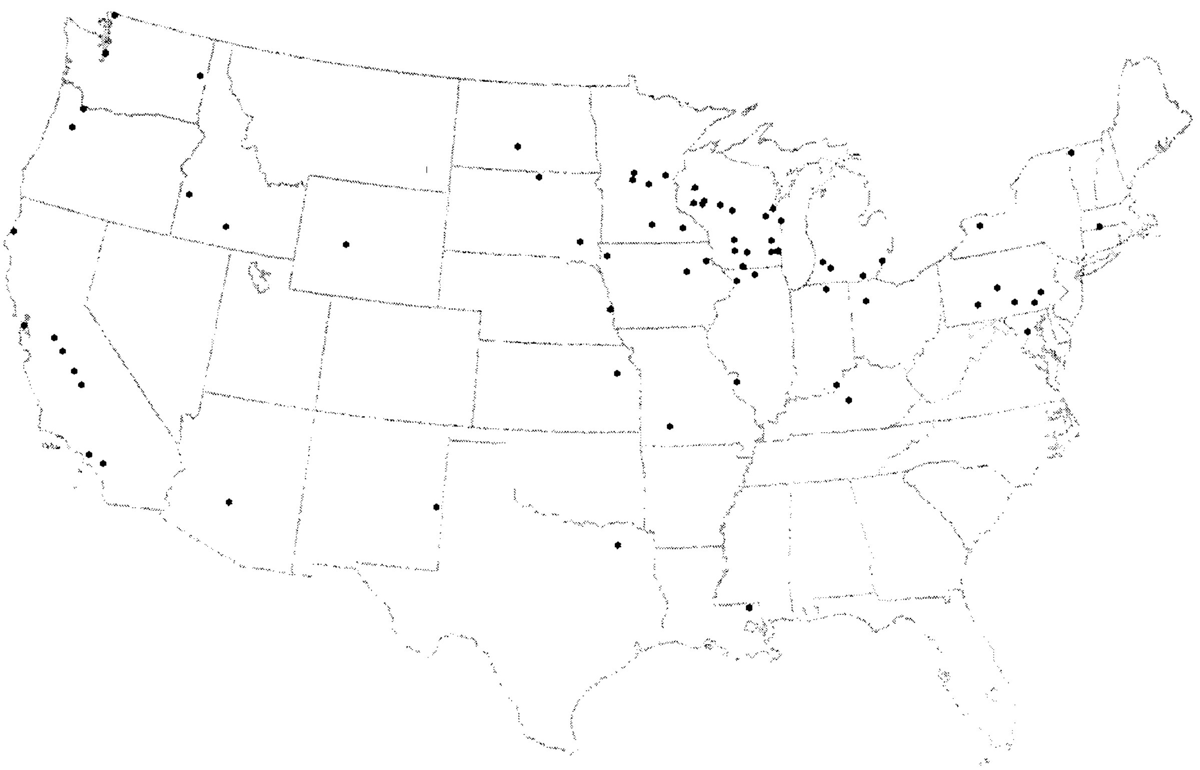

Figure 11. 71 Potential Butter Processing Sites

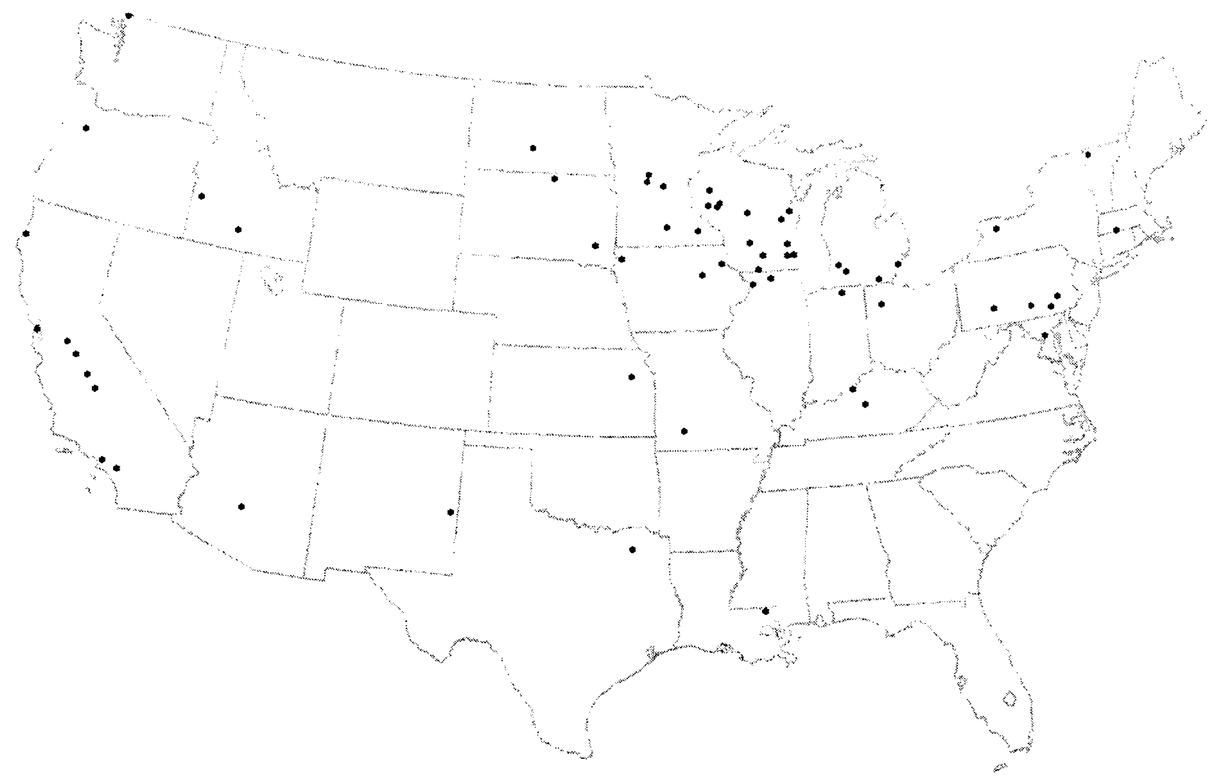

Figure 12. 60 Potential Dry, Condensed, and Evaporated Products Processing Sites

The determination of the closest city is now described. The so–called 'great circle distance' was computed from each of the 1,392 actual processing locations to each of the 622 cities. This distance describes the length of the arc between two points contained in the geodesic curve around a spheroid. In our example, the two points of interest are city locations and the spheroid is the earth. Pearson (1977), describes the trigonometric formulae necessary to accomplish this task. The procedure first requires that the spherical coordinates, *i.e.* degrees latitude and longitude, of all the cities be converted into Cartesian coordinates. Letting i denote one of the 1,392 known processing locations and j the 622 principal cities in the USDSS, the required distance in miles can be computed as follows:

$$MILES_{i,j} = 2 * \text{Arc Tangent}\left(\frac{SEG}{0.5 * \sqrt{1 - \left(\frac{SEG}{2}\right)^2}}\right) * 3{,}959$$

where:

SEG = the length, in miles, of the straight line segment, or chord, between the two points and is defined as:

$$SEG = \sqrt{(X_i - X_j)^2 + (Y_i - Y_j)^2 + (Z_i - Z_j)^2}$$

where:

X = Cosine(LAT)*Cosine(LON),
Y = Sine(LON)*Cosine(LAT),
Z = Sine(LAT),
LAT = latitude expressed in radians,
LON = longitude in radians, and

3,959 = the radius of the earth measured in miles.

Intermediate Products; Description and Composition

One of the important features of the USDSS is its ability to allow plants to ship intermediate products to other plants. This aspect of the dairy sector is one which is frequently ignored by many model builders because of the difficulties associated with its inclusion, even though its importance is readily conceded. For example, in 1994 some 60 percent of the nonfat dry milk produced in the U.S. was subsequently used in the manufacturing of another dairy product. A model which treats this as 'consumption' is almost certain to lead to erroneous conclusions. Interplant shipments of intermediate products are defined in the USDSS on the basis of the type of plant the product is shipped from, the type of plant it is shipped to, and the type of intermediate product. As such, nine transfer types are permitted and they are summarized in Table 4. Note that the composition of intermediate products was assumed to be the same in May, 1995 as it was in October of 1995. Note too that it is not necessary to specify the quantities of intermediate products that are to be shipped between plants. The USDSS will make that choice.

Table 4. Intermediate Dairy Products and Allowable Interplant Transfers

Product Type	Fat Content, %	SNF Content, %	From Plant[1]	To Plant[1]
Cream	40.0	5.4	Fluid	Soft
Cream	40.0	5.4	Fluid	Butter
Cream	40.0	5.4	DCE	Soft
Cream	40.0	5.4	DCE	Butter
Skim	0.0	9.0	Butter	DCE
NDM	0.0	96.0	DCE	Soft
NDM	0.0	96.0	DCE	Cheese
Ice cream mix	13.2	9.95	Fluid	Soft
Ice cream mix	13.2	9.95	DCE	Soft

[1] DCE = dry, condensed, and evaporated dairy products.

When defining intermediate products on the basis of composition, there are literally hundreds of different product types being moved between plants on a daily basis. The nine types listed here symbolize a representative subset and were selected after consultations with a variety of plant operators. They were chosen to represent common practices only rather than every feasible/conceivable transfer type. It is a straightforward procedure, however, to configure the USDSS to allow additional interplant possibilities.

As it happens, the composition of intermediate products is, at least partially, a conditional function of the raw milk from which they are derived. The conditional aspect recognizes both the type of final products the particular plant is producing, and the composition of the raw milk which it receives. For example, consider a fluid plant which receives raw milk at some fixed composition and only has the capability to produce a range of packaged milks and cream, which it ships out as an intermediate product. Such a plant is, by definition, operating under certain constraints. Minimum and/or maximum composition specifications must be adhered to with respect to the range of packaged milks it produces. The only options available to the plant to ensure that no components are left unused is to (a) adjust the final product mix, (b) exceed minimum composition requirements even though this has a cost associated with it, (c) produce cream with a composition such that all of the residual components are exhausted, or (d) do some combination of all three. The plant operator lacks the flexibility to choose all of these settings simultaneously; one or more must be free to adjust and attain a level that might appear to be an inefficient use of resources. The trick, of course, is for the plant to choose a combination of options which maximizes profits while using all resources. Actually, in this particular situation, the plant operator is really engaged in a cost minimization exercise rather than one of profit maximization. Specifying a mathematical programming problem in this manner would require that nonlinear constraints be employed and, of course, this can not be accommodated in an LP setting.

The assumptions and specific computations which gave rise to the above composition parameters are as follows. First, the weighted average raw milk composition parameters (see Table 3) for May and October were themselves averaged, yielding a fat content of 3.699 percent and an SNF content of 8.657 percent. From this, assuming that the skim portion of raw milk contains nonfat solids in the same proportion as the whole milk, the solids content of skim milk is estimated to be 8.657/(1-0.03699) = 9.0 percent. On the basis of conversations with plant operators, it was determined that cream at 40 percent fat was a reasonable representation of the cream generally shipped between plants. Given that cream is essentially just fat and skim milk, the SNF content in 40 percent fat cream was estimated to be 9.0*(1-0.40) = 5.4 percent. A standard specification for NDM is 96 percent nonfat solids. Finally, the composition of ice cream mix was calculated as a weighted average of the standard composition parameters for super–premium, premium, and generic mixes.

Consumption

Consumption in the USDSS refers to final demand for dairy products. It includes not only dairy products destined for human or animal consumption, but dairy products used in other food or feed manufacturing processes as well. In other words, everything leaving the 'dairy market channel,' whether it's sold in a retail store or not, is considered consumption. As such, it is a notoriously difficult statistic to estimate. For all product categories except fluid, the best statistical information available relates to production. Therefore for these categories we estimate (domestic) consumption as a residual by modifying the production figures according to the fundamental relationship:

Domestic Consumption = Production + Imports - Exports + Stock Withdrawals - StockAccumulations - Products Used in the Production of Other Dairy Products.

Estimates of consumption of fluid milk are derived from data on sales by regulated plants.

Consumption is represented in the model by 336 geographic points at which each of five product classes are consumed. Recall from the earlier discussion that one of these 336 nodes is the export transshipment point and another is the point at which stocks may accumulate. These latter two special types of consumption node do not demand all five product types—they demand cheese, butter, and dry, condensed, and evaporated products only. The consumption quantity of each of the five dairy product categories along with their associated requirements for fat and SNF content must be satisfied for the model to achieve a feasible solution. Consequently, consumption of each product category is characterized by a quantity, a fat percentage, and an SNF percentage. Consumption estimates for the months of May and October, 1995, were calculated using data from federal and state publications. Each product category is comprised of a number of products which are listed in Table 5.

Table 5. Dairy Products Contained in Five Final Dairy Product Categories

Category	Dairy products included in each category
Fluid	Packaged whole milk; 2% lowfat milk; 1% lowfat milk; skim milk; light, medium, and heavy creams; half and half; and sour cream.
Soft Products	Ice cream, frozen yogurt, yogurt, cottage cheese, ice milk, and sherbet.
Cheese	Cheddar, other American, part skim, Swiss, Muenster, brick, Limburger, mozzarella, provolone, romano, parmesan, ricotta, blue, cream, and Neufchatel.
Butter	Butter.
Dry, Condensed, and Evaporated	Evaporated whole milk, condensed whole milk, evaporated skim milk, condensed skim milk, evaporated buttermilk, condensed buttermilk, nonfat dry milk, dry whole milk, dry buttermilk.

Consumption Areas

Each of the 3,111 counties and independent cities in the continental U.S. was aggregated into one of 334 multiple–county consumption areas. County–level population estimates for

1995 were used to calculate the total population of each consumption area and within each consumption area, a single city was chosen at the population centroid to represent the demand of the entire area. The method used to aggregate counties was based primarily on information pertaining to metropolitan statistical areas (MSA). Figure 13 demonstrates the 1995 population density on a county basis. Immediately evident from this figure is the realization that population, and therefore consumption, is not uniformly distributed across the country. Although it seems rather obvious, ensuring that dairy products are made available at the places where they're consumed is an element of dairy markets which is often neglected in dairy sector models. This is particularly true, although no less important, when the focus of the investigation is on farm–level issues.

A metropolitan area (MA) is a core area containing a large population nucleus and includes adjacent communities that have a high degree of economic and social integration with that core. The United States Office of Management and Budget (OMB) defines MAs according to published standards which are applied to data collected by the Census Bureau. Standard definitions of metropolitan areas were first issued in 1949 by the Bureau of the Budget (predecessor of OMB) under the designation 'standard metropolitan area.' The current MAs are defined according to the 1990 guidelines to Census Bureau population estimates.

The general category of MAs includes additional and more specific delineation with the most fundamental component being the metropolitan statistical area. The current guidelines specify that each qualifying MSA must include at least one city with 50,000 or more inhabitants, or a Census Bureau–defined urbanized area of at least 50,000 inhabitants and a total metropolitan population of at least 100,000 (75,000 in New England). To complete an MSA designation, the OMB guidelines stipulate that the county (or counties) that contain(s) the largest city becomes the 'central county' (or counties), along with any adjacent counties that have at least 50 percent of their population in the urbanized area surrounding the largest city. Additional 'outlying counties' are included in the MSA if they satisfy specified requirements of metropolitan character such as the degree of commuting to the central counties which takes place or requirements related to population density. In New England, MSAs are defined in terms of cities and towns rather than counties. The use of cities and towns to define MSAs in New England was instituted to preserve the traditional importance of cities and towns that exists in this part of the country. However, the OMB does define New England County Metropolitan Areas (NECMAs) as a county–based alternative to the city– and town–based New England MSAs.

An area that meets the requirements for recognition as an MSA may be further recognized as a consolidated metropolitan statistical area (CMSA) if it has a population of one million or more, and separate component areas can be identified within the entire area by meeting statistical criteria specified in the guidelines. These component areas are designated as primary metropolitan statistical areas (PMSAs).

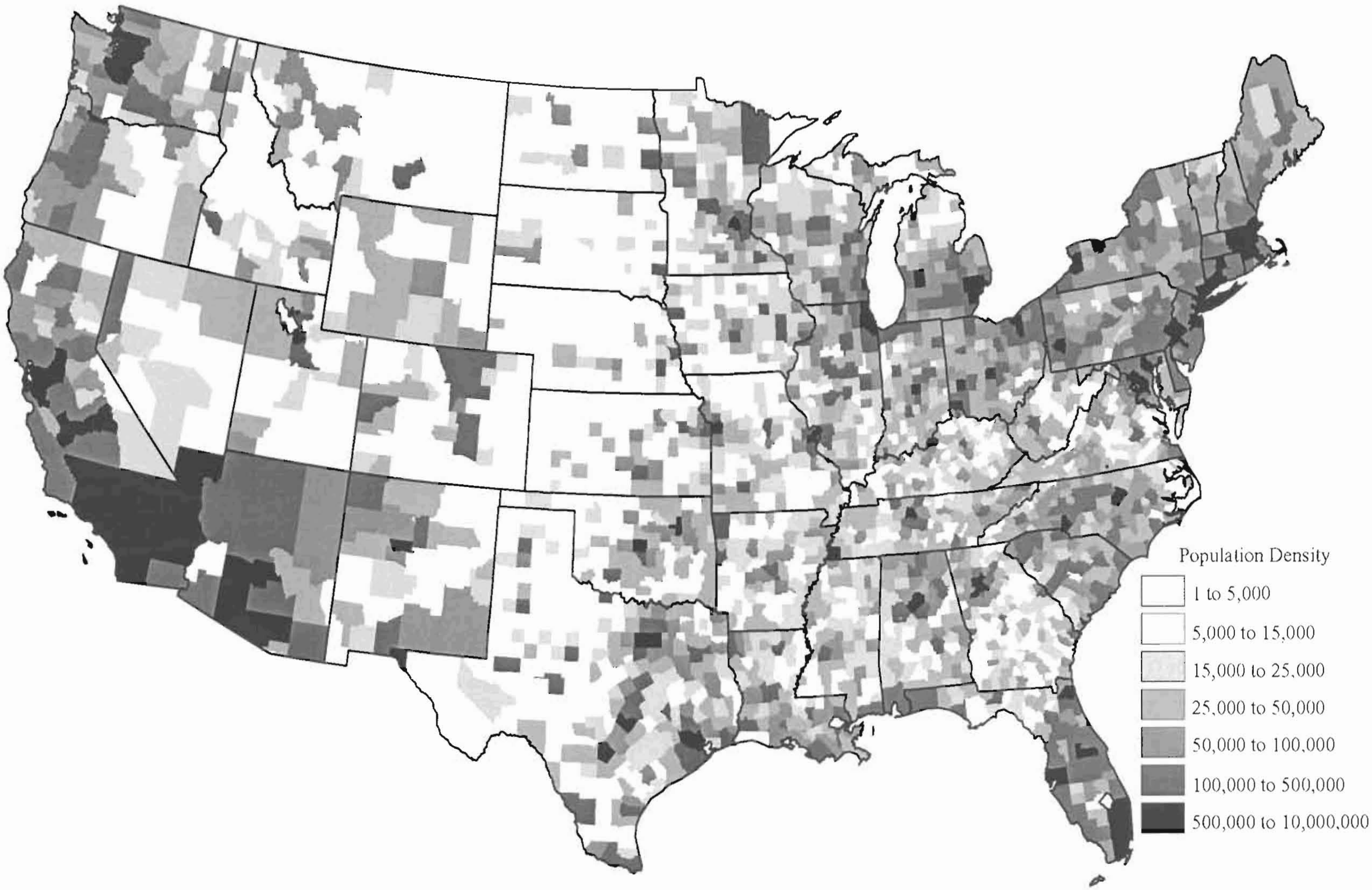

Figure 13. 1995 Population Density by County

Changes in the definitions of MAs since the 1950 census have consisted chiefly of the recognition of new areas as they attain the minimum required area population and the addition of counties to existing MAs as census population estimates verify their qualification. The dynamic nature of population has also led to changes in definitions of MAs. For example, formerly separate MAs have been merged, components of one MA have been transferred to another MA, and components have been entirely removed from MAs. As of June, 1996, the OMB recognized 255 MSAs, 18 CMSAs (comprised of 73 PMSAs), and 12 NECMAs.

In the process of defining the consumption areas for the USDSS, two deviations from the published guidelines were instituted. First, CMSAs were subdivided into smaller areas consisting of one or more PMSAs to allow for further disaggregation. This point is illustrated with the following example. The Cincinnati, OH CMSA consists of 3 consolidated PMSAs—Cincinnati, Hamilton–Middletown, and Clarksville–Hopkinsville. However, each of these PMSAs was considered a separate entity when defining the consumption areas. Second, aggregated areas were defined such that they did not cross state boundaries and occasionally this required splitting–up OMB–defined CMSAs. For example, the OMB definition for the Cincinnati CMSA includes 5 counties from Ohio, 2 counties from Indiana, and 7 counties from Kentucky. To preserve state boundaries while conforming substantially to MSA guidelines, the counties located in Ohio remained associated with Cincinnati, those in Kentucky were assigned to Covington, and those in Indiana were assigned to Evansville. In total, 290 MAs were used as the initial basis from which to determine the multiple–county consumption areas.

Unfortunately, the definition of an MA is such that not all parts of the country contain MAs, and, moreover, MAs are not uniformly distributed across the U.S. For example, the Great Plains region contains only a few MSAs yet the Great Plains land mass accounts for about one–third of the U.S. On the other hand, California contains several MAs, and combined they cover about two–thirds of the state. To account for each of the 3,111 counties and independent cities within the USDSS, counties not directly defined as part of an MA were associated with the nearest MA, or in regions where few MAs were present, groups of counties were aggregated without using an MA as the focal point. This procedure avoided collecting many counties around a single distant MA which would have given rise to a few very large aggregated areas. In total, 334 consumption areas were identified and a single city was chosen to represent each one. Figure 14 shows the 334 consumption areas as well as a proportional representation of the total consumption of dairy products associated with each area located at the representative city.

The two largest squares in Figure 14 are clearly those associated with Los Angeles and New York. Apart from the influence of some regional adjustments, which will be discussed shortly, consumption in the USDSS follows the distribution of population. The consumption area associated with the city of Los Angeles contains 15,362,165 people while the New York consumption area contains 11,229,688 people. In contrast, the marketing area in the southwest corner of North Dakota which is centered on the city of Dickinson contains just 39,006 people and is the least populated area in the model. The square located at Chicago represents 7,977,175 people and the one at Miami represents 3,443,507 people.

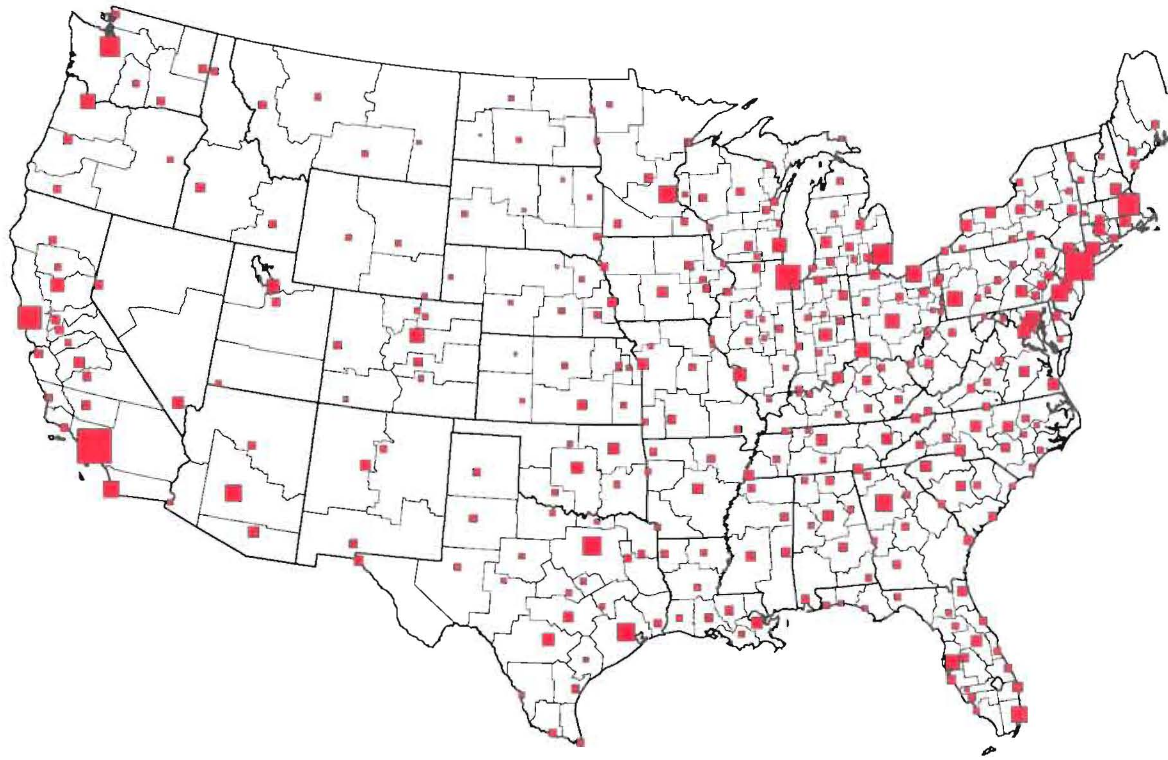

Figure 14. Proportional Representation of Final Product Consumption at 334 Multiple–County Consumption Areas

Consumption of Final Dairy Products

The ultimate goal when assembling the consumption data was to obtain an estimate of consumption for each of the 334 consumption areas as well as for the two nodes representing exports and changes in stocks. At first glance, the process appears relatively uncomplicated, *i.e.*, compute domestic consumption using the fundamental relationship stated earlier, divide total consumption by the total U.S. population to get a per capita figure, and then multiply the per capita figure by the population associated with each of the 334 consumption areas. However, as is so often the case with empirical model building, it's not that simple. In the process of adjusting the production figures to arrive at the residual consumption estimate, several key assumptions and adjustments had to be made. These are now explained before proceeding to describe the consumption data.

First of all, the figure we refer to as consumption should more correctly be termed disappearance. Nobody knows that it is actually consumed, only that it 'disappears.' Nevertheless, we use the term consumption. Second, in the context of the USDSS, what we're really interested in estimating is not even the quantity typically thought of as disappearance. Although the distinction is subtle, what we're actually estimating is consumption, or disappearance, of those milk components which we allow to enter the system within the time period covered by the data. The importance of this point will become clear momentarily.

In the past, the USDSS has been operated with either annual or quarterly data. For the present analysis, it was determined that monthly data would be appropriate and the months of May and October, 1995 were selected. These two months represent the times of the year when dairy markets are, generally, long and short, respectively. A consequence of using monthly data is that the manner in which data pertaining to stocks and trade are treated becomes critical. For example, when product is withdrawn from stocks in the current month, is it also consumed in the current month? Was it produced in the current month or did it enter storage in some preceding month? Maybe it is used to make dairy products which are consumed in some later month? Or maybe the product is withdrawn from stocks to be immediately exported. Given that we're interested in estimating uses of components that were introduced into the dairy marketing system in the current month, do we want to even consider stock withdrawals at all? Such issues become moot as the time period under consideration gets longer. The following list of assumptions were employed:

(a) Production in the current month is from raw milk produced in the current month.
(b) Current month imports are consumed in the current month. They do not become stocks and they are not available as inputs to the processing sector.
(c) Products withdrawn from stocks in the current month were produced in some preceding month, *i.e.* they are not a use of raw milk produced in the current month.
(d) Exports in the current month are either produced in the current month or come out of stocks. The latter implies they were produced in some preceding month.
(e) Stock accumulations in the current month are manufactured from raw milk produced in the current month and are neither consumed, nor exported, nor withdrawn from stocks in the current month.
(f) Stock withdrawals in the current month are either consumed domestically or are exported in the current month. More specifically, we assume that they are used to satisfy export requirements first with any remaining quantities of stock withdrawals being used to satisfy current domestic consumption needs.

Consistent with these assumptions, the fundamental relationship stated earlier can now be modified to yield a rule determining how domestic consumption is to be calculated. The rule also yields a mechanism for determining the figures we enter in the model to represent exports and changes in stocks. First, allow changes in stocks to be defined as:

$$\Delta\text{Stocks} = \text{Stocks Accumulated} - \text{Stocks Withdrawn}$$

or equivalently, the change in stocks in the current month is equal to beginning inventories minus ending inventories. Then, execute the following algorithm making sure to perform each step in the precise order in which it is listed.

START.

If ΔStocks ≥ 0 go to step (i), else jump to step (v),

(i) set Exports = Exports,

(ii) set ΔStocks = ΔStocks,

(iii) set Domestic Consumption = Production + Imports - Exports - ΔStocks - Products Used in the Production of Other Dairy Products,

(iv) set Aggregate Consumption = Domestic Consumption + Exports + ΔStocks,

END.

(v) set Exports = Max[0, Exports + ΔStocks],

(vi) set ΔStocks = 0,

(vii) set Domestic Consumption = Production + Imports - Exports - ΔStocks - Products Used in the Production of Other Dairy Products,

(viii) set Aggregate Consumption = Domestic Consumption + Exports + ΔStocks,

END.

The need to adjust for products used in the production of other classes of products arises only in the case of the dry, condensed, and evaporated products category. The adjustment is necessary to avoid the possibility of double–counting and recognizes the difference between demand for these products by non–dairy uses and the 'demand' for dairy uses. Strictly speaking, the products used by non–dairy processors would ordinarily be classified as intermediate products. However, in the USDSS, the term intermediate products refers only to those products used in a dairy–related manufacturing process. All other uses of dairy products must be considered consumption, that is, they leave the dairy market channel.

Dry, condensed, and evaporated dairy products are often used by manufacturers of other dairy products to increase the fat or SNF content during the manufacturing process. The American Dairy Products Institute (ADPI) conducts surveys of plants and reports the results as estimates of the disposition of manufactured dairy products (ADPI, 1995; ADPI, 1996). Table 6 describes the proportion of dry, condensed, and evaporated products used by dairy processors in 1994 and 1995 according to the American Dairy Products Institute. Our estimates of production for each of the products listed in Table 6 were multiplied by the corresponding percentage to arrive at a figure representing products used in the production of other dairy products. When the data for the USDSS was being put together, ADPI's 1995 results were not available so the 1994 figures were used.

Finally, our domestic per capita consumption estimates were adjusted to reflect regional differences in dietary habits. The regional adjustment was based on the daily consumption habits in the four regions described in *Food and Nutrient Intakes by Individuals in the United States, 1 Day* (Tippett *et al.*, 1995). Note that the fluid products category was not regionally adjusted because such data was not constructed from national data. Table 7 describes the states and accompanying population contained in each region.

Table 6. Proportion of Dry, Condensed and Evaporated Products Used by Dairy Processors

Product	Percent	
	1994	1995
Nonfat dry milk	68	64
Dry whole milk	2	1
Dry buttermilk	34	32
Condensed skim solids	72	59
Condensed whole solids	35	49
Condensed buttermilk solids	88	79

Table 7. Description and Population of Four Regions of the U.S.

	Northeast	Southern	Midwest	West
	Connecticut	Alabama	Illinois	Arizona
	Maine	Arkansas	Indiana	California
	Massachusetts	Delaware	Iowa	Colorado
	New Hampshire	D.C.	Kansas	Idaho
	New Jersey	Florida	Michigan	Montana
	New York	Georgia	Minnesota	Nevada
	Pennsylvania	Kentucky	Missouri	New Mexico
	Rhode Island	Louisiana	Nebraska	Oregon
	Vermont	Maryland	North Dakota	Utah
		Mississippi	Ohio	Washington
		North Carolina	South Dakota	Wyoming
		Oklahoma	Wisconsin	
		South Carolina		
		Tennessee		
		Texas		
		Virginia		
		West Virginia		
No. of states	9	17	12	11
Population, 1995	51,462,270	91,885,720	61,796,470	55,794,280
Population, %	19.7	35.2	23.7	21.4

A relative index, RI_i, of daily consumption by region was first constructed for each product class. Letting DC_i denote daily consumption in the i^{th} region and POP_i the population associated with the i^{th} region, the relative index in the i^{th} region was computed as:

$$RI_i = DC_i * \left(\frac{POP_i}{\sum_{i=1}^{4} POP_i} \right) \quad \forall i = 1,2,..,4$$

Total consumption on a regional basis, RC_i, was then computed as:

$$RC_i = \text{Aggregate Domestic Consumption} * \left(\frac{RI_i}{\sum_{i=1}^{4} RI_i} \right) \quad \forall i = 1,2,..,4$$

where Aggregate Domestic Consumption is calculated according to the modified fundamental relationship stated above. Regional per capita consumption, RPC_i, is then simply defined as RC_i/POP_i. Consumption at each of the 334 consumption areas is then computed by multiplying the appropriate regional per capita consumption figure by the population associated with each area. Recall that the 334 consumption areas observe state boundaries so it is a simple matter to assign each of the 334 areas to one of the four regions. Clearly, this procedure yields an unchanged aggregate consumption figure but allows the implied per capita consumption figure to vary regionally.

We are now ready to describe and summarize the consumption data for each of the five product categories included in the USDSS.

(i) Fluid Milk Products

Federal and state milk marketing order data from 1995 were used to construct the consumption figures for this class of products. Note that in the case of fluid products, consumption was not computed as a residual using the fundamental relationship outlined above. Instead, sales by plants were used as a proxy for consumption. The various products comprising the fluid products group essentially include all fluid milk and cream products, *i.e.*, packaged whole milk, 2% lowfat milk, 1% lowfat milk, skim milk, buttermilk, light and heavy creams, milk and cream mixtures, eggnog, and sour cream. Exports, imports, and changes in stocks are negligible or non–existent for this category and were therefore not considered.

With the exception of California, sales of fluid products for October and May were obtained from Federal Milk Market Order Statistics. Regional sales data for cream products were obtained from *Dairy Market Statistics, 1995 Annual Summary* (USDA(g), 1996). These cream sales by handlers regulated under FMMOs were then indexed by annual cream products sales to yield individual order area estimates. The sum of packaged fluid milk and cream products sales within each federal order, *i.e.* all products comprising our fluid category, was then divided by the population associated with that order to generate a per capita consumption figure. The FMMO statistics conveniently publish figures for the population reached by regulated sales within each order. The procedure was repeated for California using published state summaries (CDFA, No. 5 and No. 10, 1995) as the data source. Consistent with the available information, California was divided into three marketing areas—northern California, South Valley, and southern California, for the purpose of generating a per capita consumption figure.

Having obtained a per capita estimate of fluid products consumption for the area covered by federal orders and the state of California, *i.e.* a total area covering approximately 230 million people, the task of calculating consumption for each of the 334 consumption areas in the USDSS remained. First, to the extent possible, each of the 334 areas was associated with either a federal marketing order area or one of the marketing areas from California. Each of our consumption areas that lie within the boundaries of an order area were assigned the value associated with that order. The FMMO system does not cover all areas of the U.S., however, so areas outside the FMMO system, or the state of California, were assigned the per capita consumption figure of the nearest marketing area. Once each of the 334 consumption areas was assigned a per capita consumption figure, the per capita figure was multiplied by the population associated with that area to obtain our estimate of aggregate consumption. Table 8 lists the aggregate sales figures for the products comprising our fluid category. Note that sales from plants regulated by federal orders are itemized by individual product. The fat and SNF composition of each of these products is also presented.

Table 8. Sales of Fluid and Cream Products, Quantities and Composition; May and October, 1995

Product	May sales; million lbs.	Oct. sales; million lbs.	May/Oct. Fat %	May/Oct. SNF %
Whole milk	1,075.7	1,016.5	3.27	8.65
Flavored whole milk	53.4	50.4	3.22	8.65
2% lowfat milk, plain	1,215.0	1,148.1	1.97	8.82
2% lowfat milk, milk solids added	100.1	94.6	1.97	9.51
1% lowfat milk, plain	353.5	334.0	0.98	8.88
1% lowfat milk, milk solids added	38.4	36.2	1.00	9.50
Skim milk, plain	513.3	485.1	0.17	8.61
Skim milk, milk solids added	75.7	71.5	0.15	9.64
Flavored lowfat and skim milk	143.5	135.6	1.27	8.88
Buttermilk	41.4	39.1	1.09	8.57
Milk and cream mixtures	50.7	47.9	10.80	8.12
Light cream	7.7	7.2	18.54	6.62
Heavy cream	19.6	18.5	36.28	5.25
Sour cream	50.5	47.7	13.31	8.13
Eggnog	10.0	9.5	6.72	8.12
Sales from federally regulated Plants; million lbs.	3,748.5	3,542.2		
Plus sales from state regulated and unregulated plants; million lbs.	1,161.4	1,321.2		

			Weighted Average % Fat May	Weighted Average % Fat Oct.	Weighted Average % SNF May	Weighted Average % SNF Oct.
All Sales (Consumption); million lbs.	4,909.9	4,863.4	2.67	2.74	8.77	8.76

Figures 15 and 16 provide a look at what all of this means. Figure 15 shows how the fat percent in fluid products varies by consumption region while Figure 16 shows the variation in pounds of fat consumed from fluid products on a per capita basis. Contrast this with Figure 7 which shows how the fat content of raw milk varies by state. Areas with a relatively high raw milk fat content, such as the Midwest, are not necessarily areas with a high fat content in milk consumed. This has obvious implications for the amount of cream that must be shipped from fluid plants in these areas.

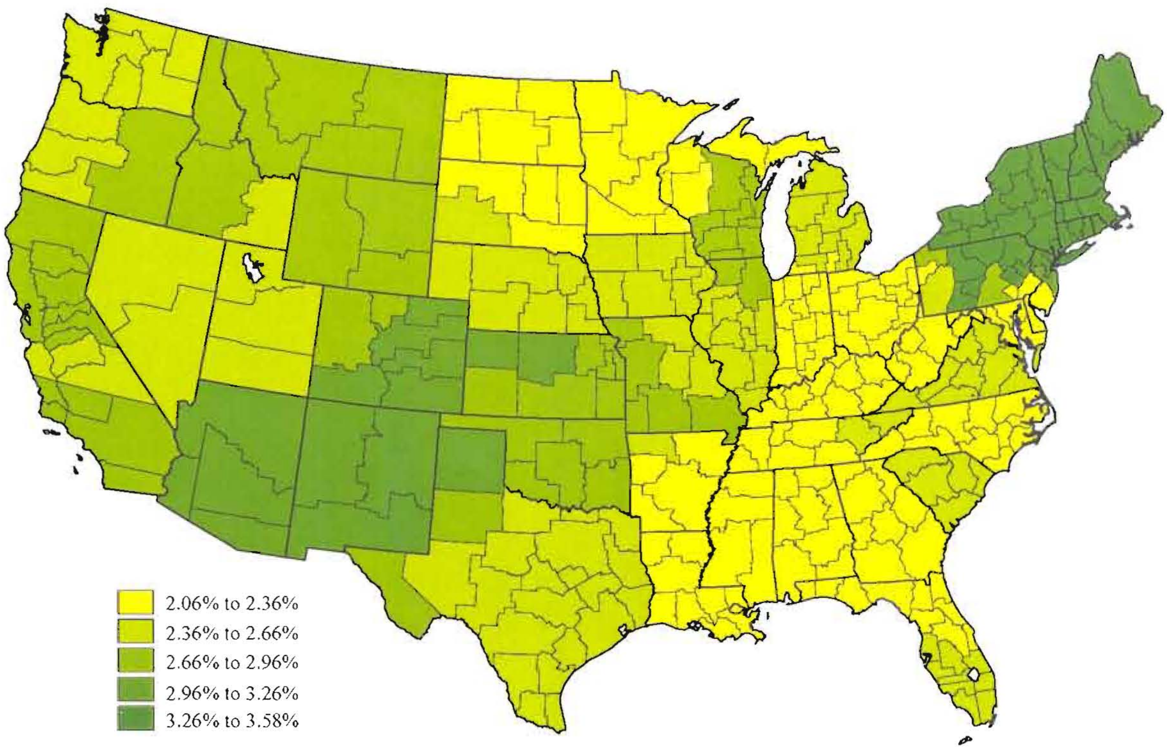

Figure 15. Percent Fat in Fluid Milk Products for 334 Consumption Areas; May 1995

Variations in composition and quantities of fluid milk products consumed were evident among the compiled data. Federal order data from the southeastern U.S. indicated that the population of those areas tended to consume smaller individual portions of fluid dairy products with lower fat contents. Conversely, the population in the Northeast tended to consume average amounts of fluid products, but the fat content was considerably higher than other areas across the U.S.

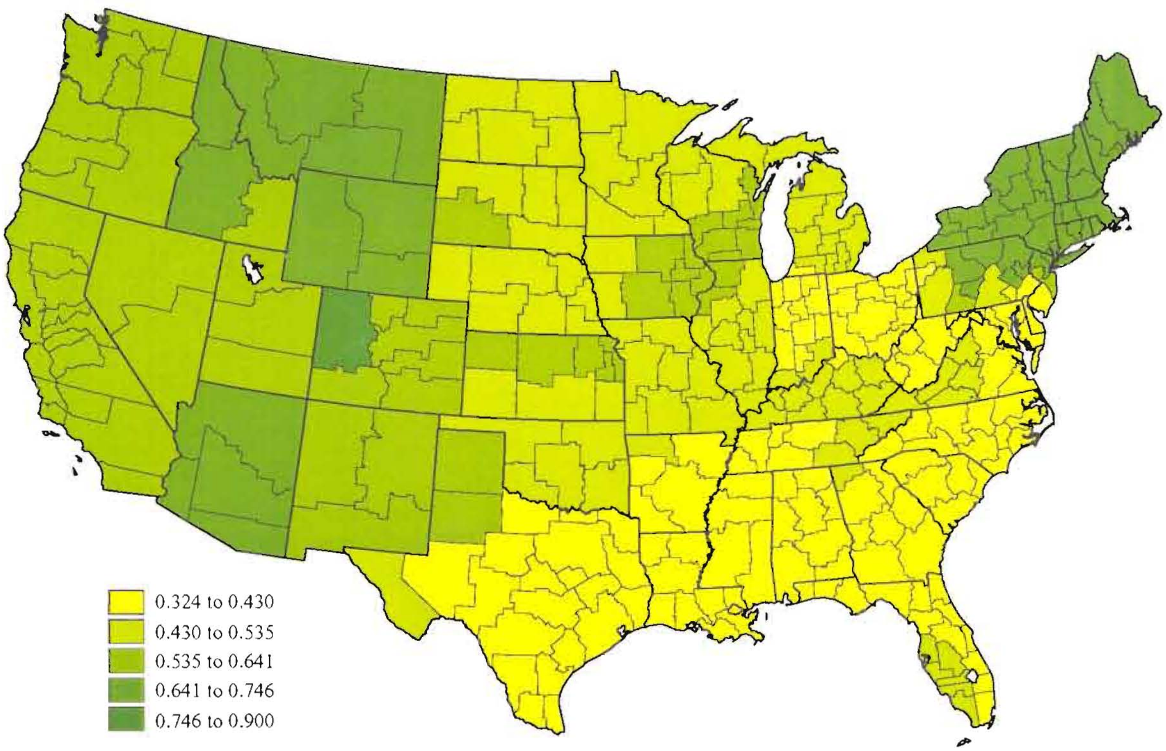

Figure 16. Pounds (per capita) of Fat Consumed from Fluid Milk Products for 334 Consumption Areas; May 1995

(ii) Manufactured Dairy Products

The term 'manufactured dairy products' refers to the four product classes excluding the fluid class. The procedure used to estimate consumption of manufactured dairy products was the same across the four product categories and has been described above. Data on monthly production and changes in stocks was taken from *Dairy Products, 1995 Annual Summary* (USDA(h), 1996) and, for the refrigerated products, from *Cold Storage Report* (USDA(b) and USDA(c), 1995). Import and export quantities were obtained from the series entitled *Dairy, Livestock, and Poultry: U.S. Trade and Prospects* (USDA(e) and USDA(f), 1995). Note that for the soft products category, changes in stocks, imports, and exports were of such small proportions that they were simply ignored. Thus, our estimate of domestic consumption was equated to monthly production.

The next four tables, Tables 9 through 12, summarize the data for the four manufactured product classes. The upper panel of each table lists the production and composition figures for the individual products comprising each product category. The center panel shows total production, imports, exports, changes in stocks, and, in the case of dry, condensed, and evaporated products, the quantity used by other dairy processors. The lower panel of each table shows the result of applying the rule described above to yield the data as it is specified in the model.

Table 9. Soft Products, Quantities and Composition; May and October, 1995

Product	May Production, 1,000 lbs.	October Production, 1,000 lbs.	Fat %	SNF %
Ice cream, hard	402,680	324,960	13.39	8.54
Ice cream, low fat, hard	167,810	135,805	3.47	11.36
Milk sherbet	34,195	23,135	1.98	13.95
Frozen yogurt	66,203	53,454	3.47	11.36
Frozen yogurt, nonfat, hard	13,188	11,339	0.50	14.33
Cottage cheese, creamed	33,601	29,758	4.51	16.53
Cottage cheese, low fat	29,294	25,037	1.94	18.76
Yogurt, plain, low fat	23,760	22,596	1.55	13.38
Yogurt, plain, whole	35,640	33,895	3.25	8.85
Yogurt, with fruit added	59,400	56,491	1.15	13.38
Production; 1,000 lbs.	865,771	716,470		
Imports; 1,000 lbs.	0	0		
Exports; 1,000 lbs.	0	0		
Changes in Stocks; 1,000 lbs.	0	0		
Domestic Consumption; 1,000 lbs.	865,771	716,470		
Exports	0	0		
Changes in Stocks	0	0		
Aggregate Consumption; 1,000 lbs.	865,771	716,470		

Dairy Product Composition

In the past, computational complexities often necessitated the use of homogeneous measurements in multiple–product dairy market models. Such models have typically relied on milk–equivalent representations as a means of reconciling milk production with dairy product consumption. Most milk–equivalent units of measurement are essentially single–component measurements. While the use of milk–equivalents can make a problem more tractable, it imposes unrealistic assumptions on the process of allocating milk to the various products produced from milk (see Bishop *et al., op. cit.*). Furthermore, if the model is to be used to assign values to milk, milk components, or dairy products, then yet another set of difficulties is encountered when milk–equivalents are used. The problem with milk–equivalent units of measurement stems from the basic fact that milk consists of several components, yet the products derived from milk utilize these components in vastly different proportions. The USDSS employs a multiple–component representation and presently uses milkfat and SNF as the principal components to be derived from milk.

For each of the five product groups, it is necessary to specify a fat and SNF percentage for domestic consumption, and, where applicable, for consumption in the form of exports and changes in stocks. It is not necessarily the case that all three consumption types will have the same composition for any given product group. In fact, it is possible that every single consumption node could have different composition parameters associated with it. The availability of data permitted regional variation in composition within the fluid milk category. The national scope of manufactured products markets, however, meant that for these products no regional variation in composition was specified.

The composition parameters are simply weighted averages of the composition of all the products that comprise a particular product group. Thus, the actual parameters in the model do not correspond exactly to any individual product. Table 13 lists the composition parameters as currently specified in the model.

Table 10. Cheese Quantities and Composition; May and October, 1995

Product	May Production, 1,000 lbs.	October Production, 1,000 lbs.	Fat %	SNF %
Cheddar	219,128	190,790	33.14	30.11
Other American	59,156	62,024	28.82	29.01
Part Skim	2,244	863	13.65	27.43
Swiss	22,551	19,883	27.45	35.34
Muenster	8,752	9,346	30.04	28.19
Brick	757	1,173	29.68	29.21
Limburger	72	68	27.10	24.25
Mozzarella	179,080	183,343	23.12	25.62
Provolone	13,679	14,006	26.62	32.43
Romano	4,559	4,669	26.94	42.15
Parmesan	9,337	9,559	27.93	48.67
Ricotta	17,154	17,563	12.98	15.39
Cream & Neufchatel	46,846	62,557	29.15	12.87
Blue	2,670	3,244	29.69	29.42
All Other Types	13,705	14,950	28.36	27.73
Production; 1,000 lbs.	599,693	594,038		
Imports, quota; 1,000 lbs.	16,711	22,192		
Imports, non–quota, 1,000 lbs.	8,675	9,233		
Exports, 1,000 lbs.	4,867	5,580		
Change in Stocks; 1,000 lbs.	-7,814	-9,811		
Domestic Consumption; 1,000 lbs.	625,079	625,463		
Exports; 1,000 lbs.	0	0		
Changes in Stocks; 1,000 lbs.	0	0		
Aggregate Consumption; 1,000 lbs.	625,079	625,463		

Table 11. Butter, Quantities and Composition; May and October, 1995

	May Production 1,000 lbs.	October Production 1,000 lbs.	Fat %	SNF %
Production; 1,000 lbs.	119,435	93,461	81.11	3.02
Production; 1,000 lbs.	119,435	93,461		
Imports; 1,000 lbs.	209	57		
Exports; 1,000 lbs.	16,687	2,870		
Changes in stocks; 1,000 lbs.	2,250	-9,213		
Domestic Consumption; 1,000 lbs.	100,707	93,518		
Exports; 1,000 lbs.	16,687	0		
Changes in Stocks; 1,000 lbs.	2,250	0		
Aggregate Consumption; 1,000 lbs.	119,644	93,518		

Table 12. Dry, Condensed, and Evaporated Products, Quantities and Composition; May and October, 1995

Product	May Production, 1,000 lbs.	October Production, 1,000 lbs.	Fat %	SNF %
Evaporated whole milk	32,435	27,681	7.56	18.40
Condensed whole milk	35,484	30,283	8.30	18.40
Evaporated skim milk	3,571	2,826	0.20	20.40
Condensed skim milk	110,703	87,616	0.20	29.80
Evaporated and condensed buttermilk	3,365	2,663	1.50	20.40
Dry whole milk	13,701	11,052	26.71	70.80
Nonfat dry milk	138,169	76,112	0.77	96.10
Dry buttermilk	5,386	3,811	5.78	91.25
Production; 1,000 lbs.	342,814	242,044		
Imports; 1,000 lbs.	1,599	256		
Exports; 1,000 lbs.	17,955	22,183		
Change in stocks; 1,000 lbs.	304	-6,221		
Other Dairy Uses; 1,000 lbs.	205,885	141,498		
Domestic Consumption; 1,000 lbs.	120,269	84,840		
Exports; 1,000 lbs.	17,955	15,962		
Changes in Stocks; 1,000 lbs.	304	0		
Aggregate Consumption; 1,000 lbs.	138,528	100,802		

Table 13. Final Products Composition Parameters; May and October, 1995

	May		October	
	Fat %	SNF %	Fat %	SNF %
Fluid Products				
Domestic Consumption[1]	2.67	8.77	2.74	8.76
Soft Products				
Domestic Consumption	7.75	10.74	7.61	10.79
Cheese				
Domestic Consumption	28.10	27.67	27.93	27.09
Exports	-	-	-	-
Changes in Stocks	-	-	-	-
Butter				
Domestic Consumption	81.11	3.02	81.11	3.02
Exports	81.11	3.02	-	-
Changes in Stocks	81.11	3.02	-	-
Dry/Condensed/Evaporated				
Domestic Consumption	5.35	52.73	3.59	53.62
Exports	8.07	59.66	20.21	52.02
Changes in Stocks	2.51	49.69	-	-

[1] Weighted average across all 334 domestic consumption nodes.

(i) Components in Fluid Milk Products

The data for calculating the fat content of fluid milk products was obtained from *Federal Milk Market Order Statistics, Annual Summary, 1995* (USDA(i), 1996) and from various issues of monthly federal order statistics (USDA(j), USDA(k), and USDA(l), 1995). In the case of California, it was obtained from published state summaries (CDFA, No. 5 and No. 10, 1995). Except for California, data regarding the SNF composition of fluid milk products, however, was unavailable. Consequently, the California SNF data was supplemented with data obtained from published food standards (Leveille *et al.*, 1983; USDA(d), 1977) to specify the SNF composition parameters. Table 14 shows the estimated average composition of the fluid products category for each of the federal orders and for California. Recall that for this class of products, the consumption data was compiled on an order basis before being allocated to the 334 consumption areas. Within each order grouping, the individual products comprising the fluid category were each assigned the annual average composition for that product. These values were then averaged using the proportion of each product within the class as weights to yield the data in Table 14.

(ii) Components in Manufactured Products

Secondary sources were used to determine the composition of manufactured products (Selinsky *et al.*, 1992; Leveille *et al.*, 1983; and USDA(d), 1977). Calculating the content of dairy components in the four manufactured products categories entailed computing a weighted average of the percent fat and the percent SNF in the various products comprising each category. With the exception of the butter category, each manufactured product category may consist of several individual products (see Tables 9–12). Due to the assumptions listed above and their implication for the way in which domestic consumption was computed, it was important to keep track of exactly which products were consumed, and from where they originated, when calculating the weighted averages. In other words, care had to be exercised to ensure that the appropriate weighting scheme was employed. For example, the varieties of cheese which are imported differ markedly from those which are manufactured within the U.S. Consequently, a different result is obtained when the weighted average composition of manufactured versus imported cheeses is computed. Tables 15 lists the average composition of various aggregate product groupings according to the origin of the products.

Table 14. Composition of Fluid Dairy Products for 33 Federal and 3 State Milk Marketing Areas; 1995

FMMO Area	Average Fat % May and Oct.	Average SNF %, May and Oct.
Black Hills	2.52	8.72
California–southern	2.73	8.99
California–South Valley	2.60	9.00
California–northern	2.77	9.03
Carolina	2.47	8.69
Central Arizona	3.15	8.66
Central Illinois	2.47	8.81
Chicago Regional	2.78	8.83
E. Ohio–W. Pennsylvania	2.16	8.76
Eastern Colorado	3.14	8.67
Eastern South Dakota	2.34	8.81
Great Basin	2.45	8.78
Greater Kansas City	2.90	8.70
Indiana	2.12	8.75
Iowa	2.39	8.74
Louis.–Lex.–Evansville	2.36	8.79
Michigan Upper Peninsula	2.26	8.73
Middle Atlantic	2.36	8.72
Nebraska–Western Iowa	2.57	8.79
New England	3.29	8.65
New Mexico–West Texas	3.02	8.74
New York–New Jersey	3.57	8.67
Ohio Valley	2.15	8.75
Pacific Northwest	2.51	8.72
S. Illinois–E. Missouri	2.54	8.73
S.W. Idaho–E. Oregon	2.82	8.71
Southeast	2.34	8.76
Southeastern Florida	2.53	8.85
Southern Michigan	2.55	8.68
Southwest Plains	2.83	8.73
Tampa Bay	2.44	8.82
Tennessee Valley	2.36	8.70
Texas	2.61	8.76
Upper Florida	2.12	8.87
Upper Midwest	2.06	8.71
Western Colorado	2.67	8.70

Table 15. Average Estimated Composition of Various Manufactured Product Types by Source of Product; May and October, 1995

	May		October	
	Fat %	SNF %	Fat %	SNF %
Soft Products,				
As Manufactured	7.75	10.74	7.61	10.79
Cheese,				
As Manufactured	28.10	27.67	27.93	27.09
Imports (quota)	28.72	31.21	28.53	31.96
Imports (non–quota)	27.11	39.01	27.56	27.77
Exports (all varieties)	32.29	29.89	32.29	29.89
Stocks	30.56	27.72	32.13	29.36
Butter				
As Manufactured	81.11	3.02	81.11	3.02
Imports	81.11	3.02	81.11	3.02
Exports	81.11	3.02	81.11	3.02
Stocks	81.11	3.02	81.11	3.02
Dry/Condensed/Evap.				
Other Dairy Uses	1.47	59.41	1.64	53.05
Nondairy Uses	5.35	52.73	3.59	53.62
Imports	6.30	65.85	3.44	24.84
Exports	8.07	59.66	14.67	66.56
Stocks	2.51	49.69	1.01	89.72

In the case of cheese, it was necessary to modify the estimated composition parameters to account for components lost to whey during the cheese manufacturing process. Whey products are not included explicitly in the USDSS. The consequence of adjusting these parameters was that more milk had to be drawn into a cheese plant to produce a unit of cheese than would be suggested by the composition of the finished cheese. Losses of fat and SNF to whey are highly dependent on a number of factors; for example, the type of cheese being produced, the type of processing equipment being used and the skill with which it is operated, and the quality and composition of the milk. Variability not withstanding, we scaled the cheese composition parameters as reported in Table 13 by 1.05 for fat and 3.0 for SNF. The implication here is that, on average, 5 percent of the fat is lost to the whey and, while it may be recovered through separation, it does not reenter the dairy marketing channel of the USDSS. A typical use of whey cream is whey butter which is used by the confectionery industry. Similarly, the factor of 3.0 used to scale the SNF parameter reflects the fact that most (approximately two-thirds) of the nonfat solids, for example whey proteins and lactose, contained in milk are not retained in the final cheese.

Cost Data

The final set of parameters to be described are the objective function coefficients, *i.e.* transportation and processing costs. The description of transportation costs is divided into three parts. First, the various transportation activities are described and the basic, or unadjusted, cost functions are presented as a function of distance only. Two items used to modify the basic functions, gross vehicle weight (GVW) limits and labor costs, are then discussed. Finally, the fully–specified transportation cost functions, as utilized in the model, are presented. The discussion then turns to processing costs.

Unadjusted Distance–Based Transportation Costs

The choice of transportation cost parameters obviously plays a major role in the USDSS; the model is designed to minimize the sum of all costs so the output generated by the model depends crucially on the cost data. Unfortunately, the task of determining transportation rates that reflect those applicable to the dairy industry is not as simple as calling a few general carriers to obtain an average rate. Many factors combine to make haulage in the dairy industry a very specialized business and as a result, few common carriers bother to get involved, let alone publish meaningful rates. This is particularly so in the case of raw milk assembly, but also holds true in the case of final product distribution.

Once a rate for each type of shipment in the model has been determined, a matrix of costs is constructed; one for each shipment type. The cost matrices, therefore, contain unit costs for every arc over which a particular shipment type can potentially occur. It is these costs which enter the objective function of the model as the objective function coefficients.

(i) Milk Assembly

Milk haulers complete the link between milk producers and milk processors by transporting raw milk in bulk–tank trucks and tractor–trailers from farms to processing facilities. From the viewpoint of producers, milk haulers are often the only point of regular contact with the organizations that market or buy their milk. In addition to transporting milk, haulers perform many important duties during milk assembly that add to the safety and, consequently, to the value of dairy products. For example, as a first check of the overall quality, the driver will make visual inspections of the farm tank and the milk it contains. Also, samples of the milk will be aseptically collected for further laboratory testing. Such tasks require a degree of specialized training and diligence as the producer's payment depends on these tests. A consequence of the ongoing attrition of both plant and dairy farm numbers is an evolving pattern of bulk milk and intermediate product shipments. Haulers are moving larger loads of raw milk over longer distances than ever before to get the milk to designated processing facilities.

The modern-day function of assembling milk is often categorized as two distinct functions—farm milk pick-up, and over-the-road delivery (the term 'over-the-road' refers to that part of the haul where the truck is fully loaded and is no longer stopping at farms to collect milk). In many long distance situations, the power units and drivers who do the initial pick-up from farms are replaced with more appropriate drivers and long-distance, highway equipment after the last farm stop. In other situations, where milk is being transferred large distances between plants, the over-the-road costs may be the most significant portion of total transportation costs.

Hauling rates are typically determined through negotiations between haulers and milk cooperatives or proprietary handlers and are generally priced in terms of dollars per hundredweight of milk. Type of equipment used, labor and fuel costs, driving conditions, maintenance policies, route mileage, number of farm stops, farm location, and point of delivery are all considerations in developing a hauling rate, and these factors are all presumably reflected in the agreed–upon hauling rate. Note that the opportunity for back–hauls is a major factor in determining rates with common carriers, yet in the bulk milk hauling business, back–hauls are rare and are therefore of little consequence.

Although proposed as an industry model, the USDSS focuses on the decisions of processors. Processors offer flat rates to haulers to cover the cost of moving bulk milk from farms to plants. In keeping with the processor orientation of the model, a flat hauling rate was selected for use in the USDSS. After simulating numerous bulk hauling scenarios with a hauling cost analysis program developed by Pratt *et al.* (1994) and consulting with industry executives, a basic, unadjusted bulk milk transportation rate of $0.004 per hundredweight per mile was chosen; equivalently, 40 cents per hundredweight per 100 miles. This single rate could not possibly apply to all milk assembly situations. It will, however, be applicable in many 'over–the–road' situations Given the geographic structure of the USDSS, the milk assembly function represents primarily over–the–road movements.

(ii) Interplant Transfers

Four intermediate product types have been specified in the USDSS; cream, skim milk, ice cream mix, and nonfat dry milk. Except for NDM, these products are shipped between plants using the same bulk–tank trucks as are used for raw milk assembly. Consequently, the bulk milk hauling rate was also applied to these products. However, an additional fixed charge (*i.e.* unrelated to distance) of 3 cents per hundredweight was added to cover the cost of handling and reloading. The interplant transfer cost for NDM was specified to be the same as was used for the dry, condensed, and evaporated products category and will be explained below. Suffice it to say, the cost in dollars per hundredweight to ship NDM between plants is $0.022*(\text{one–way miles})^{0.73}$. Unlike cream, skim milk, and ice cream mix, NDM as an intermediate product is subject to significant processing and handling costs. Hence, there is no additional fixed charge to cover costs such as handling and reloading as these would be captured by processing costs.

(iii) Final Product Distribution

Final product distribution systems are complicated and are typically unique to a particular plant. Distribution methods and costs vary considerably among firms, even when the product being shipped is relatively homogeneous. Furthermore, the results of an analysis to determine what factors affect distribution costs revealed that several variables can have significant consequences for the unit cost of distribution (Erba *et al.*, 1997). As with bulk milk assembly, the distribution cost functions take account of miles traveled, regional differences in labor costs, and differences in GVW limits among the 48 states. Unlike assembly costs however, distribution cost functions were not specified as constant functions but rather as nonlinear functions in which the per unit cost declines as the distance traveled increases.

The results from the study by Erba *et al.* (1997), which focused on fluid milk distribution costs, provided the basis for estimating all distribution cost functions used in the USDSS. Only small modifications were required to make the packaged milk distribution costs consistent with those encountered when distributing manufactured products. Fluid milk distributors typically travel fewer than 500 miles per delivery day, whereas manufactured products are distributed over much longer distances.

Historically, fluid distribution costs have been rationalized to be higher than the distribution costs of other dairy products, a difference largely attributed to the bulkiness of fluid milk products. With the adoption of modern product handling systems, such as pallets, and the relaxation of GVW limits in many states, larger trailers have grown in popularity to replace the smaller trucks and trailers traditionally used for fluid milk distribution. In the study of 35 fluid milk distribution operations, the most widely used trailer measured 45–feet in length and 102 inches wide. Many companies reported using 48– and 53–foot trailers as well. Furthermore, most milk purchased for consumption is packaged in gallon and half–gallon containers. The implication of a uniform container size is that less space within a single milk case, and therefore throughout the trailer, is left unused. Finally, the distribution vehicles of the past typically used up the cubic capacity of the smaller vehicles before reaching the maximum weight limit. Today, with larger trailers, the problem has been reversed. That is, trailers transporting packaged milk reach the maximum GVW limit before filling up the cargo space of the trailer.

Methods of product handling for butter and cheese encompass many alternatives depending on the form of the product. For example, 640 lb. barrels of cheese would be unlikely to benefit from the use of pallets, whereas 40 lb. blocks probably would. Such differences notwithstanding, butter and cheese tend to be dense products which are packaged so that load size is not limited by loading and stacking logistics. As is the case with modern fluid milk distribution methods, the transportation of butter and cheese is likely to see GVW limits reached before the available cargo space is exhausted. Fluid milk, butter, and cheese can, and frequently do, use identical tractor–trailer units because they all require refrigeration. Under these conditions, one would expect that the distribution costs of fluid milk, butter, and cheese would be nearly identical. Operators involved in the transportation of all three product types confirm this to be true. Thus, distribution costs for fluid milk, butter, and cheese were specified identically in the USDSS.

In the past, a similar logic to that described above has also been used to specify a higher cost for transporting soft products relative to the cost for other manufactured dairy products. At least some of the difference in cost has been attributed to the container sizes used to package soft dairy products. While brick ice cream containers allow for efficient packing of products, containers used for other soft products do not readily offer the same benefits. In particular, the irregularly shaped containers used to package yogurt, sour cream, and cottage cheese do not lend themselves at all well to efficient packing and space utilization. Likewise, the cylindrical containers used to package some ice creams do not use space as efficiently as brick packaging. Irregularly shaped packaging would logically lead one to question whether a trailer could be loaded with enough product to reach GVW limits. In addition, soft products appear to be the most troublesome to stack because of the possibility of product crush. Aside from product package, loading, and handling differences, distribution equipment differences may add to the cost of shipping finished dairy products. For example, trailers used to transport ice cream products require more insulation and heavy duty refrigeration units, and these additional equipment costs increase distribution costs.

However, none of these rationalizations continue to possess sufficient validity to warrant specifying soft products distribution costs any higher than other dairy products. Just as with packaged milk, cheese, and butter, the use of large trailers as the primary means of distributing finished dairy products has increased dramatically in recent years with higher GVW limits and widespread palletization. Consequently, while exhausting cubic capacity may have been a concern in the past, especially with irregular shaped containers, processors are typically more concerned now about exceeding the maximum allowable GVW within their area of distribution. Furthermore, most processors pack soft products into milk cases, as would be done with fluid milk products, eliminating any concern about product crush. Some processors use corrugated boxes to package several individual servings of yogurt or sour cream into one easily handled and stacked parcel. It would not be unusual for milk cases to be used in conjunction with this method of packaging soft products, but even without milk cases, the corrugated boxes are durable and can withstand vigorous handling and stacking. Finally, there are only minor differences in equipment costs for trailers used to distribute fluid products, cheese or butter and soft products. The cost of equipping a trailer with the extra insulation and improved cooling system to transport frozen products adds about 10 percent to the total cost of the trailer. To maintain more flexibility within the distribution fleet, distributors often choose to purchase trailers outfitted with equipment to handle frozen products. Trailers can then be devoted to hauling any dairy product requiring refrigeration without concern to the special requirements of frozen products. The cost of distributing soft products was specified to be identical to that of fluid milk, cheese, and butter.

Due primarily to equipment differences, a slightly different distribution cost function was developed for the dry, condensed, and evaporated products category. Dry, condensed, and evaporated products require no refrigeration or insulation and this reduces the cost of both purchasing and operating distribution equipment. The differences in equipment costs were estimated to reduce costs by 20 percent compared to the other four product classes. It should also be pointed–out that by far the largest portion of the dry, condensed, and evaporated product category is NDM.

The unadjusted final product distribution costs are as follows. For the product classes requiring refrigeration (*i.e.* all but the dry, condensed, and evaporated products category) the cost in dollars per hundredweight of product is $0.0245*(\text{one–way miles})^{0.73}$ while for the non–refrigerated products category it is $0.022*(\text{one–way miles})^{0.73}$. These two functions, and the basic bulk milk assembly cost function, are plotted in Figure 17. It can be clearly seen from the figure that both refrigerated and non–refrigerated distribution costs are greater than raw milk assembly costs for all routes less than about 550 miles long. Between 550 and about 820 miles, the raw milk assembly cost is greater than non–refrigerated distribution costs but less than refrigerated distribution costs. Above 820 miles, raw milk assembly is unequivocally greater than all distribution costs. Bearing in mind that these costs are as yet unadjusted for differences in GVWs and labor costs, it is clear that the length of the route will impact the choice of activity in the model. Note too that, unlike bulk milk hauling, shippers of final products often make use of back–hauls to keep costs as low as possible. This is reflected in the coefficients in the final product distribution cost functions.

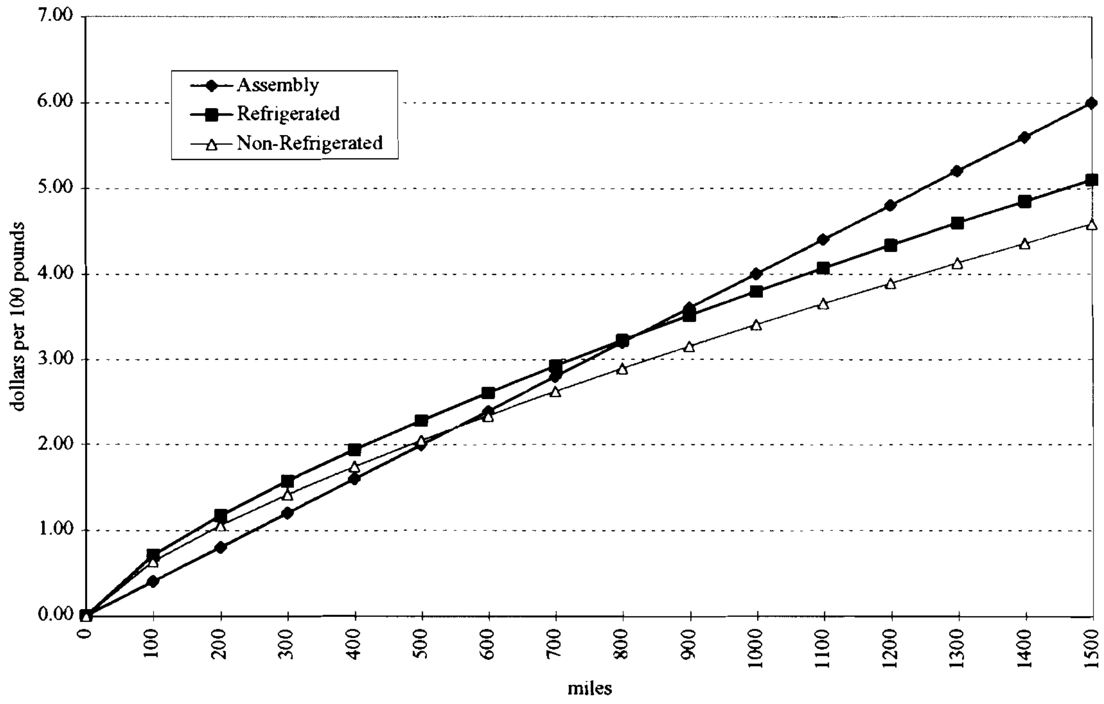

Figure 17. Transportation Costs as a Function of Distance Only

Adjustments to the Basic Transportation Cost Functions

We now discuss the two adjustments made to the basic transportation cost functions.

(i) Gross Vehicle Weight Limits

Variations in gross vehicle weight limits by state suggest that considerable differences in hauling costs could be attained depending solely on the state(s) within which a hauler operates. Allowable limits on truck size and weight have been a recurring issue at the state and federal levels since the earliest days of motor carriers and public road building. The Surface Transportation Assistance Act of 1982 increased the allowable width and length of tractor–trailer combinations and permitted the use of double trailer combinations on interstate highways and designated federal primary highways. Middendorf and Bronzini (1994) showed that increasing trailer size to achieve the maximum allowable road weight limit led to significant improvements in motor carrier productivity.

In light of the findings of Middendorf and Bronzini, the basic transportation cost functions were modified to incorporate the cost advantages imparted by higher GVW limits. State–by–state road weight limits were obtained from the Federal Highway Administration (see Figure 18).

Twenty–six states maintain minimum federal GVW limits of 80,000 lbs. However, nineteen states permit GVWs of 105,000 lbs. or more. Many of the states with high GVW limits are located in the northwestern quarter of the U.S. and the Southwest. The highest GVW limits are maintained by Michigan and New York at 164,000 lbs. and 143,000 lbs., respectively.

Incorporating the GVW data into the transportation cost functions required ascertaining the minimum GVW limit encountered along every potential route in the USDSS. Clearly, a truck originating its trip in New York state, for example, where the GVW is 143,000 lbs., and passing through Pennsylvania, where the limit is only 80,000 lbs., is unable to exploit the higher GVW limit of New York. That is, the binding GVW limit is the minimum encountered along the route. A modified version of the shortest path algorithm described earlier was used to determine the applicable GVW for all 386,884 routes in the model. It was assumed that the shortest possible route between any two cities was the one that would actually be traveled. Given this criteria, it was relatively straightforward to modify the shortest path algorithm, which, by definition, determines the path taken to get from one location to another, such that it yields not only the shortest path but the minimum GVW limit encountered along that path.

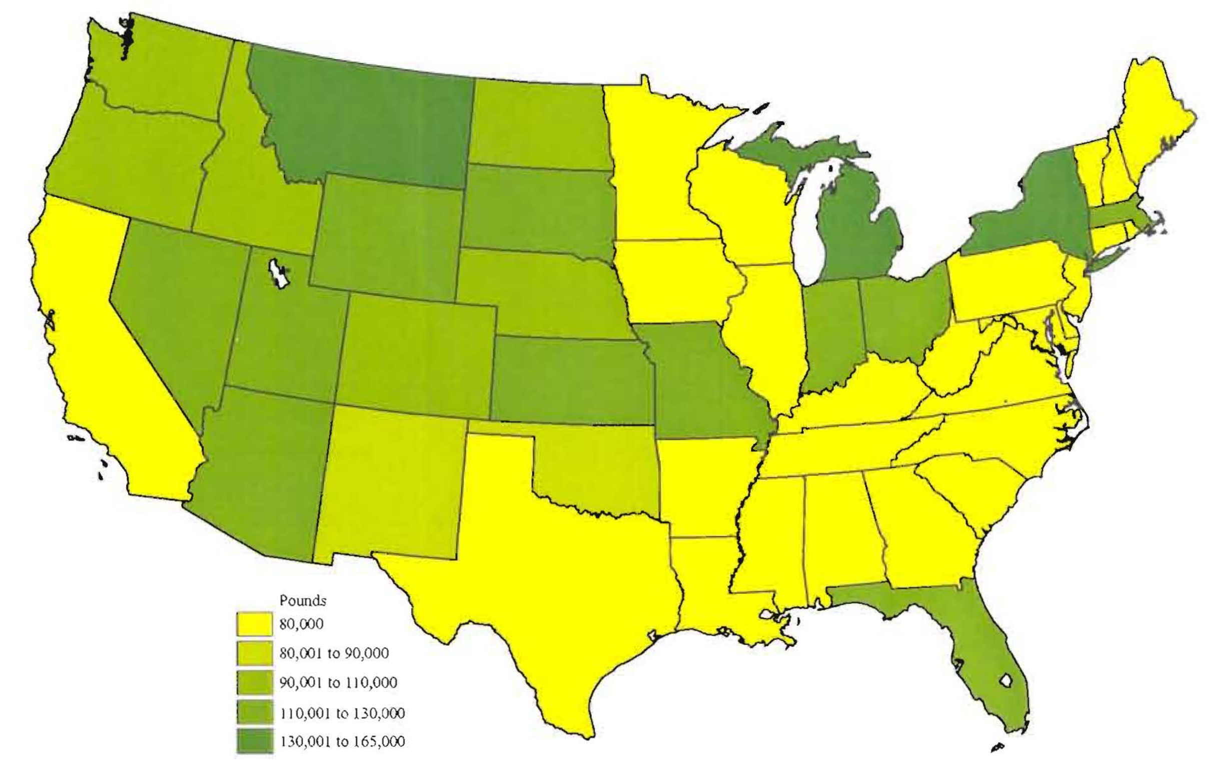

Figure 18. Gross Vehicle Weight Limits by State

The one exception to the shortest path criterion was in the case of trips to the New York city area which originate in the western or northcentral parts of New York state. In other words, all trips that both originate and terminate in New York state but which leave the state during the trip. For such routes, the shortest path follows Rte. 81 south into Pennsylvania and then takes Rte. 80 east across New Jersey and back into New York. However, for a fully laden truck, such a trip is not the least expensive due to the differences in GVW limits alluded to above. In fact, haulers making this trip will routinely take Rte. 17 from Binghamton, NY to New York city when fully laden, and will make the return trip, when the truck is empty, via New Jersey and Pennsylvania. They do this despite the fact that Rte. 17 passes through many small towns and is a considerably slower route to take. The distance and GVW matrices were adjusted to account for this anomaly. In all, 32 arcs were modified, in both directions, such that the cost of traveling them was consistent with the least cost criteria which in these 32 cases did not correspond to the shortest distance criteria.

Examination of Figure 18 will reveal that in practically all other parts of the country, the shortest route will correspond to the least expensive. It should also be apparent that as the length of the route increase, the chances of the binding GVW limit becoming 80,000 lbs. increases. Indeed, over 94 percent of the 386,884 possible routes in the model have an associated GVW limit of just 80,000 lbs.

A few final points about GVW limits need to be made. First, in states with high GVWs, it is not possible for bulk–tank trucks to come close to the maximum allowable vehicle weight when fully laden. Even using the largest trailers described by Erba *et al.* (1993), the estimated GVW of a fully loaded truck would range from 100,000 lbs. to 105,000 lbs. As such, for the purposes of calculating the cost of hauling raw milk, cream, skim milk, or ice cream mix, an 'adjusted' GVW of 100,000 was imposed on all arcs for which the actual minimum GVW limit encountered exceeded 100,000 lbs. Second, the manner in which the GVW adjustments for distribution are made is slightly different than for assembly. While distributors operating within states which permit high GVWs are able to exploit the higher road limits by using two large trailers to haul finished product, not all distributors do so. The way in which GVWs enter the function has been formulated to recognize this. In other words, the intent is to have the distribution cost functions be representative of the average cost encountered, rather than be reflective of the lowest cost haulers. The adjustment for GVW limits will have no impact on any of the transportation costs when the applicable GVW is 80,000 lbs. Finally, while distribution costs apply to finished products on a product–equivalent basis (*i.e.*, weight of actual product), the weight limit encountered recognizes the entire weight of the loaded truck—including the packaging and the truck itself.

(ii) Labor Costs

The second modification to the basic transportation functions was an effort to take account of labor costs. Labor's contribution to the overall cost of hauling in the dairy industry ranges from about 25 to 50 percent (Erba *et al.*, 1993). It also varies considerably within and between broad regions of the country. The cost of labor enters the transportation functions in the form of an index parameter and, by construction, has no effect at all on the transportation cost if the index value is equal to one, the average indexed labor cost.

The cost of labor index was developed to incorporate differences in hauling rates attributable solely to the cost of driver labor. The index was based on 1994 wages for production workers from all MAs reporting wage data (Slater and Hall, 1996). Production, or manufacturing, worker data was used because this class of labor best reflects the skill level required of drivers typically employed in the dairy hauling industry. In other words, drivers in the dairy industry tend to have a broader range of skills and be more highly paid than drivers employed by common carriers. The data encompasses both union and non–union labor and is geographically specific. An index value was computed for all 622 principal cities in the USDSS. However, not every city in the USDSS was associated with an MA reporting wage data. For those areas of the U.S. not directly associated with an MA, the great circle distance (described earlier in the section titled Processing Locations) was used to associate cities to the nearest MA, and therefore, the corresponding index value.

Under the assumption that labor in an outlying area is less costly than labor hired from a medium to large sized metropolitan area, the following regression analysis was performed to determine how to adjust the assigned labor costs for those areas of the U.S. not directly associated with an MA. The equation used was:

$$LCOST_i = \beta_0 + \beta_1 LAT_i + \beta_2 LONG_i + \beta_3 POP_i + \beta_4 CITY_i + \varepsilon_i$$

where:

$LCOST_i$ = cost of production labor at MA_i,
LAT_i = latitude of $CITY_i$ in decimal notation,
$LONG_i$ = longitude of $CITY_i$ in decimal notation,
POP_i = population density of county in which MA_i is located, and
$CITY_i$ = indicator variable denoting cities with population density of 2,500 inhabitants or more per square kilometer.

Because of the extraordinary cost of labor in the very largest MAs, an indicator variable was included in the regression model to account for any skewing effects. The regression results revealed that the true underlying factors affecting the per hour labor cost are more complex than those in the simplistic model specified here (Table 16). Nonetheless, the coefficients corresponded closely with expected results. That is, labor costs were lower in the southern U.S. than in the northern U.S., labor costs were not strongly associated with an east–west pattern, and labor costs were higher in counties with a higher population density. The effect of densely populated cities appeared to be statistically insignificant. Figure 19 describes the spatial differences in the cost of labor index across the 622 principal cities in USDSS.

All that remains now is to determine what proportion of the basic hauling cost is to be influenced by our index of labor costs. Erba *et al.*, (1997) determined that labor costs, as a percentage of direct delivery cost, were quite variable across distribution operations. Moreover, the percentage was substantially higher for distribution operations than for milk assembly. Given these results, it was determined that the transportation cost functions would be specified such that labor accounts for 35 percent of the cost of raw milk assembly and 48 percent of final product distribution. The applicable labor cost index is the value corresponding to the originating city for all routes.

Table 16. Regression Results for Labor Cost per Hour[1]

	d.f.	S.S.	M.S.	F–value
Regression	4	75.64	18.91	7.23
Residual	90	235.48	2.62	

	Coefficients	Standard Error	t–Statistic	Probability
Constant, b_0	8.0686	1.9470	4.14	<0.0001
LAT_i	0.1255	0.0385	3.26	0.0016
$LONG_i$	0.0147	0.0110	1.34	0.1830
POP_i	0.0021	0.0007	3.15	0.0022
$CITY_i$	-0.0819	0.6221	-0.13	0.8956

[1]The R^2 for the model was 0.243

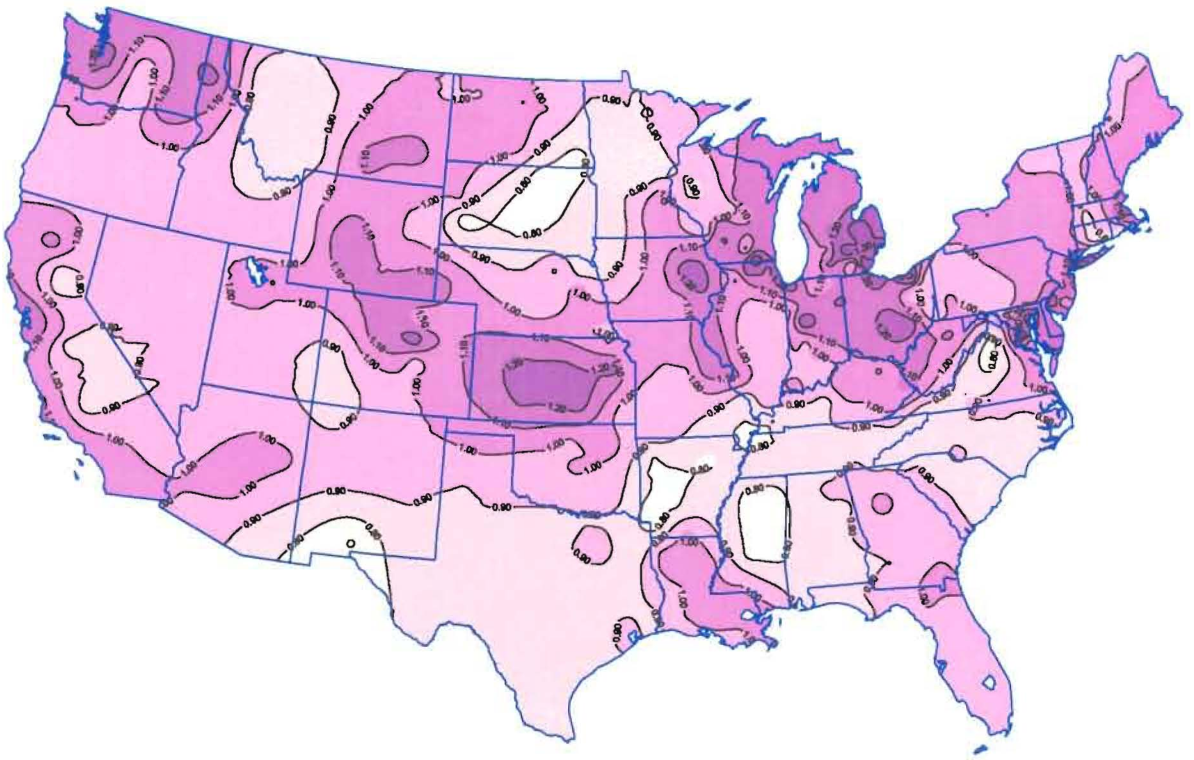

Figure 19. Spatial Differences in the Indexed Cost of Labor; Average = 1.00

Fully–Specified Transportation Costs

We now present the fully–specified transportation costs as they appear in the USDSS.

(i) Milk Assembly

$$\overline{AC}_{i,k} = 0.004 * MILES_{i,k} * \left(\frac{80{,}000}{AGVW_{i,k}} \right) * (0.65 + 0.35 * WI_i)$$

where:

$\overline{AC}_{i,k}$ = dollars per hundredweight to ship raw milk from the i^{th} supply point to the k^{th} plant location,

$MILES_{i,k}$ = one–way miles from the i^{th} supply point to the k^{th} plant location,

$AGVW_{i,k}$ = the adjusted minimum GVW encountered along the route from the i^{th} supply point to the k^{th} plant location where the adjustment entails replacing the gross vehicle weight with 100,000 in all cases where the actual gross vehicle weight exceeds 100,000, and

WI_i = wage index at the city associated with the i^{th} supply point.

(ii) Interplant Transfers

(a) Interplant transfer costs for cream, skim, and ice cream mix:

$$\overline{IC}_{k,k',m} = 0.03 + 0.004 * MILES_{k,k'} * \left(\frac{80{,}000}{AGVW_{k,k'}} \right) * (0.65 + 0.35 * WI_k)$$

where:

$\overline{IC}_{k,k',m}$ = dollars per hundredweight to ship the m^{th} intermediate product (where m = cream, skim milk, or ice cream mix) from the k^{th} plant location to the k'^{th} plant location,

$MILES_{k,k'}$ = one–way miles from the k^{th} plant location to the k'^{th} plant location,

$AGVW_{k,k'}$ = the adjusted minimum GVW encountered along the route from the k^{th} plant location to the k'^{th} plant location where the adjustment entails replacing the gross vehicle weight with 100,000 in all cases where the actual gross vehicle weight exceeds 100,000, and

WI_k = wage index at the city associated with the k^{th} plant location.

(b) Interplant transfer costs for NDM:

$$\overline{IC}_{k,k',m} = 0.022*MILES_{k,k'}^{0.73} * \left(\frac{80,000}{40,000 + 05 * GVW_{k,k'}} \right) * (052 + 048 * WI_k)$$

where:

$\overline{IC}_{k,k',m}$ = dollars per hundredweight to ship the m^{th} intermediate product (where m = NDM) from the k^{th} plant location to the k'^{th} plant location,

$MILES_{k,k'}$ = one–way miles from the k^{th} plant location to the k'^{th} plant location,

$GVW_{k,k'}$ = the minimum GVW encountered along the route from the k^{th} plant location to the k'^{th} plant location, and

WI_k = wage index at the city associated with the k^{th} plant location.

(iii) Final Product Distribution

$$\overline{DC}_{k,j,n} = S_n * MILES_{k,j}^{0.73} * \left(\frac{80,000}{40,000 + 05 * GVW_{k,j}} \right) * (052 + 048 * WI_k)$$

where:

$\overline{DC}_{k,j,n}$ = dollars per hundredweight to distribute the n^{th} final product type from the k^{th} plant location to the j^{th} consumption area,

$MILES_{k,j}$ = one–way miles from the k^{th} plant location to the j^{th} consumption area,

$GVW_{k,j}$ = the minimum GVW encountered along the route from the k^{th} plant location to the j^{th} consumption area,

WI_k = wage index at the city associated with the k^{th} plant location, and

S_n = a scalar such that S_n = 0.0245 when n = fluid milk, soft products, cheese, or butter, *i.e.* the product types requiring refrigeration; and S_n = 0.022 when n = dry, condensed, and evaporated products.

Shipping final product to the export or stocks nodes occurs at a cost of zero regardless of the nodes at which the shipments originate. The three import nodes incur a small cost to land final products at the cities with which the ports are associated. From that point, the imported products move through the road network and incur the same cost as domestically produced products.

Figure 20 demonstrates the difference that the adjustments for GVW limits and labor costs make to the basic transportation cost functions. Instead of a single cost associated with each distance, as illustrated in Figure 17, we now see a wide range of costs associated with each shipment type. Of course, for any particular arc, the same wage index and GVW limit is applicable to all cost types. Notwithstanding this, Figure 20 suggests that assembly costs do not become unambiguously greater than distribution costs until a distance of about 1,800 miles is reached. This compares with about 820 miles prior to making the spatial wage and GVW limit adjustments.

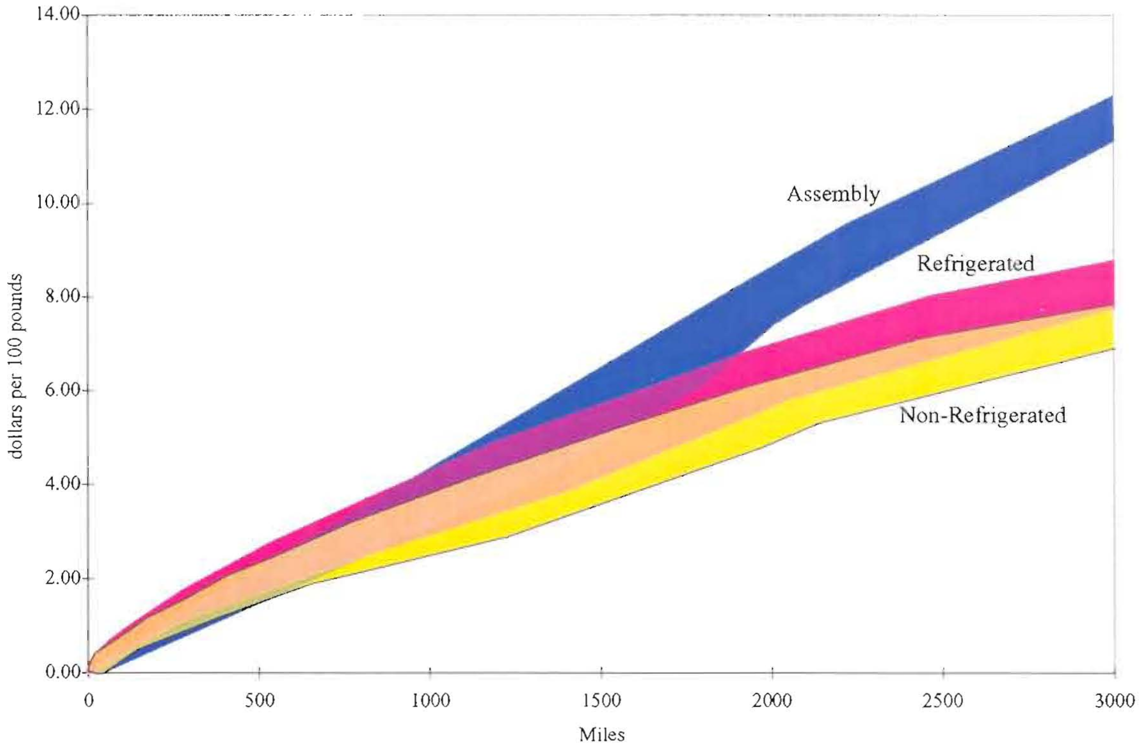

Figure 20. Fully–Specified Transportation Cost Functions

Finally, Table 17 summarizes the cost functions just presented for easy reference while Table 18 shows some examples of assembly and distribution (both refrigerated and non-refrigerated) costs for selected routes. Differences in gross vehicle weight limits and labor costs can be seen to contribute to significant variations in transportation costs. For example, if the model were to choose to do so, the cost of moving a trailer loaded with non–refrigerated products from Sandusky, MI to Effingham, IL would be 35 percent higher than moving the same product from Ephraim, UT to Denver, CO—despite the distance traveled on both routes being exactly 1,500 miles. Similarly, the cost of moving refrigerated products form New York, NY to Dallas, TX would be 24 percent higher than if the same product was transported between the same two cities but in the opposite direction.

Table 17. Transportation Cost Functions Summarized[1]

Raw Milk Assembly:

$$\$/100\text{ lb.} = 0.004 * \text{MILES}_{i,j} * \left(\frac{80{,}000}{\text{AGVW}_{i,j}}\right) * \left(0.65 + 0.35 * \text{WI}_i\right)$$

Refrigerated Product Distribution

$$\$/100\text{ lb.} = 0.0245 * \text{MILES}_{i,j}^{0.73} * \left(\frac{80{,}000}{40{,}000 + 0.5 * \text{GVW}_{i,j}}\right) * \left(0.52 + 0.48 * \text{WI}_i\right)$$

Non–Refrigerated Product Distribution

$$\$/100\text{ lb.} = 0.0220 * \text{MILES}_{i,j}^{0.73} * \left(\frac{80{,}000}{40{,}000 + 0.5 * \text{GVW}_{i,j}}\right) * \left(0.52 + 0.48 * \text{WI}_i\right)$$

where:

$\text{MILES}_{i,j}$ = one–way miles from the i^{th} location to the j^{th} location,

$\text{GVW}_{i,j}$ = the minimum gross vehicle weight encountered along the route from the i^{th} location to the j^{th} location,

$\text{AGVW}_{i,j}$ = the 'adjusted' minimum gross vehicle weight encountered along the route from the i^{th} location to the j^{th} location where the adjustment entails replacing the actual gross vehicle weight with 100,000 in all cases where the actual gross vehicle weight exceeds 100,000, and

WI_i = wage index associated with the i^{th} location.

[1] See text above for adaptations to interplant transfers.

Processing Costs

The final piece of the USDSS to be described is processing costs. Despite the homogeneity of the products processed and the methods used to process the products, costs can vary considerably across plants. The study of fluid plants cited earlier (Erba *et al.*, 1997), for example, found that several key variables have significant consequences for unit processing costs. In the present study, costs were explicitly specified to be functions of plant size, labor costs, and energy costs. Furthermore, they were specified using a functional form that permits scale economies to be realized.

Processing cost functions were developed for each of the five product categories. As was described earlier, processing costs can enter the model in two different ways depending on the formulation chosen. Objective function (1a) allows costs to be modeled strictly on a per unit basis. While such a construction implies constant returns to scale, it will be shown below that employing this formulation within an iterative solution scheme allows scale economies, or

Table 18. Transportation Costs for Selected Routes, Dollars per Hundredweight

From	To	Miles	GVW[1]	WI[2]	Assembly	Cost, $/cwt. Refrigerated	Cost, $/cwt. Non-Refrigerated
Kalamazoo, MI	Goshen, IN	50	127,400	1.23	0.17	0.37	0.33
Orlando, FL	Lakeland, FL	50	120,000	0.98	0.16	0.34	0.30
Sikeston, MO	Poplar Bluff, MO	50	120,000	0.80	0.15	0.31	0.28
Wallington, NJ	Hackettstown, NJ	50	80,000	1.17	0.21	0.46	0.41
Camden, NJ	New London, CT	250	80,000	1.20	1.07	1.51	1.36
Detroit, MI	Gary, IN	250	127,400	1.43	0.92	1.28	1.15
Ephraim, UT	Denver, CO	500	110,000	0.97	1.58	1.90	1.71
Joplin, MO	Oakley, KS	500	120,000	0.98	1.59	1.81	1.63
Knoxville, TN	Tallahassee, FL	500	80,000	0.87	1.91	2.15	1.93
Sandusky, MI	Effingham, IL	500	80,000	1.25	2.18	2.56	2.30
Glassboro, NJ	Highland, IN	750	80,000	1.20	3.21	3.37	3.03
Grand Island, NE	Glendive, MT	750	95,000	1.00	2.53	2.81	2.53
Cheyenne, WY	Granville, ND	750	105,500	1.19	2.56	2.90	2.60
Newark, OH	Bangor, ME	1,000	80,000	1.25	4.35	4.25	3.82
Yakima, WA	Saint George, UT	1,000	105,500	0.93	3.12	3.16	2.84
Philadelphia, PA	Zephyrhills, FL	1,000	80,000	1.43	4.60	4.58	4.11
Eau Claire, WI	Tampa, FL	1,500	80,000	0.91	5.81	4.88	4.38
Rapid City, SD	Petaluma, CA	1,500	80,000	0.75	5.48	4.49	4.03
Tomah, WI	Bangor, ME	1,500	80,000	1.18	6.38	5.54	4.98
Dallas, TX	New York, NY	1,528	80,000	1.01	6.13	5.20	4.67
New York, NY	Dallas, TX	1,528	80,000	1.52	7.22	6.46	5.80
Los Angeles, CA	Minneapolis, MN	1,986	80,000	1.05	8.08	6.41	5.76
Minneapolis, MN	Los Angeles, CA	1,986	80,000	1.12	8.28	6.62	5.95
Miami, FL	Seattle, WA	3,304	80,000	0.97	13.08	8.95	8.04
Seattle, WA	Miami, FL	3,304	80,000	1.24	14.33	10.13	9.09
Eureka, CA	Bangor, ME	3,669	80,000	1.05	14.93	10.06	9.03

[1] Minimum gross vehicle weight limit encountered along route; pounds.
[2] Wage Index (cost of driver labor) at route origin.

decreasing cost schedules, to be approximated. Objective function (1b), the fixed charge formulation, allows decreasing cost schedules to be modeled directly. Costs are described as being comprised of two components; a fixed cost and a variable cost. Together, these two cost components yield cost curves which decline as the quantity processed increases.

Data to specify the cost parameters for medium and large sized plants were drawn from a variety of sources. In the case of fluid products, the Erba *et al.* (1997) study cited above was used. For cheese plants, a study by Mesa–Dishington *et al.* (1987) was consulted, and for butter/powder plants the work of Stephenson and Novakovic (1990) was drawn upon. Specifying cost parameters for the soft products category was problematic due to the wide range of product types in this group. Consequently, costs for this group were estimated using those for fluid and cheese as lower and upper bounds, respectively. In all cases, the original sources reported costs as declining schedules so it was therefore necessary to estimate the fixed and variable cost parameters using a best–fit criteria. In the case of butter/powder, the various simulations (see Stephenson and Novakovic, *op. cit.*) on different plant configurations provided the basis for assuming how fixed costs would be split between butter and powder plants.

Table 19 describes the various cost parameters as well as the plant capacities and the extent to which labor and energy costs impact the overall processing cost. Note that the fixed charge formulation will result in the model choosing a discrete number of plants of a particular size and type at any location. Given the parameters in Table 19, it is not possible that more than one medium sized plant of any type will be chosen at a given location because in such a case it would be less costly to choose one large plant, or one of each size, rather than two medium sized plants. However, it is possible and, indeed, highly likely that the model would choose more than one large plant at a particular location. This is especially likely with fluid plants in densely populated areas. Observe that the fixed cost of installing a plant is incurred for every plant the model chooses to activate.

The processing cost functions are modified according to the cost of labor and energy in the same way that labor was used to modify the transportation cost functions. In fact, the labor cost index used in the case of transportation is used here as well. Two energy price indices were constructed in a similar fashion to the labor cost index; one for the cost of electricity and one for the cost of natural gas. It was necessary to distinguish between the two energy forms as the proportion of the cost attributable to each differed dramatically between fluid and manufacturing plants. For example, it was found that electricity accounted for 78 percent of the energy costs in fluid plants while in manufacturing plants the ratio was reversed; on average, 85 percent of the energy costs were attributable to natural gas. Using these ratios, the electricity and natural gas price indices were combined into plant specific energy cost indices; one for fluid plants and one for manufacturing. The regional differences in the two plant specific energy cost indices are illustrated in Figures 21 and 22.

Table 19. Fixed Costs, Variable Costs, and Plant Capacities for Processing Operations

	Fluid	Soft	Cheese	Butter	DCE[1]
Plant Size					
Medium					
Fixed Cost, $[2]	165,000	120,000	100,000	40,000	120,000
Variable Cost, $/cwt.[3]	2.43	4.50	10.40	2.91	4.89
Capacity, mil. lbs./mo.[4]	17.70	10.00	2.85	2.27	6.41
Large					
Fixed Cost, $[2]	303,000	180,000	154,000	51,000	153,000
Variable Cost, $/cwt.[3]	1.65	3.00	8.50	2.41	4.37
Capacity, mil. lbs./mo.[4]	65.00	20.00	16.45	13.10	30.00
Price Indices[5]					
Labor Cost, %	42	46	50	33	33
Energy Cost, %	9	10	10	17	17

[1] DCE = dry, condensed, and evaporated products.

[2] Equals the parameter $\overline{FC}_{k,n,r}$ in objective function (1b).

[3] Variable cost = $/cwt. of finished product. Equals the parameter $\overline{VC}_{k,n,r}$ in objective function (1b).

[4] Capacity = million pounds of finished product per month.

[5] Represents the percentage of costs total attributable to labor and energy, respectively.

The final step in preparing the data to implement the fixed charge formulation requires making the regional labor and energy cost adjustments to the variable cost parameters. To this end, the following simple modification of the variable cost parameters reported in Table 19 was undertaken.

$$\overline{VC}_{k,n,r} = \left(PW_n * WI_k + PE_n * EI_k + \left(1 - PW_n - PE_n\right)\right) * \overline{VC}^{*}_{k,n,r}$$

where:

$\overline{VC}_{k,n,r}$ = the modified variable cost of processing the n^{th} final product in a plant of size r at the k^{th} plant location,

PW_n = the proportion of the total processing cost attributable to labor for the n^{th} product type, *i.e.* the second–to–last row of Table 19,

PE_n = the proportion of the total processing cost attributable to energy for the n^{th} product type, *i.e.* the last row of Table 19,

WI_k = the wage index at the city associated with the k^{th} plant location,

EI_k = the energy cost index at the city associated with the k^{th} plant location, and

$\overline{VC}^*_{k,n,r}$ = the unadjusted variable cost of processing the n^{th} final product in a plant of size r at the k^{th} plant location, *i.e.* the values reported in Table 19.

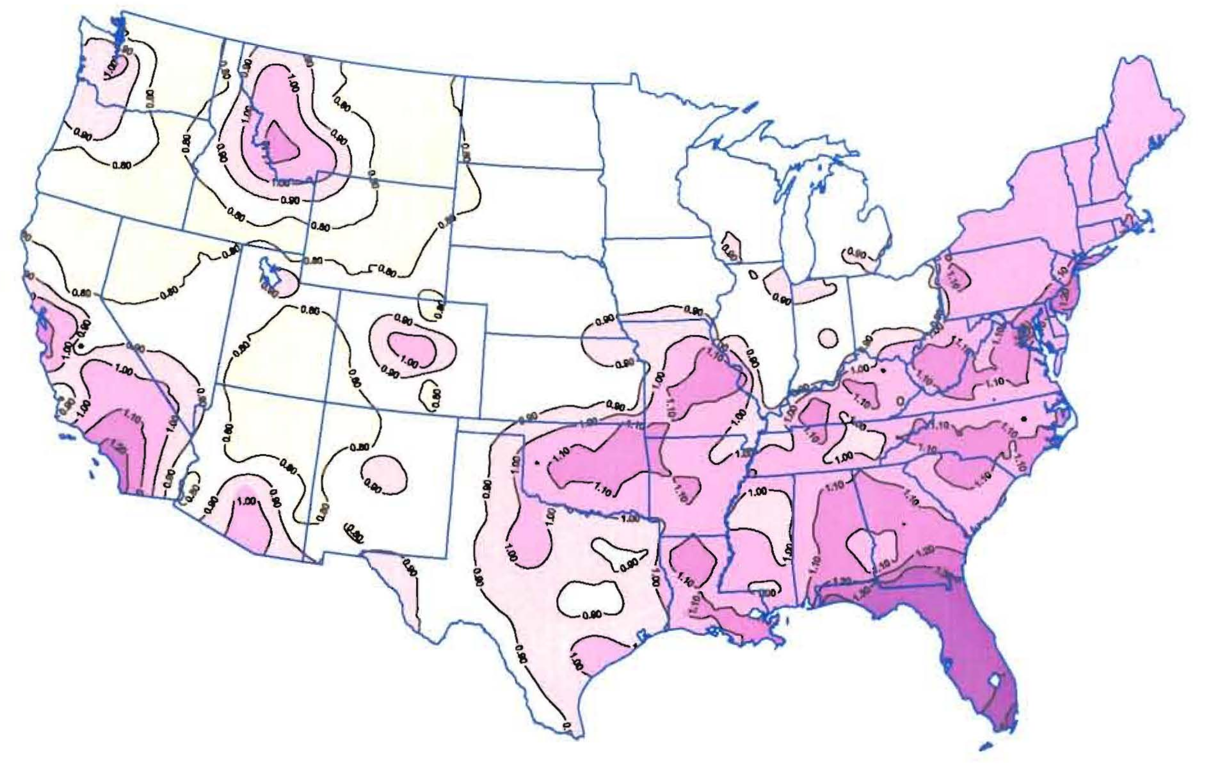

Figure 21. Spatial Differences in the Indexed Cost of Energy Used in Fluid Plants; Average = 1.00

The fixed charge formulation is mathematically elegant and intuitively appealing, but, as of this writing, fails to yield timely solutions for problems as large as the USDSS. The mathematical programming literature refers to MIP problems containing around one hundred integer variables as being large. The MIP version of the USDSS contains thousands of such variables. Consequently, for all but highly aggregated implementations of the USDSS, it is necessary to use LP approximations to the nonlinear (or more specifically, non-differentiable) fixed charge problem. Shmoys *et al.*, (1996) review many such approximation algorithms and put forth a new one. While the Shmoys *et al.* algorithm appears to hold much promise with respect to the proven convergence properties, or performance guarantees *vis-à-vis* the underlying integer problem, we have not yet fully explored its suitability for use in the USDSS. As an aside, we have recently collaborated with IBM, the GAMS Development Corporation, and the Cornell Theory Center to implement a parallel version of the OSL solution routines on Cornell's massively parallel supercomputer. The programs are currently being tested and evaluated, and hold great promise for timely solutions to this important problem. At this point we utilize a modified version of the approximation algorithm proposed by King and Logan (1964) to simulate scale economies. The convergence properties of this algorithm remain undetermined.

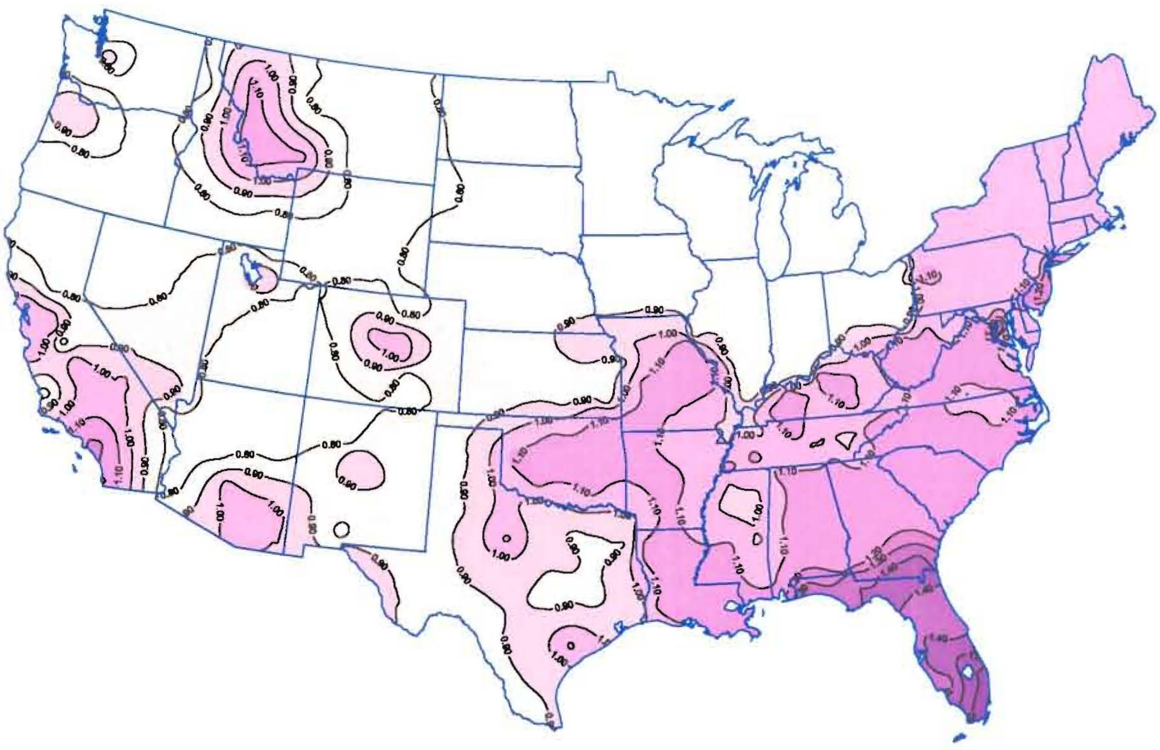

Figure 22. Spatial Differences in the Indexed Cost of Energy Used in Manufacturing Plants; Average = 1.00

The King and Logan iterative scheme uses objective function (1a) and therefore requires processing costs to enter on a per unit basis. At each iteration of the procedure, an updated per unit cost is calculated using the above cost parameters plus the quantity processed at each location in the previous iteration. The implementation of the King and Logan iterative scheme results in a cost schedule whereby the applicable cost, given any quantity to be processed, can be determined from a point along the outer envelope of the two cost curves corresponding to each of the plant sizes. Figure 23 demonstrates this concept.

Finally, a brief mention must be made of how to charge processing costs on NDM which is shipped between plants as an intermediate product when scale economies are not modeled. When the model is operated under the assumption of spatially uniform processing costs, it is a simple matter to add a fixed cost to the interplant transfer function for NDM. Note that this is analogous to the 3 cents per hundredweight added to the interplant transfer cost for cream, skim milk, and ice cream mix. A fixed cost of six dollars per hundred pounds of NDM is thus added. This equates to an NDM manufacturing cost of six cents per pound. Failure to account for this cost would cause the model to unnaturally favor NDM over raw milk as a means of supplying nonfat solids into cheese and, to a lesser extent, soft products plants.

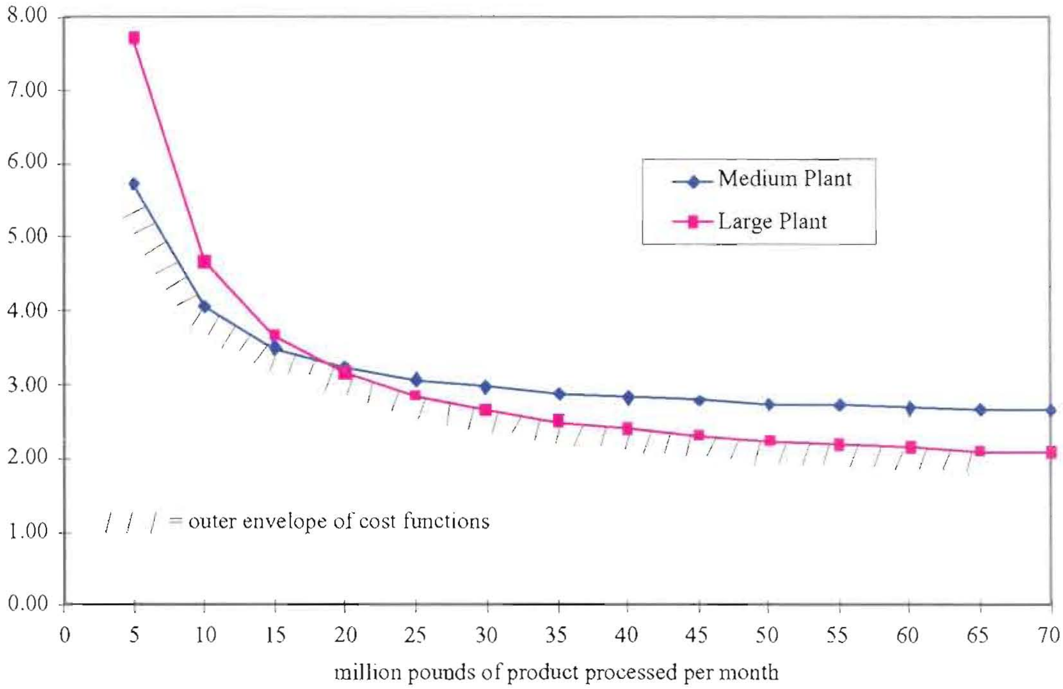

Figure 23. Example of Effective Processing Cost Function for Two Sizes of Fluid Milk Plant

CONCLUDING COMMENTS

This document has described the construction of a model known as the United States Dairy Sector Simulator (USDSS). This most recent update of the model and its data files was undertaken in support of a study to determine the appropriate regulated regional differences in the value of milk within the context of a reformed federal milk marketing order program. The base–case model for this study contained 8,112 constraints and 614,188 variables. It was solved using GAMS/OSL on an IBM RS6000 workstation. Approximately two hours of CPU time is required to obtain the optimal solution without the use of an advanced basis. Additional research bulletins describing the results of our analyses conducted as part of the research supporting the reform of federal milk marketing orders are forthcoming.

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