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## Evaluating concavity for production and cost functions

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Econometric estimation of firms' cost functions is a well-known technique, generally involving the joint estimation of the cost function with a set of cost share equations, one for each factor of production. Under common assumptions about the structure of production and cost, these equations are linear and can be consistently estimated with `sureg` subject to linear constraints. See Greene (2008, sec. 10.4.2).

A crucial assumption derived from the analytics of production and cost functions is that the cost function should be globally concave in each of the factors. Although it may not be possible to establish global concavity over all possible ranges of factor prices and levels of output, researchers have often been dissatisfied with evaluating concavity at a single point in the multidimensional space such as the multivariate point of means. Empirical rejection of concavity may indicate a misspecification of the underlying model of production and cost.

An alternative that might provide stronger evidence on the suitability of the chosen production and cost functions involves evaluating whether the estimated function is concave at each point in the sample space. In this note, we illustrate how this can readily be computed in Stata.

We illustrate with an example of a five-factor production function and consider the dual of the production problem. The cost function provides an estimate of the minimum cost ( $C$ ) of producing the level of output ( $y$ ) given the factor prices ( $P$ ) of water ( $w$ ), electricity ( $e$ ), labor ( $l$ ), capital ( $c$ ), and diesel ( $d$ ). A transcendental logarithmic, or "translog", cost function is used (Berndt and Christensen 1973), yielding the specification

$$\begin{aligned} \log C = & \alpha_0 + \alpha_w \log P_w + \alpha_e \log P_e + \alpha_l \log P_l + \alpha_c \log P_c + \alpha_d \log P_d + \alpha_y \log y + \\ & \frac{1}{2} \gamma_{ww} \log P_w^2 + \frac{1}{2} \gamma_{ee} \log P_e^2 + \frac{1}{2} \gamma_{ll} \log P_l^2 + \frac{1}{2} \gamma_{cc} \log P_c^2 + \frac{1}{2} \gamma_{dd} \log P_d^2 + \frac{1}{2} \gamma_{yy} \log y^2 + \\ & \gamma_{we} \log P_w \log P_e + \gamma_{wl} \log P_w \log P_l + \gamma_{wc} \log P_w \log P_c + \gamma_{wd} \log P_w \log P_d + \\ & \gamma_{wy} \log P_w \log P_y + \gamma_{el} \log P_e \log P_l + \gamma_{ec} \log P_e \log P_c + \gamma_{ed} \log P_e \log P_d + \\ & \gamma_{ey} \log P_e \log y + \gamma_{lc} \log P_l \log P_c + \gamma_{ld} \log P_l \log P_d + \gamma_{cy} \log P_c \log y + \\ & \gamma_{cd} \log P_c \log P_d + \gamma_{dy} \log P_d \log y \end{aligned}$$

The log(total cost) equation includes the log levels of each factor, the level of output, their squares, and their cross-products. Given differentiability of the cost function, the cost shares of all inputs can be expressed as elasticities of the cost function with respect to the input prices. Using Shephard's lemma (McFadden 1978), the cost share equations can be derived by logarithmic differentiation of the cost function:

$$S_{it}(P_i, y) = \frac{\partial \log C}{\partial \log P_i} = \alpha_i + \sum_j^n \gamma_{ij} \log P_j + \gamma_{iy} \log y \quad (1)$$

Although there are five factors of production, the adding-up conditions across factors imply that the residual covariance matrix assembled by `sureg` would be rank deficient. As discussed in Greene (2008, 278), consistent estimates can be derived by estimating any four of the five factor share equations in the system, because they sum up to unity. The `isure` option (iterated seemingly unrelated regression) ensures that the resulting estimates are insensitive to the choice of the omitted factor share.

To set up the estimation, imposing adding-up constraints and constraints of symmetry on the matrix of second partial derivatives, we use the following commands:

```
global eq1 ("lnTC lnw lne ln1 lnc lnd lny lnww lnee ln11 lncc lndd lnyy ///
lnwe lnw1 lnwc lnwd lnel lnc lnd ln1c ln1d ln1c ln1d ln1c ln1d ln1c ln1d ln1c ln1d")
global eq2 ("Sw lnw lne ln1 lnc lnd lny")
global eq3 ("Se lnw lne ln1 lnc lnd lny")
global eq4 ("Sl lnw lne ln1 lnc lnd lny")
global eq5 ("Sc lnw lne ln1 lnc lnd lny")
constraint define 1 [Sw]_cons+[Se]_cons+[Sl]_cons+[Sc]_cons+[lnTC]ln1=1
constraint define 2 [Sw]lnw+[Sw]lne+[Sw]ln1+[Sw]lnc+[lnTC]lnwd=0
constraint define 3 [Sw]lne+[Se]lne+[Se]ln1+[Se]lnc+[lnTC]lned=0
constraint define 4 [Sw]ln1+[Se]ln1+[Sl]ln1+[Sl]lnc+[lnTC]ln1d=0
constraint define 5 [Sw]lnc+[Se]lnc+[Sl]lnc+[Sc]lnc+[lnTC]ln1c=0
constraint define 6 [lnTC]lnwd+[lnTC]lned+[lnTC]ln1d+[lnTC]ln1c+[lnTC]ln1d=0
constraint define 7 [Sw]lny+[Se]lny+[Sl]lny+[Sc]lny+[lnTC]ln1y=0
constraint define 8 [Sw]lne=[Se]lnw
constraint define 9 [Sw]ln1=[Sl]lnw
constraint define 10 [Sw]lnc=[Sc]lnw
constraint define 11 [Se]ln1=[Sl]lne
constraint define 12 [Se]lnc=[Sc]lne
constraint define 13 [Sl]lnc=[Sc]ln1
constraint define 14 [Sw]lnw=[lnTC]lnww
constraint define 15 [Sw]lne=[lnTC]lnwe
constraint define 16 [Sw]ln1=[lnTC]lnw1
constraint define 17 [Sw]lnc=[lnTC]lnwc
constraint define 18 [Sw]lnd=[lnTC]lnwd
constraint define 19 [Sw]_cons=[lnTC]lnw
constraint define 20 [Se]lne=[lnTC]lnee
constraint define 21 [Se]ln1=[lnTC]lnel
constraint define 22 [Se]lnc=[lnTC]lnec
constraint define 23 [Se]lnd=[lnTC]lned
constraint define 24 [Se]_cons=[lnTC]lne
constraint define 25 [Sl]ln1=[lnTC]ln11
constraint define 26 [Sl]lnc=[lnTC]ln1c
constraint define 27 [Sl]lnd=[lnTC]ln1d
constraint define 28 [Sl]_cons=[lnTC]ln1
constraint define 29 [Sc]lnc=[lnTC]lncc
constraint define 30 [Sc]lnd=[lnTC]ln1c
constraint define 31 [Sc]_cons=[lnTC]lnc
constraint define 32 [Sw]lny=[lnTC]lnwy
constraint define 33 [Se]lny=[lnTC]lney
constraint define 34 [Sl]lny=[lnTC]lnly
constraint define 35 [Sc]lny=[lnTC]ln1y
sureg $eq1 $eq2 $eq3 $eq4 $eq5, constr(1-35) isure nolog small
(output omitted)
```



Each row of matrix `lambda` contains the five eigenvalues of `M` for that time period:<sup>1</sup>

```
. mata: mm_matlist(lambda, "%9.5f")
```

	1	2	3	4	5
1	0.13259	0.01068	0.00000	-0.00288	-0.06144
2	0.10441	0.01027	0.00000	-0.00155	-0.05348
3	0.10709	0.01655	-0.00000	-0.00247	-0.03358
4	0.14514	0.00823	-0.00000	-0.00418	-0.03001
5	0.10316	0.01127	-0.00000	-0.00150	-0.05334
6	0.13224	0.01689	-0.00000	-0.00401	-0.02283
7	0.21684	0.01486	-0.00000	-0.00653	-0.00872
8	0.16139	0.01703	-0.00000	-0.00524	-0.01880
9	0.17416	0.00939	0.00000	-0.00516	-0.03479
10	0.12426	0.01145	-0.00000	-0.00339	-0.02840
11	0.12944	0.01133	-0.00000	-0.00341	-0.03716
12	0.13342	0.00035	0.00000	-0.00388	-0.01101
13	0.21015	-0.00000	-0.00135	-0.00489	-0.01590
14	0.16252	0.01153	0.00000	-0.00391	-0.02644
15	0.14300	0.01701	0.00000	-0.00333	-0.02644
16	0.12358	0.01396	0.00000	-0.00240	-0.02924
17	0.15912	0.00914	-0.00000	-0.00324	-0.03468
18	0.10364	0.01426	-0.00000	-0.00096	-0.03496
19	0.14656	0.01169	0.00000	-0.00288	-0.03357
20	0.13079	0.00000	-0.00070	-0.00126	-0.03093
21	0.09045	0.00195	0.00000	-0.00147	-0.05131
22	0.15502	0.00835	0.00000	-0.00242	-0.02290
23	0.12108	-0.00000	-0.00010	-0.00087	-0.02347
24	0.21675	0.00000	-0.00009	-0.00389	-0.00497
25	0.17269	0.00972	0.00000	-0.00663	-0.02061
26	0.15330	0.00000	-0.00541	-0.00736	-0.01757
27	0.14008	0.00215	-0.00000	-0.00522	-0.01880
28	0.15688	0.00500	-0.00000	-0.00601	-0.01692
29	0.16513	-0.00000	-0.00518	-0.00675	-0.03441
30	0.13266	-0.00000	-0.00345	-0.00481	-0.04113
31	0.15934	0.00000	-0.00026	-0.00590	-0.04476
32	0.16523	-0.00000	-0.00512	-0.01528	-0.03634
33	0.14735	-0.00000	-0.00474	-0.01327	-0.02475
34	0.16027	-0.00000	-0.00495	-0.01862	-0.03200
35	0.15523	-0.00000	-0.00516	-0.00763	-0.01635
36	0.18057	0.00266	-0.00000	-0.00269	-0.00712
37	0.29339	0.01262	0.00365	0.00171	0.00000
38	0.15251	0.00572	0.00000	-0.01060	-0.01939
39	0.21218	0.00466	-0.00000	-0.00716	-0.01275
40	0.19018	0.00507	-0.00000	-0.00293	-0.01339
41	0.18717	0.00527	0.00052	0.00000	-0.01114
42	0.20440	0.00533	0.00146	-0.00000	-0.02895
43	0.17757	0.00648	0.00324	0.00000	-0.03411
44	0.24303	0.00480	-0.00000	-0.00217	-0.03588
45	0.18789	0.00551	-0.00000	-0.00134	-0.02136

The presence of positive eigenvalues indicates that the assumption of concavity of the cost function is not satisfied by the estimates. To comply with economic theory, either another functional form has to be chosen or necessary concavity conditions must be imposed prior to estimation by restricting matrix `M` to be negative semidefinite.

1. We use Ben Jann's `mm_matlist()` function from the `moremata` Statistical Software Components package to produce formatted matrix output.

However, in the case of the translog function, global imposition of concavity will destroy its flexibility and it should instead be imposed at a chosen reference point. According to Ryan and Wales (2000), the imposition of concavity at a single point will often lead to satisfaction of concavity at most or all data points in the sample. This can again be checked with the method described above.

## References

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