Optimal Temporal Policies in Fluid Milk Advertising

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Previous studies that have addressed the optimal allocation of generic advertising over time have assumed a symmetric demand response to increases and decreases in advertising (Bockstael, Strand, and Lipton; Liu and Forker; Kinnucan and Forker). Consumers, however, do not necessarily respond at the same pace to increases relative to decreases in advertising. In this paper, the assumption of a symmetric demand response is relaxed. An important question that naturally arises is: what are the implications of an asymmetric response to advertising for optimal advertising policies? This question has not been previously addressed in the generic commodity promotion literature and has gained only limited attention in the general marketing literature (Simon, Mesak).

Recent research has shown an asymmetric demand response to fluid milk advertising in New York City, i.e., demand decreases slowly when advertising is reduced compared to a relatively rapid expansion in demand when advertising is increased (Author Publication). Previous studies have suggested that the appearance of an advertisement is more likely to be noticed than its absence (Simon). Similarly, increased advertising may cause non-users to begin using the product while the number of users gradually decreases when advertising decreases. Both of these effects may lead to an asymmetric demand response to advertising (see Little for an excellent discussion).

The purpose of the research reported here is to determine optimal temporal patterns of advertising given an asymmetric demand response to advertising. To this end, empirical results of an asymmetric advertising-demand relationship for fluid milk are used to develop a dynamic optimization model where advertising is allocated over time so as to maximize the present value of current and future fluid milk sales. In addition to the advertising-demand response, commodity prices, seasonality in demand, and cost of advertising may also impact the optimal temporal allocation of advertising. As a result of the asymmetric demand response, and the possibility of a non-steady state solution, traditional nonlinear solution procedures have severe limitations. Therefore, a recursive methodology is employed to solve the dynamic optimization. Using this dynamic framework, the current advertising policy is evaluated, and the optimal temporal allocation of advertising is determined. This analysis is applied to generic fluid milk advertising in New York City.

Optimal advertising strategies may take several forms. Alternatives include a uniform advertising policy where advertising is the same in all periods, or a pulsing advertising policy where periods of intense advertising are alternated with periods of low or zero advertising. Many variations of pulsing advertising policies exist depending on the shape of the advertising patterns including the intensity and length of the pulses.

In a study of fluid milk advertising, both the optimal level of total advertising expenditures, and the optimal allocation of advertising over time are interesting and important issues. This analysis, however, addresses only the latter while assuming total annual advertising expenditures are maintained at historical levels. In reality, changing total annual fluid milk advertising expenditures in New York City would require either decreasing or increasing fluid milk advertising elsewhere, or changing the total funds allocated to advertising.

Previous Research

There have been few studies of optimal temporal allocation of advertising in the generic promotion and advertising literature. Kinnucan and Forker allowed seasonal variation in the demand response to advertising and found that farmer returns from fluid milk advertising in New York City were maximized when advertising expenditures followed a regular seasonal pattern. They concluded that annual advertising should be allocated as 30, 25, 20, and 25 percent for the four quarters of each year, respectively, and following this pattern would have increased demand by approximately 0.8 percent over the period 1972 to 1980. In another study, Liu and Forker used an optimal control framework to choose the optimal path of advertising, which maximized the discounted revenue stream from farm milk revenue less advertising costs. Liu and Forker also found that advertising should be more intense during the winter and less intense in late spring and early summer. The gains in demand quantity from a reallocation of advertising in Kinnucan and Forker, and Liu and Forker were modest while the increase in returns to farmers was larger. The optimal seasonal advertising pattern is driven, in large part, by seasonal variation in the milk blend price[1] paid to farmers (Liu and Forker). In other words, these studies suggest that reallocating advertising to increase demand when the farm-level milk price is highest during the year can increase returns to farmers. Both studies assumed the demand response to advertising is symmetric.

The study of optimal temporal advertising has gained limited attention in the general marketing literature. Sasieni, who also assumed a symmetric advertising-demand response, showed that with decreasing returns to scale (concave advertising-sales response function), the uniform advertising policy is optimal. This follows from the fact that the marginal return to advertising decreases as advertising increases, and it is better to allocate advertising evenly over all periods (at higher marginal returns) than use advertising pulses.

Several empirical marketing studies, however, have shown that pulsing advertising policies are superior to uniform advertising policies. Using controlled experiments, Ackoff and Emshoff tested a pulsing advertising strategy for Budweiser beer as a means to reduce advertising expenditures and found pulsing resulted in increased sales of beer relative to a uniform advertising strategy. Rao and Miller discovered evidence of increasing marginal returns to advertising at low levels of advertising for several Lever brands. As a result, they also found pulsing advertising to be more effective in increasing sales than uniform advertising.

Although some marketing tests have shown pulsing to be more effective than a constant level of advertising, advertising models have had difficulty explaining this phenomenon. To our knowledge, only two studies have investigated in depth the relationship between an asymmetric advertising-demand response and optimal advertising strategies. Simon, in a seminal paper, addressed this issue by proposing a simple model of an asymmetric sales response to advertising, where an increase in advertising resulted in an immediate increase in sales above the long-term equilibrium sales level. In the periods following the increase in advertising, sales gradually declined to a long-term level above the sales level before the increase in advertising but below the sales level immediately following the increase in advertising. Simon found empirical evidence of this form of asymmetry for several proprietary brands. By incorporating this asymmetry into a simple optimization model and investigating the marginal conditions, a pulsing advertising policy was found to dominate a uniform advertising policy in terms of sales. Mesak further developed Simon's asymmetry approach and rigorously demonstrated that pulsing is superior when the advertising response is asymmetric and the discount rate is small.

In a generic commodity promotion study, Bockstael, Strand, and Lipton used a dynamic bioeconomic model of the Maryland oyster industry and found that a pulsing advertising strategy is optimal. In this case, however, the optimal pulsing strategy did not arise from an asymmetric response to advertising, but from initial conditions and the cyclical behavior of the fish and capital stocks. Based on the empirically estimated model, advertising was found to be more effective when oyster stocks were high and oyster prices low. Bockstael, Strand, and Lipton concluded that producers would initially benefit from an optimal advertising policy consisting of three years of zero advertising, followed by five years of fairly intense advertising, followed by two years of zero or low advertising, followed by four years of intense advertising, followed by constant advertising in all future periods. However, convincing promotion program managers to follow an advertising strategy with such long cycles would likely be difficult.

While gradient-type procedures such as those commonly used in nonlinear optimization routines are very efficient, such procedures have limitations when the objective function is discontinuous, not continuously differentiable, or not strictly concave. A recursive numerical approach, which is robust to these limitations, is used in this study since asymmetric functions are not continuously differentiable. A detailed discussion of the recursive approach and examples are available in Stokey, Lucas, and Prescott.

The impacts of an asymmetric advertising demand response on the optimal advertising policy have not been considered in previous generic commodity promotion research. Accordingly, a contribution of the research reported here is the determination of the optimal pulsing or uniform advertising policy when the demand response to advertising is asymmetric. Furthermore, given the non-continuously differentiable nature of asymmetric functions, a methodological contribution of this paper is the use of an alternative procedure to solve the dynamic optimization and determine the optimal advertising policy.

Model

We assume generic advertising program managers attempt to maximize the current and future revenue for a commodity by choosing the level of advertising in each period. This can be represented by maximizing the present value of all current and future revenue subject to the amount of funds available for advertising expenditures. A fixed level of funds is assumed to be made available for advertising each month. These funds, which are earmarked for advertising, can be spent on purchasing advertising in the same month they are received, or can be saved for use in future months.

Mathematically, a promotion program's objective function can be expressed as

$$v = \sum_{t=t_0}^{\infty} b^t p_t q_t \ , \qquad (1)$$

where v is the present value of all current and future revenue, p_t is the farm level price of fluid milk, and b denotes the discount rate. The fluid milk demand in month t, q_t, depends on a number of demand shifters including current and past advertising expenditures, a_t, a_{t-1}, a_{t-2}, ...

The expenditures on advertising each month are constrained by the funds available for advertising including savings from previous months and an upper bound on advertising. The upper bound on advertising represents the maximum amount of advertising that a promotion manager would purchase in a specific month. The constraints on savings, s_t, and advertising can be written as:

$$s_{t+1} = (1+r)s_t + b - a_t, \qquad (2)$$

$$0 \le a_t \le \bar{a}, \text{ and} \qquad (3)$$

$$s_{t+1} \ge 0, \text{ for } t = t_0, t_0+1, t_0+2, \ldots \qquad (4)$$

where b denotes the fixed level of funds provided for advertising each month and r is the real rate of interest. Next period savings is computed by adding current period savings plus interest to the fixed monthly level of funds provided for advertising, minus the amount spent on advertising in the current period (equation 2). Equation (3) requires the advertising level to be nonnegative and to not exceed the upper bound on advertising. The parameter, $\bar{a}$, denotes the upper bound on advertising that could be purchased in one month without possibly inducing consumer advertising fatigue among target audiences. Also large advertising purchases during the targeted time slots in one month could reduce the availability for other advertisers and potentially "bid up" the price of advertising. Savings is assumed to be nonnegative, i.e., advertising expenditures cannot be borrowed from future months. Advertising costs are implicitly assumed to be constant in the above specification.

Combining equations 2 and 4 gives $a_t \le (1+r)s_t + b$, which constrains advertising expenditures in month t to be less than or equal to total funds available for advertising. Given starting values s_{t_0} and a_{t_0-1}, a_{t_0-2} ... at current period t_0, the problem is to choose advertising levels in all current and future months, $\{a_t\}_{t=t_0}^{\infty}$, that maximize the present value of all current and future fluid milk revenue (equation 1) subject to the feasibility conditions (equations 2-4).

We adopt a previously estimated demand function that allowed for asymmetry in the demand response to fluid milk advertising (see Author Publication for details). Advertising goodwill (Nerlove and Arrow) for milk (M) and carbonated beverages (C) are measured as

$$A_t^{\ell} = \sum_{s=0}^{12} w_s^{\ell} a_{t-s}^{\ell} \text{ for } \ell = \{M, C\},$$

where a_{t-s}^{ℓ} is the average daily per capita real advertising expenditures in month *t-s* for commodity ℓ, and w_s^{ℓ} are the empirically estimated weights which reflect the accumulation and decay of goodwill depending on current and past advertising.

To estimate the impacts on demand of increases and decreases in generic milk advertising separately, advertising goodwill was segmented into increasing and decreasing parts as

$$Z_{t-i}^{I} = \max\{\ln(A_{t-i}^{M}) - \ln(A_{t-i-1}^{M}), 0\} \text{ for } i = 0,...,3, \text{ and}$$
$$Z_{t-i}^{D} = \min\{\ln(A_{t-i}^{M}) - \ln(A_{t-i-1}^{M}), 0\} \text{ for } i = 0,...,3$$

respectively. Per capita fluid milk demand was estimated as:

$$\ln(q_t) = \underset{(-2.09)}{-2.815} + \underset{(2.02)}{0.049}\ln(A_t^M) + \sum_{i=0}^{3} a_i^I Z_{t-i}^I + \sum_{i=0}^{3} a_i^D Z_{t-i}^D \underset{(-1.00)}{- 0.227}\ln(P_t^M) + \underset{(0.95)}{0.180}\ln(I_t)$$

$$\underset{(1.43)}{+ 0.382}\ln(P_t^C) \underset{(-2.12)}{- 0.025}\ln(A_t^C) \underset{(-2.12)}{- 4.065}\ln(X_t^A) + \underset{(1.94)}{2.649}\ln(X_t^H) \underset{(-3.03)}{- 0.030}D_{jun}$$

$$\underset{(-6.18)}{- 0.069}D_{jul} \underset{(-5.82)}{- 0.065}D_{aug} + e_t, \qquad (5)$$

where q_t is average daily per capita quantity of fluid milk demand in month *t*; P_t^M is the retail price of a half gallon of whole milk; I_t is per capita food and beverage expenditures in the Northeast; P_t^C is the U.S. price index for carbonated beverages; X_t^A is the percent of the population in New York City who are African American; X_t^H is the percent of the population in New York City who are Hispanic; and D_{jun}, D_{jul}, D_{aug} are dummy variables for the summer months including June, July, and August respectively, and *t*-statistics are given in parenthesis.

A contribution of equation (5) is that asymmetry in the demand response to advertising is permitted by the separate impacts on demand of increases in goodwill and decreases in goodwill. Estimates also included $a_0^D = a_1^D = -0.049$ (-2.02), $a_2^D = a_3^D = -0.034$ (-1.79), and $a_0^I = a_1^I = a_2^I = a_3^I = 0$, and advertising goodwill for fluid milk was found to depend on the current and past seven months of advertising. Symmetry of the advertising impact on demand $(H_0 : a_i^D = a_i^I \text{ for } i = 0,...,4)$ was tested and soundly rejected concluding that fluid milk demand in New York City responds in an asymmetric manner to advertising. As the coefficients suggest, demand was found to react more rapidly to increases in advertising goodwill compared to decreases. Since this analysis focuses on the impact of fluid milk advertising on demand, the empirical result that $a_i^I = 0,\ i = 0,...,4$ can be used to collapse the demand equation to

$$\ln(q_t) = b_A \ln(A_t^M) + \sum_{i=0}^{3} a_i^D Z_{t-i}^D + \Phi W_t, \qquad (6)$$

where $b_A = 0.049$ and ΦW_t represents the factors impacting fluid milk demand other than milk advertising.

Devising a general optimal temporal strategy for future fluid milk advertising is a goal of this study. To this end, all exogenous demand shifters were assumed to remain constant at their mean monthly values computed over the sample period of January 1985 to June 1995. Prior to 1996 when the dairy support price was binding, a clear seasonal pattern existed in farm-level milk prices; however, more recently a seasonal pattern in farm-level milk prices has not been apparent. Given that future milk prices would be difficult if not impossible to predict, the real fluid milk price, p_t, was held constant at its sample mean, which is denoted as *p*.

There are several variations of an optimal temporal strategy for fluid milk advertising. The dynamic problem defined in (1) through (4) could result in a steady state solution where $a_t = a^*$ for all periods beyond the initial convergence from the starting values. This describes a uniform advertising policy. Another possibility is a pulsing policy in which case a steady state solution would not exist. The optimal policy depends on the relationship between advertising and demand described in (6).

By evaluating (1)-(4) and (6), one can gain some insights on the optimal advertising strategy and the approach to obtain the optimal strategy. Because of the asymmetric nature of the advertising response, the impact of advertising on demand will depend on whether advertising goodwill is increasing or decreasing. Based on the empirical results reported above, $\frac{\partial A_{t+\ell}}{\partial a_t} = w_\ell,\ \ell = 0,...,7$. Also define an index function as:

$$i(m) = \begin{cases} 1 \; if \;\; A_m < A_{m-1} \;\; (A_m \text{ decreases from previous period}) \\ 0 \; if \;\; A_m \geq A_{m-1} \;\; (A_m \text{ does not change or increases from previous period}) \end{cases}$$

then the partial derivative of demand with respect to advertising goodwill is:

$$\frac{\partial q_{t+k}}{\partial A_t} = \frac{q_{t+k}}{A_t} \begin{Bmatrix} b_A + i(t) \cdot a_0^D & for \;\; k = 0 \\ i(t) \cdot a_k^D - i(t+1) \cdot a_{k-1}^D & for \;\; k = 1,2,3 \\ -i(t+1) \cdot a_3^D & for \;\; k = 4 \end{Bmatrix}. \qquad (7)$$

In addition, the impact of advertising in period *t* on the present value of current and future demand is:

$$\frac{\partial v}{\partial a_t} = \sum_{\ell=0}^{7} \sum_{s=0}^{4} b^{\,t+s+\ell} \, p \, \frac{\partial q_{t+s+\ell}}{\partial A_{t+\ell}} \frac{\partial A_{t+\ell}}{\partial a_t} \, .$$

If we assume constraints (3) and (4) are nonbinding, and substitute constraint (2) into (1), the first order necessary condition is:

$$\frac{\partial v}{\partial a_t} \frac{\partial a_t}{\partial s_{t+1}} + \frac{\partial v}{\partial a_{t+1}} \frac{\partial a_{t+1}}{\partial s_{t+1}} = 0 \text{, or}$$

$$\sum_{\ell=0}^{7} \sum_{s=0}^{4} b^{\,t+s+\ell} \, \frac{\partial q_{t+s+\ell}}{\partial A_{t+\ell}} w_\ell = \sum_{\ell=0}^{7} \sum_{s=0}^{4} b^{\,t+s+\ell+1} \, \frac{\partial q_{t+s+\ell+1}}{\partial A_{t+\ell+1}} w_\ell (1+r) \, . \qquad (8)$$

This is the familiar optimality condition, where the marginal benefit of an increase in advertising in period *t* is equal to the discounted marginal benefit of saving and spending the advertising funds in period *t*+1.

In order to use a typical gradient-type approach to solve for the optimal advertising level, (8) must be continuous. From (7), however, it is clear that this condition is not satisfied. Although the demand function is continuous, it has kinks, and therefore the first order necessary condition (equation 8) is not continuous everywhere. As a result, traditional solution procedures such as those used by Bockstael, Strand, and Lipton, and Liu and Forker have severe limitations when applied to this problem. We turn to an alternative approach.

Recursive Approach

The dynamic problem presented in equations (1) through (4) can also be written as a functional equation,

$$v(s_t, \mathbf{a}_{t-1}) = \max_{a_t}\{pq(a_{t,} \mathbf{a}_{t-1}, W) + bv(s_{t+1}, \mathbf{a}_t)\} \tag{9}$$

subject to the constraints (2)-(4) where $\mathbf{a}_{t-1} = (a_{t-1}, a_{t-2}, a_{t-3}, ..., a_{t-12})$. The solution to this problem consists of a value function, $v : \mathbf{R}^{13} \rightarrow \mathbf{R}$, and a corresponding policy function, $h : \mathbf{R}^{13} \rightarrow \mathbf{R}$.[2] For any starting values for s_t and $\mathbf{a}_{t-1}$, $v(s_t, \mathbf{a}_{t-1})$ gives the maximum present value of all current and future demand, and $h(s_t, \mathbf{a}_{t-1})$ gives the optimal advertising level that satisfies (9) (i.e. $a_t = h(s_t, \mathbf{a}_{t-1})$). Using $s_{t+1} = (1+r)s_t + b - a_t$, the policy function can be applied iteratively to determine the entire future path of advertising for starting values s_t and $\mathbf{a}_{t-1}$. Under the condition that feasible demand, $q(\cdot)$, is bounded, the equivalence between the problem stated in (1) and the problem stated in (9) is established by Stokey, Lucas, and Prescott (p. 39).

The recursive formulation is more general than an approach that uses the first order necessary conditions since it does not require the objective function to be continuously differentiable. In addition, the recursive methodology can be used to solve problems that may be characterized by non-steady state solutions, which is a possibility in our case if pulsing is an optimal advertising strategy. This approach is not new and is employed in a variety of economic research including real business-cycle research (see Cooley for a discussion and examples).

Although the functional form of the demand function, $q(\cdot)$, is known, the functional form of $v(\cdot)$ is not known and, as is frequently the case, cannot be determined analytically. The commonly used approach for determining $v(\cdot)$ is the method of successive approximations (Stokey, Lucas, and Prescott). A finite grid over s_t and $\mathbf{a}_{t-1}$ is defined, and an initial guess of $v(\cdot)$, $v_0(\cdot)$, is chosen (e.g. $v_0 = 0$ for all s_t and $\mathbf{a}_{t-1}$). Then, the next estimates of $v(\cdot)$ are defined iteratively,

$$v_{n+1}(s_t, \mathbf{a}_{t-1}) = \max_{a_t}\{pq(a_t, \mathbf{a}_t, W) + bv_n(s_{t+1}, \mathbf{a}_t)\},\ n = 0, 1, 2, ..., \tag{10}$$

for all values of s_t and $\mathbf{a}_{t-1}$.

The sufficient conditions for convergence of $v_n(\cdot)$ to $v(\cdot)$ defined in (10) include continuous constraints that define a nonempty and compact valued feasible set, $q(\cdot)$ bounded and continuous over the feasible set, and $0 < b < 1$. These conditions are fully met in this problem. A proof of the convergence of $v_n(\cdot)$ for a general case can be found in Stokey, Lucas, and Prescott (p. 79) and is not restated here.

If $q(a_t, \mathbf{a}_{t-1}, W)$ is strictly concave in a_t and $\mathbf{a}_t$, the policy function $h(s_t, \mathbf{a}_{t-1})$ is a continuous single valued function (Stokey, Lucas, and Prescott, p. 81). One can observe from (5) that demand is concave in a_t, but demand can be concave, convex, or neither with respect to $\mathbf{a}_t$. When advertising goodwill is declining, the curvature of demand with respect to past advertising is data dependent because of the extended carryover effect of advertising. However, when advertising goodwill is increasing (current and recent advertising is higher than past advertising), the asymmetric effect does not exist ($Z_{t-i}^D = 0,\ i = 0,\ldots,3$), and as a result, demand is strictly concave with respect to advertising. Therefore, at high levels of current and recent advertising, demand is strictly concave with respect to advertising, and in periods of low or zero current or recent advertising, the curvature of demand cannot be determined a priori. While the concavity of $q(a_t, \mathbf{a}_{t-1}, W)$ with respect to $\mathbf{a}_t$ is not guaranteed by the functional form as discussed above, many starting values were attempted and all resulted in identical long-term solutions.

Optimization Procedure

Although not a complicated procedure, solving for the value function, $v(\cdot)$, was computationally intensive. The lagged advertising levels are state variables in this dynamic optimization problem. As a result, this analysis encountered the full brunt of the well-known "curse of dimensionality." The value function was defined on a grid of the state variables, s_t and $\mathbf{a}_{t-1}$, and linear interpolation was used to evaluate $v_n(s_{t+1}, \mathbf{a}_t)$ at values of s_{t+1} and $\mathbf{a}_t$ between the grid points. In the optimizations, $v_n(\cdot)$ was defined over six equidistant values of savings, s_t, and five equidistant values for each current and past advertising variable in $\mathbf{a}_t$. Details on this approach and other finite element techniques are provided in Judd. Interpolation allows the right hand side of (10) to be continuous in a_t, and a bisection procedure (Press et al.) was used to solve for a_t that maximizes the value function.

In the optimizations, the discount rate, b , was set to 0.995, which is 0.5 percent monthly or approximately 6 percent annually. The real monthly interest rate, *r*, was set to 0.0025 or 3 percent per annum, which is the average real rate of return on six month treasury bills in the past decade. Also, the level of real expenditures available for advertising each month, *b*, was $851 per day per million people. This value is the average real expenditure on actual fluid milk advertising in the sample period January 1986 to June 1995. The units of s_t and $\mathbf{a}_{t-1}$ were also in terms of real expenditures per day per million people.

Although the grid of state variables was fairly coarse, iterating over all possible values of s_t and $\mathbf{a}_{t-1}$ in equation (10) was very computationally intensive. To reduce computational requirements, the effects on goodwill of advertising six and seven months in the past were approximated by a geometric lag (Greene) as follows,

$$A_t^M = \sum_{s=0}^{4} w_s^M a_{t-s}^M + E_t^M \text{ , where}$$

$$E_t^M = l E_{t-1}^M + w_5^M a_{t-5}^M \text{ and } l = 0.2415 .$$

Additional details of the approximation are provided in an appendix, and the resulting implicit values for w_6 and w_7 are shown to be very close to the empirically determined weights. This approximation reduced the number of state variables and significantly improved the computational feasibility of the optimization.

The procedure for solving for the optimal value function is straightforward. First, the beginning value function, $v_0(\cdot)$, was set to an initial guess. Next, equation (10) was solved for all feasible s_t and $\mathbf{a}_{t-1}$ to get $v_1(\cdot)$. Again, using (10), $v_2(\cdot)$ was determined, and this process was repeated until the iterations converged. Convergence occurred when the distance between $v_n(\cdot)$ and $v_{n-1}(\cdot)$ over all s_t and $\mathbf{a}_{t-1}$ was very small or zero. At this point, the converged value function was the optimal value function that solved (9). The optimal policy function, $h(s_t, \mathbf{a}_{t-1})$, was determined at the same time and was given the value of the optimal advertising level in period *t* when savings was s_t and past advertising was $\mathbf{a}_{t-1}$.[3]

A number of optimizations were performed to determine the optimal advertising policy for fluid milk in New York City and test the sensitivity of the results to various assumptions. The main optimizations were performed with and without an upper bound on monthly advertising, $\bar{a}$.[4] Similar to Bockstael, Strand, and Lipton and as discussed earlier, the upper bound on advertising represents the maximum amount of advertising that can be spent in one month. Based on consultations with fluid milk advertising managers in New York, increasing advertising levels more than 300 percent may result in consumer fatigue from repetitive advertising, especially among target audiences in the television media market. Also large advertising purchases during limited time slots could increase the price of advertising.

The optimal advertising policy was determined for three possible upper bounds on advertising including two times, two and one-half times, and three times the historical average advertising level. To better understand the specific relationship between advertising and demand, results were also obtained when the upper bound on monthly advertising was removed.[5] The optimal advertising policies for these four optimizations were compared to the results from two additional advertising policies. These two include a uniform advertising policy, where advertising was held constant at *b* every month, and the actual advertising policy for the period January 1986 to June 1995. It is important to note that the sum of real advertising expenditures over time is the same for all advertising policies.

Careful analyses should evaluate the sensitivity of the optimal advertising policies to any assumptions. Therefore, sensitivity of the results to demand seasonality and changes in the advertising grid were investigated. Also several additional optimizations were performed to evaluate the impact of changes in asymmetry on the optimal advertising policy. These exercises are discussed in detail in the results.

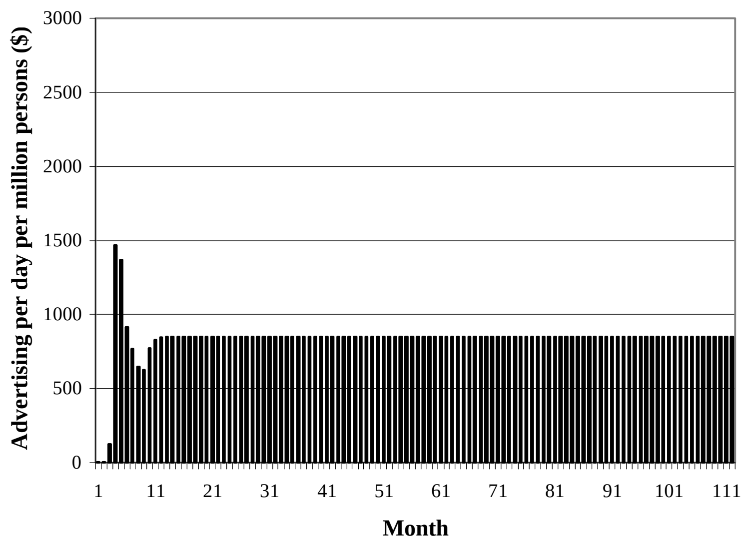
Fluid milk advertising from July 1994 to June 1995 (denoted as periods one through twelve in the optimizations) was used as the starting values for advertising, $\mathbf{a}_{12}$. Also, the starting value for savings, that is, savings in period one, was set to zero ($s_1 = 0$). For each optimization, the optimal policy function was used to determine the optimal fluid milk advertising and the corresponding fluid milk demand for one hundred months beginning in month thirteen. Using the same starting values, fluid milk demand was also determined for the uniform and the actual advertising policies.

Results

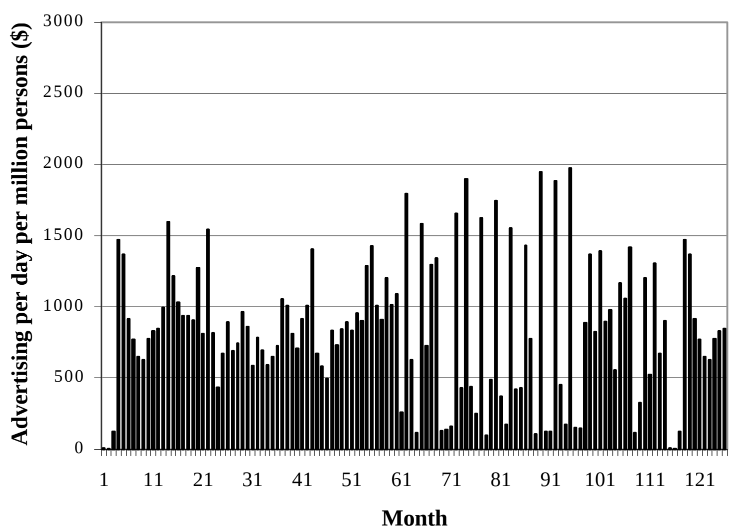
Optimal Advertising Policies

Figure 1 shows the uniform advertising policy and the actual advertising policy. The four optimal advertising policies, which were obtained using the recursive optimization procedure described above, are shown in figure 2. The sum of real advertising expenditures over months is the same for all six advertising policies.

Figure 1. Advertising Expenditures for the Uniform and Actual Advertising Policies



Uniform Advertising Policy



Actual Advertising Policy

In the uniform advertising case, advertising was constant at $851 per million people per day after the initial twelve months of starting values. The average daily advertising for January 1986 to June 1995 was used after the starting values in the actual advertising policy. One can observe that the actual advertisingexhibits some pulsing, especially between months 73 and 98 (January 1992–January 1994) where a pattern of two months of low advertising followed by one month of high advertising is prevalent.

The first optimization (top panel of figure 2) shows the optimal advertising policy when maximum advertising was two times the historical average advertising level ($\bar{a} = \$1{,}702$). A clear "steady cycle" emerges where, after the initial convergence from the starting values, the optimal policy is three months of zero advertising followed by three months of advertising at the maximum level, $\bar{a}$. Convergence to this pulsing strategy is relatively short—by the twenty-seventh month, the optimal advertising pattern has already emerged.

Figure 2. Optimal Feasible Advertising Expenditures

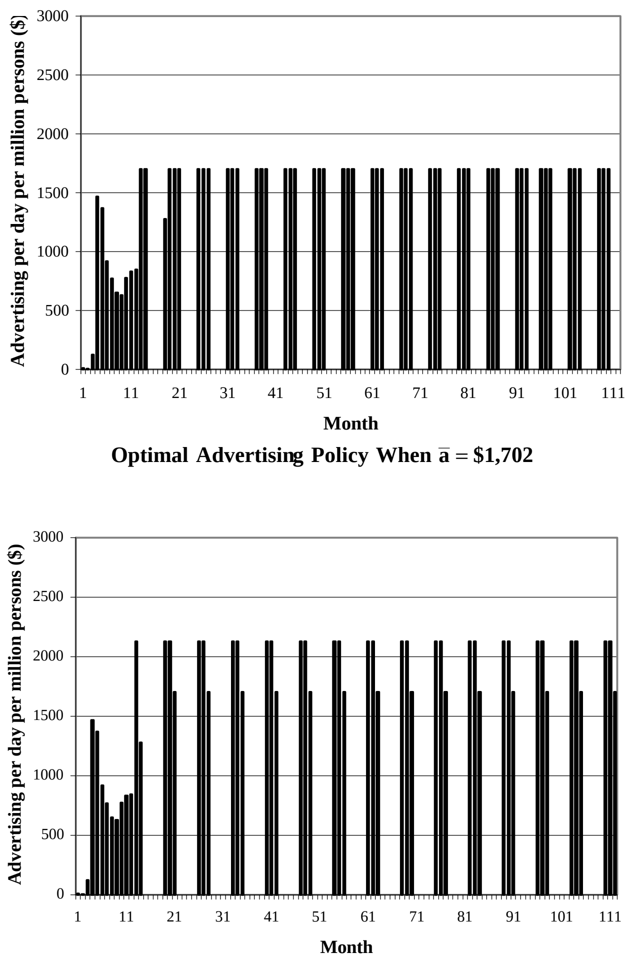
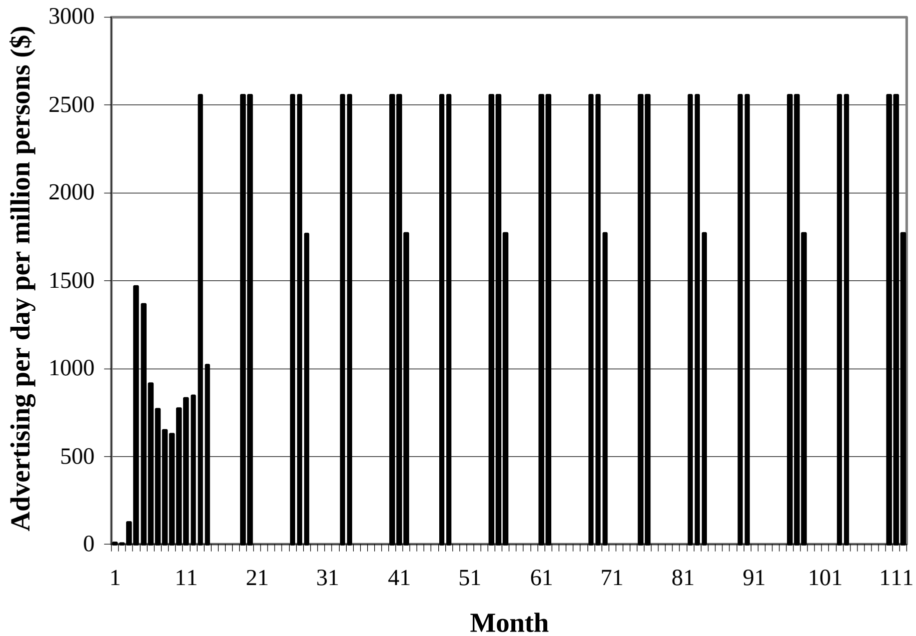
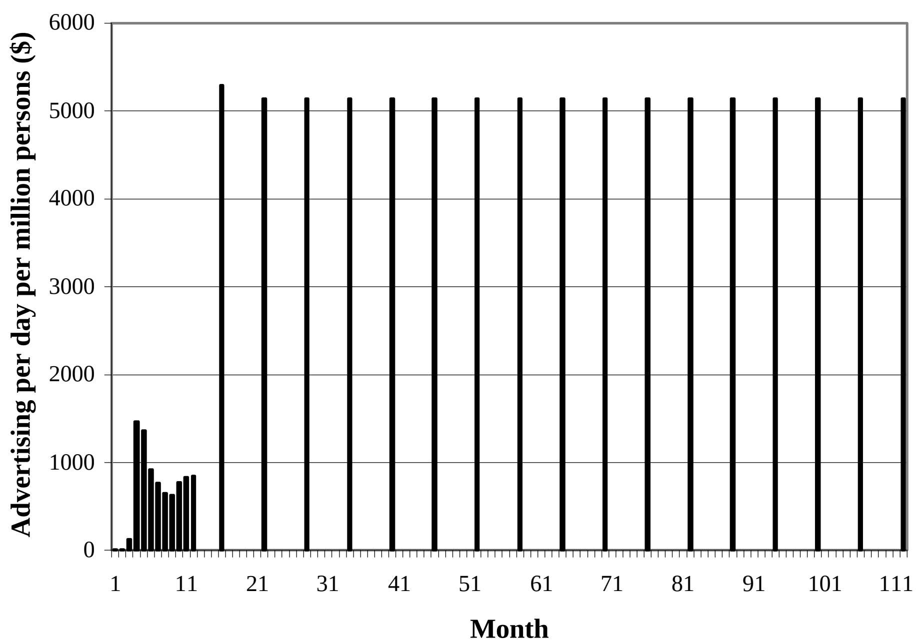


Figure 2. Optimal Feasible Advertising Expenditures (continued)



Optimal Advertising Policy When $\bar{a} = \$2{,}553$



Optimal Advertising Policy With No Upper Bound on Monthly Advertising

The second and third optimal advertising policies, also shown in figure 2, exhibit similar pulsing patterns. In the second optimal advertising policy, where the maximum advertising level was $2,128, the stable pattern includes four months of zero advertising followed by two months of advertising at $\bar{a}$, followed by one month of adverting at $1,731. The third optimal advertising policy, where the maximum advertising level was $2,553, consists of two alternating seven-month cycles. The stable pulsing pattern shows four months of zero advertising followed by two months of advertising at $\bar{a}$, followed by one month of advertising at zero or $1,769, depending on the cycle.

In the fourth optimal advertising policy, also shown in figure 2, where the upper bound on monthly advertising was removed, the optimal pulsing pattern consists of five months of zero advertising followed by one month of advertising at $5,138. While this result may not be unrealistic, it should be treated with caution since the pulse of advertising is significantly outside the range of the data, and the error associated with the corresponding fluid milk demand may be very large. Moreover, as discussed earlier, large purchases of advertising during the targeted time slots in one month could potentially drive up the price of advertising.

The optimal advertising policies show advertising to be a periodic function of time, i.e. $a(t) = a(t-k)$ for a given period (length of cycle in periodic function) k and for all months t after convergence. While the total cycle length for the first optimal policy was six months, the cycle length is seven months for the second optimal policy and an alternating seven month cycle for the third optimal policy. Similar to the first, the fourth optimal policy shows a six-month cycle length. All four optimizations show a clear convergence to a pulsing strategy.

Sensitivity Analyses

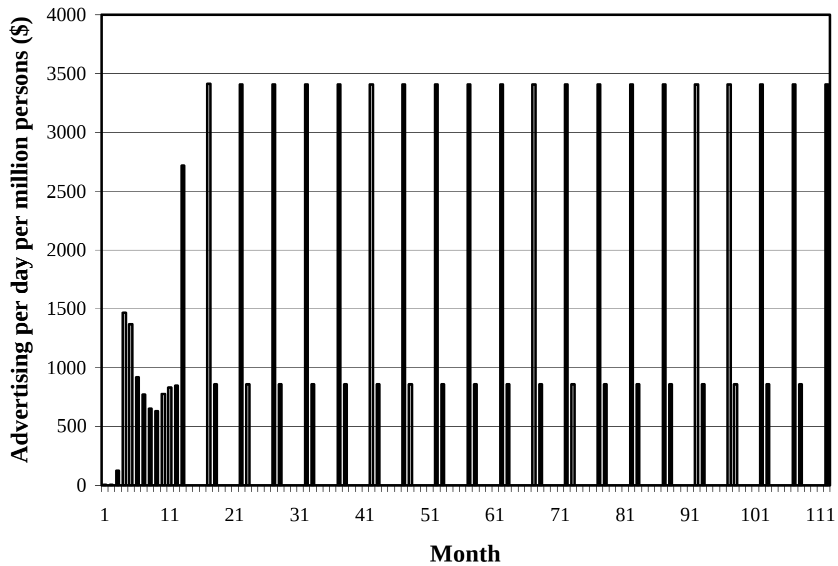
In the first set of several sensitivity analyses, seasonality in demand was incorporated into the four original optimization problems. In these optimizations, a separate value function was specified for every month of the year and the coefficients on the summer dummy variables for June, July, and August in the fluid milk demand function were explicitly incorporated into the problem.[6] One can show from the demand equation that in the summer months when demand is at a seasonal low, the marginal impact of advertising goodwill on demand is also at a seasonal low. While incorporating seasonality into the problem had a small impact on the value functions, none of the long-term optimal patterns of advertising found in our original optimizations were affected. This result is not entirely surprising since variation in the marginal impact of goodwill on demand due to seasonality (equation 5) is relatively small compared to the impacts of asymmetry on the advertising-demand response.

To evaluate the sensitivity of the results to the choice of grid points, the internal grid points for advertising were shifted up by twenty-five percent of the distance between the equidistant points for each of the four previously described optimizations. Similar to the seasonality results, slight shifts in the grid points had only small impacts on the value function and no impact on the long-term optimal patterns of advertising. In general, the optimal advertising policies were found to be quite robust to placement of the grid points as long as the distance between any two points did not become too large.

Finally, to better understand the impact of asymmetry on the optimal advertising policies, three additional optimizations were completed. For the original problem with unbounded monthly advertising, the asymmetry coefficients, a_i^D, i=0,...,3, were reduced in absolute value by twenty-five percent, fifty percent, and seventy-five percent. These changes represent a reduction in the carryover effect of advertising goodwill when goodwill is declining, and also a corresponding reduction in asymmetry in the demand response to goodwill.

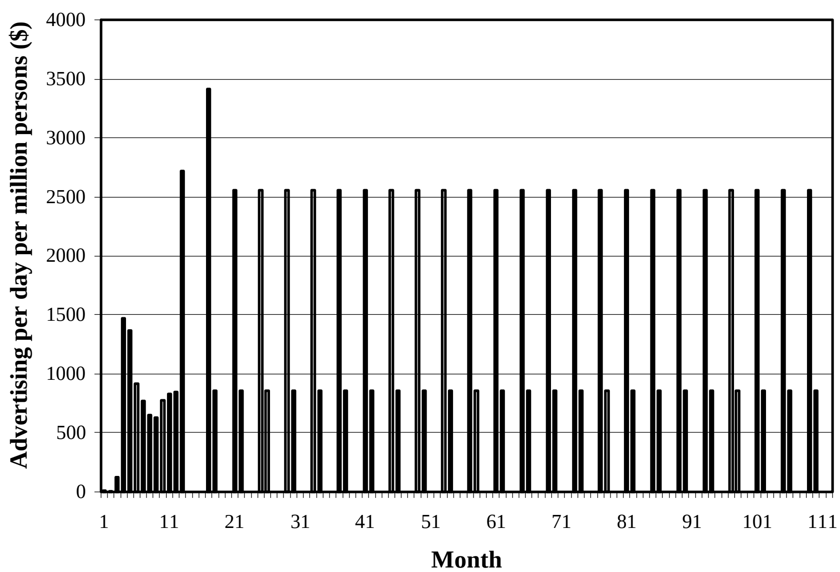
While the optimal pulsing pattern did not change for a twenty-five percent reduction in the asymmetry coefficients, a fifty percent reduction resulted in a five month pulsing cycle characterized by three months of zero advertising followed by one month of advertising at $3,410, followed by one month of advertising at $857. A seventy-five percent reduction resulted in a four month pulsing cycle consisting of two months of zero advertising followed by one month of advertising at $2,555, followed by one month of advertising at $855. These results, which are displayed in figure 3, suggest that as the asymmetric advertising carryover declines, optimal advertising cycles become shorter and pulses become more spread out.

Figure 3. Optimal Advertising Expenditures with Changes in Asymmetry



Asymmetry Coefficients Reduced 50%

$(a_0^D = a_1^D = -0.0245,\ a_2^D = a_3^D = -0.017)$



Asymmetry Coefficients Reduced 75%

$(a_0^D = a_1^D = -0.01225,\ a_2^D = a_3^D = -0.0085)$

Impacts on Demand

More important than displaying the optimal advertising strategy is the impact of alternative advertising policies on revenue. Average daily per capita fluid milk demand for the six alternative advertising strategies is displayed in table 1. For the four optimal advertising strategies, the average fluid milk demand was computed by averaging demand over the months in a cycle. For example, in the first optimization, average demand was computed by taking the mean of demand over the six-month steady cycle, which is the long-term average demand if the advertising cycle is continued into the infinite future. Averaging demand over a fixed finite time period would bias results depending on where the cycle ended in the finite period. For the uniform and actual advertising policies, however, average demand was computed by taking the mean of demand over the entire time period beginning after the starting values in month thirteen.

Table 1. Comparison of Advertising Policies

Advertising Policy	Average Demand Per Capita Per Day (lbs)	Demand Relative to Uniform Advertising Policy
Uniform	0.57003	
Actual	0.57682	1.2%
Policy 1 ($\bar{a} = \$1{,}702$)	0.59433	4.3%
Policy 2 ($\bar{a} = \$2{,}128$)	0.59763	4.8%
Policy 3 ($\bar{a} = \$2{,}553$)	0.59919	5.1%
Policy 4 (Unbounded)	0.60513	6.2%

While the actual advertising policy performs better than the uniform advertising policy, the optimal pulsing strategies give significant improvement in demand over both the uniform and the actual advertising policies. The occasional pulsing that occurs in the actual policy likely causes it to dominate the uniform advertising policy. By using pulsing strategies, demand is shown to be 4.3 to 6.2 percent greater relative to the results of the uniform advertising policy. The optimal pulsing advertising strategies are shown to give a 3.0 to 4.9 percent higher demand compared to the actual advertising policy used in the period January 1986 through June 1995. Again, caution must be applied to the results of the fourth optimal advertising policy since the policy includes advertising significantly outside the sample advertising data. Overall, however, these results suggest that managers of this program should consider a more systematic and pronounced pulsing pattern.

The optimal pulsing patterns of advertising found here differ from the optimal seasonal allocations of advertising proposed by Kinnucan and Forker, and Liu and Forker. The results in the latter two studies were heavily dependent on seasonal farm-level blend prices, whereas in the present study, the optimal patterns of advertising are driven by the demand response to advertising.

Summary and Conclusion

While most previous studies that have addressed optimal advertising over time have assumed a symmetric demand response to advertising, this paper determined the optimal advertising policy when the demand response is asymmetric. An analytical framework was developed to evaluate optimal advertising strategies, where promotion program managers were assumed to maximize the present value of all current and future fluid milk revenue. Using the empirical results of a demand equation obtained from Author Publication, the optimal temporal allocation of advertising was determined.

Due to the asymmetric nature of the demand response to advertising, the demand function for fluid milk is not continuously differentiable. As a result, traditional gradient-type solution procedures could not be used to solve the dynamic problem. An alternative, more robust recursive approach was employed to solve for the optimal advertising policy.

As an application to fluid milk demand in New York City, this study suggests that, while holding total advertising expenditures unchanged, a pulsing advertising policy is significantly more effective at increasing demand than a uniform advertising policy. The optimal advertising policy can be characterized as a six to seven month repeated cycle consisting of several months of zero advertising followed by several months of intense advertising.

Sensitivity analyses demonstrated that the results are robust to changes in the specification of the grid on which the value function is defined and also are not affected by including demand seasonality. Furthermore, additional optimizations found that reductions in the asymmetry of the demand response to advertising resulted in shorter optimal pulsing cycles and less intense advertising pulses.

Although a pulsing advertising policy clearly dominates a uniform advertising policy in this analysis, further research that analytically characterizes the relationship between demand asymmetry and optimal advertising would be beneficial. In order for analytical progress to be made, however, the obstacle of non-continuous differentiability of asymmetric functions has to be circumvented.

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Appendix

To reduce the dimensions of the dynamic optimization problem, the impact on goodwill of the advertising six and seven months in the past was approximated by an geometric lag as follows:

$$A_t^M = \sum_{s=0}^{4} w_s^M a_{t-s}^M + E_t^M \text{, where}$$

$$E_t^M = l E_{t-1}^M + w_5^M a_{t-5}^M . \qquad \text{(A1)}$$

The value of l was chosen so that the implicit approximations for w_6^M and w_7^M are very close to the values that were determined empirically. Advertising goodwill for fluid milk was originally determined as $A_t^M = \sum_{s=0}^{7} w_s^M a_{t-s}^M$. From (A1), E_t^M is by definition equal to the sum of an infinite series as follows:

$$E_t^M = \sum_{s=0}^{\infty} l^s w_5^M a_{t-5-s} .$$

l was chosen so that the sum of approximated weights on past advertising equal the sum of empirically estimated weights. This implies:

$$\sum_{s=5}^{7} w_s^M = \sum_{s=0}^{\infty} l^s w_5^M . \qquad \text{(A2)}$$

Using $\sum_{s=0}^{\infty} l^s = \frac{1}{1-l}$, rearrangement of (A2) to solve for l results in:

$$l = \frac{\frac{1}{w_5^M}\sum_{s=6}^{7} w_s^M}{1+\frac{1}{w_5^M}\sum_{s=6}^{7} w_s^M} .$$

The fluid milk advertising weights, $\{w_s^M\}_{s=0}^{12}$, normalized to sum to one, were empirically estimated to be 0.001, 0.012, 0.082, 0.255, 0.356, 0.223, 0.063, 0.008, 0.0, 0.0, 0.0, 0.0, and 0.0 respectively (Author Publication). Using the values for w_5^M, w_6^M, and w_7^M, l equals 0.2415, which was used in the simulations. As a result, the implicit values for w_6^M , and w_7^M are $l w_5^M = 0.054$ and $l^2 w_5^M = 0.013$ respectively. These, as suggested earlier, are close approximations to the empirically estimated values. Fortunately the geometric lag provides a good approximation because decreasing the dimensions of the original problem allowed the problem to become computationally feasible.

Endnotes

[1] The blend price, which is paid to farmers, is a weighted combination of the prices for fluid milk and processed dairy products, and the weights depend on the seasonally adjusted percent of raw milk attributed to fluid consumption.

[2] $h(\cdot)$ could be a correspondence. However, it was not found to be a correspondence in this study.

[3] The solution procedure was written in the C programming language and the optimizations were solved on a RISC System/6000 running IBM's UNIX operating system AIX.

[4] $\bar{a}$ is also measured in units of real expenditures per day per million people.

[5] The grid of savings for the first three optimizations was six equidistant points between and including zero and $5,106. Tests confirmed that the choice of the maximum savings did not impact results. Similarly, the grid of monthly advertising consisted of five equidistant points between and including zero and the maximum advertising level. For example, for the second optimization, the advertising grid was $0, $532, $1,064, $1,596, and $2,128. For the fourth optimization, the grid of savings was six equidistant points between and including zero and $6,808. The grid of advertising was five equidistant points also between and including zero and $6,808.

[6] In the seasonality optimizations, the grid for advertising was reduced to three equidistant points to allow computational feasibility. The seasonality results were then compared to the results

without seasonality using the same advertising grid. Although not identical, the results using the reduced grid were very similar to the results for the original optimizations.