Error-correction–based cointegration tests for panel data

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Abstract.   This article describes a new Stata command called `xtwest`, which implements the four error-correction–based panel cointegration tests developed by Westerlund (2007). The tests are general enough to allow for a large degree of heterogeneity, both in the long-run cointegrating relationship and in the short-run dynamics, and dependence within as well as across the cross-sectional units.

1   Introduction

The use of panel cointegration techniques to test for the presence of long-run relationships among integrated variables with both a time-series dimension, T, and a cross-sectional dimension, N, has received much attention recently, especially in the empirical literature. One of the most important reasons for this attention is the increased power that may be gained by accounting not only for the time-series dimension but also for the cross-sectional dimension. In spite of this, many studies fail to reject the no-cointegration null, even in cases where cointegration is strongly suggested by theory.

One explanation for this failure to reject centers on the fact that most residual-based cointegration tests, both in pure time series and in panels, require that the long-run parameters for the variables in their levels are equal to the short-run parameters for the variables in their differences. Banerjee, Dolado, and Mestre (1998) and Kremers, Ericsson, and Dolado (1992) refer to this as a common-factor restriction and show that its failure can cause a significant loss of power for residual-based cointegration tests.

As a response to this, Westerlund (2007) developed four new panel cointegration tests that are based on structural rather than residual dynamics and, therefore, do not impose any common-factor restriction. The idea is to test the null hypothesis of no cointegration by inferring whether the error-correction term in a conditional panel error-correction model is equal to zero. The new tests are all normally distributed and are general enough to accommodate unit-specific short-run dynamics, unit-specific trend and slope parameters, and cross-sectional dependence. Two tests are designed to test

the alternative hypothesis that the panel is cointegrated as a whole, while the other two test the alternative that at least one unit is cointegrated.

In this paper, we develop a new Stata command, called `xtwest`, that implements these tests.

2 The error-correction tests

The error-correction tests assume the following data-generating process:

$$\Delta y_{it} \;=\; \delta_i' d_t + \alpha_i(y_{i,t-1} - \beta_i' x_{i,t-1}) + \sum_{j=1}^{p_i} \alpha_{ij} \Delta y_{i,t-j} + \sum_{j=-q_i}^{p_i} \gamma_{ij} \Delta x_{i,t-j} + e_{it} \qquad (1)$$

where $t = 1, \ldots, T$ and $i = 1, \ldots, N$ index the time-series and cross-sectional units, respectively, while d_t contains the deterministic components, for which there are three cases. In the first case, $d_t = 0$ so (1) has no deterministic terms; in the second case, $d_t = 1$ so Δy_{it} is generated with a constant; and in the third case, $d_t = (1, t)'$ so Δy_{it} is generated with both a constant and a trend. For simplicity, we model the K-dimensional vector x_{it} as a pure random walk such that Δx_{it} is independent of e_{it}, and we further assume that these errors are independent across both i and t.[1] We will handle any dependence across i by means of bootstrap methods.

We can write (1) as

$$\Delta y_{it} \;=\; \delta_i' d_t + \alpha_i y_{i,t-1} + \lambda_i' x_{i,t-1} + \sum_{j=1}^{p_i} \alpha_{ij} \Delta y_{i,t-j} + \sum_{j=-q_i}^{p_i} \gamma_{ij} \Delta x_{i,t-j} + e_{it} \qquad (2)$$

where $\lambda_i' = -\alpha_i \beta_i'$. The parameter α_i determines the speed at which the system corrects back to the equilibrium relationship $y_{i,t-1} - \beta_i' x_{i,t-1}$ after a sudden shock. If $\alpha_i < 0$, then there is error correction, which implies that y_{it} and x_{it} are cointegrated; if $\alpha_i = 0$, then there is no error correction and, thus, no cointegration. Thus we can state the null hypothesis of no cointegration as $H_0 \colon \alpha_i = 0$ for all i. The alternative hypothesis depends on what is being assumed about the homogeneity of α_i. Two of the tests, called group-mean tests, do not require the α_is to be equal, which means that H_0 is tested versus $H_1^g \colon \alpha_i < 0$ for at least one i. The second pair of tests, called panel tests, assume that α_i is equal for all i and are, therefore, designed to test H_0 versus $H_1^p \colon \alpha_i = \alpha < 0$ for all i.

2.1 Computing the group-mean tests

We can construct the group-mean tests in three steps. The first step is to estimate (2) by least squares for each unit i, which yields

$$\Delta y_{it} \;=\; \widehat{\delta}_i' d_t + \widehat{\alpha}_i y_{i,t-1} + \widehat{\lambda}_i' x_{i,t-1} + \sum_{j=1}^{p_i} \widehat{\alpha}_{ij} \Delta y_{i,t-j} + \sum_{j=-q_i}^{p_i} \widehat{\gamma}_{ij} \Delta x_{i,t-j} + \widehat{e}_{it} \qquad (3)$$

---

1. As usual, we also require that x_{it} is not cointegrated, in case we have multiple regressors.

where the lag and lead orders, p_i and q_i, are permitted to vary across individuals and can be determined preferably by using a data-dependent rule.[2]

Having obtained $\widehat{e}_{it}$ and $\widehat{\gamma}_{ij}$, the second step is to compute

$$\widehat{u}_{it} \;=\; \sum_{j=-q_i}^{p_i} \widehat{\gamma}_{ij}\Delta x_{i,t-j} + \widehat{e}_{it}$$

which we then use to obtain $\widehat{\alpha}_i(1) = \widehat{\omega}_{ui}/\widehat{\omega}_{yi}$, where $\widehat{\omega}_{ui}$ and $\widehat{\omega}_{yi}$ are the usual Newey and West (1994) long-run variance estimators based on $\widehat{u}_{it}$ and Δy_{it}, respectively.[3]

The third step is to compute the group-mean tests in the following way:

$$G_\tau \;=\; \frac{1}{N}\sum_{i=1}^{N}\frac{\widehat{\alpha}_i}{\mathrm{SE}(\widehat{\alpha}_i)}\,, \quad G_\alpha \;=\; \frac{1}{N}\sum_{i=1}^{N}\frac{T\widehat{\alpha}_i}{\widehat{\alpha}_i(1)}$$

where $\mathrm{SE}(\widehat{\alpha}_i)$ is the conventional standard error of $\widehat{\alpha}_i$.

2.2 Computing the panel tests

We can also compute the panel tests in three steps. The first step is the same as for the group-mean tests and involves regressing Δy_{it} and $y_{i,t-1}$ onto d_t, the lags of Δy_{it}, and the contemporaneous and lagged values of Δx_{it}. This yields the projection errors

$$\Delta\widetilde{y}_{it} \;=\; \Delta y_{it} - \widehat{\delta}_i' d_t - \widehat{\lambda}_i' x_{i,t-1} - \sum_{j=1}^{p_i}\widehat{\alpha}_{ij}\Delta y_{i,t-j} - \sum_{j=-q_i}^{p_i}\widehat{\gamma}_{ij}\Delta x_{i,t-j}$$

and

$$\widetilde{y}_{i,t-1} \;=\; y_{i,t-1} - \widetilde{\delta}_i' d_t - \widetilde{\lambda}_i' x_{i,t-1} - \sum_{j=1}^{p_i}\widetilde{\alpha}_{ij}\Delta y_{i,t-j} - \sum_{j=-q_i}^{p_i}\widetilde{\gamma}_{ij}\Delta x_{i,t-j}$$

The second step is to make use of $\Delta\widetilde{y}_{it}$ and $\widetilde{y}_{i,t-1}$ in estimating the common error-correction parameter, α, and its standard error. In particular, we compute

$$\widehat{\alpha} \;=\; \left(\sum_{i=1}^{N}\sum_{t=2}^{T}\widetilde{y}_{i,t-1}^2\right)^{-1}\sum_{i=1}^{N}\sum_{t=2}^{T}\frac{1}{\widehat{\alpha}_i(1)}\widetilde{y}_{i,t-1}\Delta\widetilde{y}_{it}$$

The standard error of $\widehat{\alpha}$ is

$$\mathrm{SE}(\widehat{\alpha}) \;=\; \left((\widehat{S}_N^2)^{-1}\sum_{i=1}^{N}\sum_{t=2}^{T}\widetilde{y}_{i,t-1}^2\right)^{-1/2}$$

---

2. By adding leads and not just lags of Δx_{it}, we can allow for regressors that are weakly but not necessarily strictly exogenous.

3. This estimation procedure does not account for any deterministic terms. To correct for this, Δy_{it} in ω_{yi}^2 has to be replaced by the fitted residuals from a first-stage regression of Δy_{it} onto d_t.

where $\widehat{S}_N^2 = 1/N \sum_{i=1}^N \widehat{\sigma}_i/\widehat{\alpha}_i(1)$, with $\widehat{\sigma}_i$ being the estimated regression standard error in (3).

The third step is to compute the panel statistics as

$$P_\tau = \frac{\widehat{\alpha}}{\mathrm{SE}(\widehat{\alpha})}, \quad P_\alpha = T\widehat{\alpha}$$

2.3 Asymptotic test distribution

The asymptotic distribution of the error-correction tests is based on the sequential-limit theory, in which T is taken to infinity before N is. This implies that the tests may be justified in cases where T is substantially larger than N.

Let us define

$$C_i = \left(\int_0^1 U_i^2, \ \int_0^1 U_i dV_i \right)', \quad \widetilde{C}_i = \left(\frac{\int_0^1 U_i dV_i}{\int_0^1 U_i^2}, \ \frac{\int_0^1 U_i dV_i}{\left(\int_0^1 U_i^2 \right)^{1/2}} \right)'$$

where

$$U_i = V_i - \left(\int_0^1 V_i(W_i^d)' \right) \left(\int_0^1 W_i^d(W_i^d)' \right)^{-1} W_i^d$$

where $W_i^d = (d', W_i')'$, d is the limiting trend function, and V_i and W_i are scalar and K-dimensional standard Brownian motions that are independent of each other.[4] Let Θ and $\widetilde{\Theta}$ denote the mean values of C_i and $\widetilde{C}_i$, respectively, and let Σ and $\widetilde{\Sigma}$ denote their respective variances. Under the assumptions laid out above and the null hypothesis H_0, as $T \to \infty$ and then $N \to \infty$, sequentially,

$$H_j - \sqrt{N}(\Theta_j^H) \ \Rightarrow \ N\left(0, \Sigma_j^H\right) \tag{4}$$

where $H = \left(\sqrt{N}G_\alpha, \sqrt{N}G_\tau, \sqrt{N}P_\alpha, P_\tau \right)$ is the vector of tests, while

$$\Theta^H = \left(\widetilde{\Theta}_1, \widetilde{\Theta}_2, \frac{\Theta_2}{\Theta_1}, \frac{\Theta_2}{\sqrt{\Theta_1}} \right), \quad \Sigma^H = \left(\widetilde{\Sigma}_{11}, \widetilde{\Sigma}_{22}, \phi'\Sigma\phi, \varphi'\Sigma\varphi \right)$$

with

$$\phi = \left(-\frac{\Theta_2}{\Theta_1^2}, \frac{1}{\Theta_1} \right)', \quad \varphi = \left(-\frac{\Theta_2}{2\Theta_1^{3/2}}, \frac{1}{\sqrt{\Theta_1}} \right)'$$

as the associated mean and variance vectors. In other words, to test the null hypothesis of no cointegration based on the moments in Θ, $\widetilde{\Theta}$, Σ, and $\widetilde{\Sigma}$, we simply compute the value of the normalized test H_j with $j = 1, \ldots, 4$ so that it is in the form specified in (4). This value is then compared with the left tail of the normal distribution. Large negative values imply that the null hypothesis should be rejected.

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4. For notational simplicity, in this paper, the Brownian motions $V_i(r)$ and $W_i(r)$ defined on the interval $r \in [0,1]$ are written V_i and W_i, respectively, with the measure of integration omitted.

2.4   Cross-sectional dependence

We can generalize the results of the previous sections to account for cross-sectional dependence by using the bootstrap approach of Westerlund (2007). The method resembles that used by Chang (2004) and proceeds as follows.

The first step is to fit the least-squares regression,

$$\Delta y_{it} \;=\; \sum_{j=1}^{p_i} \widehat{\alpha}_{ij} \Delta y_{i,t-j} + \sum_{j=-q_i}^{p_i} \widehat{\gamma}_{ij} \Delta x_{i,t-j} + \widehat{e}_{it} \tag{5}$$

and then to form the vector $\widehat{w}_t = (\widehat{e}_t', \Delta x_t')'$, where $\widehat{e}_t$ and Δx_t are vectors of stacked observations on $\widehat{e}_{it}$ and Δx_{it}, respectively. We then generate bootstrap samples $w_t^* = (e_t^{*\prime}, \Delta x_t^{*\prime})'$ by sampling with replacement the centered residual vector,

$$\widetilde{w}_t \;=\; \widehat{w}_t - \frac{1}{T-1}\sum_{j=1}^{T} \widehat{w}_j$$

The next step is to generate the bootstrap sample, Δy_{it}^*. We accomplish this by first constructing the bootstrap version of the composite error, u_{it}, as

$$u_{it}^* \;=\; \sum_{j=-q_i}^{p_i} \widehat{\gamma}_{ij} \Delta x_{i,t-j}^* + e_{it}^*$$

where the least-squares estimate $\widehat{\gamma}_{ij}$ is obtained from (5). Given p_i initial values, we then generate Δy_{it}^* recursively from u_{it}^* as

$$\Delta y_{it}^* \;=\; \sum_{j=1}^{p_i} \widehat{\alpha}_{ij} \Delta y_{i,t-j}^* + u_{it}^*$$

where we again obtain $\widehat{\alpha}_{ij}$ from (5). We initiate the recursion by generating excess values of Δy_{it}^*, which we then discard. Because this makes the initiation unimportant, we may simply use zeros.

Finally, we generate y_{it}^* and x_{it}^* with the null hypothesis imposed in the following way:

$$y_{it}^* \;=\; y_{i0}^* + \sum_{j=1}^{t} \Delta y_{ij}^*, \quad x_{it}^* \;=\; x_{i0}^* + \sum_{j=1}^{t} \Delta x_{ij}^*$$

which again requires initiation through x_{i0}^* and y_{i0}^*. The value zero will do.

Having obtained the bootstrap sample y_{it}^* and x_{it}^*, we then obtain the bootstrapped error-correction test of interest. We denote this initial bootstrap test by t_1^*. If we repeat this procedure, say, S times, we obtain $t_1^*, \ldots, t_S^*$, the bootstrap distribution of the test. For a one-sided 5% nominal-level test, we then obtain the lower 5% quantile, say, t_C^*,

of this distribution. We reject the null hypothesis if the calculated sample value of the statistic is smaller than t_C^*.

Simulation results for all tests, including the bootstrapped versions, can be found in Westerlund (2007).

3 The xtwest command

3.1 Syntax

xtwest *depvar indepvars* $\begin{bmatrix} if \end{bmatrix}$ $\begin{bmatrix} in \end{bmatrix}$, lags(# $\begin{bmatrix} \# \end{bmatrix}$) leads(# $\begin{bmatrix} \# \end{bmatrix}$)
   lrwindow(#) $\begin{bmatrix}$ constant trend bootstrap(#) westerlund noisily $\end{bmatrix}$

3.2 Options

lags(# $\begin{bmatrix} \# \end{bmatrix}$) specifies the number of lags to be included in the error-correction equations. If one number is specified, it determines a fixed number of lags, p. If two numbers are specified, the Akaike information criterion (AIC) is used to determine an optimal lag length, p_i, for each separate time series, within the given limits.

leads(# $\begin{bmatrix} \# \end{bmatrix}$) specifies the number of leads to be included in the error-correction equations; this is similar to the lags() option.

lrwindow(#) sets the width of the Bartlett kernel window used in the semiparametric estimation of long-run variances.

constant adds a constant to the cointegration relationship.

trend allows for a deterministic trend in the cointegration relationship.

bootstrap(#) shows bootstrapped p-values for all four test statistics. These are robust in the presence of common factors in the time series. The argument determines the number of bootstrap replications. On Stata/IC, the number of replications must be smaller than 800.

westerlund replicates the tables in Westerlund (2007).

noisily shows the regressions for the separate series. If a range of lags or leads is given, only the regression chosen by the AIC is shown.

3.3 Empirical application

In this section, we illustrate xtwest by analyzing the influence of per capita GDP (Y_{it}) on per capita health-care expenditures (H_{it}). The data consist of a panel of 20 countries covering the period 1970 to 2001; we obtained the data from the Organisation for Economic Co-operation and Development (OECD) Health Data 2003 database. The dataset must be tsset before using it.

```
. use xtwestdata

. tsset ctr year
        panel variable:  ctr (strongly balanced)
         time variable:  year, 1 to 32
                 delta:  1 unit
```

The series are in constant 1995 prices and were transformed in logarithms. Before testing for cointegration, we need to make sure all series are integrated of order one. Westerlund (2007) used a series of unit-root tests and found strong evidence that both series are nonstationary. The postulated relationship between both variables allows for a linear time trend:

$$\ln(H_{it}) = \mu_i + \tau_i t + \beta_i \ln(Y_{it}) + e_{it} \tag{6}$$

We then used xtwest to test for cointegration, using the AIC to choose optimal lag and lead lengths for each series and with the Bartlett kernel window width set according to $4(T/100)^{2/9} \approx 3$. We used the westerlund option to replicate table 7 in Westerlund (2007).

```
. xtwest loghex loggdp, lags(1 3) leads(0 3) lrwindow(3) constant trend
> westerlund

Calculating Westerlund ECM panel cointegration tests..........

Results for H0: no cointegration
With 20 series and 1 covariate
Average AIC selected lag length: 2.8
Average AIC selected lead length: 1.65
```

Statistic	Value	Z-value	P-value
Gt	-4.082	-9.613	0.000
Ga	-27.702	-10.625	0.000
Pt	-12.969	-4.100	0.000
Pa	-22.470	-10.119	0.000

These results strongly reject the hypothesis that the series are not cointegrated.

We can use xttest2 to test for cross-sectional independence in the residuals of (2). This test requires $T > N$. As our time series are rather short and some periods are lost in the calculation of differenced variables and lags, we tested only for independence of the first five cross-sectional units. Assuming the same short-run dynamics for all series (with a single lag and lead, $p_i = q_i = 1$), we obtain the test for cross-sectional independence from

```
. xtreg d.loghex l.loghex l.loggdp ld.loghex l(-1/1).d.loggdp year if ctr < 6,
> fe
```

Fixed-effects (within) regression				Number of obs	=	145
Group variable: ctr				Number of groups	=	5

R-sq: within = 0.2794			Obs per group: min =	29
between = 0.5713			avg =	29.0
overall = 0.0001			max =	29

```
                                              F(7,133)        =        7.37
corr(u_i, Xb)  = -0.9338                       Prob > F        =      0.0000
```

D.loghex	Coef.	Std. Err.	t	P>|t|	[95% Conf. Interval]	
loghex						
L1.	-.1795043	.0328601	-5.46	0.000	-.2445004	-.1145083
loggdp						
L1.	.259809	.1191883	2.18	0.031	.0240592	.4955589
loghex						
LD.	.2265748	.0754693	3.00	0.003	.0772995	.3758501
loggdp						
FD.	.212405	.1632479	1.30	0.195	-.110493	.535303
D1.	-.1040444	.1645104	-0.63	0.528	-.4294396	.2213508
LD.	-.0926609	.1559727	-0.59	0.553	-.4011689	.2158471
year	-.0001086	.0018836	-0.06	0.954	-.0038344	.0036171
_cons	-1.265542	1.097529	-1.15	0.251	-3.436412	.9053288

sigma_u	.0705568	
sigma_e	.03372511	
rho	.81402076	(fraction of variance due to u_i)

```
F test that all u_i=0:     F(4, 133) =      0.78                Prob > F = 0.5404

. xttest2

Correlation matrix of residuals:

          __e1      __e2      __e3      __e4      __e5
__e1    1.0000
__e2    0.3375    1.0000
__e3    0.2746    0.4876    1.0000
__e4    0.2152   -0.1169    0.1840    1.0000
__e5   -0.2982   -0.0639   -0.5378   -0.3252    1.0000

Breusch-Pagan LM test of independence: chi2(10) =    29.259, Pr = 0.0011
Based on 26 complete observations over panel units
```

As these results strongly indicate the presence of common factors affecting the cross-sectional units, we bootstrapped robust critical values for the test statistics. Because the Akaike optimal lag and lead search is time-consuming when combined with bootstrapping, we held the short-term dynamics fixed.

```
. xtwest loghex loggdp, constant trend lags(1) leads(1) lrwindow(3)
> bootstrap(800)

Bootstrapping critical values under H0..........
Calculating Westerlund ECM panel cointegration tests..........

Results for H0: no cointegration
With 20 series and 1 covariate
```

Statistic	Value	Z-value	P-value	Robust P-value
Gt	−2.681	−1.731	0.042	0.046
Ga	−10.927	0.713	0.762	0.113
Pt	−12.035	−2.959	0.002	0.041
Pa	−10.524	−1.160	0.123	0.060

When we take into account cross-sectional dependencies, the tests still reject the H_0 of no cointegration.

In small datasets (such as in this application with $T = 32$), the results may be sensitive to the specific choice of parameters such as lag and lead lengths and the kernel width. When we restrict the short-run dynamics and use a shorter kernel window, the G_α statistic no longer rejects the H_0 of no cointegration.

```
. xtwest loghex loggdp, constant trend lags(1 2) leads(0 1) lrwindow(2)

Calculating Westerlund ECM panel cointegration tests..........

Results for H0: no cointegration
With 20 series and 1 covariate
Average AIC selected lag length: 1.15
Average AIC selected lead length: .05
```

Statistic	Value	Z-value	P-value
Gt	−2.736	−2.033	0.021
Ga	−11.254	0.499	0.691
Pt	−12.859	−3.902	0.000
Pa	−11.773	−2.072	0.019

4   Summary

This paper proposes a new Stata command for implementing the four panel cointegration tests developed by Westerlund (2007). The underlying idea is to test for the absence of cointegration by determining whether the individual panel members are error-correcting or not. The command, called xtwest, is very flexible and allows for an almost completely heterogeneous specification of both the long- and short-run parts of the error-correction model, where the latter can be determined from the data and hence does not require the researcher to make any difficult choices. In order to avoid misleading inference in case of cross-member correlation, xtwest also comes with a bootstrap() option.

5 References

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