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Coping with Systemic Risk in Index-based Crop Insurance

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Abstract

The implementation of index-based crop insurance is often impeded by the existence of systemic risk of insured losses. We assess the effectiveness of two strategies for coping with systemic risk: regional diversification and securitization with catastrophe (CAT) bonds. The analysis is conducted in an equilibrium pricing framework which allows the optimal price of the insurance and the number of traded contracts to be determined. We also explore the role of basis risk and risk aversion of market agents. The model is applied to a hypothetical area yield insurance for rice producers in northeast China. If yields in two regions are positively correlated, we find that enlarging the insured area leads to an increasing insurance premium. Unless capital market investors are very risk averse, a CAT bond written on an area yield index outperforms regional diversification in terms of certainty equivalents of both farmers and insurers.

Keywords: crop insurance, systemic risk, risk pooling, securitization

JEL classification: Q11, Q14.

1. INTRODUCTION

It is widely acknowledged that traditional crop insurance markets would not survive without financial subsidies in most cases. The failure of private crop insurance is mainly traced back to two causes, namely asymmetric information (Skees and Reed 1986; Just et al. 1999) and systemic risk of insurance losses (Miranda and Glauber 1997). To mitigate the first problem, index-based insurance has been proposed, such as area yield insurance or weather derivatives (Skees et al. 1997; Mahul 1999; Martin et al. 2001; Barnett et al. 2005; Vedenov and Barnett 2004; Xu et al. 2008). Unlike conventional crop insurance, payoffs from index-based insurance are based on an easily observable index that is highly correlated with actual losses and cannot be influenced by the insured.

However, the second cause of conventional crop insurance failure, systemic risk, is also an obstacle for implementing index-based insurance. Mahul (1999) indicated that index-based insurance cannot solve the problem of risk pooling and therefore cannot diversify systemic risk which occurs when a natural risk strikes simultaneously among a large number of farmers. Duncan and Myers (2000) considered the existence of systemic risk in crop yield as the main factor for the failure of the private insurance market in providing crop insurance. They show that a long-run competitive equilibrium in an insurance market can fail to exist if catastrophic or systemic risk becomes large. Their theoretical model provides insights into the factors explaining the success or failure of crop insurance markets. Among them are correlations between individual losses, the insurer's reservation utility, risk preferences of the insured, and risk preferences of the insurer. Skees and Barnett (1999) also addressed that the positive correlation across loss events increases the riskiness of the insurer's portfolio and forces the insurer to respond by increasing reserve loads and adding catastrophic loads, which might lead to a prohibitive insurance. Nonetheless, these factors inhibiting the crop insurance market have to be analyzed empirically to allow for a conclusion to be made on whether systemic risk really hampers the emergence of crop insurance in a particular market situation. Wang and Zhang (2003) investigated the magnitude of correlations of yield losses for wheat in the US at the county level. By using a spatial statistics approach, they showed that correlations between losses fade out rather quickly with growing distances between fields. As a result, the required risk premiums of insurance contracts decline when the regional size of the risk pool is extended. Moreover, risk premiums are quite close to the fair premium. The authors conclude that a private unsubsidized crop insurance market is possible in the US. On the other hand, Xu et al. (2010a/b) showed that for Germany and China, systemic weather risk cannot be regionally diversified. Goodwin (2001) observed declining correlation with increasing distance, but the correlation is more persistent in extreme yield years than normal yield years. In summary, there is a lack of consensus in the empirical literature on the impact of systemic risk on the viability of private crop insurance.

Regional diversification, as suggested by Wang and Zhang (2003) and Goodwin (2001), is not the only measure for coping with systemic risk. A direct transfer of systemic risk to the capital market through weather bonds or catastrophe (CAT) bonds has been proposed as an alternative reinsurance tool for private insurance companies underwriting crop insurance (Skees et al. 2008; Mahul 2001). In brief, the issuer of a bond grants an investor an annual return in the form of a coupon as well as principal payments in exchange for paying the bond price. In the case of an unfavorable event, the issuer retains a certain share of the principal or the coupon as a compensation for his related losses. Due to high expected returns and a low correlation with stock market returns, CAT bonds written on indices may be attractive to capital market investors. Some applications of CAT bonds and weather bonds already exist, which underpin their potential as risk management tools in agriculture (e.g., Vedenov et al. 2006; Turvey 2008). These products, however, are frequently specified on an ad hoc basis and some theoretical problems remain unsolved. In particular, pricing and the optimal design of CAT bonds require further research.

Against this background this paper assesses the viability of index-based crop insurance in China. China is one of the world's largest agricultural producers. Its share in the world production of cereals, for example, amounted to about 19.4 % in 2009. Its percentage of population engaged in agricultural activities is 38.1%. At the same time, agricultural producers in China are exposed to the pronounced yield risks, particularly weather risks (The World Bank 2007; Turvey and Kong 2010). Thus, agricultural insurance can play a vital role in expanding the agro-food economy, especially in stabilizing the income of farmers and stimulating investments in agriculture. Nevertheless, the agricultural sector in China currently appears to be under-insured: only 0.2 % of the agricultural GDP was covered by insurance in 2007 (Swiss Re, 2009). In 2005, the national agricultural insurance premium volume was $91 million, representing a mere 0.6 % of total Chinese non-life insurance premiums. The market for these products was catalyzed when the Chinese government began to subsidize insurance premiums. Nonetheless, the agricultural insurance market in China is still at an infant stage.

An assessment of the development of crop insurance markets should be based on a joint analysis of the demand and supply of the insurance. Most existing studies, however, analyze both sides separately (e.g., Mahul 1999; Vedenov and Barnett 2004; Deng et al. 2007; Wang and Zhang 2003). To our knowledge, only a few papers have discussed the viability of crop insurance markets in an equilibrium framework. Duncan and Myers (2000) used an expected-value-variance-approach for deriving necessary conditions for equilibrium in crop insurance markets facing catastrophic risk. We adopt this modeling framework but modify and extend it in several directions. First, we consider index-based insurance instead of traditional crop insurance. Thus, we have to allow for the analysis of basis risk, which is an important issue for farmers' demand for this type of insurance (Woodard and Garcia 2008; Musshoff et al. 2011). Second, we relax the restrictive assumption that the stochastic dependence of all individual losses is captured by a single correlation coefficient. This parameter is crucial for modeling the systemic risk component. Third, we extend the model from a single market to a multi-market setting, where each market represents a different region.

The contribution of this paper to the existing literature is threefold. First, we analyze the effect of regional diversification of index-based insurance theoretically in a multi-region setting. Second, we offer an empirical application of this model to the agricultural sector in China. Third, we compare the effectiveness of two alternative instruments for coping with systemic risk, namely regional diversification and securitization via CAT bonds written on area yield index. The analysis of the securitization transaction is adopted from Barrieu and El Karoui (2002) who apply variational calculus to determine the optimal structure of a weather bond. Formally, their model consists of two interrelated constrained optimization problems, each showing the structure of a principal agent model. The first part, the insurance transaction, addresses the relation between the producer and the insurer. The optimal compensation function and the optimal insurance premium are derived by maximizing the expected utility of the producer's terminal wealth given the insurer's participation constraint. The second part, the securitization transaction, models the relation between the insurer and the investor. Herein, the parameters of the bond are determined so that the expected utility for the insurer is maximized under a participation constraint for the investor and for a given optimal insurance contract.

The remainder of this article is structured as follows. In section 2 we derive the equilibrium insurance pricing model in a single region setting and then extend it to a multi-region setting. Next, we determine the optimal structure of a CAT bond written on an area yield index. In section 3 we apply these models to a hypothetical area yield insurance for rice producers in northeast China. Equilibrium prices are derived for different scenarios and the effectiveness of alternative instruments to deal with systemic risk are compared. The article ends with conclusions and suggestions for further implementation.

2. THEORETICAL FRAMEWORK

2.1. A Single Region Equilibrium Pricing Model

We consider an insurance market that is characterized by N farmers on the demand side and a single insurer on the supply side. The assumption that only one insurer exists in the market is realistic if the insurance market is at an infant stage or if one or two insurance companies are established to pilot (subsidized) crop insurance.[1] Note that this assumption rules out free market entry and thus a market equilibrium is not characterized by zero profits of the insurance supplier. We further assume a two-period economy: At t=0 all agents optimize their portfolios by buying and selling an endogenous amount of an index-based insurance to maximize the expected utility of the terminal wealth of farmers and the insurer, respectively. At t=1, the index value, crop yields and insurance payoffs are realized. Between the two periods, there is no liquid secondary market for the insurance contracts, i.e., purchased contracts cannot be resold.

Following Duncan and Myers (2000), we first derive the demand and supply for the insurance. Then, equilibrium prices are determined by a market clearing condition. All agents are assumed to be risk-averse and their preferences are expressed by an exponential utility function. The revenues R of a farmer i in one region at t=1 are defined as:

$$R_i = L_i Y_i + a_i \Theta(I) - a_i \pi(1+r) \qquad i = 1, 2, \ldots N \tag{1}$$

where L_i and Y_i denote the harvested area and the stochastic crop yield per hectare of farmer i, respectively. The size of L_i is determined prior to the hedging decision. a_i is the amount of insurance to be purchased at price π, $\Theta(I)$ is the stochastic payoff based on index I, and r is the interest rate. Since we focus on production risk, the output price is assumed to be constant and normalized to unity. Product diversification of farmers for different income sources is not considered. The expected utility maximization problem of farmer i is then given by:

$$\max_{\{a_i \geq 0\}} E[-\exp(-\lambda_f (L_i Y_i + a_i \Theta(I) - a_i \pi(1+r)))] \tag{2}$$

where λ_f denotes the absolute risk aversion coefficient. Note that farmers are not allowed to sell the contracts ($\alpha_i \geq 0$). To get a closed form solution we further assume that revenues of farmers are normally distributed. Therefore, maximizing equation (2) is equivalent to maximizing the linear certainty equivalent (CE) which is:

$$CE = E(R) - \frac{\lambda}{2} \cdot \sigma^2(R) \tag{3}$$

where $E(R)$ and $\sigma^2(R)$ denote the expected value and variance of the revenue R. This assumption is rather limiting because it rules out fat tails in the distributions of the underlying

[1] For instance, almost all crop insurance contracts in Heilongjiang Province in northeast China are offered by Sunlight Agricultural Mutual Insurance Company.

index and of farmers' income. Nevertheless, the mean-variance approach is widely accepted as a good approximation under more general utility functions and distribution assumptions (Kroll et al. 1984).

In this mean-variance framework, the optimal demand of an individual farmer purchasing index-based insurance is (see Appendix 1):

$$a_i = \max\left[0, \frac{E\big(\Theta(I)\big) - \pi(1+r) - \lambda_f cov(L_i Y_i, \Theta(I))}{\lambda_f \sigma^2_{\Theta(I)}}\right] \tag{4}$$

Equation (4) states that if the price is beyond the farmer's maximum willingness to pay, demand will be 0; otherwise the optimal demand can be expressed by the right hand side of equation (4). The optimal demand of an individual farmer increases with increasing expected indemnities and decreases with increasing insurance premium, risk aversion, and volatility of the insurance payoff. The demand is also affected by the basis risk of the index-based insurance, which is captured by the covariance term. The covariance between the production revenues and the indemnity payoffs is reasonably assumed to be negative, since hedging is otherwise impossible. The larger the covariance in absolute terms, the smaller the basis risk of the farmer and the higher the insurance demand.

The profit of the insurer, *S*, at t=1 is given by:

$$S = \beta\pi(1+r) - \beta\Theta(I) \tag{5}$$

where β represents the number of insurance contracts the insurer is willing to supply. For the sake of simplicity, we exclude the possibility of product diversification or other financial investments. The expected utility maximization problem of the insurer is therefore given by:

$$\max E[-\exp(-\lambda_s(\beta\pi(1+r) - \beta\Theta(I)))] \tag{6}$$

where λ_s denotes the risk aversion coefficient of the insurer. The first order condition yields the supply function of the insurer in a mean-variance framework:

$$\beta = \frac{\pi(1+r) - E(\Theta(I))}{\lambda_s \sigma^2_{\Theta(I)}} \tag{7}$$

The derivation is provided in Appendix 1. Equation (7) indicates that the supply of insurance increases with an increasing price of insurance contracts.

Equilibrium in the insurance market requires that aggregate demand equals supply, i.e.,

$$\beta = \sum_{i=1}^{N} a_i \tag{8}$$

Applying the market clearing condition (Eq.8) allows us to derive the equilibrium price and quantity:

$$\pi = \frac{1}{1+r}\left[E\big(\Theta(I)\big) - \frac{\lambda_f \lambda_s \sum_{i_{(a_i>0)}} cov\big(L_i Y_i, \Theta(I)\big)}{\lambda_f + \lambda_s N^*}\right] \tag{9}$$

and

$$\beta = -\frac{\lambda_f \sum_{i(a_i>0)} cov\big(L_i Y_i, \Theta(I)\big)}{(\lambda_f + \lambda_s N^*)\sigma^2_{\Theta(I)}} \tag{10}$$

The proof is in Appendix 1. Note that the market equilibrium does not depend on all N farmers. Instead, it depends on $N^* = \sum_{i=1}^{N} \mathbf{1}(a_i > 0)$ farmers who eventually purchase a positive amount of insurance contracts at the equilibrium price. Equations (9) and (10) show that the equilibrium price depends on the risk aversion of both farmers and the insurer as well as on the aggregated covariance between production revenues and insurance payoffs. High aggregated covariance (in absolute values) reflects low aggregated basis risk. This, in turn, increases the demand for insurance as well as the equilibrium price.

2.2. A Multi-Region Equilibrium Pricing Model

We now analyze regional diversification and assume that the insurer offers index-based insurances in M heterogeneous regions. We extend the single-region model of the previous section to a multi-region equilibrium pricing model. Carrying out similar steps as shown in the previous section yields equilibrium prices $\{\pi_1, \pi_2 \dots \pi_M\}$ and equilibrium quantities $\{\beta_1, \beta_2 \dots \beta_M\}$.

Analogous to equation (1) for a single region, the revenue of farmer *i* in region *m* at time 1 is:

$$R_{i,m} = L_{i,m}Y_{i,m} + a_{i,m}\Theta(I_m) - a_{i,m}\pi_m(1+r) \tag{11}$$

where *m* refers to different regions and *m*=1,2…*M*. The profit of the insurer including all regions is then:

$$S = \sum_{m=1}^{M} \beta_m \pi_m (1+r) - \sum_{m=1}^{M} \beta_m \Theta(I_m) \tag{12}$$

The individual demand function for farmer *i* in region *m* is:

$$a_{i,m} = \max\left[0, \frac{E\big(\Theta(I_m)\big) - \pi_m(1+r) - \lambda_f cov\Big(L_{i,m}Y_{i,m}, \Theta(I_m)\Big)}{\lambda_f \sigma^2_{\Theta(I_m)}}\right], i = 1,2 \dots N_m \tag{13}$$

The supply function for region *m* is:

$$\beta_m = \frac{\pi_m(1+r) - E\big(\Theta(I_m)\big) - \lambda_s \sum_{k\neq m}^{M} \beta_k cov\big(\Theta(I_m), \Theta(I_k)\big)}{\lambda_s \sigma^2_{\Theta(I_m)}} \tag{14}$$

where *k* refers to a region different from *m*. Applying the market clearing condition:

$$\sum_{i=1}^{N_m} a_{i,m} = \beta_m \tag{15}$$

This condition allows us to derive the equilibrium price in region *m* (see Appendix 2):

$$\pi_m = \frac{1}{1+r}\left[E\big(\Theta(I_m)\big) - \frac{\lambda_f \lambda_s \sum_{i_{(a_{i,m}>0)}} cov\left(L_{i,m}Y_{i,m}, \Theta(I_m)\right)}{\lambda_f + \lambda_s N_m^*} + \frac{\lambda_f \lambda_s \sum_{k\neq m}^{M} \beta_k^* cov\big(\Theta(I_m), \Theta(I_k)\big)}{\lambda_f + \lambda_s N_m^*}\right] \quad (16)$$

While the structure of the individual demand function is the same as before, the supply function for each region now includes the sum of the covariance of payoffs between region *m* and other regions. In the multi-regional equilibrium pricing model, the supply function (Eq. 14) indicates that the equilibrium supply depends not only on the premium π_m of insurance contracts within the region, but also on the other regions' equilibrium supply β_k and the covariance of payoffs between region *m* and other regions. Likewise, equilibrium prices (Eq. 16) in the M insurance markets are interdependent. Therefore, we need solve the system equations simultaneously to determine equilibrium prices and quantities for M regions.

What can be concluded for the effectiveness of regional diversification, i.e., extending the trading area of the index-based insurance? In view of equation (16), the premium will increase instead of decrease if the extended region k has a positive correlation $\rho_{m,k}$ with region *m*. This conclusion contradicts Wang and Zhang (2003) who found a decreasing buffer load even if the correlation between losses is positive. This contradiction results from applying different principles to calculate the insurance premium. While we use a mean-variance model, Wang and Zhang (2003) as well as Xu et al. (2010 a\b) derive risk premia (buffer loads) from the standard deviation of average losses which decreases with the number of insured due to the law of large numbers. This property, however, does not hold for the average variance of the total loss.

2.3. Issuance of CAT bonds

In this section, we consider issuing CAT bonds as an alternative to regional diversification for dealing with systemic risk[2] in a single region. We adopt the framework of Barrieu and El Karoui (2002) which consists of two transactions: an insurance transaction and a securitization transaction. Unlike Barrieu and El Karoui (2002), we do not solve both problems simultaneously. Instead, we proceed in two steps and assume that the insurance transaction has already been determined in accordance with the single region equilibrium model. Thus, we focus on the securitization transaction. The insurer issues a CAT bond for each contract with a maturity of one period written on the index *I* at price Φ in the capital market. This indicates that the quantity of CAT bonds equals that of the insurance contracts. Therefore, the amount of loss

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[2] Duncan and Myers (2000) examined the effect of proportional reinsurance for catastrophic risk, yet the proportion of reinsurance is rather arbitrary and is fixed. Here we determine the optimal proportion of risk sharing as part of the solution of the same problem.

from one insurance contract would be transferred into the capital market via one CAT bond. The demand side of the capital market is modeled by a representative investor who pays price Φ in t_0 and receives principal P and coupon payment c in period t_1 as a return. However, the investor commits to sacrifice a certain portion of his payoff when the index I triggers the insurance payoff $\Theta(I)$. Barrieu and El Karoui (2002) proved that the optimal amount, which is paid back, is a linear function of the payoffs. As a result, the portfolio of the insurer at t_1 is:

$$S = \beta\pi(1+r) - \beta\Theta(I) + \beta(\Phi(1+r) - c - P + \alpha\Theta(I)) \tag{17}$$

where α refers to the repayment ratio. The portfolio for the investor is:

$$\beta[-\Phi(1+r) + c + P - \alpha\Theta(I)] \tag{18}$$

where β refers to the determined optimal insurance amount in insurance transaction.

There are a few simplifying assumptions which underlie this modeling framework. First, we consider financial flows related to the CAT bond only. This means that possible diversification effects are neglected. Second, we assume that no transaction costs occur. Third, we assume that there is no liquid secondary market for either the insurance contract or the CAT bond. Hence, it is not possible for the investor to build a replicating strategy. As a result, we cannot apply a risk-neutral pricing approach. Instead, the problem of designing and pricing the CAT bond is solved in a utility maximization framework as was done in the insurance case. The representative investor is assumed to be risk-averse with exponential utility and an absolute risk aversion coefficient λ_I. Following the idea of indifference pricing, the price of the bond is determined such that the investor is indifferent in terms of his expected utility between buying and not buying the security. The optimization program of the insurer is then given by:

$$\max_{\Phi,c,\alpha} E\left[-\exp\left(-\lambda_S(\beta\pi(1+r) - \beta\Theta(I) + \beta(\Phi(1+r) - c - P + \alpha\Theta(I)))\right)\right] \tag{19}$$

$$s.t.\, E\left[-\exp\left(-\lambda_I(\beta(-\Phi(1+r) + c + P - \alpha\Theta(I)))\right)\right] \geq -1 \tag{20}$$

Equation (20) is the participation constraint of the investor, where -1 corresponds to the expected utility of the investor in the case of not buying the CAT bond. Via solving the corresponding Lagrange function, we can obtain the optimal repayment ratio:

$$\alpha^* = \frac{\lambda_S}{\lambda_S + \lambda_I} \tag{21}$$

(The derivation of equation (21) is provided in Appendix 3). The optimal repayment ratio α^* depends on the ratio of the risk aversion coefficient of the insurer to that of the investor. The binding participation constraint in (20) gives:

$$\Delta^* = -\Phi^*(1+r) + c^* + P = \frac{1}{\lambda_I\beta}\ln E(\exp(\lambda_I\alpha^*\beta\Theta(I))) \tag{22}$$

Thus, the optimal bond price is:

$$\Phi^* = \frac{1}{1+r}\left[c^* + P - \frac{1}{\lambda_I\beta}\ln E(\exp(\lambda_I\alpha^*\beta\Theta(I)))\right] \tag{23}$$

Equation (24) reveals the interesting feature that the bond can be issued at a price that is smaller than the discounted value of the expected cash flows, i.e. the fair price. Note that there is an indeterminacy concerning the optimal bond structure; the optimal repayment determines the optimal net cash flow Δ uniquely, but the relation between the optimal bond price Φ* and the optimal coupon c can be chosen arbitrarily. Here we consider the case where the bond is offered only at a discount, i.e. the bond price is equal to the discounted principal payment,

$$\Phi^* = P \cdot (1+r)^{-1} \tag{24}$$

The coupon payment for this case is:

$$c^* = \frac{1}{\lambda_I \beta} \ln E\left(\exp(\lambda_I \alpha^* \beta \Theta(I))\right) \tag{25}$$

Hitherto the principal payment P was considered as an exogenous parameter. From a marketing viewpoint, however, it might be desirable to offer a certain return r_{bond} (before stochastic repayments) to the investor which should clearly exceed the interest rate r. The definition $r_{bond} = c^*/\Phi^*$ therefore implies the bond price $\Phi^* = c^*/r_{bond}$.

3. AREA YIELD INSURANCE IN NORTHEAST CHINA

3.1. Study Area and Data

In this section, we analyze the impact of systemic risk on the demand and supply of index-based crop insurance in the three provinces of Heilongjiang, Jilin and Liaoning located in northeast China. Three provinces cover about 787,300 km^2 and are the main production areas for grain in China. Moreover, the region has a vital role for domestic food supply and food security. However, grain production in northeast China, especially in Heilongjiang, is seriously affected by agricultural risks, drought in particular. Between 2004 and 2006, 16,805 km^2 in Heilongjiang and 11,561 km^2 in Jilin were hit by drought (China Meteorological Administration 2008). In recent years, conventional and subsidized agricultural insurance in Heilongjiang has been provided mainly by Sunlight Agricultural Mutual Insurance Company. In 2009 farmers participating in agricultural insurance had to pay just 20% of the premium, while the rest was paid for by the central government (40%), provincial government (25%), and local government (15%). The question arises if these subsidies are required or if there are cheaper and more efficient ways to establish a private crop insurance market.

In the subsequent application, we specify index-based insurance into area yield insurance and design it for rice farmers in Heilongjiang and calculate its price by an equilibrium pricing model. We focus on rice because it is the most important crop in northeast China in terms of farmers' revenues. First, we calculate the market equilibrium for an area yield insurance for Heilongjiang. Next, we investigate whether systemic risk can be reduced if the index-based insurance is also traded in the provinces of Jilin and Liaoning. Finally, we analyze if a CAT bond written on the area yield is a more efficient alternative for coping with systemic yield risk.

Due to the lack of a sufficiently long time series of farm-level yield data, we use regional crop yield data and consider a region as a representative farm, i.e. farmers located in one region are assumed to have the same yield distribution. Each province consists of 9 to 14 regions. Regional yield data are available for the time period from 1994 to 2009. Area yields refer to the provincial level and are calculated by the total provincial rice production divided by the area of total rice-sown lands in each province in that year. The drawback of using aggregate yields data in farm risk analysis are well-known and have been discussed, for example, by Rudstrom et al. (2002) and Popp et al. (2005). Since farm yields are more volatile than regional yields, using the latter may cause a bias in our results: first, a farmers' idiosyncratic risk will be underestimated; second, the correlation between area yield and individual yields will be overestimated; and third, systemic risk will be overestimated in general. These biases should be recalled when interpreting the results.

3.2. Specification of the Parameters for Area Yield Insurance

The hypothetical area yield insurance resembles a put option with payoff:

$$\Theta(\bar{Y}) = T\max(K - \bar{Y}, 0) \tag{26}$$

where $\bar{Y}$ denotes the actual area yield in dt/ha, T is the tick-value, and K is the strike-level. The contract is designed per one hectare. For reasons of simplicity, the price of rice p is assumed to be constant and equals 256 ¥/dt which is the minimum purchase price policy for japonica rice (China National Development and Reform Commission, 2011). Thus the tick value T equals 256 ¥ per index point for one hectare. The strike value K equals the long-term average of the area yield. Further parameters needed for the equilibrium model are the risk-aversion parameter λ and the risk-free interest rate r. Gong et al. (2010) recently elicited the risk attitude of Chinese farmers by means of a Holt and Laury lottery. According to their experimental outcomes, farmers are risk-averse and their relative risk aversion ranges between 0.15 and 0.41. Referring to these findings we assume that farmers have a relative risk-aversion parameter of 0.4. Dividing this value by the initial wealth of farmers yields the required absolute risk aversion coefficient. We approximate the initial wealth by the annual cash income per household ($2.9 \cdot 10^4$¥) which is the per capita annual cash income multiplied by the average number of residents per household based on data from the China Statistics Yearbook (2010). The absolute risk-aversion parameter assumed for all farmers thus equals $1.4 \cdot 10^{-5}$, which is comparable to other studies (e.g., Zuhair et al. 1992). Then, the absolute risk aversion of the insurer is defined relative to this value. We assume that the insurer is also risk averse, but less risk averse than farmers. Since the choice of risk aversion is crucial in our model we will conduct sensitivity analyses with regard to these parameters. The risk-free interest rate is 3.25%, which is the average one-year deposit rate from 2008 to 2010. Table 1 summarizes the model parameters. Note that the correlations of insurance payoffs between the three provinces are positive.

Table 2 depicts the descriptive statistics of all the regions in Heilongjiang considered as representative farms in Heilongjiang. The expected yields of the regions range from 54.55 dt/ha to 79.96 dt/ha. The size of the regions in terms of their total rice area varies considerably. The number of farmers N_i, which is not recorded, is calculated by dividing the sown rice area by the average sown rice area per farm. The correlation between the regional yields and the payoff of area yield insurance, which is based on the average provincial yield, varies between -0.14 and -

0.86. These figures reflect considerable basis risk. The corresponding values for Jilin and Liaoning are provided in the Appendix 4.

Table 1: Specification of Parameters for Equilibrium Pricing in Heilongjiang, Jilin and Liaoning

	Heilongjiang	Jilin	Liaoning
Tick-size T (¥)		256/index point	
Strike-level K (dt/ha)	72.08	82.91	77.23
Expected payoff $E(\Theta)$ (¥)	660	441	611
Standard deviation σ_Θ (¥)	931	474.6	1622
Risk-free interest rate r		3.25%	
Sown area of rice (ha)	$1.55 \cdot 10^6$	$7.03 \cdot 10^5$	$5.90 \cdot 10^5$
Average sown area per farm L (ha)	3.3	1.8	1.1
Absolute risk-aversion λ_f for all farmers		$1.4 \cdot 10^{-5}$	
Absolute risk-aversion λ_s for the insurer		$1.4 \cdot 10^{-9}$	
Correlations of insurance payoffs			
Heilongjiang	1	0.10	0.39
Jilin	0.10	1	0.32
Liaoning	0.39	0.32	1

Source: own calculation

Table 2: Descriptive Statistics of Representative Farms in Heilongjiang Province

Regions	Expected yield (dt/ha)	σ_Y	Sown area of rice (ha)	No. of farmers	$\rho_{Y,\Theta}$
1 Harbin	79.96	994	473217	141902	-0.59
2 Qiqihar	60.26	755	179588	53852	-0.32
3 Jixi	72.32	912	143286	42967	-0.76
4 Hegang	54.55	970	44839	13446	-0.75
5 Shuangyashan	63.28	1396	43311	12988	-0.85
6 Daqing	61.81	1624	58896	17661	-0.14
7 Yichun	65.58	1286	31773	9528	-0.65
8 Jiamusi	67.83	867	238205	71430	-0.86
9 Qitaihe	66.33	797	17236	5169	-0.80
10 Mudanjiang	70.18	1183	44158	13242	-0.77
11 Heihe	56.13	987	9490	2846	-0.49
12 Suihua	74.88	772	266336	79865	-0.46

Source: own calculation

4. RESULTS AND DISCUSSION

We start with the discussion of the single region insurance. Table 3 displays the equilibrium prices and quantities of area yield insurances in Heilongjiang Province according to equations (4), (7), and (8) for different levels of risk aversion. For the base case explained in the previous section, the equilibrium price is 722 (¥) and $7.03 \cdot 10^4$ contracts are traded at this price.

The actuarially fair price is displayed as a benchmark. Comparing the fair price with the equilibrium price shows that a risk loading of 12.8% exists in equilibrium. At equilibrium, only three regions (i.e. representative farmers) would like to participate in the area yield insurance with an average number of 0.59 contracts per hectare. The participation ratio (i.e., insured area / total area) of the whole Province is rather small (8%).

Table 3: Equilibrium Price of Area Yield Insurance for Different Risk Aversion for Heilongjiang

Insurance	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6
Risk aversion of all farmers	$1.4 \cdot 10^{-5}$	$1.4 \cdot 10^{-5}$	$1.4 \cdot 10^{-4}$	$1.4 \cdot 10^{-6}$	$1.4 \cdot 10^{-5}$	$1.4 \cdot 10^{-5}$
Risk aversion of insurer	$1.4 \cdot 10^{-9}$	$1.4 \cdot 10^{-5}$	$1.4 \cdot 10^{-9}$	$1.4 \cdot 10^{-9}$	$1.4 \cdot 10^{-10}$	$1.4 \cdot 10^{-9}$
Fair Price $E(\Theta)/(1+r)$ (¥)			640			
Equilibrium price π (¥)	722.2	765.9	1196	651	695	765
Equilibrium quantity β	$7.03 \cdot 10^{4}$	10.7	$4.74 \cdot 10^{5}$	$9.98 \cdot 10^{3}$	$4.74 \cdot 10^{5}$	$1.04 \cdot 10^{5}$
Average amount of contract per hectare*	0.59	0.00	0.46	0.23	0.46	0.61
Participation Ratio	0.08	0.00	0.66	0.03	0.66	0.12

*The average value refers to farmers with positive insurance demand.
Source: own calculation

Varying the risk aversion has a significant impact on the equilibrium price from Case 2 to Case 5. Table 3 shows that the equilibrium price increases if the risk aversion of farmers or the insurer increases. If the risk aversion of agents decreases or even vanishes (Case 4 and Case 5), the equilibrium price converges to the fair price. The equilibrium quantity becomes small if risk aversions of farmers and of the insurer are equal.[3] In contrast, if farmers are more risk averse than the insurer (Case 3 and Case 5), the equilibrium price increases considerably and the average number of contracts per hectare and the participation ratio amount to 0.46 and 0.66, respectively.

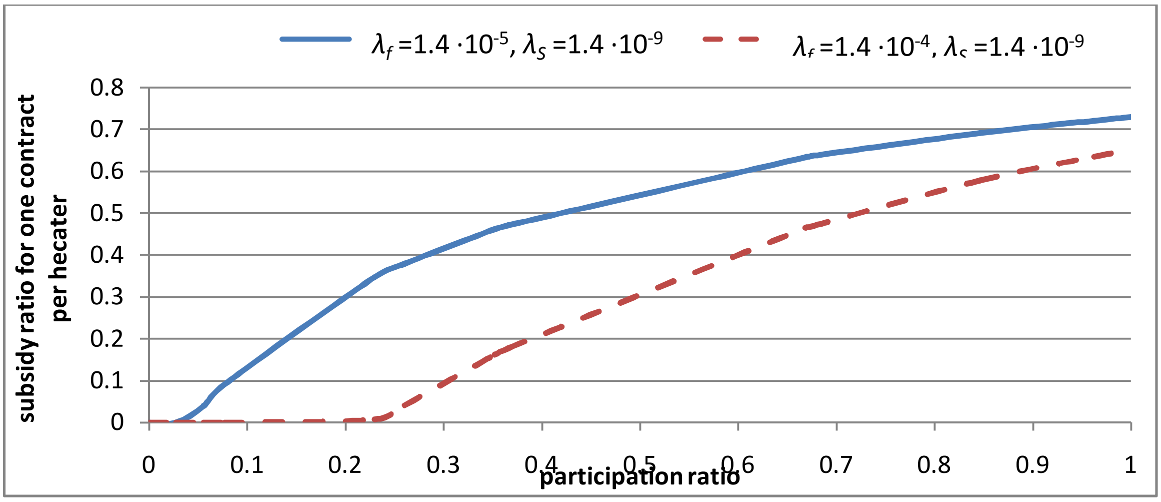
To explore the effect of basis risk we set all correlations between the individual farmer's yield and the insurance payoff to -1, i.e. we pretend a situation where no basis risk is present (Case 6). As expected, the equilibrium price and quantity increase compared with the base case (Case 1), reflecting higher insurance demand. The magnitude of this effect, however, is moderate.

Table 3 shows that the participation ratio may become quite small if the insurer is risk-averse. From an administrative perspective it might be interesting to know to what extent insurance premia have to be subsidized to ensure a desired participation in the agricultural insurance program. We can determine the average subsidy ratio required for different participation ratios as well as a given number of contracts per hectare by calculating the

1. [3] Irrespective of the degree of risk aversion, the equilibrium quantity cannot equal zero. This is a consequence of our assumption that the insurer's portfolio cannot be diversified. Without diversification, the demand curve always starts above the supply curve, leading to an intersection.

difference between the willingness to pay for farmers and the willingness to accept for the insurer. Here we analyze subsidies assuming a coverage of 1 contract per hectare. Figure 1 shows the subsidy ratio for two different risk aversions. For the base case of $\lambda_f = 1.4 \cdot 10^{-5}$ and $\lambda_s = 1.4 \cdot 10^{-9}$, a premium subsidy of 73% is required to ensure full participation in the insurance program. Clearly, the subsidy ratio is much smaller if farmers are more risk averse or if the insurer is less risk averse.

Figure 1: Premium Subsidy Ratio for Different Participation Ratios and Risk Aversions



Source: own calculation

Next, we investigate the effect of regional diversification using the multi-region model (Eq. (13) and (14)). Table 4 presents the equilibrium prices and quantities of different combinations of provinces. S1 represents the case where the insurer insures Heilongjiang only. S2 and S3 refer to scenarios in which the trading area is extended to two and three provinces, respectively. Consistent with our theoretical analysis, the insurance premia for Province Heilongjiang in S2 and S3 are higher than in S1 because correlations of area yields among the three provinces are positive. Correspondingly, the insurance quantity in Heilongjiang decreases as premium increases. Although the changes are moderate, extending the trading area does not help diversify systemic yield risk. This change is independent of the risk aversion of market participants.

Table 4: Equilibrium Prices and Quantities of Different Regions in Different Scenarios

Scenarios**	Heilongjiang (640)*		Jilin (427)		Liaoning (592)	
	Price	Quantity	Price	Quantity	Price	Quantity
S1	722.2	$7.03 \cdot 10^4$				
S2	722.8	$6.85 \cdot 10^4$	445.4	$4.75 \cdot 10^4$		
S3	723.0	$6.80 \cdot 10^4$	445.5	$4.66 \cdot 10^4$	665	$1.01 \cdot 10^3$

Source: own calculation

*The values in parentheses refer to fair prices in the provinces.

**For all the scenarios, we simply used the base case of risk aversions ($\lambda_f = 1.4 \cdot 10^{-5}$, $\lambda_S = 1.4 \cdot 10^{-9}$).

In what follows, we analyze the effect of a CAT bond that transfers part of the area yield risk of rice producers in Heilongjiang to a representative capital market investor. Again, the results are calculated for different risk preferences of the involved agents focusing on the relation between the insurer and the investor (Table 5).

Table 5: Optimal CAT Bond Design for Different Risk Aversions

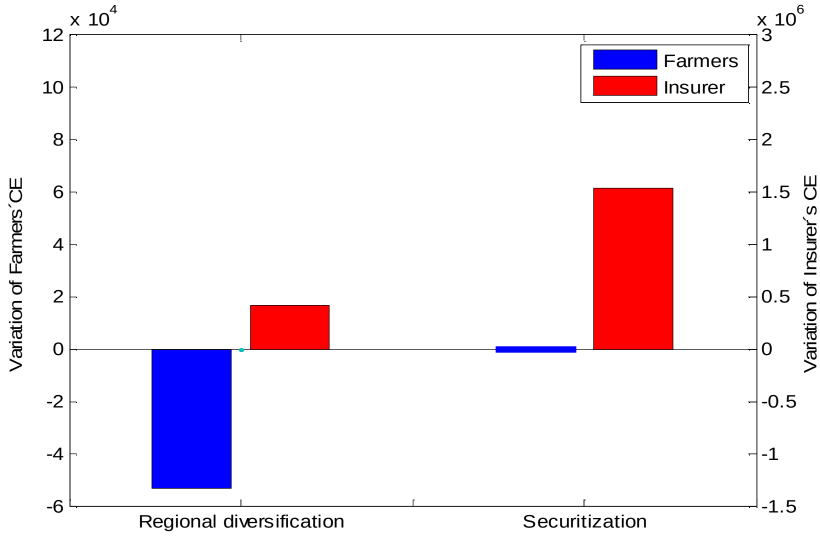
Securitization	Case 1	Case 2	Case3
Risk aversion of all farmers	$1.4 \cdot 10^{-5}$	$1.4 \cdot 10^{-5}$	$1.4 \cdot 10^{-5}$
Risk aversion of the insurer	$1.4 \cdot 10^{-9}$	$1.4 \cdot 10^{-9}$	$1.4 \cdot 10^{-9}$
Risk aversion of the investor	$1.4 \cdot 10^{-9}$	$1.4 \cdot 10^{-8}$	$1.4 \cdot 10^{-10}$
Equilibrium quantity β	$7.03 \cdot 10^{4}$	$7.03 \cdot 10^{4}$	$7.03 \cdot 10^{4}$
Risk transfer ratio α (%)	50	9.1	91
Expected repayment (¥) $E(\alpha\Theta) \cdot (1+r)^{-1}$	320	58	581
Specification		$\Phi^* = P \cdot (1+r)^{-1}$	
Principal P (¥)	7028	1310	12465
Bond price Φ (¥)	6810	1269	12072
Coupon c (¥)	340	63.4	604
Expected net return of the investor (%) $(c - E(\alpha\Theta)) \cdot (1+r)^{-1})/\Phi$	0.145	0.26	0.03

* Note: the risk-free interest rate is 3.25% and the certain return of the bond r_{bond} is fixed at 5%, which means c/Φ=5%.

In Case 1, where the risk aversion parameters of the insurer and investor are equal, the risk transfer ratio is 50%, which implies that 50% of the payoffs, i.e., 320¥ per contract, are transferred to the investor. To determine the bond price, we assume that the bond is offered at a discount (Eq. 24). The coupon, c, is then obtained from Equation (25) and amounts to 340¥, which exceeds the expected repayment. The investor's expected net return is only 0.145%. One has to recall, however, that the bond was sold at a discount, i.e. the 0.145% represents an expected return above the risk-free interest rate of 3.25%. In Case 2 (Case 3), we increase (decrease) risk aversion by a factor 10. The results are very sensitive to changes in risk aversion. In Case 2, most of the yield risk remains with the insurer, while in Case 3 almost all risk is transferred to the capital market. Also, the price of the bond, the principal, and the coupon vary considerably.

Finally, we compare the two risk coping strategies in terms of the certainty equivalent (CE) of farmers in Heilongjiang and the insurer. Figure 2 depicts the change of the CEs for the regional diversification strategy and the securitization strategy compared with a single region insurance without any strategy. It is apparent that extending the insured area leads to a reduction of the CE of farmers in Heilongjiang due to increasing insurance prices. On the other hand, the CE of the insurer increases. In contrast, CAT bonds are superior to regional diversification: The CE of the insurer increases even more while the CE of the farmers is not affected. The latter is due to the fact that the insurance transaction was assumed to be settled before designing the securitization transaction. A word of caution, however, is necessary when interpreting these results. First, the welfare change of the farmers in the acceding two provinces are not considered in this comparison. Their utility will of course increase if insurance is offered to them. Second, the bond pricing model assumed that the utility of the capital market investor does not change, i.e. the investor is indifferent between buying and not buying the bond at its "equilibrium" price. If the investor has higher opportunity costs and thus requires a higher return from the securitization, the utility of insurer issuing a CAT bond becomes, in turn, lower. A higher risk aversion of the investor has the same effect.

Figure 2: Effectiveness of Regional Diversification and Securitization



5. CONCLUSIONS

Previous research has revealed a market potential of index-based insurance products in Chinese agriculture by analyzing the demand side of the insurance market (Turvey and Kong 2010; Heimfarth and Musshoff 2011). A prediction of the market success, however, cannot be made without considering the supply side. To close this gap we analyze the feasibility of index-based insurance by means of an equilibrium pricing model. The modeling approach allows the incorporation of important factors, such as risk aversion of market agents, basis risk, and systemic risk. Moreover, we investigate two strategies to mitigate the supply reducing effect of systemic risk, namely regional diversification and securitization.

Our results shed some light on the applicability of index-based crop insurance in China and suggest some ideas for increasing market penetration. First, it is confirmed that basis risk inherent with index-based insurance curtails its advantages over traditional crop insurance. To overcome this problem, tailored products should be developed showing high correlation with actual losses of the farmers. Second, we found that indemnity payments within a province show high correlation, i.e. systemic risk is prevalent. This, in turn, leads to risk loadings of up to

12.8% in relation to the fair insurance premium depending on the assumed risk aversion. As a result, insurance coverage at equilibrium is rather low and premium subsidies of up to 80 percent would be necessary to attain full participation of farmers. An intuitive proposal to overcome this failure of risk pooling – regional diversification by enlarging the trading area of the insurance – did not provide a solution. The reason for this is that in our model, the price for insurance increases if losses across regions are positively correlated which is the normal case. One should note, however, that this conclusion is closely related to our assumption that only one insurer exists in the market so that offering more contracts increases the insurance supplier's risk exposure.

Securitization of systemic risks through issuing CAT bonds turned out to be an efficient alternative to regional diversification. This finding as well as the aforementioned results depend on some crucial assumptions. First, all quantitative results we provide are sensitive to the level and the ratio of the market participants' risk aversion. This drawback, however, is unavoidable in a utility based modeling framework. Second, simplifying assumptions about the portfolios of the farmers, the insurer, and the capital market investor affect the farmers' and investors' willingness to pay for the insurance policy and the CAT bond, respectively, and the insurer's willingness to accept the price they would receive for supplying the insurance policy. Most likely, the demand for insurance of a diversified farm is lower than that of a specialized producer. Likewise, the inclusion of other risk sources, such as price risk, will change the hedging effectiveness of the yield insurance. Finally, the equilibrium concept that we pursue here drives our results to some extent. Under the assumption of a competitive insurance market (instead of a monopolistic one) profits of insurance companies would be ruled out. We suggest the relaxation of these assumptions for further research on this subject. Despite these drawbacks, we are confident that our results provide interesting insights and advice.

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Appendices

Appendix 1: Equilibrium Pricing Model for a Single Region.

Under the mean variance criterion, the maximization problems of farmer *i* and the insurer S are given by:

$$CE_i^f = L_iE(Y_i) + a_iE\big(\Theta(I)\big) - a_i\pi(1+r) - \frac{\lambda_f}{2}(L_i^2\sigma_{Y_i}^2 + a_i^2\sigma_{\Theta(I)}^2 + 2a_iL_icov(Y_i,\Theta(I)) \quad (27)$$

$$CE^S = \beta\pi(1+r) - \beta E(\Theta(I) - \frac{\lambda_S}{2}\beta^2\sigma_{\Theta(I)}^2 \quad (28)$$

Taking the first-order conditions for Eq. (27) and Eq. (28) with respect to a_i and β, respectively, gives the individual demand function (4) and the supply function (7).

At equilibrium, some farmers will not buy any insurance. Therefore, aggregate demand is determined by $N^* = \sum_{i=1}^{N}\mathbf{1}(a_i > 0)$ farmers. Inserting Eq. (4) and Eq. (7) into Eq. (8) yields:

$$\begin{aligned}\sum_{i=1}^{N} a_i = \sum_{i=1}^{N}\max\left[0, \frac{E\big(\Theta(I)\big) - \pi(1+r) - \lambda_f cov\big(L_iY_i,\Theta(I)\big)}{\lambda_f\sigma_{\Theta(I)}^2}\right]& \\ = \frac{N^*E\big(\Theta(I)\big) - N^*\pi(1+r) - \lambda_f\sum_{i_{(a_i>0)}} cov\big(L_iY_i,\Theta(I)\big)}{\lambda_f\sigma_{\Theta(I)}^2}& \\ = \frac{\pi(1+r) - E\big(\Theta(I)\big)}{\lambda_s\sigma_{\Theta(I)}^2} = \beta&\end{aligned} \quad (29)$$

Rearranging equation (29) gives the equilibrium price (Eq. 9) and quantity (Eq. 10).

Appendix 2: Multi-Regional Equilibrium Pricing Model

In a multi-regional setting, the insurer's portfolio changes to:

$$S = \sum_{m=1}^{M}\beta_m\pi_m(1+r) - \sum_{m=1}^{M}\beta_m\Theta(I_m)$$

The corresponding certainty equivalent is then:

$$\begin{aligned}CE^S = \sum_{m=1}^{M}\beta_m\pi_m(1+r) - \sum_{m=1}^{M}\beta_mE(\Theta(I_m)& \\ -\frac{\lambda_S}{2}(\sum_{m=1}^{M}\beta_m^2\sigma_{\Theta(I_m)}^2 + 2\sum_{m\neq k}\beta_m\beta_k cov(\Theta(I_m),\Theta(I_k)))&\end{aligned} \quad (30)$$

Taking the partial derivative with respect to β_m (*m*=1…*M*) results in equation (14).

Inserting equations (13) and (14) into equation (15) yields:

$$\sum_{i=1}^{N_m} a_{i,m} = \sum_{i=1}^{N_m}\max\left[0, \frac{E\big(\Theta(I_m)\big) - \pi_m(1+r) - \lambda_f cov\left(L_{i,m}Y_{i,m},\Theta(I_m)\right)}{\lambda_f\sigma_{\Theta(I_m)}^2}\right]$$

$$= \frac{N_m^* E\big(\Theta(I_m)\big) - N_m^* \pi_m (1+r) - \lambda_f \sum_{i_{(a_{i,m}>0)}} cov\Big(L_{i,m} Y_{i,m}, \Theta(I_m)\Big)}{\lambda_f \sigma^2_{\Theta(I_m)}}$$

$$= \frac{\pi_m (1+r) - E(\Theta(I_m) - \lambda_S \sum_{m \neq k} \beta_k cov\big(\Theta(I_m), \Theta(I_k)\big)}{\lambda_S \sigma^2_{\Theta(I_m)}} = \beta_m$$

Rearranging this equation gives the equilibrium price (Eq. 16) for region m.

Appendix 3: Optimal Design of a CAT bond

When issuing a CAT bond, the insurer's portfolio becomes:

$$S = \beta\pi(1+r) - \beta\Theta(I) + \beta(\Phi(1+r) - c - P + \alpha\Theta(I))$$

The investor's portfolio is given by:

$$-\beta\Phi(1+r) + \beta c + \beta P - \alpha\beta\Theta(I)$$

The corresponding certainty equivalents of the insurer and the investor are:

$$CE^s = \beta\pi(1+r) - \beta E\big(\Theta(I)\big) - \beta\Delta + \alpha\beta E\big(\Theta(I)\big) - \frac{\lambda_s}{2}((1-\alpha)^2 \beta^2 \sigma^2_{\Theta(I)}) \tag{31}$$

$$CE^I = \beta\Delta - \alpha\beta E\big(\Theta(I)\big) - \frac{\lambda_I}{2} \alpha^2 \beta^2 \sigma^2_{\Theta(I)} \tag{32}$$

where $\Delta = -\Phi(1+r) + c + P$. The Lagrange function of the optimization problem of the insurer (Eq. (19) - (20)) is given by:

$$\begin{aligned} L = \beta\pi(1+r) - \beta E\big(\Theta(I)\big) - \beta\Delta + \alpha\beta E\big(\Theta(I)\big) - \frac{\lambda_s}{2}((1-\alpha)^2 \beta^2 \sigma^2_{\Theta(I)}) \\ + \zeta\left(\beta\Delta - \alpha\beta E\big(\Theta(I)\big) - \frac{\lambda_I}{2} \alpha^2 \beta^2 \sigma^2_{\Theta(I)}\right) \end{aligned} \tag{33}$$

Herein ζ is the Lagrange multiplier and β is a pre-determined parameter resulting from the insurance transaction. Taking the partial derivative with respect to α and Δ gives the optimality conditions:

$$\frac{dL}{d\alpha} = (1-\zeta)E\big(\Theta(I)\big) + (1-\alpha)\lambda_s \beta^2 \sigma^2_{\Theta(I)} - \zeta\lambda_I \alpha\beta^2 \sigma^2_{\Theta(I)} = 0 \tag{34}$$

$$\frac{dL}{d\Delta} = -1 + \zeta = 0 \tag{35}$$

Solving equation (35) and substituting into equation (34) yields equation (21):

$$\alpha^* = \frac{\lambda_s}{\lambda_s + \lambda_I}$$

Appendix 4

Table 9: Descriptive Statistics of Representative Farms in Jilin Province

Regions	Expected yield (dt/ha)	σ_Y	Sown area of rice (ha)	No. of farmers	$\rho_{Y,\Theta}$
1 Changchun	87.36	503	175389	95386	-0.65
2 Jilin City	82.52	923	141613	77017	-0.41
3 Siping	90.61	1076	59238	32217	-0.44
4 Liaoyuan	78.41	976	17344	9433	-0.29
5 Tonghua	90.50	981	76024	41346	-0.01
6 Baishan	62.53	669	1397	760	-0.15
7 Songyuan	96.20	1216	91627	49832	-0.16
8 Baicheng	69.62	1593	99779	54265	-0.40
9 Yanbian	50.18	1576	40498	22025	-0.51

Table 10: Descriptive Statistics of Representative Farms in Liaoning Province

Regions	Expected yield (dt/ha)	σ_Y	Sown area of rice (ha)	No. of farmers	$\rho_{Y,\Theta}$
1 Shenyang	79.31	924	126800	114375	-0.88
2 Dalian	59.28	693	29000	26158	-0.57
3 Anshan	73.81	1104	38100	34367	-0.92
4 Fushun	61.04	1045	20200	18221	-0.55
5 Benxi	59.82	690	9400	8479	-0.50
6 Dandong	64.18	768	52500	47356	-0.62
7 Jinzhou	74.79	803	29300	26429	-0.23
8 Yingkou	94.52	1157	44300	39959	-0.59
9 Fuxin	62.00	1047	5800	5232	-0.32
10 Liaoyang	69.74	1192	50400	45461	-0.97
11Panjin	94.09	674	108500	97868	-0.76
12 Tieling	75.01	1302	64700	58360	-0.91
13 Chaoyang	58.75	974	400	361	-0.24
14 Hulvdao	60.77	1050	9100	8208	-0.42