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Testing for linear and threshold cointegration under the spatial equilibrium condition

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Abstract

Economic theory states that the spatial equilibrium condition is a region where prices can be or not cointegrated. It is when prices are within such a region when they are no cointegrated, when prices are in its boundaries they are not only cointegrated but also fulfilling the Law of One Price (LOP). Nonetheless the econometric techniques assume a mean reverting process in order to test for cointegration, either linear or non linear. This research shows that in the absence of such mean reverting process by using prices in pure equilibrium, cointegration (linear and non linear) is often rejected. Such findings go in line with the Band Threshold Autoregressive Model where the neutral band is a region of no cointegration. Furthermore it can be concluded that the economic concept of perfect market integration (LOP) by itself is not sufficient for testing cointegration with some of the current econometric methods.

Keywords: Spatial Equilibrium Condition, Testing Cointegration

JEL classification: C15, E37.

1. Tests for linear and threshold cointegration

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The concept of cointegration has received large attention in different fields of the Economic Sciences. This is in part due the fact that many economic variables such as prices, GDP and exchange rates among others have specific econometric properties which makes conventional methods such as OLS regression to provide spurious results. Along with the estimation of threshold models is the task of testing threshold cointegration; nonetheless little have been done regarding the compatibility between testing for cointegration and market integration in Agricultural Economics. A piece of unique work on this regard is the research carried out by McNew & Fackler (1997). The aim of this research is to extend their work to the techniques used in threshold cointegration, thus the following work focuses not only on the classical linear tests, but also on the work done by Hansen & Seo (2002) and Seo (2006).

A core concept in cointegration analysis is the so called a unit root process: I(1) which is defined as:

$$= \quad + \quad = \quad + \sum \qquad (1)$$

where ε is a i.i.d process, furthermore $[\ \ ] =$ and $(\ \ ) =$. It has been demonstrated that performing regression analysis with I(1) variables leads to spurious regressions. Following such concern Granger (1981) and Engle & Granger (1987) introduced the concept of linear cointegration which has allowed estimating stable relationships among non-stationary economic variables (Pfaff, 2008). A common model in cointegration analysis is the so called Vector Error Correction Model which can be written as:

$$\Delta Y = \Pi Y + \Gamma \Delta Y + \cdots + \Gamma \quad \Delta Y \quad + \qquad (2)$$

where Y is a vector of variables, Π is a matrix which can be decomposed in αβ'. Although the set of variables contained on Y have a unit root, there is a linear combination of the variables which is a stationary process. The linear combination or error term can be written as

$$= \beta Y \qquad (3)$$

where β is the cointegration vector which ensures the error term to be I(0). The loading coefficient α ensures that any deviation of the equilibrium, in the short run is restored back.

In spatial price transmission analysis a limitation of the linear approach is that it neglects the so called neutral band. The neutral band is an economic theory concept which states that prices are in equilibrium. In such equilibrium, the difference of the prices for a homogeneous good in two regions is not greater than the transaction costs of trade between those two regions. The neutral band implies that within this region, prices does not adjust and therefore are not cointegrated. This issue has been addressed properly by the so called threshold cointegration concept which was introduced by Balke and Fomby (1997).

The equilibrium considered by Balke and Fomby can be denoted as

$$+ \quad = \qquad (4)$$

where the error term z_t is an autoregressive process such that

$$= {}^{(\,)} \quad + \qquad (5)$$

furthermore z_t follows a threshold autoregression such that

$$^{(\,)} = \begin{matrix} 1 & & |\quad| \leq \\ , & h\ |\ | < & |\quad| > \end{matrix} \qquad (6)$$

where θ is the threshold value. If $|\quad| \leq$ the error term z do not get back to the equilibrium, on the contrary if $|z\quad| >$ the error term z is a mean reverting process which goes back to the equilibrium. Using the first differences it is possible to write

$$\Delta \quad = \;^{(\;)} \qquad + \quad , \qquad (7)$$
$$\Delta \quad = \;^{(\;)} \qquad + \quad , \qquad (8)$$

whit $\gamma^{(\;)}$ and $\gamma^{(\;)}$ equal to zero when $|z \quad| \leq \theta$, and $\gamma^{(\;)} > 0$ and $\gamma^{(\;)} > 0$ when $|z \quad| >$. Both approaches, linear and non-linear have shown to be useful in different sets of applications. Furthermore both are grounded on the nature of the stochastic process z_t, namely if it has a unit root or if it is stationary.

As in many econometric models, the estimation of a linear or a threshold vector error correction model is not sufficient. Formally the econometric model has to be tested. Furthermore the data also has to be tested to verify its econometric properties. In linear cointegration analysis the testing procedure starts with the data (prices for instance). The first step is to verify if it has a unit root, for that some of the most common tests are the so called Augmented Dikey-Fuller Test (ADF) and the Kwiatkowske, Phillips, Schmidt & Shin test (KPSS). The ADF test has the form

$$\Delta \quad = \phi \qquad + \sum \qquad {}^{*}\Delta \qquad + \qquad (9)$$

The previous model serves to test the null hypothesis of a unit root $H \;: \phi = 0$ versus the alternative of $H \;: \phi < 0$. In case that the null cannot be rejected it is often recommended to perform the KPSS test. The KPSS test departs from considering the following model:

$$= \quad + \quad + \qquad (10)$$

with τ denoting the trend and ξ=0 when the process is level stationary, and x_t is an process such that

$$= \qquad + \qquad (11)$$

where the error term u is i.i.d $(0, \sigma \;)$. From the error ε it can be calculated S_t such that

$$S \;= \sum \quad \varepsilon \qquad (12)$$

The null hypothesis is denoted as $H \;: \quad = 0$ and the alternative is $H \;: \quad > 0$. If the null holds is not longer a random walk but a constant, therefore y is becomes a stationary process. Notice that here the null hypothesis is a stationary process I(0), while in the ADF test the null hypothesis is a unit root. The KPSS has the following test statistic

$$LM = \sum \qquad (13)$$

Both tests are not only useful for testing stationarity or unit root in the variables, but also for testing cointegration itself.

Once the variables have been tested for stationarity or a unit root, the following step is testing for cointegration. The so called Granger two-steps procedure is based first on estimating a linear combination of the variables as in equation (3), such that the resulting error term z_t is a stationary process. The second step on the Granger procedure consists on testing if the error term is stationary of it has a unit root. For that purpose the ADF and KPSS tests can be used.

Another approach different to the Granger two-step procedure is the Johansen Trace test (JTT). Introduced by Johansen (1988) he considers a VECM such as the one in equation (2), nevertheless the matrix Π is decomposed such that

$$\Delta Y + AB Y = +\Gamma \Delta Y + \cdots + \Gamma \Delta Y + \varepsilon \quad (14)$$

Where the matrix A contains the loading coefficients α and the matrix B contains the cointegration parameters β. Two auxiliary regressions are performed to eliminate the short-run dynamic effect. On the first one ΔY is regressed on the lagged differences of Y in order to obtain the residuals R_{0t}. On the second regression, Y is regressed on the same set of regressors in order to obtain the residuals R_{1t}. It happens that both residuals have a linear relationship such that

$$R = -AB R + U \quad (15)$$

where R_{0t} is vector of stationary processes and R_{1t} is a vector of non stationary processes. The Johansen test is based on finding the number of linear combinations B R which show the highest correlation with the stationary process R_0. Indeed the linear combinations is the rank of the matrix Π denoted as rk(Π). The testing procedure proposed by Johansen (1988) or JTT consist on testing the null hypothesis H_0: rk(Π)=r_0 versus the alternative of H_a: r_0< rk(Π)≤K, where K is the number of variables contained on the vector Y. The test statistic for the null can be written as:

$$Tr(r) = -T \sum \ln 1 - \lambda \quad (16)$$

where T denotes the number of observations, and λ denotes the eigenvalues.

Three possible outcomes are possible for the JTT. First if rk(Π)=K all the variables are stationary. Second if rk(Π)=0 the variables are not cointegrated. Third and last, if 0<rk(Π)<K then the variables are integrated of order r. When the variables are integrated of order r, r is the number of linear combinations which ensure B Y to be a stationary process.

The previous cointegration test; ADF, KPSS and JTT; are concerned with testing linear cointegration and do not consider a non-linear behaviour such a threshold. This problem was acknowledged by Balke & Fomby (1997). Unlike linear cointegration where there are two outcomes, cointegration and no cointegration, the threshold case has diverse possibilities. Such set of possibilities was well summarized by Balke & Fomby (1997) in the following table:

Table 1. Possible outcomes when testing for threshold cointegration

Hypothesis	Linearity	Threshold
No cointegration	(I) Linearity and no cointegration	(II)Threshold and no cointegration
Cointegration	(III) Linear cointegration	(IV) Threshold cointegration

Source: Balke & Fomby (1997)

As stated by the authors one can take any case as the null hypothesis, therefore any of the remaining three cases can be taken as the alternative hypothesis.

On his seminal paper Balke & Fomby (1997) by performing Monte Carlo simulations exanimate several linear tests such as the ADF, KPSS and JTT among other. Their aim was to address the question of how suitable such tests were for testing cointegration in artificial data generated under different threshold models. The idea behind such exercise is the assumption that even under the presence of a threshold, the error correction term globally will be a stationary process. Nevertheless in the case of the TVECM in the middle band the error term is allowed to have a unit root.

Following the idea of threshold cointegration, Hansen & Seo (2002) developed a test for threshold cointegration. Their approach was to test the null hypothesis of threshold cointegration (case IV) versus the alternative of linear cointegration (case III). Assuming that the cointegration vector β, and the threshold parameter θ are know, the model under the null is denoted as:

$$\Delta Y = A\ Y \quad (\beta) + \varepsilon \tag{17}$$

furthermore the model for the alternative hypothesis is denoted as

$$\Delta Y = A\ Y \quad (\beta) d \quad (\beta, \theta) + A\ Y \quad (\beta) d \quad (\beta, \theta) + \varepsilon$$

Hansen & Seo (2002) showed that the null can be tested with the test statistic

$$\begin{aligned} LM(\beta, \theta) = {} & \mathrm{vec}\ \ A\ (\beta, \theta) - A\ (\beta, \theta) \qquad V\ (\beta, \theta) - V\ (\beta, \theta) \\ & \times \mathrm{vec}\ \ A\ (\beta, \theta) - A\ (\beta, \theta) \end{aligned} \tag{18}$$

where $V\ (\beta, \theta)$ and $V\ (\beta, \theta)$ are the Eicker-White covariance matrix estimators for $\mathrm{vec}A\ (\beta, \theta)$ and $\mathrm{vec}A\ (\beta, \theta)$. As equation (17) is a simple OLS, it is possible to get the estimator for β under the null denoted as β, nevertheless as for θ there is not an estimator, the LM statistic has to be estimated at different values of θ such that

$$\mathrm{SupLM} = \sup \quad \mathrm{LM}\ \beta, \theta \qquad (19)$$

Equation (19) is a profile likelihood function where the search region is $[\theta\ , \theta\ ]$. The parameter θ is set at the value π , and the parameter θ is set at the value is set at the value $(1 - \pi\ )$. This imposes the constrain $\pi \leq \mathrm{Prob}(z \leq \theta) \leq 1 - \pi$.

The estimator for θ is the value that maximizes equation (19), nevertheless such an estimator will be different from the one obtained from the estimation of a TVECM using the Hansen & Seo (2002) method. The reason is that testing for threshold cointegration the estimated parameter β remains fixed and the profile likelihood is only performed for the threshold value. On the contrary for the TVECM the profile likelihood is performed over the two parameters (Hansen & Seo, 2002). Although the previous test allows testing for threshold cointegration, ideally one would like to test directly the null of no cointegration, versus the alternative of threshold cointegration (Balke & Fomby, 1997).

Seo (2006) proposed an approach that allows testing directly the null of no linear cointegration, versus the alternative of threshold cointegration. For that he proposed an error correction term with known cointegration parameter as in equation (3). Then the TVECM is of the form

$$\Delta Y = A\ (\theta) z \quad 1\ z \quad \leq \theta \quad + A\ (\theta) z \quad 1\ z \quad > \theta$$
$$+ \mu(\theta) + \Phi\ (\theta) \Delta Y \quad + \cdots + \Phi\ (\theta) \Delta Y \quad + u\ (\theta) \qquad (20)$$

with $\theta \leq \theta$, and a no adjustment region $\theta \leq z \quad (\beta) \leq \theta$.

The null hypothesis is denoted as $H : \alpha = \alpha = 0$ and the alternative $H : \alpha \neq \alpha$. Letting

$$\Sigma(\theta) = - \qquad u\ (\theta) u\ (\theta) \qquad (21)$$

the Wald statistic for testing the null when θ is fixed can be written as

$$W\ (\theta) = \mathrm{vec}\ \ A(\theta) \quad \mathrm{var}\ \ \mathrm{vec}\ \ A(\theta) \qquad \mathrm{vec}\ \ A(\theta) \qquad (22)$$

and the superior statistics is defined as

$$\mathrm{supW} = \sup_{\ \in}\ W\ (\theta) \qquad (23)$$

It is important to notice that there is a fundamental difference in the tests proposed by Hansen & Seo (2002) and Seo (2006). The first one considers a model with one threshold and two regimes, while the last one considers a model with two thresholds and three regimes. In both tests there is bootstrapping in order to get the distribution(s) of the thresholds parameter(s).

2. TESTING FOR COINTEGRATION IN THE EQUILIBRIUM

2.1. Simulating the data under the spatial equilibrium condition

The tests presented in the previous section, are the standard tools in linear and threshold cointegration analysis. So now the task is concentrated on performing such test in artificial data obtained under an economic framework. The artificial data is obtained by implementing simulation using the Takayama and Judge Models. For that the following inverse supply and demand functions are considered[1]:

$$_{,} = 5 + \sum \quad + 0.1 \quad _{,} \qquad (24) \qquad\qquad _{,} = 20 - 0.1 \quad _{,} \qquad (25)$$

$$_{,} = 2.5 + \sum \quad + 0.05 \quad _{,} \qquad (26) \qquad\qquad _{,} = 20 - 0.2 \quad _{,} \qquad (27)$$

where *p* denotes de price, *s* the supply and *d* the demand, and $\varepsilon \backsim N(0, 1)$ and a matrix of transport costs

$$T = \begin{matrix} 0 & 2 \\ 2 & 0 \end{matrix} \qquad (28)$$

The time dimension *t* is set up at 250 and 500, and for both 1000 repetitions are performed. The result of such simulation is prices always in the neutral band where either the relationship $_{,} \leq 2 + \quad _{,}$ or $_{,} \leq 2 + \quad _{,}$ hold. It is when the differences on the prices between both regions is less than the transaction costs, either $_{,} - \quad _{,} < 2$ or $_{,} - \quad _{,} < 2$, that the relationship between the prices is not stable. In other words the relationship is not a stationary process and it has a unit root. On the contrary when the difference of the two regions' prices equals to the transaction costs is when the Law of One Price (LOP) holds, either $_{,} - \quad _{,} = 2$ or $_{,} - \quad _{,} = 2$, and the relationship is stable. Nonetheless in this framework, such a relationship is not a mean reverting process but rather it is a constant.

From the previous simulation what one can obtain is prices always in equilibrium. Those prices sometimes accomplish with the LOP and sometimes not as it has been discussed. The question is how suitable are the standard test for cointegration using the artificial data.

[1] The inverse and demand functions are the same as the one used in Takayama & Judge (1964) except for the implementation of the unit root component which includes some dynamics into the model.

2.2. Implementing the tests

The second step is to test for cointegration. This can be done under the two-step procedure proposed by Engel & Granger such that

$$ {}_{,} = \quad {}_{,} + \qquad (29)$$

with denoting the error correction term (ECT). In order to test for cointegration, the ADF and KPSS are performed to evaluate if is a stationary process (cointegration), or if it has a unit-root (no cointegration). It is also interesting to test for cointegration under the restriction $= 1$, for doing so simply the prices differences between the two regions are taken as the error correction term such that :

$$ {}_{,} - \quad {}_{,} = \qquad (30)$$

As in the previous case, if is a stationary process the prices are cointegrated, otherwise price are not cointegrated.

The other way for testing cointegration is with the JTT. As the system consists of two variables, it is expected that cointegrated variables will have one cointegration relationship. That is the null H_0:rk(Π)=1 cannot be rejected. When the outcome is that the null H_0:rk(Π)=0 cannot be rejected, then the variables are no cointegrated. From an economic perspective the prices obtained from the simulation are cointegrated, as they they follow a common path and fulfil the LOP. Econometrically speaking cointegration holds when the error terms or are stable mean reverting processes. Nevertheless on the simulation for obtaining the prices there is not a mean reverting process included. The stable part of the relationship, as it was mentioned before is a constant. How this might affect the test results is an issue.

The following step is testing for threshold cointegration using the Hansen & Seo (2002) test. Such a test contrasts the null of threshold cointegration (case IV) versus the alternative of linear cointegration (case III). For such test it is possible to consider an error correction term such that $z = \quad {}_{,} - \quad {}_{,}$, with β denoting the cointegation vector obtained from the simple OLS regression, or an error correction term such that $z = \quad {}_{,} - \quad {}_{,}$.

Notice that the previous test as mentioned before considers a two regimes one threshold model. Therefore the estimated threshold obtained corresponds to the lower threshold. As the artificial data presents trade reversals the adequate specification should be a three regimes two threshold model. To what extent this can be a limitation or not can be examined when looking at the results of the test. Following this idea the final step is testing threshold cointegration with the Seo (2006) test, which considers the null hypothesis of no cointegration (Case I) versus the alternative of threshold cointegration (Case III). Unlike the Hansen & Seo test, the Seo test considers a model with three regimes and two thresholds. For this test the cointegration parameter β in this study is restricted to one, so that only the error term denoted as z $=$, $-$, is evaluated.

2.3. Results from the tests

The ADF and KPSS were performed to the equilibrium prices obtained from the simulations for a time length of 250 and 500. The results showing the rejection of the null for both tests are summarized in Table 2.

Table 2. ADF and KPSS tests: percentiles for the rejection of the null

Time periods	ADF						KPSS					
	p_1			p_2			p_1			p_2		
	α			α			α			α		
	0.01	*0.05*	*0.1*	*0.01*	*0.05*	*0.1*	*0.01*	*0.05*	*0.1*	*0.01*	*0.05*	*0.1*
250	0.024	0.078	0.155	0.037	0.091	0.157	0.993	0.994	0.994	0.992	0.992	0.992
500	0.079	0.156	0.211	0.082	0.162	0.227	1.000	1.000	1.000	0.991	0.991	0.991

Source: Own elaboration using the R package URCA developed by Pfaff (2011) v. 1.2-5

For the ADF test it can be observed that as the time length is increased and the level of confidence (α) decrease, the probability of rejecting the null of a unit root increases. On the contrary the KPSS test does not seem to be affected neither by the level of confidence or the number of time periods. Furthermore for the KPSS results consistently the null of stationarity is rejected at least in 99% of the cases. Following these results it can be confirmed that the prices obtained from the simulations have a unit root.

At this stage makes sense to recall that given the nature of the simulations, both error terms exhibit parts on which they do not vary across the time. Such periods of non-variation for both error terms lead to problems in the forthcoming analysis. As the error term is time invariant, the Gauss-Markov assumptions are violated and it is not possible to perform OLS estimation. The ADF test, KPSS test, JTT, the Hansen and Seo test and the Seo test depend on OLS regressions based on the error term. From now on whenever it is not possible to perform a test due to the non-variation on the error term, such an outcome is referred as a non-feasible solution. On the contrary the test which can be solved due to variation across the error term, are referred as feasible solutions.

The next step concerns cointegration, and for that we first focus on the Granger approach. First the error term is estimated as in equation (29) that is beta unrestricted, then the error term as in equation (30) which is with beta restricted to one. Table 3 summarizes the percentiles for the rejection of the null hypothesis. Recall that for the ADF test the null states as "No cointegration", while for the KPSS test the null states as "Cointegration".

Table 3. Percentiles of the null rejection for the ADF and KPSS test for cointegration

Restriction	Time periods	ADF				KPSS			
		Feasible Solutions	α			Feasible Solutions	α		
			0.01	*0.05*	*0.1*		*0.01*	*0.05*	*0.1*
$\beta=$	250	991	0.35	0.40	0.47	994	0.53	0.61	0.65
	500	991	0.29	0.37	0.47	1000	0.69	0.74	0.77
$\beta=1$	250	991	0.31	0.38	0.44	992	0.63	0.70	0.74
	500	991	0.27	0.34	0.41	991	0.75	0.80	0.83

Source: Own elaboration using the R package URCA developed by Pfaff (2011) v. 1.2-5

The results from the previous tests shows mixed results. On the one hand the probability that the ADF test rejects the null of "No cointegration" increases as α increases. The probability of rejecting the null for the ADF test ranges from 0.27 to 0.47, these figures do not allow to distinguish if the prices are cointegrated or not. Regarding the KPSS test the results are similar; the probability of rejecting the null of "Cointegration" goes from 0.53 to 0.83, and increases as α increases.

Following with the testing procedure for cointegration the JTT is performed. The JTT involves testing two null hypotheses, : $= 0$ and : $= 1$. When is rejected and is not rejected then the prices are cointegrated. Table 4 summarizes the percentiles for rejecting each null, and the percentiles for the cointegrated pair of prices using the JTT. The outcome from the JTT suggests weak evidence of linear cointegration, ranging from 0.21 to 0.45.

Table 4. Percentiles for the null rejection and cointegration with the JTT

Time periods	Feasible Solutions	H_0	α=0.01	α=0.05	α=0.1
250	740	*r=0*	0.22	0.28	0.33
		r=1	0.01	0.03	0.05
		Cointegration	0.21	0.25	0.28
500	909	*r=0*	0.36	0.48	0.57
		r=1	0.03	0.07	0.13
		Cointegration	0.33	0.40	0.45

Source: Own elaboration using the R package URCA developed by Pfaff (2011) v. 1.2-5

The three tests, ADF, KPSS and JTT, provide with little evidence of cointegration. Nevertheless this result might be related to structural breaks. Since the prices were generated with random walks, those random walks might lead to trade reversals. Trade reversals are indeed a structural change in the prices relationship. It is well know that the three tests quite often fail in the presence of structural breaks. As for that the analysis for the linear cointegration can be divided in two groups: prices with no trade reversals and prices with trade reversals. Table 5 summarizes the results for the ADF and KPSS test for cointegration for those two groups.

Table 5. Percentiles of the null rejection for the ADF and KPSS test for cointegration: No trade reversal and trade reversal

Group	Restriction	Time periods	ADF				KPSS			
			Feasible Solutions	α			Feasible Solutions	α		
				0.01	*0.05*	*0.1*		*0.01*	*0.05*	*0.1*
No trade reversal	$\beta=$	250	363	0.91	0.94	0.96	365	0.05	0.09	0.14
		500	282	0.87	0.89	0.92	284	0.14	0.20	0.27
	$\beta=1$	250	363	0.81	0.91	0.93	364	0.10	0.22	0.30
		500	282	0.81	0.85	0.88	282	0.20	0.29	0.40
Trade reversals	$\beta=$	250	628	0.02	0.09	0.19	629	0.81	0.91	0.95
		500	709	0.06	0.16	0.29	716	0.91	0.96	0.97
	$\beta=1$	250	628	0.01	0.08	0.15	628	0.94	0.98	0.99
		500	709	0.05	0.13	0.22	709	0.98	0.99	1.00

Source: Own elaboration using the R package URCA developed by Pfaff (2011) v. 1.2-5

The numbers in the previous table clearly show a difference between the two groups. On the one hand, in the absence of trade reversals the cointegration is rejected for most of the cases; nevertheless the rejection is higher for the ADF test and when $\beta=$. On the other hand in the presence of trade reversals, cointegration is not rejected for most of the cases. Such an outcome, as it was mentioned before, is because the structural break. Indeed the results confirm the views of Balke & Fomby (1997) for which the middle regime or neutral band, even though it might contain prices in equilibrium, is a region of no adjustment for which prices are not cointegrated. Table 6 summarizes the results of the JTT for the two groups.

Table 6. Percentiles for the null rejection and cointegration with the JTT: no trade and trade reversals

Group	Time periods	Feasible Solutions	H_0	$\alpha=0.01$	$\alpha=0.05$	$\alpha=0.1$
No structural break	250	112	*r=0*	0.92	0.95	0.97
			r=1	0.05	0.11	0.20
			Cointegration	0.87	0.84	0.78
	500	200	*r=0*	0.84	0.87	0.89
			r=1	0.10	0.23	0.27
			Cointegration	0.75	0.64	0.62
Structural break	250	628	*r=0*	0.09	0.16	0.21
			r=1	0.00	0.01	0.03
			Cointegration	0.09	0.15	0.19
	500	709	*r=0*	0.22	0.37	0.48
			r=1	0.01	0.03	0.09
			Cointegration	0.21	0.34	0.40

Source: Own elaboration using the R package URCA developed by Pfaff (2011) v. 1.2-5

The JTT provides a different panorama if compared with the ADF and KPSS tests. Now in the presence of structural breaks (trade reversals), the test suggest little evidence of cointegration. In the absence of trade reversals, the evidence of cointegration goes from the 0.62^{nd} up to the 0.87^{th} percentile.

Although at a first sight the results from the unit root test, ADF and KPSS, are counterfactual to those offered by the JTT, it is worth to recall that both are based on different approaches. On the one hand the JTT is based on finding a number of stable relationships among variables (equation 15), while the ADF and KPSS tests concentrate on finding a short memory process. Nevertheless both approaches in the end test if the long run relationship is a stable mean reverting process, which strictly speaking is not present in the artificial data used for the analyses. What it can be said so far, is that in the absence of a mean reverting process, for purely data in equilibrium the standard tests for linear cointegration provide ambiguous results.

As it was discussed before, the prices for this study are a mixture of prices accomplishing the LOP (cointegrated) and unrelated prices in equilibrium. The numbers of observations for prices which are no cointegrated might also have some influence in the linear cointegration results. Hence it makes sense to distinguish between unrelated and cointegrated (LOP for the data used on this research) when testing, in other words testing for threshold cointegration. The first test used is the proposed by Hansen and Seo (2006) which considers a two regime-one threshold model, and test the null of "Linear cointegration" versus the alternative of "threshold cointegration". The test was done for the 1000 simulations, the results are summarized in Table 7.

Table 7. Percentiles for the null rejection using the Hansen & Seo test

Time periods	*Feasible solutions*	π_0	β=			β=1		
			α=0.10	α=0.05	α=0.01	α=0.10	α=0.05	α=0.01
250	740	0.05	0.28	0.20	0.07	0.20	0.14	0.09
		0.10	0.27	0.19	0.07	0.20	0.14	0.08
		0.15	0.28	0.19	0.06	0.16	0.09	0.03
500	820	0.05	0.35	0.24	0.10	0.24	0.15	0.06
		0.10	0.36	0.23	0.10	0.25	0.16	0.06
		0.15	0.35	0.23	0.10	0.25	0.15	0.05

Source: Own elaboration using the R package tsDyn developed by Di Narzo, Aznarte & Stigler (2011) v. 0.7-60

On overall the test does suggest that prices are no threshold cointegrated, as the cointegration range from 0.36 to the 0.03 percentile. Moreover when the cointegration vector is restricted to one the evidence of threshold cointegration is even lower. It is worth to mention that this outcome might be related to the trade reversals, as in the linear test. Recall that the Hansen & Seo test is based on a model with no trade reversals which is a one threshold model. In the presence of trade reversals, the correct specification should include two thresholds. Hence using the Hansen & Seo test in prices for which markets are experiencing trade reversals is a misspecification. Following this idea the results are split in two groups as it was done before:

markets with trade reversals, and markets without trade reversals. Table 8 summarizes the outcome.

Table 8. Percentiles for the null rejection using the Hansen & Seo test: trade and no trade reversals

Group	*Time periods*	*Feasible solutions*	π_0	*β=*			*β=1*		
				α=0.10	α=0.05	α=0.01	α=0.10	α=0.05	α=0.01
No trade reversal	250	111	0.05	0.45	0.35	0.19	0.14	0.05	0.01
			0.10	0.45	0.35	0.19	0.13	0.05	0.00
			0.15	0.49	0.34	0.15	0.23	0.12	0.01
	500	111	0.05	0.51	0.39	0.22	0.35	0.22	0.07
			0.10	0.50	0.36	0.21	0.34	0.23	0.07
			0.15	0.51	0.37	0.22	0.33	0.19	0.08
Trade reversal	250	629	0.05	0.25	0.17	0.05	0.21	0.16	0.10
			0.10	0.24	0.16	0.05	0.21	0.15	0.10
			0.15	0.24	0.16	0.05	0.14	0.09	0.04
	500	709	0.05	0.33	0.22	0.08	0.23	0.15	0.05
			0.10	0.34	0.21	0.08	0.23	0.15	0.06
			0.15	0.32	0.21	0.08	0.24	0.14	0.05

Source: Own elaboration using the R package tsDyn developed by Di Narzo, Aznarte & Stigler (2011) v. 0.7-60

It is interesting to see that when the two groups are look separately the percentiles are considerable different. For instance when using markets with no structural breaks (trade reversals), the no rejection of threshold cointegration goes from the 51st to the 19th percentile, as before the rejection of the null decreases if the cointegration parameter is restricted to one. Up to this point one could conclude that the the Hansen & Seo test should be used only for its correct specification, that is markets with no trade reversals. Nonetheless it is important to keep in mind that the number of simulation for markets with those characteristics are few, for instance from both simulations time dimensions, 250 and 500, out of 1000 only 111 results for each are suitable for performing the test. With only 111 simulations, it is risky to judge that if not using the correct specification, two regime one threshold model, then the test losses power.

As an alternative for testing thereshold cointegration in markets with three regimes and two thresholds is the Seo test. For each threshold combination the Sup-Wald test as in equation (23) is obtained, this process is made for every bootstrap; therefore the computation is very time demanding (Di Narzo, Aznarte & Stigler; 2011). Before implementing the test for all the simulations, some trials were made in order to have an idea of the time needed for performing the test. When the test was performed using the estimated cointegration vector from the OLS, each test took on average one and three hours for the data with 250 and 500 observations respectively. When the cointegration parameter was restricted to one, each test took on average 30 minutes and one hour and a half for the data with time dimension 250 and 500 respectively. On the previous analysis (Hansen & Seo test) it was observed that increasing the time length from 250 to 500 did not lead to a substantial difference on the results, therefore it was decided, at this stage, to leave out the data with 500 time periods for this part of the analysis.

Furthermore since the estimation using the cointegration parameter from the OLS is time demanding, it was decided to perform the test with the restriction. Therefore the Seo test was done only for the data with time length of 250 observations, and the cointegration parameter restricted to one. The results of such test applied to the simulations are summarized in table 9.

Table 9. Percentiles for the null rejection using the Seo test

Time periods	π_0	*Feasible solutions*	$\beta=1$		
			$\alpha=0.10$	$\alpha=0.05$	$\alpha=0.01$
250	0.05	483	0.76	0.94	0.87
	0.10	488	0.78	0.94	0.92
	0.15	511	0.77	0.96	0.93

Source: Own elaboration using the R package tsDyn developed by Di Narzo, Aznarte & Stigler (2011) v. 0.7-60

The test results shows, contrary to the Hansen & Seo test, a strong evidence for threshold cointegration against no cointegration. To what extent this might be linked to the fact that many simulations consider a trade reversal, and therefore two thresholds can be seen in table 10.

Table 10. Percentiles for the null rejection using the Seo test: trade and no trade reversal

Time periods	*Group*	π_0	*Feasible solutions*	$\beta=1$		
				$\alpha=0.10$	$\alpha=0.05$	$\alpha=0.01$
250	No trade reversal	0.05	52	0.73	0.97	0.92
		0.10	56	0.73	0.93	0.95
		0.15	105	0.70	0.97	0.88
	Trade reversals	0.05	431	0.76	0.94	0.86
		0.10	432	0.79	0.94	0.92
		0.15	406	0.78	0.96	0.94

Source: Own elaboration using the R package tsDyn developed by Di Narzo, Aznarte & Stigler (2011) v. 0.7-60

The figures below do not show a major difference between using data with or without trade reversal. Both cases show a strong evidence of threshold cointegration, against no cointegration.

3. DISCUSSING THEORY AND RESULTS

The economic theory for price transmission, speaking of the equilibrium, is that prices are bounded in a region, the so called neutral band. Within the band prices are in equilibrium but no cointegrated, in the band borders prices are in a perfect equilibrium and cointegrated, in other words the LOP is fulfilled. On this regard, the spatial equilibrium condition does not contemplate at any point a mean reverting process as the econometric techniques does. Testing for cointegration, in an econometric framework, involves a short memory process, namely a unit root and a long memory process, namely a mean reverting or stationary process[2]. Recalling Balke and Fomby (1997), the relationship between the variables should be globally stationary, nonetheless within the neutral band might have a unit root. The data generated for the analyses is a perfect emulation of the neutral band as discussed in Section II.

The results from applying the ADF and KPSS test to the artificial data show that in the absence of trade reversals prices are no linearly cointegrated. For instance, economically the LOP is a representation of cointegrated prices, but econometrically the mean reverting process is necessary for testing linear cointegration; hence the rejection of linear cointegration. It is important to stress the fact that ADF and KPSS test can be fooled if trade reversals are occurring. Contrary to the ADF and KPSS test results, the JTT supports cointegration (considering no trade reversals). On this regard it can be said that the JTT provides evidence of linear cointegration even in the absence of a true mean reverting process.

By construction the data generated is a mixture of no-cointegrated prices (within the neutral band) and cointegrated prices (in the border of the neutral band), which is the threshold cointegration case. Therefore linear cointegration should be rejected not only against no-cointegration, but against linear cointegration as well. The Seo & Hansen test serves for the later purpose. As in the linear case, in testing for threshold cointegration trade reversals are relevant. In the case of the linear cointegration such phenomena can be explained by structural changes, but in the threshold case it is related to the number of regimes to include. For instance the Hansen & Seo test accounts only for one threshold, therefore having data for with trade reversals occurring (three regimes model) can affect the result. Indeed the results show that the rejection of the null (linear cointegration) is higher for the correct specification of the test (two regimes and one threshold). Nonetheless still the rejection of the null is never beyond 51%, so in this regard the evidence of threshold cointegration is weak. Likewise the Hansen & Seo test, the Hansen test results might be driven by the number of true regimes present in the data. Notwithstanding the Hansen test is based on a three regimes model. Therefore the correct specification for such test is data with trade reversals. The results suggest a strong evidence of threshold cointegration against the null of no-cointegration; furthermore there are not major differences between using data with or without trade reversals. It is important to recall that strictly speaking the data used here is missing information from the outer regime(s), as for that when selecting the trimming parameter to a certain level (0.05, 0.10 or 0.15) some data from the neutral band will be dropped out of such region. From both test it can be derived that in the absence of a true mean reverting process, there probability of rejecting linear cointegration against threshold cointegration is lower than the probability of rejecting no-cointegration against threshold cointegration.

[2] Albeit the mean reverting process is mentioned in the theory as deviations from the equilibrium which are corrected by arbitrage, to the knowledge of the author there is no much research on how to model disequilibrium in order to understand the causes behind such deviations.

The task now lays on how to conciliate the results from the cointegration tests in order to see how the neutral band fits in the concept of threshold cointegration. Ideally in the case of true threshold cointegration, both tests, Hansen & Seo and Seo, should reject their respective null hypotheses in favour of threshold cointegration. Furthermore following Balke and Fomby, the long run relationship between prices has to be globally stationary; therefore no cointegration should be rejected in favour of linear cointegration. Only when those three previous results hold it can be accounted for complete threshold cointegration, otherwise the evidence is dubious. Indeed the results show poor evidence of linear cointegration against no cointegration; the evidence of no cointegration is even stronger for the ADF and KPSS tests than for the JTT. Regarding threshold cointegration, the null of linear cointegration is often no rejected against the alternative of threshold cointegration. Nevertheless surprisingly the null of no cointegration is often rejected in favour of threshold cointegration. A reasonable question is to what extent such a finding can be accounted for threshold cointegration when the previous test does not support such hypothesis. For doing so one has to remember that the last test was performed only for the simulations with 250 observations and beta restricted to one; then it is only possible to compare the results from the test to such specific group. Table 10 summarizes the number of simulation for which the three basic test results for threshold cointegration hold.

Table 11 Number of simulations which satisfies the three conditions for threshold cointegration ($t=250$, $\beta=1$)

π_0	α	*Number of simulations*
0.05	0.10	3
	0.05	2
	0.01	1
0.10	0.10	1
	0.05	0
	0.01	0
0.15	0.10	7
	0.05	4
	0.01	0

Source: Own elaboration

4. CONCLUSIONS AND REMARKS

Overall from the tests results for cointegration, linear and threshold, it can be concluded that the equilibrium data simulations econometrically are no-cointegrated. The reason is a slight different conception between economic theory and econometric methods. For the econometric methods the perfect cointegration of prices accomplishing the Law of One price is not a representation of a mean reverting process, hence the absence of data in disequilibrium being corrected brings as a result no-cointregation. Nevertheless such findings fit into the so called Band Threshold Vector Error Correction models which consider a regime where no adjustment is taking place (Lo & Zivot, 2001), namely the neutral band on which the long run relationship is a unit root process. The results from the linear part support the findings from McNew & Fackler (1997) which found that even prices in equilibrium do not necessarily exhibit a cointegration relationship. Moreover the threshold cointegration tests do not provide enough evidence of threshold cointegration. Therefore it can be said that the LOP, although being a representation of cointegrated prices, it is not sufficient for testing cointegration, linear and non-linear, using some of the current econometric techniques. In view that the spatial equilibrium condition is not sufficient for testing cointegration, it should not be surprised that estimating a threshold model with such sort of data will bring bad results as showed by Araujo-Enciso & v.Cramon-Taubadel (2011).

It is important to stress that the trade reversals have shown to cause problems in testing for cointegration. In the case of linear cointegration it is possible to find a reasonable number of test which accounts for that. In the case of threshold cointegration, it was shown that selecting the incorrect number of regimes can affect substantially the results.

In general it is recommended to explore the results here obtained more in detail, for instance by applying the Seo test to the whole number of simulations, and by increasing or balancing the number of simulation with and without trade reversals.

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