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Forum

Box-Jenkins Forecasting Models: Comment

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Two recent papers in this *Review* (Vol 47, No.2, August 1979) set out to evaluate alternative forecasting techniques applied in the Australian beef market. The two papers: "Forecasting NSW Beef Production: An Evaluation of Alternative Techniques," by C. Gellatly, and "Comparing the Box-Jenkins and Econometric Techniques for Forecasting Beef Prices" by I.J. Bourke both give rise for some concern at the choice of Box-Jenkins models which were estimated. In consequence the subsequent comparative model evaluations may reflect rather more poorly on the technique than they otherwise might.

Although my comments have some features common to both papers, it will be appropriate to deal with each individually.

(i) The "Gellatly" Quarterly Box-Jenkins Model of N.S.W. Beef Production

The model finally selected for fitting was

$$(1 - \phi_1 B)(1 - \Phi_1 B^4)(X_t - \overline{X}) = a_t$$

which when substituting parameter estimates, gives

$$\underset{(0.061)}{(1 - 0.908B)}\underset{(0.125)}{(1 - 0.238B^4)}(X_t - \overline{X}) = a_t$$

with standard errors in parentheses.

Although the data are not given, Appendix B does present the autocorrelation function (a.c.f.) and partial autocorrelation function (p.a.c.f.) for the series. From these, the following can be observed.

(a) the a.c.f. does not decay exponentially, as might be expected of a stationary first order autoregressive process, such as that estimated above.

(b) neither the a.c.f. nor p.a.c.f. exhibit the seasonal peak at lag 4, that a quarterly seasonal model might be expected to give. The lack of evidence of seasonality in the undifferenced series is supported by the fact that the estimate of Φ is non-significant.

(c) the a.c.f. dies out slowly. This suggests that the series is non-stationary and that differencing should be applied. Such a conclusion would, for example, have been reached if the

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autocorrelation coefficients had been tested against a null hypothesis that they were not significantly different from zero after lag 1; this would only have been accepted at lag 14. Additional evidence for first differencing is contained in the estimate of ϕ_1 and its standard error. At the 5 per cent significance level, it is not significantly different from unity. Hence the operator $(1-0.908B)$ could well be represented by $(1-B)$, which is of course, equivalent to the first differences of the data.

It would have been interesting to see the a.c.f. and p.a.c.f. for first differences of the data. Additionally one might enquire whether the author examined a range-mean plot of the data to assess whether there was any need for transformation. It is my experience in constructing univariate models for UK beef production, prices and cattle slaughterings that a logarithmic-transformation is necessary, if only to deal with the greater variability in the 1972-5 period (Revell 1977, 1978).

The above comments in relation to the estimated model would imply a more parsimonious equivalent form:

$$\nabla X_t = a_t$$

or, in other words, that beef production is a random walk process. This is not particularly helpful for forecasting purposes. Nevertheless, it does explain why the estimated model did no better than a naive alternative on a U statistic comparison. It also partially explains why the Q statistic showed no evidence of non-randomness in the residual autocorrelations.[1]

In conclusion, it is suggested that either a simpler model, or a different diagnosis with appropriate differencing and transformation are required. The latter might clearly affect the relative ranking of the Box-Jenkins technique compared with the alternative approaches of regression and the Committee.

(ii) The "Bourke" Quarterly and Monthly Models of Cow/Cutter Beef Prices

A general comment related to visual inspection of Figure 1. The price-plot clearly suggests non-stationarity. It is therefore difficult to reconcile the quarterly and monthly models, which essentially consist of the same data, in that seasonal differencing are applied to the monthly data, but not to the quarterly observations. A second point relates to the appropriate transformation of the data. Again the greater variability in the period 1972-5 in relation to the level of the series might require use of a logarithmic transformation; an examination of the residuals would indicate whether it was necessary.

There are some specific comments in relation to the two models, which will be considered separately.

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[1] It should be noted that the Q statistic is a weak test of non-randomness in residual autocorrelations, and that it is often more fruitful to examine individual residual autocorrelations for indications of model inadequacy.

(a) *The Quarterly Model*
The model selected for fitting was[2]

$$(1-\phi_1 B)(1-\Phi_S B^4)Z_t = (1-\Theta_S B^4)U_t$$

with parameter estimates as follows (standard errors are given in parentheses below)

$$\underset{(0.081)}{(1-0.938B)}\ \underset{(0.035)}{(1-0.960B^4)}Z_t = \underset{(0.094)}{(1-1.021B^4)}U_t$$

First, none of the parameter estimates are significantly different from unity. Hence the operator $(1-\phi_1 B)$ can be approximated by $(1-B)=\nabla$, suggesting the need for first differencing. Secondly the seasonal autoregressive and moving average parameters are redundant, owing to their near cancellation.

In short, quarterly canner/cutter cow beef prices might well be represented by the random walk process $\nabla X_t = U_t$.

(b) *The Monthly Model*
The specified model

$$(1-\phi_1 B-\phi_2 B^2)\ \nabla_{12}Z_t = (1-\Theta_1 B^{12})U_t$$

which yielded estimates as follows

$$\underset{(0.0803)}{(1-1.3175B}\ \underset{(0.0792)}{+\ 0.3423B^2)}\ \nabla_{12}Z_t = \underset{(0.0440)}{(1-1.0137B^{12})}U_t$$

also exhibits parameter redundancy. The seasonal moving average coefficient Θ_1 is not significantly different from unity. Hence it can be written as $(1-B^{12})$ and cancels with the ∇_{12} operator. The second order autoregressive operator $(1-\phi_1 B-\phi_2 B^2)$ can, to a reasonable approximation (in view of the standard errors of the coefficients) be written as $(1-1.3B+0.3B^2)$ which factorises to $(1-B)(1-0.3B)$ or $(1-0.3B)\nabla$. Thus the model can be parsimoniously written as $(1-0.3B)\ \nabla Z_t = U_t$

which is a first-order autoregressive process in first differences with no seasonality. It would be interesting to know whether the author had explored this possibility.

At least the non-stationarity of both the monthly and quarterly models under the parsimonious model forms are represented consistently by differencing in the unit interval.

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[2] Note the AR parameters ϕ and Φ and the MA parameter Θ have been transposed in accordance with the customary notation used by Box and Jenkins (1970).

In conclusion, it is clear that there is often a 'non-uniqueness' in Box-Jenkins model diagnosis from the sample economic time series data with which we must work. However, a careful analysis of the tentatively estimated models will reveal any model inadequacies, and may suggest either more complex models, or simpler alternatives. In the models described above, it is suggested that equivalent, more parsimonious forms than those estimated exist, although ones which are perhaps less useful for the forecaster. The choice of alternative differencing and transformation might, however, produce more acceptable models.

References

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