Paper prepared for the 123rd EAAE Seminar

PRICE VOLATILITY AND FARM INCOME STABILISATION

Modelling Outcomes and Assessing Market

and Policy Based Responses

Dublin, February 23-24, 2012

DYNAMIC CONTROL AS MEASURE TO STABILIZE AGRICULTUTAL MARKETS:

On Theory and Options to Correct Cyclical Movements

Nuppenau E.-A.[1]

1 Institut fuer Agrarpolitik und Marktforschung, Senckenbergstrasse 3, Giessen, Germany

ernst-august.nuppenau@agrar.uni-giessen.de

Dynamic control as measure to stabilize agricultural markets: On theory and options to correct cyclical movements

Nuppenau E.-A.

Abstract

This paper deals with a control theory approach to stabilize cyclical price movements. Firstly, we pursue a welfare economist's approach, delineating a public objective function derived from consumer and producer surplus as well as trade budget at world market price. Trade is a control variable and the market price is a state variable. Moreover we assume adaptive formation of price expectation at the producer side. Secondly the control problem is outline and solved by discrete control theory. Thirdly, the paper makes suggestions how to translate the optimal control framework and results into a trade policy of the envisaged country. Finally, limitations are discussed and an outlook for broadening the concept for international concerns is offered.

Keywords: dynamic control, expectations, cyclical prices, stabilization policy

JEL classification: C54, C61, F61.

1. INTRODUCTION

Price volatility on agricultural product markets has short and long term effects. Whereas the discussion on short term effects and stabilization of income normally are given priority in the debate on farmers' impact for political reason (FAO et al. 2011), the long run effects are usually of less concern. Also needs for stabilization of negative dynamic effects, resulting from expectations, are discussed sometimes. At least this happens when a new phase of volatility on international markets occurs. If there is great uncertainty on future prices and expectations matter, market participants and policy makers mostly, after observing instability over years, discover negative dynamics on food markets, eventually running out of self-regulations; i.e. in terms of signalling scarcities. For example, if prices are high farmers welcome it (but how transitory is it?). Experience from past phases of high price volatility is that markets, after a certain time, have difficulties in forming price expectations. Especially from the supply side the well known phenomena of cycles (hogs) may result if prices go up and down with a certain frequency. To the knowledge of the author, even the contest on "efficient market hypothesis" vs. "counter cyclical behaviour recommendation and "to be competitive in the long run" has not completely eradicated the medium term discussion (Johnson and Plott, 1989). So what can we do to gear expectations and stabilize markets if they are characterized by cyclical behaviour?

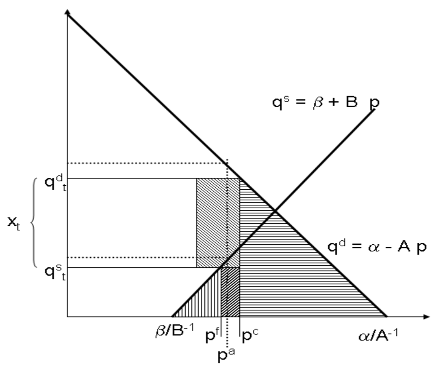
The paper deals with dynamic control theory as an option to stabilize volatile agricultural markets which are characterized by cyclical behaviour. Markets may be based on specific modes of expectations (adaptive expectations). Referring to the assumption that agricultural market may be not be always efficient because price expectations are formed according to a special rationality of farmers (learning as departing from "rational expectations"), we develop a framework which rests upon minimizing negative welfare effects. The aim of the paper is to derive a social welfare oriented stabilization scheme which enables policy makers to correct expectation failures. Moreover, the analysis does not only take farmers' income or producer surplus into account, because this would imply that consumers would not gain from lower food prices (if world markets are down); rather we conduct a social welfare approach including consumer surplus. In general we first specify an objective function, discuss dynamic price formation (due to expectations), and deliver a control theory approach (result) for stabilization.

The paper is organized in 4 sections. 1. We derive a welfare function. 2. A system analysis is conducted which works with flexible price expectations and shows which parameters are needed in the analysis and how cycles emerge. 3. A modified version of dynamic control is outlined. 4. The model is analytically solved. The model encounters conditions of stationary markets and markets showing trends. Policy instruments are indicated later as tariff changes. To avoid a big, and immediately raising discussion on the issue of introducing protection (contrary to WTO), we discuss how dynamic control based intervention can be made WTO conform.

2. WELFARE ANALYSIS

To derive an objective function for dynamic control one can analyse welfare effects as deviation from an optimal condition. In a first step we assume a steady state or optimal equilibrium. For that case we state that long-run, average prices are world market prices. It is a result from static welfare optimization in partial equilibrium analysis (see Just et al. 2008).

Figure 1: Welfare Analysis of Trade



Source: own design

The welfare function is determined as an integral over demand, supply and the trade budget:

$$W_t = \int_{-B/b}^{p_t^e} [\beta + B \cdot p] dp + \int_{p_t^c}^{A/a} [\alpha - A \cdot p] dp + [p_t^c - p_t^f] q_t^s + [p_t^c - p_W] x_t \, dp \quad (1)$$

It is a quadratic expression of expected prices for producers and imports translated into a divergence from a norm (norm: local price equals world market prices). Additionally the analysis can be expanded to several crops and we can expand analyses to vectors and matrices.

By a detailed analysis of welfare using the integration procedures discussed in a cost benefit analysis outline of Just et al. (2008) it can be proved that linear supply and demand functions result in a quadratic expression of welfare (2); including expected price and imports. To establish the objective function of control one can derive an expression of welfare such as:

$$\begin{aligned}\Delta W_t &= -.5[\Delta p_t^e + [A+B]^{-1}\Delta x_t]' A^{-1}[A+B]B[\Delta p_t^e + [A+B]^{-1}\Delta x_t] \\ &- .5[x_t - [A+B][[A+B]^{-1}(\alpha-\beta) - p_w]]' [A+B]^{-1}[x_t - [A+B][[A+B]^{-1}(\alpha-\beta) - p_w]] \\ &+ .5[x_0 - [A+B][[A+B]^{-1}(\alpha-\beta) - p_w]]' [A+B]^{-1}[x_0 - [A+B][[A+B]^{-1}(\alpha-\beta) - p_w]]\end{aligned} \tag{2}$$

In this description welfare is given as function of consumer and expected price and it is dependent on trade (in the Diagram an import case is depicted). Since the market equilibrium is:

$$p_t^e = B^{-1}(\alpha - \beta) - B^{-1}A^{-1}p_t^c - B^{-1}x_t \tag{3}$$

Welfare can be also expressed in terms of the consumer price, only, which is then the prevailing price. Note it is not the optimal price. In formulating (2) and (3) we do not follow the strategy that trade is endogenous and the world market price is always the prevailing price in the county. This would be a type of policy which is classified as zero intervention. We see dynamics and expectation and time for trade to adjust. The problem with a pure free trade policy, as indicated, is that price fluctuations from the world market will be transmitted without any check into the country. The fluctuations may be of only stochastic nature, i.e. independent; but they can also generate cyclical movements. So, why should a country import “wrong” signals”? In contrast, a simple policy such as decoupling will not work. We have to take the world market as a reference (if not by arguing for welfare reason then at least because of the WTO). The case we discuss is one in which the country may wish to have only a partly coupling for which we want to find the optimal policy. We envisage fluctuating world markets prices which are the cause for internal changes in price expectations. Moreover, we assume that price transmission shall be not perfect and hence the indicated situation is that internal prices will not be stable. They induce cyclical movements. The normal thing, though, is that expected price can not be observed and directly corrected. The logic in dynamic price analysis (Johnson and Plott, 1989) is that prices are determined by demand of a following period, whereas supply is predetermined in the current period (prices lagged). Using the above relationship the expected prices translate into the market (consumer) prices later. (So far we have not clarified on modes of expectation and stock-piling).

For the consecutive optimization of an adjustment path, derived from an inter-temporal optimization and taking a fixed component, the most appropriate final version of welfare is:

$$\begin{aligned}\Delta W_t &= -.5[p_t^c - [A+B]^{-1}[(\alpha-\beta) - x_t]]' A^{-1}[p_t^c - [A+B]^{-1}[(\alpha-\beta) - x_t]] - .5[x_t - \\ &[A+B][[A+B]^{-1}(\alpha-\beta) - p_w]]' [A+B]^{-1}[x_t - [A+B][[A+B]^{-1}(\alpha-\beta) - p_w]] + c_0\end{aligned}$$

The welfare is dependent on market prices which have to be outlined as adjusting in future periods and it is quadratic. Price is considered state variable and trade is the control variable.

3. PRICE FORMATION

We are quite aware that, in a static case, which serves as a reference and special case, the domestic market price should be the world market price (Just et al. 2008). In this regard, the approach can (must) follow the general wisdom of trade policy analysis. The difference is that we deal with a dynamic problem which becomes evident, when we look at market price formation including expectations and fluctuations in domestic equilibriums. In contrast to the theory of "rational" expectation, which eventually works in case of static markets, our approach is about learning in a dynamic market environment. An often used hypothesis about expectations is this regard, in case of learning, emanates from adaptive price expectations (Pashigian, 1970). It was shown by this author that this type of expectation corresponds by all means with postulates of a rational expectation formation if a disruptive term is a part of a lag-structure. Formally the process of expectation formation can be described by the equation (4):

$$p_t^e = \Omega\, p_{t-1}^e + [1 - \Omega]\left[p_{t-1} - p_{t-1}^e\right] \tag{4}$$

As remark (if we work with more than one price) matrices Ω can be retrieved from observations and econometrics corresponding to prices following a differential equation (Nuppenau, 1987).

$$p_t = p_{t-1} + u_t - [I - \Omega]u_{t-1} \tag{5}$$

Like Nerlove et al. (1979) have shown the econometrics is well established. The condition of "optimality" of expectations (for an early critique see also Pashigian (1970) is another topic. It is suggested that producers, who rely on this type of the expectations, in principle, use a special technique which tries to minimize the mean squared error of the prognosis (see Nerlove et al. 1979 at p. 92ff). Alternatively Ω can also be interpreted as a model parameter which has to be calculated out of empirical research, i.e. applying time lagged model structure, especially in supply functions. Then, Ω reflects the intensity of adaptation and lags. For example, if Ω is I this would be naive expectation as special case. In general differential equations for price development can be reduced for taking equilibriums (3); though it includes stochastic terms.

$$-\mathrm{A}\, p_t = \beta - \alpha + \mathrm{B}\, p_{t,t-1}^e + x_t + u_t - v_t \tag{6}$$

Multiplied by [I-Ω] equation (6) is displaced by a variant with a lag of 1 period:

$$-[\mathrm{I} - \Omega]\mathrm{A}\, p_{t-1} = [\mathrm{I} - \Omega][\beta - \alpha] + [\mathrm{I} - \Omega]\mathrm{B}\, p_{t-1,t-2}^e + [\mathrm{I} - \Omega]x_{t-1} + [\mathrm{I} - \Omega][u_{t-1} - v_{t-1}] \tag{6'}$$

If matrices are symmetric the subtraction of the second equation from the first is providing a price formation subject to a lag structure (see Turnosky, 1974).

$$-\mathrm{A}p_t + [\mathrm{I} - \Omega]\mathrm{A}p_{t-1} = -\Omega[\alpha - \beta] + \mathrm{B}\Omega\, p_{t-1} + x_t - [\mathrm{I} - \Omega]x_{t-1} + u_t - v_t + [\mathrm{I} - \Omega][u_{t-1} - v_{t-1}]$$

$$p_t = [\mathrm{I} - \Omega - \mathrm{A}^{-1}\mathrm{B}\Omega]p_{t-1} - \mathrm{A}^{-1}x_t - \mathrm{A}^{-1}[\mathrm{I} - \Omega]x_{t-1} + \mathrm{A}^{-1}[u_t - v_t] - \mathrm{A}^{-1}[\mathrm{I} - \Omega][u_{t-1} - v_{t-1}] + \mathrm{A}^{-1}\Omega[\alpha - \beta] \tag{7}$$

Combining matrices and displaying (8) as reduced form a differential equation with the structure of M and N (see Appendix for definition of M and N) gives:

$$p_t = \mathrm{M}\, p_{t-1} + \mathrm{A}^{-1}\mathrm{N}_t x_{t-1} + \mathrm{A}^{-1}\theta u_{t-1} - \mathrm{A}^{-1}x_t + \mathrm{A}^{-1}[u_t - v_t] \tag{8}$$

Notice we introduced a rather generalized system which can be used for multiple market analysis. It is derived from a combination of expectation and linear supply and demand and price determining functions. In fact it provides the constraint for the control problem. The limitation is that markets are depicted with static responses to price changes. This could be amended through flexible supply and demand responses, i.e. making them time dependent. A simple version is to see the absolute level of marginal functions subject to change:

$$\begin{aligned} p_t = & [\mathrm{I} - \Omega - \mathrm{AB}^{-1}\Omega]p_{t-1} + \mathrm{A}^{-1}x_t - \mathrm{A}^{-1}[\mathrm{I} - \Omega]x_{t-1} + \mathrm{A}^{-1}[u_t - v_t] - \mathrm{A}^{-1} \\ & [\mathrm{I} - \Omega][u_{t-1} - v_{t-1}] + \mathrm{A}^{-1}[\alpha_t - \beta_t] - \mathrm{A}^{-1}[\mathrm{I} - \Omega][\alpha_{t-1} - \beta_{t-1}] \end{aligned} \tag{9}$$

With this approach it is feasible to clarify on empirical questions with regards to trends, etc.

4. Formulating the Control Problem

By the specification of the objective function for the control problem (equation 2) and the declaration of the dynamic constraints (equation 8) the preparatory work for the calculation of the control (getting a solution) is ready. We combine minimization of losses from a virtual optimum which is characterized by the average world market price and the dynamics which is due to expectation lags. From the given model formulation, in discrete periods, it should be already apparent that a special method of control theory is envisaged: discrete control. In general, control theory is part of dynamic optimization (Chow, 1975). Given that dynamic optimization operates as approach which considers equality conditions it follows that the used method is a special case of this technique. The related special approach with Lagrange multipliers for inequality conditions is only advantageous, given analytic results with minimum income, for instance, of farmers. Then dynamic optimization works with complex functions and inequality conditions; and it relies on numeric solutions (Chow, 1975). Here we use an analytical solution.

4.1. Determination of the deterministic component of the control problem

The essential background for control theory is an extended version of Lagrange optimization which is now working in a dynamic framework. The constraint becomes part of the target function, to be optimized, in mode of a Lagrange constraint. Thence our deterministic, dynamic optimization problem reads as follows (whereby the most general approach is used):

$$\begin{aligned} L = & -\frac{1}{2}\sum_{t=1}^{T}\left\{ \begin{array}{l} \left[p_t^v - [\mathrm{A}_t + \mathrm{B}_t]^{-1}[\alpha_t - \beta_t - x_t]\right]' \mathrm{B}_t^{-1}[\mathrm{A}_t + \mathrm{B}_t]\mathrm{A}_t\left[p_t^v - [\mathrm{A}_t + \mathrm{B}_t]^{-1}[\alpha_t - \beta_t - x_t]\right] + \\ \left[x_t - [\alpha_t - \beta_t] + [\mathrm{A}_t + \mathrm{B}_t]p_{w,t}\right]'[\mathrm{A}_t + \mathrm{B}_t]^{-1}\left[x_t - [\alpha_t - \beta_t] + [\mathrm{A}_t + \mathrm{B}_t]p_{w,t}\right] - \\ \left[x_0 - [\alpha_t - \beta_t] + [\mathrm{A}_t + \mathrm{B}_t]p_{w,t}\right]'[\mathrm{A}_t + \mathrm{B}_t]^{-1}\left[x_0 - [\alpha_t - \beta_t] + [\mathrm{A}_t + \mathrm{B}_t]p_{w,t}\right] \end{array} \right\} \\ & -\sum_{t=1}^{T}\lambda_t'\left[\begin{array}{l} p_t - \mathrm{A}_t^{-1}\mathrm{B}_t[\mathrm{B}_{t-1}^{-1}[\mathrm{I} - \Omega_t]\mathrm{A}_{t-1} - \Omega_t]p_{t-1} + \mathrm{A}_t^{-1}x_t - \mathrm{A}_t^{-1}\mathrm{B}_t\mathrm{B}_{t-1}^{-1}[\mathrm{I} - \Omega_t] \\ x_{t-1} - \mathrm{A}_t^{-1}[\alpha_t - \beta_t] + \mathrm{A}_t^{-1}\mathrm{B}_t\mathrm{B}_{t-1}^{-1}[\mathrm{I} - \Omega_t][\alpha_{t-1} - \beta_{t-1}] \end{array} \right] \end{aligned} \tag{10}$$

The variable λ_t which is, in contrast, to a static Lagrange-approach formulated as a time dependent vector can be interpreted as an evaluation measurement for the value of losses. Losses remain after the optimization which is a minimization of divergences. It gives the periodic marginal losses in consequence of the difference from the optimal situation because of adaptation based on expectations, only. The inclusion of the world market prices in the adaptation makes it possible to be flexible with changing world market prospects.

Then, according to generic control theory a control, variable or reaction function has to be found (Chow, 1975). The task at the beginning is to find a result for T and T-1 as end-state-behavior (terminal condition), notably, from first derivatives. Finally after iterations one gets the current reaction at t. The background is that shadow prices change over time. We get a closed loop analysis of periodical behavior. This behavior is specified as response and dependents always on the last period (past) realization of observable p and x at t-1 (equation 11). The policy recommendation is to adjust the trade regime such as to determine the instrument or control variable (trade: x_t) in a response to a prevailing price of the previous period as well as introducing a moderation of the prevailing trade. The trade in t is moderated (adjusted) from what it was in the previous period t-1 according to a calculus (equation 11) taking the welfare implication into account. Technically (i.e. mathematically) we describe the control problem, in the case of cyclical price formation, as operation or function of past realizations of x_{t-1} and p_{t-1}. In this regard one can state a reaction function as "G_is":

$$x_t = G_{1,t}\, p_{t-1} + G_{2,t}\, x_{t-1} + G_{3,t}\left[\alpha_t - \beta_t\right] + G_{4,t}\, p_{w,t} \tag{11}$$

Equation (11) follows a typical control outline (Chow, 1975) in which the G_i's, respectively $G_{i,t}$'s (if the control varies) as those in the period t, must be calculated. The task is now to determine by optimization the G_{it}'s.

4.2. *Deduction of the adjustment policy*

For equation (10) as control problem the task is to determine the coefficients $G_{i,t-1}$ as dependent on optimization. To do so one has to start with a general optimization (12). For any period the first derivatives deliver optimal response, which is ensured by zeroing them as a necessary condition: then the necessary conditions are:

$$\frac{\partial L}{\partial p_t} = -\mathrm{A}_t\left[\mathrm{A}_t + \mathrm{B}_t\right]\mathrm{B}_t^{-1}\left[p_t - \left[\mathrm{A}_t + \mathrm{B}_t\right]^{-1}\left[\alpha_t - \beta_t\right] + \left[\mathrm{A}_t + \mathrm{B}_t\right]^{-1} x_t\right] - \lambda_t + \mathrm{A}_{t+1}^{-1}\mathrm{B}_{t+1}\mathrm{B}_t^{-1}$$
$$\left[\left[\mathrm{I} - \Omega_{t+1}\right]\mathrm{A}_t - \Omega_{t+1}\right]\lambda_{t+1} = 0 \tag{12a}$$

$$\frac{\partial L}{\partial x_t} = -\mathrm{A}_t\left[\mathrm{A}_t + \mathrm{B}_t\right]\mathrm{B}_t^{-1}\left[p_t - \left[\mathrm{A}_t + \mathrm{B}_t\right]^{-1}\left[\alpha_t - \beta_t\right] + \left[\mathrm{A}_t + \mathrm{B}_t\right]^{-1} x_t\right]\left[\mathrm{A}_t + \mathrm{B}_t\right]^{-1\prime} - \left[\mathrm{A}_t + \mathrm{B}_t\right]^{-1}$$
$$\left[x_t - \left[\alpha_t - \beta_t\right] + \left[\mathrm{A}_t + \mathrm{B}_t\right] p_{w,t}\right] - \mathrm{A}_t^{-1}\lambda_t + \mathrm{A}_{t+1}^{-1}\mathrm{B}_{t+1}\mathrm{B}_t^{-1}\left[\mathrm{I} - \Omega_{t+1}\right]\lambda_{t+1} = 0 \tag{12b}$$

$$\frac{\partial L}{\partial \lambda_t} = p_t - \mathrm{A}_t^{-1}\mathrm{B}_t\left[\mathrm{B}_{t-1}^{-1}\left[\mathrm{I} - \Omega_t\right]\mathrm{A}_{t-1} - \Omega_t\right] p_{t-1} + \mathrm{A}_t^{-1} x_t - \mathrm{A}_t^{-1}\mathrm{B}_t\mathrm{B}_{t-1}^{-1}\left[\mathrm{I} - \Omega_t\right] x_{t-1} -$$
$$\mathrm{A}_t^{-1}\left[\alpha_t - \beta_t\right] + \mathrm{A}_t^{-1}\mathrm{B}_t\mathrm{B}_{t-1}^{-1}\left[\mathrm{I} - \Omega_t\right]\left[\alpha_{t-1} - \beta_{t-1}\right] = 0 \tag{12c}$$

However, we are not finished. The conditions do not help to establish a single solution since the conditions comprise elements of t+1. Because of that, in analogy to Chow (1975) who, in this regard, analyzed a special case in which the instrument variable is not part of the target function, the calculation starts with the end condition. By starting with the final G´s, for every period, the optimal strategy is retrieved for the next, depending on the last period (closest future). Again, because of terminal conditions at the end of adjustment $\lambda_T=0$ we retrieve the terminal conditions (13a-13 c). Previous λ´s (λ_{T-1}, λ_{T-2}, etc.) can be calculated in recursive steps until λ_{t+1} is reached. In particular if we see T as terminal three optimality conditions for the end period T are characterized by the fact that $\lambda_{T+1}= 0$. Hence the derivatives (12) reduce to (13):

$$\frac{\partial L}{\partial p_T} = \mathrm{A}_T [\mathrm{A}_T + \mathrm{B}_T] \mathrm{B}_T^{-1} p_T + \mathrm{A}_T \mathrm{B}_T^{-1} x_T - \mathrm{A}_T \mathrm{B}_T^{-1} [\alpha_T - \beta_T] + \lambda_T = 0 \tag{13a}$$

$$\frac{\partial L}{\partial x_T} = \mathrm{A}_T [\mathrm{A}_T + \mathrm{B}_T] \mathrm{B}_T^{-1} p_T + \mathrm{A}_T \mathrm{B}_T^{-1} x_T + x_T - \mathrm{A}_T \mathrm{B}_T^{-1} [\alpha_T - \beta_T] - [\alpha_T - \beta_T] + \\ [\mathrm{A}_T + \mathrm{B}_T] p_{w,T} + \mathrm{A}_T^{-1} [\mathrm{A}_T + \mathrm{B}_T] \lambda_T = 0 \tag{13b}$$

$$\frac{\partial L}{\partial \lambda_T} = p_T - \mathrm{A}_T^{-1} \mathrm{B}_T [\mathrm{B}_{T-1}^{-1} [\mathrm{I} - \Omega_T] \mathrm{A}_{T-1} - \Omega_T] p_T + \mathrm{A}_T^{-1} x_T - \mathrm{A}_T^{-1} \mathrm{B}_T \mathrm{B}_{T-1}^{-1} [\mathrm{I} - \Omega_T] x_{T-1} + \\ \mathrm{A}_T^{-1} [\alpha_T - \beta_T] + \mathrm{A}_T^{-1} \mathrm{B}_T \mathrm{B}_{T-1}^{-1} [\mathrm{I} - \Omega_T] [\alpha_{T-1} - \beta_{T-1}] = 0 \tag{13c}$$

5. EXCURSION

Before we continue our analysis within the control framework, it should (can) be shown that the just derived result (13) corresponds with the optimality criteria in a “static world”. In other words terminal condition coincides with the general proposition that domestic prices must be the same as world market prices. For dogmatic reason it means that no protection should be involved in adjustment; trade intervention should be done solely in a dynamic framework finally leading to free trade. To prove this the second equation is deducted from the first: (13a- 13b):

$$\lambda_T - x_T + (\alpha_T - \beta_T) - [\mathrm{A}_T + \mathrm{B}_T] p_{w,T} + \mathrm{A}_T^{-1} [\mathrm{A}_T + \mathrm{B}_T] \lambda_T = 0 \tag{14a}$$

By rearranging one gets

$$\mathrm{A}_T^{-1} \mathrm{B}_T \lambda_T = -x_T + [\alpha_T - \beta_T] - [\mathrm{A}_T + \mathrm{B}_T] p_{w,T} \tag{14b}$$

Then, since the first equation (13a) is:

$$[\mathrm{A}_T + \mathrm{B}_T] p_T - [\alpha_T - \beta_T] + x_T + \mathrm{A}_T \mathrm{B}_T^{-1} \lambda_T = 0 \tag{14c}$$

it follows

$$[\mathrm{A}_T + \mathrm{B}_T] p_T - [\mathrm{A}_T + \mathrm{B}_T] p_{w,T} = 0 \tag{14d}$$

For an interpretation: Within the approach it is guaranteed that the dogma of the identity of domestic price and world market price holds for the end period and which policy should aim.

6. RETRIEVING THE G (CONTROL)-COEFFICIENT

In contrast to the long term optimality which is characterized by the world market price regime, the control problem asks for short term deviations of domestic prices from world market prices and implies a temporal deviation from free trade. A question is how to calculate the instrument variable x_T which relies on the situation in T-1. This is important because in period T-1 the trade x_{T-1} has to be known (as derived from the first derivate of the objective function for any period t which has to be shown). According to the "to be optimized" number of periods, i.e. t to T (or T minus t) which is every period after t to T, the subsequent x_t, x_{t+1}, …x_{T-1}, x_T values have to be calculated (planned) for an optimal adjustment until terminal period T is reached.

However, since the present problem comprises, in contrast to the analysis of Chow, also the instrument variable which is now part of the configuration of the target function, it is necessary to use a more complex approach. This approach includes matrix algebra and generally applies to multiple markets (Nuppenau, 1986). To sketch the solution, we work with the analogy between the control function and recombination of the three optimality conditions. For more detail see again Nuppenau (1986). Applying matrix algebra to system (13) makes sense also because of the most likely simultaneous analysis of several markets if, for instance, interactions of crops prevail, which use the same resources. In principle it can be shown that a reduced form of (13) delivers x_T as a function of p_{T-1}, x_{T-1}, α_T, β_T, α_{T-1}, β_{T-1}, and $p_{w,T}$ (Nuppenau, 1986):

$$x_T = \left[F_T^{21}\mathrm{K}_T\left[\mathrm{A}_T - \mathrm{B}_T\right]^{-1} + F_T^{22}\mathrm{K}_T\mathrm{A}_T^{-1} + F_T^{23}\mathrm{A}_T^{-1}\right]\left[\alpha_T - \beta_T\right] + F_T^{23}\mathrm{A}_T^{-1}\mathrm{N}_T\left[\alpha_{T-1} - \beta_{T-1}\right] + \tag{16}$$
$$F_T^{23}\mathrm{M}_T p_{T-1} + F_T^{23}\mathrm{A}_T^{-1}\mathrm{N}_T x_{T-1} + F_T^{22}\mathrm{A}_T^{-1}\mathrm{K}_T\mathrm{B}_T p_{w,T}$$

Hereby matrices F^{ij}, M, N and K are calculated internally and given in the Appendix. The consecutive argument is taking the analogy of (16) and (11). This enables us a determination of G within the suggested framework for a policy response:

$$x_T = G_T^{11} p_{T-1} + G_T^{12} x_{T-1} + G_T^{13} p_{w,T} + G_T^{14}\left[\alpha_T - \beta_T\right] + G_T^{15}\left[\alpha_{T-1} - \beta_{T-1}\right] \tag{11}$$

The G^{ij} matrices become determined. The problem, however, remains that only the final policy has been described yet. From the final policy in period T one has to recur (by feedback loop) to the period before, T-1, and then T-2, T-3; etc. coming finally to t. In other words to get the right policy x_t being contingent on previous prices and control we need information on p_{T-t} and x_{T-t} as coming from the end.

By this dynamic view the adjustment strategy is expressed in as a policy plan. The same applies to state variables p_{T-1}; an iteration is needed. It begins with first derivatives p_{T-1}, x_{T-1} and λ_{T-1}. As in general the recursive problem is that the new variables depend on λ_T. This becomes evident if we look at again at period t-1, next to last period and its optimality. The conditions are:

$$\frac{\partial L}{\partial p_{T-1}} = \mathrm{A}_{T-1}\left[\mathrm{A}_{T-1} + \mathrm{B}_{T-1}\right]\mathrm{B}_{T-1}^{-1} p_{T-1} + \mathrm{A}_{T-1}\mathrm{B}_{T-1}^{-1} x_{T-1} - \mathrm{A}_{T-1}\mathrm{B}_{T-1}^{-1}\left[\alpha_{T-1} - \beta_{T-1}\right] + \tag{17a}$$
$$\lambda_{T-1} + \mathrm{A}_{T-1}^{-1}\mathrm{B}_T\left[\mathrm{B}_{T-1}^{-1}\left[\mathrm{I} - \Omega_T\right]\mathrm{A}_{T-1} - \Omega_T\right]\lambda_T = 0$$

$$\frac{\partial L}{\partial x_{T-1}} = \mathrm{A}_{T-1}[\mathrm{A}_{T-1}+\mathrm{B}_{T-1}]\mathrm{B}_{T-1}^{-1}p_{T-1} + [\mathrm{A}_{T-1}+\mathrm{B}_{T-1}]\mathrm{B}_{T-1}^{-1}x_{T-1} - [\mathrm{A}_{T-1}+\mathrm{B}_{T-1}]\mathrm{B}_{T-1}^{-1} [\alpha_{T-1}-\beta_{T-1}] + [\mathrm{A}_{T-1}+\mathrm{B}_{T-1}]p_{w,T-1} + \mathrm{A}_{T-1}^{-1}[\mathrm{A}_{T-1}+\mathrm{B}_{T-1}]\lambda_{T-1} + [\mathrm{A}_{T-1}+\mathrm{B}_{T-1}] \mathrm{A}_{T}^{-1}\mathrm{B}_T\mathrm{B}_{T-1}^{-1}[\mathrm{I}-\Omega_T]\lambda_T = 0 \tag{17b}$$

$$\frac{\partial L}{\partial \lambda_{T-1}} = p_{T-1} - \mathrm{A}_{T-1}^{-1}\mathrm{B}_{T-1}[\mathrm{B}_{T-2}^{-1}[\mathrm{I}-\Omega_{T-1}]\mathrm{A}_{T-2}-\Omega_{T-1}]p_{T-2} + \mathrm{A}_{T-1}^{-1}x_{T-1} - \mathrm{A}_{T-1}^{-1}\mathrm{B}_{T-1} \mathrm{B}_{T-2}^{-1}[\mathrm{I}-\Omega_{T-1}]x_{T-2} - \mathrm{A}_{T-1}^{-1}\mathrm{B}_{T-1}\mathrm{B}_{T-2}^{-1}[\mathrm{I}-\Omega_{T-1}]\,[\alpha_{T-2}-\beta_{T-2}] + \mathrm{A}_{T-1}^{-1}[\alpha_{T-1}-\beta_{T-1}] = 0 \tag{17c}$$

These optimality conditions are different from the end-period derivatives (or transversal condition) since λ_T appears in addition to λ_{T-1}. Because the matrices, previous to the pre-period variable, differ in one factor $A_T B_T^{-1}$, the decision rules x_{T-1} can be used for λ_T.

$$\lambda_T = -\mathrm{K}_T\, p_T - \mathrm{K}_T[\mathrm{A}_T+\mathrm{B}_T]^{-1}x_T + \mathrm{K}_T[\mathrm{A}_T+\mathrm{B}_T]^{-1}[\alpha_T-\beta_T] \tag{18}$$

Within (17a) λ_T has to be adopted as a result depending on p_{T-1} and x_{T-1}.

Notice after the insertion of the third equation of the first optimization condition (13a) for p_T one gets λ_T which just depends on p_{T-1}, x_T, and x_{T-1}:

$$\lambda_T = -\mathrm{K}_T\begin{bmatrix} \mathrm{A}_T^{-1}\mathrm{B}_T[\mathrm{B}_{T-1}^{-1}[\mathrm{I}-\Omega_T]\mathrm{A}_{T-1}-\Omega_T]p_{T-1} - \mathrm{A}_T^{-1}x_T - \mathrm{A}_T^{-1}\mathrm{B}_T \\ \mathrm{B}_{T-1}^{-1}[\mathrm{I}-\Omega_T]x_{T-1} + \mathrm{A}_T^{-1}[\alpha_T-\beta_T] - \mathrm{A}_T^{-1}\mathrm{B}_T\mathrm{B}_{T-1}^{-1}[\mathrm{I}-\Omega_T] \\ [\alpha_{T-1}-\beta_{T-1}] \end{bmatrix} - \mathrm{K}_T[\mathrm{A}_T+\mathrm{B}_T]^{-1}x_T + \mathrm{K}_T[\mathrm{A}_T+\mathrm{B}_T]^{-1}[\alpha_T-\beta_T] \tag{18'}$$

Again using (11)

$$x_T = G_T^{11}p_{T-1} + G_T^{12}x_{T-1} + G_T^{13}p_{w,T} + G_T^{14}[\alpha_T-\beta_T] + G_T^{15}[\alpha_{T-1}-\beta_{T-1}] \tag{11}$$

it follows that λ_T by coevally is:

$$\lambda_T = [G_T^{11} - \mathrm{K}_T[\mathrm{A}_T^{-1}\mathrm{B}_T[\mathrm{B}_{T-1}^{-1}[\mathrm{I}-\Omega_T]\mathrm{A}_{T-1}-\Omega_T]]]p_{T-1} + [G_T^{12} - \mathrm{K}_T\mathrm{A}_T^{-1}\mathrm{B}_T\mathrm{B}_{T-1}^{-1}[\mathrm{I}-\Omega_T]] x_{T-1} + G_T^{13}p_{w,T} + [G_T^{14}+\mathrm{I}]\,[\alpha_T-\beta_T] + [G_T^{15} + [\mathrm{A}_T+\mathrm{B}_T]\mathrm{B}_{T-1}^{-1}[\mathrm{I}-\Omega_T]]\,[\alpha_{T-1}-\beta_{T-1}] \tag{19}$$

This equation can be expressed as a combination of two types of matrices G^{ij} and Γ^{ij}, so that λ_T becomes:

$$\lambda_T = [G_T^{11}-\Gamma_T^{11}]p_{T-1} + [G_T^{12}-\Gamma_T^{12}]x_{T-1} + G_T^{13}p_{w,T} + [G_T^{14}+\mathrm{I}]\,[\alpha_T-\beta_T] + [G_T^{15}+\Gamma_T^{15}[\alpha_{T-1}-\beta_{T-1}]] \tag{19'}$$

The Γ^{ij} are depicting the second part. This simplification allows us to display the equation system (17) for the pre-period more narrowly after substitution for λ_T as based on previous periods. To determine the Γ^{ij} s compare them between the equations!

In principle a system (2)

$$\begin{bmatrix} \mathrm{K}_{T-1}+\Gamma_T^{11}\left[G_T^{11}-\Gamma_T^{11}\right] & \mathrm{K}_{T-1}\left[\mathrm{A}_{T-1}+\mathrm{B}_{T-1}\right]^{-1}+r_T^{11}\left[G_T^{12}-\Gamma_T^{12}\right] & \mathrm{I} \\ \mathrm{K}_{T-1}+\Gamma_T^{12}\left[G_T^{11}-\Gamma_T^{11}\right] & \mathrm{K}_{T-1}\mathrm{A}_{T-1}^{-1}+\Gamma_T^{12}\left[G_T^{12}-\Gamma_T^{12}\right] & \mathrm{A}_{T-1}^{-1}\mathrm{K}_{T-1}\mathrm{B}_{T-1}\mathrm{A}_{T-1}^{-1} \\ \mathrm{I} & \mathrm{A}_{T-1}^{-1} & 0 \end{bmatrix} \begin{bmatrix} p_{T-1} \\ x_{T-1} \\ \lambda_{T-1} \end{bmatrix} =$$

$$\begin{bmatrix} \Gamma_T^{11}\left[G_T^{14}+\mathrm{I}\right] & \mathrm{K}_{T-1}\left[\mathrm{A}_{T-1}+\mathrm{B}_{T-1}\right]^{-1}+\Gamma_T^{11}\left[G_T^{15}+\mathrm{K}_T\Gamma_T^{12}\right] & 0 & \Gamma_T^{11}G_T^{13} & 0 & 0 & 0 \\ \Gamma_T^{12}\left[G_T^{14}+\mathrm{I}\right] & \mathrm{K}_{T-1}\mathrm{A}_{T-1}^{-1}+\Gamma_T^{12}\left[G_T^{15}+\mathrm{K}_T\Gamma_T^{12}\right] & 0 & \Gamma_T^{12}G_T^{13} & \mathrm{A}_{T-1}^{-1}\mathrm{K}_{T-1}\mathrm{B}_{T-1} & 0 & 0 \\ 0 & \mathrm{A}_{T-1}^{-1} & \Gamma_{T-1}^{12} & 0 & 0 & \Gamma_T^{11}\mathrm{K}_{T-1}^{-1} & \Gamma_T^{12}\mathrm{K}_{T-1}^{-1} \end{bmatrix} \begin{bmatrix} \alpha_T-\beta_T \\ \alpha_{T-1}-\beta_{T-1} \\ \alpha_{T-2}-\beta_{T-2} \\ p_{w,T} \\ p_{w,T-1} \\ p_{T-2} \\ x_{T-2} \end{bmatrix} \tag{20}$$

is representing the optimality at T-1. Summarizing matrices gives a new expression:

$$\mathrm{H}_{T-1}y_{T-1}=\mathrm{M}_{T-1}y_{T-2}+h_{T-1} \tag{20´}$$

or

$$\begin{bmatrix} H^{11} & H^{12} & H^{13} \\ H^{21} & H^{22} & H^{23} \\ H^{31} & H^{32} & H^{33} \end{bmatrix} \begin{bmatrix} p_{T-1} \\ x_{T-1} \\ \lambda_{T-1} \end{bmatrix} = M_{T-1}y_{T-2}+h_{T-1} \tag{20´´}$$

where $y_{T-1} = [p_{T-1}, x_{T-1}, \lambda_{T-1},]´$ stands as vector for the endogenous variables and $x_{T-1} = [p_{T-2}, x_{T-2},]´$ for lagged variables and $h_{T-1} = [\alpha_T,\ldots, p_{w,t-1}]$ for exogenous variables. Exogenous variables include world market prices, changing conditions in supply and demand, etc.

7. RECURSIVE DETERMINATION OF COEFFICIENTS

In subsequence to this detailed illustration of calculations for particular optimal conditions of each period, in a recursive way as well as the determination of activity matrices G, it is important to find a general concept for the recursive calculation of matrices. For an easier understanding: a conceptualization will be exemplified for one matrix element H^{11}_t in system (20); others are similar. This construction of the periodic optimality conditions for a numeric calculation is done in analogy to Chow (1975). As the matrix notation (10), which is established as optimality condition for the pre-period, shows it devotes its construction to elements H^{ij}_{T-1}. For each element the recursive character can and has to be reflected and it is a difference equation. In the following consideration the general way of calculation is exemplified for H^{11}_{T-1}.

$$\mathrm{H}_{T-1}^{11}=\mathrm{K}_{T-1}-K_T\mathrm{A}_T^{-1}\mathrm{B}_T\left[\mathrm{B}_{T-1}^{-1}\left[\mathrm{I}-\Omega_T\right]\mathrm{A}_{T-1}-\Omega_T\right]\left[G_T^{11}-\mathrm{K}_T\mathrm{A}_T^{-1}\mathrm{B}_T\left[\mathrm{B}_{T-1}^{-1}\left[\mathrm{I}-\Omega_T\right]\mathrm{A}_{T-1}-\Omega_T\right]\right] \tag{21}$$

with

$$G_T^{11}=\mathrm{B}_T\left[\mathrm{B}_{T-1}^{-1}\left[\mathrm{I}-\Omega_T\right]\mathrm{A}_{T-1}-\Omega_T\right]$$

Equation (21) can be re-written as (21). For a general H^{11}_{t-1} (recursion from T if K_{t-1}, it looks similar as H^{11}_{t-1} of the pre-period and if matrices are symmetric we get:

$$H^{11}_{t-1} = K_{t-1} - H^{11}_{t} A^{-1}_{t} B_{t} \left[B^{-1}_{t-1}[I - \Omega_{t}]A_{t-1} - \Omega_{t}\right]' \left[B^{-1}_{t-1}[I - \Omega]_{t} A_{t-1} - \Omega_{t}\right] A_{t} B^{-1}_{t} \tag{22}$$

Note the iteration is a backward loop. For the numerical calculation, it moreover becomes apparent that the development of the matrix H^{11}_{t-1}follows a differential equation (now for matrices) in modified fashion of starting with the future to receive the current coefficients.

$$H^{11}_{t-1} = K_{t-1} - H^{11}_{t} Z_{t} \tag{22´}$$

with

$$Z_{t} = \left[B^{-1}_{t-1}[I - \Omega_{t}]A_{t-1} - \Omega_{t}\right]' \left[B^{-1}_{t-1}[I - \Omega_{t}]A_{t-1} - \Omega_{t}\right]$$

Starting with T-1 and getting H^{11}_{T-1}, which is based on the terminal (transversal) calculation of H^{11}_{T} the process of finding a value of H^{11}_{t}, which is based on H^{11}_{t+1}, is a way of backward iteration. In a similar fashion it is possible to delineate other H^{ij}_{t-1}. By this process the determination of the policy variables G^{ij}_{t} is accomplished. From the knowledge of the G^{ij}_{t}´s we can calculate the "optimal" trade policy in terms of quantitative imports or exports, respectively. However, this raises the question of numerical application in a changing world of parameters.

8. TRADE REGIME

Another, important question refers to a translation of the technical results into acceptable trade regimes (eventually being WTO accepted). A possibility is to specify the result in terms of a trade regime based on tariff equivalents. A tariff equivalent results in a price regime instead of a quantitative control of trade (imports). The internal calculation of tariff is a simple application of the model. It delivers a tariff based on policy instrument. To do so we again take the control

$$x_t = G^*_1 p^c_{t-1} + G^*_2 p^w_{t-1}$$

and apply it to the price formation. In numerical applications, for which we calculated G matrices the price equation and price determination is combined; then we get:

$$p^e_t = B^{-1}(a-b) - B^{-1}A^{-1}p^c_t - B^{-1}G^*_1 p^c_{t-1} + G^*_2 p^w_{t-1} \tag{23}$$

Using the notation

$$p^e_t = p^e_{t-1} + \Omega[p_{t-1} - p^e_{t-1} \Leftrightarrow p^e_t = [1-\Omega]p^e_{t-1} + \Omega p_{t-1}$$

and use a description with a lag operator "L"

$$[I - [1-\Omega]L]p^e_t = \Omega p_{t-1} \Leftrightarrow p^e_t = [I - [1-\Omega]L]^{-1}\Omega p_{t-1}$$

which makes the price calculation possible, we get:

$$[I - [1-\Omega]L]^{-1}\Omega p_{t-1}^{c} = B^{-1}(a-b) - B^{-1}A^{-1}p_t^c - B^{-1}G_1^* p_{t-1}^c + G_2^* p_{t-1}^w \quad (24)$$

or

$$\Omega p_{t-1}^{c} = [I-[1-\Omega]L][B^{-1}(a-b) - B^{-1}A^{-1}p_t^c - B^{-1}G_1^* p_{t-1}^c + G_2^* p_{t-1}^w] \quad (24`)$$

The envisaged price p^c_t can calculated based on p^c_{t-1} and p^w_{t-1}; and then the difference to the foresee world market price determines the needed tariff. Such regime is technical feasible.

Another issue is how to treat the import/export control in terms of trade arrangements with trading partners as well as to derive a joint optimal policy instrument (in other word how to get political support or agreements). To clarify on these issues, even such as conformity with WTO regulations and trade fitting in bilateral agreements in general, it has to be highlighted that: (1) the use of trade as instrument to stabilize markets from cyclical movements does not intended to create protectionism. We merely work with deviations from a “steady state” trade situation which is characterized by the prevalence of world market conditions for trade within a country. (2) Surely, the paper, to a certain extend, is technical. It calculates import reactions as a pathway to stabilise internal market situations. It shows also how a country should behave for reasons of stabilization, not protection. (3) As shown, it is feasible to offer an analysis of a well-defined pathway for adjustment. This pathway shows how to come back to a “normal situation” of “in-country-prices equal to world market prices” which is the result of static trade policy and which trade economists prefer. It has to be mentioned in this regard that a final debate for getting “excuse” in terms of trade regimes as measure to regulate market imperfection, can be only part of negotiations; the concept of control theory enables policy makers to determine pathways for negotiations on adjustments and show impacts on trading partners in rational manner.

In that regard the analysis can be broadened to the subject of cyclical world markets and we suggest studies on game theoretical propositions, i.e. how to react to partner control. In this case we need information on the systematic development of the world market prices of concern and to be implemented in structural world market price movement. A simple case would be an anticipated decline or increase of world markets which can be mathematically expressed as a first order difference equation. In such case of a cyclical movement of world markets, a second order difference equation can be a way to anticipate the world market in the optimization.

9. SUMMARY AND OUTLOOK

This paper presents a control theory approach as a response to cyclical price movements on national and internal food markets. It works with a welfare economic related objective function based on consumer and producer surplus as well as trade balances. The adjustment path for prices as state variables and trade as control variable is optimized. Finally, some remarks on the trade regime and translation into feasible model application highlighted the relevance and potential for trade negotiation. However, it is important to notice that such choices of policy regimes require an approval from trade organizations as well as the control can be modelled further as a game between trading partners. Another expansion would be an inclusion of adjustment costs. This implies a dynamic supply function and new equilibrium which modifies the result of the price dynamics such as:

$$\alpha - \mathrm{A}p_t = \beta + \mathrm{B}p^e_{t,t-1} + Cq_{t-1} + x_t + u_t - v_t$$
$$\alpha - \mathrm{A}p_t = \beta + Ca^{-1} + \mathrm{B}p^e_{t,t-1} + CA^{-1}p_{t-1}] + x_t + u_t - v_t \tag{25}$$

The difference of the approach with the lagged equilibrium is given as a new price formation which includes adjustment costs. However, in this case, a second order lag structure appears which is not really problematic because also a variant with a matrix expression exists for such structure (Tu, 1982). The remaining question is should we recognize adjustment costs in the target function? Mathematically it is feasible though increases the complexity.

REFERENCES

FAO et al. (2011). Price Volatility in Food and Agricultural Markets: Policy Responses Policy Report. www.oecd.org/dataoecd/40/34/48152638.pdf

Chow, G. (1975). *Analysis and Control of Dynamic Economic Systems*. New York.

Johnson, M.D., Plott, C.R. (1989). The effects of two trading institutions on price expectations and the stability of supply response lag markets. *Journal Economic Psychology* 10: 189-216.

Just, R.E., Hueth, D., Schmitz, A. (2008), *Applied welfare economics*. Cheltenham.

Nerlove, M. Grether, D.M., Carvalho, J.L. (1979). *Analysis of Economic Time Series. A Synthesis Analysis.* New York.

Nuppenau, E.-A. (1987). *Darstellung und Weiterentwicklung von Modellen zur Analyse, Bewertung und Politik-empfehlung bei Instabilitäten auf Agrarmärkten – Diskutiert am Beispiel des Kartoffelmarktes*. Agrarökonomische Studien Band 13, Kiel, Germany: Vauk Verlag.

Pashigian, B.P. (1970). Rational Expectations and the Cobb-Web Theory. *Journal of Political Economy* 78: 338-352.

Turnovsky, S.J. (1974). Price Expectations and the Welfare Gains from Price Stabilization. *American Journal of Agricultural Economics* 56/4: 706-716.

Tu, P.V.N. (1982). *Dynamical Systems. An Introduction with Applications in Economics and Biology.* Heidelberg, Germany: Springer.

APPENDIX

The target function is composed of frequently recurring sequences of the combinations:

$$\mathrm{A}_T[\mathrm{A}_T + \mathrm{B}_T]\mathrm{B}_T^{-1} = \mathrm{K}_T$$

so that it is easier to equalize the particular coefficients with a central matrix. By using this standard form it is possible to write the system of equation as:

$$5.45)\begin{bmatrix} \mathrm{K}_T & \mathrm{K}_T[\mathrm{A}_T+\mathrm{B}_T]^{-1} & \mathrm{I} \\ \mathrm{K}_T & \mathrm{K}_T\mathrm{A}_T^{-1} & \mathrm{A}_T^{-1}\mathrm{K}_T\mathrm{B}_T\mathrm{A}_T^{-1} \\ \mathrm{I} & \mathrm{A}_T^{-1} & 0 \end{bmatrix} \begin{bmatrix} p_T \\ x_T \\ \lambda_T \end{bmatrix} =$$

$$\begin{bmatrix} \mathrm{K}_T[\mathrm{A}_T+\mathrm{B}_T]^{-1} & 0 & 0 & 0 & 0 \\ \mathrm{K}_T\mathrm{A}_T^{-1} & 0 & 0 & 0 & \mathrm{A}_T^{-1}\mathrm{K}_T\mathrm{B}_T \\ \mathrm{A}_T^{-1} & \mathrm{A}_T^{-1}\mathrm{N}_T & \mathrm{M}_T\mathrm{A}_T^{-1}\mathrm{N}_T & & 0 \end{bmatrix} x \begin{bmatrix} \alpha_T - \beta_T \\ \alpha_{T-1} - \beta_{T-1} \\ p_{T-1} \\ x_{T-1} \\ p_{w,T} \end{bmatrix}$$

Furthermore we use the definition:

$$\mathrm{M}_T = \mathrm{A}_T^{-1}\mathrm{B}_T\left[\mathrm{B}_{T-1}^{-1}\left[\mathrm{I} - \Omega_T\right]\mathrm{A}_{T-1} - \Omega_T\right]$$

and

$$\mathrm{N}_T = \mathrm{B}_T\mathrm{B}_{T-1}^{-1}\left[\mathrm{I} - \Omega_T\right]$$

Is the left matrix inverted, the system

$$\mathrm{K}_T^* x_T = \mathrm{M}_T^* \alpha_T$$

The values of the F- matrices can be calculated by the left multiplication with the K matrices so that the unit matrix results

$$F_T \mathrm{K}_T^* = \mathrm{I}$$

Or written

$$\begin{bmatrix} F_T^{11} & F_T^{12} & F_T^{13} \\ F_T^{21} & F_T^{22} & F_T^{23} \\ F_T^{31} & F_T^{32} & F_T^{33} \end{bmatrix} \begin{bmatrix} \mathrm{K}_T & \mathrm{K}_T\left[\mathrm{A}_T - \mathrm{B}_T\right]^{-1} & \mathrm{I} \\ \mathrm{K}_T & \mathrm{K}_T\mathrm{A}_T^{-1} & \mathrm{A}_T^{-1}\mathrm{K}_T\mathrm{B}_T\mathrm{A}_T^{-1} \\ \mathrm{I} & \mathrm{A}_T^{-1} & 0 \end{bmatrix} = \begin{bmatrix} \mathrm{I} & 0 & 0 \\ 0 & \mathrm{I} & 0 \\ 0 & 0 & \mathrm{I} \end{bmatrix}$$