Paper prepared for the 123rd EAAE Seminar

PRICE VOLATILITY AND FARM INCOME STABILISATION

Modelling Outcomes and Assessing Market and Policy Based Responses

Dublin, February 23-24, 2012

Catastrophic crop insurance effectiveness: does it make a difference how yield losses are conditioned?

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Abstract

The study evaluates the effectiveness of a catastrophic drought-index insurance developed by applying two alternative methods - the standard regression analysis and the copula approach. Most empirical analyses obtain estimates of the dependence of crop yields on weather by employing linear regression. By doing so, they assume that the sensitivity of yields to weather remains constant over the whole distribution of the weather variable and can be captured by the effect of the weather index on the yield conditional mean. In our study we evaluate, whether the prediction of farm yield losses can be done more accurately by conditioning yields on extreme realisations of a weather index. Therefore, we model the dependence structure between yields and weather by employing the copula approach. Our preliminary results suggests that the use of copulas might be a more adequate way to design and rate weather-based insurance against extreme events.

Keywords: catastrophic insurance, weather-based insurance, copula

JEL classification: C18, Q14

1. INTRODUCTION

In recent decades weather-based insurance has been considered as a valuable alternative for traditional crop insurance. The main advantage of the former is that it is better suited to combat asymmetric information problems, i.e. adverse selection and moral hazard. An additional important advantage of weather-based insurance is that it reduces considerably transaction costs and thus allows a faster settlement of claims. The latter characteristic of weather-based insurance makes it particularly relevant in the context of catastrophic event management when the help must be provided within few days to a large number of affected farms.

Another important empirical aspect is a rather low participation of farmers in weather-based insurance programs. There are many factors which influence the farmer decision to buy a weather-based insurance contract. In addition to factors evaluated in the context of traditional agricultural insurance such as farm's socio-economic characteristics, risk aversion, level of production diversification, etc., for weather-based insurance the literature discusses the effect of informal insurance, basis risk and model prediction uncertainties on the farmers' demand for insurance (Akter, 2011; Barnett, 2010; Bokusheva and Breustedt, 2012).

The presence of informal insurance might reduce demand for formal insurance against moderate risks due to a certain capacity to share risk among members of a rural community. However, the financial reserve within a community might be not sufficient to cope with catastrophic risks. Catastrophic risks usually have a systemic character, i.e. they affect simultaneously a large group of producers. Accordingly, wealthier households of a community might be themselves affected by a catastrophic event, which would reduce their capacity to provide informal insurance. In this context, weather-based insurance designed to cope with catastrophic risks might be better targeted to the needs of rural households compared to insurance products insuring against both extreme and moderate yield losses.

The main reasons for basis risk are: (i) a low correlation of farm yields with a weather index due to presence of other important risks, namely those beyond the hazard to be insured by a particular weather-based insurance product, i.e. so called loss-specific basis risk; (ii) a low sensitivity of farm yields to weather data of meteorological stations situated at a considerable distance to the farm, i.e. spatial basis risk; and (iii) a low correlation between a weather index and yields due to the timing of the occurrence of the insured event, i.e. temporal basis risk (Skees et al., 2007). Evaluating the first type of basis risk is in general relevant if the decision in favor of a particular weather-based insurance should be made, but this type of basis risk can hardly be reduced by improving the design of weather-based insurance.

However, considering two other types of basis risk, a solid enhancement of the weather-based insurance can be achieved by improving its design. In this paper we evaluate such one option by conditioning the insurance pay-out to the occurrence of a catastrophic event determined by a weather index.[1] Empirical evidence suggests that extreme events have higher extent of spatial correlation. Thus, catastrophic insurance should be less affected by spatial basis risk. Furthermore, we assume that considering extreme events only might reduce the scope of temporal basis risk, as sensitivity of crop yields to weather might be much stronger for extreme realisations of the weather index.

1. [1] There are surely other ways to improve weather-based insurance design; however, in this paper we purposely reduce our focus on its enhancement by modifying to catastrophic weather-based insurance.

We evaluate the effectiveness of a catastrophic weather-based insurance against drought developed by applying two alternative methods - the standard regression analysis and the copula approach. Most empirical analyses obtain estimates of the dependence of crop yields on a weather index by employing linear regression. Consequently, they assume, first, that the sensitivity of yields to weather remains constant over the whole distribution of the weather variable; second, that it can be measured by the effect of weather on the yield conditional mean. We argue that, when insuring against catastrophic events, the prediction of farm yield losses can be done more accurately by measuring and employing tail dependence of the joint distribution of weather and yield variables. Therefore, we develop a copula-based approach for the design and rating of catastrophic insurance. Finally, we compare risk-reducing effectiveness of insurance contracts, derived by means of these alternative approaches, by employing the Expected Utility model (EU model).

In our study we define the catastrophic drought insurance as an insurance which pays indemnity if the weather index, chosen to indicate the drought occurrence, falls below the third decile q_{30} of its probability distribution. Given that the lower values of an index correspond with lower yields as in the case of the cumulative rainfall, the third decile represents the 30 per cent of the years, in particular those with lowest rainfall observations.

The remainder of the paper is structured as follows. Section 2 provides an overview of the methodology and data. Section 3 presents our preliminary results. Conclusions are drawn in the final section.

2. METHODOLOGY AND DATA

2.1. Methods and empirical procedure

In our empirical analysis we employ two approaches to determine sensitivity of crop yields to weather. The first one is based on linear regression, and the other one characterizes dependence between crop yields and weather by means of a copula.

Linear regression is a standard tool used in empirical studies to estimate dependence of crop yields on weather. However, the use of linear regression implies restricting the measurement of sensitivity to the effect of weather on the mean of the dependent variable, in our case farm yields. Accordingly, when employing linear regression, researchers implicitly assume that the sensitivity of crop yields to weather remains constant over the whole distribution of the yield variable and can be captured by the effect of weather on the yield conditional mean. Additionally, since regression analysis is based on linear correlation, it provides consistent estimates only in the case of multivariate normal distributions. But, empirical evidence shows that most empirical distributions do not follow the Gaussian law. Moreover, linear correlation is not adequate for representing dependency in the tails of multivariate distributions (McNeil et al, 2005). This quality makes it hardly applicable for the assessment of extreme losses.

In our study, we analyse the capacity of linear regression to capture dependence of crop yields on weather in the context of catastrophic weather-based insurance. Therefore, for our empirical data we design weather-based insurance contracts by employing estimates of the linear regression and compare them we estimates obtained from copula model estimations.

Copulas

A copula allows to link marginal distributions together to form the joint distribution. A *d*-dimensional copula $\mathrm{C}(\mathbf{u}) = \mathrm{C}(\mathrm{u}_1,\ldots,\mathrm{u}_\mathrm{d})$ is a multivariate distribution function on $[0,1]^\mathrm{d}$ with standard uniform marginal distributions (McNeil et al., 2005).

According to the Sklar's theorem (1959), if *F* is a joint distribution function with marginal distributions $\mathrm{F}_1,\ldots,\mathrm{F}_\mathrm{d}$, then there exists a copula $\mathrm{C}: [0,1]^\mathrm{d} \to [0,1]$ such that for all $\mathrm{x}_1,\ldots,\mathrm{x}_\mathrm{d}$ in $\overline{R} = [-\infty,\infty]$,

$$\mathrm{F}(\mathrm{x}_1,\ldots,\mathrm{x}_\mathrm{d}) = \mathrm{C}(\mathrm{F}_1(\mathrm{x}_1),\ldots,\mathrm{F}_\mathrm{d}(\mathrm{x}_\mathrm{d})). \tag{1}$$

Consequently, the Sklar's theorem states that any continuous multivariate distribution can be uniquely described by two parts: the marginal distributions F_i and the multivariate dependence structure captured by the copula *C*. This definition explains the usefulness of copulas for modeling multivariate dependence.

In general, there are many families of copulas. Most important distinction is done between parametric and nonparametric (e.g. kernel) copulas. Most empirical investigations employ copulas for model simulations and thus employ primarily parametric copulas which are better suited for simulation purposes. Parametric copulas consist of implicit and explicit types of copulas. Implicit copulas are defined by well-known multivariate distribution functions, e.g. the Gaussian copula and Student's t copula (McNeil et al, 2005). These two copulas are often referred as elliptical copulas and demonstrate radial symmetry. The latter characteristic makes their application limited to the joint distributions with the same tail dependence in the left and right tails. An important property of the explicit copulas is that they possess a simple closed form. The mostly known class of explicit copulas is Archimedean copulas.

In our study we evaluate several copulas on the subject of their adequacy to model the dependence between crop yields and a weather index for our study objects. We employ two implicit copulas – the Gaussian copula and Student's t copula, and three explicit Archimedean copulas – the Clayton, Gumbel and Frank copulas. While the Gaussian, Student's t and Frank copulas assume radial symmetry, Clayton and Gumbel copulas allow to model joint distributions with asymmetric dependence structure. In particular, the Clayton copula exhibits strong left tail dependence and relatively weak right tail dependence, whereas the Gumbel copula exhibits strong right tail dependence and relatively weak left tail dependence. As in our analysis we are interested in the lower tails of yield distribution, we employ survival Gumbel copula, i.e. the Gumbel copula applied to survival functions of marginal distributions (Bokusheva, 2011, p. 128). A description of single copula models can be found in the Appendix.

The comparison of different copula models was conducted by employing the goodness-of-fit test based on the Cramer-vonMises statistics (Genest et al., 2009).

Insurance contract parameters

For the design and rating of weather-based insurance contract we estimate the expected shortfall (ES) of the weather index (W) at the confidence level $\alpha = 0.7$ which corresponds with 3rd decile of the weather index distribution, i.e.:

$$ES_{1-\alpha}(W) = VaR_{1-\alpha}(W) - \frac{1}{1-\alpha}\int_{l=0}^{1-\alpha} q_l(F_W)dl \qquad (2)$$

where q_l is the l-th quantile of the weather index distribution F_W. $VaR_{1-\alpha}$ is the Value-at-Risk (VaR) of the weather variable W at the level $1-\alpha$. Thus, (2) allows us to calculate the expected deviation of the weather index from its VaR, which is used to determine the occurrence of a catastrophic event.

In the next step, the tick size for the weather-based insurance contract has to be defined. It can be estimated by considering conditional distribution of the yield variable (Y). Therefore, we consider a bivariate distribution of the weather index and crop yield (W, Y) ~ H with the distribution function $H(w, y)$. The respective marginal distributions are $u=F(w)$ and $v=G(y)$.

In the linear regression framework the mean of the dependant variable, in our case – the yield, are conditioned on single realisations of the independent variable, in our analysis – the weather index, as follows:

$$\tilde{\mu} = E(Y|W = w) = \mu_Y + \frac{\sigma_{WY}}{\sigma_W^2}(w - \mu_W) \tag{3}$$

where $\tilde{\mu}$ is the conditional mean of *Y*, μ_Y and μ_W are the mean of the marginal distributions of the yield and weather variables, respectively; σ_{WY} is the covariance between *W* and *Y*, and σ_W^2 is the variance of *W*.

As it is shown in (3), the expectation parameter of the yield variable is defined conditioned on single realizations of the weather index. Additionally, the dependence parameter is restricted to be constant over all realisations of the weather index. In our analysis, we however, seek to determine the expected value of the yield given the weather index will fall below its critical level, i.e. *VaR*(*W*):

$$\tilde{\mu}^* = \tilde{\mu}_{|W \le VaR_{1-\alpha}(W)} = E(Y|W \le VaR_{1-\alpha}(W)) \tag{4}$$

Furthermore, we assume that the dependence structure between yields and weather index can be different in the tails of the distribution. Thus, to estimate the expression in (4), we suggest to express the conditional distribution of the yield variable in terms of a copula:

$$\mathrm{H}_{\mathrm{Y|W=w}}(y) = c_{G(Y)|F(W=w)}(v)\Big|_{\substack{G(y)=v \\ F(w)=u}} \tag{5}$$

where $c_{G(Y)|F(W=w)}(v) = \frac{\partial}{\partial u} C(u,v)\Big|_{F(w)=u}$ (6)

which is the first derivative of the copula with respect to *u*.

To consider all realisations of the weather index below its VaR, we rewrite (5) to obtain:

$$c_{G(Y)|F(W)\le 1-\alpha}(v) = \frac{1}{P(F(W) \le 1-\alpha)} \int_{l=0}^{1-\alpha} \frac{\partial}{\partial u} C(u,v)\Big|_{\substack{F(w)=l \\ G(y)=v}} \partial l \tag{7}$$

Taking the inverse of the expression in (7) considering the distributions of the yield variable *G*(*Y*) allows to generate single realizations of the yield variable. Accordingly, $\tilde{\mu}^*$ can be estimated by taking the inverse of (7) integrated over *v*, which distributed uniformly in the interval [0,1] , i.e.:

$$\tilde{\mu}^* = G^{\leftarrow}\left(\int_{k=0}^{1} c_{G(Y)|F(W)\leq 1-\alpha}(s)dk \right) = \\ G^{\leftarrow}\left(\frac{1}{P(F(W)\leq 1-\alpha)} \int_{k=0}^{1}\int_{l=0}^{1-\alpha} \frac{\partial}{\partial u} C(u,v)\bigg|_{\substack{F(w)=l \\ G(y)=k}} \partial l \partial k \right) \tag{8}$$

where $G^{\leftarrow}$ denotes the generalized inverse with respect to the yield marginal distribution function.

Subsequently, by employing the estimates of the ES of the weather index and the respective value of the $\tilde{\mu}^*$, the tick size can be determined as:

$$tick\ size = \frac{y_{critical} - \tilde{\mu}^*}{ES_{1-\alpha}(W)}$$

where $y_{critical}$ is critical level of the yield to be insured, i.e. to be larger than $\tilde{\mu}^*$. This parameter is to specify in the insurance contract.

Accordingly, the fair premium is calculated as:

$$fair\ premium = (1-\alpha)(y_{critical} - \tilde{\mu}^*).$$

The design of the insurance contract based on the linear regression analysis was done in accordance with Skees et al. (1997). The strike value of the weather index was set to the third decile of the weather variable, i.e. the insurance pay-out was conditioned on the same realisations of the weather index as in the copula-based approach.

EU model

The evaluation of two types of insurance contracts – one based on the linear regression analysis and the other one designed by using the copula approach was done by employing the Expected Utility (EU) model.

In the first step we obtain insurance contracts parameters – the tick size and fair premium. Based on that we calculate the farmer's insured yield as:

$$\mathrm{y}_{\mathrm{t}}^{\mathrm{insured}} = y_t + tick\ size * W_t\Big|_{W_t \leq VaR_{1-\alpha}(W)} - fair\ premium$$

where y_t is the farmer's uninsured yield.

To determine the farmer's certainty equivalent the negative exponential utility function was employed:

$$U(x) = 1 - \exp(-r_a x)$$

where r_a is the absolute risk aversion[2].

2.2. Data

The study employs wheat yield data for 47 large grain producers from five different administrative units, so called rayons, located in Northern Kazakhstan. These data were provided by the rayon statistical offices. The weather data originates from the National Hydro-Meteorological Agency of the Republic of Kazakhstan – Kazgydromet. Both yield and weather data cover the period from 1980 to 2010. The yield time series are tested for the presence of structural breaks and were adjusted for technological trends (s. for details Conradt et al, 2012).

We define the catastrophic drought insurance as an insurance which pays indemnity if the weather index, chosen to indicate the drought occurrence, falls below the third decile q_{30} of its probability distribution. In this study, we determine drought occurrence by employing cumulative rainfall amount from April to July. Given that cumulative rainfall corresponds with lower wheat yields, the third decile represents 30 per cent of total observations, in particular those with lowest rainfall amounts.

The Kolmogorov-Smirnov test showed that both the farm detrended yields and the cumulative rainfall can be well described by the Weibull distribution. Only for yields of two farms the Weibull distribution could not be rejected at 5 per cent significance level, for all remaining farms the Weibull distribution provided good fit, i.e. at a significance level higher than 10 per cent.

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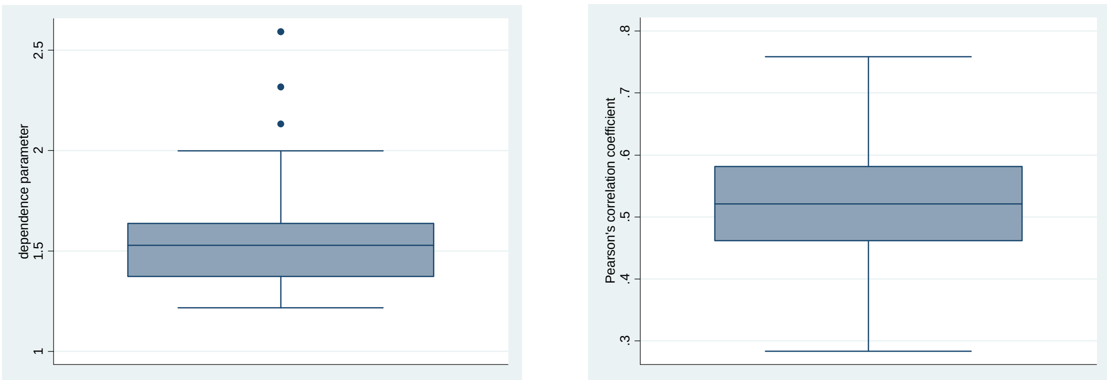
[2] The absolute risk aversion coefficient was chosen to represent a rather risk averse decision maker and correspond with the value of the relative risk aversion coefficient r_r=1.

3. PRELIMINARY RESULTS

To model the dependence between farm wheat yields and cumulative rainfall, we have chosen the survival Gumbel copula. Compared to other selected copulas, this copula could not be rejected by the goodness-of-fit test for any single study farm in our sample. The sample average value of the dependence parameter is 1.57, which suggests quite pronounced left-tail dependence in the joint distribution of the farm wheat yields and the cumulative rainfall from April to July. However, as it can be seen in Figure 1a, the dependence varies considerably across study farms. The estimates of the Pearson's correlation coefficient are also exhibit rather high variation (Figure 1b).

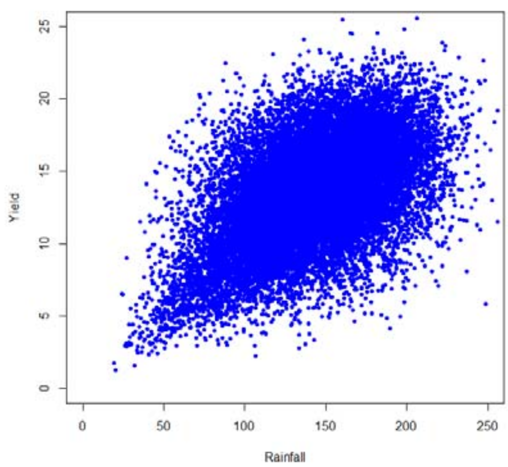


1a) 1b)

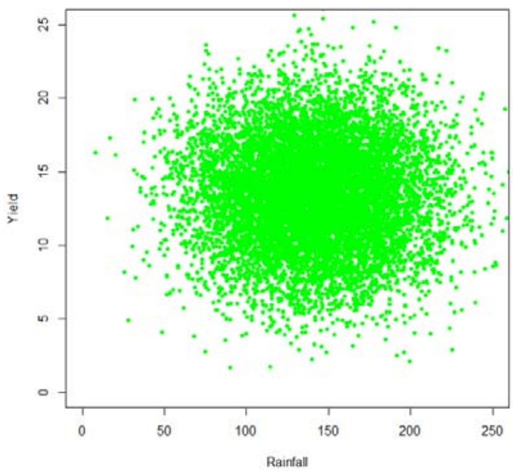
Figure 1. Estimates of yield-weather dependence: 1a) θ -parameter for the survival Gumbel copula, 1b) Pearson's coefficient of correlation; 47 study farms (1980-2010 period).

Source: authors' own estimates

Figure 2 shows the dependence structure modelled for the same empirical bivariate distribution of wheat yield and cumulative rainfall by means of two different copulas – the Gumbel and Gaussian copulas, respectively. These model simulations were done for the study farm, for which both dependence parameters – the copula dependence parameter and the coefficient of correlation - were close to their sample average values. Figure 2 makes it obvious that using the Gaussian copula, which correspond to the dependence structure captured by linear correlation, can cause a serious undervaluation of the dependence structure in the left tail of the joint distribution of yield and weather variables.



2a)



2b)

Figure 2. Two bivariate distributions with the same margins (study farm 10 in rayon 1): 1a) survival Gumbel copula, θ =1.52 1b) Gaussian copula, ρ=0.48.

Source: authors' own estimates and representation

Our EU-model estimation results show that copula-based insurance contracts would allow an increase in the EU for 29 of totally 47 study farms and would rather decrease the EU for 18 sample farms, while the insurance contracts based on linear regression estimates would not allow any improvement in the EU for any farm (Table 1). As it can be seen from results presented in Table 1, the performance of the catastrophic weather-based insurance varies across five study rayons. While all farms in rayons 1, 4 and 5 could benefit from purchasing catastrophic insurance based on copula estimates, no farm in rayons 2 and 3 would be able to reduce their risk by purchasing such insurance contracts. This result suggests that independent of the approach chosen for the insurance contract design, an important aspect for the development of the catastrophic weather-based insurance is the selection of an appropriate weather parameter.

Table 1. Change in the expected utility compared to uninsured yields: results from regression-based approach (LR) and copula-based approach (copula), 47 study farms

	Rayon 1		Rayon 2		Rayon 3		Rayon 4		Rayon 5	
	LR	copula	LR	copula	LR	copula	LR	copula	LR	copula
Farm 1	1.000	1.011	0.964	0.991	0.993	0.965	1.000	1.033	0.991	1.021
Farm 2	1.000	1.010	0.975	0.991	0.991	0.964	1.000	1.020	0.977	1.045
Farm 3	1.000	1.012	0.972	0.986	0.992	0.954	1.000	1.055	0.968	1.038
Farm 4	1.000	1.016	0.983	0.992	1.000	0.977	1.000	1.016	0.972	1.038
Farm 5	1.000	1.012	0.992	0.993	0.992	0.960	1.000	1.021	0.974	1.047
Farm 6	1.000	1.014	0.963	0.982	0.992	0.943	1.000	1.028	0.977	1.037
Farm 7	1.000	1.010	0.985	0.992	0.992	0.941	1.000	1.026	0.977	1.038
Farm 8	1.000	1.021	0.948	0.986	--	--	1.000	1.041	--	--
Farm 9	1.000	1.013	0.975	0.990	--	--	1.000	1.025	--	--
Farm 10	1.000	1.003	0.969	0.990	--	--	1.000	1.027	--	--
Farm 11	1.000	1.014	0.983	0.992	--	--	--	--	--	--
Farm 12	1.000	1.015	--	--	--	--	--	--	--	--
on average	1.000	1.013	0.974	0.990	0.993	0.958	1.000	1.029	0.977	1.038

Source: authors' own estimates

CONCLUSIONS

The paper presents a copula-based approach for the design and rating of weather-based catastrophic insurance. The effectiveness of this approach is compared with the common approach based on linear regression by evaluating EU-model estimates of insured yields against uninsured yields.

Our preliminary results suggest that the application of the copula approach might seriously improve the performance of catastrophic insurance. However, a prerequisite for the insurance effectiveness is the selection of a weather indicator which enables an adequate representation of plant growing conditions. So far, the study employed only one weather index to indicate strong drought. Thus, more investigations are required to validate our first results.

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APPENDIX

Copula models

The Gaussian copula is given by:

$$C_P^{Ga}(\mathbf{u}) = P(\Phi(X_1 \le u_1),\ldots,\Phi(X_d \le u_d)) = \mathbf{\Phi}_P(\Phi^{-1}(u_1),\ldots,\Phi^{-1}(u_d)), \quad \text{(A1)}$$

where P is a linear correlation matrix, Φ denotes the standard univariate normal distribution function and $\boldsymbol{\Phi}$ denotes the joint distribution function of the vector $\boldsymbol{X} \sim N^d(0, P)$.

The d-dimensional t copula takes the form:

$$C_{v,P}^{t}(\mathbf{u}) = \mathbf{t}_{v,P}(t_v^{-1}(u_1),\ldots,t_v^{-1}(u_d)), \quad \text{(A2)}$$

where t_v is the distribution function of a standard univariate t distribution, $\mathbf{t}_{v,P}$ is the joint distribution function of the vector $\boldsymbol{X} \sim t^d(v, 0, P)$ and P is a linear correlation matrix.

The Clayton copula has the following closed form:

$$C_\theta^{Cl}(u_1,\ldots u_d,\theta) = \left(u_1^{-\theta} + \ldots + u_d^{-\theta} - (d-1)\right)^{-1/\theta} \text{ with } 0 < \theta < \infty, \quad \text{(A3)}$$

where θ denotes the dependence parameter to be estimated. As $\theta \to 0$ it represents independence, while as $\theta \to \infty$ it describes perfect dependence.

In contrast to the Clayton copula, the Gumbel copula exhibits strong right tail dependence and relatively weak left tail dependence. It is determined as:

$$C_\theta^{Gu}(u_1,\ldots u_d,\theta) = \exp\left\{-\left((-\ln u_1)^\theta + \ldots + (-\ln u_d)^\theta\right)^{1/\theta}\right\} \text{ with } 1 \le \theta < \infty. \quad \text{(A4)}$$

A detailed overview of different copula families can be found in the textbook by Nelsen (1999).