REVIEW OF MARKETING AND AGRICULTURAL ECONOMICS
VOL. 50, NO. 2 (*August*, 1982)

Economic Surplus Concepts and Their Use in Benefit Cost Analysis

Alan Randall*

The conceptual bases for project evaluation, the use of Hicksian consumer's surplus concepts of value, and the implementation of the currently accepted techniques for valuing non-marketed goods and services, are developed in some detail. While the primary focus is on partial equilibrium analysis of changes pertaining to a single good, some of the complications introduced by multiple changes and general equilibrium conditions are considered.

1 Benefit Cost Analysis in the Decision Process

Benefit cost analysis (BCA) emerged as a response to the notion that there ought to be some analysis of the relative magnitudes of benefits and costs of public investments: perhaps, a public sector counterpart of the feasibility studies which prudent private investors perform prior to undertaking long-term development projects. The idea was to provide for some kind of *a priori* test that public projects proposed for economic development purposes were in fact likely to make positive contributions to that goal.

Such a test, once performed, may be put to any of several uses in the public decision process. One of the earliest attempts to institutionalize BCA occurred in the United States where the Flood Control Act of 1936 required that federal flood control projects may be authorized if and only if it is shown that "the benefits to whomsoever they accrue" exceed the costs. This requirement was later applied to all federal projects dealing with water and related land resources. Clearly, the purpose of these requirements was to provide a filter which would systematically eliminate, prior to consideration in the political domain, those projects deleterious to national economic development.

While BCA is always an evaluative tool, there is no inherent reason why it should be used as a filter. On one hand, it could be more than a mere filter. A large and often sophisticated literature explores, develops and sometimes promotes the use of BCA in ranking projects and determining optimal project size. The goal is the optimal package of optimally sized projects, all from the perspective of economic efficiency. To my knowledge, no political jurisdiction has ever mandated that public works programmes or regulatory policy packages be assembled in strict obedience to that criterion. However, many agencies use BCA as a more or less strict ranking device for internal planning.

On the other hand, BCA can be used as less than a filter. In many jurisdictions, it is performed as one component of an effort to assemble a more comprehensive set of information about proposed projects and policies. In this role, BCA serves to identify and document in efficiency terms the opportunity

---

* University of Kentucky, Lexington, KY 40546, U.S.A. This article was prepared while I was visiting at the University of New England and later at the University of Chicago. Portions of this material were presented to the 1980 conference of the Australian Agricultural Economics Society, and to seminars at Latrobe University, the University of Chicago, the University of New South Wales and the University of Newcastle.

My substantial debt to John R. Stoll, who was my colleague on much of the research which led to this article and my co-author on several papers reviewed herein, is gratefully acknowledged. John P. Hoehn and D. R. Gallagher offered helpful comments on a previous draft.

costs of pursuing other kinds of goals. BC information is not necessarily compelling, but is weighted by various parties to the decision process in accordance with the priority each places on economic efficiency relative to other objectives. While failure to make a positive contribution to national economic efficiency does not boost a proposal's chances, neither is it (nor should it be, according to this role model) the death knell for a proposal which serves other worthwhile goals.

In any of these roles (optimizing decision rule, filter, or contribution to a multidimensional information system), BCA carries some imperative or suggestive force. So, parties having a stake in the success or failure of a proposal are interested in the outcome of its BCA. For this reason, the quality control problems inherent in any empirical economic analysis are magnified in BCA[1].

There are two obvious approaches to quality control. First, the theory and methods relevant to BCA can be developed and refined to promote their conceptual and empirical validity, and then codified in a set of rules for the performance of BCA. Second, BCAs and supporting documents may be opened to public scrutiny, to permit the process of criticism which is one of the necessary conditions for scientific objectivity (Popper 1957). With respect to US water resources projects, where BCA is a mandatory filter, both approaches have been pursued. The current *Principles and Standards* and their implementation rules (U.S. Water Resources Council 1973, 1979) require evaluation procedures generally consistent with competent economic analysis. BC statements are public information and may be challenged directly in Congressional hearings and indirectly in the courts (under the National Environmental Policy Act) and in required public hearings under various licensing and permitting laws[2].

Few other agencies and jurisdictions have gone so far down the "mandatory BCA subject to quality control and public scrutiny" route for evaluating water resources projects[3]. On the other hand, the domain of BCA (to serve as one component of a multidimensional information system) has spread far beyond the water resources field into, *inter alia,* public finance and the economics of public utilities, and efficiency evaluation of complex regulatory programmes to promote

1. For example, Clark (1980), arguing against mandatory BCA for Australian water resources projects, has claimed that several decades of mandatory BCA in the U.S. have not produced dramatic improvements in decision making there. On the contrary, he argues, fair means and foul have been used to boost the estimated benefits of proposed projects relative to costs and thus subvert the watchdog function of BCA.

2. There are, however, some limits to impartiality in the U.S. approach to water resources project evaluation. Congressional opposition in 1979 led President Carter to withdraw his proposal that all required BCAs be conducted, not by the agencies proposing projects, but by an independent evaluation team assigned to the Water Resources Council.

3. While Australia has no counterpart to the (U.S.) Flood Control Act, the Australian Water Resources Council in its role as a reviewer of project proposals has encouraged improvements in their supporting documents. Yet, Australian water resources authorities have been reluctant to adopt detailed guidelines for the performance and review of BCA's: no Australian analogue to the *Principles and Standards* (U.S. Water Resources Council 1973) has yet appeared. Such policy statements as exist (e.g., Australian Water Resources Council 1978; and Newman 1979) place considerable emphasis on enlightened notions of multiple objective planning, but avoid specific reference to BCA in general and in particular. Documents intended to guide the implementation of current water resource planning policy (e.g., Ministry of Water Resources and Water Supply, Victoria 1977) make it clear that while projected benefits and costs of proposed projects are to be duly considered, the exhaustive analyses mandated in the United States (U.S. Water Resources Council 1973, 1979) are neither required nor expected.

environmental quality and human health and safety[4]. In some cases, public agencies, or research organizations under contract to them, have performed BCAs. In other cases, BCA has been undertaken by researchers in academic and similar organizations. The findings of these efforts, while having no "official" status, may be influential if they are disseminated so as to attain notice in professional and political circles.

Given the expanded domain of BCA the analyst encounters many kinds of goods and services which are not customarily marketed. Such goods are not merely unpriced, which is inconvenience enough to the BC analyst, but often exhibit economic characteristics which complicate the analysis: e.g., indivisibility in production, nonrivalry in consumption, and/or nonexclusiveness. The professional literature has reflected the increasing complexity of situations in which BCA is applied. Witness the progression from Eckstein (1958) to Mishan (1971), Maler (1974), and Freeman (1979*a*) in the book literature, and the profusion of journal articles on the theory of welfare change measurement and its empirical application in project and programme evaluation.

Sinden (1980), writing in this *Review*, has drawn attention to the conceptual issues and practical problems which are encountered in BCA, and to the confusion which an interested non-specialist might derive from an increasingly sophisticated and contentious specialized literature.

This paper is specifically addressed to two of the issues raised by Sinden (1980): the use of economic surplus concepts of value, and their application in the valuation of non-marketed cost and benefit items. First, a general conceptual model of the typical BCA problem situation is developed. A proposed project or programme is conceived as modifying a given complex environment, changing, perhaps qualitatively and quantitatively, the flow of services that environment provides. Then, the application of economic surplus concepts to the valuation of service flows is considered in substantial detail. In particular, Hicksian consumer's surplus concepts are applied to partial equilibrium analyses of marginal and nonmarginal changes in the quantity of goods and services provided. The emphasis on nonmarginal changes and total value concepts effectively adapts consumer's surplus concepts for application to goods and services which are indivisible in production, nonrival in consumption, and/or nonexclusive. The conceptual bases for empirical valuation of such goods are then developed. To this point, all analyses are for changes pertaining to a single good, in a partial equilibrium context. Toward the conclusion, some of the complications which arise from relaxing these restrictive assumptions are briefly considered.

2 The Typical Project Evaluation Problem Situation: Non-marginal Changes in Complex Environments

Let us now turn to an examination of the typical project or programme evaluation problem.

4. In 1981, U.S. President Reagan sought by administrative fiat (Executive Order 12 291) to impose a BCA filter for all new regulatory initiatives and reauthorization of existing regulations. EO 12 291 also contains language suggesting that BCA should be used in establishing regulatory priorities. Thus, this edict required a dramatic elevation of BCA from its more customary role—providing merely one among many kinds of information—in regulatory matters.

Consider a complex environment, E, producing a vector of services $\underline{S} = (s_1, \ldots, s_k, \ldots, s_m)$, valued by people. The services (or goods, or amenities) are likely to be diverse—e.g., support services for human, animal and plant life; aesthetic services including atmospheric visibility, landscape amenities, and diversity of flora and fauna; recreation opportunities; waste disposal services; etc.—and many of them are likely to be nonmarketed. The supply of each of these services in any time period, t, is a function—uniquely determined by geological, hydrological, atmospheric and ecological relationships—of the attributes, $\underline{A} = (a_1, \ldots, a_r, \ldots, a_s)$ of the environment.

$$(1) \quad \begin{aligned} s_{1t} &= f_1\,(\underline{A}_t) \\ &\vdots \\ s_{mt} &= f_m\,(\underline{A}_t) \end{aligned}$$

Man enters the system as a modifier of resource attributes. He may do this directly, e.g., by reassigning land to other uses, diverting water, removing vegetation, disturbing soil for mining, etc. He may also modify the resource as a side effect (expected or unexpected) of some other decision, e.g., disturbing land elsewhere for cultivation or mining, deposition of wastes in water upstream, etc. For each kind of resource attribute,

$$(2) \quad \begin{aligned} a_{1t} &= g_1\,(\underline{N}_t, \underline{X}_t) \\ &\vdots \\ a_{st} &= g_s\,(\underline{N}_t, \underline{X}_t) \end{aligned}$$

where $\underline{N}$ is a vector of "natural systems inputs", e.g., geological, hydrological, atmospheric and ecological, and $\underline{X}$ is a vector of man-controlled inputs and activities, including harvesting effort.

Both $\underline{N}$ and $\underline{X}$ are subject to scarcity, and the attribute production functions are determined by the laws which govern natural systems and by man's technology. The production system for environmental services is now complete, if it is remembered that the level of demand for some kinds of services, s_k, influences the level of $\underline{X}$ and that interactions between $\underline{X}$ and $\underline{N}$ are possible and likely. For example, the attempt to enjoy high levels of waste assimilation services involves high levels of pollution inputs, which may modify $\underline{N}$, the 'natural' characteristics of the system.

Now consider the value of resource services. Each individual, j, enjoys utility in each time period, t:

$$(3) \quad U_{jt} = U_{jt}[\underline{S}^a_{jt}, \underline{Z}^b_{jt}\,(\underline{S}_{jt})^b, \underline{Z}^a_{jt}],$$

where $\underline{Z}$ is a vector of valued goods and services which are not directly provided by the environment E (for example, $\underline{Z}$ includes things bought at the local shopping centre); and the service vector $\underline{S}$ is divided in $\underline{S}^a$ which are enjoyed directly and $\underline{S}^b$ which are inputs into the production of $\underline{Z}^b$; finally, $\underline{Z}^a$ are those $\underline{Z}$ which are produced independently of $\underline{S}$. This formulation permits both direct and derived demand for environment services.

By minimizing the individual's expenditure, subject to the constraint that his utility must be maintained at a level equal to or greater than that which he enjoys with the existing environment, *E*, Hicksian compensated demand curves for each of the services s_{jkt} or for the whole vector $\underline{S}_{jt}$ can be derived. Taking the latter route, the individual's total net valuation of the service vector, $V_{jt}(\underline{S}_{jt})$, may be directly calculated. Alternatively, the V_{jkt} for each s_{jkt} may be estimated independently and then aggregated across service types[5].

The capital value of the environment *E* is obtained by summing the net values of service flows, discounted at the rate, *r*, across time periods and individuals:

$$(4) \quad PV(E) = \sum_t \sum_j V_{jt}(\underline{S}_{jt})/(1+r)^t.$$

Thus, the environment, *E*, is seen as a capital good acquiring value to the extent that the services it provides are valued by man. Those services are determined by the environment's attributes, which are themselves determined by the characteristics of the natural system and by the activities of man. If that environment were to be disturbed—that is, if the $\underline{X}$ vector of man-controlled inputs were to be modified—its attributes could change, changing the $\underline{S}$ vector of services it provides and its capital value.

Consider a project Δ, which would change $\underline{X}$ to $\underline{X}^\Delta$, thus converting the environment, *E*, to some "with project" state, E^Δ, at some conversion cost, $C^\Delta = \Sigma_t C_t^\Delta/(1+r)^t$. The proposed project would replace the "without project" stream of services, $\underline{S}$, with some "with project" stream, $\underline{S}^\Delta$. The net present value of such a project is

$$(5) \quad PV(\Delta) = PV[(E^\Delta - C^\Delta) - E] = \frac{\Sigma_t\, [V_t(\underline{S}_t^\Delta) - C_t^\Delta - V_t(\underline{S}_t)]}{(1+r)^t}$$

where $V_t(\underline{S}_t^\Delta) = \Sigma_j V_{jt}(\underline{S}^\Delta{}_{jt})$ and $V(\underline{S}_t) = \Sigma_j V_{jt}(\underline{S}_{jt})$.

Thus, $PV(E^\Delta)$ is calculated by summing the net discounted value of "with project" services across service types, individuals and time periods, while $PV(E)$ is calculated by handling "without project" services similarly. Given that the basic value data were derived by constraining the individual's utility to the without project level, the net present value of the project Δ can exceed zero if and only if the sum of individual welfare gains from its implementation exceed the sum of individual welfare losses. In other words, BCA is an empirical test for potential Pareto-improvements.

5. It is a common practice to value service types independently and aggregate these values by simple summation to calculate $V_{jt}(\underline{S}_{jt})$. In a recent working paper, Hoehn and Randall (1982) show that, for this aggregation procedure to be valid, it is necessary that substitution and complementarity relationships among s_{jkt} be entirely absent. However, there are good reasons to expect, especially, substitution relationships to be frequently encountered. Hoehn and Randall provide valid aggregation rules for the general case in which substitution and complementarity are permitted, but these rules are not always easy to implement.

2.1 Benefit Cost Analysis and the Market

Voluntary exchange among individuals in markets satisfies the actual Pareto-improvement criterion. Individuals endowed with rights to keep what they have engage in voluntary exchange only if assured they will receive something of greater (or, at least equal) value to themselves than they give up. Buyer's best offer must at least equal seller's reservation price. Thus individual gains exceed (or, are at least equal to) individual losses for each and every individual. Trade which does not satisfy that condition will not take place.

On the other hand, the potential Pareto-improvement criterion is a "what if?" test. *If* those who would gain from implementation of the project, *could* offer enough to buy the acquiescence of those who would lose, the project passes the test. But, the gainers are not required to buy the project's implementation from the losers. The losers are not, therefore, assured of compensation for their losses. The benefit cost criterion values S^{Δ} and S as a market would—i.e., $PV(E^{\Delta} - C^{\Delta})$ is identical to the buyer's best offer for the project, and $PV(E)$ is identical to the seller's reservation price for the "without project" state—but gainers are not required to compensate losers, as they would be in a market.

3 A General Model for Valuing Single Period Service Flows

The typical project or programme promises (or threatens, according to one's perspective) nonmarginal changes in a complex environment. Its evaluation calls for a nonmarginal microeconomics. The project is often indivisible or lumpy in production: the dam (will/will not) be constructed, the open-cut mine (will/will not) be established, the smelter (will/will not) be built, the designated land area (will/will not) be developed as a national park, or the proposed petroleum exploration (will/will not) take place. Engineering considerations often require that, if the project is to be implemented, it must be implemented at one of a small number of discrete sizes. Some kinds of project outputs are nonrival in consumption (air and water quality, for example), while many kinds are lumpy in consumption (once project specification or programme rules have been established, the individual must take the level of output provided: opportunities for post-project adjustments in his consumption bundle may not exist). Some kinds of project outputs are non-exclusive (air and water quality, again), so that post-project trading opportunities are not available. Finally, some decisions (e.g., to inundate an area with distinct geological and ecological features, to permit extinction of an endangered species, or to create near indestructable hazardous or toxic wastes) made in one time period effectively remove some options from the opportunity set for all subsequent time periods. Thus, many projects and programmes are proposals for change which is nonmarginal in aggregate, and important categories of benefit and cost items must be considered nonmarginal from the perspective of the affected individual.

Accordingly, a conceptual model for valuation is needed which is not limited to the economics of marginal change. The model developed immediately below, for conceptualizing the value to an individual of one time period's flow of a single service, treats total valuation as the general case. The well-known results for marginal valuation are derived as a special case from the general model.

For notational convenience, drop the subscripts identifying the individual and time period, and let $\underline{Z}$ represent the vector of all goods and services except for the single service Q which is of particular interest. The utility function (3) may then be rewritten as

$$(6) \quad U = U(Q,\underline{Z})$$

If $\underline{P}$ is the vector of prices for $\underline{Z}$ and $\underline{Y}$ is the individual's income, the utility function is expressed as

$$(7) \quad U = U(\underline{P},Q,\underline{Y})$$

Alternatively, if Y is the numeraire value of $\underline{Z}$, the utility function, implicit in prices, may be expressed as[6]

$$(8) \quad U = U(Q,Y) = U[\underline{P}(Q,Y)]$$

It is, in general, conceivable that a project or programme could directly and simultaneously change $\underline{P}$, Q and $\underline{Y}$ (or Y). However, it is useful to focus on a more restricted analysis at the outset. As Mishan (1977) has argued, most public investments or regulatory initiatives do not change income directly; rather, they change individual utility by first modifying $\underline{P}$ or Q. While most authors (including Mishan) focus on the $\underline{P}$ vector, I believe it is often useful to focus initially on Q. That is, the individual is most immediately affected, as a result of a public project, by a change in the flow of services the impacted environment provides him.

Consider an individual who enjoys the levels Q° and Y° of the service and the "all other goods" numeraire. His initial welfare level is $U(Q^{\circ}, Y^{\circ})$, at the origin (figure 1). To the right of the origin the level of provision of Q to the individual increases; to the left of the origin, it decreases. From the origin, a movement up the vertical axis indicates a decrease in the numeraire, while a movement down indicates an increase in the numeraire[7]. The total value (TV) curve, or bid curve, is of positive slope, given that the service is a commodity and the individual is not satiated in the range under consideration. For decreases in Q, the TV curve lies in the southwest quadrant; for increases in Q, it lies in the northeast quadrant. If it is possible to define the quantity of the service in unidimensional, cardinal terms, the assumption of diminishing rates of commodity substitution is sufficient to ensure the curvature shown[8].

---

6. Note that in general $\underline{Y} \neq Y$, and when Q is a priced commodity $\underline{Y} > Y$. However, the case where Q is unpriced to the individual and $\underline{Y} = Y$ is often of interest in BCA.

7. The orientation of the Y axis here, the inverse of its customary orientation, may seem a little strange at first. However, its usefulness soon becomes apparent. It permits easy reading of total value (TV), which is positive for increments in Q and negative for decrements. Further, working in the north-east quadrant, the first derivative of TV with respect to Q, i.e., marginal value, (in cases where it exists, see text at footnote 8, below) can be immediately recognised as an income-compensated demand curve for Q.

8. If "quantity" is multidimensional, or if it cannot be accurately defined in cardinal terms, no *a priori* assumption can be made concerning the curvature of the TV curve (Bradford 1970).

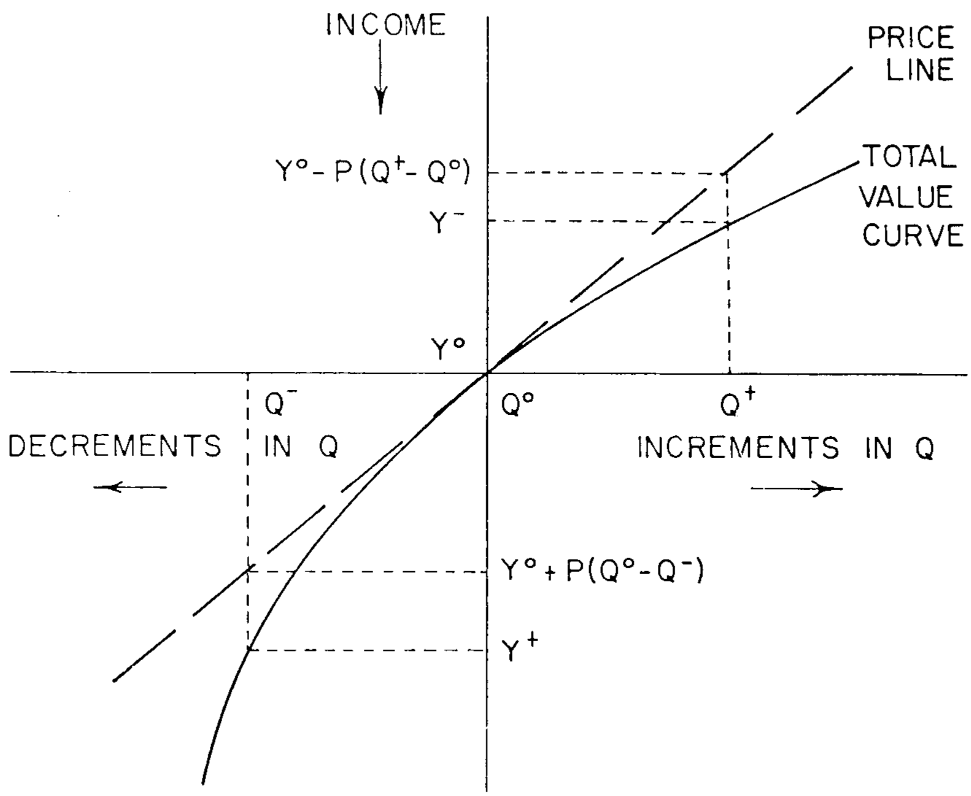

Figure 1. The Total Value Curve for Increments and Decrements in the Level of Provision of Service, Q, for an Individual who Initially Enjoys the Level Q° and the Income Y°.

The *TV* curve is an indifference curve, passing through the individual's initial state. That is,

$$(9) \quad U(Q^\circ,Y^\circ) = U(Q^-,Y^+) = U(Q^+,Y^-).$$

Starting at the origin, $Y^\circ - Y^-$ is the individual's willingness to pay (*WTP*) to obtain an increment in the level of provision of the service from Q° to Q^+. Willingness to accept (*WTA*), i.e., $Y^+ - Y^\circ$, is the amount of money which would induce the individual to accept voluntarily a decrease in the level of provision of the service from Q° to Q^-. *WTP* is the total value to the individual of an increment from Q° to Q^+; *WTA* is the total value to the individual of a decrement from Q° to Q^-. Restating Equation (9),

$$(10) \quad U(Q^\circ,Y^\circ) = U(Q^-,Y^\circ + WTA) = U(Q^+,Y^\circ - WTP).$$

3.1 WTP, WTA and Economic Surplus

By derivation, *WTP* and *WTA* represent the total value to the individual of a given increment or decrement, respectively, in the level of *Q* provided. In market terms, *WTP* is the individual's best offer to buy the increment, taken as a "lump"; and *WTA* is the same individual's reservation price to sell the decrement, similarly conceived. How do *WTP* and *WTA* relate to the more customary measures of value, economic surplus and market price?

Hicks (1943) showed that there are four measures of consumer's surplus[9], none of which is conceptually identical to the Marshallian measure. These are now usually called equivalent surplus (*ES*), equivalent variation (*EV*), compensating surplus (*CS*), and compensating variation (*CV*) (Currie, Murphy and Schmitz 1971). The surpluses differ from the variations in that the latter are calculated after the consumer has made optimizing adjustments in his consumption set, while the former do not permit such adjustments. In general, the variations should be used when such optimizing adjustments are possible. However, benefit cost analysts often encounter situations where optimizing adjustments are prohibited: once project or programme specifications have been determined, the individual must take these as given. Similarly, where the services under consideration are indivisible in production or nonrival in consumption, individual quantity adjustments are impossible. Thus there will be circumstances in which the Hicksian surpluses are appropriate.

The difference between the Hicksian compensating and equivalent measures of consumer's surplus is considerably more significant.

The equivalent measure is defined as the amount of compensation, paid or received, which would bring the consumer to his subsequent welfare level if the change did not take place. The compensating measure is defined as the amount of compensation, paid or received, which would keep the consumer at his initial welfare level if the change did take place.

The Hicksian compensating and equivalent measures of consumer's surplus are both measures of the welfare impacts of changes, but they differ with respect to the reference level of welfare. The compensating measure, by using the initial welfare level as the reference level, measures the welfare impact of changes as if the individual had a right to his initial level of welfare (that is, as if he had the choice of keeping what he has or voluntarily trading for changes). The equivalent measure, by using the subsequent welfare level as the reference level, treats the individual as if he had only a right to his subsequent level of welfare (that is, as if he must accept his subsequent situation, or seek to trade his way back toward his initial situation). Clearly, Hicksian compensating measures are consistent with the potential Pareto-improvement criterion, while Hicksian equivalent measures are not.

To clarify the relationship between Hicksian compensating and equivalent measures of value, *WTA* and *WTP*, and the total value curve introduced in figure 1, consider the following example. The benefit cost analyst is evaluating a proposed project which would, among other things, divert a specified area of natural habitat (perhaps a terrestrial wildlife habitat, an inland wetland, or a coastal marsh) to some alternative use, effectively destroying its usefulness as habitat. The benefit cost analyst needs to know, among other things, the value

9. Benefits and costs are properly valued as *economic surplus*, i.e., the value of a good or service in excess of its factor costs (the latter not including resource rents). Economic surplus may be partitioned into consumer's surplus and "producer's surplus" or resource rents (Mishan 1968), but the relative size of these two subsets is a distributional issue which, for given levels of output and demand, does not affect the magnitude of economic surplus. Mishan (1968) and Currie *et al.* (1971) show that the analytics of consumer's surplus and "producer's surplus" are identical. Thus, I frame the following discussion in terms of the customary consumer's surplus language; all of the results obtained can be easily generalized to economic surplus.

of the losses which would be suffered by an individual who currently enjoys the services and amenities provided by that habitat. In the "without project" situation, the individual has the utility level $U(Q^\circ, Y^\circ)$. To keep the example simple, assume that this individual gains no benefits from the project. Thus his "with project" utility level would be $U(Q^-, Y^\circ)$. Given a specified level of Q, either the "without project" level Q° or the "with project" level Q^-, optimizing adjustments in Q are impossible.

What is the welfare impact of the proposed change on this individual? One could determine his WTA to accept the proposed change. Let us call this $WTA^C_{Q^\circ, Y^\circ; Q^\circ, Y^\circ; Q^-}$. The superscript C indicates that this is a Hicksian compensating measure of value, the first subscript pair, Q°, Y°, indicates that the individual's reference level of welfare (or, if you will, his presumed right) is Q°, Y°. The second subscript pair indicates that Q°, Y° is also his initial welfare level. The third subscript, Q^-, indicates the level of provision of habitat-related services the individual would enjoy after he has accepted the compensation and the change in services; if it turned out that he was compensated with an amount exactly equal to his WTA, his after compensation income would equal $Y^\circ + WTA^C$. This measure of WTA for a reduction in the quantity of wildlife-related amenities from Q° to Q^-, which we shall denote by the abbreviated notation WTA^C, was derived from a total value curve passing through the individual's initial state at Q°, Y° (figure 2).

However, there is another value measure which is sometimes used to estimate the individual amenity user's loss: the amount of money he would be willing to pay to avoid a reduction in the provision of amenities. What kind of value measure is WTP to avoid a less preferred situation? It assumes the individual must accept the less preferred situation, or pay to avoid it. Thus, the reference level of welfare is not the initial situation, but the proposed (or subsequent, in Hicksian terminology) welfare level. So, this second measure of the individual's welfare loss can be denoted $WTP^E_{Q^-, Y^\circ; Q^\circ, Y^\circ; Q^\circ}$. The superscript indicates that it is a Hicksian equivalent measure of value. The first subscript pair indicates that the reference level of welfare (or, if you will, the individual's presumed right) is taken to be Q^-, Y°. The second subscript pair indicates that the individual's initial state is Q°, Y°. The third subscript indicates that, after the individual has paid, he will be permitted to enjoy the Q° level of amenities; if he pays exactly his WTP, his final income will be $Y^\circ - WTP^E$.

Notice that the initial welfare level is different from the reference welfare level. That is the distinguishing feature of Hicksian equivalent measures. Compensating measures on the other hand assume that the initial situation is the reference welfare level. WTP^E cannot be found using a total value curve passing through the individual's initial state. It can only be found using another total value curve, which passes through the individual's reference level of welfare, Q^-, Y° (figure 2).

At this point, notice that the pair of total value curves shown in figure 2 may also be used to estimate the value to the same individual of a different project: a project which would increase the level of provision of habitat-related amenities from an initial level Q^- to a "with project" level Q°. For evaluating this project, the individual's initial situation is Q^-, Y°. His willingness to pay for the increment in amenities the project would provide is $WTP^C_{Q^-, Y^\circ; Q^-, Y^\circ; Q^\circ}$. It is a compensating measure, since the reference level of welfare is the same as the individual's initial welfare level. A *WTA* measure can also be defined: $WTA^E_{Q^\circ,Y^\circ;Q^-,Y^\circ;Q^-}$. This is the individual's willingness to accept compensation in lieu of a promised increment in amenities from Q^- to Q°. It is an equivalent measure, since the reference level of welfare is not the same as the individual's initial welfare level. It cannot be estimated from the total value curve passing through the individual's initial welfare level, Q^-, Y°, but must be estimated from a new total value curve passing through the individual's reference welfare level, Q°, Y°.

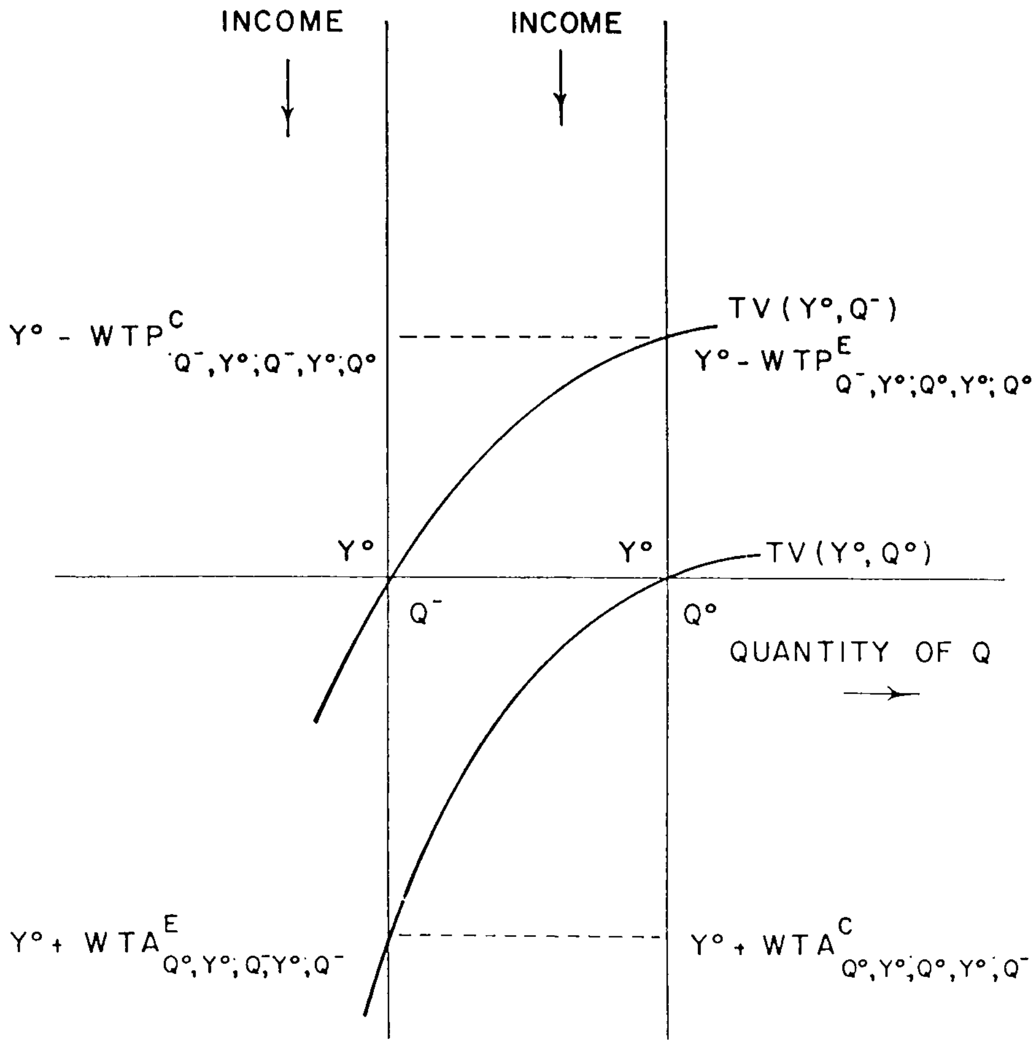

Figure 2. The Relationships between WTP *and* WTA, *and Hicksian Compensating and Equivalent Measures of Consumer's Surplus.*

The foregoing example makes a number of important points. The Hicksian compensating measures of value of alternative levels of provision of a good, service, or amenity can be determined from a single total value curve, while equivalent measures of value must be estimated using a series of different total value curves, one passing through each of the possible levels of provision under consideration. When comparing two alternative levels of provision of a good (as in figure 2), there are four relevant Hicksian value measures: WTP^C to obtain the preferred level; WTP^E to avoid the less preferred level; WTA^C to accept the less preferred level; and WTA^E to forego a promised increment to the preferred level. There is a compensating and equivalent version of *WTP*, as there is of *WTA*. Figure 2 suggests that, when comparing any pair of alternative levels of provision of a good, service, or amenity, WTP^C is equal in value to WTP^E, while WTA^C is equal in value to WTA^E.

3.2 The Relative Magnitudes of Various Value Measures: Compensating, Marshallian, and Equivalent Measures of Consumers' Surplus, and Price Multiplied by the Quantity Change

The concept of consumer's surplus is most commonly used, as by Currie, Murphy and Schmitz (1971), to analyze the welfare impacts of price changes. It is generally concluded that (in absolute value terms) for price increases, $EV \leq M \leq CV$ and, for price decreases, $CV \leq M \leq EV$ (where M is Marshallian consumer's surplus). Willig (1976) rigorously derived empirically operational bounds on the magnitude of the differences between compensating and equivalent variation measures of the welfare impact of price changes.

In benefit cost analysis, however, the immediate concern is the evaluation of the welfare impact of changes in the levels of goods, services, or amentities provided, rather than changes in price levels. In such cases, it is convenient to work with the terms *WTP* and *WTA* (thus rendering the absolute value terminology unnecessary). A general rule can be stated: $WTP \leq M \leq WTA$. Returning to the specific situation of the example used in figure 2,

(11) $WTP^E_{Q^-,Y^\circ;Q^\circ,Y^\circ;Q^\circ} = WTP^C_{Q^-,Y^\circ;Q^-,Y^\circ;Q^\circ} \leq WTA^C_{Q^\circ,Y^\circ;Q^\circ,Y^\circ;Q^-} = WTA^E_{Q^\circ,Y^\circ;Q^-,Y^\circ;Q^-}$.

Two questions remain: (1) under what conditions are *WTP* and *WTA* equal, and (2) when $WTP < WTA$, can bounds on the difference be rigorously defined?

(a) Assume that the resource service under valuation is a perfectly divisible good, traded in infinitely large markets at zero transactions costs at the unit price p. Consider a proposed programme which if implemented would reduce the amount held by an individual from Q to Q^- while leaving the individual's numeraire holdings of Y at Y°. In figure 3, the programme would bump the individual from point E to B, lowering his welfare level from I° to I^-. However, there is no good reason for the individual to remain at B. Instead, frictionless markets will permit him to trade along his new budget line until he reaches D and achieves the welfare level I^* while holding Q^* of the good. Given this frictionless adjustment, his WTP^E is EF, which is equal to $Y''Y'$ while his WTA^C is BC, which is equal to $Y'Y''$. Thus, WTP^E is equal to WTA^C and both are equal to $p(Q^\circ - Q^-)$.

This conclusion can be grasped intuitively by considering that a good traded in infinitely large markets at a constant unit price with zero transactions costs has all the important characteristics of money (i.e., currency). Thus, the well-known result, that willingness to accept compensation and permit imposition of a lump sum tax is equal to willingness to pay to avoid the same tax and both are equal to the tax itself, applies to this quite restrictive case.

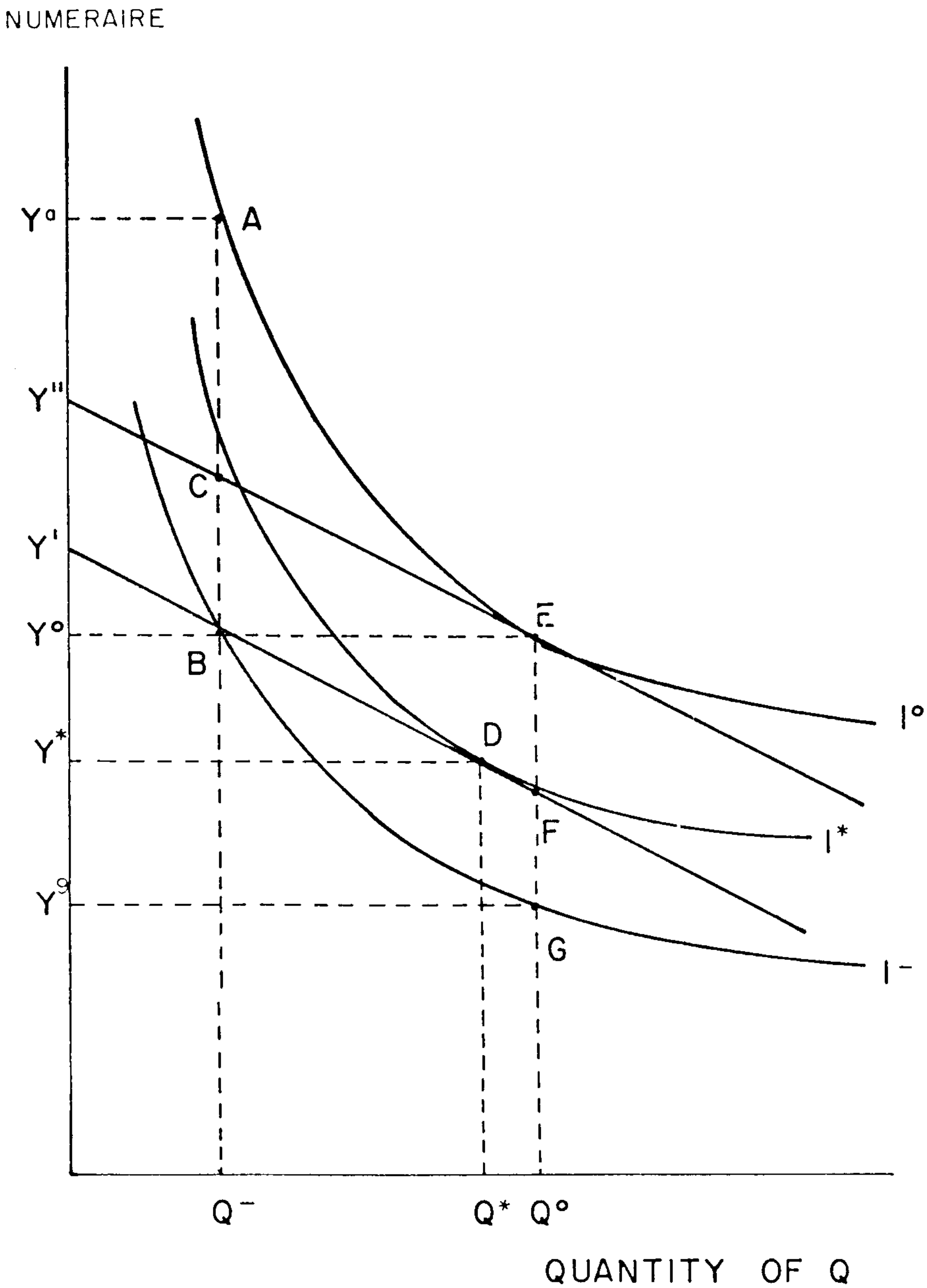

Figure 3. The Welfare Impact of a Change in the Level of Provision of a Service from Q° to Q⁻

In this case, it is appropriate to replace the total value curve with a linear price line tangent to the *TV* curve at the origin (figure 1). Thus, the typical result of partial equilibrium micro-economic theory—that price is equal to unit value at the margin—is derived as a special case from the more general valuation framework developed here.

(b) Now, assume that Q is a lumpy good and can be held only in the amounts Q° and Q^-. Observe immediately that in the case of indivisible or lumpy goods, since intermediate adjustments in commodity holdings are not permissible, the Hicksian compensating and equivalent measures in commodity space are analogous to the Hicksian surpluses, not the variations, defined over price changes.

In this case and returning to figure 3, the price lines become meaningless. WTP^E is EG which is equal to $Y^g Y^\circ$, and WTA^C is BA, which is equal to $Y^\circ Y^a$, and larger in absolute value than WTP^E.

(c) Now, assume that Q is a divisible good, which can be traded, but at a positive parametric transactions cost. This case can be analysed—and is, by Randall and Stoll (1980*b*)—with the help of the kinked budget constraint introduced by Neihans (1971). The results lie in the range bounded by the results of cases (a) and (b), above. When transactions costs are low, the positive-transactions-costs result will approach the zero-transactions-costs result of case (a). When transactions costs are high, the positive-transactions-costs result will approach the "no trade possible" result of case (b). This is only to be expected: prohibitive transactions costs simply prevent trade and thus have the same effect as anything else which would prevent trade.

3.3 Bounds on the Difference between WTA and WTP

$WTA - WTP \geq 0$, for normal goods. In addition, $WTA - WTP$ for a given increment or decrement in natural resource service flows is never greater than when post-change trade is prohibited. Randall and Stoll (1980*a*), building on the work of Willig (1976), rigorously derived bounds on $WTA - WTP$ for this latter case. Where ζ is the price flexibility of income (i.e., $\frac{\partial P(Q,Y)}{\partial Y} \cdot \frac{Y}{P(Q,Y)}$) for the good or service in question, and when $\zeta M/Y^\circ$ is small (say, $\leq .05$),

$$(12) \quad WTA - WTP = \frac{\zeta M^2}{Y^\circ}$$

provides a serviceable approximation. Rigorous bounds, appropriate when M/Y° is large and ζ is not constant, are derived by Randall and Stoll (1980*a*), who also provide a guide for adapting Willig's numerical coefficients (1976, table 1) to the "quantity change" case.

These bounds permit derivation of accurate estimates of the theoretically correct value measures (WTP^C for increments and WTA^C for decrements) from empirical data in the form of WTP^E, WTA^E, or M. In general, for goods and services which are relatively unimportant in the individual consumption bundle (i.e., M/Y° is small), the empirical value of $WTA - WTP$ is small (i.e., only a few dollars, or cents), and the calculation of the correct value measures is indeed, as Hicks (1943) suggested, "a fiddling business". However, as M grows large (becoming a substantial fraction of Y°) and ζ increases, $WTA - WTP$ becomes large (table 1). In evaluating proposals involving significant changes in unique and treasured environments or human health and safety, to provide just two examples, substantial error may be introduced by the use of inappropriate value measures.

*Table 1: Estimated WTA-WTP, using the Approximate Bounds**

M	Y°	M/Y°	ζ	WTA − WTP
($)	($)			($)
50	20,000	.0025	.3	.04
100	20,000	.005	.3	.15
100	20,000	.005	1.0	.50
100	20,000	.005	2.0	1.00
500	20,000	.025	.5	6.25
500	20,000	.025	1.5	18.75
1,000	20,000	.05	.5	25.00
1,000	20,000	.05	2.0	100.00
1,000	20,000	.05	5.0	500.00
5,000	20,000	.25	1.0	1,250.00
5,000	20,000	.25	5.0	6,250.00
10,000	20,000	.5	2.0	10,000.00
10,000	20,000	.5	5.0	25,000.00

*The approximate bounds are quite reliable when $M/Y^{\circ} \leq .05$, but become increasingly unreliable as M/Y° becomes large.

4 Empirical Estimation of Consumer's Surplus Values for Non-Marketed Goods and Services

Many of the services of complex environments are not marketed directly, for reasons already discussed. In these cases, value information—in the form of unit competitive prices for divisible goods, or buyer's best offer or seller's reservation price for lumpy changes—is not directly observable in organized markets. Considerable ingenuity has been devoted, in recent years, to devising and implementing methods of value estimation for non-marketed goods (*see*, e.g., Sinden and Worrell 1979). The various approaches which have been developed differ substantially with respect to conceptual underpinnings, empirical sophistication, data sources, and method of analysis.

Of the "mainstream" methods (i.e., those firmly grounded in consumer's surplus concepts of value), various taxonomies have been suggested but none is universally accepted. To my mind, the most satisfactory classification of methods is based upon the conceptual approach to estimating consumer's surplus. Two distinct approaches can be identified: (1) *income compensation approaches*, which measure value directly in terms of the compensation required to restore some reference level of utility, and (2) *expenditure function approaches*, which use data on expenditures for related goods to infer the value of the non-marketed good in question. The first-mentioned group of methods includes the various versions of contingent valuation (willingness-to-pay surveys, iterative bidding methods, and experimental methods). The latter group includes the travel cost, property value and hedonic methods.

Below, each group of methods is introduced with a brief conceptual development, followed by a concise review of some applications. The theoretical development shows that both groups of methods yield, at the conceptual level, identical measures of value. The choice of valuation method, then, revolves around practical issues: the reliability of obtainable data, and the inaccuracies which may be introduced when necessary analytical assumptions diverge from reality.

4.1 The Income Compensation Approach

Working with the utility function (8) implicit in prices, $U=U[\underline{P}(Q,Y)]$, the income compensation function, $\mu(Q \mid Q^*,Y^\circ)$, represents the least amount of the numeraire the individual would require with Q to achieve the same level of utility as with Q^* and Y° (Hurwicz and Uzawa 1971). A system of partial differential equations may be derived for various reference levels of Q,

$$(13) \quad \frac{\partial\mu(Q|Q^*,Y)}{\partial Q} = \underline{P}[Q,\mu(Q|Q^*,Y^\circ)].$$

For a change from Q'' to Q', the Hicksian equivalent measure of the welfare impact upon the individual—in this case, his WTP to avoid the change—is

$$(14) \quad WTP = \int_{Q'}^{Q''} \underline{P}[Q,\mu(Q|Q',Y^\circ)]dQ.$$

The compensating measure for the same change is

$$(15) \quad WTA = \int_{Q'}^{Q''} \underline{P}[Q,\mu(Q|Q'',Y^\circ)]dQ,$$

that is, both WTP and WTA are defined as areas under (different) Hicksian compensated demand curves for Q. WTP and WTA may be directly observed using any technique which permits estimation of (relevant points on) the respective indifference surfaces passing through

$$(16) \quad U'(Q',Y^\circ) = U'(Q'', Y^\circ - WTP), \text{ for } WTP, \text{ and}$$
$$U''(Q'',Y^\circ) = U''(Q',Y^\circ + WTA), \text{ for } WTA.$$

Since indifference surfaces are not directly observable in the ordinary course of events, estimation methods using this approach necessarily involve the researcher in conscious creation of opportunities to observe (16). Ideally, markets would be created in which individuals would reveal their Lindahl prices for non-marketed, nonrival environmental services. But, that ideal has thus far been unattainable. A quite bewildering variety of methods has been proposed and tested in prototype form, each having some but not all of the desirable characteristics of the elusive Lindahl-price-revealing market.

Bishop and Heberlein (1979) established an experimental market in which they purchased away from licensed hunters the right to hunt Canada geese. Real money and real goods (goose permits) changed hands. Within their experimental population, different strata received different-valued offers, to be accepted or rejected. Thus, WTA was not directly observed. However, a logit analysis permitted estimation of aggregate consumer's surplus for the experimental population.

Bohm (1972) experimentally observed WTP to watch television programs, examining the effect of various alternative incentives on WTP. Again, money changed hands and the goods were delivered.

Groves and Ledyard (1977) have identified, in concept, the general characteristics of "incentive-compatible mechanisms", that is, markets or market-like situations in which each individual's optimal strategy is to reveal his true Lindahl price for nonrival goods. The earlier-circulated Clarke Tax (Tideman and Tullock 1976) is a special case of the more general Groves-Ledyard formulation. In general, these mechanisms will yield true Lindahl prices in situations where real money prices (and the necessary side-taxes) are collected and the goods are delivered.

Unfortunately, most research contexts involving nonrival environmental goods do not permit the goods to be delivered. In these cases, the researcher is confronted with a cruel dilemma: to obtain value data, incentive-compatibility must be sacrificed; alternatively, individual valuations can be "induced" (in effect, the individual is told what his valuation for the good is) and the role of incentive-compatibility in facilitating group agreement about provision of nonrival goods can be explored. To choose the first-mentioned approach exposes the researcher to the possibility of obtaining value data distorted by strategic behaviour, while to choose the latter sacrifices the goal of gathering value data. Both approaches have been pursued and, as it happens, the latter approach has generated some evidence which is supportive of the former approach.

During the 1960's, surveys in which respondents were asked if (and sometimes how much) they would be willing to pay for various (carefully, or loosely, defined) environmental amenities gained currency. Among what mostly amounted to rather unimpressive opinion-pollstering, the work of Davis (1963) stood out like a beacon. During the 1970's, a series of articles (notably Randall, Ives and Eastman 1974; Brookshire, Randall and Stoll 1980; and Schulze, d'Arge and Brookshire 1981) has formally developed both the conceptual bases and the method of applying a direct asking approach consistent with the valuation theory outlined in the earlier sections of the present article. Hypothetical markets are established, in which respondents reveal their valuations of nonrival environmental goods. Such valuations are thus contingent upon the existence of the hypothetical markets described (hence, the general term, contingent valuation, for this kind of research). Randall has preferred to collect value data via an iterative bidding routine, while others (e.g., Hammack and Brown 1974; Gramlich 1977; and Bishop and Heberlein 1979) have used one-shot questions asking maximum *WTP* or, sometimes, minimum *WTA*.

While many contingent valuation methods are not incentive-compatible, there is a body of psychological evidence to the effect that strategic behaviour is likely to be encountered infrequently (Hebert, Shikiar and Perry 1980). Smith (1980), working with induced valuations, and using an experimental approach in which various kinds of incentives (some incentive-compatible, and some not) were compared, found that, while the extent of strategic behaviour was reduced by incentive-compatibility, most experimental subjects did not behave strategically in the absence of incentive-compatibility. This latter finding was especially strong in one-shot experiments in which the direct rewards for strategic behaviour were small (i.e., the conditions under which most willingness-to-pay surveys are conducted). Thus, it appears that contingent valuation methods cannot be dismissed out-of-hand, simply on the charge of susceptibility to strategic bias.

Following a somewhat different approach, based on the Australian tradition of decision analysis under uncertainty (Anderson, Dillon and Hardaker 1977), Sinden (1974, 1978) has estimated indifference curves from which value information for environmental amenities can be generated. While Sinden has developed some methods of internally validating his results, it remains true that perfect devices for observing Lindahl prices for goods which cannot be delivered within the experiment are as yet unavailable.

4.2 The Expenditure Function Approach[10]

An alternative formulation of the empirical valuation problem starts with a utility function of the form (6), that is, $U = U(Q,\underline{Z})$. Maximizing (6) subject to the budget constraint $\Sigma_i\ p_i z_i = \underline{Y}$, generates a set of Marshallian demand functions

(17) $z_i = z_i(\underline{P},Q,\underline{Y})$.

The possibility that Q is an argument in the demand for private goods suggests that market data—i.e., prices and quantities taken—for z_i may be used to reveal the welfare impact of changes in Q. Let us explore this possibility. First, we establish the theoretical equivalence of the expenditure function and income compensation approaches. Then, implementation of the expenditure function approach is considered.

The utility maximization problem yields ordinary demand equations (17). The dual of the same problem minimizes expenditure, $\Sigma_i p_i z_i$ subject to the constraint that utility must be at least equal to some specified level, U. Solution of the dual problem yields the expenditure function. Considering a proposed change from Q'' to Q', where $U'(Q',\underline{Z}) < U''(Q'',\underline{Z})$, the relevant expenditure functions are, respectively

(18) $E'(\underline{P},Q,U')$, and
$E''(\underline{P},Q,U'')$.

The derivative of any expenditure function with respect to any price, p_i, yields a Hicksian compensated demand function for z_i. For the expenditure functions (18), the compensated demand functions are

(19) $Z_i^{h'} = \frac{dE'}{dp_i} = E'_{p_i}\ (\underline{P},Q,U')$, and

$Z_i^{h''} = \frac{dE''}{dp_i} = E''_{p_i}\ (\underline{P},Q,U'')$.

The inverse Hicksian compensated demand curves for Q are given by

(20) $\frac{dE'}{dQ} = -\ E'_q\ (\underline{P},Q,U')$, and

$\frac{dE''}{dQ} = -\ E''_q\ (\underline{P},Q,U'')$.

Thus, the equivalent and compensating measures of the welfare impact of the proposed change are respectively,

(21) $WTP = -\int_{Q'}^{Q''} E'_q\ (\underline{P},Q,U')dQ$, and

(22) $WTA = -\int_{Q'}^{Q''} E''_q\ (\underline{P},Q,U'')dQ$.

10. This section makes considerable use of Freeman's (1979*b*) excellent review.

Equation (21) is, of course, equivalent to equation (14) and equation (22) is equivalent to equation (15). This alternative formulation, however, offers the prospect of empirically estimating *WTP* and *WTA* without directly observing (relevant points on) indifference curves in (Q,Y) space. Instead, under favorable conditions, it should be possible to estimate *WTP* and *WTA* via appropriate manipulation of readily accessible market data for private goods, z_i, expressed in forms suitable, initially, for estimating (17). A number of techniques have been developed which use this approach. Examples include methods which analyze travel costs incurred in recreation site visits, property values, and hedonic prices.

Now, let us consider the theoretical prerequisites for successful application of these methods.

4.2.1. If utility functions are separable . . . : when the utility functions are strongly separable in Q, i.e.,

$$(23) \quad U(Q,\underline{Z}) = U_z(\underline{Z}) + U_q(Q),$$

the demand functions for z_i will all be of the form

$$(24) \quad z_i = z_i(\underline{P},\underline{Y}),$$

that is, completely independent of the level of Q. Certain commonly used functional forms for utility functions (e.g., the Cobb-Douglas and CES forms) have this property, and Freeman (1979) argues that some important classes of environmental services may in fact be separable. While Freeman mentions various unpriced amenities of urban living and the option value of unique natural resources, even better examples may be the so-called non-user values (existence and intrinsic values) which, by definition, arise from enjoyment of natural resource services without the simultaneous use of any complementary inputs.

In the case of strong separability, empirical valuation methods based on the expenditure function approach are without any prospects, and valuation is performed via the income compensation approach or not at all.

4.2.2 When utility functions are nonseparable in $\underline{Z}$ and Q: In many cases, demands for z_i may not be separable from Q, as in equation (17). If such a system of demand equations has been estimated, and it satisfies the Slutsky conditions for integrability, it may be possible to solve for the underlying expenditure function. If so, equations (21) and (22) can be estimated and the value of Q at the margin, or the welfare impact of $Q'' - Q'$, can be estimated. Unfortunately, it is generally necessary to impose additional conditions on the problem in order to solve the system completely (Maler 1974). However, there are two kinds of assumptions—each of which is benign (i.e., consistent with reality) in some particular cases—which permit satisfactory solution: (i) weak complementarity and (ii) perfect substitution.

Weak complementarity occurs if when the quantity of z_i demanded is zero, the marginal utility of Q is zero (Maler 1974). In such cases, when Q increases the demand for z_i shifts out, and the value of $Q'' - Q'$ is approximated by the integral between $z_i(\underline{P},Q'',\underline{Y})$ and $z_i(\underline{P},Q',\underline{Y})$, to the extent that Marshallian consumer's surplus approximates the Hicksian compensating measure (Willig 1976; Randall and Stoll 1980*a*).

The assumption of weak complementarity provides the basis for the travel cost method of valuing recreation amenities (Clawson and Knetsch 1966; Stevens 1966; Burt and Brewer 1971) and the land value method of valuing increments in air quality, view quality, and other residential amenities (Freeman 1974; Brown and Pollakowski 1977). It should be noted, however, that Maler (1977) expresses doubts as to whether the weak complementarity assumption is generally satisfied in the housing market and (by extension) in other markets frequently used to provide the basic data for implementation of these methods.

Perfect substitution: If we can identify some good z_i which is a perfect substitute for Q, while Q and $\underline{Z}^j$ (z_i is not in $\underline{Z}^j$) are independent in the utility and demand functions, the marginal demand price of Q reduces to the price p_i of z_i multiplied by the substitution ratio between z_i and Q (Maler 1974). If the elasticity of substitution between z_i and Q is less than infinite, this method would underestimate the value of Q.

While I know of no competent published application of this concept, its prospects and its limitations seem obvious. Foremost among the latter must be the fact that many kinds of environmental services become policy-relevant simply because they have no good substitutes. Where existing substitutes are markedly expensive, the issue of market clearance must be faced: if the environmental service were unit priced at p_i multiplied by the substitution ratio, would all of Q be demanded? This must be demonstrated, not merely assumed[11].

4.2.3 Hedonic prices: Assume first that z_i and Q are not independent in the utility function. Second, assume that z_i can be defined in terms of a vector of characteristics $\underline{C}_i = (c_{i1},\ldots,c_{in})$. Third, assume that a purchaser, j, of z_i can vary $\underline{C}_i$ by choosing a particular unit, z_{ij}. That is, z_i is not our customary homogeneous good, but a class of goods like "house" or "automobile" such that different members of the class may possess different packages of characteristics. Finally, suppose that one of the characteristics in $\underline{C}_i$ is c_{iq}, the amount of Q enjoyed along with z_i. Therefore, as the consumer selects, for example, a particular house or car, the amount of residential air quality he enjoys along with his house or the amount of safety associated with his car is also determined. For any unit of z_i, say z_{ij}, its price, $p_{z_{ij}}$, is

$$(25) \quad p_{z_{ij}} = p_{z_i}(c_{ij1}, \ldots, c_{ijq}, \ldots, c_{ijn}),$$

where p_{z_i} is the hedonic price function for z_i. If p_{z_i} can be estimated from observation of the prices $p_{z_{ij}}$ and the characteristics $\underline{C}_{ij}$ of different z_{ij}, then the price of any $z_{ik}(k \neq j)$ can be calculated given knowledge of its characteristics. The implicit price of the characteristic, c_{ijq}, for individual j can be found by differentiation:

11. In their analysis of wetland values, Gosselink, Odum and Pope (1974) make this error, among others.

$$(26) \quad p_{c_{ijq}} = \frac{dp_{zi}}{dc_{ijq}}$$

Under favourable conditions, it is possible to use information in the implicit price function to identify the demand for c_{iq}, that is, the demand for Q if Q is enjoyed only as a characteristic of z_i. Assume the individual purchases only one unit of z_i (or, if more than one unit, only identical units) and the utility function is separable in z_i and $\underline{Z}^j$ (z_i is not in $\underline{Z}^j$) so that the marginal rate of substitution between any pair of characteristics of z_i is independent of $\underline{Z}^j$. Then, depending on the form of the hedonic price function (Rosen 1974), it might be possible to estimate the inverse demand curve[12] for Q. In such a case, the integral between the inverse demand curves for Q'' and Q' would approximate (Willig 1976; Randall and Stoll 1980*a*) the appropriate Hicksian compensating measure of value.

In the brief period since publication of Rosen (1974), many attempts to use hedonic price functions in valuing non-marketed goods have been initiated. Applications have included many aspects of residential amenities (e.g., Harrison and Rubenfeld 1978; Abelson 1979) and work-place safety (Thaler and Rosen 1975).

4.3 Comparing Two Kinds of Empirical Methods: Some Comments

The value estimation methods briefly discussed can be classified into two quite different kinds, those conceptually based on income compensation, and those based on the expenditure function. While the formal equivalence of the two conceptual approaches is easily demonstrated, the strengths and weaknesses of the two kinds of empirical valuation methods derived therefrom are almost polar opposites. Expenditure function approaches start with market-generated price and quantity data for some z_i and, by using various more or less benign assumptions and various more or less contrived mathematical manipulations, may eventually arrive at value estimates for associated nonrival environmental goods. With income compensation approaches, the analysis is usually straightforward and consistent with the relevant theory, but the data come from "markets" which are in some ways contrived and seldom incentive-compatible. Thus, it is not plausible to argue on conceptual grounds that either class of technique is perfect, or that either one should be summarily dismissed in favor of the other.

In every case, the above-cited publications reporting applications of these various methods include some kinds of evidence for the success of the undertaking: plausible value estimates are obtained and, for example, estimates may be replicated within or across studies; or it may be demonstrated that (some of) the variation in individual valuations is explained by estimated economic relationships with robust coefficients of the expected sign. The body of such evidence is, by now, quite impressive. There seems little doubt that, in appropriate applications, experimental methods, contingent valuation, travel cost, property value, and hedonic price studies—to name only the more common

12. If the hedonic price function is linear, it is not possible to identify an inverse demand curve for Q. However, the marginal implicit price can be interpreted as marginal *WTP* for small changes in Q. If the hedonic price function is nonlinear, identification of the inverse demand curve for Q depends on model specification and functional form.

methods—can cast considerable light on the economic value of nonrival environmental goods.

However, the "crucial experiment"—that is, one which tests a refutable hypothesis to the effect that estimated values are (are not) equal to the real values—is seldom permitted. In this situation, demonstrations that the results obtained with one kind of method are consistent with those of another provide supportive, but not conclusive, evidence of reliability. Various researchers have attempted such comparisons, with considerable success: Knetsch and Davis (1966) and, more recently, Bishop and Heberlein (1979) report that contingent valuation estimates of *WTP* for recreational amenities are consistent with those obtained with the travel cost method[13]. Brookshire *et al.* (1982) report that contingent valuation estimates of WTP for visual air quality in a residential environment are consistent with hedonic prices estimated from the market in residential land.

Looking to the future, there seems little reason to expect any slow-down in the rapid development (of both concepts and methods) in expenditure function approaches which has occurred in the past two decades. Recent developments in the theory and application of discrete choice models (Small and Rosen 1981) are suggestive of a new wave of progress.

While it seems likely that economists will always be uneasy with contingent valuation and similar approaches, given their lack of incentive-compatibility, one may predict that they will be used increasingly. For recreation benefit evaluation in the context of federal water resource projects, the contingent valuation method has been approved, along with the travel cost method (U.S. Water Resources Council 1979). Further, there are many valuation contexts in which there is currently no available option representing the expenditure function approaches: e.g., where the levels of provision of Q under consideration go beyond the currently observable range; where the relationship between z_i and Q in consumption cannot be satisfactorily rendered in a model capable of solution for consumer's surplus values of Q; and where—the extreme example of the problem just mentioned—Z and Q are strongly separable in the utility and demand functions. In cases like these, income compensation approaches remain feasible. For example, Greenley, Walsh and Young (1981) used contingent valuation methods to estimate the option value of preventing an irreversible degradation of in-stream water quality.

5 Some Complications

To this point, the discussion of valuation concepts and methods has been framed in terms of a simple case: partial equilibrium analysis of the welfare impact of proposed changes in the quantity (or with some straightforward modifications to the analysis, the price) of a single good. However, it is commonly believed that serious difficulties arise in some more complex cases: (a) when the

13. There does appear to be a problem, however, with contingent valuation estimates of *WTA*, in cases where compensation for diminished amenities seems "fair" but is nevertheless not customary (*see* Bishop and Heberlein 1979; and Meyer 1979).

proposed change eventually effects prices or quantities of other goods (i.e., the general equilibrium situation), (b) when several prices or quantities are simultaneously changed, and (c) when the proposed change directly changes the price or quantity of one good and the consumer's income. For completeness, the recent literature pertaining to these cases is now briefly reviewed.

5.1 General Equilibrium

Harberger (1971) notes that, contrary to fairly widespread opinion, there are no conceptual impediments to general equilibrium consumer's surplus analysis. Small and Rosen (1981), in a discrete choice framework and using some theoretical developments from the intervening decade re-iterate that conclusion. Small and Rosen include a re-calculation of Harberger's "excess burden", in Hicksian compensating terms, for discrete choices.

It remains true, however, that the calculation of general equilibrium consumer's surplus in empirical studies is not yet routine.

5.2 Multiproduct Consumer's Surplus

Where a proposed change simultaneously affects quantities or prices of several goods, the analysis discussed in section 3 is complicated a little, but not in unexpected ways. Willig (1979) extends his single-product analysis of price changes (1976) to this case, again providing bounds on the error introduced by using Marshallian consumer's surplus instead of the appropriate Hicksian measure.

At this point, it can be noted that path-dependency[14] is an attribute of Marshallian measures of multiproduct consumer's surplus, but not of the Hicksian measures (Silberberg 1978; Willig 1979; Just, Hueth and Schmitz 1982).

5.3 Changes Which Simultaneously Affect Price (or Quantity) and Income

Mishan (1977) argues that the typical proposal subject to BCA would change the price—or, the quantity—of some good. Such a change would surely affect the individual's welfare, that is, his real income. However, the effect on real income is the *result* of the change in price or quantity of the relevant good.

Silberberg (1978), Hause (1975), and Chipman and Moore (1980) consider a different case: that where the price (or quantity) of some good and income (in terms of some numeraire commodity whose price is fixed) are simultaneously changed. For example, the individual may be offered a chance to pay some amount (less than his gain in consumer's surplus) to get a price decrease. By construction, this opportunity offers the individual a welfare improvement, i.e., a positive net consumer's surplus such that $U(p',Y') > U(p^{\circ},Y^{\circ})$, where in every

14. That is, the notion that the value of consumer's surplus estimates depends on the path of integration across multiple price change.

case the quantity of numeraire is measured at its axis by converting all holdings of Q to the numeraire at the prevailing unit price of Q. The question is, how much is WTP for that opportunity, where $U(p',Y'-WTP) = U(p^\circ,Y^\circ)$? Compensating variation provides an unambiguous answer.

Now, imagine there are two such opportunities, ranked $U(p'',Y'') > U(p',Y') > U(p^\circ,Y^\circ)$. WTP for each of the two opportunities is given by $U(p'',Y''-WTP'') = U(p',Y'-WTP') = U(p^\circ,Y^\circ)$. Since $U(p'',Y'') > U(p',Y')$, one would expect $WTP'' > WTP'$. Indeed, that relationship must hold, if WTP is to satisfy the conditions for a proper, cardinal indicator of indirect utility (Chipman and Moore 1980). However, it has been argued that when the compensating variation is used to measure WTP there is, in general, no assurance that measured $WTP'' > WTP'$ (Hause 1975; Silberberg 1978; Chipman and Moore 1980). In other words, the compensating variation may provide *inconsistent rankings* of two or more alternatives each of which involves simultaneous changes in p and Y.

This phenomenon is quite easily illustrated (figure 4). Starting with situation 1 (i.e., price p° and $Y^\circ = A$), compare situation 2 (i.e., price p' and $Y' = B'$) and situation 3 (i.e., price p'' and $Y'' = C''$). In terms of utility, these situations are ranked $U(1)<U(2)<U(3)$. However, when WTP is estimated as the compensating variation, WTP' (i.e., the interval $B'B$) exceeds WTP'' (i.e., the interval $C''C$). Thus WTP' to obtain the less preferred of two alternative improvements exceeds WTP'' to obtain the preferred alternative. Chipman and Moore (1980) demonstrate that, to obtain consistent rankings using the compensating variation, it is necessary and sufficient that preferences be parallel with respect to the numeraire.

The equivalent variation is not susceptible to this problem. By that measure, $U(p^\circ,A''-EV'') = U(p^\circ,A'-EV') = U(p^\circ,A)$ and EV'' (i.e., the interval $A''A$) exceeds EV' (i.e., the interval $A'A$). In diagrammatic terms, all EV measurements are made using lines of equal slope (in this case, slope $-p^\circ$). Thus, convexity of indifference curves is sufficient for EV to yield consistent rankings.

Some authors—notably, Silberberg (1978) and Chipman and Moore (1980)—have been so impressed by the susceptibility of compensating variation to inconsistent rankings that they suggest there is little reason for its use. On the other hand, Mishan (1977) defends the compensating variation and Small and Rosen (1981) note the issue but choose to work with compensating variation, anyway. What is one to make of all this?

Consistency of rankings is not the only requirement of a welfare change measure. It remains true that the compensating measures correctly identify individual offer prices. Thus, they are entirely consistent with the potential Pareto-improvement criterion (section 3.1) which, I have argued (section 2) is the appropriate criterion for BCA. Further, the compensating value measures precisely identify those changes which could occur via voluntary exchange if the status quo was secured by nonattentuated property rights (section 2.1). These are important attributes of a welfare change measure.

The equivalent measure is incapable of precisely measuring offer and reservation prices and thus incapable of precisely identifying proposals which

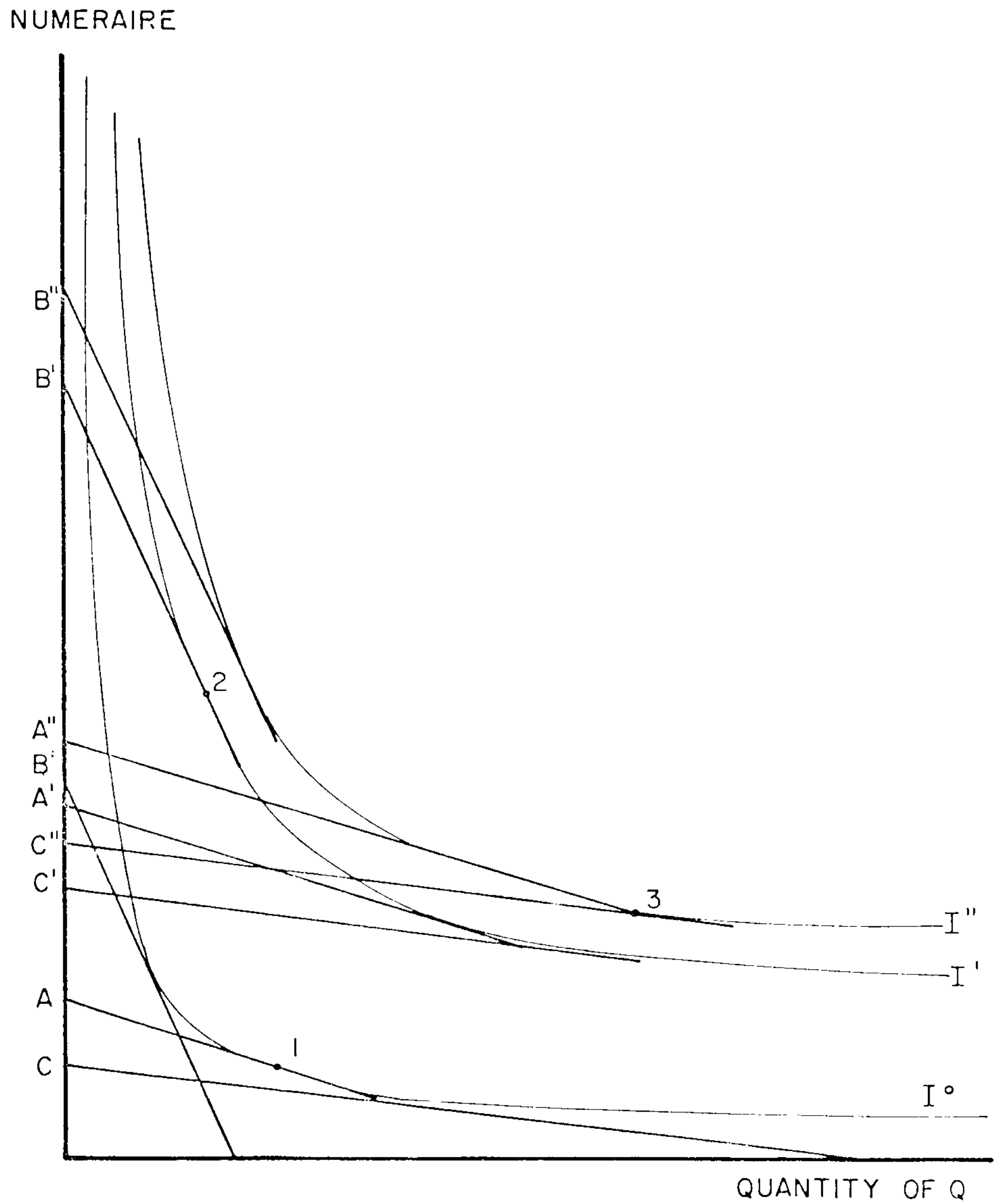

Figure 4. An Initial Situation, 1, and two Alternatives, 2 and 3, which differ in p and Y.

represent potential Pareto-improvements. Starting at situation 1, the individual's offer price would be more than $EV' = A'A$ to get situation 2 but less than $EV'' = A''A$ to get situation 3 (figure 4). Why does the equivalent variation have this undesirable property? Because it ignores information about the prices which would prevail in the alternative situations. But, surely, an individual considering trading to an alternative situation considers not just the amount of numeraire he would have but also its purchasing power in terms of the non-numeraire commodity Q. That is, he considers both the p and Y dimensions of each alternative. The compensating measure considers both the p and Y dimensions, and thus correctly identifies offer and reservation prices.

So, the equivalent measures correctly rank alternatives, but do not correctly identify potential Pareto-improvements. The compensating measures correctly identify potential Pareto-improvements, but may inconsistently rank alternatives differing in both p and Y. In terms of the discussion in section 1, the compensating measure is an accurate filter, while the equivalent measure is an accurate ranking device (i.e., a proper, cardinal indicator of indirect utility), but not vice versa.

Supposing one wanted a test (of benefits versus costs to the individual) capable of both filtering and ranking, how should one proceed?

One option is to use the equivalent measure to rank the alternatives, and the compensating measure to determine offer and reservation prices of preferred alternatives and to identify the opportunities for potential Pareto-improvements.

Alternatively, one could use only the compensating measures. Taking this route, the filtering task is directly handled while the ranking task proceeds pairwise. The compensating measure provides a correct ranking of any two alternatives. Thus, a sequence of pairwise rankings using compensating measures will identify the preferred alternative. Returning to figure 4, *WTP* for a change from 1 to 2 is $B'B$ and *WTP* for a move from 2 to 3 is $C''C'$. Both are positive. However, *WTP* for a change from 1 to 3 is $C''C$, which is positive; but *WTP* for a move from 3 to 2 is $B'B''$, which is negative. By the pairwise compensating test, 3 is unambiguously preferred to 2 for the individual.

BCA involves aggregation of unit-weighted individual benefits and costs across the affected population. A proposed undertaking is a potential Pareto-improvement if the algebraic sum of compensating measures of individual welfare change is positive. In spite of a claim by Boadway (1974) to the contrary, analyses by Randall and Stoll (1980*a*) and Just, Hueth and Schmitz (1982) show that the sum of compensating measures, properly defined, is an accurate filter in the general equilibrium case. With respect to rankings, the general equilibrium case is yet to be analysed. I would conjecture that pairwise ranking on the basis of the sum of compensating measures would provide an accurate ranking of projects, such that the project to which no other is preferred is the "best" project in potential Pareto-improvement terms. If the "sum of equivalent measures" criterion generated the same ranking as an exhaustive pairwise comparison of summed compensating measures, another conjecture, then the sum of equivalent measures would be a proper ranking device. These conjectures are worthy of theoretical analysis in the general equilibrium case. The findings of such analysis would have obvious and important implications for BCA.

6 Concluding Comments

This has been a lengthy article. The BCA problem was conceptualized for non-marginal changes in a complex environment with production indivisibilities and nonrival and nonexclusive commodities. Then, a conceptual model for valuation of goods or amenity-service flows in such a context was developed, for the partial equilibrium case. In that section, the relationships among Hicksian and Marshallian consumer's surplus concepts were explored, and the Hicksian compensating measure was identified as uniquely consistent with the potential Pareto-improvement criterion and with the prices relevant to voluntary exchange, i.e., buyer's best offer and seller's reservation price. The discussion in these sections was quite exhaustive.

Then in section 4, the conceptual bases for empirical value estimation were developed and, in section 5, some complications which arise in the application of consumer's surplus concepts to multiple changes and general equilibrium conditions were explored. In these sections the discussion is more suggestive than exhaustive. It is the author's hope that one can find therein a comment or two to get one's thinking started, and a useful reference or two to follow up, for many of the conceptual or empirical problems which may arise in the use of economic surplus values in BCA.

One final comment is essential. Rigorous analysis of economic surplus value concepts applies specifically to impacts upon individual welfare. Strictly speaking, there is no justification for aggregating welfare gains and losses across individuals which does not rely on explicit ethical judgements. The recommendation of this paper—that benefit cost analysis proceed by interpersonal aggregation of individual gains and losses measured with Hicksian compensating concepts of welfare change—combines a preference for the potential Pareto-improvement criterion with an interpersonal aggregation rule that (signed) gains and losses "to whomsoever they accrue" be summed. This aggregation rule is fundamental to benefit cost analysis: a quarrel with that rule is a quarrel with the benefit cost criterion *per se,* not with this attempt to provide a framework for implementation of the criterion.

I recognize that a benefit cost decision rule, in the absence of additional rules protecting individuals from loss, is not Pareto-safe. In the context of, e.g., Fishkin (1979), such a decision rule is tyrannical. But, that is the subject for another treatise.

References

ABELSON, P. W. (1979), "Property prices and the value of amenities", *Journal of Environmental Economics and Management* 6, 11–28.

ANDERSON, J. R., DILLON, J. L. and HARDAKER, J. B. (1977), *Agricultural Decision Analysis*, Iowa State University Press, Ames.

AUSTRALIAN WATER RESOURCES COUNCIL (1978), *A National Approach to Water Resources Management*, Canberra.

BISHOP, R. C. and HEBERLEIN, T. (1979), "Measuring values of extra-market goods: are indirect measures biased?", *American Journal of Agricultural Economics* 61, 926–30.

BOADWAY, R. W. (1974), "The welfare foundations of cost-benefit analysis", *Economic Journal* 84, 926–39.

BOHM, P. (1972), "Estimating demands for public goods: an experiment", *European Economic Review* 3, 111–130.

BRADFORD, D. F. (1970), "Benefit-cost analysis and the demand for public goods", *Kyklos* 23, 775–91.

BROOKSHIRE, D. S., RANDALL, A. and STOLL, J. R. (1980), "Valuing increments and decrements in natural resource service flows", *American Journal of Agricultural Economics* 62, 478–88.

BROOKSHIRE, D. S., THAYER, M. A., SCHULZE, W. D. and D'ARGE, R. C. (1982), "Valuing public goods: a comparison of survey and hedonic approaches", *American Economic Review* 72, 165–77.

BROWN G. M. and POLLAKOWSKI, H. O. (1977), "Economic valuation of shoreline", *Review of Economics and Statistics* 69, 273–8.

BURT, O. R. and BREWER, D. (1971), "Evaluation of net social benefits from outdoor recreation", *Econometrica* 39, 813–27.

CLARK, S. D. (1980), Water law and scope for change, presented to Workshop on the Economics of Water Allocation, Australian Rural Adjustment Unit, Armidale, November 11–12.

CLAWSON, M. and KNETSCH, J. L. (1966), *Economics of Outdoor Recreation*, John Hopkins University Press, Baltimore.

CHIPMAN, J. S. and MOORE, J. C. (1980), "Compensating variation, consumer's surplus, and welfare", *American Economic Review* 70, 933–49.

CURRIE, J. M., MURPHY, J. A. and SCHMITZ, A. (1971), "The concept of economic surplus and its use in economic analysis", *Economic Journal* 81, 741–99.

DAVIS, R. K. (1963), "Recreation planning as an economic problem", *Natural Resources Journal* 3, 239–49.

FISHKIN, J. S. (1979), *Tyranny and Legitmacy: A Critique of Political Theories*, Johns Hopkins University Press, Baltimore.

FREEMAN, A. M. (1979*a*), *The Benefits of Environmental Improvement: Theory and Practice*, Johns Hopkins University Press, Baltimore.

FREEMAN, A. M. (1979*b*), "Approaches to measuring public goods demands", *American Journal of Agricultural Economics* 61, 915–20.

FREEMAN, A. M. (1974), "On estimating air pollution control benefits from land value studies", *Journal of Environmental Economics and Management* 1, 74–83.

GOSSELINK, J. E., ODUM, E. P. and POPE, R. M. (1974), *The Value of the Tidal Marsh*, LSU-SG-74-03, Louisiana State University, Center for Wetland Resources, Baton Rouge.

GRAMLICH, R. W. (1977), "The demand for clean water: the case of the Charles River", *National Tax Journal* 30, 183–94.

GREENLEY, D. A., WALSH, R. G. and YOUNG, R. A. (1981), "Option value: empirical evidence from a case study of recreation and water quality", *Quarterly Journal of Economics* 95, 657–73.

GROVES, T. E. and LEDYARD, J. (1977), "Optimal allocation of public goods: a solution to the free-rider problem", *Econometrica* 45, 783–809.

HAMMACK, J. and BROWN, G. M. (1974), *Waterfowl and Wetlands: Toward Bioeconomic Analysis*, Johns Hopkins University Press, Baltimore.

HARBERGER, A. C. (1971), "Three basic postulates for applied welfare economics: an interpretive essay", *Journal of Economic Literature* 9, 785–97.

HARRISON, D. and RUBINFELD, D. (1978), "Hedonic housing prices and the demand for clean air", *Journal of Environmental Economics and Management* 5, 81–102.

HAUSE, J. C. (1975), "The theory of welfare cost measurement", *Journal of Political Economy* 83, 1145–82.

HEBERT, J., SHIKIAR, R. and PERRY, R. (1980), "Valuing environment via bidding games: a psychological perspective", in Adams, R. C. *et al.* (eds.), *The Visual Aesthetic Impact of Alternative Closed Cycle Cooling Systems*, NUREG/CR-0989, Battelle Pacific Northwest Laboratory, Richland, WA.

HICKS, J. R. (1943), "The four consumer's surpluses", *Review of Economic Studies* 11, 31–41.

HOEHN, J. P. and RANDALL, A. (1982). Aggregation and disaggregation of programme benefits in a complex policy environment, Staff Paper 140, Department of Agricultural Economics, University of Kentucky, Lexington.

HURWICZ, L. and UZAWA, H. (1971), "On the integrability of demand functions", in Chipman, J. S. *et al.* (eds.), *Preferences, Utility and Demand,* Harcourt Brace Jovanovich, New York.

JUST, R. E., HUETH, D. L. and SCHMITZ, A. (1982), *Applied Welfare Economics and Public Policy,* Prentice-Hall, Englewood Cliffs.

KNETSCH, J. L. and DAVIS, R. K. (1966), "Comparisons of methods for recreation evaluation", in Kneese, A. V. and Smith, S. C., (eds.), *Water Research,* Johns Hopkins University Press, Baltimore.

MALER, K-G. (1977), "A note on the use of property values in estimating marginal willingness to pay for environmental quality", *Journal of Environmental Economics and Management* 4, 355—69.

MALER, K-G. (1974), *Environmental Economics: A Theoretical Inquiry,* Johns Hopkins University Press, Baltimore.

MEYER, P. A. (1979), "Publicly vested values for fish and wildlife: criteria for economic welfare and interface with the law", *Land Economics* 55, 223–35.

MINISTRY OF WATER RESOURCES AND WATER SUPPLY, VICTORIA (1977), *Guidelines for Water Resources Planning,* Melbourne.

MISHAN, E. J. (1977), "The plain truth about consumer's surplus", *Zeitschrift fur Nationalokonomie* 37, 1–24.

MISHAN, E. J. (1971; 2nd ed., 1976), *Cost Benefit Analysis,* George Allen and Unwin, London.

MISHAN, E. J. (1968), "What is producer's surplus?" *American Economic Review* 58, 1269–82.

NEWMAN, K. (1979), *Commonwealth Water Policy,* House of Representatives, Canberra, March 18.

NEIHANS, J. (1971), "Money and barter in general equilibrium with transactions costs", *American Economic Review* 61, 773–83.

POPPER, K. (1957), "Philosophy of science: a personal report", in C. Mace (ed.), *British Philosophy in Mid-Century,* Allen and Unwin, London.

RANDALL, A., IVES, B. C. and EASTMAN, C. (1974), "Bidding games for valuation of aesthetic environmental improvements", *Journal of Environmental Economics and Management* 1, 132–49.

RANDALL, A. and STOLL, J. R. (1980*a*), "Consumer's surplus in commodity space", *American Economic Review* 70, 449–55.

RANDALL, A. and STOLL, J. R. (1980*b*), *Economic Surplus and Benefit Cost Analysis,* Agricultural Economics Research Report 35, University of Kentucky, Lexington.

ROSEN, S. (1974), "Hedonic prices and implicit markets: product differentiation in pure competition", *Journal of Political Economy* 82, 34–55.

SCHULZE, W. D., D'ARGE, R. C. and BROOKSHIRE, D. S. (1981), "Valuing environmental commodities: some recent experiments", *Land Economics* 57, 151–72.

SILBERBERG, E. (1978), *The Structure of Economics: A Mathematical Analysis,* McGraw-Hill, New York.

SINDEN, J. A. (1980), "Pangloss, Pandora and Pareto for the aspiring benefit cost analyst", *Review of Marketing and Agricultural Economics* 48, 99–110.

SINDEN, J. A. (1978), "Estimation of consumer's surplus values for land policies", *Australian Journal of Agricultural Economics* 22, 175–93.

SINDEN, J. A. (1974), "A utility approach to the valuation of aesthetic and recreational experiences", *American Journal of Agricultural Economics* 56, 61–72.

SINDEN, J. A. and WORRELL, A. C. (1979), *Unpriced Values: Decisions Without Market Prices,* Wiley-Interscience, New York.

SMALL, K. A. and ROSEN, H. S. (1981), "Applied welfare economics with discrete choice models", *Econometrica* 49, 105–30.

SMITH, V. L. (1980), "Experiments with a decentralized mechanism for public good decisions", *American Economic Review* 70, 584–99.

STEVENS, J. B. (1966), "Recreation benefits from water pollution control". *Water Resources Research* 2, 167–82.

THALER, R. H. and ROSEN, S. (1975), "The value of saving a life: a market estimate", in Terleckyj, N., (ed.), *National Bureau of Economic Research Studies in Income and Wealth,* 40.

TIDEMAN, T. N. and TULLOCK, G. (1976), "A new and superior process for making public choices", *Journal of Political Economy* 84, 1145–59.

U.S. WATER RESOURCES COUNCIL (1979), "Procedures for evaluation of national economic development benefits and costs in water resources planning (level C): final rule", *Federal Register* 44 (242, Part IX), December 14.

U.S. WATER RESOURCES COUNCIL (1973), "Water and related land resources: establishment of principles and standards for planning", *Federal Register* 38 (174, Part III), September 10.

WILLIG, R. D. (1979), "Consumer's surplus without apology: reply", *American Economic Review* 69, 469–74.

WILLIG, R. D. (1976), "Consumer's surplus without apology", *American Economic Review* 66, 587–97.