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Abstract

In this study, we develop and estimate a censored LA/AIDS model using household-level purchase data. In addition to imposing non-negativitity constraints, we account for the endogeneity of unit value. We address the non-negativity issue using an Amemiya-Tobin approach, which imposes the adding-up condition on both observed and latent shares. We address the endogeneity of unit value by estimating share equations and unit value equations simultaneously. Given the need to evaluate high-order probability integrals, we use a simulated probability method in the model estimation. This model is applied to estimate and analyze a demand system featuring six fish and three meat commodities, using Norwegian household data.

JEL Classification: C34, D12.

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I. Introduction

The estimation of demand systems using household-level data is more challenging than the conventional time-series (market-level) data approach, for two reasons. First, the use of household data often produces a significant proportion of zero-purchase observations. If these are not appropriately accounted for, biased estimates can result. Second, household data are usually highly disaggregated across products, and it is next to impossible to estimate a completely disaggregated system because of the large number of products. Therefore, product aggregation is inevitable.

Product aggregation raises the issue of price unobservability.[1] Goods are purchased in elementary products and each product has its unique price. However, after these elementary products are aggregated into commodity categories, the price of the aggregated commodity is not defined and is unobservable. Researchers use the unit value of the aggregated commodity as its price, which is derived by dividing expenditures by the aggregated quantity. This derived unit value of the commodity varies not only with its genuine price, but also with its quality. Quality is determined by the composition of the quantities of the elementary products chosen by households.

The zero-purchase issue has been the subject of research in the econometrics literature for the past two decades (e.g., Wales and Woodland (1983), Lee and Pitt (1986), and Golan, Perloff, and Shen (2001)). However, within the demand systems framework, control of endogenously determined quality components in the specification of commodity unit values has not been adequately investigated. A recent study by

Crawford, Laisney, and Preston (2003) addressed this issue, but not for a censored system. In this study we estimate a demand system model in the presence of significant censoring, while accounting for unit-value endogeneity. The model is applied to a set of Norwegian household data on meat and fish purchases.

The remaining sections of this paper are organized as follows. First, we discuss the two approaches frequently used to estimate censored demand system applications. Next, using consumer utility maximization theory, we describe the endogeneity of unit values. Then we present the econometric model used in the analysis of Norwegian household fish and meat purchase. Finally, we present the empirical results.

II. Estimation Methods for Censored Demand Systems

One method of estimating a censored demand system is the Kuhn-Tucker approach proposed by Wales and Woodland (1983), and its associated dual suggested by Lee and Pitt (1986). The Kuhn-Tucker primal approach derives demand (share) equations from the maximization of an explicitly specified random utility function along with a set of non-negativity and budget constraints. Lee and Pitt's (1986) dual methodology derives the demand (share) equations using Roy's Identity from a random indirect utility function, and assumes that consumers compare virtual (reservation) prices to actual market prices in making their purchase decisions.

The main issue that must be addressed when using the Kuhn-Tucker approach and its dual is the derivation of an estimable demand system. For some system specifications, it is not an easy task to specify a direct or indirect utility function that allows for system estimation. Furthermore, one must address the problem of incoherency, which is

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[1] Other issues associated with aggregation, such as utility properties, are not addressed in this paper.

characterized by the sum of purchase regime probabilities not equaling one. As noted by van Soest and Kapteyn (1993), van Soest and Kooreman (1990), and Ransom (1987), an incoherent system will lead to inconsistent parameter estimates. No proper solution to the incoherency problem has yet been developed in the literature.

Another estimation method is the Amemiya-Tobin approach proposed by Amemiya (1974) and operationalized by Wales and Woodland (1983). In this approach, unlike in the Kuhn-Tucker method, demand (share) equations are derived from a non-stochastic utility function and latent expenditures (shares) are hypothesized to differ from observed expenditures because of a number of factors, including errors in maximization by the consumer, errors in measurement of the observed shares, or random disturbances that influence the consumer's decisions (Wales and Woodland, 1983). To account for these differences, error terms are added to the deterministic shares. Given assumed normality of equation error terms, observed expenditures (shares) are normally distributed about the deterministic expenditures (shares). Non-negativity constraints are incorporated via truncation of the above equation error terms, as in the censored multivariate Tobit model proposed by Amemiya (1974). Unlike in the Kuhn-Tucker approach, incoherency is not a problem in this approach, given the structure of the mapping from latent to observed expenditure shares.

However, when using the Amemiya-Tobin approach, one has to deal with the specific adding-up issue embedded in the system of censored share equations. The adding-up condition is required not only in the latent shares, but also in the observed shares. The non-purchase of some commodities increases the amount households can reallocate to other purchased commodities. Such reallocation complicates the mapping from latent to observed shares, but this issue must be addressed to avoid model misspecification. Under the Lee and Pitt (1986) specification, such reallocation is fulfilled by virtual prices.[2]

This adding-up issue was not addressed by Golan, Perloff and Shen (2001). Using the Amemiya-Tobin framework, they developed a maximum entropy (non-parametric) approach to estimate a censored demand system. In their model, the computational burden of evaluating the multivariate probability integral using the maximum likelihood estimator was avoided. However, the issue of adding-up was not appropriately addressed and will not be easily addressed given the structure of their model.

In this paper we extend the Amemiya-Tobin approach to demand system estimation via the use of a linearized Almost-Ideal Demand System (AIDS) specification. We selected the AIDS model because it has been widely used in the estimation of non-censored demand systems, and it is relatively easy to estimate within the Amemiya-Tobin framework. It would be difficult, if not impossible, to estimate such a model under the Kuhn-Tucker and Lee-Pitt approaches because of the functional form of the underlying utility functions associated with the AIDS specification. In addition to accounting for non-negativity, and the imposition of the adding-up restriction on both latent and observed shares, the proposed LA/AIDS framework also involves endogenization of unit

[2] This reallocation by virtual prices is conducted as follows. Suppose a household purchases all the commodities in the system. Expenditure shares are determined by the marketing prices of *all* the commodities in the system and they sum to one. Suppose the household changes its demand later by not purchasing some of the commodities. Under the new situation, if we still use all the marketing prices to determine the purchased commodities, the sum of the shares of the purchased commodities will be less than one. However, for this situation under the Lee-Pitt model, the virtual prices for the non-purchased commodities are derived, and are used to replace their corresponding marketing prices in determining the demand for the purchased commodities. Since these virtual prices are smaller than the associated marketing prices, shares for these purchased commodities then increase and sum to one.

values.

III. Quality Issue in Composite Commodity Demand

Goods are purchased in elementary (basic) products and each product is homogeneous and has its own unique price. Consider the conventional utility (U) maximization problem faced by a household, expressed in terms of the elementary goods (q_r):

$$(1) \qquad Max\, U(q_1, q_2, \cdots, q_R) \qquad s.t. \sum_{r=1}^{R} p_r q_r = Y$$

where p_r is the price of the r^{th} elementary good, R is the number of goods, and Y is household income. To simplify the problem, we assume that all q's are measured in the same unit.

The large number of elementary goods available in the market prohibits the estimation of a system of all of them. Instead, these elementary products are typically aggregated into certain commodity categories, at the stage of data survey and/or at the later stage of economic analysis. How they're aggregated depends mainly on the focus of the specific problem of interest, but the theoretical legitimacy for aggregation rests on the separability requirement for the underlining consumer's preferences. Suppose the total R elementary goods in the household utility function are allowed, under the weak separability condition, to aggregate into M ($M < R$) commodity categories, and each elementary good belongs to only one aggregated commodity. Equation (1) can be written as follows in terms of the M composite commodities (Q_j):

$$(2) \qquad Max\, U(Q_1, Q_2, \cdots, Q_M) \qquad s.t. \sum_{j=1}^{M} V_j Q_j = Y,$$

where $Q_j = \sum_{i \in j} q_i$, $V_j = \frac{E_j}{Q_j}$, i.e., the unit value of the composite commodity Q_j, and

$E_j = \sum_{i \in j} p_i q_i$, i.e., the expenditure devoted to Commodity Q_j. All the variables in (2) are observable, and the solution of (2) gives:

(3) $Q_j = Q_j(V_1, V_2 \cdots, V_M, Y)$, $j = 1, 2, \ldots, M$.

Equation (3) cannot be estimated directly, because unit values are endogenous. Following Theil and Deaton, we define V using Hicks's composite commodity theorem, as follows.

If we assume that the prices of all elementary goods vary proportionally within the composite commodity j, then the following holds:

(4) $p_i = P_j \cdot p_i^*, \quad i \in j$,

where p_i^* is a quality indicator for commodity q_i (Thiel (1955) and Deaton (1988)), which is determined by its attributes, and P_j is the price index for j. Both P_j and p_i^* are unobservable and exogenous to consumers.

Expenditures (E_j) and the resulting unit values, V_j, of the j[th] composite commodity may be related to expenditures on the associated elementary goods via the following:

(5) $E_j = \sum_{i \in j} p_i q_i = \sum_{i \in j} P_j \cdot p_i^* q_i = P_j \sum_{i \in j} p_i^* q_i$

A quantity-weighted sum of elementary goods base prices can be used as a measure of average quality (R_j) of a particular composite commodity j:

(6) $\psi_j = \sum_{i \in j} \left(\frac{q_i}{Q_j} \right) \cdot p_i^* = \frac{\sum_{i \in j} p_i^* q_i}{Q_j}$

After combining (5) and (6), the relationship between unit value and quality is:

(7) $\ln V_j = \ln P_j + \ln \psi_j$,

where it is assumed that the first component of (7) is constant within the j^{th} composite commodity via (4), and the second term depends upon the endogenously determined quality of the j^{th} composite commodity. Consequently, these unit values are not exogenous to consumers.

Because of the endogeneity of unit values, equations (3) and (7) need to be estimated simultaneously. Since both P_j and R_j in (7) are not observable, proxies are required: regional/seasonal variables serve as proxies for P_j and household characteristics serve as proxies for household preferences for R_j.

When using household purchase data to estimate a demand system, one needs to address both the endogeneity issue and the selectivity bias problem arising from the fact that some households make zero purchases of some commodities. Furthermore, there is the problem of missing unit values when zero purchases occur. Numerous studies on censored demand system estimation have been conducted in the past two decades, with Wales and Woodland (1983) and Lee and Pitt (1986) as pioneers. However, with only one exception, none of these studies have addressed the endogeneity issue of unit values; instead they have simply used the sample mean of unit values as the missing prices for nonpurchase occasions. The single exception was a firm-level energy use study by Bousquet and Ivaldi (1998), who estimated price equations together with a demand system. However, their rationale for this approach was that they were addressing the missing data problem, which is not the same as addressing the quality issue.

IV. The Censored LA/AIDS Demand Systems with Endogenous Unit-Values

Following Deaton and Muellbauer (1980) and Pollack and Wales (1992), we can derive an AIDS model based on the latent shares for $M + 1$ commodities as follows:

(8) $$S^* = U^\varepsilon + \varepsilon,$$

where S^* is a $(M + 1)$ column vector of latent expenditure shares,

$U^\varepsilon = A + \gamma \ln V + \eta \ln Y$, $A = \alpha + \beta X$, $Y = \frac{y^*}{P^*}$, V is an $[M + 1]$ column vector of commodity unit values, X is a $[L \text{ x } 1]$ vector of demographic characteristics, y^* is total expenditures, ε is an $[(M + 1) \text{ x } 1]$ vector of equation error terms, and P^* is a translog price index. Equation parameters are α $[(M+1) \text{ x } 1]$, η $[(M+1) \text{ x } 1]$, and β $[(M+1) \text{ x } L]$, and γ is an $[(M+1) \text{ x } (M+1)]$ symmetric matrix. Equation (8) can be viewed as the empirical version of (3) expressed in the expenditure shares.

Given the complexity of the problem, instead of using the non-linear AIDS specification we use the linear approximate specification (LA/AIDS), where a linear approximation to $\ln P^*$ is used as the expenditure deflator. In their Monte Carlo analysis of the use of alternative indices, Buse and Chan (2000) recommend the use of a Tornqvist index to approximate $\text{Ln}P^*$ when prices exhibit a mixture of positive and negative collinearity, as is the case in our empirical application.

Therefore, following Buse and Chan (2000) and Moschini (1995), the following invariant Tornqvist price index is used as a total expenditure deflator:

(9) $$\ln P^* \approx \ln P_i^T = \frac{1}{2}(S_i + S_0)'(\ln P_i - \ln P_0),$$

where the subscript "0" represents some base observation (in our application it is the sample mean value) and "i" represents households.

As we noted in the previous section, the price vectors P_i and P_0 in (9) are not observed. Instead, the observed unit values are used as substitutes for the prices in (9), and the price index is still treated as exogenous, as are the shares in (9). However, the unit value in (8) is endogenized and defined as:

(10) $\ln V = \delta Z + e$,

where Z is a [H x 1] vector of variables that influence the household's choice on the commodity's unit values (quality), δ is an [(M+1) x H] vector of parameters, and e is a vector of the error terms. Equation (10) can be viewed as the empirical version of (7).

Given the budget constraint, we know the latent shares must sum to one. This can be attained through parameter restrictions. Theoretical constraints such as homogeneity and symmetry can also be imposed on (8). However, there are no non-negativity constraints imposed on these latent shares, and there is nothing in the formulation to ensure that the elements of S^* lie between 0 and 1.

The adding-up restriction implies that the joint density function of ε is singular. Consequently, one of the $[M + 1]$ latent share equations must be dropped during estimation. In dropping any equation from the estimation, we assume that the remaining M share equations' error terms, ε in (8), are distributed multivariate normal with a joint probability density function (*PDF*).

The mapping of the vector of latent shares, S^*, to observed shares, S, must take into account that the elements of S lie between 0 and 1, and sum to unity for each observation. The following mapping rule, from Wales and Woodland (1983), imposes these two characteristics:

(11) $$S_i = \begin{cases} \dfrac{S_i^*}{\sum_{j\in\Psi} S_j^*}, & \text{if} \quad S_i^* > 0, \\ 0, & \text{if} \quad S_i^* \le 0, \end{cases} \qquad (i = 1, 2, \cdots, M+1\ ),$$

where ψ is a set of all positive shares' subscripts. As Wales and Woodland point out, the way (11) maps S^* to S both is simple and has the property that the resulting density function is independent of whatever set of S^*'s is used in its derivation. If any latent share happens to be negative, (11) will force the associated observed share to be zero and re-value all the positive shares.

Assuming that at least one commodity is purchased, we can partition any observed purchase patterns into three general purchase regimes: (i) at least one commodity is purchased, but the total number of purchased commodities is less than M, (ii) M commodities are purchased, and (iii) all $M+1$ commodities are purchased. For each regime we can develop regime-specific likelihood functions that can be used to obtain system parameter estimates. Since a particular household is associated with only one purchase regime, the likelihood function appropriate for its purchase pattern determines the contribution this household makes to the overall sample likelihood function value.

Regime I Likelihood Function: Some Commodities Not Purchased

For households where k commodities are purchased and $M > k \geq 1$, we can rearrange the ordering of the $M + 1$ commodities so that the first k are purchased. We drop the last share equation. In this case equation (8) can be written as $S^* = U^{\omega} + \omega$ with

(12) $\quad U^{\omega} = A + \gamma_1 \ln V_1 + \gamma_0 (\delta_0 Z) + \eta \ln Y$,

where γ_1 [k x 1] is associated with the positive purchases, γ_0 [(M+1-k) x 1] is associated with the zero purchases, V_1 is a vector of the observed unit values, and $(\delta_0 Z)$ is a vector of the predicted unit values for the non-purchased commodities, and $\omega = \varepsilon + \delta_0 e_0$

represents the new error terms, where e_0 is the error term of unobserved unit values. We assume $\varepsilon \sim MN\,(0, \Sigma_{\varepsilon\varepsilon})$, where $\Sigma_{\varepsilon\varepsilon}$ is an [M x M] error covariance matrix and is defined as:

$$(13) \qquad \Sigma_{\varepsilon\varepsilon} = \begin{bmatrix} \Sigma_{\varepsilon_1\varepsilon_1} & \Sigma_{\varepsilon_1\varepsilon_0} \\ \Sigma'_{\varepsilon_1\varepsilon_0} & \Sigma_{\varepsilon_0\varepsilon_0} \end{bmatrix},$$

where $\Sigma_{\varepsilon_1\varepsilon_1}$ is a k x k error term covariance submatrix associated with the purchased commodities, $\Sigma_{\varepsilon_0\varepsilon_0}$ is a (M-k) x (M-k) covariance submatrix associated with the non-purchased commodities, and $\Sigma_{\varepsilon_1\varepsilon_0}$ is a (M-k) x k submatrix of covariance across purchased and non-purchased commodities. Considering the unit value equations, we further assume that the two sets of errors in (8) and (10) are jointly distributed normal with zero mean vector and variance covariance matrix as:

$$(14) \qquad \Sigma_{\varepsilon} = \begin{bmatrix} \Sigma_{\varepsilon\varepsilon} & \Sigma_{\varepsilon e} \\ \Sigma'_{\varepsilon e} & \Sigma_{ee} \end{bmatrix},$$

where $\Sigma_{\varepsilon e}$ is the covariance across share and unit value equations and is defined as:

$$(15) \qquad \Sigma_{\varepsilon e} = \begin{bmatrix} \Sigma_{\varepsilon_1 e_1} & \Sigma_{\varepsilon_0 e_1} \\ \Sigma_{\varepsilon_1 e_0} & \Sigma_{\varepsilon_0 e_0} \end{bmatrix},$$

and Σ_{ee} is the variance covariance matrix of the error terms of unit value equations and is defined as:

$$(16) \qquad \Sigma_{ee} = \begin{bmatrix} \Sigma_{e_1 e_1} & \Sigma_{e_1 e_0} \\ \Sigma'_{e_1 e_0} & \Sigma_{e_0 e_0} \end{bmatrix}.$$

Then the joint distribution of ω and e is $MN\,(0, \Sigma)$, where Σ is an [(2M+1) x (2M+1)] error covariance matrix:

$$(17) \qquad \Sigma = \begin{bmatrix} \Sigma_{\omega\omega} & \Sigma_{\omega e} \\ \Sigma'_{\omega e} & \Sigma_{ee} \end{bmatrix},$$

where $\Sigma_{\omega e} = (\Sigma_{\omega e_1}, \Sigma_{\omega e_0})$ with $\Sigma_{\omega e_1} = \Sigma_{\varepsilon e_1} + \delta_0 \Sigma_{e_0 e_1}$, $\Sigma_{\omega e_0} = \Sigma_{\varepsilon e_0} + \delta_0 \Sigma_{e_0 e_0}$,

$\Sigma_{\varepsilon e_1} = (\Sigma_{\varepsilon_1 e_1}, \Sigma_{\varepsilon_0 e_1})$, $\Sigma_{\varepsilon e_0} = (\Sigma_{\varepsilon_1 e_0}, \Sigma_{\varepsilon_0 e_0})$, and

(18) $$\Sigma_{\omega\omega} = \Sigma_{\varepsilon\varepsilon} + \delta_0 \Sigma_{e_0 \varepsilon} + \Sigma_{\varepsilon e_0} \delta_0^{'} + \delta_0 \Sigma_{e_0 e_0} \delta_0^{'} = \begin{bmatrix} \Sigma_{11} & \Sigma_{10} \\ \Sigma_{10}^{'} & \Sigma_{00} \end{bmatrix},$$

where Σ_{11} is a *k* x *k* error term covariance submatrix associated with the purchased commodities, Σ_{00} is a (*M-k*) x (*M-k*) submatrix associated with the non-purchased commodities, and Σ_{10} is a (*M-k*) x *k* submatrix of covariance across purchased and non-purchased commodities.

Given (13)-(18), the likelihood of a household's being in a purchase regime where the first *k* commodities are positive and the remaining are zero can be represented via the following: [3]

(19) $$L(S_1, S_2, \cdots, S_k > 0; S_{k+1} = S_{k+2} \cdots = S_M = 0)$$
$$= \varphi(e_{1|S}, \Sigma_{e_{1|S}}) \int_{S_1}^{+\infty} \int_{1-\frac{S_1^*}{S_1}}^{0} \int_{1-\frac{S_1^*}{S_1}-S_{k+1}^*}^{0} \cdots \int_{1-\frac{S_1^*}{S_1}-S_{k+1}^*-\cdots-S_{M-1}^*}^{0} \phi(S_1^*, S_2^*, \cdots, S_M^*; U^{\omega}, \Sigma_{\omega\omega}) dS_M^* \cdots dS_{k+1}^* dS_1^*,$$

where $\phi(\cdot)$ is the *M*-dimension *PDF* of latent shares, and φ(.) is the *k*-dimension *PDF* of the errors of the observed unit values given the *k* positive shares with the mean vector of

(20) $$e_{1|S} = \ln V_1 - \delta_1 X - \Sigma'_{\omega e_1} \Sigma_{\omega\omega}^{-1} S^1,$$

and the error covariance matrix,

(21) $$\Sigma_{e_{1|S}} = \Sigma_{e_1 e_1} - \Sigma'_{\omega e_1} \Sigma_{\omega\omega}^{-1} \Sigma_{\omega e_1},$$

where δ_1 is the vector of parameters associated with the [*k* x 1] observed unit values defined in (10), and $S^1 = (S_1, S_2, \cdots, S_K)'$, the [k x 1] vector of the positive shares.

Equation (19) is based on the mapping defined by (11). The integral in (19) is [M–k+1]-fold, i.e., the number of non-purchased commodities plus one. As noted above, if the demand system encompasses a large number of commodities and there are a large number of non-purchased commodities for a particular household, the conventional method for numerically evaluating (19) is impractical. However, (19) can be evaluated using a number of alternative simulation procedures. For the present analysis we use the smooth recursive conditioning simulator (GHK) suggested by Geweke (1991), Hajivasiliou, McFadden and Ruud (1997), and Keane(1994). The GHK procedure requires that (19) be a rectangular standard multivariate normal probability. The current representation of (19) does not satisfy this requirement. However, (19) can be stated in a form that can be simulated using the GHK algorithm, as follows:

$$(22)\qquad L(S_1, S_2, \cdots, S_k > 0; S_{k+1} = S_{k+2} \cdots = S_{M+1} = 0) = B \cdot \varphi(e_{1|S}, \Sigma_{e_{1|S}}) \cdot \Phi_{M-k+1}(b; R_C),$$

where $\Phi_{M-k+1}(b; R_C)$ is a [M-k+1] dimensional multivariate standard normal *cdf* evaluated at vector b with correlation coefficient matrix R_C. Note that $\Phi_{M-k+1}(b; R_C)$ is an [M-k+1]-fold probability integral. The detailed transformation of (19) to (22), with the definitions of the matrices b, R_C, and B, is presented in the appendix.

Regime II Likelihood Function: One Commodity Not Purchased

In Regime II, the number of commodities actually purchased, k, equals M. Under this special case, equation (19) can be simplified as:

$$(23)\qquad L(S_1, S_2, \cdots, S_M > 0; S_{M+1} = 0) = B_1 \cdot \varphi(e_{1|S}, \Sigma_{e_{1|S}}) \int_{S_1}^{+\infty} \phi(S_1^*; U^*, \Omega_{11}) dS_1^* .$$

[3] The Jacobian between latent shares (S*) and observed shares (S) is ignored because it is independent of model parameters. For details, see Wales and Woodland (1983).

The appendix also shows the derivation of (23) and the definition of B_1, U^*, and Ω_{11}. Equation (23) implies that under Regime II, the likelihood function requires only the integration of a univariate *PDF*.

Regime III Likelihood Function: All Commodities Purchased

For households where all commodities are purchased ($k = M+1$), the likelihood function is just the [($2M$+1) x 1] multivariate *PDF* of joint error terms, ε and e, which are defined in (8) and (10), and distributed as $MN(0,\Sigma_\varepsilon)$. That is:

(24) $$L(S_1, S_2, \cdots, S_{M+1} > 0) = \phi(\varepsilon, e).$$

Consistent and efficient parameter estimates can be obtained by maximizing the sum of log likelihood function over all households, where each household has been associated with one of the above regime-specific likelihood functions.

Evaluation of Predicted Shares and Demand Elasticities

Expected values of observed expenditure shares can be obtained from our censored demand system by summing the product of each regime's probability and the expected conditional share values over all possible regimes. Let R_s represent a particular purchase regime:

$$R_s = (S_1 = S_2 = \cdots = S_s = 0; S_{s+1} > 0, \cdots, S_{M+1} > 0).$$ [4]

The expected value of the j[th] observed expenditure share is:

(25) $$E(S_j) = \sum_{s=1}^{M} \alpha_{R_s} E(S_j \mid R_s),$$

where α_{R_s} is the probability that regime R_s occurs.

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[4] This is the regime where the first s shares are zero. Given s zero-valued shares, other possible purchase patterns can be transformed to this pattern by rearranging the share ordering. Under this definition, regime R_s is actually the sum of all the purchase patterns with s zero-valued shares.

The expected share value conditional on purchase regime R_s can be represented as:

$$(26) \qquad E(S_j \mid R_s) = \begin{cases} \dfrac{E(S_j^* \mid R_s)}{\sum_{i=s+1}^{M+1} E(S_i^* \mid R_s)}, & if \;\; j > s, \\ 0, & if \;\; j \le s; \end{cases}$$

with $E(S_j^* \mid R_s) = U_j^\varepsilon + E(\varepsilon_j \mid R_s) = U_j^\varepsilon + \dfrac{\alpha_{R_s}^{\varepsilon_j}}{\alpha_{R_s}}$, where $\alpha_{R_s}^{\varepsilon_j}$ is the first moment of ε_j given R_s.

From (25) the impact of changes in prices, demographic characteristics or expenditures on food demand can be obtained, but one needs to evaluate M-dimension integrals. Given that there are 2^{M+1}-1 purchase regimes, one may need to evaluate these integrals a large number of times for a reasonably sized demand system.

Phaneuf, Kling, and Herriges (2000) developed a simulation procedure to evaluate the elasticities for a censored demand system applied to recreation choices. We adapt their procedure to our application. Assume we have R replicates of the [M+1] error term vector, ε in (8). The r^{th} simulated latent share, S_r^*, evaluated at the sample means of our exogenous variables (indicated by a bar over a variable) is:

$$(27) \qquad S_r^* = \alpha + \gamma \ln \overline{V} + \beta \ln \frac{\overline{Y}}{\overline{P^*}} + \varepsilon_r$$

where ε_r is the r^{th} replicate of ε. The r^{th} replicate of the i^{th} observed share then is:

$$(28) \qquad S_{ir} = \begin{cases} \dfrac{S_{ir}^*}{\sum_{j \in \Psi} S_{jr}^*}, & \text{if} \quad S_{ir}^* > 0, \\ 0, & \text{if} \quad S_{ir}^* \le 0, \end{cases}$$

The expected observed share vector for R replicates is then calculated as a simple average of these simulated values:

$$(29) \qquad E(S) = \frac{1}{R}\sum_{r=1}^{R} S_r \ .$$

If there is a small change in unit value j, ΔV_j, then the elasticity vector with respect to this unit value change is:

$$(30) \qquad \eta_j = -\Lambda_j + \frac{\Delta E(S)}{\Delta V_j} \cdot \frac{V + \Delta V_j / 2}{E(V) + \Delta E(V)/2},$$

where Λ_j is a vector of 0's with the j^{th} element equal to 1, and $\Delta E(S)$ is the change in the simulated $E(S)$ given the change of unit value, ΔV_j.

V. An Analysis of Fish and Meat Purchases by Norwegian Households

Per capita consumption of fish and meat in Norway has increased in the past two decades, though it is still lower than in the other Nordic countries. Household preference for fish and meat in Norway has also shifted to higher quality products. For example, pork is substantially leaner than it used to be. This shift may be due to increases in household income, nutrition and health concerns, or advertising. In this section, we investigate the structure of fish and meat purchases by Norwegian households, and the quality effects determined by household characteristics.

Description of the Data

The data used in this paper (provided by GfK Norge, a marketing research company) comes from a panel survey of more than 1,500 Norwegian households. These households report on a weekly basis the expenditure and quantity of each item purchased at every shopping trip made within the given week. Each household also provides its demographic characteristics, such as size, age, location and income.

The data used for estimation contains household purchase information for fish and meat products for 1999-2000 (aggregated by month), including total expenditures and quantities, social economic variables, and annual demographic information for the households. The final system consists of nine aggregated commodities: Cod, Salmon, Farmed Fish (Fishfarm), Prawns, Canned Mackerel (Macibx), Canned or Bucket Herring (Heribx), Pork, Beef, and Other Meats (Other). Purchase statistics for these commodities are provided in Table 1. Of these nine, beef is purchased most frequently (93%), followed by pork (59%); canned or bucket herring is purchased the least (14%). The three meat commodities account for about 80% of the total expenditures, while the six fish commodities account for only 20%. The observed unit values vary from 3.68 paid for farm-raised fish to 9.21 paid for salmon. The unit value of salmon has the largest variation, as evidenced by its standard deviation, while canned mackerel has the least. Since not all the households participated in the survey over this two-year period, and about 80% of the observations (on a monthly basis) are non-purchase occasions for most of the commodities in the system, we do not have enough information to conduct a formal panel structure analysis. However, the data can be pooled in such a way that it gives enough observations to handle the heavily censored problem. Outliers corresponding to commodity expenditures greater than five standard deviations from the average observed value are deleted from the sample (about 1.8%). The final data contain 6,017 observations of 1,347 households covering the years 1999-2000.

Table 2 provides an overview of the explanatory variables used in share and unit value equations. As defined in the AIDS specification, total expenditure and unit values (in place of the unobserved prices) are included in the share equations. Household demographic variables are incorporated through the intercept, as suggested by Pollack and Wales (1992). The same set of household demographic variables is adopted in the unit value equations.

Empirical Results

With nine commodity categories, and ten demographic variables in the share and unit value equations, a total of 295 parameters were estimated using the GAUSS software system and BHHH maximum likelihood procedure (Berndt, Hall, Hall and Hausman, 1974). Two hundred replicates are used to simulate the multiple probability integrals. Table 3 shows the maximum likelihood coefficients estimates for the demographic variables in share and unit value equations. Table 4 presents the estimates for the symmetry-restricted unit value coefficients and the total expenditure coefficients in the share equations. The equation omitted during estimation corresponds to the commodity Other Meats.[5] The associated parameters for this omitted equation are retrieved from the LA/AIDS adding-up, symmetry, and homogeneity constraints. Because of space constraints, estimated coefficient values of the error term variance/covariance matrix are not presented, but they may be obtained from the authors.

Of the 81 demographic-related parameters estimated in share equations, 30 (37%) are statistically significant at the 0.05 level of significance. Of the same set of parameters

[5] The results obtained here when other meat is the omitted commodity are compared to results obtained when other commodities are omitted from the analysis. We confirmed that these alternative results asymptotically converge regardless of which commodity is omitted.

estimated in unit value equations, 27 (33%) are significant. The impact of total expenditures is significant for all commodity shares except for prawns.

Table 4 also shows the estimated own- and cross-unit value coefficients of share equations. Six of the own-price coefficients are found to be statistically different from zero at the 0.05 level of significance, while three (cod, salmon, and canned herring) are not significant. Of the 36 cross-unit value coefficients estimated, 11 are statistically significant at the level of 0.05 (31%).

Elasticities

The estimated parameters themselves are of little interest. From these parameters, however, we estimated uncompensated, unconditional own- and cross-unit value, total expenditure, and demographic elasticities by the simulation procedure defined by equations (27) to (30). The resulting elasticity estimates are shown in Tables 5 to 8. All of the own-unit value elasticities are found to be negative, as expected, with a range of –0.44 for canned mackerel to –1.24 for pork (Table 5). Turning to the expenditure elasticities, we find that cod, salmon, prawns, pork, and other meat have values greater than one, indicating that they are luxury commodities, while farm-raised fish, canned fish, and beef prove to be necessities.

Of the six fish commodities, cod and salmon are found to be complements. Cod is a complement for prawns and canned fish, but salmon is a substitute for all other fish commodities. Farm-raised fish is found to be a substitute for all other fish commodities except canned mackerel. Of the three meat commodities, beef and pork are found to be substitutes, while other meat is a substitute for pork, but a complement for beef. The cross-price elasticities between fish and meat commodities are reported in Table 5.

Endogenizing unit values allows us to decompose the demographic effects on quantity purchased into two types: direct and indirect. The indirect effect is through the change in unit values, which in turn has a direct effect on purchases. These direct and indirect effects are provided in Tables 6 and 7, respectively. Table 6 shows the estimated demand elasticities for the continuous demographic variables used in our analysis, holding constant the unit values.[6] This table also shows the percentage point change in shares due to a discrete change in the set of dichotomous demographic characteristics. Table 7 lists the results that allow the unit values to vary. The age of the head of the household is found to be positively related for fish commodities and other meat, and negatively related for beef regardless of whether the unit values are allowed to vary. The household head's age is negatively related for pork if unit values are not allowed to vary, but positive if unit values can vary. Other variables do not have any pattern; their effects depend on the specific commodity.

Table 8 gives the elasticities of demographic variables for the unit values. As noted above, the unit value has two parts: exogenously determined price and endogenously determined quality. The change of a household's characteristics can only affect quality, but the regional dummies may capture price variations. From Table 8, we find that household income significantly increases the quality of purchased meat and canned fish products, while household size significantly decreases the quality of purchased pork and beef but has no significant effect on other commodities. The older the head of the household, the lower the quality of salmon, pork, and beef purchased. The proportion of persons under age 16 decreases the quality of beef purchased, but has no significant effects on other commodities. We also find that people in metropolitan

[6] Except for the exogenous variable of concern, all exogenous variables are set at their mean values.

areas pay lower prices for salmon, canned mackerel, and other meat, but higher prices for farm-raised fish. Regions are found to have significant effects on prices for all commodities except prawns and canned fish.

VI. Conclusions

In this study, we developed and estimated a censored LA/AIDS model using household-level purchase data. In addition to imposing non-negativitity constraints, we accounted for the endogeneity of unit value. We addressed the non-negativity issue using an Amemiya-Tobin approach, which imposes the adding-up condition on both observed and latent shares. We addressed the endogeneity of unit value by estimating share equations and unit value equations simultaneously. Given the need to evaluate high-order probability integrals, we used a simulated probability method in the model estimation. The model is based on a LA/AIDS model, but can be applied to other systems as long as the unit values are linear (or logarithm linear) to the shares.

We estimated and analyzed a demand system featuring six fish and three meat commodities, using Norwegian household data. We found the effects of unit value, total expenditure, and demographic characteristics on household purchases all to be within the theoretical expectation.

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Table 1. Overview of Norwegian Household Fish and Meat Purchases

Commodity	% Purchasing Occasions	Mean Expenditure Share	Standard Deviation of Exp. Share	Mean Unit Value over Purchasing Occasions	Standard Deviation of UV over Pur.
Cod	0.2092	0.0389	0.1247	7.6882	2.8891
Salmon	0.1968	0.0388	0.1339	9.2108	6.1466
Fishfarm	0.4753	0.0644	0.1668	3.6805	1.9467
Prawns	0.2153	0.0342	0.1152	6.7890	3.6806
Macibx	0.1958	0.0139	0.1027	5.3711	0.8227
Heribx	0.1380	0.0144	0.0769	4.6978	4.7435
Pork	0.5945	0.1956	0.2275	7.0324	3.9637
Beef	0.9307	0.5445	0.3087	6.1118	2.2606
Other	0.1898	0.0552	0.1510	6.3752	5.3700

Table 2. Demographic Variables Used in Share and Unit Value Equations

Name	Description (unit)	In Share Equations	In Unit Value Equations	Mean	Standard Deviation
UNIT VALUE	*unit value*	Yes	No	(see Table 1)	
TOTEXP	*Total expenditure*	Yes	No	33,752	28,246
*INCOME**	*hh income*	No	Yes	333.66	162.16
*HSIZE**	*hh size*	Yes	Yes	2.2090	1.2134
*AGE_HEAD**	*head age*	Yes	Yes	52.578	14.345
KID16_PROP	*proportion of persons under 16*	Yes	Yes	0.1836	0.1750
METRO	*dummy versus rural (0/1)*	Yes	Yes	0.7574	0.4287
NORTH	*region dummy (0/1)*	Yes	Yes	0.1031	0.3042
CENTRAL	*region dummy (0/1)*	Yes	Yes	0.1557	0.3626
WEST	*region dummy (0/1)*	Yes	Yes	0.1843	0.3878
OSLO	*region dummy (0/1)*	Yes	Yes	0.1335	0.3402
*EAST***	*region dummy (0/1)*	Yes	Yes	0.4234	1.3881

Note:
*In estimation we use the logarithm of this variable.
**Indicates region used as the base.

Table 3. Censored Demand System Parameter Estimates

	Cod		Salmon		Fishfarm		Prawns		Macibx		Heribx		Pork		Beef		Other	
	Share	Unit Value	Share	Unit Value	Share	Unit Value	Share	Unit Value	Share	Unit Value	Share	Unit Value	Share	Unit Value	Share	Unit Value	Share	Unit Value
INTERCEPT	**-1.1951**	**1.9428**	**-1.3689**	**4.0951**	**-0.4477**	**0.5134**	**-0.3011**	**2.2524**	0.0186	**1.7704**	**-0.4928**	**1.8017**	**0.1876**	**2.7104**	**5.8068**	**1.8493**	**-1.2074**	**1.3092**
INCOME	-0.0065	-0.0040	**0.0326**	-0.0498	**0.0176**	0.0198	0.0152	0.0238	-0.0045	**0.0220**	0.0031	**0.1280**	0.0091	**0.0908**	**-0.0519**	**0.0598**	-0.0117	**0.1389**
HSIZE	-0.0064	-0.0179	-0.0260	-0.1211	**0.0376**	-0.0205	0.0114	0.0319	**0.0279**	-0.0349	0.0017	-0.1115	**-0.1295**	**-0.1972**	**0.1070**	**-0.0324**	-0.0236	-0.0302
AGE_HEAD	**0.1384**	0.0294	**0.2257**	**-0.3556**	**0.2222**	**0.1439**	**0.0504**	-0.1088	**0.0388**	-0.0428	**0.1371**	-0.2292	**-0.1987**	**-0.2980**	**-0.7666**	**-0.0947**	**0.1526**	-0.0739
KID16_PROP	-0.0099	-0.0425	0.0033	0.0139	**0.0380**	-0.0051	**-0.0213**	-0.0373	0.0035	0.0122	-0.0116	-0.1358	-0.0264	0.0320	**0.0564**	**-0.0313**	-0.0321	-0.0051
METRO	0.0103	0.0268	**0.0545**	-0.1086	0.0146	**0.0515**	0.0098	-0.0106	0.0002	**-0.0424**	-0.0044	0.0008	**-0.1141**	-0.0176	-0.0580	0.0180	**0.0872**	**-0.1111**
NORTH	**-0.0700**	**-0.2086**	-0.0127	-0.0868	0.0025	**0.2179**	**-0.1239**	0.1911	**-0.0399**	-0.0215	**-0.0507**	-0.0765	**0.1752**	0.0391	**0.0848**	**0.0576**	0.0347	**0.1851**
CENTRAL	-0.0423	-0.0769	-0.0055	0.0512	0.0268	**0.1815**	**-0.0575**	-0.0889	**-0.0334**	0.0029	0.0025	-0.0779	**0.1409**	**0.0692**	0.0062	-0.0127	-0.0376	0.0710
WEST	0.0069	-0.0764	0.0105	0.0416	-0.0109	**0.0915**	-0.0090	0.0446	**-0.0508**	-0.0156	-0.0019	0.1344	**0.1106**	**0.1084**	-0.0476	**0.0323**	-0.0076	**0.1652**
OSLO	-0.0079	0.0329	**0.0864**	-0.0580	0.0228	-0.0012	0.0166	0.0133	0.0159	-0.0239	**0.0268**	0.1067	-0.0412	**0.1136**	**-0.1652**	**0.0416**	0.0457	0.0576
TOTEXP	**0.0580**	--	**0.0194**	--	**-0.0505**	--	0.0014	--	**-0.0213**	--	**-0.0134**	--	**0.0995**	--	**-0.1406**	--	**0.0475**	--

Note: The bold cells indicate coefficients with the ratio of the estimated coefficient to the coefficient standard error exceeding 2.0.

Table 4. Estimates for Restricted Symmetric Unit Value Coefficients

	Cod	Salmon	Fishfarm	Prawns	Macibx	Heribx	Pork	Beef	Other
Cod	-0.0033								
Salmon	**-0.0636**	-0.0018							
Fishfarm	0.0122	**0.0567**	**0.0588**						
Prawns	-0.0176	0.0015	**0.0299**	**-0.0295**					
Macibx	-0.0297	**0.0215**	-0.0251	-0.0136	**0.0629**				
Heribx	**-0.0497**	0.0106	0.0112	**0.0295**	0.0128	0.0123			
Pork	**0.1217**	**-0.0350**	0.0051	-0.0021	-0.0018	-0.0220	**-0.1870**		
Beef	**0.1170**	0.0155	**-0.1530**	-0.0110	-0.0266	-0.0146	**0.0822**	**0.0545**	
Other	**-0.0871**	-0.0055	0.0041	0.0129	-0.0004	0.0100	0.0389	**-0.0641**	**0.0912**

Note: The bold cells indicate coefficients with the ratio of the estimated coefficient to the coefficient standard error exceeding 2.0.

Table 5. Simulated Unit Value and Expenditure Elasticities

		Cod	Salmon	Fishfarm	Prawns	Macibx	Heribx	Pork	Beef	Other
Quantity Impacts	**Cod**	-1.0492	-0.2300	0.0760	-0.0620	-0.0935	-0.1695	0.4113	0.4068	-0.2898
	Salmon	-0.1875	-1.0109	0.1421	0.0071	0.0505	0.0235	-0.0466	0.0200	0.0018
	Fishfarm	0.0027	0.1850	-0.7744	0.1029	-0.0771	0.0412	0.0131	-0.5263	0.0329
	Prawns	-0.1312	0.0002	0.1684	-1.1417	-0.0615	0.1456	0.0080	-0.0658	0.0781
	Macibx	-0.3040	0.1812	-0.1970	-0.1175	-0.4411	0.1139	-0.0065	-0.2427	0.0138
	Heribx	-0.4289	0.0751	0.1135	0.2336	0.1073	-0.9009	-0.1773	-0.1211	0.0988
	Pork	0.1227	-0.0520	0.0341	0.0012	0.0030	-0.0281	-1.2406	0.0953	0.0644
	Beef	0.0564	0.0107	-0.0952	-0.0082	-0.0110	-0.0062	0.0635	-0.9649	-0.0452
	Other	-0.1393	-0.0066	0.0439	0.0160	0.0115	0.0200	0.0246	-0.0883	-0.8818
Exp. Elasticity		1.2389	1.0664	0.8629	1.0353	0.8449	0.9296	1.1614	0.9063	1.0817

Table 6: Unconditional Demand Impacts of Changes in Demographic Characteristics With Fixed Unit Value

	Cod	Salmon	Fishfarm	Prawns	Macibx	Heribx	Pork	Beef	Other
	Elasticities								
INCOME	-0.0195	0.0725	0.0656	0.0779	-0.0360	0.0299	0.0192	-0.0326	0.0026
HSIZE	-0.0478	-0.0629	0.1058	0.0444	0.2282	-0.0100	-0.1867	0.0758	-0.0472
AGE_HEAD	0.6666	0.6599	0.9198	0.4197	0.5102	1.2375	-0.0776	-0.4491	0.3767
KID16_PROP	-0.0021	-0.0005	0.0070	-0.0040	0.0002	-0.0017	-0.0090	0.0159	-0.0060
	Percentage Point Change in Shares From Discrete Change in Dichotomous Exogenous Variable								
METRO	0.23	1.10	0.46	0.26	0.05	-0.02	-2.35	-1.15	1.42
NORTH	-1.05	-0.46	-0.14	-1.77	-0.51	-0.57	3.61	0.91	-0.02
CENTRAL	-0.63	-0.30	0.47	-0.94	-0.44	0.01	3.14	-0.65	-0.66
WEST	0.12	0.11	-0.21	-0.16	-0.60	-0.02	2.72	-1.90	-0.06
OSLO	0.06	1.73	0.74	0.40	0.28	0.41	-0.37	-4.52	1.27

Table 7: Unconditional Demand Impacts of Changes in Demographic Characteristics With Changing Unit Value

	Cod	Salmon	Fishfarm	Prawns	Macibx	Heribx	Pork	Beef	Other
	Elasticities								
INCOME	-0.0101	0.0781	0.0411	0.1033	-0.0369	0.0410	0.0112	-0.0349	0.0209
HSIZE	-0.0860	-0.0568	0.0936	0.0228	0.1886	0.0149	-0.1378	0.0640	-0.0524
AGE_HEAD	0.6678	0.6823	0.9127	0.4227	0.3951	1.2188	0.0136	-0.4810	0.3668
KID16_PROP	-0.0011	-0.0004	0.0078	-0.0039	0.0006	-0.0018	-0.0100	0.0140	-0.0053
	Percentage Point Change in Shares From Discrete Change in Dichotomous Exogenous Variable								
METRO	0.45	1.03	0.35	0.22	-0.07	-0.07	-1.95	-1.10	1.14
NORTH	-0.88	0.06	0.06	-1.49	-0.71	-0.38	3.23	-0.61	0.73
CENTRAL	-0.51	-0.04	0.87	-0.73	-0.52	0.06	2.53	-1.36	-0.30
WEST	0.09	0.32	0.01	0.02	-0.72	0.10	2.01	-2.27	0.44
OSLO	0.22	1.68	0.67	0.43	0.27	0.41	-0.65	-4.34	1.32

Table 8: Quality Impacts of Changes in Demographic Characteristics

	Cod	Salmon	Fishfarm	Prawns	Macibx	Heribx	Pork	Beef	Other
	Elasticities								
INCOME	-0.0040	-0.0498	0.0198	0.0238	0.0220	0.1280	0.0908	0.0598	0.1389
HSIZE	-0.0179	-0.1211	-0.0205	0.0319	-0.0349	-0.1115	-0.1972	-0.0324	-0.0302
AGE_HEAD	0.0294	-0.3556	0.1439	-0.1088	-0.0428	-0.2292	-0.2980	-0.0947	-0.0739
KID16_PROP	-0.2858	0.4562	-0.0109	-0.3330	0.0656	-0.6343	0.3385	-0.1885	-0.0194
	Percentage Change in Unit Value From Discrete Change in Dichotomous Exogenous Variable								
METRO	0.18	-3.70	0.11	-0.09	-0.23	0.01	-0.19	0.11	-0.43
NORTH	-1.27	-2.71	0.51	1.80	-0.11	-0.32	0.43	0.35	0.73
CENTRAL	-0.41	1.56	0.51	-0.89	0.02	-0.31	0.79	-0.08	0.31
WEST	-0.38	1.31	0.29	0.44	-0.08	0.53	1.32	0.20	0.80
OSLO	0.16	-1.83	-0.01	0.13	-0.12	0.47	1.52	0.27	0.31

Appendix: Derivation of the Estimable Likelihood Functions

The likelihood function in (19) can be decomposes into two components, in a procedure similar to that shown by Pudney (A3.5, pp. 327-328, 1989). Our case is more complicated because of adding-up restrictions on both S^* and S. Below is a simplification of (19) in which we reduce the dimension of $\phi(\cdot)$ from M to $[M–k+1]$:

$$
\begin{aligned}
(A.1)\quad & L(S_1, S_2, \cdots, S_k > 0;\, S_{k+1} = S_{k+2} \cdots = S_M = 0) \\
& = B \cdot \varphi(e_{1|S}, \Sigma_{e_{1|S}}) \int_{S_1}^{+\infty} \int_{1-\frac{S_1^*}{S_1}}^{0} \int_{1-\frac{S_1^*}{S_1}-S_{k+1}^*}^{0} \cdots \int_{1-\frac{S_1^*}{S_1}-S_{k+1}^*-\cdots-S_{M-1}^*}^{0} \phi(S_1^*, S_{M+1}^*, \cdots, S_M^*; U^*, \Omega_{11})\, dS_M^* \cdots dS_{k+1}^* dS_1^*
\end{aligned}
$$

where $U^* = \begin{pmatrix} U_1^* \\ U_{k+1}^* \\ \vdots \\ U_M^* \end{pmatrix} = \Omega_{11}\Omega_{10}^{-1} \begin{pmatrix} U_1 \\ U_{k+1} \\ \vdots \\ U_M \end{pmatrix}$, an [($M$-$k$+1) x 1] vector, and

$B = (2\pi)^{\frac{1-k}{2}} \cdot |\Sigma|^{-\frac{1}{2}} \cdot |\Omega_{11}|^{\frac{1}{2}} \cdot e^{-\frac{1}{2}\left\{ \begin{pmatrix} U_1 \\ U_{k+1} \\ \vdots \\ U_M \end{pmatrix}' \Omega_{00}^{-1} \begin{pmatrix} U_1 \\ U_{k+1} \\ \vdots \\ U_M \end{pmatrix} - \begin{pmatrix} U_1^* \\ U_{k+1}^* \\ \vdots \\ U_M^* \end{pmatrix}' \Omega_{11}^{-1} \begin{pmatrix} U_1^* \\ U_{k+1}^* \\ \vdots \\ U_M^* \end{pmatrix} \right\}}$. Vector $\begin{pmatrix} U_1 \\ U_{k+1} \\ \vdots \\ U_M \end{pmatrix}$ is from U^ω defined in (12).

The above Ω_{ij}'s are [(M-k+1) x (M-k+1)] matrixes, and defined as:

$$
\Omega_{11} = \begin{bmatrix} I'\sigma_{11}I & I'\sigma_{10} \\ \sigma_{10}'I & \sigma_{00} \end{bmatrix},\ \Omega_{00} = \begin{bmatrix} J'\sigma_{11}J & J'\sigma_{10} \\ \sigma_{10}'J & \sigma_{00} \end{bmatrix},\ \text{and } \Omega_{10} = \begin{bmatrix} I'\sigma_{11}J & I'\sigma_{10} \\ \sigma_{10}'J & \sigma_{00} \end{bmatrix},
$$

where I is a [k x 1] vector of ones, and J is a [k x 1] vector with the elements:

$\left(1, \frac{U_2}{(\frac{S_2}{S_1})U_1}, \frac{U_3}{(\frac{S_3}{S_1})U_1}, \frac{U_4}{(\frac{S_4}{S_1})U_1}, \cdots, \frac{U_k}{(\frac{S_k}{S_1})U_1}\right)'$. The σ_{ij}'s are defined via the following [M x M]

matrix: $\begin{bmatrix} \sigma_{11} & \sigma_{10} \\ \sigma_{10}' & \sigma_{00} \end{bmatrix} = \begin{bmatrix} A\Sigma_{11}^{-1}A' & A\Sigma_{10}^{-1} \\ \Sigma_{10}^{-1'}A' & \Sigma_{00}^{-1} \end{bmatrix}^{-1}$. The [$k$ x k] matrix A is a diagonal matrix with

elements: $\left(1, \frac{S_2}{S_1}, \frac{S_3}{S_1}, \cdots, \frac{S_k}{S_1}\right)$. Finally, the Σ_{ij}^{-1} matrices are obtained from the full error

variance matrix, $\Sigma_{\omega\omega}$, in (18).

From the results shown in Tallis (1965), the likelihood function represented by (A.1) can be further transformed to:

$$(A.2)\quad L(S_1, S_2, \cdots, S_k > 0; S_{k+1} = S_{k+2} \cdots = S_{M+1} = 0) = B \cdot \varphi(e_{1|S}, \Sigma_{e_{1|S}}) \cdot \Phi_{M-k+1}(b; R_C),$$

where $\Phi_{M-k+1}(b; R_C)$ is a [M-k+1] dimensional multivariate standard normal *cdf* with correlation coefficient matrix as R_C, and evaluated at vector b. Vector b is [(M-k+1) x 1] and can be shown to be equal to $E \cdot G$, where E is a [M-k+1] diagonal matrix with diagonal elements equal to $((C_1RC_1')^{-1/2}, (C_{k+1}RC_{k+1}')^{-1/2}, \cdots, (C_MRC_M')^{-1/2})$; where

$$C = \begin{pmatrix} C_1 \\ C_{k+1} \\ \vdots \\ C_M \end{pmatrix} = H \cdot D^{\frac{1}{2}},\ H = \begin{bmatrix} \frac{1}{W_1} & 1 & 1 & \cdots & 1 \\ 0 & -1 & 0 & \cdots & 0 \\ 0 & 0 & -1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & 0 & \cdots & -1 \end{bmatrix}$$, a [M-k+1] square matrix, R is the correlation coefficient matrix derived from Ω_{11}, and D the diagonal elements of Ω_{11}. Term

$$G = \begin{pmatrix} 1 - H_1U^* \\ -U_{k+1}^* \\ \vdots \\ -U_M^* \end{pmatrix}$$, where H_1 is the first row of matrix H. The new correlation coefficient

matrix (R_C) is given as $R_C = ECRC'E'$ (Tallis, 1965).

Equation (A.2) represents a rectangular standard multivariate normal probability, which can be conveniently evaluated using standard simulation procedures such as *GHK*. This equation is represented as (22) in the text.

Derivation of Equation (23)

Equation (23) is the likelihood for Regime II, in which where the number of commodities actually purchased, *k*, equals M. Under this special case, equation (A.2) can be restated as:

$$(A.3) \quad L(S_1, S_2, \cdots, S_M > 0; S_{M+1} = 0) = B_1 \cdot \varphi(e_{1|S}, \Sigma_{e_{1|S}}) \cdot \int_{S_1}^{+\infty} \phi(S_1^*; U^*, \Omega_{11}) dS_1^*,$$

where $U^* = U_1^* = \Omega_{11}\Omega_{10}^{-1}U_1$, and $\Omega_{11} = I'\sigma_{11}I$, $\Omega_{00} = J'\sigma_{11}J$, $\Omega_{10} = I'\sigma_{11}J$ are all scalars now with $\sigma_{11} = (A\Sigma_{\omega\omega}{}^{-1}A')^{-1}$, A is an M x M diagonal matrix with diagonal elements: $\left(1, \frac{S_2}{S_1}, \frac{S_3}{S_1}, \cdots, \frac{S_k}{S_1}\right)$, and I a [M x 1] vector of ones,

$$J = \left(1, \frac{U_2}{(\frac{S_2}{S_1})U_1}, \frac{U_3}{(\frac{S_3}{S_1})U_1}, \frac{U_4}{(\frac{S_4}{S_1})U_1}, \cdots, \frac{U_M}{(\frac{S_M}{S_1})U_1}\right)', \text{ and}$$

$$B_1 = (2\pi)^{\frac{1-M}{2}} \left|\Sigma_{\omega\omega}\right|^{-\frac{1}{2}} \left|\Omega_{11}\right|^{\frac{1}{2}} e^{-\frac{1}{2}\{U_1'\Omega_{00}^{-1}U_1 - U_1^{*'}\Omega_{11}^{-1}U_1^*\}}.$$

Thus, under purchase regime *II*, the likelihood function requires only the integration of a univariate *PDF*. Equation (A.3) is represented as (23) in the text.

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