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## ZEF-Discussion Papers on Development Policy No. 163

Oded Stark

### **Policy responses to a dark side of the integration of regions and nations**

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## **Abstract**

In this paper I study policy responses to an increase in post-merger distress. I consider the integration of regions and nations as a merger of populations which I view as a revision of social space, and I identify the effect of the merger on aggregate distress. The paper is based on the premise that the merger of groups of people alters their social landscape and their comparators. Employing a specific measure of social distress that is based on the sensing of relative deprivation, a merger increases aggregate distress: the social distress of a merged population is greater than the sum of the social distress of the constituent populations when apart. In response, policies are enacted to ensure that aggregate distress and/or that of individuals does not rise after a merger. I consider two publicly-financed, cost-effective policies designed so as not to reduce individuals' incomes: a policy that reverses the negative effect of the merger on the aggregate level of relative deprivation, bringing it back to the sum of the pre-merger levels of aggregate relative deprivation of the two populations when apart; and a policy that is aimed at retaining the relative deprivation of each individual at most at its pre-merger level. These two policies are developed as algorithms. Numerical examples illustrate the application of the algorithms.

*Keywords:* Merger of populations; Revision of social space; Aggregate relative deprivation; Societal distress; Policy responses

*JEL classification:* D04; D63; F55; H53; P51

## 1. Introduction

I study policy responses to an increase in aggregate social distress brought about by the integration of regions and nations, which I view as a merger of populations and the revision of social space and the comparison set. Specifically, I look at the merger of populations as a merger of income vectors; I measure social distress by aggregate relative deprivation; and I maintain that (except in the special case in which the merged populations have identical income distributions) a merger increases aggregate relative deprivation. Given this increase, I assess how a budget-constrained policy-maker can reverse the increase by means of least-cost post-merger increases in individual incomes.

When populations merge, the social environment and the social horizons faced by the individuals who constitute the merged population change: people who were previously outside the individuals' social domain are brought in. Mergers of populations occur in many spheres of life, at different times and places. They arise as a result of administrative considerations or naturally, they are imposed or chosen. With the help of specific examples, Stark (2010), and Stark, Rendl, and Jakubek (2012) raise the possibility that the revision of social space associated with the integration of societies can chip away at the sense of wellbeing of the societies involved. If integration also brings in its wake social distress, then greater economic gain is required to make integration desirable.

In Section 2 I present measures of individual and aggregate relative deprivation and I claim that the aggregate relative deprivation of merged populations is larger than or equal to the sum of the pre-merger levels of the aggregate relative deprivation of the constituent populations (a superadditivity result). In Section 3 I study policy responses to the increase in post-merger discontent. Section 4 provides discussion and conclusions.

## 2. A measure of deprivation and the superadditivity of aggregate relative deprivation (*ARD*) with respect to the merger of two populations

I measure the distress of a population by the sum of the levels of distress experienced by the individuals who constitute the population. I refer to this sum as the aggregate relative deprivation (*ARD*) of the population. I measure the distress of an individual by the extra income units that others in the population have, I sum up these excesses, and I normalize by the size of the population. This approach tracks the seminal work of Runciman (1966) and its

articulation by Yitzhaki (1979), and Hey and Lambert (1980); a detailed description is in Stark and Hyll (2011). In my definition of relative deprivation I resort to income-based comparisons, namely, an individual feels relatively deprived when others in his comparison group earn more than him. To concentrate on essentials, I assume that the comparison group of each individual consists of all members of his population.

Formally, for an ordered vector of incomes in population  $P$  of size  $n$ ,  $x = (x_1, \dots, x_n)$ , where  $x_1 \leq x_2 \leq \dots \leq x_n$ , I define the relative deprivation of the  $i$ -th individual whose income is  $x_i$ ,  $i = 1, 2, \dots, n$ , as

$$RD(x_i, x) \equiv \frac{1}{n} \sum_{j=i}^n (x_j - x_i). \quad (1)$$

To ease the analysis that follows, an alternative representation of the relative deprivation measure is helpful.

**Lemma 1.** Let  $F(x_i)$  be the fraction of those in the population  $P$  whose incomes are smaller than or equal to  $x_i$ . The relative deprivation of an individual earning  $x_i$  in population  $P$  with an income vector  $x = (x_1, \dots, x_n)$  is equal to the fraction of those whose incomes are higher than  $x_i$  times their mean excess income, namely,

$$RD(x_i, x) = [1 - F(x_i)] \cdot E(x - x_i \mid x > x_i). \quad (2)$$

**Proof.** I multiply  $\frac{1}{n}$  in (1) by the number of the individuals who earn more than  $x_i$ , and I divide  $\sum_{j=i}^n (x_j - x_i)$  in (1) by this same number. I then obtain two ratios: the first is the fraction of the population who earn more than the individual, namely  $[1 - F(x_i)]$ ; the second is mean excess income, namely  $E(x - x_i \mid x > x_i)$ .  $\square$

The aggregate relative deprivation is, in turn, the sum of the individual levels of relative deprivation

$$ARD(x) = \sum_{i=1}^n RD(x_i, x) = \sum_{i=1}^n \frac{\sum_{j=i}^n (x_j - x_i)}{n}. \quad (3)$$

$ARD(x)$  is my index of the level of “distress” of population  $P$ . (For several usages of this measure in recent related work see Stark, 2010; Fan and Stark, 2011; Stark and Fan, 2011; Stark and Hyll, 2011; Stark, Hyll, and Wang, 2012; Stark, Rendl, and Jakubek, 2012.)

I now consider two populations,  $P_1$  and  $P_2$ , with ordered income vectors  $x^1 = (x_i^1)$  and  $x^2 = (x_i^2)$  of dimensions  $n_1$  and  $n_2$ , respectively. Total population size is  $n = n_1 + n_2$ . The ordered income vector of the merged population is denoted  $x^1 \circ x^2$ , and is the  $n$ -dimensional income vector obtained by merging the two income vectors and ordering the resulting  $n$  components from lowest to highest.<sup>1</sup>

In the following claim I state that the difference  $ARD(x^1 \circ x^2) - ARD(x^1) - ARD(x^2)$  is in fact non-negative: a merger increases aggregate relative deprivation or leaves it unchanged. Namely, if I conceptualize the merger of two income vectors as an addition operator, then  $ARD$  is a superadditive function of the income vectors. (A function  $H$  is superadditive if for all  $x, y$  it satisfies  $H(x + y) - H(x) - H(y) \geq 0$ .)

**Claim 1.** Let  $P_1$  and  $P_2$  be two populations with ordered income vectors  $x^1$  and  $x^2$ , and let  $x^1 \circ x^2$  be the ordered vector of merged incomes. Then

$$ARD(x^1 \circ x^2) - ARD(x^1) - ARD(x^2) \geq 0.$$

**Proof.** A proof for the case of the merger of populations with two incomes each is in Stark (2010); proof for the case of the merger of any two populations is in Stark (2012).  $\square$

*Example 1:* consider the merger of populations  $P_1$  and  $P_2$  with income vectors  $x^1 = (1, 2)$  and  $x^2 = (3, 4)$ , respectively. The pre-merger levels of aggregate relative deprivation are  $ARD(x^1) = 1/2$  and  $ARD(x^2) = 1/2$ . In the merged population with income vector  $x^1 \circ x^2 = (1, 2, 3, 4)$ , I have that  $ARD(x^1 \circ x^2) = 5/2 > 1 = ARD(x^1) + ARD(x^2)$ . This example vividly illustrates further why a formal proof of the superadditivity result is needed. Even in the simple case in which the two populations do not overlap and a relatively poor, two-person population  $x^1 = (1, 2)$  merges with a relatively rich, two-person population  $x^2 = (3, 4)$ , the overall relative deprivation effect cannot be pre-ascertained. In such a case, it is quite clear that upon integration members of the poorer population are subjected to more

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<sup>1</sup> The operator  $\circ$  is commutative and associative on the set of ordered vectors, and satisfies the closure property.



relative deprivation, whereas members of the richer population other than the richest are subjected to less relative deprivation. Because one constituent population experiences an increase of its *ARD* while another experiences a decrease, whether the *ARD* of the merged population is higher than the sum of the *ARDs* of the constituent populations cannot be determined without formal analysis. Put differently, in a setting in which others could only bring negative externalities, a smaller population will always experience less aggregate relative deprivation. But in a setting such as mine when others joining in can confer both negative externalities (of 3 and 4 upon 1 and 2) and positive externalities (of 1 and 2 upon 3), it is impossible to determine without proof whether the expansion of a population will entail a reduction in aggregate relative deprivation or an increase.<sup>2</sup>

Because throughout I have kept incomes unchanged, the incomes of the members of a constituent population are not affected by its merger with another population: in my setting, a merger changes the social comparisons space that governs the sensing and calculation of relative income (relative deprivation), but it leaves absolute incomes intact. If I assume that individuals' wellbeing depend positively on absolute income and negatively on the experienced relative deprivation, a merger leads to a deterioration of the aggregate wellbeing of at least one of the merged populations.

I next ask how a government that is concerned about the increase of the aggregate level of social distress will be able to respond in a cost effective manner. Governments must be well aware that an increase in social distress could translate into social unrest, and there have been plenty of episodes, historical and current, to remind governments of the short distance between social distress and social protest.

### **3. Policy responses to the post-merger increase in *ARD***

The unwarranted repercussions of a merger on the wellbeing of populations and individuals invite design and assessment of policies aimed at counteracting the increase in *ARD* or in individuals' *RD*.

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<sup>2</sup> To see the variation in the externality repercussion even more starkly, note that when 3 joins 1 and 1, he confers a negative externality on the incumbents; when 3 joins 5 and 5 he confers neither a negative externality nor a positive externality on the incumbents; and when 3 joins 4 and 5, he confers a positive externality on incumbent 4.

I study publicly-financed, cost-effective policies that are constrained not to reduce individuals' incomes. I consider two targets of governmental policy aimed at reversing the deleterious effect of merger:

Bringing down the aggregate level of relative deprivation to a level equal to the sum of the pre-merger levels of the aggregate relative deprivation in the two populations when apart; I refer to this problem as  $\Pi_1$ .

Seeing to it that no individual in the integrated population senses higher relative deprivation than the relative deprivation he sensed prior to the merger; I refer to this problem as  $\Pi_2$ .

Naturally, the government is keen to minimize the cost of implementing its chosen policy, which it enacts subject to the condition that in the process, no income is allowed to decrease.<sup>3</sup> Under the first policy, individual levels of relative deprivation may increase, decrease, or remain unchanged. Under the second policy, individual relative deprivations cannot increase. This added constraint implies that the budgetary allocation needed to solve the second problem will be larger than the corresponding one needed to solve the first problem.

The cost of the solutions to these two problems can be interpreted as lower bounds on the additional income that the process of economic (income) growth has to yield in order to retain the aggregate relative deprivation or the individual levels of relative deprivation at their pre-merger levels.

### 3.1 Solving problem $\Pi_1$

Clearly, the basic requirement of problem  $\Pi_1$  can be satisfied by a trivial solution: lifting the incomes of all the individuals to the highest level of income in the merged population. In general, such a solution will not, however, be optimal.<sup>4</sup> It will be possible to achieve

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<sup>3</sup> I resort here to this last condition because of an implicit assumption that an individual's utility depends positively on his income and negatively on his relative deprivation. Because I do not know the exact rate of substitution between decrease in relative deprivation and decrease in income, I do not know how much income I could take away from an individual whose relative deprivation decreased in the wake of the merger. Therefore, to guarantee that the utility of an individual will not be decreased in the process, I impose the requirement that incomes cannot be lowered. Put differently, seeing to it that the individual's post-merger relative deprivation is not higher than his pre-merger relative deprivation while holding the individual's income constant constitutes a sufficient condition for retaining the individual's wellbeing at its pre-merger level.

<sup>4</sup> There are, however, specific cases where this solution is optimal such as when, for example, the merged populations consists each of one individual, with one individual earning less than the other.

optimality by choosing carefully a subset of individuals for whom the marginal increase in incomes yields the highest marginal decrease in aggregate relative deprivation.

Let  $x^1 \circ x^2 = (x_1, \dots, x_n)$  be the ordered vector of incomes in the merged population. Consider the subset in the merged population of the individuals who earn the lowest income; I denote this subset by  $\Omega$ . I now analyze what happens when marginally and by the same amount I increase the incomes of the individuals in  $\Omega$ , where marginal increase refers to such an increase that the incomes of these individuals will not become higher than the income of any individual outside the set  $\Omega$ .

*First*, suppose that the set  $\Omega$  consists of just one individual out of the  $n$  members of the merged population, and that the government appropriates a sum  $\varepsilon$  to increase his income, where  $\varepsilon$  is small enough to satisfy my definition of a marginal increase in income. Using (2), this individual's relative deprivation decreases by  $\frac{n-1}{n}\varepsilon$  because the mean excess income of the fraction of  $\frac{n-1}{n}$  individuals earning more than him is reduced by the amount  $\varepsilon$ . At the same time, as this individual's income was, and continues to be, the lowest in the population, this disbursement does not increase the relative deprivation of any other individual and therefore, the change in aggregate relative deprivation is

$$-\Delta ARD = \frac{n-1}{n}\varepsilon. \quad (4)$$

I next show that (4) is the highest marginal decrease in  $ARD$  achievable upon spending  $\varepsilon$  on a single individual. I do this by contradiction. Suppose that I were to increase by  $\varepsilon$  not the income of the lowest-earning individual but the income of an individual earning  $x_i > x_1$ . Then, the relative deprivation of this  $i$  individual will decrease as a result of his income getting closer to the incomes of the individuals earning more than him, but the relative deprivation of those individuals who earn less than him will increase. Namely, when  $\bar{n}_i$  ( $\tilde{n}_i$ ) is the number of individuals earning strictly more (less) than  $x_i$ , the marginal change in aggregate relative deprivation will be

$$-\Delta ARD = \frac{\bar{n}_i}{n}\varepsilon - \frac{\tilde{n}_i}{n}\varepsilon = \frac{\bar{n}_i - \tilde{n}_i}{n}\varepsilon, \quad (5)$$

because the mean excess income of the fraction of  $\frac{\bar{n}_i}{n}$  individuals earning more than  $x_i$  falls by the amount  $\varepsilon$ , yet at the same time, the relative deprivation of each of the  $\tilde{n}_i$  individuals earning less than  $x_i$  increases by  $\frac{\varepsilon}{n}$ . Because  $\tilde{n}_i \geq 1$  and  $\bar{n}_i < n$ , comparing (4) and (5) yields

$$\frac{\bar{n}_i - \tilde{n}_i}{n} \varepsilon < \frac{n-1}{n} \varepsilon.$$

Thus, channeling the transfer to an individual who is not the lowest income recipient in the population yields a lower decrease in aggregate relative deprivation than increasing the income of the individual who earns the lowest income.

*Second*, I allow the set  $\Omega$  to expand to include more than one individual. I denote by  $|\Omega|$  the size of the set  $\Omega$ . Suppose again that the government appropriates the sum  $\varepsilon$  to increase the earnings of each member of the subset  $\Omega$  by  $\frac{\varepsilon}{|\Omega|}$ . The fraction of the individuals who are earning more than members of the  $\Omega$  set is equal to  $\frac{n-|\Omega|}{n}$ , and the mean excess income of these individuals falls by  $\frac{\varepsilon}{|\Omega|}$ . Therefore, each of the members of  $\Omega$  will experience a decrease in relative deprivation equal to  $\frac{n-|\Omega|}{n} \frac{\varepsilon}{|\Omega|}$ . Again, because no individual experiences an increase in his relative deprivation, this disbursement yields a change in aggregate relative deprivation

$$-\Delta ARD = |\Omega| \frac{n-|\Omega|}{n} \frac{\varepsilon}{|\Omega|} = \frac{n-|\Omega|}{n} \varepsilon.$$

As in the case of the set  $\Omega$  consisting of a single individual, this is obviously the optimal use of  $\varepsilon$  for any subset of the merged population.

Drawing on the preceding protocol, I present the optimal solution to problem (policy response)  $\Pi_1$  in the form of an algorithm as follows.

Algorithm  $A_1$ :

1. Include in the set  $\Omega$  all the individuals who earn the lowest income in the merged population.

2. Proceed to increase simultaneously the incomes of the members of the set  $\Omega$ , until either
  - a. the aggregate relative deprivation is brought down to the pre-merger level
  - or
  - b. the incomes of the members of the set  $\Omega$  reach the income of the first individual(s) who is (are) not a member (members) of this set, in which case start from step 1 once again.

It is easy to ascertain the optimality of Algorithm  $A_1$ : at each step, I increase the incomes of those individuals who earn the lowest, therefore the decrease in aggregate relative deprivation is the most effective, and the relative deprivation of no individual increases in the process. Heuristically, I start “pumping” incomes from the bottom, and I simultaneously gauge the aggregate relative deprivation response. The two processes move in tandem, and in opposite directions. The pumping from below is ratcheted up the hierarchy of the individuals, and it ceases when aggregate relative deprivation reaches the level at which it was prior to the merger.

*Example 2:* consider the merger of populations  $P_1$  and  $P_2$  with income vectors  $x^1 = (1, 2)$  and  $x^2 = (3, 4)$ , respectively. The pre-merger levels of aggregate relative deprivation are  $ARD(x^1) = 1/2$  and  $ARD(x^2) = 1/2$ . Because in the merged population with income vector  $x^1 \circ x^2 = (1, 2, 3, 4) \equiv z$  I have that  $ARD(x^1 \circ x^2) = 5/2 > 1 = ARD(x^1) + ARD(x^2)$ , the government seeks to lower the aggregate relative deprivation of the merged population back to  $1/2 + 1/2 = 1$ . Applying Algorithm  $A_1$ , I first include in the set  $\Omega$  the individual earning 1, and I increase his income. Upon his income reaching the income of the next individual who earns 2, I obtain the income vector  $z^1 = (2, 2, 3, 4)$ , with  $ARD(z^1) = 7/4$ . Thus, giving the individual earning 1 an additional unit of income is insufficient to bring down aggregate relative deprivation to its pre-merger level. I therefore add the next individual (the individual whose pre-merger income was 2) to the set  $\Omega$ , and I proceed to further increase the incomes of each of the two individuals who now constitute the set  $\Omega$  and whose incomes are, for now, 2 each. At the point where these two incomes are elevated to  $11/4$  each, I obtain  $z^2 = (11/4, 11/4, 3, 4)$  with  $ARD(z^2) = 1$ . Thus, in order to

bring the aggregate relative deprivation in the merged population to the sum of the pre-merger level, I have to transfer  $7/4$  units of income to the individual earning 1, and  $3/4$  units of income to the individual earning 2, which gives  $10/4$  as the total cost of implementing the policy.

### 3.2 Solving problem $\Pi_2$

In order to solve problem  $\Pi_2$ , I first present a simple link between the levels of relative deprivation and the levels of income in a population.

**Lemma 2.** If an individual has the  $i$ -th highest income in a population, he has the  $i$ -th lowest level of relative deprivation in the population.<sup>5</sup>

**Proof.** It is easy to see that individuals earning the highest level of income have zero relative deprivation, which is the lowest possible level, whereas the order of the other individuals in the relative deprivation hierarchy is obtained from the two relationships

$$RD(x_j, x) > RD(x_k, x) \text{ for } x_j < x_k$$

and

$$RD(x_j, x) = RD(x_k, x) \text{ for } x_j = x_k. \quad \square$$

Lemma 2 tells me that the relative deprivation of an individual is inexorably related to his rank in the income hierarchy. The procedure of solving problem  $\Pi_2$  builds on the simple fact that the hierarchy of the levels of relative deprivation mimics in reverse the hierarchy of incomes.

The following algorithm solves problem  $\Pi_2$ .

Algorithm  $A_2$ :

1. Starting with the post-merger income vector  $x^1 \circ x^2$ , I construct a vector  $w$  by arranging the elements of the  $x^1 \circ x^2$  vector in descending order with respect to the pre-merger levels of relative deprivation. (If two or more individuals have the same

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<sup>5</sup> By  $i$ -th highest I mean an ordering that allows for (co-)sharing a position, that is, in a population with incomes  $(1, 2, 2, 3)$ , the individual earning 3 has the 1st highest income, the individuals earning 2 have the 2nd highest incomes, and the individual earning 1 has the 3rd highest income.

pre-merger level of relative deprivation, I place leftmost the one with the lower income.)

2. I pick the individuals one at a time according to their placement in the  $w$  vector starting from the rightmost end and proceeding leftwards. If an individual has higher relative deprivation than prior to the merger, I increase his income to the minimal level that brings down his relative deprivation to the pre-merger level. If the relative deprivation of an individual is the same as or is lower than prior to the merger, I do not raise his income.

To establish the rationale and optimality of Algorithm  $A_2$ , I implement 1 above by numbering the elements in the  $w$  vector in a descending order, namely as  $w = (w_n, \dots, w_2, w_1)$ , such that the leftmost individual earning  $w_n$  is the individual who had the highest pre-merger level of relative deprivation, and the rightmost individual earning  $w_1$  is the individual whose pre-merger level of relative deprivation was the lowest.

The optimality of Algorithm  $A_2$  hinges on the property that an individual's relative deprivation never increases as a result of changes made after his "turn" has come, given that I am proceeding leftwards in the  $w$  vector. To see this, I denote the vector of incomes after  $i$  steps,  $1 \leq i < n$ , with  $i$  incomes  $w'_i, \dots, w'_1$  being dealt with, as  $w^i = (w_n, \dots, w_{i+1}, w'_i, \dots, w'_1)$ . When I proceed then to the next income  $w_{i+1}$ , one of two possibilities arises.

First, the current relative deprivation of the individual with income  $w_{i+1}$ ,  $RD(w_{i+1}, w^i)$ , can be lower or equal to the relative deprivation that he had prior to the merger; in such a circumstance, I do not increase his income. Therefore, the relative deprivations of other individuals, in particular those with incomes to the right of this individual,  $w'_i, \dots, w'_1$ , do not increase. The second possibility is that the current relative deprivation of the individual with income  $w_{i+1}$ ,  $RD(w_{i+1}, w^i)$ , is higher than his pre-merger relative deprivation. In such a circumstance, I increase his income to the level  $w'_{i+1}$ , which is the minimal income that equalizes the pre-merger relative deprivation and  $RD(w'_{i+1}, w^i)$ . I note that this change in income cannot affect the relative deprivation of those having incomes  $w'_i, \dots, w'_1$  because, according to Lemma 2,  $w'_{i+1} \leq w'_j$  for  $j = 1, \dots, i$ . It is a trivial feature of the relative deprivation

index that the relative deprivation of an individual earning  $v$  does not increase when incomes that are lower than  $v$  are raised, as long as the raised incomes do not surpass  $v$ .

The preceding reasoning leads me to conclude that for  $i = 1, \dots, n$ , the  $w'_i$  income is the lowest possible level of income which guarantees, first, that the relative deprivation of an individual will be no higher than prior to the merger and, second, that this individual's relative deprivation will not be affected by the process of adjusting the incomes of individuals to his left in the  $w$  vector whose incomes are  $w_n, \dots, w_{i+1}$ . This protocol delivers the optimality of Algorithm  $A_2$ .

Heuristically, in order to address problem  $\Pi_2$  I first raise the incomes at the top of the constructed hierarchy of the levels of relative deprivation; I do so in order to equate the levels of relative deprivations of the top-income individuals with the pre-merger levels of relative deprivation. Then, because the comparisons that yield relative deprivation are with incomes on the right in the income hierarchy, the changes made at the top determine by how much incomes that are further down the hierarchy have to be raised as I move leftwards.

*Example 3:* consider the merger of populations  $P_1$  and  $P_2$  with income vectors  $x^1 = (1, 2)$  and  $x^2 = (3, 4)$ , respectively. The pre-merger levels of relative deprivation are  $RD(1, x^1) = 1/2$ ,  $RD(2, x^1) = 0$ ,  $RD(3, x^2) = 1/2$ , and  $RD(4, x^2) = 0$ . Therefore, in the merged population with income vector  $x^1 \circ x^2 = (1, 2, 3, 4)$ , I have that the vector  $w$ , ordered according to the descending pre-merger levels of relative deprivation (with the lower of two incomes associated with the same level of relative deprivation placed leftmost) is  $w = (w_4, w_3, w_2, w_1) = (1, 3, 2, 4)$ . I pick first “for treatment” the individual with income  $w_1 = 4$ . Noting that his relative deprivation was not increased as a result of the merger,  $w'_1 = 4$  and thus,  $w^1 = (1, 3, 2, 4)$ . Moving leftwards, I next attend to the individual with income  $w_2 = 2$ . Because  $RD(2, w^1) = 3/4 > 0 = RD(2, x^1)$ , I need to raise income  $w_2$  to the level  $w'_2 = 4$ , because then  $RD(4, w^1) = 0 = RD(2, x^1)$ . Consequently, I obtain  $w^2 = (1, 3, 4, 4)$ . Proceeding further leftwards to  $w_3 = 3$ , I have that  $RD(3, w^2) = 1/2 = RD(3, x^2)$ , and so no increase in income is needed in this case. Thus, I obtain  $w^3 = (1, 3, 4, 4)$ . Because for the remaining individual with income  $w_4 = 1$  I have that  $RD(1, w^3) = 2 > 1/2 = RD(1, x^1)$ , I need to increase



his income to  $w'_4 = 3$  as then,  $RD(3, w^3) = 1/2 = RD(1, x^1)$ . Thus, the final income vector is  $w^4 = (3, 3, 4, 4)$ , which gives 4 as the total cost of implementing the policy.

Pulling together the results of *Example 2* and *Example 3*, I have:

*Example 2*: Income vector  $z^2 = (2.75, 2.75, 3, 4)$ , cost of implementation 2.5;

*Example 3*: Income vector  $w^4 = (3, 3, 4, 4)$ , cost of implementation 4.

Not surprisingly, because the constraint on implementing policy  $\Pi_2$  is stricter than the constraint on implementing policy  $\Pi_1$ , enacting policy  $\Pi_2$  is costlier.

#### 4. Discussion and conclusions

Processes and policies that integrate economic entities also revise the social landscape of the people who populate the entities. I have considered the case in which the form that the revision takes is an expansion - be it the result of closer proximity to others, more intensive social interactions, or reduced barriers to the flow of information. I have argued that a consequence of the changing social milieu is the casting of a shadow on the anticipated economic gains.

An increase in aggregate relative deprivation is a down side to the integration of regions and nations. It puts a strain on the individuals in the merged population, casting a shadow over the production and trade (scale and scope) benefits anticipated from integration. An increase in relative deprivation can itself cause an adverse physiological reaction such as psychosomatic stress, and could lead to social unrest and a collective response in the form of public protest. To aid a social planner who seeks cost-effectively to counter this negative effect, I analyzed policy measures in a setting in which incomes are not allowed to fall. In this setting, the policy measure to be adopted depends on whether the policy objective is to bring the aggregate level of relative deprivation down to the sum of the pre-merger levels, or to ensure that no individual experiences more relative deprivation than prior to the merger. I formulated algorithms to guide the implementation of these policy measures, and in illustrative examples I calculated the associated cost that the social planner would need to bear.

However, my analysis did not take into account all the possible effects of a merger. As already mentioned, the integration of regions and nations is expected to increase efficiency.

When the possibility of a merger is contemplated, an interesting question to address would be whether the anticipated boost in productivity will suffice to pay for the cost of the policies discussed above.

My analysis is essentially of the “comparative statics” type, with the revision of the social landscape occurring at the time of the merger, and the expected increase in incomes in the wake of the merger yet to come. Introducing dynamics need not erode my main argument, however. The revision of the comparison group could be gradual and coincide with the processes of scale economies and scope economies taking hold. Still, as long as the latter processes do not result in sufficient convergence of incomes, the former process could still damage the post-merger sense of wellbeing. (“Sufficiency” stands for, say in the case of *Example 2*, convergence of  $z = (1, 2, 3, 4)$  to  $z^2 = (2.75, 2.75, 3, 4)$ ).

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