Concession Bidding Rules and Investment Time Flexibility

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Summary

We study the competition to operate an infrastructure service by developing a model where firms report a two-dimensional sealed bid: the price to consumers and the concession fee paid to the government. Two alternative bidding rules are considered in this paper. One rule consists of awarding the exclusive right of exercise to the firm that reports the lowest price. The other consists of granting the franchise to the bidder offering the highest fee. We compare the outcome of these rules with reference to two alternative concession arrangements. The former imposes the obligation to immediately undertake the investment required to roll-out the service. The latter allows the winning bidder to optimally decide the investment time. The focus is on the effect of bidding rules and managerial flexibility on expected social welfare. We find that the two bidding rules provide the same outcome only when the contract restricts the autonomy of the franchisee, and we identify the conditions under which time flexibility can provide a higher social value.

Keywords: Concessions, Auctions, Bidding Rules, Managerial Flexibility

JEL Classification: L51, D44, D92

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1 Introduction

One way of bringing competitive forces into natural monopoly industries is to delineate a monopoly franchise and auction it off to the bidder offering the best proposal (Desmetz, 1968; Dnes, 1995; Klein and Gray, 1997).

There are a wide variety of "concessions"[1]and different types of competitive bidding rules. As far as concession arrangements are concerned, one key difference is whether the conceding authority imposes specific obligations regarding the means to be used by the operator, namely the required investment.[2] At one extreme, the government can eliminate almost all scope for discretion, by imposing investment plans which rule out any time flexibility. At the other, contracts can be designed so as to leave a large degree of autonomy to the winning bidder, by simply assigning the right, as distinct from the obligation, to supply the market.

Another key issue relates to the bid evaluation process, namely which specifications to include for the technical and financial proposals.[3] As far as the financial offers are concerned, when the concession does not involve sale of existing assets, awarding authorities frequently base the bidding on the highest (one-time or annual) fee paid to the government, or on the lowest price charged to consumers (World Bank, 1998).

The debate about concession design and award procedures is not new. For example, Alfred Marshall argued that "[...] the competition for the franchise shall turn on the price or the quality, or both, of the services or the goods, rather than on the annual sum paid for the lease"[4]. However, the modern literature on franchise bidding has not explored in depth the effects

[1]Throughout the paper we use the term *concession* broadly to refer to "any arrangement in which a firm obtains from the government the right to provide a particular service under conditions of significant market power" (World Bank, 1998, p.10).

[2]"In 1993 Argentina's national freight rail network was partitioned and concessioned under 30-year contracts. As part of the concession agreements, winning bidders agreed to invest about $1.2 billion in the rail network over 15 years [...] Despite substantial efficiency gain in service, however, traffic levels have fallen short of expectations, reaching only 60 to 70 percent of projected traffic [...] Given the lower-than-expected traffic levels, the investment amounts agreed in the contracts are likely to be unnecessary and uneconomic" (World Bank, 1998, p.75).

[3]Conceding authorities often adopt a two-stage process whereby technical proposals are evaluated before proceeding to the financial offers. The winning bidder is then selected on the basis of the best financial proposal from among those who passed the technical evaluation (World Bank, 1998).

[4]Quoted in Ekelund and Hebert (1981), p.471.

of alternative bidding rules, and the relationship between the outcome of the award process and concession arrangements.

Our model is novel in that it treats the choice both of bidding rules and of concession design. The purpose is twofold. First, we analyze the outcome of the above-mentioned bidding rules ("highest concession fee" *vs* "lowest price"), with reference to two alternative concession arrangements. The former imposes the obligation to immediately undertake the investment required to roll-out the service. The latter involves investment time flexibility, by simply assigning the right to supply the market. Since the two bidding rules involve different outcomes when the contract does not restrict the autonomy of the franchisee, the second issue addressed in the paper is, Which combination (bidding method *and* concession arrangement) performs best in terms of expected social welfare?

While this paper focuses on concession contracts, our analysis is related to the literature on procurement, in particular to the branch of the literature which considers the question of how to include quality other than sale price in the procurement process (Laffont and Tirole, 1987; Che, 1993). In particular, Che (1993) shows that the optimal buying mechanism distorts the quality provided by the suppliers downwards relative to the first best levels. In other words, the buyer, acting as if he does not care about the quality, may reduce the dispersion between suppliers and thus increase the level of procurement competition. Hence, if we interpret the construction time as the procured project quality (Herbesman et al., 1995), Che's result implies that the government may benefit from a reduced sale price in exchange for a project completion delay.

Our paper contributes to the literature in two ways. First, our findings suggest that concessioning an infrastructure service without imposing the obligation to immediately supply the market (i.e. acting as if "quality" does not matter) does not increase *per se* the level of competition. For instance, if such a contract is awarded to the bidder offering the highest concession fee, firms will not exploit the delay option, and will submit the same bids as those they would have reported to acquire a contract which imposes the obligation to immediately roll-out the service. Second, similarly to Che (1993), we find that a contract which does not impose such an obligation may prove to be welfare-improving, provided the franchise is awarded to the bidder that reports the lowest tariff.

The rest of the paper is organized as follows. Section 2 outlines the model and describes the concession value. Section 3 looks at the outcome of the two bidding rules. Section 4 focuses on the effect of bidding rules and concession arrangements on expected social welfare. Section 5 concludes and the Appendix contains the proofs omitted in the text.

2 The concession value

Consider a natural monopoly industry facing demand uncertainty which is beyond the supplier's control. To supply the market, the operator must afford capital costs, without being able to exercise any degree of discretion with respect to the type of investment to be undertaken and product quality.[5]

The standardized service under consideration can be operated only by acquiring an exclusive right of exercise auctioned off by a public authority (hereafter "the government"). For the sake of simplicity, we assume that the franchise term is sufficiently long to be approximated by infinite.[6] Depending on the auction formats (Section 3), the franchise will be awarded to the bidder reporting the lowest price to consumers, or to the firm offering the highest up-front payment (concession fee) to the government.

Before focussing on the bidding rules, let's describe the value of the concession, by taking the price as given and by ignoring the fee. We make the following assumptions.

Assumption 1 The new infrastructure can be built instantly, at a cost I. The investment is sunk, it can neither be changed, nor temporarily stopped, nor shut down. Operating and maintenance costs are comparatively small and set to zero.

Assumption 2 The price of the service (p) reported by the winning bidder is constant over the franchise term.

Assumption 3 At any time $t \geq 0$ there is a mass y_t of identical consumers, each of whom has an inelastic demand for one unit of the service up to some reservation price $p^{\max}$.

[5] An example is provided by toll roads. Demand for a highway is largely beyond the franchise holder, traffic forecasts are notoriously imprecise, and it is difficult to make accurate traffic predictions especially in the long term (Engel, Fisher, and Galetovic, 2001). Moreover, the service is fairly standard, and there is a limited scope for creativity on the part of an operator.

[6] For the effect of concession length on the concession value see Engel, Fischer and Galetovic (2001) and D'Alpaos, Dosi and Moretto (2006).

Assumption 4 The timing of the demand is as follows. Current demand ($t=0$) is y_0, but at $t=1$ it may either rise to $(1+u)y_0$ with probability q , or decrease to $(1-d)y_0$ with probability $1-q$ ($u>0$ and $0<d<1$):

$$
\begin{array}{llll}
 & \nearrow & y_1^+ = (1+u)y_0 & \text{with probability } q \\
y_0 & & & \\
 & \searrow & y_1^- = (1-d)y_0 & \text{with probability } 1-q
\end{array}
$$

From $t>1$, the demand will rise (decrease) at the constant rate u (d).

By assumptions 1-4, we first derive the concession value at $t=0$ when the franchisee must immediately operate the service. Since the flow of profits that the firm will receive once the investment is undertaken is py_t for all $t \geq 0$, the discounted value of profit flows from time 1 onward, evaluated at time zero, is given by $py_0 \sum_{t=1}^{\infty} \frac{(1+u)^t}{(1+\rho)^t} \equiv py_0 \frac{1+u}{\rho-u}$, with probability q and $py_0 \sum_{t=1}^{\infty} \frac{(1-d)^t}{(1+\rho)^t} \equiv py_0 \frac{1-d}{\rho+d}$, with probability $1-q$ respectively, where $\rho>u$ is the constant discount rate.

Lemma 1 *The expected Net Present Value at* $t=0$ *is :*

$$NPV^0 = (p-\tilde{p})K_0 \tag{1}$$

where:

$$\tilde{p} \equiv \frac{I}{K_0}, \qquad \textit{and} \qquad K_0 \equiv \left[1+q\frac{1+u}{\rho-u}+(1-q)\frac{1-d}{\rho+d}\right] y_0$$

Proof. See Appendix A ■

In (1), $\tilde{p}K_0$ represents the minimal discounted expected total cash flow for which the concession has a positive value.

Consider now the case where the winning bidder is allowed to keep open the option to invest for one period. In this case, the conditon $NPV^0>0$ is no longer sufficient for immediately building the new infrastructure, since it does not account for the franchisee's ability to react to unfavorable market conditions (*e.g.* traffic levels falling short of expectations).

Since in our setting a period is sufficient for obtaining information on the investment profitability, the decision to wait is economically significant only if operating the service becomes profitable under the upward realization of the demand level (y_1^+). From now on we restrict the analysis only to this case, i.e. we assume that $py_0 \frac{1-d}{\rho+d} < \frac{I}{1+\rho} < py_0 \frac{1+u}{\rho-u}$ (Dixit and Pindyck, 1994).

Lemma 2 *The expected Net Present Value at $t = 1$ as of today is:*

$$NPV^1 = (p - \tilde{p})K_0 + (\hat{p} - p)K_1 \tag{2}$$

where:

$$\hat{p} \equiv \frac{1 + \rho - q}{1 + \rho}\frac{I}{K_1} \quad \textit{and} \quad K_1 \equiv \left[1 + (1 - q)\frac{1 - d}{\rho + d}\right] y_0$$

Proof. See Appendix B ■

By putting together (1) and (2), we get the concession value, which accounts for how much the option to delay the investment is worth.

Proposition 1 *For any given p, the concession value is:*

$$\begin{aligned} V(p) &= \max\left[NPV^0, NPV^1\right] \\ &\equiv (p - \tilde{p})K_0 + \max\left[(\hat{p} - p)K_1, 0\right] \end{aligned} \tag{3}$$

Proof. Straightforward from Lemma 1 and 2. ■

The second term on the r.h.s. of (3) represents the option value embedded in a contract which does not impose the obligation to immediately afford sunk capital costs. Since $K_0 - K_1 > 0$, by defining $\bar{p} \equiv \phi\tilde{p} + (1 - \phi)\hat{p}$, where $\phi \equiv \frac{K_0}{K_0 - K_1} > 1$ and $(1 - \phi) \equiv -\frac{K_1}{K_0 - K_1} < 0$,[7] equation (3) can be rewritten as follows:

$$V(p) = \max[(p - \tilde{p})K_0, (p - \bar{p})(K_0 - K_1)]. \tag{4}$$

Finally, for the rest of the paper we add the following assumption:

Assumption 5 $\frac{\hat{p}}{\tilde{p}} \equiv \frac{1+\rho-q}{1+\rho}\frac{K_0}{K_1} > 1$ and $\frac{\bar{p}}{\tilde{p}} \equiv \frac{q}{1+\rho}\frac{K_0}{K_0 - K_1} < 1$.

which ensures that $0 < \bar{p} < \tilde{p} < \hat{p}$, and provides the following optimal investment rule (See Figure 1):

if $p > \hat{p}$ it is optimal to invest at $t = 0$
if $\bar{p} < p < \hat{p}$ it is optimal to invest at $t = 1$
if $p < \bar{p}$ it is never optimal to invest.

[7] It is easy to see that $\bar{p} \equiv \frac{I}{1+\rho}\frac{q}{K_0 - K_1} > 0$

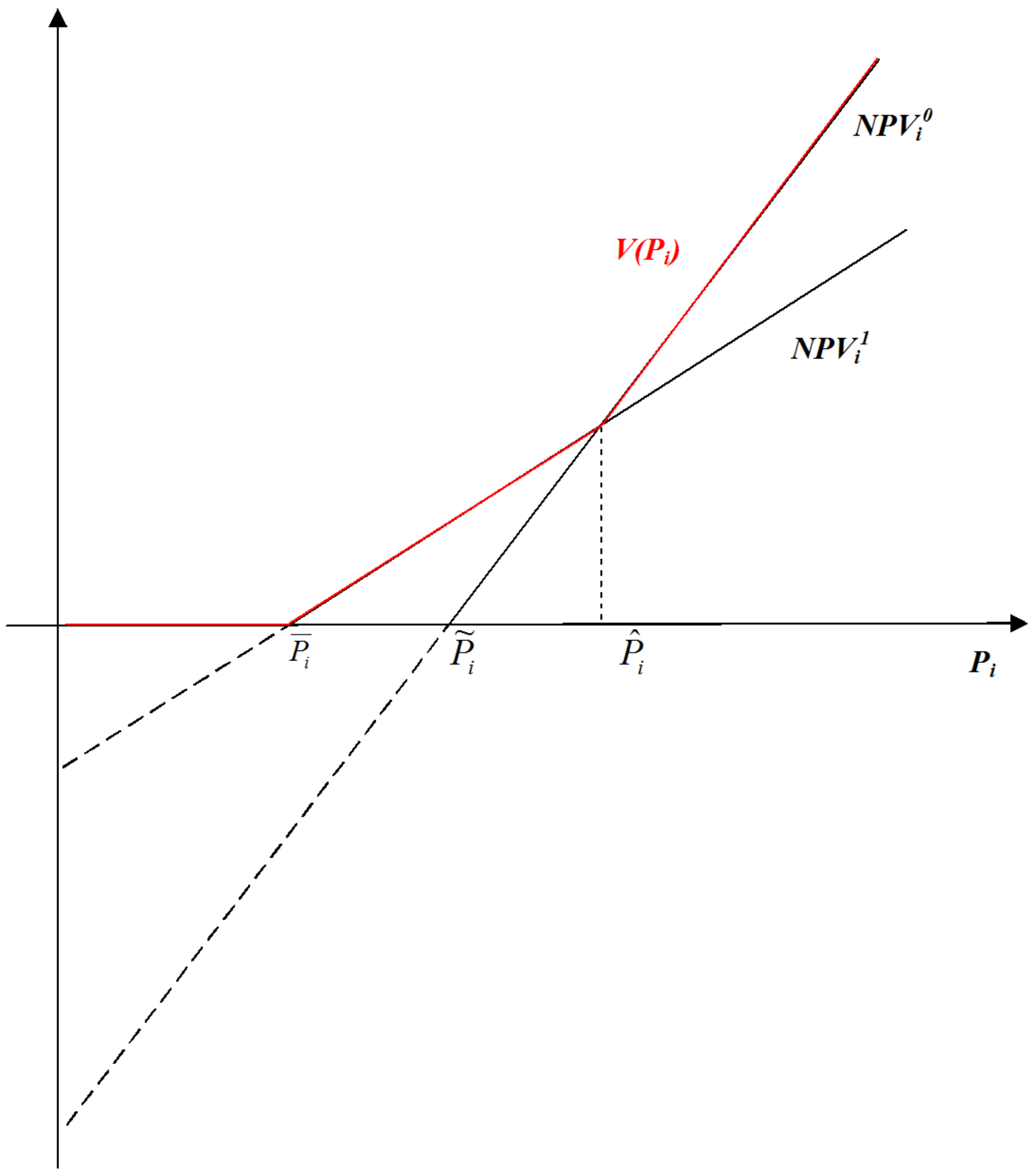

Figure 1: The NPV with time flexibility

3 Auction formats and concession design

A firm can operate the service only after submitting a two-dimensional successful bid. In particular, each firm must report the price at which it will commit itself to supply the market (p), and the up-front ($t = 0$) payment to the government (R). Two alternative sealed-auction formats are considered in this paper:

- The concession is awarded to the bidder offering the lowest price. Should two or more firms report the same tariff, the franchise will be awarded to the bidder offering the highest fee (*LPHF auction format*).
- The concession is awarded to the bidder offering the highest fee. Should two or more firms report the same payment, the franchise will be awarded to the bidder offering the lowest price (*HFLP auction format*).

The above formats allow the government to break a tie voluntarily, by awarding the concession to the firm that reports the highest fee (LPHF format) or the lowest price (HFLP format).[8]

We analyze the effects of these bidding rules by considering two alternative contracts:

- The franchisee is not allowed to delay the investment, i.e. the service must be operated at $t = 0$ (*Case 1*).
- The franchisee is allowed to keep the option to operate the service alive for one period (*Case 2*).

We conclude the model set-up by adding the following assumptions:

Assumption 6 There are N competing firms.

Assumption 7 Each bidder i ($i = 1, 2, ...N$) observes y_0 and the multiplicative parameters (u, d), knows the distribution $(q, 1 - q)$ and the realization of the investment cost I_i,and only knows that I_j, $j \neq i$ are independent random variables, with the same absolutely continuous

[8] If there are more firms that submit the same two-dimensional bid (p, R), then a random drawing determines the winner.

distribution G, with positive density g over the interval $I = [I^l, I^u] \subseteq \mathbb{R}$. For the sake of simplicity, we assume that capital costs are uniformly distributed on I with $I^l = 0$.[9]

Assumption 8 $p^{\max} \geq \tilde{p}^u \equiv \frac{I^u}{K_0}$, i.e. the consumers' reservation price is such that even the most inefficient firm would be interested in operating the service.

Assumption 9 Bidders are not subject to any liquidity or budget constraint, so that each firm i has sufficient resources to pay the up-front fee after winning the auction.

Assumption 10 The investment time is verifiable by the government and contract terms cannot be renegotiated.

Finally, all aspects of the bidding situation are known to the government except for the investment costs I_i $(i = 1, 2, ...N)$ known only by each firm itself.

3.1 Case 1

Consider the outcome of the two auction formats when the government design the concession so as to impose the obligation to immediately undertake the investment.

Since bidders will play so as to avoid being involved in ties with a positive probability, under the *LPHF* format the firms' optimal strategy is to choose first the lowest price that maximizes their probability of winning and then, conditional on this tariff, report the highest fee. This is indeed an application of the invariance result established by Jackson and Swinkels (2004) which states that if a "strategy profile forms an equilibrium for one omniscient tie-breaking rule, it remains an equilibrium for any other trade-maximizing omniscient tie-breaking rule" (p.2). In other words, how bidders behave in the event of a tie and the tie-breaking then used are irrelevant for the existence of a pure strategy equilibrium.[10]

[9] None of the results depend on the assumption that $G(I)$ is a uniform distribution as long as $I + \frac{G(I)}{g(I)}$ is a monotone increasing function.

[10] Jackson and Swinkels's approach is to show that an equilibrium exists in an auxiliary game in which tie-breaking is endogenously chosen and then to show that the sharing rule is, in fact, irrelevant. See also Simon and Zame (1990) for a full formal analysis of

According to the invariance result, the bidders' pricing problem reduces to a Bertrand game where each firm picks up the lowest price p that maximizes the expected net present value NPV^0 as defined in (1).

$$\max_{p_i} NPV^0(p_i; \tilde{p}_i) \Pr\left[\min_{j \neq i} p_j \geq p_i\right] \tag{5}$$

Further, as the firm reporting the lowest price is the one with the highest NPV^0, it will minimize the probability of loosing by offering the highest fee. Then, conditionally on $p_i(\tilde{p}_i)$, we obtain the fee by maximizing:

$$\max_{R_i^0} \left[NPV^0(p(\tilde{p}_i); \tilde{p}_i) - R_i^0\right] \Pr\left[\max_{j \neq i} R^0{}_j \leq R_i^0\right] \tag{6}$$

The equilibrium strategy for the *LPHF* format is summarized in the following Lemma.

Lemma 3 *When the concessionaire is not allowed to delay the investment, the LPHF auction involves the following unique symmetric equilibrium strategy rules:*

$$p_i = p(\tilde{p}_i) \equiv (1 - \frac{1}{N})\tilde{p}_i + \frac{1}{N}\tilde{p}^u \leq \tilde{p}^u \tag{7}$$

$$R_i^0 = \frac{N-1}{N} NPV_i^0 \equiv \frac{N-1}{N}\left[\frac{1}{N}(\tilde{p}^u - \tilde{p}_i)K_0\right] \tag{8}$$

Proof. See Appendix C ■

Going back to the definition of NPV^0, since by assumption 7 the threshold levels $\tilde{p}_i$ are distributed uniformly within the support $\tilde{P} = [0, \tilde{p}^u]$, equation (7) implies that also NPV_i^0 are uniformly distributed over the interval $[0, NPV_u^0]$, with interim profits positive for all types but the weakest firm, which never wins and whose NPV_l^0 is equal to zero even if it does win.

By substituting back (7) in the NPV_i^0, (1) can be rewritten as a function of the reported price:

$$NPV_i^0 \equiv \frac{1}{N-1}(\tilde{p}^u - p(\tilde{p}_i))K_0$$

endogenous sharing rule in discontinuous games. In the spirit of Simon and Zame we can think of the $LPHF$ auction format as a two-stage game where bidders choose the price in the first stage and then the fee in the second stage in order to prevent tie (the reverse holds for the $HFLP$ format).

In other words, the bidder reporting the lowest price is indeed the one with the highest NPV^0. Then, besides the fact that the concession is awarded to the bidder that reports the lowest tariff, it is a dominant strategy for all firms to offer the highest fee in order not to increase the rivals' probability of winning.

The same line of reasoning applies for the *HFLP* format.

Proposition 2 *When the concessionaire is not allowed to delay the investment, the two auction formats involve the same outcome: the concession will be awarded to the most efficient firm which will report the two-dimensional bid* (p, R^0) *defined by (7) and (8).*

Proof. See Appendix D. ■

The above result is not surprising. In fact, as long as the government imposes the obligation to immediately invest, the same outcome can be replicated by a third auction format, where the government selects the winning bidder according to a scoring rule (a first-score auction). More specifically let's assume that the government is committed to awarding the franchise to the firm that obtains the highest score $s^0(p_i, R_i^0)$, defined as:

$$s_i^0 = R_i^0 - \Delta(p_i) \tag{9}$$

where $\Delta(p_i) \equiv \int_0^{p_i} (N-1)\frac{(x-p^{-1}(x))}{(\tilde{p}^u - x)} K_0 dx$ and $p(.)$ is the optimal pricing rule defined as in (7). Since $\frac{1}{N-1}(x - p^{-1}(x))K_0$ is the NPV^0 evaluated under the optimal price of the service, the term $\Delta(p_i)$ is increasing in p_i, and the score increases as the fee increases and/or the price reduces.

Proposition 3 *A unique symmetric equilibrium of the first-score auction is one in which each firm offers the two-dimensional bid* (p_i, R^0) *defined by (7) and (8).*

Proof. See Appendix E ■

Similarly to Che (1993), we find that the scoring rule (9) involves systematic distortion against the concession fee. In other words, since in order to win the auction the bidders must compete both in the price and in the fee, an optimal scoring rule should reduce the fee below the level that the firm would have reported if the price had been imposed by the government. In fact, letting:

$$s_0^0(\tilde{p}_i) = \max_{p_i} \left[NPV^0(p_i; \tilde{p}_i) - \Delta(p_i) \right] \tag{10}$$

the problem can be seen as one in which each firm, indexed by its adjusted expected project value $s_0^0(\tilde{p}_i)$, proposes to meet the level of score s_i^0, i.e.:

$$\left[s_0^0(\tilde{p}_i) - s_i^0\right] \Pr(\max_{j \neq i} s_j^0 \leq s_i^0)$$

or substituting (10) and (9)

$$\left[NPV^0(p(\tilde{p}_i); \tilde{p}_i) - R_i^0\right] \Pr\left[\max_{j \neq i} R^0{}_j \leq R_i^0\right]$$

which is equivalent to (6).

3.2 Case 2

In the previous section we have shown that when the contract imposes the obligation to immediately operate the service, the two auction formats involve identical outcomes in terms of price to consumers and concession fee. Does this equivalence still hold when the franchisee is allowed to postpone the investment?

We begin by identifying the equilibrium strategy under the *LPHF* auction. As in section 3.1, by the Jackson and Swinkels' invariance result, bidders' optimal strategy is to choose first the lowest price and then report the fee. The firms' pricing problem is still a Bertrand game where the project value to be maximized is now given by $V(p_i) = \max\left[NPV^0, NPV^1\right]$. In other words, each bidder selects two prices contingent to the investment time, and reports the one that maximizes the probability of winning the auction. As the firm reporting the lowest tariff is also the one with the highest $V(p_i)$, it will minimizes the probability of being involved in a tie by reporting the highest fee.

The equilibrium strategy for the *LPHF* auction is summarized in the following Lemma.

Lemma 4 *When the concessionaire is allowed to delay the investment, the LPHF format involves the following unique symmetric equilibrium strategy rules:*

$$p(\bar{p}_i) = (1 - \frac{1}{N})\bar{p}_i + \frac{1}{N}\bar{p}^u \leq \bar{p}^u \tag{11}$$

$$R_i^1 = \frac{N-1}{N} NPV_i^1 \equiv \frac{N-1}{N}\left[\frac{1}{N}(\bar{p}^u - \bar{p}_i)(K_0 - K_1)\right] \tag{12}$$

Proof. See Appendix F ∎

By direct inspection of (7) and (11), it is easy to show that:

$$p(\bar{p}_i) \leq p(\tilde{p}_i), \quad \text{for all } i. \tag{13}$$

and then:

$$R_i^1 \leq R_i^0, \quad \text{for all } \ i \tag{14}$$

Disequality (13) implies that competing by maximizing NPV_i^1 is a dominant strategy when the price plays a key role in winning the auction, as occurs under the *LPHF* format. For instance, by exploiting the time flexibility, bidders are able to submit a price $(p(\bar{p}_i))$ lower than the one they would be able to announce if they adopted NPV_i^0 as a reference, as occurs when agents compete to acquire a contract which transfers all risks to the concessionaire, by ruling out time flexibility.

By contrast, (14) suggests that bidders will not find it profitable to exploit time flexibility when the concession fee plays a key role in the auction *(HFLP)*. For instance, by referring to NPV_i^0, bidders will report a fee (R_i^0) higher than the payment they would have reported if they referred to NPV_i^1.

Proposition 4 *When the concessionaire is allowed to delay the investment, the two auction formats involve different outcomes:*

- *Under HFLP the concession will be awarded to the most efficient firm that reports the two-dimensional bid* $(p(\tilde{p}_i), R_i^0)$
- *Under LPHF the concession will be awarded to the most efficient firm that reports the two-dimensional bid* $(p(\bar{p}_i), R_i^1)$

Proof. Straightforward from Lemma 3 and 4 ∎

4 Welfare comparison

4.1 The welfare function

We found that the two auction formats involve the same outcome in terms of price to consumers and concession fee when the contract rules out investment time flexibility. Moreover, this outcome is equal to the one which would emerge if the government awarded a contract which does not impose the obligation to immediately operate the service by using the *HFLP* format. Consequently, the government's choice reduces to the following alternatives: i) impose the obligation to invest immediately (in this case the bidding rule is irrelevant), ii) allow the winning bidder to delay the investment, awarding the concession by using the *LPHF* format.

In order to provide a decision rule, we assume that from the government's point of view a euro in the pocket of consumers and a euro in the hand of a public authority are equally valuable, and that the objective function does not include the winning bidder's net profits: [11]

$$W = E(\mathbf{S}) + E(\mathbf{R})$$

where $E(S)$ and $E(\mathbf{R})$ are the expected discounted consumer surplus and the expected government's revenue respectively. In particular, for the former, we need to distinguish between the consumer surplus if the concessionaire operates the service at $t = 0$ ($\mathbf{S}^0$) from the consumer surplus if the firm invests at $t = 1$ ($\mathbf{S}^1$):

$$\mathbf{S}^0 = \sum_{t=0}^{\infty} \frac{1}{(1+\rho)^t} \int_{p_i(\tilde{p}_i)}^{p^{\max}} E_0(y_t)dp \,, \quad \text{and} \quad \mathbf{S}^1 = q \left\{ \sum_{t=1}^{\infty} \frac{1}{(1+\rho)^t} \int_{p_i(\tilde{p}_i)}^{p^{\max}} y_t^+ dp \right\}$$

where $\mathbf{S}^1$ is evaluated at $t = 1$ as of today and only for y_t^+.

The following Lemma gives the values of the consumer surplus and the government's revenue under the two auction formats with and without time flexibility.

Lemma 5 *i) LPHF (without investment time flexibility) and HFLP (with or without flexibility) provide the following expected consumer surplus and government's revenue:*

$$E(\mathbf{S}^0) = \left[p^{\max} - \frac{1}{2}\frac{N+1}{N}\tilde{p}^u \right] K_0$$

$$E(\mathbf{R}^0) = \frac{N-1}{N(N+1)}\tilde{p}^u K_0$$

[11] Since the fee is a constant fraction of the concession value, in qualitative terms the results of the comparative welfare analysis would not change if the welfare were defined as the sum of the consumer surplus and the (firm's) project value.

ii) LPHF (with investment time flexibility) provides the following expected consumer surplus and government's revenue:

$$E(\mathbf{S}^1) = \left[p^{\max} - \frac{1}{2}\frac{N+1}{N}\bar{p}^u\right](K_0 - K_1)$$

$$E(\mathbf{R}^1) = \frac{N-1}{N(N+1)}\bar{p}^u(K_0 - K_1)$$

Proof. See Appendix G ■

From Lemma 5 it is easy to show that:

$$E(\mathbf{R}^1) - E(\mathbf{R}^0) \equiv -\frac{N-1}{N(N+1)}\frac{1+\rho-q}{1+\rho}I^u < 0$$

and:

$$E(\mathbf{S}^1) - E(\mathbf{S}^0) \equiv -p^{\max}K_1 + \frac{1}{2}\frac{N+1}{N}\frac{1+\rho-q}{1+\rho}I^u$$

Thus, investment time flexibility, by inducing the bidders to reduce the price, raises the consumer surplus but has a detrimental effect on the government's revenue. Then, by defining $\Delta W^{1,0}$ as:

$$\begin{aligned} \Delta W^{1,0} &= [E(\mathbf{S}^1) + E(\mathbf{R}^1)] - [E(\mathbf{S}^0) + E(\mathbf{R}^0)] \qquad (15) \\ &\equiv -p^{\max}K_1 + \frac{1+\rho-q}{1+\rho}\frac{N^2+1}{2N(N+1)}I^u \end{aligned}$$

we get the following proposition.

Proposition 5 *i) If* $\Delta W^{1,0} > 0$*, a contract which allows the concessionaire to optimally decide the investment timing involves the highest expected welfare value, provided the franchise is awarded according to the LPHF bidding rule.*

ii) If $\Delta W^{1,0} < 0$*, investment time flexibility does not provide any higher welfare value.*

Proof. *Straightforward from Lemma 5.* ■

The second part of the proposition deserves some comments. Since $\Delta W^{1,0} < 0$ means that allowing the winner bidder to decide the investment time does not increase the expected welfare value, from the government's point of view, imposing the obligation to invest immediately or allowing the franchise holder to decide when to roll-out the service becomes irrelevant. However, whereas in the former case the overall welfare value is not affected by the bidding rule, in the latter case it becomes more socially profitable to award the concession through the *HFLP* format.

4.2 Comparative statics analysis

Comparative statics analysis provides insights into the effect of some key parameters upon the payoff of alternative concession arrangements and bidding rules. In particular, let's consider how $\Delta W^{1,0}$ is affected by demand volatility (d), the number of bidders (N) and the upper boundary of the investment cost (I^u).

$$\frac{\partial \Delta W^{1,0}}{\partial d} > 0 \tag{16}$$

$$\frac{\partial \Delta W^{1,0}}{\partial N} > 0 \tag{17}$$

$$\frac{\partial \Delta W^{1,0}}{\partial I^u} > 0 \tag{18}$$

The interpretation of (16) is straightforward if we refer to the Real Option Theory. For instance, an increase in demand volatility makes the option of waiting for new information before affording sunk costs more valuable; this, in turn, increases the value of a contract which does not impose the obligation to immediately invest. Under the *LPHF* format, bidders will exploit this option value by further reducing the price. This involves an increase in consumer surplus which more than compensates for the fall in expected government revenue.

As for the number of competitors, an increase in N tends to make a flexible contract and, consequently, the *LPHF* format more socially appealing. We get a similar result when the upper boundary of the investment cost (I^u) increases. This is because the *LPHF* format allows a larger number of inefficient firms to report relatively low prices which still assure a positive expected net present value. In effect, since the upper boundary I^u plays the role of "reserve price", regardless of the auction format, an increase in I^u, although it reduces the government revenue, involves an increase in the consumer surplus. However, since the *LPHF* format induces a level of competition on the price that is higher than the level of competition induced by the *HFLP* auction, the expected consumer surplus gain $E(\mathbf{S}^1) - E(\mathbf{S}^0)$ exceeds the fall in expected government revenue $E(\mathbf{R}^1) - E(\mathbf{R}^0)$.

Remark 1 *If the volatility of the demand increases, the level of competition increases, or firms' heterogeneity increases, the LPHF format tends to out-*

perform the HFLP format, provided the concessionaire is allowed to optimally decide the investment timing.

4.3 Demand elasticity

Since infrastructure services often exhibit a very low demand elasticity, our analysis has been carried out by assuming an inelastic demand. With a downward sloping demand curve, it seems plausible that the expected welfare benefits arising from awarding contract which gives the franchisee the right to decide when to operate the service tend to drop as the elasticity of demand increases.

For instance, since an increase in elasticity makes the profit function "more concave" in the price, firms will become more risk-averse (Spulberg, 1995). This causes an increase in equilibrium bids (Krishna, 2002) which, under the *LPHF* auction, takes on the form of a decrease in equilibrium prices involving an increase in the expected consumer surplus which is likely to more than compensate for the fall in public revenue.

Although the price competition generated by a downward sloping demand curve is present whether the contract allows or rules out investment time flexibility, the price reduction tends to be more marked in the second case since the managerial flexibility lessens the effects of risk aversion. Put another way, if the contract rules out any time flexibility, firms will be induced to bid more aggressively in order to "buy" insurance against the possibility of losing the franchise.

Remark 2 *An increase in demand elasticity tends to reduce the potential welfare gains arising from awarding a concession which allows the winning bidder to optimally decide the investment time.*

5 Final remarks

Concession arrangements and award procedures can take different forms and entail various legal and economic issues. In this paper we have focussed on the effects of bidding rules, by comparing the outcome of two sealed-auction formats which approximate actual practices:

- the concession is awarded to the bidder offering the lowest price charged to consumers; should two or more firms report the same price, the franchise will be awarded to the bidder offering the highest fee for the lease (*LPHF format*)

- the concession is awarded to the bidder offering the highest fee; should two or more firms report the same payment, the franchise will be awarded to the bidder offering the lowest price (*HFLP format*).

Our findings suggest that the choice between these auction formats can have a definitive effect on the price charged to consumers and the concession fee when the conceding authority gives the winning bidder the right to undertake the investment required to roll-out the service at a date of his choosing. By contrast, when the government imposes the obligation to immediately operate the service, the outcome of the award process is not affected by the bidding rule.

Another issue addressed in this paper is the effect of time flexibility on the expected social value. Although the effect is not univocal, the analysis has shown that when the volatility of the demand increases, the number of competitors increases, or the firms' heterogeneity increases, a contract allowing the franchisee to optimally decide the investment timing tends to outperform concession arrangements which transfer all risks to the concessionaire, by ruling out investment time flexibility.

However, in order to capture these potential welfare benefits, the contracts which give the option to delay the investment should be awarded by using a bidding rule which emphasizes the price charged to consumers rather than the fee paid to the government (*LPHF* format). For instance, if the option-to-delay were awarded via the *HFLP* format, firms would report the same two-dimensional bid which they would have reported if the conceding authority had imposed the obligation to immediately operate the service. In other words, the *HFLP* format would annul the effects of the greater competitive pressure deriving from the awarding of a contract which does not restrict the managerial autonomy of the franchisee.

Appendix

A Proof of Lemma 1

Assumptions 3 and 4 allow us to write the time evolution of demand as:

$$y_t \begin{array}{ll} \nearrow & y_t^+ = (1+u)^t y_0 \quad \text{with probability } q \\ \searrow & y_t^- = (1-d)^t y_0 \quad \text{with probability } 1-q \end{array} \quad \text{for all } t \geq 0 \tag{19}$$

The flow of profits that the concessionaire will receive once the investment is undertaken is simply:

$$\pi(y_t) = p y_t \quad \text{for all } t \geq 0 \tag{20}$$

Substituting (19) into (20), we are able to write the instantaneous profit function as:

$$\pi_t \begin{array}{ll} \nearrow & \pi_t^+ = (1+u)^t p y_0 \quad \text{with probability } q \\ \searrow & \pi_t^- = (1-d)^t p y_0 \quad \text{with probability } 1-q \end{array} \tag{21}$$

and the discounted value of profit flows from time 1 evaluated at time zero becomes:

$$\sum_{t=1}^{\infty} \frac{\pi_t}{(1+\rho)^t} \begin{array}{ll} \nearrow & \sum_{t=1}^{\infty} \frac{\pi_t^+}{(1+\rho)^t} = \frac{1+u}{\rho-u} p y_0 \quad \text{with probability } q \\ \searrow & \sum_{t=1}^{\infty} \frac{\pi_t^-}{(1+\rho)^t} = \frac{1-d}{\rho+d} p y_0 \text{ with probability } 1-q \end{array} \quad \text{for all } t \geq 0 \tag{22}$$

with $\rho - u > 0$. Referring to (21) and (22), the project's Net Present Value (NPV^0) is given by:

$$NPV^0 = \left[1 + q\frac{1+u}{\rho-u} + (1-q)\frac{1-d}{\rho+d}\right] p y_0 - I \tag{23}$$

from which it is easy to get the expression in the text:

$$NPV^0 = (p - \tilde{p}) K_0$$

where $\tilde{p} \equiv \frac{I}{K_0}$ and $K_0 \equiv \left[1 + q\frac{1+u}{\rho-u} + (1-q)\frac{1-d}{\rho+d}\right] y_0$. This concludes the proof.

B Proof of Lemma 2

As stated, if the firm is able to postpone the investment decision of one period, $NPV^0 > 0$ no longer constitutes a condition for immediately building the new infrastructure. As a result, in evaluating the NPV at time zero the firm has to consider this option value that must be included as part of the total cost of the investment. Operatively, the firm will compare the NPV^0 with the NPV^1 at $t = 1$ as of today, evaluated only for π_t^+:

$$NPV^1 = q\left[\sum_{t=1}^{\infty}\frac{\pi_t^+}{(1+\rho)^t} - \frac{I}{1+\rho}\right] \equiv q\left[\frac{1+u}{\rho-u}py_0 - \frac{I}{1+\rho}\right] \tag{24}$$

The overall project value is then given by:

$$\max\left[NPV^0, NPV^1\right] \tag{25}$$

Further, by (25), it is possible to calculate the value of the firm's i option to wait as:

$$OP^0 = \max\left[NPV^0, NPV^1\right] - NPV^0 = \max\left[NPV^1 - NPV^0, 0\right] \tag{26}$$

If $NPV^1 - NPV^0 > 0$ it is optimal to wait one period and decide to invest at $t = 1$ only in the case of good news. If, on the contrary, $NPV^1 - NPV^0 < 0$ it is optimal to invest at $t = 0$. Then, by imposing $NPV^0(\hat{p}) = NPV^1(\hat{p})$, (26) can be rewritten as follows:

$$OP^0 = \max\left[(\hat{p} - p)K_1, 0\right] \tag{27}$$

where $\hat{p} \equiv \frac{1+\rho-q}{1+\rho}\frac{I}{K_1}$ and $K_1 \equiv \left[1 + (1-q)\frac{1-d}{\rho+d}\right]y_0$. Substituting (27) back into (26) and solving for NPV^1 we get:

$$NPV^1 = NPV^0 + OP^0 \equiv (p - \tilde{p})K_0 + \max\left[(\hat{p} - p)K_1, 0\right]$$

This concludes the proof.

C Proof of Lemma 3

Before proving the Lemma let's formally set the problem. Consider the bidding decision of the firm i and suppose that all other firms use the symmetric strategy $(p(\tilde{p}_j)\ R^0(\tilde{p}_j))\ \forall j \neq i$ that specifies every bidder's willingness to pay. Further, let $H_i(p_i, R_i^0)$ denote the probability that firm i will win the auction with the two-dimensional bid (p_i, R_i^0) and the specified tie-breaking rule. Formally:

$$H_i(p_i, R_i^0) = \Pr\left[\min_{j\neq i} p(\tilde{p}_j) \geq p_i\right] + \Pr\left[\min_{j\neq i} p(\tilde{p}_j) = p_i\right] \times \tag{28}$$

$$\times \left\{\Pr\left[\max_{j\neq i} R^0(\tilde{p}_j) \leq R_i^0\right] + \frac{1}{1+k}\Pr\left[\max_{j\neq i} R^0(\tilde{p}_j) = R_i^0\right]\right\}$$

where k is the number of other bidders that bid exactly (p_i, R_i^0). The first term on the r.h.s. of (28) comes from events in which the firm i is the outright winner. The second term comes from events in which there is more than one firm that bids p_i and ties are resolved according to a second bid on the fee. Then, according to the tie-breaking rule, the firm i is the winner if it reports the highest fee R_i^0. Finally, if there is still more than one firm that bids the same (p_i, R_i^0), the winner is determined randomly from among those with the highest bid.

A bid (p_i, R_i^0) is a best response at $\tilde{p}_i$ (i.e. I_i) by the firm i if it maximizes its expected payoff against the rivals' strategies $(p(\tilde{p}_j), R^0(\tilde{p}_j), \forall j \neq i)$, that is, if for any feasible bid (p, R^0) we get:

$$\left[NPV^0(p_i;\tilde{p}_i) - R_i^0\right] H_i(p_i, R_i^0) \geq \left[NPV^0(p;\tilde{p}_i) - R_i^0\right] H_i(p, R^0)$$

Note that if $p(\tilde{p}_j)$ is a strictly monotone increasing function and $R^0(\tilde{p}_j)$ a strictly monotone decreasing function, then $H_i(p_i, R_i^0)$ is strictly increasing in the two arguments.

We solve the problem by exploting the invariance result established by Jackson and Swinkels (2004). The invariance result states that: 1) if a bidding strategy forms an equilibrium for one "omniscient" tie-breaking rule, it remains an equilibrium for any other trade-maximizing omniscient tie-breaking rule; 2) if a player has an improving deviation relative to some bidding strategy and tie-breaking rule, then there is a slight modification of the deviation strategy which is still improving but which in addition allows the player to avoid ties (Theorem 3 p. 24).

Since bidders prefer to avoid ties and the tie-breaking rules are not important in establishing the existence of the bidding equilibrium, the problem can be splitted into two sub-problems. First, we determine the pricing rule assuming zero probability of having a tie:

$$p_i = \arg\max NPV^0(p_i;\tilde{p}_i)\Pr\left[\min_{j\neq i} p(\tilde{p}_j) \geq p_i\right] \tag{29}$$

Second, since in the case of a positive probability of having a tie the players will strictly benefit by reporting a positive fee, conditionally on $p_i(\tilde{p}_i)$, we derive the optimal fee as:

$$R_i^0 = \arg\max\left[NPV^0(p(\tilde{p}_i);\tilde{p}_i) - R_i^0\right]\Pr\left[\max_{j\neq i} R^0(\tilde{p}_j) \leq R_i^0\right] \tag{30}$$

Let's begin with (29). We show that a price strategy for firm i is a symmetric function $p(\tilde{p}_i)$ mapping from the set of firm types $\tilde{P} = [0, \tilde{p}^u]$ to the set of possible prices $P \subset \mathbb{R}_+$.Yet, for each firm i this function is continuously differentiable and strictly increasing with the property that $p'(\tilde{p}_i) < 1$ and $p(\tilde{p}^u) = \tilde{p}^u$.

Let's assume that each bidder makes rational conjectures about the distribution of the rivals' prices represented by a common distribution function $F(p)$, which is strictly increasing on the interval $P \subset \mathbb{R}_+$, and the hazard rate $h(p) \equiv \frac{f(p)}{1-F(p)}$ is increasing in p. This assumption allows definition of $F^{(N-1)}(p_i) \equiv 1-(1-F(p_i))^{N-1}$ as the cumulative distribution (with density $f^{(N-1)}(p_i)$) of the minimum of the $N-1$ rivals' price, i.e. the probability that all the other bidders set lower tariffs than i on the same support P. We can then write the firm i 's expected payoff (29) as:

$$(p_i - \tilde{p}_i)K_0(1-F(p_i))^{N-1} \tag{31}$$

Maximizing (31) with respect to p_i yields the necessary condition:

$$(1-F(p_i))^{N-1}[1-(N-1)(p_i-\tilde{p}_i)h(p_i)] = 0$$

from which we get:

$$p_i = \tilde{p}_i + \frac{1}{(N-1)h(p_i)} \tag{32}$$

By the assumption $h'(p_i) > 0$ the second order condition is always satisfied, i.e.: $-(p_i - \tilde{p}_i)h'(p_i) - h(p_i) < 0$.

Since the costs are uniformly distributed on $I = [0, I^u]$, also $\tilde{p}_i$ are distributed uniformly within the support $\tilde{P} = [0, \tilde{p}^u]$. Furthermore, the less efficient firm knows for certain that it will lose the auction, then $h(p) \to \infty$ and from

(32) we get $p_i \longrightarrow \tilde{p}^u$: i.e. the firm has a project value that is too low to win and then fixes as price $p = \tilde{p}^u$. Finally, $\frac{dp_i}{d\tilde{p}_i} = -\frac{-1}{1+\frac{h'(p_i)}{(N-1)h(p_i)^2}} > 0$ and < 1.

So far we have assumed that I_i (i.e. $\tilde{p}_i$) is private information, but used the distribution $F(.)$ over the rivals' price strategies to derive the firm i optimal price. To characterize the link between the distribution of I_i ($\tilde{p}_i$) and the firm's conjecture on output prices we impose:

$$F(p_i) = G(\tilde{p}_i) = \frac{\tilde{p}_i}{\tilde{p}^u} \equiv \frac{I_i}{I^u} \tag{33}$$

This is a problem of statistical inference. We need to ensure that the function $p_i(.)$ of the random variable I_i (i.e. $\tilde{p}_i$) is itself a random variable and to induce the distribution of p_i from the distribution of I_i (i.e. $\tilde{p}_i$). This procedure is an example of the distributional strategies approach introduced by Milgrom and Weber (1985). Since the investment costs are uniformly distributed over $I = [0, I^u]$, by (33) and the hazard rate we get:

$$h(p_i) \equiv \frac{f(p_i)}{1 - F(p_i)} = \frac{\frac{1}{\tilde{p}^u}}{1 - \frac{\tilde{p}_i}{\tilde{p}^u}} \frac{d\tilde{p}_i}{dp_i}$$

from which:

$$\frac{dp_i}{d\tilde{p}_i} = \frac{1}{h(p_i)} \frac{1}{\tilde{p}^u - \tilde{p}_i}$$

By (32):

$$(\tilde{p}^u - \tilde{p}_i)\frac{dp_i}{d\tilde{p}_i} = \frac{1}{h(p_i)} \equiv (N-1)(p_i - \tilde{p}_i) \tag{34}$$

The above equality can be expressed as a first order differential equation in $p(\tilde{p})$ as:

$$p'(\tilde{p})(\tilde{p}^u - \tilde{p}_i) - p(\tilde{p})(N-1) + \tilde{p}(N-1) = 0 \tag{35}$$

with the boundary condition that $p(\tilde{p}^u) = \tilde{p}^u$. By the linearity of (35) we can try a solution of type:

$$p(\tilde{p}) = A\tilde{p} + B \tag{36}$$

Substituting (36) in (35) and rearranging we obtain :

$$\begin{aligned} A(\tilde{p}^u - \tilde{p}_i) - (A\tilde{p} + B)(N-1) + \tilde{p}(N-1) &= 0 \\ [-A - A(N-1) + (N-1)]\tilde{p} + A\tilde{p}^u - B(N-1) &= 0 \end{aligned}$$

from which, defining $A = \frac{N-1}{N}$ and $B = \frac{\tilde{p}^u}{N}$, we get:

$$p(\tilde{p}_i) = (1 - \frac{1}{N})\tilde{p}_i + \frac{1}{N}\tilde{p}^u \tag{37}$$

This proves the first part of the proposition.

Let's now turn to the second sub-problem. Since the firms know in advance that in the event of a tie the regulator will break the tie basing on the reported fee, it is a dominant strategy for all firms to offer the highest fee in order not to increase the rivals' probability of winning. Substituting (37) into (1), the NPV_i^0 becomes:

$$NPV_i^0 \equiv (p_i - \tilde{p}_i)K_0 \equiv \frac{1}{N}(\tilde{p}^u - \tilde{p}_i)K_0 \tag{38}$$

From (38) the weakest firm does not give any value to the project, i.e. $NPV_l^0 \equiv \frac{1}{N}(\tilde{p}^u - \tilde{p}^u)K_0 = 0$. Since the thresholds $\tilde{p}_i$ are distributed uniformly within $\tilde{P}= [0, \tilde{p}^u]$, the bidding problem becomes equivalent to the case where each bidder i assigns a value to the project which is also distributed uniformly over the interval $[0, NPV_u^0]$. The equilibrium strategy form (30) calls upon a firm to bid a constant fraction of its NPV (Krishna, 2002, p. 19), i.e.:

$$R_i^0 = \frac{N-1}{N} NPV_i^0 \equiv \frac{N-1}{N}\left[\frac{1}{N}(\tilde{p}^u - \tilde{p}_i)K_0\right] \equiv \frac{1}{N}[\tilde{p}^u - p(\tilde{p}_i)]K_0$$

This concludes the proof of the Lemma.

D Proof of Proposition 2

To prove Proposition 2 it is sufficient to show that by reversing the proof of Lemma 3, we get the same result. Let's first assume that there is a symmetric price rule $p : [0, \tilde{p}^u] \to [0, p^u]$ which is strictly increasing with $p'(\tilde{p}_i) < 1$ and boundary condition $p(\tilde{p}^u) = \tilde{p}^u$. By (1), the project value can be expressed as $NPV^0(\tilde{p}_i) \equiv (p(\tilde{p}_i) - \tilde{p}_i)K_0$, where $NPV^0 : [0, \tilde{p}^u] \to [NPV_u^0, 0]$ is a strictly decreasing function.

Let's now consider the bidding decision of firm i. Assuming that all other firms use a strictly monotone decreasing bid function $R^0(\tilde{p}_i) : [0, \tilde{p}^u] \to$

$[R^0(0), R^0(\tilde{p}^u)]$ $\forall i$ that specifies every bidder's willingness to pay, the firm i's expected payoff from bidding R_i^0 is:

$$\left[NPV^0(\tilde{p}_i) - R_i^0\right] \Pr\left[\max_{j \neq i} R^0(\tilde{p}_j) \leq R_i^0\right]$$

Since $R^0(\tilde{p}_i)$ is monotone in $[0, \tilde{p}^u]$, the probability of winning when bidding the amount R_i^0 against rivals who play the strategy $R^0(\tilde{p}_j)$, $j \neq i$ is $\Pr\{R^0(\tilde{p}_j) \leq R_i^0) \; \forall j \neq i\} = \Pr(R^{0^{-1}}(R_j^0) \geq \tilde{p}_i \mid \forall j \neq i) = 1 - G^{(N-1)}(\tilde{p}_i) \equiv \left(\frac{\tilde{p}^u - \tilde{p}_i}{\tilde{p}^u}\right)^{N-1}$. That is, since $R^0(\tilde{p}_i)$ is one-to-one in $[0, \tilde{p}^u]$, choosing a bid in $[R^0(0), R^0(\tilde{p}^u)]$ is equivalent to choosing a $\tilde{p}_i$ in $[0, \tilde{p}^u]$. We can then write the firm i 's expected payoff as:

$$U(\tilde{p}_i) \equiv \left[NPV^0(\tilde{p}_i) - R^0(\tilde{p}_i)\right](1 - G^{(N-1)}(\tilde{p}_i)) \tag{39}$$

from which it is deduced that $NPV^0(\tilde{p}_i) - R^0(\tilde{p}_i)$ must be non-negative to guarantee a positive expected payoff (otherwise winning the auction would be unprofitable). Let's suppose that bidder i submits a bid $R^0(\check{p}_i)$ when his or her true trigger is $\tilde{p}_i$. Maximizing (39) with respect to $\check{p}_i$ and imposing the truth-telling condition $\check{p}_i = \tilde{p}_i$ yields the necessary condition:

$$0 = \frac{\partial U(\check{p}_i, \tilde{p}_i)}{\partial \check{p}_i} \mid_{\check{p}_i = \tilde{p}_i} = -R'^0(\tilde{p}_i)(1 - G^{(N-1)}(\tilde{p}_i)) - \left[NPV^0(\tilde{p}_i) - R^0(\tilde{p}_i)\right] g^{(N-1)}(\tilde{p}_i). \tag{40}$$

By (40), the maximization problem can be reduced to the following first-order linear differential equation:

$$R'^0(\tilde{p}_i)(1 - G^{(N-1)}(\tilde{p}_i)) = -\left[NPV^0(\tilde{p}_i) - R^0(\tilde{p}_i)\right] g^{(N-1)}(\tilde{p}_i)$$

and rearranging we get: $NPV^0(\tilde{p}_i)d(1 - G^{(N-1)}(\tilde{p}_i)) = R'^0(\tilde{p}_i)(1 - G^{(N-1)}(\tilde{p}_i)) - R^0(\tilde{p}_i)g^{(N-1)}(\tilde{p}_i) \equiv dR^0(\tilde{p}_i)(1 - G^{(N-1)}(\tilde{p}_i))$. Since $G^{(N-1)}(\tilde{p}^u) = 1$, integration yields:

$$-R^0(\tilde{p}_i)(1 - G^{(N-1)}(\tilde{p}_i)) = \int_{\tilde{p}_i}^{\tilde{p}^u} NPV^0(y)d(1 - G^{(N-1)}(y)), \tag{41}$$

and

$$R^0(\tilde{p}_i) = (N-1)\int_{\tilde{p}_i}^{\tilde{p}^u} NPV^0(y)\frac{(\tilde{p}^u - y)^{N-2}}{(\tilde{p}^u - \tilde{p}_i)^{N-1}}dy \quad \text{for any } \tilde{p}_i < \tilde{p}^u$$

By standard arguments, it easy to show that if the bidder i's private trigger is equal to the upper value $\tilde{p}^u$, his or her bid must be equal to the current value of the project, i.e. $R^0(\tilde{p}^u) = NPV^0(\tilde{p}^u) = 0$. This makes zero expected profit for the worst bidder and ensures that the proposed equilibrium is unique in $[0, \tilde{p}^u]$ (Krishna, 2002, p. 17). Furthermore, differentiating (41) with respect to $\tilde{p}_i$ confirms the assumed monotonicity of the optimal strategy $R^0(\tilde{p}_i)$:

$$\frac{d}{d\tilde{p}_i} R^0(\tilde{p}_i) = \frac{(N-1)}{(\tilde{p}^u - \tilde{p}_i)} \left[R^0(\tilde{p}_i) - NPV^0(\tilde{p}_i) \right] < 0 \text{ for all } \tilde{p}_i \in [0, \tilde{p}^u) \tag{42}$$

and by continuity for $\tilde{p}_i = \tilde{p}^u$ as well. Finally, the monotonicity of $NPV^0(\tilde{p}_i)$ also assures the sufficiency of (40).

So far we have assumed the existence of the price rule $p(\tilde{p}_i)$ and its properties. However it can be easily derived on the lines of Lemma 3. It is useful to note that since $p(\tilde{p}_i)$ is one-to-one in $[0, \tilde{p}^u]$, choosing a price p_i in $[0, p^u]$ is equivalent to choosing a trigger $\tilde{p}_i$ in $[0, \tilde{p}^u]$. Then the bidder i's direct utility function (under the truth-telling condition) can be written as:

$$\begin{aligned} U(\tilde{p}_i) &\equiv \left[(p(\tilde{p}_i) - \tilde{p}_i) K_0 - R^0(\tilde{p}_i) \right] (1 - G^{(N-1)}(\tilde{p}_i)) \\ &= \left[(p_i - \tilde{p}_i) K_0 - R^0(\tilde{p}_i) \right] (1 - F^{(N-1)}(p_i)) \end{aligned} \tag{43}$$

where $F(p_i) = G(\tilde{p}_i)$ stands for the firm i rational conjecture about the distribution of the rivals' prices. For any $R^0(\tilde{p}_i) < (p_i - \tilde{p}_i)K_0$, the firm will maximize (43) by choosing p_i such that the expected revenue $(p_i - \tilde{p}_i)K_0(1 - F^{(N-1)}(p_i))$ is maximum. Thus, Lemma 3 confirms that $p(\tilde{p}_i)$ is linear in $\tilde{p}_i$ with $p'(\tilde{p}_i) < 1$ and $p(\tilde{p}^u) = \tilde{p}^u$. This concludes the proof.

E Proof of Proposition 3

To prove this proposition we follow Che (1993, Proposition 2). The first step is to show that under the first-score auction the price is chosen independently of the score and it is given by:

$$p_i = \arg\max \left\{ NPV^0(p_i; \tilde{p}_i) - \Delta(p_i) \right\} \tag{44}$$

In addition, since

$$\begin{aligned} \frac{dNPV^0(p_i; \tilde{p}_i)}{dp_i} - \frac{d\Delta(p_i)}{dp_i} &= K_0 - (N-1) \frac{(p_i - \tilde{p}_i)}{(\tilde{p}^u - p_i)} K_0 \\ &= [1 - (N-1)(p_i - \tilde{p}_i) h(p_i)] K_0 \end{aligned}$$

$h(p_i) = \frac{1}{(\tilde{p}^u - p_i)}$ is equal to zero if $p_i = p(\tilde{p}_i)$ as in (7), the scoring rule is able to implement the optimal bid.

To do this it is sufficient to show that for any couple of bids that give the same score, the one that contains the price p_i always outperforms the other. Let's suppose that there are two equilibrium bids (p_i^+, R_i^0) and $(p_i', R_i'^0)$ with $p_i^+ \neq p_i$, $p_i' = p_i$ and $R_i'^0 = R_i^0 + \left[\Delta(p_i') - \Delta(p_i^+)\right]$. It is easy to show that the two bids perform the same score, i.e. $s^0(p_i^+, R_i^0) = s^0(p_i', R_i'^0)$.

$$
\begin{aligned}
s^0(p_i', R_i'^0) &= R_i'^0 - \Delta(p_i') \\
&= R_i^0 + \left[\Delta(p_i') - \Delta(p_i^+)\right] - \Delta(p_i') \\
&= R_i^0 - \Delta(p_i^+) = s^0(p_i^+, R_i^0)
\end{aligned}
$$

Although the two bids give the same score, the expected profit of $(p_i', R_i'^0)$ is higher than the expected profit of (p_i^+, R_i^0), that is:[12]

$$
\begin{aligned}
U(p_i', R_i'^0) &= \left[(p_i' - \tilde{p}_i)K_0 - R_i'^0\right] \Pr\left(\text{win; } s^0(p_i', R_i'^0)\right) \\
&= \left\{(p_i - \tilde{p}_i)K_0 - R_i^0 - \left[\Delta(p_i) - \Delta(p_i^+)\right]\right\} \Pr\left(\text{win; } s^0(p_i^+, R_i^0)\right) \\
&= \left\{(p_i^+ - \tilde{p}_i)K_0 - (p_i^+ - \tilde{p}_i)K_0 + (p_i - \tilde{p}_i)K_0 - R_i^0 - \left[\Delta(p_i) - \Delta(p_i^+)\right]\right\} \Pr\left(\text{win; } s^0(p_i^+, R_i^0)\right) \\
&= \left\{(p_i^+ - \tilde{p}_i)K_0 - R_i^0 + \left[(p_i - \tilde{p}_i)K_0 - \Delta(p_i) - ((p_i^+ - \tilde{p}_i)K_0 - \Delta(p_i^+)\right]\right)\right\} \Pr\left(\text{win; } s^0(p_i^+, R_i^0)\right. \\
&\geq U(p_i^+, R_i^0)
\end{aligned}
$$

where the last inequality follows from (44). Next, since the price is chosen independently from the score, substituting $p_i = p(\tilde{p}_i)$ we can rewrite the above firm i's expected payoff as:

$$
\begin{aligned}
U(p_i, R_i^0) &= \left[(p(\tilde{p}_i) - \tilde{p}_i)K_0 - R_i^0\right] \Pr(\text{win; } s^0(p_i, R_i^0) \\
&= \left[NPV^0(\tilde{p}_i) - R^0(\tilde{p}_i)\right] (1 - G^{(N-1)}(\tilde{p}_i))
\end{aligned}
$$

which is equivalent to (39). The optimal fee then follows in the usual way. This concludes the proof.

F Proof of Lemma 4

Lemma 4 can be proved following the proof of Lemma 3. The pricing rule is obtained by maximizing the expected project value. In particular, each bidder should maximize the project value as defined in (4):

[12] See Che (1993, p. 678) for a formal proof that Pr(win; $s^0(p_i', R_i'^0)$ = Pr(win; $s^0(p_i^+, R_i^0)) > 0$.

$$\max_{p_i} V(p_i)(1 - F(p_i))^{N-1}$$

or equivalently:

$$\max_{p_i} \{\max[(p_i - \tilde{p}_i)K_0, (p_i - \bar{p}_i)(K_0 - K_1)]\} (1 - F(p_i))^{N-1}.$$

The optimal price strategy is then given by:

$$p_i^{option} = \min [p(\tilde{p}_i), p(\bar{p}_i)] \tag{45}$$

where $p(\tilde{p}_i)$ is the price when the firm maximizes the NPV_i^0 and $p(\bar{p}_i)$ stands for the price when it maximizes the NPV_i^1. Since Lemma 3 provides $p(\tilde{p}_i)$, we need to derive the pricing rule that maximizes:

$$\max_{p_i} [(p_i - \bar{p}_i)(K_0 - K_1)](1 - F(p_i))^{N-1}$$

The first order condition for this case is:

$$(1 - F(p_i))^{N-1}[(K_0 - K_1) - (N - 1)[(p_i - \tilde{p}_i)K_0 + (\hat{p}_i - p_i)K_1]h(p_i)] = 0$$

from which we obtain:

$$\begin{aligned} p_i &= \frac{K_0}{K_0 - K_1}\tilde{p}_i - \frac{K_1}{K_0 - K_1}\hat{p}_i + \frac{1}{(N-1)h(p_i)} \\ &= \bar{p}_i + \frac{1}{(N-1)h(p_i)} \end{aligned} \tag{46}$$

Since $h'(p_i) > 0$, the second order condition is always satisfied, i.e.: $-[(p_i - \tilde{p}_i)K_0 + (\hat{p}_i - p_i)K_1]h'(p_i) - (K_0 - K_1)h(p_i) < 0$. As the costs are uniformly distributed on $I = [0, I^u]$ also $\bar{p}_i$ are distributed uniformly in $\bar{P} = [0, \bar{p}^u]$. The firm with $\bar{p}^u$ has a project value that is too low to win, i.e. the less efficient firm knows for certain that it will lose the auction, then $h(p) \to \infty$ and from (46) $p_i \to \bar{p}^u$.Finally, we get $\frac{dp_i}{d\bar{p}_i} = -\frac{-1}{1+\frac{h'(p_i)}{(N-1)h(p_i)^2}} > 0$ and < 1.

Simple verification shows that from (33) we obtain a first order differential equation in $p(\bar{p})$ similar to (35), from which it is easy to get the price rule (11) in the text. Substituting $p(\bar{p})$ into (2) the NPV_i^1 becomes:

$$NPV_i^1 = (p_i - \bar{p}_i)(K_0 - K_1) \equiv \frac{1}{N}(\bar{p}^u - \bar{p}_i)(K_0 - K_1) \tag{47}$$

which is also distributed uniformly in $[0, NPV_u^1]$, with $NPV_l^1 \equiv \frac{1}{N}(\bar{p}^u - \bar{p}^u)(K_0 - K_1) = 0$. It follows that the bidding equilibrium strategy requires reporting of a fee that is a constant fraction of the NPV^1 (Krishna, 2002, p. 19):

$$R_i^1 = \frac{N-1}{N} NPV_i^1 \equiv \frac{N-1}{N}\left[\frac{1}{N}(\bar{p}^u - \bar{p}_i)(K_0 - K_1)\right] \equiv \frac{1}{N}[\bar{p}^u - p(\bar{p}_i)](K_0 - K_1)$$

Finally, recalling that by assumption 5 we get $\bar{p}_i \leq \tilde{p}_i \leq \hat{p}_i$, the following disequality $p(\bar{p}_i) < p(\tilde{p}_i)$ is always satisfied for all i, i.e.:

$$\begin{aligned}(1-\frac{1}{N})\left[\phi\tilde{p}_i + (1-\phi)\hat{p}_i\right] + \frac{1}{N}\left[\phi\tilde{p}^u + (1-\phi)\hat{p}^u\right] &< (1-\frac{1}{N})\tilde{p}_i + \frac{1}{N}\tilde{p}^u \\ (\phi - 1)\left\{\left[(1-\frac{1}{N})\tilde{p}_i + \frac{1}{N}\tilde{p}^u\right] - \left[(1-\frac{1}{N})\hat{p}_i + \frac{1}{N}\hat{p}^u\right]\right\} &< 0\end{aligned}$$

It therefore follows that reporting $p(\bar{p}_i)$ and offering $R_i^1 = \frac{N-1}{N} NPV_i^1$ is a dominant strategy for each firm. This concludes the proof.

G Proof of Lemma 5

Let's first consider the expected revenue. Defining $V_i = \max\left[NPV_i^0, NPV_i^1\right]$, the bidder i's expected payment is given by:

$$\mathcal{E}(R_i) = R_i \Pr(\text{win}) \equiv \frac{N-1}{N} V_i (\frac{V_i}{V^u})^{N-1}$$

The regulator earns from each bidder an expected payment $\mathcal{E}(R_i)$. Since he does not know the bidders' valuations, he takes an expected value:

$$\begin{aligned}E[\mathcal{E}(R_i)] &= \int_0^{V^u} \mathcal{E}(R^1(V_i))\frac{1}{V^u} dV_i \\ &\equiv \frac{N-1}{N}(\frac{1}{V^u})^N \int_0^{V^u} V_i^N dV_i \\ &\equiv \frac{N-1}{N(N+1)} V^u\end{aligned}$$

from which we get:

$$E[\mathbf{R}] = NE[\mathcal{E}(R_i)] \equiv \frac{N-1}{N+1} V^u \tag{48}$$

Substituting (38) and (47) into (48), we obtain:

$$E[\mathbf{R}^0] = \frac{N-1}{N(N+1)}\tilde{p}^u K_0$$

if the firms cannot postpone the decision, and:

$$E[\mathbf{R}^1] = \frac{N-1}{N(N+1)}\bar{p}^u (K_0 - K_1)$$

if they can. We are now able to calculate the difference:

$$E[\mathbf{R}^1] - E[\mathbf{R}^0] = \frac{N-1}{N(N+1)}\left[\bar{p}^u (K_0 - K_1) - \tilde{p}^u K_0\right] \equiv -\frac{N-1}{N(N+1)}\hat{p}^u K_1 < 0 \tag{49}$$

Let's now turn to the consumers' surplus. We need to distinguish between the *HFLP* and the *LPHF* format. Indicating the surplus for the first and second cases by S^0 and S^1 respectively, we get:

$$\begin{aligned}
\mathbf{S}^0 &= E\left\{\sum_{t=0}^{\infty}\frac{1}{(1+\rho)^t}\int_{p_i(\tilde{p}_i)}^{p^{\max}} y_t dp\right\} = \sum_{t=0}^{\infty}\frac{1}{(1+\rho)^t}\int_{p_i(\tilde{p}_i)}^{p^{\max}} E(y_t) dp \\
&= (p^{\max} - p_i(\tilde{p}_i))(y_0 + \sum_{t=1}^{\infty}\frac{1}{(1+\rho)^t}E(y_t) = (p^{\max} - p_i(\tilde{p}_i))K_0
\end{aligned}$$

and:

$$\begin{aligned}
\mathbf{S}^1 &= q\left\{\sum_{t=1}^{\infty}\frac{1}{(1+\rho)^t}\int_{p_i(\bar{p}_i)}^{p^{\max}} y_t^+ dp\right\} = (p^{\max} - p_i(\bar{p}_i))q\sum_{t=1}^{\infty}\frac{y_t^+}{(1+\rho)^t} \\
&= (p^{\max} - p_i(\bar{p}_i))qY_u y_0 = (p^{\max} - p_i(\bar{p}_i))(K_0 - K_1)
\end{aligned}$$

Since the consumers do not know the winning bidder, the ex-ante surplus is given by:

$$E[\mathbf{S}^0] = (p^{\max} - Ep_i(\tilde{p}_i))K_0 \equiv (p^{\max} - \frac{1}{2}\frac{N+1}{N}\tilde{p}^u)K_0$$

and:

$$E[\mathbf{S}^1] = (p^{\max} - Ep_i(\bar{p}_i))(K_0 - K_1) \equiv (p^{\max} - \frac{1}{2}\frac{N+1}{N}\bar{p}^u)(K_0 - K_1)$$

where:

$$E\left[p_i(\tilde{p}_i)\right] = \int_0^{\tilde{p}^u} p_i(\tilde{p}_i)\frac{1}{\tilde{p}^u}d\tilde{p}_i \equiv \frac{1}{2}\frac{N+1}{N}\tilde{p}^u$$

and:

$$E\left[p_i(\bar{p}_i)\right] = \int_0^{\bar{p}^u} p_i(\bar{p}_i)\frac{1}{\bar{p}^u}d\bar{p}_i \equiv \frac{1}{2}\frac{N+1}{N}\bar{p}^u$$

The difference between the two consumer's surplus therefore becomes:

$$\begin{aligned} E[\mathbf{S}^1] - E[\mathbf{S}^0] &\equiv (p^{\max} - \frac{N+1}{N}\frac{\bar{p}^u}{2})(K_0 - K_1) - (p^{\max} - \frac{N+1}{N}\frac{\tilde{p}^u}{2})K_0 \quad (50) \\ &\equiv \left[-p^{\max} + \frac{1}{2}\frac{N+1}{N}\hat{p}^u\right]K_1 \end{aligned}$$

Finally, by (49) and (50), the difference between the welfare value resulting from the *LPHF* auction format and the welfare value resulting from the *HFLP* is given by:

$$\begin{aligned} \Delta W^{1,0} &= \left[-p^{\max} + \frac{1}{2}\frac{N+1}{N}\hat{p}^u\right]K_1 - \frac{N-1}{N(N+1)}\hat{p}^u K_1 \\ &= -p^{\max}K_1 + \frac{1+\rho-q}{1+\rho}\frac{N^2+1}{2N(N+1)}I^u \end{aligned}$$

This concludes the proof.

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