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# **Valence Advantages and Public Goods Consumption: Does a Disadvantaged Candidate Choose an Extremist Position?**

## **Summary**

Does a disadvantaged candidate always choose an extremist program? When does a less competent candidate have an incentive to move to extreme positions in order to differentiate himself from the more competent candidate? If the answer to these questions were positive, as suggested in recent work (Ansolabehere and Snyder (2000), Aragonès and Palfrey (2002), Groseclose (1999), and Aragonès and Palfrey (2003)), this would mean that extremist candidates are bad politicians. We consider a two candidates electoral competition over public consumption, with a two dimensional policy space and two dimensions of candidates heterogeneity. In this setting, we show that the conclusion depends on candidates relative competences over the two public goods and distinguish between two types of advantages (an absolute advantage and comparative advantage in providing the two public goods).

**Keywords:** Candidate Quality, Extremism, Public Goods Consumption

**JEL Classification:** C72, D72

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Does a disadvantaged candidate always choose an extremist program? When does a less competent candidate have an incentive to move to extreme positions in order to differentiate himself from the more competent candidate? If the answer to these questions were positive, as suggested in recent work, this would mean that extremist candidates are bad politicians.

Our objective is to answer these questions, and in so doing, to reexamine the results obtained in the recent literature on the competence of politicians. We consider a two candidates electoral competition over public consumption, with a two dimensional policy space and two dimensions of candidates heterogeneity. In this setting, we show that the conclusion depends on candidates relative competences over the two public goods and distinguish between two types of advantages (an absolute advantage and comparative advantage in providing the two public goods).

The closest works to this paper are Ansolabehere and Snyder (2000), Aragonés and Palfrey (2002), Groseclose (1999), and Aragonés and Palfrey (2003). These papers focus on variations of the spatial model of election, introduced by Downs (1957), where two candidates have to choose a position on the unit interval. In all these works, candidates have an unidimensional personal characteristic that determines their (dis)advantage. In these analyzes, voters utility is separable in policy and politician personal characteristic. They study the existence of the equilibrium and conclude that the advantaged candidate locates more centrally than the disadvantaged one.

Ansolabehere and Snyder (2000) show that, in the absence of uncertainty, the advantaged candidate locates at the center, and that the disadvantaged candidate always loses and locates anywhere on the unit interval. As noticed by Aragonés and Palfrey (2002), the existence of equilibrium becomes a problem when there is uncertainty or when candidates maximize their share of votes. In this last case, the advantaged candidate always wants to choose the same program as the disadvantaged candidate to get all the votes, whereas the disadvantaged candidate has an incentive to differentiate his platform in order to get at least some votes. Aragonés and Palfrey (2002) examine

the existence of mixed strategy equilibria in this electoral competition. They consider a discrete unit interval, and show that, when the advantage is small enough, the advantaged candidate chooses a probability distribution with a single peak in the center, whereas the disadvantaged candidate chooses a probability distribution with two peaks, one on each side of the center. In the present work, as in these two papers, voters utility function can be written as additively separable in policy and valence, but candidates scores on the valence dimension differs among voters. In the public goods consumption model, if a candidate benefits from an absolute advantage, our results are close to Ansolabehere and Snyder (2000); when an equilibrium exists, a candidate with an absolute advantage generally locates centrally, and the disadvantaged candidate locates anywhere in his policy set.

Groseclose (2001) and Aragonés and Palfrey (2003) show that the existence problem can disappear when candidates have policy preferences. Groseclose (1999) shows that when candidates put sufficiently high weight on policy, a pure strategy equilibrium may exist and the advantaged candidate chooses a more moderate position than the disadvantaged candidate. Aragonés and Palfrey (2003) consider two candidates who privately know their ideal point and their tradeoffs between policy preferences and winning and show that a pure strategy equilibrium always exists. They also show that the result of Aragonés and Palfrey (2002) is the limit case when policy preferences goes to zero.

One stream of the political economy literature, reviewed by Persson and Tabellini (2000, chapter 4, section 4.7), assumes that candidates differ in their ability to deliver services to citizens<sup>1</sup>. These papers investigate electoral accountability when voters have incomplete information on politicians. Since we focus on candidates locations with incomplete information about voter types, we consider that there is no uncertainty on candidates competences.

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<sup>1</sup>Rogoff and Siebert (1988) study a model of adverse selection; Rogoff (1990) and Banks and Sundaram (1993, 1996) study politicians accountability in models with moral hazard and adverse selection.

Other scholars consider different asymmetries between the candidates<sup>2</sup>. Several analyzes show that Republican and Democrat have different effects on the economy<sup>3</sup>, and study the impact of real or perceived economic performance on elections outcomes<sup>4</sup>.

However, none of these papers considers candidates with a two dimensional competence. In section 3.1, we propose a political competition model where the candidates propose two public goods. The two opportunistic candidates have different competences to provide two public goods. They share the same beliefs on the uncertain median voter preferences and maximize their probability of winning. In section 3.2, we define two kinds of advantages in this model, the absolute advantage (one candidate is better in the provision of both goods) and the comparative advantage (each candidate is better in the provision of one of the two goods). We show that this model is equivalent to a non-spatial valence model, with two orthogonal dimensions, a policy dimension and a non-policy dimension. In this valence model, we define the Unanimity Valence advantage (one candidate as a higher "score" for all voters on the non-policy dimension). We show that the absolute advantage and the Unanimity Valence advantage are two similar definitions. In section 3, we focus on the case where one candidate has an absolute advantage; our results are similar to those of spatial valence models, that is, an equilibrium exists if and only if the advantage is large enough, the advantaged

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<sup>2</sup>See Ansolabehere and Snyder (2000) and Groseclose (2001) for a review of this literature.

<sup>3</sup>Hibbs (1977), Beck (1982), and Chappel and Keech (1986) show that Democrat and Republican governments have different influences on the unemployment rate. Alesina and Sachs (1988) and Tabellini and La Via (1989) show that parties are associated with different monetary policies.

<sup>4</sup>Fiorina (1981) and Austen-Smith and Banks (1989) assume that citizens vote retrospectively conditioned to the difference between platforms and performance. Aragonés (1997) surveys and contributes to the literature on the "negativity effect" where voters vote on past performances and weight more negative than positive informations. See also Kernell (1977), Lau (1982), Klein (1991), Abelson and Levi (1985), Mueller (1973), Bloom and Price (1975), and Key (1966).

candidate wins with certainty, and he generally locates more centrally than the disadvantaged candidate. In section 3.4, we analyze the situation of comparative advantages; the results are sensibly different: in the public goods consumption model, platforms do not converge, whereas they converge to the non-ideological voter preferred program in the valence model. We show that a pure strategy equilibrium generally exists. Finally, candidate's equilibrium probability of winning increases with the candidate competences. Furthermore, we show that a mixed Nash equilibrium exists whatever the kind of advantage considered.

## 1 The model

The model is inspired by the "Multidimensional Public Consumption Model" introduced in Tabellini and Alesina (1990). We first define the two types of agents, voters and candidates:

**Voters:** Let assume a population of voters of mass 1. Citizens have the same income  $w_i = w$  and face a tax rate  $\tau$ . Let  $c$  be a representative citizen's private consumption level. All citizens face the same budget constraint:  $c = (1 - \tau)w$ . The government provides two public goods,  $x \geq 0$  and  $y \geq 0$ . Citizens disagree on the importance of the two public goods and citizen  $i$ 's preferences are parametrized by the weight  $\alpha_i \in [0, 1]$  he places on public good  $x$ . If  $1 < \alpha_i < 0$ , his preferences are summarized in the following utility function:

$$\begin{aligned} W_i() &= u(c) + \alpha_i \ln(x) + (1 - \alpha_i) \ln(y) \text{ if } x, y > 0, \\ &= -\infty \text{ if } xy = 0, \end{aligned} \tag{1}$$

If  $\alpha_i = 0$ ,

$$\begin{aligned} W_i() &= u(c) + \ln(y) \text{ if } y > 0, \\ &= -\infty \text{ if } y = 0 \end{aligned} \tag{2}$$

And, if  $\alpha_i = 1$ ,

$$\begin{aligned} W_i() &= u(c) + \ln(x) \text{ if } x > 0, \\ &= -\infty \text{ if } x = 0, \end{aligned} \tag{3}$$

Since citizens have the same private consumption level,  $u(c)$  does not play any role in the analysis and will be dropped from the model. These preferences belong to the set of intermediate preferences defined by Grandmont (1978), and satisfy the single crossing property. Hence, a Condorcet winner exists and it is given by the preferred policy of the median voter  $\alpha_m$ .

**Candidates:** We consider two office motivated candidates  $A$  and  $B$ . When a candidate is elected, he gets an exogenous ego-rent  $R$ . Candidates share the same beliefs over the distribution of voters, and suppose that  $\alpha_i$  is distributed on  $[0, 1]$  with the cumulative distribution function  $F$ . In the seminal model of multidimensional public consumption, the two candidates have the same competencies to provide both public goods. And, if there is no debt (as in our model), both candidates platforms converge to the median voter preferred policy.

We relax this assumption and suppose that each candidate has different competencies associated to each public good. Candidates are heterogeneous on two dimensions. Let  $(\eta_x^A, \eta_y^A)$  respectively be candidate  $A$  competencies to provide  $x$  and  $y$ . Symmetrically,  $(\eta_x^B, \eta_y^B)$  denotes candidate  $B$  competencies to provide  $x$  and  $y$ . These competencies will determine the candidates' efficiency in providing each public good, and are inversely related to the cost of providing each public good. With these assumptions, candidates face different budget constraints when they are in power. We consider linear costs to provide both public goods and normalize the government budget,  $\tau w$ , to 1. Hence, if candidate  $A$  is elected, his budget constraint is given by <sup>5</sup>:

$$\frac{x}{\eta_x^A} + \frac{y}{\eta_y^A} = 1, \tag{4}$$

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<sup>5</sup>Since rents from power are exogeneous, candidates have an incentive to exhaust their entire budget.



with  $\eta_x^A, \eta_y^A > 0$  and  $x, y \geq 0$ . Symmetrically, if candidate  $B$  is elected, he must respect:

$$\frac{x}{\eta_x^B} + \frac{y}{\eta_y^B} = 1, \quad (5)$$

with  $\eta_x^B, \eta_y^B > 0$  and  $x, y \geq 0$ .

Since we suppose that platforms must be credible and there is no debt, candidates have different policy sets. Let  $z^A = (x^A, y^A)$  and  $z^B = (x^B, y^B)$  respectively denote one candidate  $A$  platform and one candidate  $B$  platform.

Remark that if we put all the competencies to 1, then the model is exactly identical to the multidimensional public consumption model. The policy set becomes unidimensional and there exists a unique equilibrium where both platforms converge to the expected median voter preferred program. Now we show that results are affected when competencies differ among goods and candidates.

## 2 Link with valence models

### 2.1 Link with valence models

Recall that when  $A$  proposes  $z^A = (x^A, y^A)$ , the platform must respect:

$$\frac{x^A}{\eta_x^A} + \frac{y^A}{\eta_y^A} = 1, \quad (6)$$

Symmetrically, when  $B$  proposes  $z^B = (x^B, y^B)$ , the platform must respect:

$$\frac{x^B}{\eta_x^B} + \frac{y^B}{\eta_y^B} = 1, \quad (7)$$

To compare the public consumption model to valence models, we propose two variable changes. Let  $s^A = \frac{x^A}{\eta_x^A}$  and  $s^B = \frac{y^B}{\eta_y^B}$  denote the share invested in good  $x$  by candidate  $A$  and candidate  $B$  respectively. After this transformation, the strategies  $s^A$  and  $s^B$  belong to  $[0, 1]$ . With the budget constraints, we can redefine voter  $i$  utility function as follows:

$$V_i(s^C) = u_i(s^C) + \delta_i^C, \quad (8)$$

with  $C = A$  or  $B$ ;  $u_i(s^C) = \alpha_i \ln(s^C) + (1 - \alpha_i) \ln(1 - s^C)$  and  $\delta_i^C = \alpha_i \ln(\eta_x^C) + (1 - \alpha_i) \ln(\eta_y^C)$ <sup>6</sup>.

We will refer to this non-spatial model as the "valence model". Indeed, voters utility functions are then separable in the policy and valence dimensions. We now turn to define two different kinds of advantages in the initial model with public goods, and translate them into advantages in the valence model.

## 2.2 Definitions

We define absolute and comparative advantages in the context of public goods consumption. A candidate has an absolute advantage when he outperforms his opponent over the two policy dimensions. A natural definition of an absolute advantage is the following:

**Definition 1** *A candidate A has an absolute advantage on another candidate B to provide both public goods, if and only if  $\eta_x^A \geq \eta_x^B$  and  $\eta_y^A \geq \eta_y^B$ , with at least one strict inequality.*

We define the comparative advantages situation where each candidate is relatively better than his opponent in providing one of the public goods. Formally,

**Definition 2** *A candidate A has a comparative advantage to provide x and B has a comparative advantage to provide y if and only if  $\frac{\eta_x^A}{\eta_x^B} > 1 > \frac{\eta_y^A}{\eta_y^B}$ .*

We will now consider the equivalent of the absolute advantage in a valence model. Say that a candidate has a Unanimity Valence Advantage (UVA) when all voters consider him best on the valence dimension:

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<sup>6</sup>Notice that  $\delta_i^C$  may be negative. The important argument is the difference between both candidates images  $\delta_i^A - \delta_i^B$ . If the latter is positive, then  $i$  prefers  $A$  to  $B$  on the non-policy dimension.

**Definition 3** *Candidate A has a Unanimity Valence Advantage (UVA) if and only if:  $\forall i, \delta_i^A \geq \delta_i^B$  with, for at least one voter  $j$ ,  $\delta_j^A > \delta_j^B$ .*

The following proposition confirms the intuition that the UVA and the absolute advantage are, in our context, two similar definitions:

**Proposition 1** *Candidate A has a UVA if and only if he has an absolute advantage.*

**Proof of Proposition 1:**

The necessary condition is straightforward: if Candidate A has an absolute advantage, then  $\eta_x^A \geq \eta_x^B$  and  $\eta_y^A \geq \eta_y^B$ , with at least one strict inequality, and it directly follows that:

$$\forall \alpha_i \in ]0, 1[, \alpha_i \ln(\eta_x^A) + (1 - \alpha_i) \ln(\eta_y^A) > \alpha_i \ln(\eta_x^B) + (1 - \alpha_i) \ln(\eta_y^B),$$

and, for  $\alpha_i \in \{0, 1\}$ ,

$$\alpha_i \ln(\eta_x^A) + (1 - \alpha_i) \ln(\eta_y^A) \geq \alpha_i \ln(\eta_x^B) + (1 - \alpha_i) \ln(\eta_y^B).$$

Regarding the sufficient condition, suppose that Candidate A has a UVA, then:

$$\forall \alpha_i \in [0, 1], \alpha_i \ln(\eta_x^A) + (1 - \alpha_i) \ln(\eta_y^A) \geq \alpha_i \ln(\eta_x^B) + (1 - \alpha_i) \ln(\eta_y^B),$$

Notice that for  $\alpha_i = 0$ , the inequality becomes  $\eta_y^A \geq \eta_y^B$ , and, for  $\alpha_i = 1$ , it becomes  $\eta_x^A \geq \eta_x^B$ .

Now, we claim that  $\eta_y^A = \eta_y^B = \eta_y$  and  $\eta_x^A = \eta_x^B = \eta_x$ . By definition of the UVA, there exists  $\alpha$  in  $[0, 1]$  such that:

$$\alpha \ln(\eta_x) + (1 - \alpha) \ln(\eta_y) > \alpha \ln(\eta_x) + (1 - \alpha) \ln(\eta_y),$$

this is impossible.

## 2.3 Payoff functions

In this section, we derive the candidates payoff functions. Candidates maximize their probability of victory. Let  $\pi^A$  and  $\pi^B$  denote candidate  $A$  and candidate  $B$ 's expected payoff. Furthermore as  $\pi^B = 1 - \pi^A$ , it is sufficient to compute candidate  $A$ 's payoff function. Considering the probability of winning in the election is equivalent to considering that candidates maximize their expected number of votes. Hence, candidate  $A$ 's payoff is given by:

$$\pi^A(z^A, z^B) = \int_{\{\alpha_i \in [0,1] : W_i(z^A) \geq W_i(z^B)\}} R dF(\alpha_i), \quad (9)$$

If all quantities are strictly positive<sup>7</sup>, voter  $i$  prefers  $z^A$  to  $z^B$  if and only if:

$$\alpha_i \ln\left(\frac{x_A y_B}{x_B y_A}\right) \geq \ln\left(\frac{y_B}{y_A}\right), \quad (10)$$

Let  $\hat{\alpha}$  be the type of the voter indifferent between  $z^A$  and  $z^B$ :

$$\hat{\alpha} \ln(x_A) + (1 - \hat{\alpha}) \ln(y_A) = \hat{\alpha} \ln(x_B) + (1 - \hat{\alpha}) \ln(y_B), \quad (11)$$

We deduce from this expression:

$$\hat{\alpha} = 1 - \frac{\ln\left(\frac{x_A}{x_B}\right)}{\ln\left(\frac{x_A y_B}{x_B y_A}\right)}, \quad (12)$$

Hence, candidate  $A$  gets votes from left (small  $\alpha_i$ ) or votes from right (high  $\alpha_i$ ), depending on the candidates' relative positions. Formally, if  $\frac{x_A y_B}{x_B y_A} > 1$ , candidate  $A$ 's payoff is given by:

$$\pi^A(z^A, z^B) = (1 - F(\hat{\alpha})) R \quad (13)$$

If  $\frac{x_A y_B}{x_B y_A} = 1$ , then all voters prefer  $z^A$  to  $z^B$  if and only if  $y_A \geq y_B$ :

$$\begin{aligned} \pi^A(z^A, z^B) &= R \text{ if } y_A > y_B, \\ &= \frac{R}{2} \text{ if } y_A = y_B, \\ &= 0 \text{ if } y_B > y_A. \end{aligned} \quad (14)$$

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<sup>7</sup>Cases where candidates propose only of one good are considered in the proofs.

And, if  $\frac{x_A}{x_B} \frac{y_B}{y_A} < 1$ , candidate  $A$ 's payoff is given by:

$$\pi^A(z^A, z^B) = F(\hat{\alpha}) R \quad (15)$$

This payoff function seems to have many discontinuities, but, in the proofs of propositions 5 and 9, we show that discontinuities only arise for situations where at least one of the candidates only proposes one of the two goods. We now turn to the determination of equilibrium when one of the candidates has an absolute advantage.

### 3 Absolute advantage of one of the candidates

Not surprisingly, since the situation of an absolute advantage is similar to the unidimensional spatial model, our results are comparable to those of spatial models with uncertainty over the median voter preferences. When the advantage is small, as in spatial models<sup>8</sup>, there is no pure strategy equilibrium.

**Proposition 2** *Suppose that  $A$  has an absolute advantage (equivalently, a UVA in the valence model). If  $\frac{\eta_x^B}{\eta_x^A} + \frac{\eta_y^B}{\eta_y^A} > 1$ , then there does not exist a pure strategy equilibrium.*

In the case where relative competencies are equal to 1, the condition of proposition 2 is  $\ln 2 > \delta^A - \delta^B$ . The intuition of this result is the same as in the spatial model. The advantaged candidate gets all votes when he imitates the disadvantaged candidate. Since the advantage is small, the disadvantaged candidate can differentiate himself from the advantaged candidate and get a positive share of votes. There is thus no pure strategy equilibrium. Now, when the advantage is large enough, the advantaged candidate can locate to

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<sup>8</sup>see Groseclose (1999), Ansolabehere and Snyder (2000) and Aragones and Palfrey (2002) for similar results in spatial models.

a central position so that the disadvantaged candidate gets no vote, whatever his policy choice<sup>9</sup>:

**Proposition 3** *Suppose that A has an absolute advantage (equivalently, a UVA in the valence model). If  $\frac{\eta_x^B}{\eta_x^A} + \frac{\eta_y^B}{\eta_y^A} \leq 1$ , then there exists a continuum of pure strategy equilibria where payoffs are  $\pi^{A*} = 1$  and  $\pi^{B*} = 0$ , and platforms are given by:*

*In the public goods consumption model:*

$$z^{A*} = \left( \mu, \left( 1 - \frac{\mu}{\eta_x^A} \right) \eta_y^A \right),$$

*with  $\mu \in \left[ \eta_x^B, \left( 1 - \frac{\eta_y^B}{\eta_y^A} \right) \eta_x^A \right]$ , and  $z^{B*}$  is any candidate B feasible program, and,*

*in the valence model:*

$$s^{A*} = \nu,$$

*with  $\nu \in \left[ \frac{\eta_x^B}{\eta_x^A}, \left( 1 - \frac{\eta_y^B}{\eta_y^A} \right) \right]$ , and  $s^{B*}$  is any real in  $[0, 1]$ .*

In this situation, the advantaged candidate is always certain to win the election, because he always provides more of both goods than the disadvantaged candidate. We now analyze the relation between absolute advantage and the location of the electoral platform.

### 3.1 Absolute advantage and location on the policy space

In our context, we need to specify what we call a central position in the valence model and a symmetric platform in the public goods consumption model. We suppose from now on that  $F$  is the cumulative of the uniform distribution on  $[0, 1]$ .

**Definition 4** *In the public goods consumption model, a platform  $z = (x, y) \in [0, 1]^2$  is **symmetric** if and only if  $x = y$ .*

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<sup>9</sup>See Ansolabehere and Snyder (2000) for a similar result in a spatial model with no uncertainty about the voters distribution.

**Definition 5** In the valence model, a platform  $s \in [0, 1]$  is central if and only if  $s = \frac{1}{2}$ .

Now, we define the following order relation to compare candidates positions:

**Definition 6** In the public goods consumption model, a platform  $z = (x, y)$  is (weakly) **more symmetric** than a platform  $z' = (x', y')$  if and only if  $I(z) = \left| \frac{x}{x+y} - \frac{1}{2} \right| \leq I(z') = \left| \frac{x'}{x'+y'} - \frac{1}{2} \right|$ .

We call  $I(z)$  the position index of policy  $z$ . The more a platform is asymmetric, the higher the position index. We use this index to compare the candidates equilibrium positions. In the valence model, we consider the following criteria:

**Definition 7** In the valence model, a platform  $s$  is (weakly) **more moderate** than a platform  $s'$  if and only if  $\left| s - \frac{1}{2} \right| \leq \left| s' - \frac{1}{2} \right|$

In the case where candidate  $A$  has an absolute advantage, these definitions do not allow to make a clear comparison, because of the multiplicity of equilibria. We thus consider the average candidates equilibrium positions of the candidates. Let  $\mathbf{S}^{C*}$  be the set of candidate  $C$  equilibrium platforms.

**Definition 8** In the public goods consumption model, if the equilibrium payoffs are identical for every equilibrium, candidate  $C'$  platform is said to be (weakly) **generally more symmetric** than candidate  $C''$  platform in equilibrium if:  $\int_{z \in \mathbf{S}^{C*}} I(z) dz \leq \int_{z \in \mathbf{S}^{C''*}} I(z) dz$ .

**Definition 9** In the valence model, if the equilibrium payoffs are identical for every equilibrium, candidate  $C'$  platform is said to be (weakly) **generally more moderate** than candidate  $C''$  platform in equilibrium if:  $\int_{s \in \mathbf{S}^{C*}} \left| s - \frac{1}{2} \right| ds \leq \int_{s \in \mathbf{S}^{C''*}} \left| s - \frac{1}{2} \right| ds$ .

When a candidate has an absolute advantage, he always wins with probability 1, and his opponent always loses. Our definitions suppose that each candidate plays one of the equilibrium strategies with equal probability. If candidate  $A$  has an absolute advantage, we obtain the following result:

**Proposition 4** *Candidate  $A$ ' platform is generally more moderate and generally more symmetric than candidate  $B$ .*

This result is similar to Ansolabehere and Snyder (2000). The advantaged candidate locates more centrally than the disadvantaged candidate. Now, we complete the analysis to give insight on the predictability of the election outcome.

### 3.2 Mixed strategy equilibrium: existence

We have shown that, when the absolute advantage is large enough, a pure strategy equilibrium exists, then it ensures the existence of a mixed strategy equilibrium. In the case where the advantage is not large, that is  $\frac{\eta_x^B}{\eta_x^A} + \frac{\eta_y^B}{\eta_y^A} > 1$ , then there does not exist a pure strategy equilibrium. We know however that a mixed strategy equilibrium exists:

**Proposition 5** *If a candidate has an absolute advantage, then there exists a mixed strategy equilibrium.*

The proof of this proposition is similar to Aragones and Palfrey (2002, Theorem 5), and uses the Dasgupta and Maskin (1986) theorem on the existence of a mixed strategy equilibrium for games with discontinuous payoffs.

Since we focus on the different types of advantage, we do not characterize the mixed strategy equilibrium. We focus now on the situation where candidates have comparative advantages.



## 4 Comparative advantage

In this section, we derive the unique equilibrium when candidates have comparative advantages, and provide necessary and sufficient conditions for existence.

### 4.1 Equilibrium

We suppose that the distribution of the median voter type is uniform,  $F(\alpha) = \alpha$ , that  $A$  has a comparative advantage to provide  $x$  and  $B$  has a comparative advantage to provide  $B$ . Let  $\theta_x = \frac{\eta_x^A}{\eta_x^B}$  and  $\theta_y = \frac{\eta_y^B}{\eta_y^A}$  be the respective strength of candidate  $A$  and candidate  $B$  comparative advantage (in this case, definition 2 states that  $\theta_x, \theta_y > 1$ ). The following result holds:

**Proposition 6** *Suppose that candidate  $A$  has a comparative advantage in good  $x$  and candidate  $B$  a comparative advantage in good  $y$ . Then, there exists at most one pure strategy equilibrium, where the equilibrium payoffs are:*

$$\pi^{A*} = 1 - \hat{\alpha}^*, \text{ and } \pi^{B*} = \hat{\alpha}^*.$$

with  $\hat{\alpha}^* = \frac{\ln \theta_y}{\ln(\theta_x \theta_y)}$ , and the equilibrium platforms are:

*In the public goods consumption model:*

$$z^{A*} = (\eta_x^A \hat{\alpha}^*, \eta_y^A (1 - \hat{\alpha}^*)),$$

$$z^{B*} = (\eta_x^B \hat{\alpha}^*, \eta_y^B (1 - \hat{\alpha}^*)),$$

*And, in the valence model:*

$$s^{A*} = s^{B*} = \hat{\alpha}^*.$$

The intuition for the proof is as follows. Candidates cannot both choose platforms specializing in one of the public goods. If it were true, one of them would have an absolute advantage, and by the same reasoning as in the previous Section, a pure strategy equilibrium may fail to exist. Candidates cannot

specialize in the public good for which they don't have a comparative advantage, since they would then have an incentive to use their advantage and provide more of both good than their opponent. Hence, candidates must be specializing in the public good for which they have a comparative advantage. In the valence model, platforms converge to  $\hat{\alpha}^*$  which is different from the median voter preferred position ( $\frac{1}{2}$ ) and this equilibrium corresponds to platforms divergence in the public goods consumption model, since candidates propose different quantities of public goods.

However, when the comparative advantage of a candidate is not high enough, the other candidate may want to imitate it. As in the case of a small absolute advantage, one cannot guarantee existence of a pure strategy equilibrium. This leads to the following result (here,  $\theta_x, \theta_y > 1$  is always true).

**Proposition 7** *The equilibrium exists if and only if  $\theta_x \ln(\theta_x) \geq \frac{\ln(\theta_y)}{\theta_y}$  and  $\theta_y \ln(\theta_y) \geq \frac{\ln(\theta_x)}{\theta_x}$ .*

The following graph represents the area where a pure strategy equilibrium exists:

We now present two comparative statics results on the equilibrium. First we show, not surprisingly, that a candidate who has a higher comparative advantage, obtains a higher payoff.

**Corollary 1** *A candidate payoff increases with his comparative advantage:*

$$\frac{\partial \pi^{A*}}{\partial \theta_x} > 0, \text{ and } \frac{\partial \pi^{B*}}{\partial \theta_y} > 0.$$

However, we also obtain the less obvious result that, when candidate A becomes better at providing  $x$ , his equilibrium quantity of  $x$  does not necessarily increase:

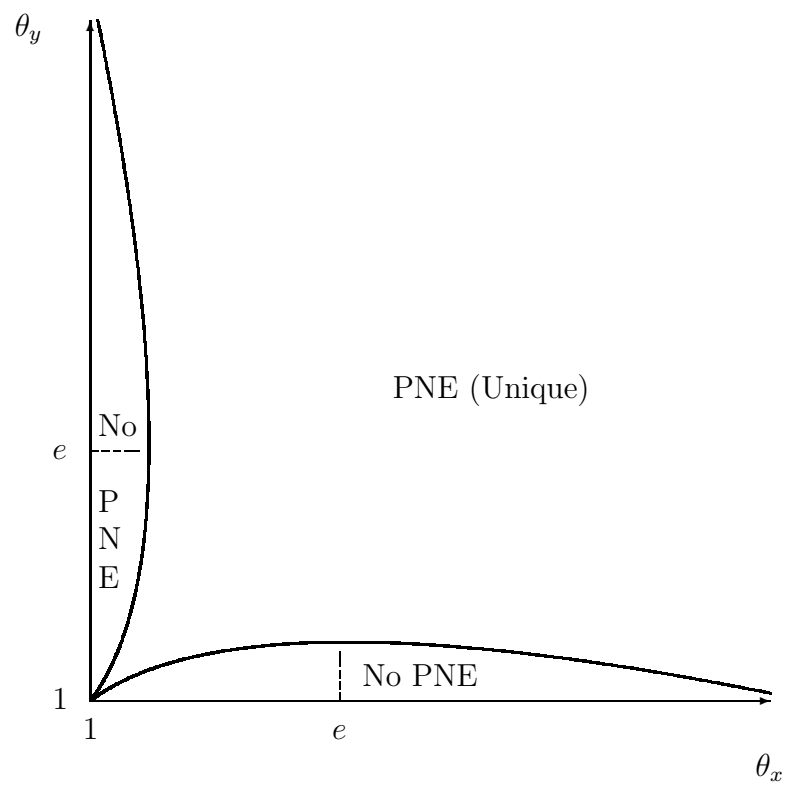


Figure 1: Pure Nash Equilibrium and Comparative Advantages

### Corollary 2

- (i) *The sign of  $\frac{\partial x^{A*}}{\partial \eta_x^A}, \frac{\partial y^{B*}}{\partial \eta_y^B} \propto \ln(\theta_x \theta_y) - 1$  can be positive or negative*  
(ii)  *$\frac{\partial y^{A*}}{\partial \eta_y^A}, \frac{\partial x^{B*}}{\partial \eta_x^B} > 0$ .*

Corollary 2 shows that an increase in a candidate's competence does not necessarily translate into an increase in the public good provision in the equilibrium platform. This result stems from two countervailing effects. On the one hand, when  $\eta_x^A$  increases, candidate  $A$  substitutes public good  $x$  to public good  $y$  (a substitution effect). But, on the other hand, candidate  $A$  can increase his number of votes by increasing  $y^{A*}$ . And, if the comparative advantages are strong, he may have an incentive to increase his provision of public good  $y$  (an income effect which may dominate the substitution effect).

## 4.2 Comparative advantage and platform symmetry

In this section, we provide a sufficient condition under which candidate  $B$  chooses a more symmetric platform than candidate  $A$  when both candidates have comparative advantages in one of the public goods (remember  $\theta_x = \frac{\eta_x^A}{\eta_x^B} > 1$  and  $\theta_y = \frac{\eta_y^B}{\eta_y^A} > 1$ ).

**Proposition 8** *If  $A$  has a comparative advantage in  $x$  and  $B$  a comparative advantage in  $y$  then:*

*In the public consumption model,  $z^{B*}$  is always more symmetric than  $z^{A*}$  if and only if  $\frac{\eta_x^A \eta_x^B}{\eta_y^A \eta_y^B} \geq \left( \frac{\ln \frac{\eta_x^A}{\eta_x^B}}{\ln \frac{\eta_y^B}{\eta_y^A}} \right)^2$ , whereas,*

*in the valence model platforms converge:  $s^{A*} = s^{B*} = \frac{\ln \frac{\eta_y^B}{\eta_y^A}}{\ln \left( \frac{\eta_y^B}{\eta_y^A} \frac{\eta_x^A}{\eta_x^B} \right)}$ .*

Proposition 8 provides a necessary and sufficient condition for the platform of candidate  $B$  to be more balanced than that of candidate  $A$ . This condition holds when  $\frac{\eta_x^A \eta_x^B}{\eta_y^A \eta_y^B}$  is large enough. The natural question arising at this point can be, does there exist a link between competencies symmetry

and candidate's platform symmetry? Formally, does  $|\eta_x^A - \eta_y^A| \geq |\eta_x^B - \eta_y^B|$  means that  $\frac{\eta_x^A \eta_y^B}{\eta_y^A \eta_x^B} \geq \left( \frac{\ln \frac{\eta_x^A}{\eta_y^B}}{\ln \frac{\eta_y^A}{\eta_x^B}} \right)^2$ ? The answer is no. Indeed, consider the following numerical example; let  $\eta_x^A = 10$ ,  $\eta_y^A = 5$ ,  $\eta_x^B = 6$  and  $\eta_y^B = 6$ , then  $|\eta_x^A - \eta_y^A| \geq |\eta_x^B - \eta_y^B| = 0$  and  $\frac{\eta_x^A \eta_y^B}{\eta_y^A \eta_x^B} = 2 \leq \left( \frac{\ln \frac{5}{6}}{\ln \frac{6}{5}} \right)^2$ . Then  $B$  has more balanced competencies but its program is more asymmetric than candidate  $A$ 's one.

### 4.3 Mixed strategy equilibrium: existence

As in the case of an absolute advantage, when candidates have comparative advantages, a mixed strategy equilibrium always exists:

**Proposition 9** *If candidates have comparative advantages, then a mixed strategy equilibrium always exists.*

## 5 Conclusion

We have shown that when candidates have two-dimensional competences, two kinds of advantages can be defined. The Absolute advantage is similar to the Unanimity Valence Advantage, and the disadvantaged candidate generally adopts a more extremist equilibrium position than the advantaged candidate. The conclusion is different when the candidates have comparative advantages. Platforms converge in the valence model form, but candidates provide different quantities of public goods and their probability of winning increases with their competencies. Furthermore, we have given necessary and sufficient conditions for the existence of a (unique) pure strategy equilibrium. We also show existence of a mixed strategy equilibrium when the pure strategy equilibrium fails to exist. A natural following research would be to characterize such mixed strategy equilibria and to consider public goods production functions with return to scale different from constant.  $\frac{\eta_x^B}{\eta_x^A} + \frac{\eta_y^B}{\eta_y^A} > 1$ . We distinguish two cases. Suppose  $((x_A^*, y_A^*), (x_B^*, y_B^*))$  is an equilibrium:

## 6 Appendix

**Case 1** If not  $\frac{x_A^*}{x_B^*} \geq 1, \frac{y_A^*}{y_B^*} \geq 1$ , with at least one inequality being strict and  $(x_A^*, y_A^*) \neq (x_B^*, y_B^*) : A$  can propose  $x'_A = x_B^*$  and  $y'_A = \eta_y^A \left(1 - \frac{x_B^*}{\eta_x^A}\right) > \eta_y^B \left(1 - \frac{x_B^*}{\eta_x^B}\right) = y_B^*$  because he has an absolute advantage. Then, it is not an equilibrium.

**Case 2** If  $\frac{x_A^*}{x_B^*} \geq 1, \frac{y_A^*}{y_B^*} \geq 1$ , with at least one inequality being strict and  $(x_A^*, y_A^*) \neq (x_B^*, y_B^*)$ . Candidate  $B$ ' payoff is nul ( $\pi_B^* = 0$ ), because he proposes smaller quantities of both public goods than his adversary. We distinguish the following subcases:

If  $x_A^* > \eta_x^B$ , then  $y_A^* = \eta_y^A \left(1 - \frac{x_A^*}{\eta_x^A}\right) < \eta_y^A \left(1 - \frac{\eta_x^B}{\eta_x^A}\right) < \eta_y^B$ .  $B$  can propose  $y'_B > y_A^*$ , hence  $\pi_B = F(\hat{\alpha}) R > 0$ .

If  $y_A^* > \eta_y^B$ , then  $x_A^* = \eta_x^A \left(1 - \frac{y_A^*}{\eta_y^A}\right) < \eta_x^A \left(1 - \frac{\eta_y^B}{\eta_y^A}\right) < \eta_x^B$ .  $B$  can propose  $x'_B > x_A^*$ , hence  $\pi_B = [1 - F(\hat{\alpha})] R > 0$ .

If  $x_A^* < \eta_x^B$  and  $y_A^* < \eta_y^B$ , then  $B$  can move to  $y''_B > y_A^*$  with  $\tilde{x}_B > x_A^*$  and he gets a strictly positive payoff. Finally, it cannot be an equilibrium.

**Proof of Proposition 3:**  $\frac{\eta_x^B}{\eta_x^A} + \frac{\eta_y^B}{\eta_y^A} \leq 1$  : The proof is in two steps. In the first step, we show that the situations described in proposition 3 are equilibria. In the second step, we show that there is no other equilibrium.

*Step 1:* Let us prove that  $((x_A^*, y_A^*), (x_B^*, y_B^*)) = \left(\left(\mu, \eta_y^A \left(1 - \frac{\mu}{\eta_x^A}\right)\right), (x_B, y_B)\right)$  with  $\mu \in \left[\eta_x^B, \eta_x^A \left(1 - \frac{\eta_y^B}{\eta_y^A}\right)\right]$  is an equilibrium. Here,  $x_A^* \geq \eta_x^B \geq x_B$ ,  $\forall x_B \in [0, \eta_x^B]$  and  $y_A^* \geq y_B$ ,  $\forall y_B \in [0, \eta_y^B]$ , with at least one inequality being strict. Hence, candidate  $B$  cannot be strictly better. Furthermore,  $A$  gets the maximum payoff,  $\pi_A^* = R$ .

*Step 2:* Now, let us show that  $((x_A^*, y_A^*), (x_B^*, y_B^*)) = \left(\left(\mu, \eta_y^A \left(1 - \frac{\mu}{\eta_x^A}\right)\right), (x_B, y_B)\right)$  with  $\mu \notin \left[\eta_x^B, \eta_x^A \left(1 - \frac{\eta_y^B}{\eta_y^A}\right)\right]$  is not an equilibrium. Since  $\mu < \eta_x^B$  or  $\mu > \eta_x^A \left(1 - \frac{\eta_y^B}{\eta_y^A}\right)$ ,  $B$  can not receive a strictly positive payoff. Finally, it cannot be an equilibrium.

**Proof of Proposition 4:** Candidate  $A$ 's mean equilibrium position index is:

$$\bar{I}_A = \frac{\left| \frac{\eta_x^B}{\left(1 - \frac{\eta_x^B}{\eta_x^A}\right)\eta_y^A + \eta_x^B} - \frac{1}{2} \right| + \left| \frac{\left(1 - \frac{\eta_y^B}{\eta_y^A}\right)\eta_x^A}{\eta_y^B + \left(1 - \frac{\eta_y^B}{\eta_y^A}\right)\eta_x^A} - \frac{1}{2} \right|}{2},$$

and, candidate  $B$ 's mean index is:

$$\bar{I}_B = \frac{1}{2},$$

Furthermore, by definition of an absolute advantage,  $\frac{\eta_x^B}{\eta_x^A} \leq 1$  and  $\frac{\eta_y^B}{\eta_y^A} \leq 1$  with at least one strict inequality, so that  $\bar{I}_A < \frac{1}{2} = \bar{I}_B$ .

**Proof of Proposition 5:** To prove the result, we rely on the main Theorem of Dasgupta and Maskin (1986, p.14):

**Theorem 1** (*Dasgupta and Maskin, 1986*) Let  $[(S_i, U_i); i = 1, \dots, N]$  be a game. Let  $S_i \subseteq R^1$  ( $i = 1, \dots, N$ ) a closed interval and  $U_i : S \rightarrow R^1$  ( $i = 1, \dots, N$ ) continuous except on a subset  $S^{**}(i)$  of  $S^*(i)$ , where  $S^*(i)$  is defined by:

$$S^*(i) = \{(s_1, \dots, s_N) \in S : \exists j \neq i, \exists d, 1 \leq d \leq \Delta(i) \text{ such that } s_j = f_{ij}^d(s_i)\},$$

where  $\Delta(i)$  is a positive integer and for each integer  $d$ , with  $1 \leq d \leq \Delta(i)$ , and  $f_{ij}^d : R^1 \rightarrow R^1$  is a one-to-one continuous function. Suppose that  $\sum_{i=1}^N U_i(\mathbf{s})$  is upper semi-continuous and  $U_i(s_i, \mathbf{s}_{-i})$  is bounded and weakly lower semi-continuous in  $s_i$ . Then, the game  $[(S_i, U_i); i = 1, \dots, N]$  possesses a mixed-strategy equilibrium.

Let reformulate our game to be able to apply the Dasgupta-Maskin Theorem. Let  $x_P = s_P \eta_{xP}$  and  $y_C = \eta_{yP} (1 - s_P)$ , with  $P = A, B$ , and  $p \in [0, 1]$ . When candidate  $A$  has an absolute advantage our electoral game is equivalent to the game defined by  $[(S_i, \pi_i); i = A, B]$ , with  $S_i = [0, 1]$ ,  $i = A, B$  and the payoffs functions are:

If  $s^A, s^B \notin \{0, 1\}$ ,

$$\pi_A(s^A, s^B) = \begin{cases} (1 - F(\hat{\alpha})) R & \text{if } s^A > 1 - \frac{\eta_y^B}{\eta_y^A} (1 - s^B) \\ R & \text{if } s^A \in \left[ \frac{\eta_x^B}{\eta_x^A} s^B, 1 - \frac{\eta_y^B}{\eta_y^A} (1 - s^B) \right] \\ F(\hat{\alpha}) R & \text{if } s^A < \frac{\eta_x^B}{\eta_x^A} s^B \end{cases},$$

and,

$$\pi_B(s^A, s^B) = \begin{cases} F(\hat{\alpha}) R & \text{if } s^A > 1 - \frac{\eta_y^B}{\eta_y^A} (1 - s^B) \\ 0 & \text{if } s^A \in \left[ \frac{\eta_x^B}{\eta_x^A} s^B, 1 - \frac{\eta_y^B}{\eta_y^A} (1 - s^B) \right] \\ (1 - F(\hat{\alpha})) R & \text{if } s^A < \frac{\eta_x^B}{\eta_x^A} s^B \end{cases}, \text{ and,}$$

if  $s_P \notin \{0, 1\}$  and  $s_{-P} \in \{0, 1\}$ ,

$$\pi_P = R \text{ and } \pi_{-P} = 0, \text{ and,}$$

if  $s_P = s_{-P} \in \{0, 1\}$ ,

$$\pi_P = \pi_{-P} = \frac{R}{2},$$

With

$$\hat{\alpha} \equiv \hat{\alpha}(s^A, s^B) = \frac{Ln \left( \frac{1-s^B}{1-s^A} \frac{\eta_y^B}{\eta_y^A} \right)}{Ln \left( \frac{s^A}{s^B} \frac{1-s^B}{1-s^A} \frac{\eta_y^B}{\eta_y^A} \frac{\eta_x^A}{\eta_x^B} \right)},$$

Let us verify the conditions of the Dasgupta-Maskin Theorem:

1.  $S_i = [0, 1] \subseteq R^1$  ( $i = A, B$ ) is a closed interval.
2.  $\pi_A(s^A, s^B)$  and  $\pi_B(s^A, s^B)$  are continuous excepted on a set of mass 0. We have to choose  $\Delta(A) = \Delta(B) = 4$ . Furthermore,  $f_{AB}^1(0) = f_{BA}^1(0) = 0$ ,  $f_{AB}^2(0) = f_{BA}^2(0) = 1$ ,  $f_{AB}^3(1) = f_{BA}^3(1) = 0$ , and  $f_{AB}^4(1) = f_{BA}^4(1) = 1$ . In our case,  $S^*(A) = S^*(B)$ .  $\pi_A(s^A, s^B)$  is continuous in  $s^A$  on the rest of the strategy space, because, if  $s^B \notin \{0, 1\}$ ,  $\lim_{s^A \rightarrow 0} \hat{\alpha} = \lim_{s^A \rightarrow 1} \hat{\alpha} = 1$ , if



$s^B \neq 1$ ,  $\lim_{s^A \rightarrow 1 - \frac{\eta_B}{\eta_A}(1-s^B)} \hat{\alpha} = 1$ , and if  $s^B \neq 0$ ,  $\lim_{s^A \rightarrow \frac{\eta_B}{\eta_A} s^B} \hat{\alpha} = 1$ . In the same way  $\pi_B(s^A, s^B)$  is also continuous on the same subset.

3.  $\pi_A(s^A, s^B) + \pi_B(s^A, s^B) = R$ , is a constant function, hence  $\sum_{i=1}^N U_i(\mathbf{s})$  is upper semi-continuous.

4.  $0 \leq \pi_i(s^A, s^B) \leq 1$ , ( $i = A, B$ ) then  $U_i(s_i, \mathbf{s}_{-i})$  is bounded.

5. Let us prove that  $\pi_i(s^A, s^B)$  is weakly lower semi-continuous in  $s_i$ . Let  $S_i^{**}(i) = \{s_i \in S_i : \exists \mathbf{s}_{-i} \in S_{-i} \text{ such that } (s_i, \mathbf{s}_{-i}) \in S^{**}(i)\}$ , the definition of weak lower semi-continuity is given by:

**Definition 10**  $U_i(s_i, \mathbf{s}_{-i})$  is weakly lower semi-continuous in  $s_i$  if  $\forall \bar{s}_i \in S_i^{**}(i)$ ,  $\exists \lambda \in [0, 1]$  such that  $\forall \mathbf{s}_{-i} / (\bar{s}_i, \mathbf{s}_{-i}) \in S^{**}(i)$ ,

$$\lambda \liminf_{s_i \rightarrow -\bar{s}_i} U_i(s_i, \mathbf{s}_{-i}) + (1 - \lambda) \liminf_{s_i \rightarrow +\bar{s}_i} U_i(s_i, \mathbf{s}_{-i}) \geq U_i(\bar{s}_i, \mathbf{s}_{-i})$$

First consider  $\pi_A(s^A, s^B)$  and let  $s^B$  be fixed.

**Case 1**  $s^B = 0$ : here, there are two discontinuities, when  $s^A = 0$  and  $s^A = 1$ . If  $s^A$  is strictly between these two bounds, then  $\pi_A = R$ . In the first discontinuity,  $\pi_A(0, 0) = \frac{R}{2}$ ,  $\liminf_{s_i \rightarrow -0} \pi_i(s_i, 0) = \frac{R}{2}$  and,  $\liminf_{s_i \rightarrow +0} \pi_i(s_i, 0) = R$ . If we choose  $\lambda = 1$ , it is true that  $\frac{R}{2} \geq \frac{R}{2}$ . In the second discontinuity,  $s^A = 1$ . Since  $\pi_A(1, 0) = \frac{R}{2}$ ,  $\liminf_{s_i \rightarrow -1} \pi_i(s_i, 0) = R$  and  $\liminf_{s_i \rightarrow +1} \pi_i(s_i, 0) = \frac{R}{2}$ , if we take  $\lambda = 0$ , it is true that  $\frac{R}{2} \geq \frac{R}{2}$ .

**Case 2**  $s^B = 1$ : here, there are also two discontinuities. The first when  $s^A = 0$ , and the second when  $s^A = 1$ . If  $s^A$  is strictly between these two bounds, her payoff is constant and equal to  $R$ . In the first discontinuity,  $\pi_A(0, 1) = \frac{R}{2} = \liminf_{s_i \rightarrow -0} \pi_i(s_i, 1)$ . Let us choose  $\lambda = 1$ , we verify  $\frac{R}{2} \geq \frac{R}{2}$ . In the second discontinuity,  $\pi_A(1, 1) = \frac{R}{2} = \liminf_{s^A \rightarrow +1} \pi_A(s^A, 1)$ . Let choose  $\lambda = 0$ , it is true that  $\frac{R}{2} \geq \frac{R}{2}$ .

Let us now consider  $\pi_B(s^A, s^B)$  and fix  $s^A$ . There are two discontinuity values of  $s^A$ , 0 and 1. For the two values, there are two discontinuities, when  $s^B = 0$

and when  $s^B = 1$ . Since candidate  $B$ 's payoff is equal to  $R$  when  $s^A \in ]0, 1[$  and it is equal to  $\frac{R}{2}$  in the two bounds, the reasoning is exactly the same as for  $\pi_A(s^A, s^B)$ . Finally,  $\pi_B(s^A, s^B)$  is weakly lower semi-continuous in  $s^B$ .

**Proof of Proposition 6:** Let consider the modified model where the utility of voter  $i$  is given by:

$$V_i(s^C) = u_i(s^C) + \delta_i^C,$$

with  $s^C \in [0, 1]$ ,  $C = A, B$ . The indifferent voter is given by (if  $s^C \neq 0, 1$ ;  $C = A, B$ ):

$$\hat{\alpha}(s^A, s^B) = \frac{N(s^A, s^B)}{D(s^A, s^B)},$$

where  $N(s^A, s^B) = \ln \theta_y + \ln \frac{1-s^B}{1-s^A}$  and  $D(s^A, s^B) = \ln \theta_x \theta_y + \ln \frac{s^A}{s^B} \frac{1-s^B}{1-s^A}$ .

Suppose  $0 < \hat{\alpha}(s^A, s^B) < 1$ , then in an interior equilibrium  $(s^{A*}, s^{B*})$ , the first order conditions are:

$$\frac{\partial \hat{\alpha}(s^{A*}, s^{B*})}{\partial s^A} \propto s^{A*} D(s^{A*}, s^{B*}) - N(s^{A*}, s^{B*}) = 0,$$

and,

$$\frac{\partial \hat{\alpha}(s^{A*}, s^{B*})}{\partial s^B} \propto N(s^{A*}, s^{B*}) - s^{B*} D(s^{A*}, s^{B*}) = 0,$$

then,

$$s^{A*} = s^{B*} = \hat{\alpha}(s^{A*}, s^{B*}),$$

Hence,

$$\hat{\alpha}(s^{A*}, s^{B*}) = \frac{\ln \theta_y}{\ln \theta_x \theta_y},$$

with  $\frac{\ln \theta_y}{\ln \theta_x \theta_y} \in [0, 1]$ , because the definition of comparative advantages ensures that  $\theta_x, \theta_y > 1$ . To complete the proof, we have to show that situations where  $\hat{\alpha}(s^A, s^B)$  is not defined or does not belong to  $]0, 1[$  cannot correspond to an equilibrium.

First remark that all situations where one candidate gets a nul payoff cannot be an equilibrium. Indeed, this candidate can always imitate his opponent and then  $\hat{\alpha}(s^A, s^B) = \frac{\ln \theta_y}{\ln \theta_x \theta_y}$  and both players payoffs become strictly positive.

Now suppose that  $\hat{\alpha}(s^{A*}, s^{B*})$  is not defined, i.e., either  $D(s^{A*}, s^{B*}) = 0$  (equivalent to  $\frac{x_A^* y_B^*}{x_B^* y_A^*} = 1$ ), or  $s^{A*}$  or  $s^{B*}$  is in  $\{0, 1\}$ . If  $D(s^{A*}, s^{B*}) = 0$ , then candidate A's payoff is given by:

$$\begin{aligned}\pi^A(s^{A*}, s^{B*}) &= R \text{ if } s^{A*} < 1 - \theta_y(1 - s^{B*}), \\ &= \frac{R}{2} \text{ if } s^{A*} = 1 - \theta_y(1 - s^{B*}), \\ &= 0 \text{ otherwise.}\end{aligned}$$

Suppose  $(s^{A*}, s^{B*})$  such that  $s^{A*} \leq 1 - \theta_y(1 - s^{B*})$  is an equilibrium. Then  $\pi^B(s^{A*}, s^{B*}) \in \{0, \frac{R}{2}\}$ , whereas  $\pi^B(s^{A*}, s^B) = R$  until  $0 \leq s^B \leq \frac{\theta_y - 1 + s^{A*}}{\theta_y} \leq 1$ . Hence  $B$  has an incentive to deviate, this is a contradiction. If  $s^{A*}$  or  $s^{B*}$  is in  $\{0, 1\}$ , but not both of them. Then one of the candidate gets a nul payoff and this cannot be an equilibrium. Now, if  $s^{A*}$  and  $s^{B*}$  are in  $\{0, 1\}$ , then  $\pi^A(s^{A*}, s^{B*}) = \pi^B(s^{A*}, s^{B*}) = \frac{R}{2}$ . If one of the candidate deviates and locates in  $]0, 1[$ , he gets all the votes, then this is not an equilibrium.

Suppose that  $\hat{\alpha}(s^A, s^B) \leq 0$  or  $\hat{\alpha}(s^A, s^B) \geq 1$ , then one of the two players gets a null payoff. We have already proved that this cannot be an equilibrium.

**Proof of Proposition 7:** We first show the following lemma (remember that  $\theta_x, \theta_y > 1$  here):

**Lemma 1**  $\frac{\ln \theta_y}{\ln \theta_x \theta_y} < \frac{\theta_x(\theta_y - 1)}{\theta_x \theta_y - 1}$

**Proof of Lemma 1:** Let  $\theta_x = \theta$  and  $\theta_y = \lambda\theta$  with  $\frac{1}{\theta} < \lambda$ . Then the inequality can be written as follows:

$$h(\lambda) = \lambda\theta^2 \ln \theta - (\theta - 1) \ln \lambda - (2\theta - 1) \ln \theta > 0,$$

The differentiate of  $h$  is  $h'(\lambda) = \theta^2 \ln \theta - \frac{(\theta - 1)}{\lambda} > \theta l(\theta) = \theta^2 \ln \theta - \theta(\theta - 1)$ . The function  $l$  is increasing ( $l'(\theta) = \ln \theta$ ) and  $l(1) = 0$ , then  $h'(\lambda) > 0$ . Furthermore,  $h(1) = 0$ , then the inequality is always true.

Without loss of generality, we focus on candidate  $A$  incentives to deviate from  $(s^{A*}, s^{B*}) = \left(\frac{\ln \theta_y}{\ln \theta_x \theta_y}, \frac{\ln \theta_y}{\ln \theta_x \theta_y}\right)$ . There are many situations where  $A$  may obtain a higher payoff. Straightforwardly, candidate  $A$  has no incentive to play  $s^A \in \{0, 1\}$ , otherwise,  $\pi^A(s^A, s^{B*}) = 0$ .

**Case 1** If  $A$  can deviate by playing  $s^A$  such that its payoff is given by equation 13, i.e.  $D(s^A, s^{B*}) > 0$  (equivalent to  $\frac{x_A y_B^*}{x_B^* y_A} > 1$ ). Suppose  $\hat{\alpha}(s^A, s^{B*}) \leq 0$ , then his payoff  $\pi^A(s^A, s^{B*}) = R$ . The two conditions imply that  $\theta_x \theta_y \frac{s^A}{s^{B*}} \frac{1-s^{B*}}{1-s^A} > 1$  and  $\theta_y \frac{1-s^{B*}}{1-s^A} \leq 1$  (it means that  $N(s^A, s^{B*}) < 0$ ). This is equivalent to  $\frac{s^{B*}}{s^{B*} + (1-s^{B*})\theta_x \theta_y} < s^A \leq 1 - \theta_y(1 - s^{B*})$ . Such a value of  $s^A$  exists if and only if  $\frac{\theta_x(\theta_y - 1)}{\theta_x \theta_y - 1} < s^{B*} < 1$ . Since  $s^{B*} = \frac{\ln \theta_y}{\ln \theta_x \theta_y - 1}$ , lemma 1 ensures that this cannot be true. Then candidate  $A$  cannot play this kind of deviation. Now, suppose  $0 < \hat{\alpha}(s^A, s^{B*}) < 1$ , then  $\pi^A(s^A, s^{B*}) = 1 - F(\hat{\alpha}(s^A, s^{B*}))$ . Here, the second order derivative of candidate  $A$ 's payoff is:

$$\begin{aligned} \frac{\partial^2 \pi^A(s^A, s^{B*})}{(\partial s^A)^2} &= -\frac{\partial^2 \hat{\alpha}(s^A, s^{B*})}{(\partial s^A)^2} \\ &= \frac{1 - 2s^A}{(s^A(1 - s^A))^2} [s^A D(s^A, s^{B*}) - N(s^A, s^{B*})] - \frac{1}{s^A(1 - s^A)} D(s^A, s^{B*}), \end{aligned}$$

Hence,

$$\frac{\partial^2 \pi^A(s^{A*}, s^{B*})}{(\partial s^A)^2} \propto (\hat{\alpha}(s^{A*}, s^{B*}))^2 - \hat{\alpha}(s^{A*}, s^{B*}) < 0,$$

Then  $s^{A*}$  maximizes the payoff of candidate  $A$  in this case.

**Case 2** Suppose candidate  $A$  deviates such that its payoff is given by equation (14), i.e.  $\theta_x \theta_y \frac{s^A}{s^{B*}} \frac{1-s^{B*}}{1-s^A} = 1$  (equivalent to  $\frac{x_A y_B^*}{x_B^* y_A} = 1$ ). Then  $\frac{s^{B*}}{s^{B*} + (1-s^{B*})\theta_x \theta_y} = s^A$ . In this case,

$$\begin{aligned} \pi^A(s^A, s^{B*}) &= R \text{ if } s^A < 1 - \theta_y(1 - s^{B*}), \\ &= \frac{R}{2} \text{ if } s^A = 1 - \theta_y(1 - s^{B*}), \\ &= 0 \text{ if } s^A > 1 - \theta_y(1 - s^{B*}). \end{aligned}$$

In the previous case, we have seen that  $1 - \theta_y(1 - s^{B*}) < \frac{s^{B*}}{s^{B*} + (1-s^{B*})\theta_x \theta_y}$ , then this deviation is not profitable ( $\pi^A(s^{A*}, s^{B*}) > \pi^A(s^A, s^{B*}) = 0$ ).

**Case 3** If  $A$  can deviate by playing  $s^A$  such that its payoff is given by equation 15, i.e.  $D(s^A, s^{B*}) < 0$  (equivalent to  $\frac{x_A y_B^*}{x_B^* y_A} < 1$ ). Suppose that  $A$  can deviate

by playing  $s^A$  such that  $\hat{\alpha}(s^A, s^{B*}) \geq 1$ , then his payoff  $\pi^A(s^A, s^{B*}) = R$ . The two conditions imply that  $\theta_x \theta_y \frac{s^A}{s^{B*}} \frac{1-s^{B*}}{1-s^A} < 1$  and  $\frac{s^{B*}}{\theta_x} \leq s^A$  (it means that  $N(s^A, s^{B*}) \leq D(s^A, s^{B*})$ ). These two conditions are equivalent to  $\frac{s^{B*}}{\theta_x} \leq s^A < \frac{s^{B*}}{s^{B*} + (1-s^{B*})\theta_x \theta_y}$ . Such a deviation exists if and only if  $s^{B*} > \frac{\theta_x(\theta_y-1)}{\theta_x \theta_y - 1}$ , and lemma 1 states this cannot be true. Now suppose that A deviates by playing  $s^A$  such that  $0 < \hat{\alpha}(s^A, s^{B*}) < 1$  (then  $D(s^A, s^{B*}) < N(s^A, s^{B*}) < 0$ ). Then  $s^A < \frac{s^{B*}}{\theta_x}$  and  $s^A < 1 - \theta_y(1 - s^{B*})$ . It is easy to show that  $1 - \theta_y(1 - s^{B*}) < \frac{s^{B*}}{\theta_x}$  with lemma 1, then  $s^A < 1 - \theta_y(1 - s^{B*})$ . The first derivative of candidate A' payoff is:

$$\begin{aligned} \frac{\partial \pi^A(s^A, s^{B*})}{\partial s^A} &= \frac{\partial \hat{\alpha}(s^A, s^{B*})}{\partial s^A} \\ &= \frac{1}{1-s^A} D(s^A, s^{B*}) - \frac{1}{s^A(1-s^A)} N(s^A, s^{B*}), \end{aligned}$$

The roots of this equation are given by  $\hat{\alpha}(\bar{s}^A, s^{B*}) = \bar{s}^A$ . The second order derivative verifies:

$$\frac{\partial^2 \pi^A(\bar{s}^A, s^{B*})}{(\partial s^A)^2} \propto (\hat{\alpha}(\bar{s}^A, s^{B*}))^2 - \hat{\alpha}(\bar{s}^A, s^{B*}) < 0,$$

Finally,  $\bar{s}^A = \hat{\alpha}(\bar{s}^A, s^{B*})$  with  $\theta_x \theta_y \frac{\bar{s}^A}{s^{B*}} \frac{1-s^{B*}}{1-\bar{s}^A} < \theta_y \frac{1-s^{B*}}{1-\bar{s}^A} < 1$  is the only remaining possible deviation. Candidate A has an incentive to deviate if and only if  $\pi^A(\bar{s}^A, s^{B*}) > \pi^A(s^{A*}, s^{B*})$ , i.e. if and only if  $\bar{s}^A > 1 - s^{A*}$ . Let  $\tilde{s}^A = 1 - s^{A*}$ , then A has an incentive to deviate iff  $\tilde{s}^A D(\tilde{s}^A, s^{B*}) > N(\tilde{s}^A, s^{B*})$  and  $\tilde{s}^A < 1 - \theta_y(1 - s^{B*})$ . These inequalities are equivalent to:

$$\begin{aligned} \ln \frac{\theta_x}{\theta_y} \ln \left[ \theta_x \theta_y \frac{\ln \theta_x}{\ln \theta_y} \right] &> 0, \text{ and,} \\ \ln \theta_x &< \frac{\ln \theta_y}{\theta_y}, \end{aligned}$$

By a symmetry argument, candidate B has an incentive to deviate iff:

$$\begin{aligned} \ln \frac{\theta_y}{\theta_x} \ln \left[ \theta_x \theta_y \frac{\ln \theta_y}{\ln \theta_x} \right] &> 0, \text{ and,} \\ \ln \theta_y &< \frac{\ln \theta_x}{\theta_x}, \end{aligned}$$

Suppose  $\theta_x \geq \theta_y$ , then the equilibrium exists iff  $\theta_y \geq \frac{\ln \theta_y}{\ln \theta_x}$  and  $(\frac{\ln \theta_y}{\ln \theta_x} \geq \frac{1}{\theta_x \theta_y}$  or  $\frac{\ln \theta_y}{\ln \theta_x} \geq \frac{1}{\theta_x})$ , i.e. iff  $\frac{1}{\theta_x \theta_y} \leq \frac{\ln \theta_y}{\ln \theta_x}$ . If  $\theta_y \geq \theta_x$  the equilibrium exists iff  $\frac{\ln \theta_y}{\ln \theta_x} \geq \frac{1}{\theta_x}$  and  $(\theta_x \theta_y \geq \frac{\ln \theta_y}{\ln \theta_x}$  or  $\theta_y \geq \frac{\ln \theta_y}{\ln \theta_x})$ , i.e. iff  $\theta_x \theta_y \geq \frac{\ln \theta_y}{\ln \theta_x}$ . Finally, the equilibrium exists iff  $\theta_x \ln \theta_x \geq \frac{\ln \theta_y}{\theta_y}$  and  $\theta_y \ln \theta_y \geq \frac{\ln \theta_x}{\theta_x}$ .

**Proof of Proposition 8:** First notice that  $\hat{f}(X) = \frac{X\hat{\alpha}^*}{X\hat{\alpha}^*+1-\hat{\alpha}^*} - \frac{1}{2} \geq 0$  if and only if  $X \geq \frac{1-\hat{\alpha}^*}{\hat{\alpha}^*}$  and is a strictly increasing function of  $X$ , because  $\hat{\alpha}^* \in ]0, 1[$ . Since  $\frac{\eta_x^B}{\eta_y^B} < \frac{\eta_x^A}{\eta_y^A}$ , we consider three cases:

**Case 1** Suppose  $\frac{\eta_x^B}{\eta_y^B} < \frac{\eta_x^A}{\eta_y^A} \leq \frac{1-\hat{\alpha}^*}{\hat{\alpha}^*}$ , then  $I(z^{A*}) - I(z^{B*}) = \hat{f}\left(\frac{\eta_x^B}{\eta_y^B}\right) - \hat{f}\left(\frac{\eta_x^A}{\eta_y^A}\right) < 0$ .

**Case 2** Suppose  $\frac{1-\hat{\alpha}^*}{\hat{\alpha}^*} \leq \frac{\eta_x^B}{\eta_y^B} < \frac{\eta_x^A}{\eta_y^A}$ , then  $I(z^{A*}) - I(z^{B*}) = \hat{f}\left(\frac{\eta_x^A}{\eta_y^A}\right) - \hat{f}\left(\frac{\eta_x^B}{\eta_y^B}\right) > 0$ .

**Case 3** Suppose  $\frac{\eta_x^B}{\eta_y^B} \leq \frac{1-\hat{\alpha}^*}{\hat{\alpha}^*} \leq \frac{\eta_x^A}{\eta_y^A}$ , then  $I(z^{A*}) - I(z^{B*}) = \hat{f}\left(\frac{\eta_x^A}{\eta_y^A}\right) + \hat{f}\left(\frac{\eta_x^B}{\eta_y^B}\right) - 1$ . With simple computations, we find that this last expression is positive if and only if  $\frac{\eta_x^A}{\eta_y^A} \frac{\eta_x^B}{\eta_y^B} \geq \left(\frac{1-\hat{\alpha}^*}{\hat{\alpha}^*}\right)^2$ .

**Proof of Proposition 9:** Candidates have comparative advantages, then the game is symmetric in  $A$  and  $B$ . Hence, we will only consider candidate  $A$ . We apply the Dasgupta et Maskin Theorem (1986, p.14), presented in the Proof of Proposition 5. Let suppose that  $x_P = \eta_{xP} * s_P$  and  $y_P = \eta_{yP} * (1 - s_P)$ , ( $s_P \in [0, 1]$ ;  $P = A, B$ ). We define  $(\tilde{s}^A, \tilde{s}^B) = \left(\frac{\tilde{x}}{\eta_x^A}, \frac{\tilde{x}}{\eta_x^B}\right)$  with  $\tilde{x} = \eta_x^A \frac{\theta_y - 1}{\theta_y \theta_x - 1}$ . The game with comparative advantages is equivalent to the following game:  $[(S_i, \pi_i); i = A, B]$  the electoral competition game, with  $S_i = [0, 1]$ ,  $i = A, B$  and:

If  $s^A, s^B \notin \{0, 1\}$  and  $(s^A, s^B) \neq (\tilde{s}^A, \tilde{s}^B)$ ,

$$\pi_A(s^A, s^B) = \left\{ \begin{array}{l} (1 - F(\hat{\alpha})) R \text{ if } s^A > \max\left(\frac{s^B}{\theta_x}, 1 - \theta_y(1 - s^B)\right) \text{ (1A)} \\ R \text{ if } s^A \in \left[\frac{s^B}{\theta_x}, 1 - (1 - s^B)\theta_y\right] \text{ (2A)} \\ F(\hat{\alpha}) R \text{ if } s^A < \min\left(1 - \theta_y(1 - s^B), \frac{s^B}{\theta_x}\right) \text{ (3A)} \\ 0 \text{ if } s^A \in \left[1 - (1 - s^B)\theta_y, \frac{s^B}{\theta_x}\right] \text{ and } s^B \neq s^A \text{ (4A)} \end{array} \right\}, \text{ and,}$$

if  $s_P \notin \{0, 1\}$  and  $s_{-P} \in \{0, 1\}$ ,

$$\pi_P = R \text{ and } \pi_{-P} = 0, \text{ and,}$$

if  $s_P = s_{-P} \in \{0, 1\}$ ,

$$\pi_P = \pi_{-P} = \frac{R}{2},$$

and,

$$\pi_P(\tilde{s}^A, \tilde{s}^B) = \pi_{-P}(\tilde{s}^A, \tilde{s}^B) = \frac{R}{2},$$

with,

$$\hat{\alpha} \equiv \hat{\alpha}(s^A, s^B) = \frac{Ln\left(\frac{1-s^B}{1-s^A}\theta_y\right)}{Ln\left(\frac{s^A}{s^B}\frac{1-s^B}{1-s^A}\theta_y\frac{\frac{1}{\eta_x^B}}{\frac{1}{\eta_x^A}}\right)}$$

Let us verify the conditions of the Dasgupta-Maskin Theorem:

1.  $S_A = [0, 1] \subseteq R^1$  ( $i = A, B$ ) is a closed interval.
2.  $\pi_A(s^A, s^B)$  is continuous excepted on a set of measure 0. We have to choose  $\Delta(A) = 5$ . Furthermore,  $f_{AB}^1(0) = 0$ ,  $f_{AB}^2(0) = 1$ ,  $f_{AB}^3(1) = 0$ ,  $f_{AB}^4(1) = 1$ ,  $f_{AB}^5(\tilde{s}^A) = \tilde{s}^B$ . To prove that  $\pi_A(s^A, s^B)$  is continuous on the rest of the strategy space, we distinguish between two cases:

**Case 4** If  $0 < s^B < \frac{\theta_y - 1}{\theta_y - \frac{1}{\theta_x}}$ . Here,  $\frac{1}{\theta_x}s^B > 1 - \theta_y(1 - s^B)$ , then  $\pi_A(s^A, s^B)$  is defined by (1A), (3A) and (4A). We have to show that  $\pi_A(s^A, s^B)$  is

continuous in  $s^A = \frac{1}{\theta_x} s^B$  and in  $s^A = 1 - \theta_y (1 - s^B)$ . Regarding the first value, since:

$$\lim_{s^A \rightarrow \frac{1}{\theta_x} s^B} \hat{\alpha}(s^A, s^B) = 1,$$

then:

$$\lim_{s^A \rightarrow \frac{1}{\theta_x} s^B} \pi(s^A, s^B) = 0 = \pi\left(\frac{1}{\theta_x} s^B, s^B\right),$$

because  $s^B \neq 0$ . Regarding the second value, since:

$$\lim_{s^A \rightarrow 1 - \theta_y(1 - s^B)} \hat{\alpha}(s^A, s^B) = 0,$$

then:

$$\lim_{s^A \rightarrow 1 - \theta_y(1 - s^B)} \pi(s^A, s^B) = 0 = \pi(1 - \theta_y(1 - s^B), s^B).$$

**Case 5** If  $\frac{\theta_y - 1}{\theta_y - \theta_x} \leq s^B < 1$ ,  $s^B \neq s^A$ . Here  $\frac{1}{\theta_x} s^B \leq 1 - \theta_y(1 - s^B)$ , then  $\pi_A(s^A, s^B)$  is defined by (1A), (2A) and (4A). We have to show that  $\pi_A(s^A, s^B)$  is continuous in  $s^A = \frac{1}{\theta_x} s^B$  and in  $s^A = 1 - \theta_y(1 - s^B)$ . Regarding the first value, since:

$$\lim_{s^A \rightarrow \frac{1}{\theta_x} s^B} \hat{\alpha}(s^A, s^B) = 1,$$

we obtain:

$$\lim_{s^A \rightarrow \frac{1}{\theta_x} s^B} \pi(s^A, s^B) = R = \pi\left(\frac{1}{\theta_x} s^B, s^B\right),$$

because  $s^B \neq 0$ . Regarding the second value, since:

$$\lim_{s^A \rightarrow 1 - \theta_y(1 - s^B)} \hat{\alpha}(s^A, s^B) = 0,$$

we obtain:

$$\lim_{s^A \rightarrow 1 - \theta_y(1 - s^B)} \pi(s^A, s^B) = R = \pi(1 - \theta_y(1 - s^B), s^B).$$

3.  $\pi_A(s^A, s^B) + \pi_B(s^A, s^B) = R$ , is a constant function, hence  $\sum_{i=1}^N U_i(\mathbf{s})$  is upper semi-continuous.



4.  $0 \leq \pi_i(s^A, s^B) \leq 1$ , ( $i = A, B$ ) then  $U_i(s_i, \mathbf{s}_{-i})$  is bounded.
5. We have to show that  $\pi_A(s^A, s^B)$  is weakly lower semi-continuous in  $s^A$ . We have presented the definition of the weakly lower semi-continuity in the Proof of Proposition 5. There are discontinuities only when  $s^B = 0$  or 1 or  $\tilde{s}^B$ . We distinguish these three cases:
- Cases 1 and 2: identical to Cases 1 et 2 of the Proof of Proposition 5.
- Case 3:  $s^B = \tilde{s}^B$ . Here, there is a discontinuity in  $s^A = \tilde{s}^A$ . Since  $\frac{1}{\theta_x} \tilde{s}^B = 1 - \theta_y (1 - \tilde{s}^B) = \tilde{s}^A$ . Candidate A payoff is given by:

$$\pi_A(s_A, s_B) = \begin{cases} (1 - F(\hat{\alpha})) R & \text{if } s_A > \tilde{s}_A \\ \frac{R}{2} & \text{if } s_A = \tilde{s}_A \\ F(\hat{\alpha}) R & \text{if } s_A < \tilde{s}_A \end{cases}$$

To prove the lower semi-continuity in this point, we need to compute  $\lim_{s^A \rightarrow +\tilde{s}^A} \hat{\alpha}(s^A, s^B)$  and  $\lim_{s^A \rightarrow -\tilde{s}^A} \hat{\alpha}(s^A, s^B)$ . We know that:

$$\hat{\alpha}(s^A, \tilde{s}^B) = \frac{1}{\frac{Ln(1-\tilde{s}^A) - Ln(1-s^A)}{Ln(s^A) - Ln(\tilde{s}^A)} + 1}.$$

Furthermore, when  $s^A > \tilde{s}^A$ , we obtain:

$$\frac{\tilde{s}^A}{1 - s^A} \leq \frac{Ln(1 - \tilde{s}^A) - Ln(1 - s^A)}{Ln(s^A) - Ln(\tilde{s}^A)} \leq \frac{\tilde{s}^A}{1 - \tilde{s}^A}.$$

Hence,

$$\lim_{s^A \rightarrow +\tilde{s}^A} \hat{\alpha}(s^A, s^B) = \lim_{s^A \rightarrow -\tilde{s}^A} \hat{\alpha}(s^A, s^B) = \tilde{s}^A,$$

We deduce the limits:

$$\lim_{s^A \rightarrow +\tilde{s}^A} \inf \pi_A(s^A, \tilde{s}^B) = (1 - F(\tilde{s}^A)) R, \text{ and } \lim_{s^A \rightarrow -\tilde{s}^A} \pi_A(s^A, \tilde{s}^B) = F(\tilde{s}^A) R$$

Furthermore,  $\pi_A(\tilde{s}^A, \tilde{s}^B) = \frac{R}{2}$ , then if we choose  $\lambda = \frac{1}{2}$ , we verify that:

$$\lambda F(\tilde{s}^A) R + (1 - \lambda) (1 - F(\tilde{s}^A)) R \geq \frac{R}{2}.$$

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