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## Coalition Governments in a Model of Parliamentary Democracy


#### Abstract

Summary We analyze the relative importance of party ideology and rents from office in the formation of coalitions in a parliamentary democracy. In equilibrium, the types of coalitions that are formed may be minimal winning, minority or surplus and they may be ideologically `disconnected'. The coalitions that form depend upon the relative importance of rents of office and seat shares of the parties. If rents are high, governments cannot be surplus. With low rents or the formateur close to the median, minority governments occur for a wider ideological dispersion. Further, there is a nonmonotonic relationship between connectedness of coalitions and rents.


Keywords: Coalitions, Ideology, Rents
JEL Classification: C72, D72, H19


#### Abstract

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## 1. Introduction

We analyze what kind of governments will form in a parliamentary democracy where no single party has an absolute majority of seats. This scenario is of interest since in many parliamentary democracies it is unusual for a single party to control more than half the seats in parliament. In a study of 313 elections in 11 democracies in Europe from 1945-1997, (see Diermeier and Merlo (2004)) it was found that only 20 of the elections returned a single party with more than half the seats in parliament. Hence, coalition governments naturally emerge. Furthermore, coalition governments are fairly diverse in nature. A study of 15 post World War II European democracies by Gallagher, Laver and Mair (1995) finds that about $35 \%$ of coalitions are minimal winning (i.e. they have just the required number of parties needed to constitute a majority), $36 \%$ are minority coalitions (i.e. the coalition members constitute less than half the seats in the legislature) and the rest of the coalitions (29\%) are surplus (i.e. they have more than the required number of parties needed to constitute a majority). ${ }^{1}$ From the point of view of ideological closeness such democracies also show diversity in that coalitions are not necessarily ideologically 'connected' ${ }^{2}$ and may often leave out the median party. ${ }^{3}$

[^0]We construct a game theoretic model where parties care both for ideology and rents to examine the way equilibrium coalitions change as parties care more or less for ideology relative to the value they attach to the rents from office. ${ }^{4}$ In doing so we provide an explanation for the diverse coalitions seen in the data. Before comparing our paper with others in the recent literature that also generate coalitional diversity, we briefly describe our model.

There are a given number of parties in a parliament or legislature where no party has more than half the seat shares. Parties have preferences over ideology as well as rents. The preferences of parties and their seat shares are given, as is a status quo policy. Following the recent literature we assume each party is chosen to be the formateur with the probability of being chosen equal to its seat share (see Diermeier and Merlo (2004) for empirical validity of this procedure which was originally suggested by Baron and Ferejohn (1989)). The formateur invites any subset of parties in the legislature to form the government. If all the invitees agree to join the government and if the prospective government wins the support of a majority of legislators, then the government assumes office. Otherwise a status quo policy is implemented. We assume that under a coalition government, the implemented policy is given by the seatweighted average of the ideal policies of the coalition partners and each party in the coalition receives a share of power in proportion to its seats ${ }^{5}$. Thus, we rule out side transfers between parties and commitment to a policy at the government

[^1]formation stage. All a formateur can commit to is a choice of coalition partners and the compromise over policy and rents is then given by this weighted average rule if the coalition forms a government. Parties agree to be in a coalition depending on whether their utility from being in the coalition is greater than their utility from the status quo. We impose no assumptions on the nature of the status quo policy. We solve for the sub-game perfect equilibrium of this game with the additional refinement of weakly undominated strategies. The refinement rules out Pareto dominated equilibria which arise from co ordination failure.

We now summarize our main results. When rents are low or the formateur is close to the median, minority governments occur for a wider ideological dispersion. Surplus coalitions are often the result of a formateur trying to 'balance' policy by including parties on either end of the ideological spectrum. Further, there is a non monotonic relationship between connectedness of coalitions and rents i.e. equilibrium coalitions can be disconnected at low rents, they can be connected at moderate rents and again disconnected at high rents. A general result that we get is that when the rents from office are large enough and the status quo is given by a seat weighted average of the ideal points of the legislature with rents being shared in the same proportion, equilibrium governments are minimum winning subject to the inclusion of the formateur. ${ }^{6}$ Essentially these are comparative static results, looking at how equilibrium coalitions vary with the choice of formateur, party size as well as with changes in the value of ideology to rents from office.

[^2]We now briefly discuss the related literature to make clear our contribution. The empirical diversity of coalition governments in terms of size has received attention in the recent game theoretic models of legislative bargaining, notably in two papers by Diermeier and Merlo (2000) and Baron and Diermeier (2001), (collectively referred to as BDM). In their models, the formateur can buy the support of other parties by adapting a compromise policy position or by making side payments in return for support. They put no limit on the amount of side transfers, hence bargaining always leads to an efficient outcome. Using this approach they are able to generate equilibrium governments which can be minority, minimum winning, or surplus. The previous theoretical literature had consistently predicted minimum winning (or minimum winning connected) coalitions in equilibrium which was at odds with the data $^{7}$. However, the BDM papers predict that coalitions do not vary with a change in the relative value of ideology and rents. Hence, unlike our model, for a given set of ideal points and a given formateur, coalitions in the BDM model are invariant to how parties trade off ideology for rents. In particular, their limiting coalitions need not be minimal winning. Nor do these papers shed light on the issue of disconnected coalitions. (Brams et. al (2002) is one of the few papers which addresses this issue at a theoretical level).

Both the question of how coalitions change with the relative valuation that parties place on ideology and rents and the phenomenon of disconnected coalitions have received attention in the empirical literature as well. Indridason ((2003) and (2005))

[^3]in two papers empirically tests specific hypotheses about the nature of governments as the ideological component becomes more or less important compared to holding political office. Kalandrakis (2003) tests for whether minority governments occur more frequently when the overall utility of cabinet positions is low. Our model (in contrast to BDM) does predict coalitions changing with the relative importance of office holding to ideology and are thus consistent with the findings in Indridason and Kalandrakis. The non monotonic relation between connectedness of coalitions and the importance of rents that we predict is also borne out by the data in Indridason's paper. We make two assumptions that allow us to generate such comparative static results. First, we do not allow for commitment at the coalition formation stage, second the rents are divided only among the coalition members, no other transfers are allowed. This enables us to analyze how equilibrium coalitions change as the relative value that parties place on rents change. ${ }^{8}$ Hence our model provides a theoretical foundation for these empirical findings.

The remainder of the paper is organized as follows. In Section 2 we set up the model. In Section 3 we solve the model to analyze coalitional diversity and provide a discussion of the comparative statics in Section 4. In Section 5 we do a two dimensional analysis and explicitly compare our setup with previous papers in the literature. In Section 6 we discuss the empirical relevance of our work as well as possible extensions and conclude.

## 2. The Model

There are $N$ parties in a parliament where each party is denoted by its ideal point and seat share. Let $\left(N,\left\{S_{i}, x_{i}\right\}\right)$ denote a parliament comprised of $N$ parties where $S_{i}$ denotes party $i$ 's seat share and $x_{i}$ its ideal point. We assume that there is a 'hung'

[^4]parliament, i.e. $S_{i} \leq \frac{1}{2}$ for all $i \in N$. Let $x \in X$ be the policy implemented, where $X$ $\subset \mathbb{R}$ is the policy space. Denote by $x_{i}$ the ideal policy of party $i$. Parties have convex distance preferences over policy. Formally, define $u$ as a twice differentiable function with $u(0)=0, u^{\prime}>0, u^{\prime \prime} \geq 0$. The payoff to party $i$ is given by
$$
-u\left(\left|x_{i}-x\right|\right)+\pi_{i} P
$$
if party $i$ is in government and
$$
-u\left(\left|x_{i}-x\right|\right)
$$
if it is not where $\pi_{i}$ is the relative size (seat share) of the party in terms of the number of parties in the government and $P$ is the total rents that can be had by being in office. Thus, the main distinction between being in government and out of government is that only parties in government get a share of $P .{ }^{9}$

The sequence of actions can be described as follows.
(1) Government Formation The process of government formation is comprised of three stages: formateur selection, coalition formation and the vote of confidence.
(a) Formateur Selection Each party is asked to become the formateur with probability $S_{i}$.
(b) Coalition Selection The formateur asks any subset of parties in the legislature, $D$, to form a government. All the members invited by the formateur (which may or may not include itself) must simultaneously decide whether or not to accept the offer. If the offer is unanimously accepted, then $D$

[^5]goes on to seek the vote of confidence, otherwise a caretaker government is instituted i.e. the status quo is implemented.
(c) Vote of Confidence If the members of $D$ decide to accept the formateur's offer, it must seek the vote of confidence from the legislature. Each member of the legislature simultaneously votes to approve or disapprove of the coalition $D$ (we assume that when indifferent, a party member votes for the proposed government). If the coalition wins more than $50 \%$ of the votes, then it goes on to form a government, otherwise we assume that a status quo policy is implemented by a caretaker government.
(2) Policy Selection With a slight abuse of notation let $D$ also denote the government in office. Depending upon the outcome of the government formation stage, there could either be a single party government, coalition government or a caretaker government in power. There are two cases to consider.

- Single Party or Coalition Government: Let $\pi_{i}$ denote the relative seat share of party $i$ in the government. We assume that the policy chosen $x_{D}$ by such a government is given by $\sum_{i \in D} \pi_{i} x_{i}$ and if party $i$ is part of the government it gets a transfer equal to $\pi_{i} P=\frac{s_{i}}{\sum_{k \in D} s_{k}} P$.
- Caretaker Government: This is the same as the status quo policy. We analyze government formation under different assumptions about the nature of the caretaker government. In one, the policy implemented is given by $\sum_{i \in C} S_{i} x_{i}$ and each member of the legislature gets a transfer $S_{i} P^{10}$. In the other, we assume that the status quo policy $x_{\phi}$ is exogenously given and no rents are distributed.

[^6]2.1. Equilibrium. The solution concept we employ is subgame perfection, in (weakly) undominated strategies. Thus, we will solve the game backwards. There are two stages in the legislative game viz. government formation and policy making. We assume that each party in the legislature acts as a cohesive decision making unit which tries to maximize the total payoff of the party ${ }^{11}$. As noted, each coalition is uniquely associated with a policy and division of perks: the implemented policy is a seat weighted average of the members ideal points and $P$ shared in the same proportion ${ }^{12}$.

Let $v_{i}(D)$ denote the payoff of party $i$ when $D$ is the ruling coalition. If $i \notin D$, then $v_{i}(D)=-u\left(\left|x_{i}-x_{D}\right|\right)$ and if $i \in D$, then $v_{i}(D)=-u\left(\left|x_{i}-x_{D}\right|\right)+\pi_{i} P$. Let $v_{i}(\phi)$ denote the payoff of party $i$ when there is a caretaker government. At the vote of confidence stage, the members of party $i$ will vote for the proposed government $D$ if $v_{i}(D) \geq v_{i}(\phi)$. Let $A(D)$ denote the set of parties that would vote for the proposed government $D$ and let $s_{A(D)}$ denote its size. If $s_{A(D)}>\frac{1}{2}$, then $D$ forms the government. Let $W$ denote the set of coalitions that will win the vote of confidence. Formally, $W \equiv\left\{D \in 2^{C}\right.$ s.t. $\left.s_{A(D)}>\frac{1}{2}\right\}$. Now we come to the coalition selection stage. At this stage the formateur $k$ must choose the parties it invites to form a government. Let $Y$ denote the set of coalitions that are unanimously preferred by its constituents over the status quo. Formally, $Y \equiv\left\{D \in 2^{C}\right.$ s.t. $\left.v_{i}(D) \geq v_{i}(\phi)\right\}$. Thus, every coalition member has a veto power in that it can decide not to be in the coalition. Hence, unanimity is required among the selected members for a coalition to be formed. Let $D_{k}$ denote the coalition most preferred by party $k$, i.e. $D_{k}=\operatorname{argmax}_{D \in W \cap Y} v_{k}(D)$. If $D_{k}$ is not unique any of them are chosen with equal probability. Thus, associated with

[^7]each formateur $k$ we have an equilibrium coalition (or government) $D_{k}{ }^{13}$. Formally, a legislative equilibrium can be defined as follows

Definition 1. A legislative equilibrium is a collection of coalitions $D_{1}, D_{2}, \ldots, D_{N}$ such that $\forall k \in C, D_{K}=\arg \max _{D \epsilon W \cap Y} v_{k}(D)$

Note that existence is not a problem as the sets $W$ and $Y$ are non empty and so is their intersection. Hence, $D_{k}$ is non empty.
2.2. Defining different coalitions. Before stating our main results it is useful to make precise the types of coalitions we had described in the introduction. Recall that ( $N,\left\{S_{i}, x_{i}\right\}$ ) denotes a parliament comprised of $N$ parties where $S_{i}$ denotes party $i$ 's seat share and $x_{i}$ its ideal point. Let $D \subseteq N$ denote the coalition in power. Some special cases of interest are

- $|D|=1$ - a single party is in power.
- $|D|=\phi$ - no party is in power, there is a caretaker government in place.
- $\sum_{k \in D} S_{k} \leq \frac{1}{2}-D$ is a minority government.
- $\sum_{k \in D} S_{k}>\frac{1}{2}$ and $\exists i \in D$ such that $\sum_{k \in D \backslash i} S_{k}>\frac{1}{2}-D$ is a surplus or super-majority government.
- $\sum_{k \in D} S_{k}>\frac{1}{2}$ and for any $i \in D, \sum_{k \in D \backslash i} S_{k} \leq \frac{1}{2}-D$ is a minimal winning coalition government.
- $D=N$ - a consensus (or national) government.
- Let $C\left(\left\{x_{i}\right\}_{i \in D}\right)$ denote the convex hull of the ideal points of the coalition partners. If $\exists j \notin D$ such that $x_{j} \in C\left(\left\{x_{i}\right\}_{i \in D}\right)$ then $D$ is a disconnected coalition. Otherwise $D$ is a connected coalition.


## 3. Explaining Coalitional Diversity

As we have noted, different kinds of coalitions can occur in a parliamentary democracythe wide range of coalition governments can be minority , minimal winning, or even

[^8]super-majority (surplus). Moreover, we have also noted that coalitions need not be ideologically connected. In this section we study conditions under which we can get different coalition governments as equilibria of our model. To fix ideas let us focus on a hung parliament comprised of 3 parties, 1,2 and 3 . Let the policy space be $\mathbb{R}$ with $x_{1}=0, x_{2}=x \leq \frac{1}{2}$ and $x_{3}=1$. Assume further that party $i$ 's preferences over the policy are captured by a quadratic loss function such that when the policy chosen is $y$, party $i$ 's policy specific payoff is $-\left(x_{i}-y\right)^{2}$. We shall vary our illustrations based on 1) the relative party sizes, 2) the relative dispersion of parties' ideal points and 3) the nature of the status-quo arrangement. It will become clear during the following analysis that this particular set up is employed for tractability alone and the qualitative results go through for a general set up. In the following part of the section we provide only illustrative examples of different coalition governments and relegate a fuller characterization to the Appendix.
3.1. Minority Government. In order for a minority government to emerge in our set up two conditions must be satisfied: 1) the government must have the support of at least one party outside the government, 2) the formateur must prefer the party in government (which may be different from itself) to any other feasible government.

The above conditions will be satisfied in a variety of situations. For instance, note that any party would always prefer a minority government comprised of itself to any other form of government. Hence, as long as a party other than the formateur prefers the formateur over the status quo, we will get a minority government comprised of the formateur alone. For instance consider the case where party 2 is the formateur. Suppose that the status quo, $x_{\emptyset}$ lies to the left of $x$. In that case, party 3 prefers 2's ideal policy $x$ over $x_{\emptyset}$. Hence, as long as 3 's share of $P$ under the status-quo, $P_{\emptyset 3}$, is small, it would prefer a minority government of 2 over the status-quo.

Note that a minority government need not be a centrist government. If the $x_{\emptyset}$ lies outside of $[0,1]$, it is possible to have an extremist party forming a minority
government. For instance, suppose that party 3 (which is furthest from the two parties) is the formateur and $x_{\emptyset}>1$. Then, as long as $P_{\emptyset 2}$ is small, 2 would be willing to support a minority government of party 3 .

It is also possible to have a situation in which we get a minority government that does not include the formateur. For instance, suppose that party 1 is the formateur and all the parties are of equal size. Assume, further, that $x$ is close to $\frac{1}{2}$ and $P$ is close to 0 . If the status quo lies in $\left[x, \frac{3 x}{2}\right]$, then 1 cannot form a minority government $\{1\}$ nor can it form a government $\{1,2\}$, since the status quo is preferred by both 2 and 3 over either of these governments. In this case, 1's best choice is to ask 2 to form a minority government! The outcome under such a government will be $x$ which is preferred to $x_{\emptyset}$ by both 1 and 2 .

It is of interest to note that the empirical predictions (see section 6 for a detailed analysis) about the greater likelihood of minority governments occurring at low rents from office as well as when the party is more centrally located is borne out. In terms of our setup, what it means is that the range of $x$ at which you get minority governments is greater when $P$ is low or when party 2 (the central party) is the formateur. Also, note that unless the status quo is outlying i.e. $x_{\emptyset}$ lies outside of $[0,1]$ party 3 cannot form a minority government.
3.2. Surplus government. When we have moderate rents from office, surplus governments occur due to two reasons. First, a formateur may wish to balance the policy his way by including parties on either side of the ideological spectrum. Second, a formateur may be able to induce a party to join the government only if another party is also asked to be in the government. In the latter case we have the interesting scenario where if one of the parties (say party $i$ ) is left out, the conditions for a viable government are not satisfied i.e. $D-i \notin W \cap Y$.

To see the way a formateur may balance the policy, let us consider the case when 2 is the formateur and $x$ is close to $\frac{1}{2} .2$ can form a government with either 1 or 3 .

However, by including both 1 and 3, it balances the policy to be close to its own ideal point.

To understand why a surplus coalition may be formed because two parties need to be invited together, consider the scenario where 3 is the formateur and $P$ is low. $\{2,3\} \succ_{3}\{1,2,3\}$. However, as long as $x_{\phi} \succ_{2}\{2,3\},\{1,2,3\} \succ_{3}\{1,3\}$ and both 2 and 3 prefer $\{1,2,3\}$ to $x_{\phi}, 3$ will form the surplus coalition $\{1,2,3\}$. Here 3 is forced to form a surplus government as its preferred coalition $\{2,3\}$ is not viable. Of course, a surplus government does not occur when $P$ is high.
3.3. Minimal winning governments. While minimal winning coalitions can occur in a variety of situations, the 'limiting case' i.e. when $P$ is large enough always leads to minimal winning coalitions as long as all parties enjoy a fraction $\beta$ of the total rents from office under the status quo. In particular, minimal winning coalitions are also of minimum size, subject to the inclusion of the formateur. However, where $P_{\phi i}=0$ for some $i$, minimal winning coalitions do not necessarily emerge even when $P$ is large.

As we have noted before, at low $P$ minority governments occur for a larger range of $x$. As $P$ increases and $x$ is not too close to $\frac{1}{2}$ we see minimal winning coalitions emerging both with 2 and 1 as formateur. Further, with 3 as formateur, as $P$ increases we move from surplus to minimal winning coalitions. The intuition behind this is simple. At low $P$ the coalition $\{2,3\}$ is not viable. However, when $P$ increases, the value to being part of the coalition in government goes up so $\{2,3\}$ becomes viable. Also, when $x$ is close to $\frac{1}{2}$ the minimum winning coalitions that form can be disconnected, at moderate increases it can become connected and become disconnected again. We now turn our attention to the issue of ideological connectedness of coalitions.
3.4. Connected vs. disconnected governments. Disconnected coalitions occur in our setup in one of two ways. Either party 2 is very central and does not prefer to form a coalition with 1 or 3 (this occurs at low $P$ ), or the ideologically closer party is
larger in size and hence at high $P$ the smaller parties get together leaving the larger party out.

Both these effects can be understood by looking at an instance where there is a large central party with two smaller parties on either side. Further, assume that the status quo policy $x_{\phi}$ is close to party 2 's ideal policy. It is easy to see that at low $P$ we get disconnected coalitions whenever 1 or 3 is the formateur. As long as $P_{\emptyset_{1}}$ and $P_{\emptyset 3}<\frac{P}{2}$ even at low $P$ both 1 and 3 prefer $\{1,3\}$ over $x_{\phi}$ as $x_{13}$ i.e. the policy under the coalition $\{1,3\}$ is close to $x_{\phi}$ so there is little difference in the policy implemented but the rents are larger for 1 and 3 under $\{1,3\}$. As $P$ increases, we see that the coalition $\{1,2\}$ (as well as $\{2,3\}$ ) become viable as 2 is willing to form a coalition with its ideologically connected partners to get part of the rents from office. However, when $P$ increases even more, 1 and 3 prefer to form a coalition between themselves as they will be able to share rents equally whereas with 2 in the coalition, its higher seat share will cause it to get a larger share of the rents. Hence, we see this non monotonic relation between connectedness of coalitions and rents from office. So far empirical work (discussed in the introduction and concluding section) has focussed on the inverse relation between rents and connectedness i.e. they have tried to see if connected coalitions become less likely as rents increase. It is of importance that the empirical hypothesis has been rejected suggesting that there may be a non monotonic relation. Our setup suggests why we may see this non monotonicity i.e. disconnected coalitions occurring at low rents and again at high rents. Also, note that a median party may be left when the benefits from office are high as we show. This has also received attention in the empirical studies on coalitions.

## 4. Rents and Coalition structure: Understanding the comparative STATICS

The different coalitions that emerge when we vary the seat share as well as the rents from office give us some general insight into the main question that this paper
tries to address. What effect does the change in the value of rents from office $P$ have on the outcome of the government formation game? An increase in $P$ makes it more lucrative for all the political parties to be a part of the government. Hence the set of coalitions that are acceptable to members within the coalition at a lower level of rents are acceptable at a higher level as well. However, when the status quo is given by the seat weighted average of all the members of the parliament (or more generally by some function of the total rents available) they may want to block a coalition they are not in at a higher level of rents as their benefit from the status quo increases. Hence, $W \cap Y$ is non monotonic in $P$. This is evident from the characterization for both the symmetric and non symmetric cases we analyzed. However, we do know that Riker's size principle holds in the limit. For the exogenously given status quo when rents drop to 0 when the status quo is implemented, the analysis is somewhat different. Here, the set of coalitions that are feasible at a lower level of rents are feasible at a higher level of rents as well since people outside the coalition are unaffected by changes in $P$. Formally we have,

Lemma 1. If $P=0$ under $x_{\phi}, W \cap Y$ is weakly increasing in $P$, i.e. if $D \in W \cap Y$ for some $P$, then $D \in W \cap Y$ for all $P^{\prime}>P$.

Proof. It suffices to show that both $W$ and $Y$ are non-decreasing in $P$.
First, let us consider $D \in W$. For any party $i$, such that $i \in A(D)$ but $i \notin D$, the payoff from voting for $D$ is unaffected by changes in $P$. However, for any party $i \in D \cap A(D), v_{i}(D)$ is strictly increasing in $P$. This establishes that the number of parties voting for $D$ is weakly increasing in $P$.

Now consider $Y$. For any $i \in D(\neq \emptyset), v_{i}(D)$ is strictly increasing in $P$. Hence, $Y$ i.e. the set of coalitions that are unanimously preferred by its constituents over the status quo cannot go down.

The above lemma leads to a useful insight: an increase in political rents may give the formateur a greater set of potential coalitions. Hence, the payoff of the formateur
may be higher with higher $P$, even after controlling for the direct effect of a greater $P$.

The only remaining case to consider is when the $P$ is vested to a particular coalition (say, the last coalition in power) in case the formateur fails to form a coalition. It is easy to see that either we will get a minimum winning coalition (subject to inclusion of the formateur) beyond a certain $P$ or we get minority governments. Minority governments occur for instance if a median party proposes a single party government and the status quo is such that $P$ is vested with the median party if it fails to form a government. We can summarize the several possibilities by the following proposition

Proposition 1. There exists an upper bound to $P$ beyond which the equilibrium coalition $D_{k}$ for any formateur $k$ does not change with changes in $P$. Moreover, these 'limiting coalitions' cannot be surplus.

Proof. When the status quo is given by the seat weighted average of all parties in parliament, the proof goes as follows. Consider party $i \in N$ and let $M$ denote the smallest minimum winning coalition subject to inclusion of $i$ and denote by $s \geq \frac{1}{2}$ the relative size of the coalition (i.e. the fraction of seats it has in the parliament). Denote an alternative larger coalition by $M^{\prime}$ and its relative size by $s^{\prime}$. Clearly, the relative seat share for the formateur under $M>M^{\prime}$. As $u$ is bounded, there exists $P$ for which $u_{M}^{i}+\pi_{i} P>u_{M^{\prime}}^{i}+\pi_{i}^{\prime} P$ for all $i$. Thus, a formateur will never propose $M^{\prime}$. Hence, a larger coalition is ruled out. Now we need to show that a smaller coalition will be voted down by a majority. Denote the smaller coalition by $M^{\prime \prime}$ and its relative size by $s^{\prime \prime}<\frac{1}{2}$. It will be voted against by all members not included in $s^{\prime \prime}$ as long as $u_{\phi}+S_{i} P>u_{M^{\prime \prime}}$ for all $i$ not in $M^{\prime \prime}$ The value of $P$ which satisfies both inequalities is the one beyond which all equilibrium coalitions are minimal winning. When $x_{\phi}$ is arbitrary and $P=0$ if $x_{\phi}$ is implemented, we know that $v_{i}(D)$ is strictly increasing in $P$. Moreover, $v_{i}(D)$ is not bounded. Hence, for sufficiently large $P$ we have $v_{i}(D)>v_{i}(\emptyset)$ for all $i \in D$. Hence, as $P$ increases, any $D$ belongs to $Y$. We have
also proved that $W$ is non decreasing in $P$. Given that $v_{i}(D)$ is strictly increasing in $P$ and not bounded this implies that beyond some $P$ all majoritarian coalitions are in $W$. Hence, beyond a critical $P$ the set $W$ does not change. This means that $W \cap Y$ does not change. Thus, for every $k, D_{k}=\operatorname{argmax}_{D \in W \cap Y} v_{k}(D)$ does not change.

The only remaining case to consider is when the $P$ is vested to a particular coalition (say, the last coalition in power) in case the formateur fails to form a coalition. It is easy to see that surplus coalitions cannot occur. Consider any surplus coalition and let $D-i$ denote the coalition without member $i$. The change in the ideological component in utility of the formateur $k$ is $\left|u_{k}\left(\left|x_{i}-x_{D}\right|\right)-u_{k}\left(\left|x_{i}-x_{D-i}\right|\right)\right|$ which is finite. The gain in rents is $\frac{s_{i}}{s_{D-i}} P-\frac{s_{i}}{s_{D}} P$. For sufficiently large $P$ this gain is greater than any ideological loss. Now, this also implies that the coalition which guarantees the largest share of $P$ will determine the coalition for each formateur $k$ beyond a certain level of $P$.

However, at intermediate levels of rents coalition structure is responsive to changes in $P$ as we have seen. The following propositions (written in terms of three parties for gleaning the main intuition) gives us an indication of what kind of coalitions occur as $P$ changes

Proposition 2. Let there be 3 parties, 1, 2 and 3. Let the policy space be $\mathbb{R}$ with $x_{1}=0, x_{2}=x \leq \frac{1}{2}$ and $x_{3}=1$. Denote by $S_{i}$ the seat share of party $i$ and assume that $\max S_{i}<\frac{1}{2}$. Let the status quo policy $x_{\phi} \in[0,1)$. Party 3 can never form a minority government.

Proof. As $x_{\phi}<1$, both party 1 and party 2 prefer the status quo. Hence, $\{3\}$ does not belong to $W$.

Proposition 3. Let there be 3 parties, 1, 2 and 3. Let the policy space be $\mathbb{R}$ with $x_{1}=0, x_{2}=x \leq \frac{1}{2}$ and $x_{3}=1$. For each party $i$ let $S_{i}$ the seat share of party $i$ $=\frac{1}{3}$. Let the status quo policy be $x_{\phi} \in[0,1)$ and let $P_{\phi i}=P_{\phi}$ for all $i$. The range of
$P$ for which 2 is able to form a minority government is greater than that for which 1 can form a minority government. Further, for a given $P$, the range of $x$ at which 1 can form a government is greater than that for which 2 can form a government. The range is higher the lower is $P$.

Proof. We need to consider two cases, $x_{\phi} \leq x$ and $x_{\phi}>x$. When $x_{\phi} \leq x, 3$ supports $\{2\}$ as long as $u(|x-1|)>u\left(\left|x_{\phi}-1\right|\right)+P_{\phi}$. However, 3 never supports $\{1\}$. Moreover, 2 never supports $\{1\}$. When $x_{\phi}>x,\{2\}$ has 1's support whenever $u\left(\left|x_{\phi}\right|\right)+P_{\phi}<u(|x|)$ while $\{1\}$ has 2's support when $u\left(\left|x_{\phi}-x\right|\right)+P_{\phi}<u(|x|)$. As $u\left(\left|x_{\phi}-x\right|\right)>u\left(\left|x_{\phi}\right|\right)$, the range of $P$ for which 2 is able to form a minority government is greater than that for which 1 can form a minority government. The proof for the second and third parts of the proposition is similar and the interested reader can see the characterisation in the Appendix to confirm the intuition.

Proposition 4. Let there be 3 parties, 1, 2 and 3. Let the policy space be $\mathbb{R}$ with $x_{1}=0, x_{2}=x \leq \frac{1}{2}$ and $x_{3}=1$. Denote by $S_{i}$ the seat share of party $i$ and assume that $\max S_{i}<\frac{1}{2}$. Let the status quo policy $x_{\phi} \in[0,1]$ and each party enjoy a fraction $S_{i} P$ of the rents under the status quo. Further, let $S_{2}=\max S_{i}$. There exists a threshold $P$ beyond which the equilibrium coalitions are disconnected whenever 1 or 3 is the formateur. Below this threshold, coalitions connectedness varies nonmonotonically with $P$.

Proof. The result on the limiting coalitions follows from Proposition 1 which says that the limiting coalitions when parties enjoy a fraction of the rents under the status quo must be minimum size, subject to inclusion of the formateur. In this case the equilibrium coalition is $\{1,3\}$ whenever 1 or 3 are the formateurs. See the characterisation in the Appendix for the non monotonic relation.

We now analyze some special cases. In particular, we analyze what happens when the status quo policy is not endogenously determined but instead depends on some
exogenous factors like the policy that the previous government had chosen as is often the case. Moreover, we assume that no rents are distributed in that case. This can be thought of in one of two ways-either the failure to form a government affects the ability to extract rents (i.e. there is a spending freeze) or the old patronage appointments made by the previous government does not change so the $P$ stays with whichever coalition was in power. In this setup, we examine two special scenarios. We shall show that if parties are ideologically motivated (i.e. $P=0$ ) consensus governments cannot form. The other case is when the status quo is outlying as in the case where policy preferences of all parties have shifted as in moving from a protectionist trade policy to a more liberal one or more generally in a regime change (from socialism to free market).

Proposition 5. (Purely Ideological parties) Assume that $P=0$ and that $x_{\emptyset} \in$ $\left[x_{\min }, x_{\max }\right]$, then there cannot be a consensus government unless the implemented policy of the national government exactly coincides with $x_{\emptyset}$.

Proof. To see this let $\bar{x}$ denote the seat-share weighted average of all parties' policy positions. There are two possibilities to consider, 1) $\bar{x}>x_{\emptyset}$, and 2) $\bar{x}<x_{\emptyset}$. In the first case the party with ideal point $x_{\text {min }}$ will refuse to be a part of the coalition whereas in the second case the party with ideal point $x_{\max }$ will refuse to join the government. Hence the consensus government option, i.e., set $N$ is not in $Y$ and therefore not in $W \cap Y$.

Proposition 6. (Outlying status quo) With an outlying status quo, i.e. if $x_{\emptyset} \notin$ $\left[x_{\min }, x_{\max }\right]$, then one can get a consensus government even if $P=0$.

Proof. It suffices to consider the following example. Let the ideal points of the parties be $0,0.5$ and $0.5+\varepsilon$ with $\varepsilon \approx 0$. Let each party have an equal seat share and let the party with ideal point 0 be the formateur. Assume that $x_{\emptyset}=0.7$. It is clear that if the party with ideal point 0 invites either of the parties with ideal points 0.5 or $0.5+\varepsilon$
to form a coalition, then they refuse it as the outside option is closer to them. But if the party with ideal point 0 invites both of them together, then the implemented policy is $0.33+\varepsilon / 3$ which both the invitees prefer over 0.7 . Hence, the party with ideal point 0 would propose a consensus government.

Note that for small values of $P$ the consensus government is still the equilibrium. Again, if the outside option were to be too unattractive (e.g. 0.8) then there would be no consensus government. Thus, we see that a consensus government can form for moderately unattractive outside option and low $P$. This illustrates another point as well. Here, we have an example of a scenario where we have a surplus government but if we take equilibrium considerations the government is not surplus as the formateur with ideal point 0 can form a government only if both the parties are invited. Thus, we also provide a new explanation for why we may see coalitions which have additional members who may be left out without the government losing a majority. Also, if $P \neq 0$ as noted earlier we can have national governments even when the status quo $x_{\emptyset} \in\left[x_{\text {min }}, x_{\text {max }}\right]$

The results above give us a number of insights. First, we note that surplus coalitions cannot occur when rents are high. However, depending on the status quo arrangement, coalitions can be either minority or minimum winning. Minority governments occur with greater frequency when a median party is the formateur and also when rents are low (Proposition 3 demonstrates that). Extreme parties find it difficult to form minority governments unless the status quo is very outlying (as shown in Proposition 2). Another important result to comment on is the relation between connectedness of coalitions and political rents. Following Axelrod (earlier cit.) one would predict that at low rents parties would form ideologically connected coalitions. However, that is not necessarily the case as we have seen. This non monotonic relation arises as a consequence of central parties refusing to be part of the coalition at low rents. Furthermore, size of seats in parliament may cause the coalition to be
disconnected even at low rents (see Proposition 4). As we have pointed out, the data also does not support the hypothesis about disconnected coalitions being more frequent at low rents (see Indridason (2003)). It is also worth noting that coalition size is non monotonic in $P$. As we have seen we could either move from larger coalitions to smaller coalitions (surplus to minimal winning) when $P$ increases or conversely from smaller coalitions to larger ones (minority to minimal winning).

## 5. Multidimensional Analysis and comparison with Baron Diermeier <br> MERLO's EFFICIENT BARGAINING FRAMEWORK

So far we have done a one dimensional analysis. The models of Baron Diermeier and Diermeier Merlo (BDM when referring to them together) consider scenarios where there are two policy dimensions and characterize a 'symmetric case' with three parties which are equidistant from each other. While the two dimensional model they use can be analyzed in our framework as well, the one dimensional analysis suffices to look at the issue of 'connectedness' ${ }^{14}$ of coalitions and develop comparative static results with regard to how equilibrium coalitions could change (from connected to disconnected or vice versa) as a function of seat share, ideological closeness and the value of the rents from office. In this section we provide a two dimensional characterization for a symmetric 3 party case which would also facilitate comparison with BDM. We also comment on the differences in the results that they obtain with our results for the same set of ideal points. Following BDM, let the ideal points of party 1,2 and 3 be located on the vertices of an equilateral triangle with the coordinates being respectively $(0,0),(1,0)$ and $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$. To make things comparable we assume that $S_{i}=\frac{1}{3}$ for all $i$. We divide the regions by drawing lines from the party's ideal points which cross at the centroid of the triangle

[^9]

Figure 1. Possible coalitions

When the status quo is the seat weighted average, we have $x_{\phi}=\left(\frac{1}{2}, \frac{\sqrt{3}}{6}\right)$. We show that regardless of the identity of the formateur and the value of $P$ minimal winning coalitions emerge. Further, the formateur is indifferent to the choice of a coalition partner. The intuition is simple enough. Any formateur is unable to form a minority government as the other partners prefer the status quo policy to the formateur's ideal point. However, any party asked by the formateur is willing to join as the compromised policy (which is midway between them) is preferred by both to the status quo. This goes through for any value of $P$ as the seat shares are the same and there is thus no issue of wanting to take a smaller party. We formalize this in the following proposition.

Proposition 7. Let there be three parties 1, 2, 3 and two policy dimensions $x$ and $z$. Further, let the parties have policy positions given by $(0,0),(1,0)$ and $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ respectively. Let the utility function for party $i$ be given by $-\frac{1}{2}\left(y_{1}-x_{i}\right)^{2}-\frac{1}{2}\left(y_{2}-z_{i}\right)^{2}+\pi_{i} P$, if party $i$ is in government and by $-\frac{1}{2}\left(y_{1}-x_{i}\right)^{2}-\frac{1}{2}\left(y_{2}-z_{i}\right)^{2}$ otherwise, where $y=$ $\left(y_{1}, y_{2}\right)$ is the implemented policy. The status quo policy is given by $x_{\phi}=\left(\frac{1}{2}, \frac{\sqrt{3}}{6}\right)$. Then, for any formateur $i \in 1,2,3$ the equilibrium government will be minimum winning. Further, $i$ will be indifferent to the choice of a coalition partner.

Proof. As the parties are located symmetrically, we can consider the case where 1 is the formateur without loss of generality. In that case 1 can propose to either 2 or 3 and in each case the utility both to the formateur and the other party will be $-\frac{1}{4}+\frac{P}{2}$. Now the utility from the status quo which is $\left(\frac{1}{2}, \frac{\sqrt{3}}{6}\right)$ (to both parties) will be $-\frac{1}{3}+\frac{P}{3}<-\frac{1}{4}+\frac{P}{2}$. Hence this proposal will be accepted by whoever the formateur invites. Clearly, this also rules out supermajoritarian governments as the policy (in this three party case) is the same as the status quo policy. Now, we only need to check that a formateur cannot propose a minority government. This is also easy to see as the minority government yields a utility of -1 for the two excluded parties which is less than $-\frac{1}{3}+\frac{P}{3}$. Hence, the minority government will be voted against by the two excluded parties. Notice, that the results are the same whoever is chosen by the formateur in the minimum winning coalition as the three parties are equidistant from each other. Hence, the formateur is indifferent to a choice of coalition partner.

This result contrasts with both the papers. In the static version of Diermeier and Merlo, only minority and surplus (supermajoritarian) coalitions occur. That is because of the particular status quo they take. Diermeier and Merlo constrain the status quo $x_{\phi}$ to lie in the set $\left(\left(x_{1}, y_{1}\right),\left(x_{2}, y_{2}\right),\left(x_{3}, y_{3}\right)\right)$. Baron and Diermeier (2001) have an arbitrary status quo, in general their status quo can lie in one of the six partitions of the triangle or even outside it. They get minimum winning coalitions except when the status quo is very outlying. The reason is that with an outlying status quo, coalition partners (in their framework) are willing to make large transfers to be in the government.

An important distinction in our approach as compared to BDM is worth mentioning. For large enough $P$ we necessarily get minimum winning coalition. In other words when the value that parties place on rents (or directed transfers to their own groups) as compared to ideology increases, we converge to the prediction of Riker. On the other hand, the choice of coalition in BDM is driven by the status quo and is
independent of the value of $P$ increasing. Hence, even when $P$ is very large they do not converge to Riker's prediction. Both approaches have merit, however because of 'efficient bargaining' BDM are unable to analyze how coalition structure varies as the relative importance of ideology to rents from office changes. It is also worth noting that they also ignore any endowment constraints. They analyze $P$ as perks which can be freely transferred but they do not analyze what happens when the value is small so that the transfers required for the efficient outcome cannot be made .In other words, their results would vary when the constraint becomes binding (i.e. $P$ is low).

A comparison can also be carried out for an arbitrary status quo in the BDM framework to see what happens if we replace their assumption of full commitment and perfect transferability with the 'no commitment' assumption that we use in our model. To rule out issues of how $P$ would be divided we set it to 0 . We find that even when we consider the status quo chosen by BDM in the two papers we get different results if we assume no commitment. We first analyze the case. where the status quo is the policy of any one of the parties. In that scenario, there are two cases to consider, one where the policy of the formateur $i$ is the status quo and another where $j \neq i$ is the formateur. In the first case it is easy to see that $i$ proposes a minority government including only itself and all parties support it as they are indifferent to the government's policy and the status quo $x_{\phi}$ which are the same. In the second case the proposer $j$ also proposes a minority government and is supported by the party which is different to $j$ 's policy and the status quo as they are equidistant. Hence, the results in the static version of their paper gives only minority governments under our assumptions.

Now, consider the more general case. It is easy to show that only minority or minimal winning governments can form. In each of these six regions (and assuming $\left.x_{\phi} \neq x_{i}\right)$ if the party closest to the status quo is the proposer a minimal winning coalition forms and the proposer is indifferent to the identity of the other coalition partner.

## 6. Empirical relevance, future work and concluding remarks

We have presented a model of parliamentary democracy which predicts political coalition formation as a function of seat share and the importance of rents relative to ideology. In particular, two limiting cases arise, one when parties care only for ideology and another where the rents from office become very large. We find that in the limit Riker's size principle (in its modified form) provides an upper bound to the size of coalitions when the value of rents from office become very big. This contrasts with the efficient bargaining models of BDM. We explicitly consider the role of party size and show that 'disconnected coalitions' can occur even with parties that are purely driven by ideology. Thus, when parties care only for ideology they may leave out an ideologically close partner because a large party can tilt the policy too close towards its ideal point because of its greater bargaining strength.

We can compare our paper to the papers by BDM which use efficient bargaining and perfect commitment within the coalition. Given any coalition, our model predicts a policy orientation that is skewed towards larger parties while BDM predicts that seat share will be irrelevant. This stems from the bargaining procedure in BDM giving equal weights to all parties. With weights in proportion to size, our results in this regard will match. The more fundamental difference comes from the assumption of no commitment which leads to substantive differences in predictions for coalitions for a given composition of the legislature. Empirically, we do not believe that perfect commitment or the converse are observed. However, our results are robust to some degree of imperfect commitment. We believe that apart from the usefulness of analyzing the polar opposite of BDM, no commitment at the coalition formation stage is often a good approximation-this implies that the proposer cannot make a take it or leave it offer such that anything in the status quo's majority 'win set' would be accepted. This seems consistent with the observed phenomenon of power sharing that is seen in coalitions around the world. As we had pointed out earlier, cabinet
seats being allotted roughly in proportion to seat shares is one of the most observed empirical regularities of coalition governments. Laver and Schofield (1990) also provide a discussion for why at the coalition formation stage the manifestos written are not binding and that actual policies are a result of intricate bargaining inside the coalition. ${ }^{15}$ Hence, both as an approximation to what happens in the real world and in terms of predictive power we argue that no commitment is not a very unrealistic assumption.

A crucial issue that we address, but which BDM cannot (because of their assumption of efficient bargaining) is looking at the size and ideological connectedness of coalitions as functions of rents. The two recent papers by Indridason ((2003) and (2005)) go some distance towards looking at this. In one paper he examines the nature of disconnected coalitions in the context of a few northern European countries and tentatively concludes that this is related to whether the office seeking model is more appropriate or whether the coalition is governed by ideological considerations. However, he does not find support for the hypothesis that as the office motive of parties become more important, connected coalitions occur with less frequency. Our model provides a theoretical framework for explaining why in fact there may be a non monotonic relation between coalition connectedness and rents. In particular, it may explain why disconnected coalitions occur at low rents. It is certainly worth looking at some of our predictions using the same dataset ${ }^{16}$. Kalandrakis (2002) tests the prediction that minority governments occur less frequently when rents are high, a hypothesis that finds support in the dataset of Indridason as well. Our model provides a theoretical foundation for this claim. We also find that central parties are able to

[^10]form minority governments than parties which are further from the median which is supported in the tests carried out by Indridason and Kalandrakis.

An extremely important issue is the formateur selection procedure. Diermeier and Merlo (2004) provide the first systematic empirical study but they study only the first stage i.e. they look at whether it is the case that the largest party is always asked to form the government against the alternate that this is roughly in proportion to seat shares and finds empirical support for the latter. This does not matter particularly in this paper but would definitely have important implications in analyzing party formation.

Another challenging extension on the theoretical side is to integrate these models of coalitional bargaining with endogenous party formation. There are also a fair number of models of party formation/strategic entry starting from the 'citizen candidate' models of endogenous candidate entry to more recent papers by Riviere (2000), Morelli (2001) and Osborne and Tourky (2002) (in the recent papers a distinction is made between a candidate and a party). Hamlin and Hjortland (2000) for example integrate the citizen candidate model with a legislature where by assumption the policy is a seat weighted average of the entire legislature (this seat weighted average with two exogenously given parties had been first analyzed by Ortuño-Ortín (1997)) and De Sinopoli and Iannantuoni (2001) consider a model with non strategic parties but strategic voters. Both papers predict two parties under proportional representation contrary to empirical evidence. These papers thus miss one of the most important aspects of parliamentary democracy which is coalition formation ${ }^{17}$ (see Dhillon (2003) for a comprehensive survey of the recent literature). Baron and Diermeier (2001) and Austen Smith and Banks (1988) do look at endogenous policy with fixed candidates

[^11]while Bandyopadhyay and Oak (2004) look at endogenous party formation but with sincere voting. A fully strategic model of a parliamentary democracy is certainly something which would add to our understanding of how legislative bargaining affects political competition.

## Appendix: Characterising equilibrium coalitions ${ }^{18}$

I: A symmetric three party characterisation when the status quo is given by a seat weighted average and $P$ is shared in proportion to seats. Consider a legislature comprised of 3 parties, 1,2 and 3 , with $x_{1}=0, x_{2}=x \leq \frac{1}{2}$ and $x_{3}=1$. We shall further assume that $S_{1}=S_{2}=S_{3}=\frac{1}{3}$. The status quo is given by $x_{\phi}=\frac{1+x}{3}$ and each party gets rents $\frac{P}{3}$ in that case. Each party has well defined preferences denoted by a weak ordering $\succ_{i}$ over the set of possible coalitions. If two or more parties prefer a coalition $D$ over the status quo $\{1,2,3\}$, then coalition $D$ succeeds in forming a government. To simplify the characterisation, we will assume a quasilinear utility function of the form

$$
u_{i}=-\left(x_{i}-x\right)^{2}+z
$$

We will completely characterize the set of equilibrium coalitions.
First suppose that party 2 is chosen as the formateur. It obviously prefers $\{2\}$ over any other $D$ and will succeed in forming the government if $\{2\} \succ_{1}\{1,2,3\}$. 1's payoff from $\{2\}$ in power is $-x^{2}$ while his payoff from $\{1,2,3\}$ is $-\left(\frac{1+x}{3}\right)^{2}+\frac{P}{3}$. Hence, 1 will support the coalition if

$$
-x^{2} \geq-\left(\frac{1+x}{3}\right)^{2}+\frac{P}{3}
$$

which simplifies to $8 x^{2}-2 x \leq 1-3 P$. Hence, if the above condition holds, 2 will successfully propose a minority government comprised only of itself. If this condition does not hold, then 2 's other alternatives are $\{1,2\}$ or $\{2,3\}$ or $\{1,2,3\}$ or $\{1,3\}$. Since $x<\frac{1}{2},\{1,2\} \succ_{2}\{2,3\}, 2$ will propose $\{1,2\}$ if $\{1,2\} \succ_{2}\{1,2,3\}$ which is equivalent to

$$
-\left(\frac{x}{2}\right)^{2}+\frac{P}{2} \geq-\left(x-\frac{1+x}{3}\right)^{2}+\frac{P}{3}
$$

[^12]which simplifies to $\frac{P}{6}+\frac{1}{9} \leq \frac{4 x}{9}-\frac{7 x^{2}}{36}$. Note that 1 will always support $\{1,2\}$ over $\{1,2,3\}$ and hence 2 is assured of winning the vote of confidence if 2 proposes $\{1,2\}$. Conversely, if $\frac{P}{6}+\frac{1}{9} \geq \frac{4 x}{9}-\frac{7 x^{2}}{36}$, then 2's next best alternative would be $\{1,2,3\}$. Notice that the reason 2 proposes the surplus coalition $\{1,2,3\}$ is to balance the policy towards 2's ideal point by having two parties with preferences on either side of 2 's ideal point. This happens when 2 is more distant from 1 and $P$ is low.

The following diagram summarizes the various possible coalitions in the ( $P, x$ ) space.


Figure 2. Coalitions with 2 as Formateur

Let party 1 be the formateur. Its most preferred government is $\{1\}$, which it will succeed in forming if $\{1\} \succ_{2}\{1,2,3\}$, which is same as

$$
-x^{2} \geq-\left(x-\frac{1+x}{3}\right)^{2}+\frac{P}{3}
$$

which simplifies to $\frac{5 x^{2}}{9}+\frac{4 x}{9} \leq \frac{1}{9}-P$. If the above condition fails to hold, the next best feasible alternative for party 1 is $\{1,2\}$. Party 2 will accept 1 's proposal to form $\{1,2\}$ if

$$
-\left(\frac{x}{2}\right)^{2}+\frac{P}{2} \geq-\left(x-\frac{1+x}{3}\right)^{2}+\frac{P}{3}
$$

which simplifies to $\frac{P}{6}+\frac{1}{9} \leq \frac{4 x}{9}-\frac{7 x^{2}}{36}$. If neither $\{2\}$ nor $\{1,2\}$ are feasible, then 1 could propose either $\{2\},\{1,3\}$ or $\{1,2,3\}$.

Note that $\{2\}$ will always get party 2 's support. Hence, 1 would propose $\{2\}$ if $\{2\} \succ_{1}\{1,3\}$ and $\{2\} \succ_{1}\{1,2,3\}$. The former condition is equivalent to $-x^{2} \geq$ $-\left(\frac{1}{2}\right)^{2}-\frac{P}{2}$ and latter is equivalent to $x \geq-\left(\frac{1+x}{3}\right)^{2}+\frac{P}{3}$. Similar conditions can be obtained for the range over which $\{1,3\}$ is the best feasible combination. The following diagram shows the various equilibrium coalitions when party 1 is the formateur. In particular, notice that when 1 proposes $\{2\}$ we have an example where a formateur proposes a government excluding itself. It can be interpreted as a party opting out of forming a government when it knows it will not be able to form one ${ }^{19}$.


Figure 3. Coalitions with 1 as Formateur
To study the possible coalitions when party 3 is the formateur, we do a similar exercise of deriving 3's best feasible coalitions. The following diagram shows the various equilibrium coalitions when party 3 is the formateur. The interesting thing to note here is how disconnected coalitions occur over a wider range, and minority governments never occur as either 3 or the reaming two parties always prefer the $\{1,2,3\}$ coalition.

Connected vs. disconnected coalitions. An important insight that the recent empirical work on coalitions has revealed (see Indridason, earlier cit.) is that disconnected

[^13]

Figure 4. Coalitions with 3 as Formateur
coalitions may be seen more frequently when ideology is less important as compared to rents (or what we also interpret as special transfers as opposed to policies which affect all groups). However, as we noted, this is not necessarily the case. To see the intuition for how rents affect the ideological composition of coalitions, consider two particular cases, one where there is a large party which is centrally located and two smaller parties on either side and another where there are two ideologically similar parties with a large party further away from them. We can, to fix ideas, assume, as in the symmetric case, that we have parties $1,2,3$ with ideal points $(0, x, 1) . S_{2} \geq$ $\max \left(S_{1}, S_{3}\right)$. Thus, essentially we now introduce asymmetry in party size to see how that affects coalition formation. The closer $x$ is to $\frac{1}{2}$ the lower is the value of $P$ to disconnected coalitions. Again, as $S_{2}$ gets bigger chances of a disconnected coalition increases (until $S_{2}=1$ ). This captures the intuitive phenomenon that the centrist party is left out as it is asking for too much (in terms of share of $P$ ). However, this is a special case of a more general result, namely as the value of $P$ gets larger (and rents are distributed even under a status quo), the equilibrium coalitions are minimum winning (subject to the formateur being in the coalition). Hence, in this case the two smallest parties form a coalition and if they are on either side the coalition is disconnected. On the other hand, when they are on the same side, we can get again
get disconnected coalitions when $P$ is very high as the far extreme party will call on the smallest partner which may be farthest from it. However, as we have pointed out earlier, the relation is non monotonic in $P$.

We now characterise a case where there are two small parties on either side of a large party. The characterisation for the other cases are similar.

## II: A 'central' large party and two small parties with the status quo given

 by a seat weighted average and $P$ shared in proportion to seats. We now characterise equilibrium coalitions when there is a centrally positioned large party and two smaller parties on either side of it on the ideological plane. We analyze the tradeoff between ideological distance and party size.The legislature is composed of three parties $1,2,3$ with , $S_{1}=S_{3}=S, \frac{1}{4}<S<\frac{1}{3}$. Thus, $S_{2}=1-2 S$. As before, each party has well defined preferences denoted by a weak ordering $\succ_{i}$ over the set of possible coalitions and the status quo is given by a seat weighted average of party members' ideal points and $P$ shared in proportion to seats. If two or more parties prefer a coalition $D$ over the status quo $\{1,2,3\}$, then coalition $D$ succeeds in forming a government. We assume that the status quo is given by a seat weighted average and $P$ shared in proportion to seats.

We look at how equilibrium coalitions are affected as party size and rents from office changes i.e. we demarcate regions on the $(S, P)$ plane holding the distance of the parties from each other constant. To focus on this tradeoff, we consider the case where the smaller parties are symmetrically placed on either side of the central large party. WLOG assume the ideal points to be $0, \frac{1}{2}, 1$.

Party 1 as formateur. Note that party 1 would like to propose $\{1\}$. However, $\{1,2,3\}$ is preferred by both 2 and 3 at all $P$. The other alternatives that 1 has is to either propose $\{1,2\}$ or $\{1,3\}$. This is because $\{1,2,3\}$ is dominated by $\{1,3\}$ for both 1 and 3 (except when $P=0$ when the $\{1,2,3\}$ coalition is indistinguishable from the $\{1,2\}$ coalition.) and the single party coalitions do not belong to $W \cap Y$ either.

Thus, we need to check only whether $\{1,2\}$ or $\{1,3\}$ will form. Under the $\{1,2\}$ coalition policy is given by $\frac{S}{1-S} 0+\frac{1-2 S}{1-S} \frac{1}{2}=\frac{1-2 S}{2(1-S)}$ and for the $\{1,3\}$ coalition the policy is $\frac{1}{2}$. Hence, the equilibrium coalition with 1 as formateur is the disconnected coalition $\{1,3\}$ as long as

$$
-\left(\frac{1}{2}\right)^{2}+\frac{P}{2}>-\left(\frac{1-2 S}{2(1-S)}\right)^{2}+\frac{S}{1-S} P
$$

or

$$
P>\frac{2-3 S}{(1-3 s)(1-s)}
$$

However, when the opposite inequality holds i.e.

$$
P<\frac{2-3 S}{(1-3 S)(1-S)}
$$

the $\{1,2\}$ coalition forms if 2 prefers that to the status quo (i.e. the $\{1,2,3\}$ coalition). The condition for that is

$$
-\left(\frac{S}{2(1-S)}\right)^{2}+\frac{1-2 S}{1-S} P>\frac{P}{3}
$$

or

$$
P>\frac{3 S^{2}}{4(1-S)(2-5 S)}
$$

Combining these conditions we get

$$
\frac{3 S^{2}}{4(1-S)(2-5 S)}<P<\frac{2-3 S}{(1-3 S)(1-S)}
$$

Hence, we see that at low $P$ we get a disconnected coalition whenever 1 or 3 is the formateur as 2 is unwilling to accept an offer to form a coalition with either, as $P$ increases we get connected coalitions. However as $P$ increases even further, we get disconnected coalitions as the smaller parties prefer to leave the large centrist party out so that they get a larger share of the rents. Hence, the relationship between connectedness of coalitions and rents is non monotonic in rents.


Figure 5. Coalitions with 1 as Formateur


Figure 6. Coalitions with 2 as Formateur
Party 2 as formateur. With 2 as formateur, we always get connected coalitions, either the $\{1,2,3\}$ coalition when $P$ is small or the $\{1,2\}$ (or $\{2,3\}$ ) coalition as $P$ increases. The coalitions with party 3 as formateur is symmetric to that with party 1 so we do not analyze it separately.

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[^0]:    ${ }^{1}$ See also Laver and Schofield (1990) chapters 4 and 5 for details. The distinction they make between the members of government and members who support the government is that the members of the government are directly involved in policy making and enjoy additional benefits of office by having control over various departments of the government.
    ${ }^{2}$ A left right coalition leaving a centrist out is an example of a disconnected coalition i.e a coalition that is not ideologically connected.
    ${ }^{3}$ The most famous example is the Dutch elections in 1994 when the PvdA emerged as the biggest party, even though it lost 12 seats in parliament. With the PvdA, the right-liberal VVD and leftliberal Democraten 66 being of a roughly equal size in parliament, a disconnected coalition was formed without the centrist Christian Democrats (as reported in Wikipedia. Indridason (2003) and (2005) provide detailed analysis of such cases.)

[^1]:    ${ }^{4}$ The motivation of political parties i.e. ideological or office seeking is an old debate in political economy though how the relative importance of the two affects government formation has not received much formal attention. Office motivated parties or candidates have been formally analyzed starting from Downs (1957). Wittman (1977) analyzed ideologically motivated parties. Citizen candidate models (see Besley and Coate (1997) and Osborne and Slivinski (1996)) look at endogenous candidate entry with ideological candidates who cannot commit to policies other than their ideal point.
    ${ }^{5}$ The particular assumption about the bargaining outcome has strong empirical support. Empirical studies of power sharing among coalition partners (see Browne and Fendreis (1980) and Laver

[^2]:    and Schofield, (1990)) have found substantial evidence that coalition partners share cabinet portfolios in proportion to their relative seat shares. Since a large bulk of political power is vested in various ministerial offices, the politician in charge of a particular ministry is entitled to that power as well as the right to make a policy in the relevant area.
    ${ }^{6}$ This is a modified version of Riker's size principle that governments should be minimum winning (see Riker (1962)).

[^3]:    ${ }^{7}$ The study of equilibrium coalitions dates back to Riker (1962). He predicted a minimum winning coalition i.e. the minimal winning coalition made of the smallest number of members. With consideration of ideology following Axelrod (1970), the natural thing to predict would be a 'minimum winning connected' coalition-i.e. the smallest winning coalition that is ideologically connected. Until the papers by BDM, most of the theoretical papers did make such predictions. (See Austen Smith and Banks (1988), Baron (1989, 1991,1998), Lupia and Strom (1995) and Roemer (2001) for instance).

[^4]:    ${ }^{8}$ Note that with budget constraints (i.e. bounds on how much monetary transfers parties are able to make) the coalitions in BDM do change. However, they do not analyze what happens in the presence of such constraints

[^5]:    ${ }^{9}$ Note that we could alternately have assumed that people care for policy and rents (or directed transfers) in the ratio $\alpha$ and $1-\alpha$. Hence, payoff for a citizen of group $i$ can be written as $-\alpha u\left(\left|x_{i}-x\right|\right)+(1-\alpha) z$. The comparative statics that we do would have been in terms of the marginal rate of substitution between policy benefits and rents i.e. varying $\alpha$. This makes no qualitative change, so we keep the simpler formulation.

[^6]:    ${ }^{10}$ This is similar to Austen Smith and Banks (1988). It would make no qualitative difference if we assumed that the total rents in a caretaker government are only a fraction $\beta$ of what would be available otherwise i.e. each party in the event of a caretaker government gets $\beta_{i} P$.

[^7]:    ${ }^{11}$ We could instead have assumed that each party tries to maximize the payoff of its representative member. The results essentially remain unchanged.
    ${ }^{12}$ The results remain unchanged if the policy (and rents sharing) is given by any function of the seat weighted average, as long as higher seats get higher weights.

[^8]:    ${ }^{13}$ The definition of $D_{k}$ ensures that weakly dominated strategies are not played.

[^9]:    ${ }^{14}$ In multidimensional space, even though our definition of 'connectedness' is applicable, connectedness is most easily understood for a one dimensional issue space.

[^10]:    ${ }^{15}$ See also Laver and Shepsle (1995).
    ${ }^{16}$ Different people have used different ways to estimate the rents from office. Diermeier, Keane and Merlo (2002) estimate the 'value' to being a politician but for the US senate. It would be interesting to do an analysis with the measures they use in the context of a parliamentary democracy.

[^11]:    ${ }^{17}$ There are also papers dealing with strategic entry under more general outcome functions most notably Dutta, Jackson and LeBreton (2000) but the generality of the paper makes it impossible for them to generate any sharp predictions except that under complete information at least one player (candidate) will behave strategically.

[^12]:    ${ }^{18}$ The programme used to generate the different coalitions can be used to characterise coalitions for arbitrary values of the status quo as well as any arbitraty division of $P$.

[^13]:    ${ }^{19}$ In the last few elections in India this has happened when a party has indicated that it does not wish to form a government.

