AgEcon SEARCH
RESEARCH IN AGRICULTURAL & APPLIED ECONOMICS

The World's Largest Open Access Agricultural & Applied Economics Digital Library

This document is discoverable and free to researchers across the globe due to the work of AgEcon Search.

Help ensure our sustainability.

Give to AgEcon Search

AgEcon Search
<http://ageconsearch.umn.edu>
aesearch@umn.edu

*Papers downloaded from **AgEcon Search** may be used for non-commercial purposes and personal study only. No other use, including posting to another Internet site, is permitted without permission from the copyright owner (not AgEcon Search), or as allowed under the provisions of Fair Use, U.S. Copyright Act, Title 17 U.S.C.*

Applications of Relations and Graphs to Coalition Formation

Rudolf Berghammer, Agnieszka Rusinowska
and Harrie de Swart

NOTA DI LAVORO 77.2006

MAY 2006

CTN – Coalition Theory Network

Rudolf Berghammer, *Institute of Computer Science, University of Kiel*
Agnieszka Rusinowska, *Nijmegen School of Management, Radboud University Nijmegen and
Department of Mathematical Economics, Warsaw School of Economics*
Harrie de Swart, *Department of Philosophy, Tilburg University*

This paper can be downloaded without charge at:

The Fondazione Eni Enrico Mattei Note di Lavoro Series Index:
<http://www.feem.it/Feem/Pub/Publications/WPapers/default.htm>

Social Science Research Network Electronic Paper Collection:
<http://ssrn.com/abstract=904196>

The opinions expressed in this paper do not necessarily reflect the position of
Fondazione Eni Enrico Mattei
Corso Magenta, 63, 20123 Milano (I), web site: www.feem.it, e-mail: working.papers@feem.it

Applications of Relations and Graphs to Coalition Formation

Summary

A stable government is by definition not dominated by any other government. However, it may happen that all governments are dominated. In graph-theoretic terms this means that the dominance graph does not possess a source. In this paper we are able to deal with this case by a clever combination of notions from different fields, such as relational algebra, graph theory, social choice and bargaining theory, and by using the computer support system RelView for computing solutions and visualizing the results. Using relational algorithms, in such a case we break all cycles in each initial strongly connected component by removing the vertices in an appropriate minimum feedback vertex set. So, we can choose an un-dominated government. To achieve unique solutions, we additionally apply social choice rules. The main parts of our procedure can be executed using the RelView tool. Its sophisticated implementation of relations allows to deal with graph sizes that are sufficient for practical applications of coalition formation.

Keywords: Graph Theory, RELVIEW, Relational Algebra, Dominance, Stable Government

JEL Classification: D85, C63, C88, D71, D72

Co-operation for this paper was supported by European COST Action 274 "Theory and Applications of Relational Structures as Knowledge Instruments" (TARSKI).

"This paper was presented at the 11th Coalition Theory Network Workshop organised by the University of Warwick, UK on behalf of the CTN, with the financial support of the Department of Economics of the University of Warwick, UNINET, the British Academy, and the Association for Public Economic Theory, Warwick, 19-21 January 2006."

Address for correspondence:

Agnieszka Rusinowska
Radboud University Nijmegen
Nijmegen School of Management
P.O. Box 9108
6500 HK Nijmegen
The Netherlands
Phone: + 31 24 3612033 (12028)
Fax: + 31 24 3611933
E-mail: a.rusinowska@fm.ru.nl

1 Introduction

In Rusinowska et al. [13] a government is defined as a pair consisting of a coalition (a set of parties) and a policy. Different governments may have different utilities (values) for the different parties. In Berghammer et al. [4] we have shown how the notion of ‘government g dominates government h ’ can be described in terms of relational algebra. This enabled us to use the Kiel RELVIEW tool for computing the dominance relation. The governments that are un-dominated are by definition the stable ones.

In this paper we deal with the problem what to do when there is no un-dominated government. Using graph-theoretic terms this means that the dominance graph does not possess a source. By a clever combination of well known concepts from different domains (relational algebra, the RELVIEW tool for their manipulation, graph theory, social choice rules and bargaining) we are able to deal with this case and to choose a government which is as close as possible to stable. As in Berghammer et al. [4], the decisive parts of our procedure are formulated as relational expressions and programs, respectively, so that RELVIEW can be used for executing them and for visualizing the results.

The remainder of the paper is organized as follows. In Section 2 we present the model of coalition formation. Section 3 introduces some preliminaries from relational algebra, gives an overview on RELVIEW, and recalls the method of Berghammer et al. [4] for computing the dominance relation with the help of this tool. Section 4 forms the core of the paper. We describe a general procedure for choosing a government in the case that there is no stable one. In the graph theoretical part we compute initial strongly connected components and minimum feedback vertex sets. If our procedure results in more than one government, we apply bargaining and social choice rules to select one of them.

2 The Model of Coalition Formation

In this section, we briefly recall some of the main ideas of the model of coalition formation presented in Rusinowska et al. [13], i.e., the notions essential for the application of relational algebra and RELVIEW to the model. Let N be the finite set of political parties and P be the finite set of all policies. A set of parties, i.e., an element of the powerset 2^N , is called a *coalition*. We define a *government* as a pair consisting of a coalition and a policy. Hence,

$$G := \{ (S, p) \mid S \in 2^N \wedge p \in P \}$$

denotes the set of all governments. Usually, we assume that only a majority coalition (i.e., a coalition with more than half of the total number of seats in Parliament) can form a government. Nevertheless, one may easily imagine a government formed by a minority coalition.

Each party is assumed to have preferences on all policies and on all coalitions. Then a coalition is called *feasible* if it is acceptable to all its members. A policy is *feasible for a given coalition* if it is acceptable to all members of that coalition and

a government is said to be *feasible* if it consists of a feasible coalition and a policy feasible for that coalition. By G^* we denote the set of all feasible governments:

$$G^* := \{ g \in G \mid G \text{ is feasible} \}.$$

For each $i \in N$, we assume a utility function $U^{(i)} : G \rightarrow \mathbb{R}$, where $U^{(i)}(g)$ denotes the utility (or value) of the government $g \in G$ to party $i \in N$. A precise description of the utility of a government to a party has been given in Rusinowska et al. [13]. In Roubens et al. [11], the MacBeth technique has been applied to determine these utilities.

A feasible government $g = (S, p) \in G^*$ *dominates* a feasible government $h \in G^*$ (denoted as $g \succ h$) if the property

$$(\forall i \in S : U^{(i)}(g) \geq U^{(i)}(h)) \wedge (\exists i \in S : U^{(i)}(g) > U^{(i)}(h))$$

holds. We call “ $\succ$ ” the *dominance relation* and the directed graph $(G^*, \succ)$ the *dominance graph*. A feasible government is said to be *stable* if it is dominated by no feasible government. By

$$SG^* := \{ g \in G^* \mid \neg \exists h \in G^* : h \succ g \}$$

we denote the set of all (feasible) stable governments. Using graph-theoretic terminology, SG^* is the set of sources (or initial vertices) of the dominance graph.

3 Computing the Dominance Relation with RELVIEW

In this section, we first present the basics of relational algebra and indicate how sets can be modeled. For more details, see e.g., Schmidt and Ströhlein [14] or Brink et al. [10]. After a short introduction to the RELVIEW tool, we then recall how the dominance relation can be computed and visualized with this tool.

3.1 Relational Algebra and RELVIEW

If X and Y are sets, then a subset R of the Cartesian product $X \times Y$ is called a (binary) relation with *domain* X and *range* Y . We denote the set (in this context also called type) of all relations with domain X and range Y by $[X \leftrightarrow Y]$ and write $R : X \leftrightarrow Y$ instead of $R \in [X \leftrightarrow Y]$. If X and Y are finite sets of size m and n respectively, then we may consider a relation $R : X \leftrightarrow Y$ as a Boolean matrix with m rows and n columns. The Boolean matrix interpretation of relations is well suited for many purposes and also used as one of the graphical representations of relations within the RELVIEW tool. Therefore, in this paper we often use Boolean matrix terminology and notation. In particular, we write $R_{x,y}$ instead of $\langle x, y \rangle \in R$ or $x R y$.

We assume the reader to be familiar with the basic operations on relations, viz. R^T (*transposition*), $\overline{R}$ (*complement*), $R \cup S$ (*union*), $R \cap S$ (*intersection*),

$R; S$ (*composition*), R^* (*reflexive-transitive closure*), and the special relations O (*empty relation*), L (*universal relation*), and I (*identity relation*). If R is included in S we write $R \subseteq S$ and equality of R and S is denoted as $R = S$.

Relational algebra offers some simple and elegant ways to describe subsets of a given set or, equivalently, predicates on this set. In this paper we will use vectors, membership-relations, and injective embeddings for this task.

A *vector* v is a relation v with $v = v; L$. In the Boolean matrix model this condition means that each row either consists of ‘true’ entries only or consists of ‘false’ entries only. As for a vector, therefore, the range is irrelevant, we consider in the following mostly vectors $v : X \leftrightarrow \mathbf{1}$ with a specific singleton set $\mathbf{1} := \{\perp\}$ as range and omit in such cases the second subscript, i.e., write v_x instead of $v_{x,\perp}$. Analogously to linear algebra we will use lower-case letters to denote vectors. A vector $v : X \leftrightarrow \mathbf{1}$ can be considered as a Boolean matrix with exactly one column, i.e., as a Boolean column vector, and *describes* (or is a description of) the subset $\{x \in X \mid v_x\}$ of X . If a vector describes a singleton set, i.e., an element of its domain, it is called a *point*.

As a second way to model sets we will use the relation-level equivalents of the set-theoretic symbol “ $\in$ ”, i.e., *membership-relations* $M : X \leftrightarrow 2^X$. These specific relations are defined by $M_{x,Y}$ if and only if $x \in Y$, for all $x \in X$ and $Y \in 2^X$. A Boolean matrix representation of M requires exponential space. However, in Berghammer et al. [2] an implementation of M using ordered binary decision diagrams is presented, the number of vertices of which is linear in the size of X .

If the vector v describes a subset Y of X , then $\text{inj}(v) : Y \leftrightarrow X$ denotes the *injective embedding* of Y into X . This means that for all $y \in Y$ and $x \in X$ we have $\text{inj}(v)_{y,x}$ if and only if $y = x$. A combination of injective embeddings and membership-relations allows a *column-wise enumeration* of sets of subsets. More specifically, if v describes a subset $\mathfrak{S}$ of 2^X in the sense defined above, then for all $x \in X$ and $Y \in \mathfrak{S}$ we have $(M; \text{inj}(v)^T)_{x,Y}$ if and only if $x \in Y$. Using matrix terminology this means that the elements of $\mathfrak{S}$ are described precisely by the columns of the relation $M; \text{inj}(v)^T$ of type $[Y \leftrightarrow X]$.

Relational algebra has a fixed and surprisingly small set of constants and operations which (in the case of finite carrier sets) can be implemented very efficiently. At Kiel University we have developed a computer system for the visualization and manipulation of relations and for relational prototyping and programming, called RELVIEW. The tool is written in the C programming language, uses ordered binary decision diagrams for implementing relations, and makes full use of the X-windows graphical user interface. Details and applications can be found, for instance, in Berghammer et al. [3], Behnke et al. [1], Berghammer et al. [2], and Berghammer et al. [5].

The main purpose of RELVIEW is the evaluation of relation-algebraic expressions. These are constructed from the relations of its workspace using pre-defined operations and tests, user-defined relational functions, and user-defined relational programs. A relational program is much like a function procedure in the programming languages Pascal or Modula 2, except that it only uses relations as data type. It starts with a head line containing the program name and the for-

mal parameters. Then the declaration of the local relational domains, functions, and variables follows. Domain declarations can be used to introduce projection relations and pairings of relations in the case of Cartesian products, and injection relations and sums of relations in the case of disjoint unions, respectively. The third part of a program is the body, a while-program over relations. As a program computes a value, finally, its last part consists of a return-clause, which is a relation-algebraic expression whose value after the execution of the body is the result.

3.2 Computing and Visualizing Dominance

In Berghammer et al. [4] we have developed a relation-algebraic specification of dominance and stability. To this end, we supposed a relational description of government membership and the parties' utilities to be given. The first means that we have a relation $M : N \leftrightarrow G^*$ at hand such that for all $i \in N$ and $g \in G^*$

$$M_{i,g} \iff \text{party } i \text{ is a member of government } g;$$

the second means that we have for each party $i \in N$ a relation $R^{(i)} : G^* \leftrightarrow G^*$ at hand such that for all $g, h \in G^*$.

$$R_{g,h}^{(i)} \iff U^{(i)}(g) \geq U^{(i)}(h).$$

Based on the relations $R^{(i)}, i \in N$, we first introduced a global utility (or comparison) relation $C : N \leftrightarrow G^* \times G^*$ by demanding for all $i \in N$ and $g, h \in G^*$

$$C_{i,\langle g,h \rangle} \iff R_{g,h}^{(i)}$$

and transformed this component-based specification into a relation-algebraic (i.e., component-free) one. Then we proved the following fact: If $\pi : G^* \times G^* \leftrightarrow G^*$ and $\rho : G^* \times G^* \leftrightarrow G^*$ are the projection relations of the direct product $G^* \times G^*$ and the vector $\text{DomVec}(M, C) : G^* \times G^* \leftrightarrow \mathbf{1}$ is defined by

$$\text{DomVec}(M, C) = \overline{(\pi; M^\top \cap \overline{C}^\top); \mathbf{L} \cap (\pi; M^\top \cap E; \overline{C}^\top); \mathbf{L}}, \quad (1)$$

where $E := \rho; \pi^\top \cap \pi; \rho^\top : G^* \times G^* \leftrightarrow G^* \times G^*$ is the so-called exchange relation¹, then we have for all $\langle g, h \rangle \in G^* \times G^*$ that $\text{DomVec}(M, C)_{\langle g,h \rangle}$ if and only if $g \succ h$. Hence, equation (1) is a relation-algebraic specification of the dominance relation with the government membership relation M and the global utility relation C as its input.

Strictly speaking, according to (1) dominance is specified as a vector of type $[G^* \times G^* \leftrightarrow \mathbf{1}]$. But what we really wanted is a specification as a relation of type $[G^* \leftrightarrow G^*]$. So, we additionally had to apply the technique of Schmidt and Ströhlein [14] for transforming a vector with a direct product as domain

¹ This name stems from the fact that for all $u, v \in G^* \times G^*$ we have $E_{u,v}$ if and only if $u_1 = v_2$ and $u_2 = v_1$.

into the corresponding relation. Doing so, we obtained a relation-algebraic specification $DomRel(M, C) : G^* \leftrightarrow G^*$ of the dominance relation by

$$DomRel(M, C) = \pi^T; (\rho \cap DomVec(M, C); L). \quad (2)$$

Both equations (1) and (2) can be used for specifying relation-algebraically the vector description $StabVec(M, C) : G^* \leftrightarrow \mathbf{1}$ of the set SG^* of all stable governments. We used (1) and arrived after some steps at

$$StabVec(M, C) = \overline{\rho^T; DomVec(M, C)}. \quad (3)$$

We immediately could transform the three relation-algebraic specifications (1), (2), and (3) into the programming language of RELVIEW. In the first case the result is the following program:

```
DomVec(M,C)
  DECL Prod = PROD(M^*M,M^*M);
    pi, rho, E
  BEG  pi = p-1(Prod);
    rho = p-2(Prod);
    E = rho*pi^ & pi*rho^
    RETURN -dom(pi*M^ & -C^ ) & dom(pi*M^ & E*-C^ )
  END.
```

Here the first declaration introduces *Prod* as a name for the direct product $G^* \times G^*$. Using *Prod*, the projection relations and the exchange relation are then computed by the three assignments of the body and stored as *pi*, *rho*, and *E*, respectively. The return-clause of the program consists of a direct translation of (1) into RELVIEW-syntax, where $\wedge$, $-$, $\&$, and $*$ denote transposition, complement, intersection, and composition, and, furthermore, the operation *dom* computes for a relation $R : X \leftrightarrow Y$ the vector $R; L : X \leftrightarrow \mathbf{1}$.

Similarly, by straightforward translations we obtained as RELVIEW-implementations of the relation-algebraic specifications (2) and (3) the following two relational programs:

```
DomRel(M,C)
  DECL Prod = PROD(M^*M,M^*M);
    pi, rho
  BEG  pi = p-1(Prod);
    rho = p-2(Prod)
    RETURN pi^ * (rho & DomVec(M,C) * L1n(C))
  END.

StabVec(M,C)
  DECL Prod = PROD(M^*M,M^*M);
    rho
  BEG  rho = p-2(Prod)
    RETURN -(rho^ * DomVec(M,C))
  END.
```

Fig. 1. Dominance without a stable government

The operation **L1n** of the RELVIEW-program **DomRel** computes for a relation $R : X \leftrightarrow Y$ the universal relation L of the specific type $[1 \leftrightarrow Y]$, in matrix terminology hence a Boolean universal row vector.

4 The Case of no Stable Government

Based on the situation in Poland after the 2001 elections, in Berghammer et al. [4] we obtained a dominance graph with three sources, representing the stable governments. In such a non-unique case one might allow negotiations in order to choose a government from among the stable ones; see Rusinowska and de Swart [12]. If there is exactly one stable government, obviously this one has to be chosen. In this paper we consider the remaining case that the dominance graph has no source, like in the RELVIEW-picture of Fig. 1. The situation described by this graph appears if we change the utilities of the example of Berghammer et al. [4] a little bit. As in the original case, for reasons of clearness the picture shows a transitive reduction of the dominance graph only.

Assuming that a computed dominance graph has no source, in this section we first describe our procedure to select a government in this case as a whole. After that we go into details and show how to compute initial strongly connected components and minimum feedback vertex sets relation-algebraically. We also sketch the application of techniques of social choice theory.

4.1 The General Approach

If the computed dominance graph has no source, i.e., there exists no stable government, the central question is which government should be chosen. In this section we answer this question by proposing a procedure for choosing a government that can be considered as rather stable.

As a whole, our proposal is presented below. In it, we apply some well-known concepts from graph theory. First, we use *strongly connected components* (SCCs), i.e., maximal sets of vertices such that each pair of vertices is mutually reachable. Especially we are interested in SCCs without arcs leading from outside into them. These SCCs are said to be *initial*. We also use minimum feedback vertex sets, where a *feedback vertex set* (FVS) is a set of vertices that contains at least one vertex from every cycle of the graph. And here is our proposal:

1. Compute the set $\mathcal{I}$ of all initial SCCs of the dominance graph.
2. For each SCC C from $\mathcal{I}$ do:
 - a) Compute the set $\mathcal{F}$ of all minimum FVSs of the subgraph generated by the vertices of C .
 - b) Select from all sets of $\mathcal{F}$ with a maximal number of ingoing arcs one with a minimal number of outgoing arcs. We denote this one by F .
 - c) Break all cycles of C by removing the vertices of F from the dominance graph.
 - d) Select an un-dominated government from the remaining graph. If there is more than one candidate, use bargaining or social choice rules in order to choose one.
3. If there is more than one set in $\mathcal{I}$, select the final stable government from the results of the second step by applying bargaining or social choice rules again.

An outgoing arc of the dominance graph denotes that a government dominates another one and an ingoing arc denotes that a government is dominated by another one. Hence the governments of an initial SCC can be seen as a cluster which is not dominated from outside. The application of the second step to such a set of ‘candidates’ corresponds to a removal of those candidates which are ‘least attractive’ for two reasons: because they are most frequently dominated and they dominate other governments least frequently. In Section 4.4 we will apply this approach to the example given in Fig. 1.

4.2 Computing Initial Strongly Connected Components

Given a finite graph (V, R) with relation $R : V \leftrightarrow V$ for the arcs, the SCCs of (V, R) are precisely the equivalence classes of the equivalence relation $R^* \cap (R^T)^*$. The following RELVIEW-program **Classes** for column-wisely enumerating the equivalence classes of an equivalence relation $S : X \leftrightarrow X$ has been published in Berghammer and Fronk [6]. In it, the calls **Ln1(S)** and **On1(S)** compute the universal vector $\mathbf{L} : X \leftrightarrow \mathbf{1}$ and the empty vector $\mathbf{O} : X \leftrightarrow \mathbf{1}$, respectively, the call **point(v)** yields one of the points contained in the non-empty vector $\mathbf{v}$, and the operation $+$ computes the relational sum. In matrix terminology the latter means that it puts the matrices one upon the other, so that the RELVIEW-expression

$(C^\sim + c^\sim)^\sim$ of `Classes` ‘concatenates’ the matrix C and the vector c .

```

Classes(S)
  DECL C, v, c
  BEG  C = On1(S);
        v = Ln1(S);
        WHILE -empty(v) DO
          c = S * point(v);
          IF isempty(C) THEN C = c
                                ELSE C = (C^\sim + c^\sim)^\sim FI;
          v = v & -c OD
        RETURN C
      END

```

Using the operation `rtc` for computing reflexive-transitive closures, from the above remark we obtain that the call `Classes(rtc(R) & rtc(R^\sim))` column-wisely enumerates the SCCs of R .

In Berghammer and Fronk [6] the authors also refine the program `Classes` to a RELVIEW-program that computes the initial SCCs of R . Essentially this refinement consists of an additional assignment in front of the hitherto first assignment to compute `rtc(R) & rtc(R^\sim)` and to store the result as S (which now is a local variable instead of the formal parameter), and it simply checks after the computation of the next equivalence class c via the assignment $c = S * \text{point}(v)$ whether c is initial and executing only in that case the conditional of the original while-loop. It leads to the following program:

```

InitSccs(R)
  DECL S, C, v, c
  BEG  S = rtc(R) & rtc(R^\sim);
        C = On1(S);
        v = Ln1(S);
        WHILE -empty(v) DO
          c = S * point(v);
          IF incl(R*c, c) THEN
            IF isempty(C) THEN C = c
                                ELSE C = (C^\sim + c^\sim)^\sim FI FI;
          v = v & -c OD
        RETURN C
      END

```

The RELVIEW-expression `incl(R*c, c)` of `InitSccs` tests whether the vector $R*c$ (describing the predecessors of c with respect to R) is contained in c which, in words, exactly means that the SCC described by c is initial.

4.3 Computing Minimum Feedback Vertex Sets

The following relation-algebraic computation of minimum FVSs follows the lines of Berghammer and Fronk [7]. As in Section 4.2 we assume that (V, R) is a finite graph with relation $R : V \leftrightarrow V$.

Let $M : V \leftrightarrow 2^V$ be the membership-relation on vertices. In a first step we reduce the computation of the FVSs to the computation of *simple chordless cycles*, i.e., simple cycles c which do not contain a pair x, y of vertices that forms an arc in (V, R) but not an arc in c . Since a set F of vertices is a FVS if and only if it contains a vertex from every simple chordless cycle (Berghammer and Fronk [7]), it suffices to enumerate column-wisely the vertex sets of the simple chordless cycles of (X, R) via a relation $K : V \leftrightarrow \mathfrak{C}$, where $\mathfrak{C}$ denotes the set of vertex sets of the simple chordless cycles. Assuming K to be at hand, for all $F \in 2^V$ we are able to calculate as follows (where c ranges over the simple chordless cycles and S ranges over $\mathfrak{C}$):

$$\begin{aligned}
F \text{ is a FVS} &\iff \forall c : \exists x : x \in F \wedge x \text{ vertex of } c \\
&\iff \forall S : \exists x : M_{x,F} \wedge K_{x,S} \\
&\iff \neg \exists S : \overline{M^T}; \overline{K}_{FS} \wedge L_S \\
&\iff \overline{\overline{M^T}; \overline{K}}; L_F
\end{aligned}$$

This calculation yields $\overline{\overline{M^T}; \overline{K}}; L : 2^V \leftrightarrow \mathbf{1}$ as the vector representation of all FVSs of (V, R) . Next we apply that the vector $v \cap \overline{Q}; v$ describes the least elements of the set described by the vector v with respect to the preorder Q ; see e.g., Schmidt and Ströhlein [14]. If we use the above vector as v , the size comparison relation on 2^V as Q , and implement the expressions developed so far in RELVIEW, we obtain the following program for computing the vector description of the minimum FVSs from the relation K :

```

MfvsVec(K)
  DECL LeEl(Q,v) = v & -(-Q * v);
  DECL M
  BEG  M = epsi(0(K))
        RETURN LeEl(cardrel(0(K)), -dom(-(M^*K)))
  END.

```

From this program we obtain a program for the column-wise enumeration of the minimum FVSs by applying the technique described in Section 3.1.

We call a set S of vertices of (V, R) *progressively infinite* if it is non-empty and for each vertex $x \in S$ there exists a successor $y \in S$. Fundamental for obtaining a relation-algebraic specification of the relation $K : V \leftrightarrow \mathfrak{C}$ (a task we still have to solve) is the following fact (Berghammer and Fronk [7]): S is the vertex set of a simple chordless cycle if and only if it is a minimal progressively infinite set. Thus, our next goal is identified. We have to develop a RELVIEW-program, say **MprinfVec**, that computes the vector description of the minimal progressively infinite sets. Then the technique of Section 3.1 shows that K is computed by

$$M * \text{inj}(\text{MprinfVec}(R))^\sim.$$

In order to obtain a vector that describes the minimal progressively infinite sets, we first neglect minimality and calculate for a set S of vertices as follows:

$$\begin{aligned}
S \text{ progr. infinite} &\iff (\exists x : x \in S) \wedge (\forall x : x \in S \rightarrow \exists y : y \in S \wedge R_{x,y}) \\
&\iff (\exists x : M_{x,S}) \wedge (\forall x : M_{x,S} \rightarrow \exists y : M_{y,S} \wedge R_{x,y}) \\
&\iff (\exists x : L_{\perp,x} \wedge M_{x,S}) \wedge (\forall x : M_{x,S} \rightarrow (R; M)_{x,S}) \\
&\iff (L; M)_{\perp,S} \wedge (\neg \exists x : L_{\perp,x} \wedge M_{x,S} \wedge \overline{R}; M_{x,S}) \\
&\iff (L; M)_S^\top \wedge L; (\overline{M \cap R}; \overline{M})_{\perp,S}^\top \\
&\iff ((L; M)^\top \cap L; (\overline{M \cap R}; \overline{M})^\top)_S
\end{aligned}$$

Hence, $(L; M)^\top \cap L; (\overline{M \cap R}; \overline{M})^\top : 2^V \leftrightarrow \mathbf{1}$ is a vector description of the progressively infinite sets of the graph (V, R) . Minimalization now is obtained by using two well known results: $\overline{M}^\top; \overline{M} : 2^V \leftrightarrow 2^V$ relation-algebraically specifies set inclusion on 2^V and the vector $v \cap (Q^\top \cap \bar{I})v$ describes the minimal elements of the set described by the vector v with respect to the preorder Q ; see again Schmidt and Ströhlein [14]. If we combine these facts with the vector description of the progressively infinite sets and formulate the result in RELVIEW-syntax, we arrive at the following RELVIEW-program:

```

MprintfVec(R)
  DECL Min(Q,v) = v & -((Q^ & -I(Q)) * v);
  M, SI, L
  BEG  M = epsi(0(R));
       SI = -(M^ * -M);
       L = L1n(R)
       RETURN Min(SI, (L*M)^ & -(L * (M & -(R*M)))^ )
  END.

```

The bottleneck of this program is the use of set inclusion since the size of the ordered binary decision diagrams of this relation is exponential in the size of the base set. Using the present RELVIEW-version it can be only applied to graphs with up to approximately 30 vertices. As we apply it, however, only to initial SCCs, this usually suffices for practical applications of coalition formation. It still should be mentioned that Berghammer and Fronk in [6] develop a refinement of our programs that avoids the use of set inclusion and can be used for graphs consisting of about 100 vertices in general and even more in advantageous cases.

4.4 The Example Revisited

In the following, we want to demonstrate an application of the RELVIEW-programs we have developed so far. As input we assume the dominance relation, the transitive reduction of which graphically is depicted in Fig. 1.

Following the general procedure of Section 4.1, in the first step we have to compute the initial SCCs using the RELVIEW-program `InitSCCs` of Section 4.2. The graph of Fig. 1 possesses exactly one initial SCC. Its RELVIEW-represen-

Fig. 2. SCCs and initial SCC of the former example

tation as Boolean vector is shown on the right-hand side of Fig. 2. To give an impression how a column-wise enumeration of sets of subsets looks in RELVIEW, on the left-hand side of the figure we additionally show the six SCCs of the input as 17×6 matrix. In both cases labels are added to rows and columns by a specific feature of the tool for illustration purposes. In RELVIEW a black square of a Boolean matrix means ‘true’ and a white square means ‘false’. Hence, the SCCs of the input are $\{1, 4, 5, 9\}$, $\{2\}$, $\{3, 8, 12, 13, 15, 16, 17\}$, $\{6\}$, $\{7, 10, 14\}$, and $\{11\}$. The only initial SCC is $C_3 = \{3, 8, 12, 13, 15, 16, 17\}$.

Next, we perform Step a) of the general procedure to the initial SCC. C_3 contains the cycles $\{12, 16\}$, $\{3, 8\}$, $\{8, 12, 15\}$, $\{3, 8, 12, 15\}$, $\{8, 12, 16, 13\}$, $\{8, 12, 16, 15\}$ and $\{3, 8, 12, 16, 15\}$. By means of the RELVIEW-program *MfvsVec* of Section 4.3 we obtain two minimum FVSs, viz. $\{8, 16\}$ and $\{8, 12\}$. The Boolean RELVIEW-matrix of Fig. 3 column-wisely enumerates these sets.

Fig. 3. Minimum FVSs of the initial SCC

Since Step a) of the general procedure of Section 4.1 demands to compute the minimum FVSs of the subgraph generated by the initial SCC C_3 , strictly speaking we first get a relation of type $[C_3 \leftrightarrow \mathfrak{F}]$ as result, which means that the elements of $\mathfrak{F}$ are considered as subsets of C_3 . The matrix of Fig. 3 is obtained from this result by multiplying it from the left with $\text{inj}(v)^T$, where the vector $v : G^* \leftrightarrow \mathbf{1}$ describes the SCC C_3 . Thus, the computed minimum FVSs become subsets of the set G^* .

Fig. 4. The original graph marked with the minimum FVSs

That $\{8, 16\}$ and $\{8, 12\}$ are indeed the only minimum FVS hopefully becomes clear if we consider Fig. 4. It shows two copies of the input graph of Fig. 1. In both cases we have instructed RELVIEW to draw the vertices of the initial SCC as squares and additionally to indicate a minimum FVS by the colour black. In the graph on the left-hand side we identify the minimum FVS $\{8, 16\}$ and in the other graph the minimum FVS $\{8, 12\}$. From Fig. 4 we also see that five arcs lead from outside into the FVS $\{8, 12\}$, but only four arcs lead from outside into $\{8, 16\}$. Hence, by Steps b) and c) of the general procedure we have to remove the vertices 8 and 12 from the graph, which leads to 16 and 17 as new sources, i.e., as governments that can be considered as rather stable. What government finally is chosen depends on specific circumstances. Here social choice rules or bargaining can help; we discuss this point in the next subsections.

4.5 Application of Social Choice Theory

According to the procedure described in Section 4.1, if the application of graph theory does not give a unique solution, we select the final government from among the ‘graph-theoretical’ results by applying social choice rules or bargaining theory. This subsection concerns an application of some well-known social choice rules, like *Plurality Rule* (*Most Votes Count*), *Majority Rule* (*Pairwise Comparison*), *Borda Rule*, and *Approval Voting*, to our choice problem. For an overview and comparison of social choice rules see, for instance, Brams and Fishburn [9], and de Swart et al. [15].

The input for an application of social choice theory consists of: (at least two) selected governments (from which we have to choose one), parties forming these governments, and preferences of the parties over the governments. Moreover,

for each government each party either accepts (approves of) or does not accept (disapproves of) it. We consider four rules:

1. *Plurality Rule:* Under this rule only the first preference of a party is considered. A government g is collectively preferred to a government h if the number of parties that prefer g most is greater than the number of parties that prefer h most. The government chosen under the plurality rule is the government which is put first by most parties.
2. *Majority Rule:* This rule is based on the majority principle. A government g is collectively preferred to h if g defeats h , i.e., the number of parties that prefer g to h is greater than the number of parties that prefer h to g . If there is a government that defeats every other government in a pairwise comparison, this government is chosen, and it is called a Condorcet winner.
3. *Borda Rule:* Here weights are given to all the positions of the governments in the individual preferences. For n governments, every party gives n points to its most preferred government, $n - 1$ points to its second preference, etc., and 1 point to its least preferred government. A decision is made based on the total score of every government in a given party profile.
4. *Approval Voting Rule:* Under Approval Voting (Brams and Fishburn [8]), each party divides the governments into two classes: the governments it approves of and the ones it disapproves of. Each time a government is approved of by a party is good for one point. The government chosen is the one that receives most points.

Let us apply these rules to our example. We have two governments chosen by the ‘graph theoretical part’ of the procedure described in Section 4.1: governments 16 and 17, and denote them by g_{16} (formed by parties A and C) and g_{17} (formed by parties A and B), respectively. Let $\succ_i$ denote the preference relation $R^{(i)}$ of party $i \in \{A, B, C\}$ over the set $\{g_{16}, g_{17}\}$. In our example, we have:

$$g_{17} \succ_A g_{16} \qquad g_{16} \succ_B g_{17} \qquad g_{16} \succ_C g_{17}$$

Moreover, all three parties accept both governments, except party C which does not approve of g_{17} . In the case of two alternatives, the Plurality Rule, the Majority Rule and the Borda Rule give the same result: government g_{16} . Moreover, g_{16} is approved of by all three parties in question, while g_{17} is approved of only by two parties.

4.6 Application of Bargaining Theory

We also like to mention another way for choosing one final government from among (at least two) governments selected by the ‘graph-theoretical part’ of our procedure. This alternative method is based on bargaining theory. In Rusinowska and de Swart [12], the authors define six bargaining games in which parties belonging to stable governments (it is assumed that there are at least two stable ones) bargain over the choice of one stable government. Subgame perfect equilibria of the games are investigated. Of course, the result of a bargaining game

depends not only on the bargaining procedure, but also on the order in which parties bargain. In Rusinowska and de Swart [12], a procedure for choosing the order of parties for a given game is also proposed.

In this paper, we apply only some bargaining games analyzed in Rusinowska and de Swart [12], to show how such an application of bargaining to our choice problem may look like. Since we have only two governments and only three parties, the games are very simple. There are several common assumptions for our bargaining games. First of all, it is assumed that a party, when submitting an offer, may propose only one government. Moreover, the same offers are not repeated: a party cannot propose a government which has been already proposed before. Finally, it is assumed that choosing no government is the worst outcome for each party. Our bargaining games differ from each other with respect to the bargaining procedures and the bargaining costs. Here we consider the games in which a party prefers to form a government it likes most with a delay, rather than to form immediately (with no delay) a less preferred government.

Let us consider one of the bargaining games for which the parties' order chosen by a special procedure is (A, B, C) . This is the order of parties according to the number of seats in Parliament. It is assumed in this game that a party, when submitting an offer, may propose only a government the party belongs to. The bargaining procedure for this game is the following. First, party A proposes either government g_{16} or government g_{17} . If g_{16} is proposed, then party C (which is involved in g_{16}) either accepts or rejects the proposal. Since there are no more parties 'responsible' for g_{16} , if party C accepts the offer, government g_{16} is chosen. Otherwise, no government is created, since party C is involved in no more governments. On the other hand, if A proposes g_{17} , it is party B which has to react. Similarly, the acceptance of this offer causes g_{17} to be formed, and the rejection leads to no government formed. There is only one subgame perfect equilibrium for this game and it leads to the choice of government g_{17} , the most preferred result of party A .

The other two games we like to mention are less profitable for the strongest party A . One of them gives more room for parties other than the strongest one. Let us assume that a party does not have to belong to the government it proposes, and all parties have to react to each offer. This means that if A submits an offer (g_{16} or g_{17}), both parties B and C must either accept or reject the offer. For some orders of the parties, this game has more than one subgame perfect equilibria, but they always lead to the creation of g_{16} , i.e., the government most preferred by parties B and C .

Finally, let us assume that only the strongest party, i.e., party A , may submit an offer, and the other party forming the proposed government has to react. The subgame perfect equilibrium of this game also results in the choice of government g_{16} .

5 Conclusions

The central concepts of the coalition formation model are the notion of (feasible) government and the notion of stable government. The latter is defined as a feasible government dominated by no feasible government. In the present paper, we aim to answer the question which government should be chosen if there is no stable government (that is, if the dominance graph has no source). The attractiveness and novelty of our approach consists in: 1. the clever combination of notions from partly different domains (relational algebra, graph theory, social choice theory and bargaining), and 2. the immediate and easy support by the computer system RELVIEW for computing solutions and for visualizing the results. Given a dominance graph without a source, first we compute all initial strongly connected components. The governments of an initial strongly connected component can be seen as a cluster which is not dominated from outside. Next, for each initial strongly connected component, we compute the set of all minimum feedback vertex sets, where a minimum feedback vertex set is a minimal set of vertices which breaks all cycles. Next, we choose a specific minimum feedback vertex set according to the following rule. First, we choose the set(s) for which the number of ingoing arcs is maximal. Since an ingoing arc denotes that a government is dominated, such a choice means selecting governments dominated most frequently. Next, if there are at least two such sets, we choose one for which the number of outgoing arcs is minimal, meaning the choice of the governments which dominate other governments least frequently. Next, we break all cycles by removing the chosen set of governments. One may say that we remove governments which are least attractive for two reasons: because they are most frequently dominated and they dominate other governments least frequently. According to our procedure, if there is more than one initial strongly connected component, we select the final stable government (from the results of the procedure described above) by applying bargaining or some well-known social choice rules. Concerning the application of bargaining, we construct several bargaining games and choose the government which is a subgame perfect equilibrium result. Concerning the application of social choice theory, we apply the plurality rule, the majority rule, the Borda rule, or approval voting. Of course, some of these applications may also lead to a non-unique solution. In this case, we propose to combine several techniques and to apply a several-steps method consisting of, for instance, a social choice rule in the first step, and a bargaining game in the second step.

References

1. Behnke, R., Berghammer, R., Meyer, E., Schneider, P., 1998. RELVIEW – A system for calculation with relations and relational programming. In: Astesiano, E., (Ed.), Proc. Conf. “Fundamental Approaches to Software Engineering (FASE ’98)”, LNCS 1382, Springer, pp. 318-321.
2. Berghammer, R., Leoniuk, B., Milanese, U., 2002. Implementation of relational algebra using binary decision diagrams. In: de Swart, H., (Ed.), Proc. 6th Int.

- Workshop “Relational Methods in Computer Science”. LNCS 2561, Springer, pp. 241-257.
3. Berghammer, R., von Karger, B., Ulke, C., 1996. Relation-algebraic analysis of Petri nets with RELVIEW. In: Margaria, T., Steffen, B., (Eds.), Proc. 2nd Workshop “Tools and Applications for the Construction and Analysis of Systems (TACAS ’96)”. LNCS 1055, Springer, pp. 49-69.
 4. Berghammer, R., Rusinowska, A., de Swart, H., 2005. Applying relational algebra and RELVIEW to coalition formation. Submitted for publication, available via URL <http://www.uvt.nl/faculteiten/fww/onderzoek/skt/pdf/RelVieW.pdf>
 5. Berghammer, R., Schmidt, G., Winter, M., 2003. RELVIEW and RATH – Two systems for dealing with relations. In: de Swart, H., Orlowska, E., Schmidt, G., Roubens, M., (Eds.), Theory and Applications of Relational Structures as Knowledge Instruments. LNCS 2929, Springer, pp. 1-16.
 6. Berghammer, R., Fronk, A., 2004. Considering design tasks in OO-software engineering using relations and relation-based tools. Journal on Relational Methods in Computer Science 1, 73-92.
 7. Berghammer, R., Fronk, A., 2005. Exact computation of minimum feedback vertex sets with relational algebra. Submitted for publication.
 8. Brams, S.J., Fishburn, P.C., 1983. Approval Voting. Birkhäuser, Boston.
 9. Brams, S.J., Fishburn, P.C., 2002. Voting Procedures. In: Arrow, K., Sen, A., Suzumura, K., (Eds.), Handbook of Social Choice and Welfare. Elsevier Science, Amsterdam.
 10. Brink, C., Kahl, W., Schmidt, G., (Eds.), 1997. Relational Methods in Computer Science. Advances in Computing Science, Springer.
 11. Roubens, M., Rusinowska, A., de Swart, H., 2005. Using MacBeth to determine utilities of governments to parties in coalition formation. Forthcoming in the European Journal of Operational Research.
 12. Rusinowska, A., de Swart, H., 2004. Negotiating a stable government - an application of bargaining theory to a coalition formation model. Submitted for publication.
 13. Rusinowska, A., de Swart, H., van der Rijt, J.W., 2005. A new model of coalition formation. Social Choice and Welfare 24, 129-154.
 14. Schmidt, G., Ströhlein, T., 1993. Relations and Graphs. Discrete Mathematics for Computer Scientists, EATCS Monographs on Theoret. Comput. Sci., Springer.
 15. de Swart, H., van Deemen, A., van der Hout, E., Kop, P., 2003. Categorical and ordinal voting: an overview. In: de Swart, H., Orlowska, E., Schmidt, G., Roubens, M., (Eds.), Theory and Applications of Relational Structures as Knowledge Instruments. LNCS 2929, Springer, pp. 147-196.

NOTE DI LAVORO DELLA FONDAZIONE ENI ENRICO MATTEI

Fondazione Eni Enrico Mattei Working Paper Series

Our Note di Lavoro are available on the Internet at the following addresses:

<http://www.feem.it/Feem/Pub/Publications/WPapers/default.html>

<http://www.ssrn.com/link/feem.html>

<http://www.repec.org>

<http://agecon.lib.umn.edu>

NOTE DI LAVORO PUBLISHED IN 2006

SIEV	1.2006	<i>Anna ALBERINI</i> : <u>Determinants and Effects on Property Values of Participation in Voluntary Cleanup Programs: The Case of Colorado</u>
CCMP	2.2006	<i>Valentina BOSETTI, Carlo CARRARO and Marzio GALEOTTI</i> : <u>Stabilisation Targets, Technical Change and the Macroeconomic Costs of Climate Change Control</u>
CCMP	3.2006	<i>Roberto ROSON</i> : <u>Introducing Imperfect Competition in CGE Models: Technical Aspects and Implications</u>
KTHC	4.2006	<i>Sergio VERGALLI</i> : <u>The Role of Community in Migration Dynamics</u>
SIEV	5.2006	<i>Fabio GRAZI, Jeroen C.J.M. van den BERGH and Piet RIETVELD</i> : <u>Modeling Spatial Sustainability: Spatial Welfare Economics versus Ecological Footprint</u>
CCMP	6.2006	<i>Olivier DESCHENES and Michael GREENSTONE</i> : <u>The Economic Impacts of Climate Change: Evidence from Agricultural Profits and Random Fluctuations in Weather</u>
PRCG	7.2006	<i>Michele MORETTO and Paola VALBONESE</i> : <u>Firm Regulation and Profit-Sharing: A Real Option Approach</u>
SIEV	8.2006	<i>Anna ALBERINI and Aline CHIABAI</i> : <u>Discount Rates in Risk v. Money and Money v. Money Tradeoffs</u>
CTN	9.2006	<i>Jon X. EGUIA</i> : <u>United We Vote</u>
CTN	10.2006	<i>Shao CHIN SUNG and Dinko DIMITRO</i> : <u>A Taxonomy of Myopic Stability Concepts for Hedonic Games</u>
NRM	11.2006	<i>Fabio CERINA</i> (lxxviii): <u>Tourism Specialization and Sustainability: A Long-Run Policy Analysis</u>
NRM	12.2006	<i>Valentina BOSETTI, Mariaester CASSINELLI and Alessandro LANZA</i> (lxxviii): <u>Benchmarking in Tourism Destination, Keeping in Mind the Sustainable Paradigm</u>
CCMP	13.2006	<i>Jens HORBACH</i> : <u>Determinants of Environmental Innovation – New Evidence from German Panel Data Sources</u>
KTHC	14.2006	<i>Fabio SABATINI</i> : <u>Social Capital, Public Spending and the Quality of Economic Development: The Case of Italy</u>
KTHC	15.2006	<i>Fabio SABATINI</i> : <u>The Empirics of Social Capital and Economic Development: A Critical Perspective</u>
CSRM	16.2006	<i>Giuseppe DI VITA</i> : <u>Corruption, Exogenous Changes in Incentives and Deterrence</u>
CCMP	17.2006	<i>Rob B. DELLINK and Marjan W. HOFKES</i> : <u>The Timing of National Greenhouse Gas Emission Reductions in the Presence of Other Environmental Policies</u>
IEM	18.2006	<i>Philippe QUIRION</i> : <u>Distributional Impacts of Energy-Efficiency Certificates Vs. Taxes and Standards</u>
CTN	19.2006	<i>Somdeb LAHIRI</i> : <u>A Weak Bargaining Set for Contract Choice Problems</u>
CCMP	20.2006	<i>Massimiliano MAZZANTI and Roberto ZOBOLI</i> : <u>Examining the Factors Influencing Environmental Innovations</u>
SIEV	21.2006	<i>Y. Hossein FARZIN and Ken-ICHI AKAO</i> : <u>Non-pecuniary Work Incentive and Labor Supply</u>
CCMP	22.2006	<i>Marzio GALEOTTI, Matteo MANERA and Alessandro LANZA</i> : <u>On the Robustness of Robustness Checks of the Environmental Kuznets Curve</u>
NRM	23.2006	<i>Y. Hossein FARZIN and Ken-ICHI AKAO</i> : <u>When is it Optimal to Exhaust a Resource in a Finite Time?</u>
NRM	24.2006	<i>Y. Hossein FARZIN and Ken-ICHI AKAO</i> : <u>Non-pecuniary Value of Employment and Natural Resource Extinction</u>
SIEV	25.2006	<i>Lucia VERGANO and Paulo A.L.D. NUNES</i> : <u>Analysis and Evaluation of Ecosystem Resilience: An Economic Perspective</u>
SIEV	26.2006	<i>Danny CAMPBELL, W. George HUTCHINSON and Riccardo SCARPA</i> : <u>Using Discrete Choice Experiments to Derive Individual-Specific WTP Estimates for Landscape Improvements under Agri-Environmental Schemes: Evidence from the Rural Environment Protection Scheme in Ireland</u>
KTHC	27.2006	<i>Vincent M. OTTO, Timo KUOSMANEN and Ekko C. van IERLAND</i> : <u>Estimating Feedback Effect in Technical Change: A Frontier Approach</u>
CCMP	28.2006	<i>Giovanni BELLA</i> : <u>Uniqueness and Indeterminacy of Equilibria in a Model with Polluting Emissions</u>
IEM	29.2006	<i>Alessandro COLOGNI and Matteo MANERA</i> : <u>The Asymmetric Effects of Oil Shocks on Output Growth: A Markov-Switching Analysis for the G-7 Countries</u>
KTHC	30.2006	<i>Fabio SABATINI</i> : <u>Social Capital and Labour Productivity in Italy</u>
ETA	31.2006	<i>Andrea GALLICE</i> (lxxix): <u>Predicting one Shot Play in 2x2 Games Using Beliefs Based on Minimax Regret</u>
IEM	32.2006	<i>Andrea BIGANO and Paul SHEEHAN</i> : <u>Assessing the Risk of Oil Spills in the Mediterranean: the Case of the Route from the Black Sea to Italy</u>
NRM	33.2006	<i>Rinaldo BRAU and Davide CAO</i> (lxxviii): <u>Uncovering the Macrostructure of Tourists' Preferences. A Choice Experiment Analysis of Tourism Demand to Sardinia</u>
CTN	34.2006	<i>Parkash CHANDER and Henry TULKENS</i> : <u>Cooperation, Stability and Self-Enforcement in International Environmental Agreements: A Conceptual Discussion</u>
IEM	35.2006	<i>Valeria COSTANTINI and Salvatore MONNI</i> : <u>Environment, Human Development and Economic Growth</u>
ETA	36.2006	<i>Ariel RUBINSTEIN</i> (lxxix): <u>Instinctive and Cognitive Reasoning: A Study of Response Times</u>

ETA	37.2006	<i>Maria SALGADO</i> (lxxix): <u>Choosing to Have Less Choice</u>
ETA	38.2006	<i>Justina A.V. FISCHER and Benno TORGLER</i> : <u>Does Envy Destroy Social Fundamentals? The Impact of Relative Income Position on Social Capital</u>
ETA	39.2006	<i>Benno TORGLER, Sascha L. SCHMIDT and Bruno S. FREY</i> : <u>Relative Income Position and Performance: An Empirical Panel Analysis</u>
CCMP	40.2006	<i>Alberto GAGO, Xavier LABANDEIRA, Fidel PICOS And Miguel RODRÍGUEZ</i> : <u>Taxing Tourism In Spain: Results and Recommendations</u>
IEM	41.2006	<i>Karl van BIERVLIET, Dirk Le ROY and Paulo A.L.D. NUNES</i> : <u>An Accidental Oil Spill Along the Belgian Coast: Results from a CV Study</u>
CCMP	42.2006	<i>Rolf GOLOMBEK and Michael HOEL</i> : <u>Endogenous Technology and Tradable Emission Quotas</u>
KTHC	43.2006	<i>Giulio CAINELLI and Donato IACOBUCCI</i> : <u>The Role of Agglomeration and Technology in Shaping Firm Strategy and Organization</u>
CCMP	44.2006	<i>Alvaro CALZADILLA, Francesco PAULI and Roberto ROSON</i> : <u>Climate Change and Extreme Events: An Assessment of Economic Implications</u>
SIEV	45.2006	<i>M.E. KRAGT, P.C. ROEBELING and A. RUIJS</i> : <u>Effects of Great Barrier Reef Degradation on Recreational Demand: A Contingent Behaviour Approach</u>
NRM	46.2006	<i>C. GIUPPONI, R. CAMERA, A. FASSIO, A. LASUT, J. MYSLIAK and A. SGOBBI</i> : <u>Network Analysis, Creative System Modelling and DecisionSupport: The NetSyMoD Approach</u>
KTHC	47.2006	<i>Walter F. LALICH</i> (lxxx): <u>Measurement and Spatial Effects of the Immigrant Created Cultural Diversity in Sydney</u>
KTHC	48.2006	<i>Elena PASPALANOVA</i> (lxxx): <u>Cultural Diversity Determining the Memory of a Controversial Social Event</u>
KTHC	49.2006	<i>Ugo GASPARINO, Barbara DEL CORPO and Dino PINELLI</i> (lxxx): <u>Perceived Diversity of Complex Environmental Systems: Multidimensional Measurement and Synthetic Indicators</u>
KTHC	50.2006	<i>Aleksandra HAUKE</i> (lxxx): <u>Impact of Cultural Differences on Knowledge Transfer in British, Hungarian and Polish Enterprises</u>
KTHC	51.2006	<i>Katherine MARQUAND FORSYTH and Vanja M. K. STENIUS</i> (lxxx): <u>The Challenges of Data Comparison and Varied European Concepts of Diversity</u>
KTHC	52.2006	<i>Gianmarco I.P. OTTAVIANO and Giovanni PERI</i> (lxxx): <u>Rethinking the Gains from Immigration: Theory and Evidence from the U.S.</u>
KTHC	53.2006	<i>Monica BARNI</i> (lxxx): <u>From Statistical to Geolinguistic Data: Mapping and Measuring Linguistic Diversity</u>
KTHC	54.2006	<i>Lucia TAJOLI and Lucia DE BENEDICTIS</i> (lxxx): <u>Economic Integration and Similarity in Trade Structures</u>
KTHC	55.2006	<i>Suzanna CHAN</i> (lxxx): <u>"God's Little Acre" and "Belfast Chinatown": Diversity and Ethnic Place Identity in Belfast</u>
KTHC	56.2006	<i>Diana PETKOVA</i> (lxxx): <u>Cultural Diversity in People's Attitudes and Perceptions</u>
KTHC	57.2006	<i>John J. BETANCUR</i> (lxxx): <u>From Outsiders to On-Paper Equals to Cultural Curiosities? The Trajectory of Diversity in the USA</u>
KTHC	58.2006	<i>Kiflemariam HAMDE</i> (lxxx): <u>Cultural Diversity A Glimpse Over the Current Debate in Sweden</u>
KTHC	59.2006	<i>Emilio GREGORI</i> (lxxx): <u>Indicators of Migrants' Socio-Professional Integration</u>
KTHC	60.2006	<i>Christa-Maria LERM HAYES</i> (lxxx): <u>Unity in Diversity Through Art? Joseph Beuys' Models of Cultural Dialogue</u>
KTHC	61.2006	<i>Sara VERTOMMEN and Albert MARTENS</i> (lxxx): <u>Ethnic Minorities Rewarded: Ethnostratification on the Wage Market in Belgium</u>
KTHC	62.2006	<i>Nicola GENOVESE and Maria Grazia LA SPADA</i> (lxxx): <u>Diversity and Pluralism: An Economist's View</u>
KTHC	63.2006	<i>Carla BAGNA</i> (lxxx): <u>Italian Schools and New Linguistic Minorities: Nationality Vs. Plurilingualism. Which Ways and Methodologies for Mapping these Contexts?</u>
KTHC	64.2006	<i>Vedran OMANOVIĆ</i> (lxxx): <u>Understanding "Diversity in Organizations" Paradigmatically and Methodologically</u>
KTHC	65.2006	<i>Mila PASPALANOVA</i> (lxxx): <u>Identifying and Assessing the Development of Populations of Undocumented Migrants: The Case of Undocumented Poles and Bulgarians in Brussels</u>
KTHC	66.2006	<i>Roberto ALZETTA</i> (lxxx): <u>Diversities in Diversity: Exploring Moroccan Migrants' Livelihood in Genoa</u>
KTHC	67.2006	<i>Monika SEDENKOVA and Jiri HORAK</i> (lxxx): <u>Multivariate and Multicriteria Evaluation of Labour Market Situation</u>
KTHC	68.2006	<i>Dirk JACOBS and Andrea REA</i> (lxxx): <u>Construction and Import of Ethnic Categorisations: "Allochthones" in The Netherlands and Belgium</u>
KTHC	69.2006	<i>Eric M. USLANER</i> (lxxx): <u>Does Diversity Drive Down Trust?</u>
KTHC	70.2006	<i>Paula MOTA SANTOS and João BORGES DE SOUSA</i> (lxxx): <u>Visibility & Invisibility of Communities in Urban Systems</u>
ETA	71.2006	<i>Rinaldo BRAU and Matteo LIPPI BRUNI</i> : <u>Eliciting the Demand for Long Term Care Coverage: A Discrete Choice Modelling Analysis</u>
CTN	72.2006	<i>Dinko DIMITROV and Claus-JOCHEN HAAKE</i> : <u>Coalition Formation in Simple Games: The Semistrict Core</u>
CTN	73.2006	<i>Ottorino CHILLEM, Benedetto GUI and Lorenzo ROCCO</i> : <u>On The Economic Value of Repeated Interactions Under Adverse Selection</u>
CTN	74.2006	<i>Sylvain BEAL and Nicolas QUÉROU</i> : <u>Bounded Rationality and Repeated Network Formation</u>
CTN	75.2006	<i>Sophie BADE, Guillaume HAERINGER and Ludovic RENOU</i> : <u>Bilateral Commitment</u>
CTN	76.2006	<i>Andranik TANGIAN</i> : <u>Evaluation of Parties and Coalitions After Parliamentary Elections</u>
CTN	77.2006	<i>Rudolf BERGHAMMER, Agnieszka RUSINOWSKA and Harrie de SWART</i> : <u>Applications of Relations and Graphs to Coalition Formation</u>

(lxxviii) This paper was presented at the Second International Conference on "Tourism and Sustainable Economic Development - Macro and Micro Economic Issues" jointly organised by CRENoS (Università di Cagliari and Sassari, Italy) and Fondazione Eni Enrico Mattei, Italy, and supported by the World Bank, Chia, Italy, 16-17 September 2005.

(lxxix) This paper was presented at the International Workshop on "Economic Theory and Experimental Economics" jointly organised by SET (Center for advanced Studies in Economic Theory, University of Milano-Bicocca) and Fondazione Eni Enrico Mattei, Italy, Milan, 20-23 November 2005. The Workshop was co-sponsored by CISEPS (Center for Interdisciplinary Studies in Economics and Social Sciences, University of Milan-Bicocca).

(lxxx) This paper was presented at the First EURODIV Conference "Understanding diversity: Mapping and measuring", held in Milan on 26-27 January 2006 and supported by the Marie Curie Series of Conferences "Cultural Diversity in Europe: a Series of Conferences.

2006 SERIES

CCMP	<i>Climate Change Modelling and Policy</i> (Editor: Marzio Galeotti)
SIEV	<i>Sustainability Indicators and Environmental Valuation</i> (Editor: Anna Alberini)
NRM	<i>Natural Resources Management</i> (Editor: Carlo Giupponi)
KTHC	<i>Knowledge, Technology, Human Capital</i> (Editor: Gianmarco Ottaviano)
IEM	<i>International Energy Markets</i> (Editor: Anil Markandya)
CSRM	<i>Corporate Social Responsibility and Sustainable Management</i> (Editor: Sabina Ratti)
PRCG	<i>Privatisation Regulation Corporate Governance</i> (Editor: Bernardo Bortolotti)
ETA	<i>Economic Theory and Applications</i> (Editor: Carlo Carraro)
CTN	<i>Coalition Theory Network</i>