Applications of Relations and Graphs to Coalition Formation

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Summary

A stable government is by definition not dominated by any other government. However, it may happen that all governments are dominated. In graph-theoretic terms this means that the dominance graph does not possess a source. In this paper we are able to deal with this case by a clever combination of notions from different fields, such as relational algebra, graph theory, social choice and bargaining theory, and by using the computer support system RelView for computing solutions and visualizing the results. Using relational algorithms, in such a case we break all cycles in each initial strongly connected component by removing the vertices in an appropriate minimum feedback vertex set. So, we can choose an un-dominated government. To achieve unique solutions, we additionally apply social choice rules. The main parts of our procedure can be executed using the RelView tool. Its sophisticated implementation of relations allows to deal with graph sizes that are sufficient for practical applications of coalition formation.

Keywords: Graph Theory, RELVIEW, Relational Algebra, Dominance, Stable Government

JEL Classification: D85, C63, C88, D71, D72

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1 Introduction

In Rusinowska et al. [13] a government is defined as a pair consisting of a coalition (a set of of parties) and a policy. Different governments may have different utilities (values) for the different parties. In Berghammer et al. [4] we have shown how the notion of 'government g dominates government h' can be described in terms of relational algebra. This enabled us to use the Kiel RelView tool for computing the dominance relation. The governments that are un-dominated are by definition the stable ones.

In this paper we deal with the problem what to do when there is no un-dominated government. Using graph-theoretic terms this means that the dominance graph does not possess a source. By a clever combination of well known concepts from different domains (relational algebra, the RelView tool for their manipulation, graph theory, social choice rules and bargaining) we are able to deal with this case and to choose a government which is as close as possible to stable. As in Berghammer et al. [4], the decisive parts of our procedure are formulated as relational expressions and programs, respectively, so that RelView can be used for executing them and for visualizing the results.

The remainder of the paper is organized as follows. In Section 2 we present the model of coalition formation. Section 3 introduces some preliminaries from relational algebra, gives an overview on RelView, and recalls the method of Berghammer et al. [4] for computing the dominance relation with the help of this tool. Section 4 forms the core of the paper. We describe a general procedure for choosing a government in the case that there is no stable one. In the graph theoretical part we compute initial strongly connected components and minimum feedback vertex sets. If our procedure results in more than one government, we apply bargaining and social choice rules to select one of them.

2 The Model of Coalition Formation

In this section, we briefly recall some of the main ideas of the model of coalition formation presented in Rusinowska et al. [13], i.e., the notions essential for the application of relational algebra and RelView to the model. Let N be the finite set of political parties and P be the finite set of all policies. A set of parties, i.e., an element of the powerset 2^N, is called a *coalition*. We define a *government* as a pair consisting of a coalition and a policy. Hence,

$$G := \{ (S, p) \mid S \in 2^N \land p \in P \}$$

denotes the set of all governments. Usually, we assume that only a majority coalition (i.e., a coalition with more than half of the total number of seats in Parliament) can form a government. Nevertheless, one may easily imagine a government formed by a minority coalition.

Each party is assumed to have preferences on all policies and on all coalitions. Then a coalition is called *feasible* if it is acceptable to all its members. A policy is *feasible for a given coalition* if it is acceptable to all members of that coalition and a government is said to be *feasible* if it consists of a feasible coalition and a policy feasible for that coalition. By G^* we denote the set of all feasible governments:

$$G^* := \{ g \in G \mid G \text{ is feasible} \} .$$

For each $i \in N$, we assume a utility function $U^{(i)} : G \to \mathbb{R}$, where $U^{(i)}(g)$ denotes the utility (or value) of the government $g \in G$ to party $i \in N$. A precise description of the utility of a government to a party has been given in Rusinowska et al. [13]. In Roubens et al. [11], the MacBeth technique has been applied to determine these utilities.

A feasible government $g = (S, p) \in G^*$ *dominates* a feasible government $h \in G^*$ (denoted as $g \succ h$) if the property

$$(\forall\, i \in S : U^{(i)}(g) \geq U^{(i)}(h)) \wedge (\exists\, i \in S : U^{(i)}(g) > U^{(i)}(h))$$

holds. We call "$\succ$" the *dominance relation* and the directed graph $(G^*, \succ)$ the *dominance graph*. A feasible government is said to be *stable* if it is dominated by no feasible government. By

$$SG^* := \{g \in G^* \mid \neg \exists\, h \in G^* : h \succ g\}$$

we denote the set of all (feasible) stable governments. Using graph-theoretic terminology, SG^* is the set of sources (or initial vertices) of the dominance graph.

3 Computing the Dominance Relation with RelView

In this section, we first present the basics of relational algebra and indicate how sets can be modeled. For more details, see e.g., Schmidt and Ströhlein [14] or Brink et al. [10]. After a short introduction to the RelView tool, we then recall how the dominance relation can be computed and visualized with this tool.

3.1 Relational Algebra and RelView

If X and Y are sets, then a subset R of the Cartesian product $X \times Y$ is called a (binary) relation with *domain* X and *range* Y. We denote the set (in this context also called type) of all relations with domain X and range Y by $[X \leftrightarrow Y]$ and write $R : X \leftrightarrow Y$ instead of $R \in [X \leftrightarrow Y]$. If X and Y are finite sets of size m and n respectively, then we may consider a relation $R : X \leftrightarrow Y$ as a Boolean matrix with m rows and n columns. The Boolean matrix interpretation of relations is well suited for many purposes and also used as one of the graphical representations of relations within the RelView tool. Therefore, in this paper we often use Boolean matrix terminology and notation. In particular, we write $R_{x,y}$ instead of $\langle x, y\rangle \in R$ or $x\, R\, y$.

We assume the reader to be familiar with the basic operations on relations, viz. R^{T} (*transposition*), $\overline{R}$ (*complement*), $R \cup S$ (*union*), $R \cap S$ (*intersection*),

$R; S$ (*composition*), R^* (*reflexive-transitive closure*), and the special relations O (*empty relation*), L (*universal relation*), and I (*identity relation*). If R is included in S we write $R \subseteq S$ and equality of R and S is denoted as $R = S$.

Relational algebra offers some simple and elegant ways to describe subsets of a given set or, equivalently, predicates on this set. In this paper we will use vectors, membership-relations, and injective embeddings for this task.

A *vector* v is a relation v with $v = v; \mathsf{L}$. In the Boolean matrix model this condition means that each row either consists of 'true' entries only or consists of 'false' entries only. As for a vector, therefore, the range is irrelevant, we consider in the following mostly vectors $v : X \leftrightarrow \mathbf{1}$ with a specific singleton set $\mathbf{1} := \{\perp\}$ as range and omit in such cases the second subscript, i.e., write v_x instead of $v_{x,\perp}$. Analogously to linear algebra we will use lower-case letters to denote vectors. A vector $v : X \leftrightarrow \mathbf{1}$ can be considered as a Boolean matrix with exactly one column, i.e., as a Boolean column vector, and *describes* (or is a description of) the subset $\{x \in X \mid v_x\}$ of X. If a vector describes a singleton set, i.e., an element of its domain, it is called a *point*.

As a second way to model sets we will use the relation-level equivalents of the set-theoretic symbol "$\in$", i.e., *membership-relations* $\mathsf{M} : X \leftrightarrow 2^X$. These specific relations are defined by $\mathsf{M}_{x,Y}$ if and only if $x \in Y$, for all $x \in X$ and $Y \in 2^X$. A Boolean matrix representation of M requires exponential space. However, in Berghammer et al. [2] an implementation of M using ordered binary decision diagrams is presented, the number of vertices of which is linear in the size of X.

If the vector v describes a subset Y of X, then $inj(v) : Y \leftrightarrow X$ denotes the *injective embedding* of Y into X. This means that for all $y \in Y$ and $x \in X$ we have $inj(v)_{y,x}$ if and only if $y = x$. A combination of injective embeddings and membership-relations allows a *column-wise enumeration* of sets of subsets. More specifically, if v describes a subset $\mathfrak{S}$ of 2^X in the sense defined above, then for all $x \in X$ and $Y \in \mathfrak{S}$ we have $(\mathsf{M}; inj(v)^{\mathsf{T}})_{x,Y}$ if and only if $x \in Y$. Using matrix terminology this means that the elements of $\mathfrak{S}$ are described precisely by the columns of the relation $\mathsf{M}; inj(v)^{\mathsf{T}}$ of type $[Y \leftrightarrow X]$.

Relational algebra has a fixed and surprisingly small set of constants and operations which (in the case of finite carrier sets) can be implemented very efficiently. At Kiel University we have developed a computer system for the visualization and manipulation of relations and for relational prototyping and programming, called RelView. The tool is written in the C programming language, uses ordered binary decision diagrams for implementing relations, and makes full use of the X-windows graphical user interface. Details and applications can be found, for instance, in Berghammer et al. [3], Behnke et al. [1], Berghammer et al. [2], and Berghammer et al. [5].

The main purpose of RelView is the evaluation of relation-algebraic expressions. These are constructed from the relations of its workspace using predefined operations and tests, user-defined relational functions, and user-defined relational programs. A relational program is much like a function procedure in the programming languages Pascal or Modula 2, except that it only uses relations as data type. It starts with a head line containing the program name and the formal parameters. Then the declaration of the local relational domains, functions, and variables follows. Domain declarations can be used to introduce projection relations and pairings of relations in the case of Cartesian products, and injection relations and sums of relations in the case of disjoint unions, respectively. The third part of a program is the body, a while-program over relations. As a program computes a value, finally, its last part consists of a return-clause, which is a relation-algebraic expression whose value after the execution of the body is the result.

3.2 Computing and Visualizing Dominance

In Berghammer et al. [4] we have developed a relation-algebraic specification of dominance and stability. To this end, we supposed a relational description of government membership and the parties' utilities to be given. The first means that we have a relation $M : N \leftrightarrow G^*$ at hand such that for all $i \in N$ and $g \in G^*$

$$M_{i,g} \iff \text{party } i \text{ is a member of government } g;$$

the second means that we have for each party $i \in N$ a relation $R^{(i)} : G^* \leftrightarrow G^*$ at hand such that for all $g, h \in G^*$.

$$R^{(i)}_{g,h} \iff U^{(i)}(g) \geq U^{(i)}(h)\,.$$

Based on the relations $R^{(i)}, i \in N$, we first introduced a global utility (or comparison) relation $C : N \leftrightarrow G^* \times G^*$ by demanding for all $i \in N$ and $g, h \in G^*$

$$C_{i,\langle g,h\rangle} \iff R^{(i)}_{g,h}$$

and transformed this component-based specification into a relation-algebraic (i.e., component-free) one. Then we proved the following fact: If $\pi : G^* \times G^* \leftrightarrow G^*$ and $\rho : G^* \times G^* \leftrightarrow G^*$ are the projection relations of the direct product $G^* \times G^*$ and the vector $DomVec(M, C) : G^* \times G^* \leftrightarrow \mathbf{1}$ is defined by

$$DomVec(M,C) = \overline{(\pi; M^{\mathsf{T}} \cap \overline{C}^{\mathsf{T}}); \mathsf{L}} \cap (\pi; M^{\mathsf{T}} \cap E; \overline{C}^{\mathsf{T}}); \mathsf{L}\,, \tag{1}$$

where $E := \rho; \pi^{\mathsf{T}} \cap \pi; \rho^{\mathsf{T}} : G^* \times G^* \leftrightarrow G^* \times G^*$ is the so-called exchange relation[1], then we have for all $\langle g, h\rangle \in G^* \times G^*$ that $DomVec(M, C)_{\langle g,h\rangle}$ if and only if $g \succ h$. Hence, equation (1) is a relation-algebraic specification of the dominance relation with the government membership relation M and the global utility relation C as its input.

Strictly speaking, according to (1) dominance is specified as a vector of type $[G^* \times G^* \leftrightarrow \mathbf{1}]$. But what we really wanted is a specification as a relation of type $[G^* \leftrightarrow G^*]$. So, we additionally had to apply the technique of Schmidt and Ströhlein [14] for transforming a vector with a direct product as domain

[1] This name stems from the fact that for all $u, v \in G^* \times G^*$ we have $E_{u,v}$ if and only if $u_1 = v_2$ and $u_2 = v_1$.

into the corresponding relation. Doing so, we obtained a relation-algebraic specification $DomRel(M,C) : G^* \leftrightarrow G^*$ of the dominance relation by

$$DomRel(M,C) = \pi^{\mathsf{T}}; (\rho \cap DomVec(M,C); \mathsf{L})\,. \tag{2}$$

Both equations (1) and (2) can be used for specifying relation-algebraically the vector description $StabVec(M,C) : G^* \leftrightarrow \mathbf{1}$ of the set SG^* of all stable governments. We used (1) and arrived after some steps at

$$StabVec(M,C) = \overline{\rho^{\mathsf{T}}; DomVec(M,C)}\,. \tag{3}$$

We immediately could transform the three relation-algebraic specifications (1), (2), and (3) into the programming language of RelView. In the first case the result is the following program:

```
DomVec(M,C)
  DECL Prod = PROD(M^*M,M^*M);
       pi, rho, E
  BEG  pi = p-1(Prod);
       rho = p-2(Prod);
       E = rho*pi^ & pi*rho^
       RETURN -dom(pi*M^ & -C^) & dom(pi*M^ & E*-C^)
  END.
```

Here the first declaration introduces `Prod` as a name for the direct product $G^* \times G^*$. Using `Prod`, the projection relations and the exchange relation are then computed by the three assignments of the body and stored as `pi`, `rho`, and `E`, respectively. The return-clause of the program consists of a direct translation of (1) into RelView-syntax, where `^`, `-`, `&`, and `*` denote transposition, complement, intersection, and composition, and, furthermore, the operation `dom` computes for a relation $R : X \leftrightarrow Y$ the vector $R; \mathsf{L} : X \leftrightarrow \mathbf{1}$.

Similarly, by straightforward translations we obtained as RelView-implementations of the relation-algebraic specifications (2) and (3) the following two relational programs:

```
DomRel(M,C)
  DECL Prod = PROD(M^*M,M^*M);
       pi, rho
  BEG  pi = p-1(Prod);
       rho = p-2(Prod)
       RETURN pi^ * (rho & DomVec(M,C) * L1n(C))
  END.

StabVec(M,C)
  DECL Prod = PROD(M^*M,M^*M);
       rho
  BEG  rho = p-2(Prod)
       RETURN -(rho^ * DomVec(M,C))
  END.
```

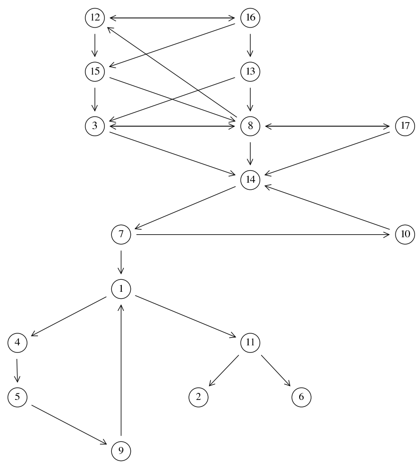

Fig. 1. Dominance without a stable government

The operation `L1n` of the RelView-program `DomRel` computes for a relation $R : X \leftrightarrow Y$ the universal relation L of the specific type $[\mathbf{1} \leftrightarrow Y]$, in matrix terminology hence a Boolean universal row vector.

4 The Case of no Stable Government

Based on the situation in Poland after the 2001 elections, in Berghammer et al. [4] we obtained a dominance graph with three sources, representing the stable governments. In such a non-unique case one might allow negotiations in order to choose a government from among the stable ones; see Rusinowska and de Swart [12]. If there is exactly one stable government, obviously this one has to be chosen. In this paper we consider the remaining case that the dominance graph has no source, like in the RelView-picture of Fig. 1. The situation described by this graph appears if we change the utilities of the example of Berghammer et al. [4] a little bit. As in the original case, for reasons of clearness the picture shows a transitive reduction of the dominance graph only.

Assuming that a computed dominance graph has no source, in this section we first describe our procedure to select a government in this case as a whole. After that we go into details and show how to compute initial strongly connected components and minimum feedback vertex sets relation-algebraically. We also sketch the application of techniques of social choice theory.

4.1 The General Approach

If the computed dominance graph has no source, i.e., there exists no stable government, the central question is which government should be chosen. In this section we answer this question by proposing a procedure for choosing a government that can be considered as rather stable.

As a whole, our proposal is presented below. In it, we apply some well-known concepts from graph theory. First, we use *strongly connected components* (SCCs), i.e., maximal sets of vertices such that each pair of vertices is mutually reachable. Especially we are interested in SCCs without arcs leading from outside into them. These SCCs are said to be *initial*. We also use minimum feedback vertex sets, where a *feedback vertex set* (FVS) is a set of vertices that contains at least one vertex from every cycle of the graph. And here is our proposal:

1. Compute the set $\mathfrak{I}$ of all initial SCCs of the dominance graph.
2. For each SCC C from $\mathfrak{I}$ do:
   a) Compute the set $\mathfrak{F}$ of all minimum FVSs of the subgraph generated by the vertices of C.
   b) Select from all sets of $\mathfrak{F}$ with a maximal number of ingoing arcs one with a minimal number of outgoing arcs. We denote this one by F.
   c) Break all cycles of C by removing the vertices of F from the dominance graph.
   d) Select an un-dominated government from the remaining graph. If there is more than one candidate, use bargaining or social choice rules in order to choose one.
3. If there is more than one set in $\mathfrak{I}$, select the final stable government from the results of the second step by applying bargaining or social choice rules again.

An outgoing arc of the dominance graph denotes that a government dominates another one and an ingoing arc denotes that a government is dominated by another one. Hence the governments of an initial SCC can be seen as a cluster which is not dominated from outside. The application of the second step to such a set of 'candidates' corresponds to a removal of those candidates which are 'least attractive' for two reasons: because they are most frequently dominated and they dominate other governments least frequently. In Section 4.4 we will apply this approach to the example given in Fig. 1.

4.2 Computing Initial Strongly Connected Components

Given a finite graph (V, R) with relation $R : V \leftrightarrow V$ for the arcs, the SCCs of (V, R) are precisely the equivalence classes of the equivalence relation $R^* \cap (R^{\mathsf{T}})^*$. The following RelView-program `Classes` for column-wisely enumerating the equivalence classes of an equivalence relation $S : X \leftrightarrow X$ has been published in Berghammer and Fronk [6]. In it, the calls `Ln1(S)` and `On1(S)` compute the universal vector $\mathsf{L} : X \leftrightarrow \mathbf{1}$ and the empty vector $\mathsf{O} : X \leftrightarrow \mathbf{1}$, respectively, the call `point(v)` yields one of the points contained in the non-empty vector `v`, and the operation `+` computes the relational sum. In matrix terminology the latter means that it puts the matrices one upon the other, so that the RelView-expression

`(C^ + c^)^` of `Classes` 'concatenates' the matrix `C` and the vector `c`.

```
Classes(S)
  DECL C, v, c
  BEG  C = On1(S);
       v = Ln1(S);
       WHILE -empty(v) DO
         c = S * point(v);
         IF isempty(C) THEN  C = c
                       ELSE  C = (C^ + c^)^ FI;
         v = v & -c OD
       RETURN C
  END
```

Using the operation `rtc` for computing reflexive-transitive closures, from the above remark we obtain that the call `Classes(rtc(R) & rtc(R^))` column-wisely enumerates the SCCs of `R`.

In Berghammer and Fronk [6] the authors also refine the program `Classes` to a RelView-program that computes the initial SCCs of `R`. Essentially this refinement consists of an additional assignment in front of the hitherto first assignment to compute `rtc(R) & rtc(R^)` and to store the result as `S` (which now is a local variable instead of the formal parameter), and it simply checks after the computation of the next equivalence class `c` via the assignment `c = S * point(v)` whether `c` is initial and executing only in that case the conditional of the original while-loop. It leads to the following program:

```
InitSccs(R)
  DECL S, C, v, c
  BEG  S = rtc(R) & rtc(R^);
       C = On1(S);
       v = Ln1(S);
       WHILE -empty(v) DO
         c = S * point(v);
         IF incl(R*c,c) THEN
           IF isempty(C) THEN C = c
                         ELSE C = (C^ + c^)^ FI FI;
         v = v & -c OD
       RETURN C
  END
```

The RelView-expression `incl(R*c,c)` of `InitSccs` tests whether the vector `R*c` (describing the predecessors of `c` with respect to `R`) is contained in `c` which, in words, exactly means that the SCC described by `c` is initial.

4.3 Computing Minimum Feedback Vertex Sets

The following relation-algebraic computation of minimum FVSs follows the lines of Berghammer and Fronk [7]. As in Section 4.2 we assume that (V, R) is a finite graph with relation $R : V \leftrightarrow V$.

Let $\mathsf{M} : V \leftrightarrow 2^V$ be the membership-relation on vertices. In a first step we reduce the computation of the FVSs to the computation of *simple cordless cycles*, i.e., simple cycles c which do not contain a pair x, y of vertices that forms an arc in (V, R) but not an arc in c. Since a set F of vertices is a FVS if and only if it contains a vertex from every simple chordless cycle (Berghammer and Fronk [7]), it suffices to enumerate column-wisely the vertex sets of the simple chordless cycles of (X, R) via a relation $K : V \leftrightarrow \mathfrak{C}$, where $\mathfrak{C}$ denotes the set of vertex sets of the simple chordless cycles. Assuming K to be at hand, for all $F \in 2^V$ we are able to calculate as follows (where c ranges over the simple chordless cycles and S ranges over $\mathfrak{C}$):

$$
\begin{aligned}
F \text{ is a FVS} &\iff \forall\, c : \exists\, x : x \in F \wedge x \text{ vertex of } c \\
&\iff \forall\, S : \exists\, x : \mathsf{M}_{x,F} \wedge K_{x,S} \\
&\iff \neg\exists\, S : \overline{\mathsf{M}^{\mathsf{T}}; K}_{FS} \wedge \mathsf{L}_S \\
&\iff \overline{\overline{\mathsf{M}^{\mathsf{T}}; K}; \mathsf{L}}_F
\end{aligned}
$$

This calculation yields $\overline{\overline{\mathsf{M}^{\mathsf{T}}; K}; \mathsf{L}} : 2^V \leftrightarrow \mathbf{1}$ as the vector representation of all FVSs of (V, R). Next we apply that the vector $v \cap \overline{\overline{Q}; v}$ describes the least elements of the set described by the vector v with respect to the preorder Q; see e.g., Schmidt and Ströhlein [14]. If we use the above vector as v, the size comparison relation on 2^V as Q, and implement the expressions developed so far in RelView, we obtain the following program for computing the vector description of the minimum FVSs from the relation K:

```
MfvsVec(K)
  DECL LeEl(Q,v) = v & -(-Q * v));
  DECL M
  BEG  M = epsi(O(K))
       RETURN LeEl(cardrel(O(K)),-dom(-(M^*K)))
  END.
```

From this program we obtain a program for the column-wise enumeration of the minimum FVSs by applying the technique described in Section 3.1.

We call a set S of vertices of (V, R) *progressively infinite* if it is non-empty and for each vertex $x \in S$ there exists a successor $y \in S$. Fundamental for obtaining a relation-algebraic specification of the relation $K : V \leftrightarrow \mathfrak{C}$ (a task we still have to solve) is the following fact (Berghammer and Fronk [7]): S is the vertex set of a simple chordless cycle if and only if it is a minimal progressively infinite set. Thus, our next goal is identified. We have to develop a RelView-program, say `MprinfVec`, that computes the vector description of the minimal progressively infinite sets. Then the technique of Section 3.1 shows that K is computed by

```
M * inj(MprinfVec(R))^.
```

In order to obtain a vector that describes the minimal progressively infinite sets, we first neglect minimality and calculate for a set S of vertices as follows:

$$
\begin{aligned}
S \text{ progr. infinite} &\iff (\exists\, x : x \in S) \wedge (\forall\, x : x \in S \rightarrow \exists\, y : y \in S \wedge R_{x,y}) \\
&\iff (\exists\, x : \mathsf{M}_{x,S}) \wedge (\forall\, x : \mathsf{M}_{x,S} \rightarrow \exists\, y : \mathsf{M}_{y,S} \wedge R_{x,y}) \\
&\iff (\exists\, x : \mathsf{L}_{\perp,x} \wedge \mathsf{M}_{x,S}) \wedge (\forall\, x : \mathsf{M}_{x,S} \rightarrow (R;\mathsf{M})_{x,S}) \\
&\iff (\mathsf{L};\mathsf{M})_{\perp,S} \wedge (\neg\exists\, x : \mathsf{L}_{\perp,x} \wedge \mathsf{M}_{x,S} \wedge \overline{R;\mathsf{M}}_{x,S} \\
&\iff (\mathsf{L};\mathsf{M})^{\mathsf{T}}_{S} \wedge \overline{\mathsf{L};(\mathsf{M} \cap \overline{R;\mathsf{M}})}_{\perp,S} \\
&\iff ((\mathsf{L};\mathsf{M})^{\mathsf{T}} \cap \overline{\mathsf{L};(\mathsf{M} \cap \overline{R;\mathsf{M}})}^{\mathsf{T}})_{S}
\end{aligned}
$$

Hence, $(\mathsf{L};\mathsf{M})^{\mathsf{T}} \cap \overline{\mathsf{L};(\mathsf{M} \cap \overline{R;\mathsf{M}})}^{\mathsf{T}} : 2^V \leftrightarrow \mathbf{1}$ is a vector description of the progressively infinite sets of the graph (V, R). Minimalization now is obtained by using two well known results: $\overline{\mathsf{M}^{\mathsf{T}};\overline{\mathsf{M}}} : 2^V \leftrightarrow 2^V$ relation-algebraically specifies set inclusion on 2^V and the vector $v \cap \overline{(Q^{\mathsf{T}} \cap \bar{\mathsf{I}})v}$ describes the minimal elements of the set described by the vector v with respect to the preorder Q; see again Schmidt and Ströhlein [14]. If we combine these facts with the vector description of the progressively infinite sets and formulate the result in RelView-syntax, we arrive at the following RelView-program:

```
MprinfVec(R)
  DECL Min(Q,v) = v & -((Q^ & -I(Q)) * v);
       M, SI, L
  BEG  M = epsi(O(R));
       SI = -(M^ * -M);
       L = L1n(R)
       RETURN Min(SI, (L*M)^ & -(L * (M & -(R*M)))^)
  END.
```

The bottleneck of this program is the use of set inclusion since the size of the ordered binary decision diagrams of this relation is exponential in the size of the base set. Using the present RelView-version it can be only applied to graphs with up to approximately 30 vertices. As we apply it, however, only to initial SCCs, this usually suffices for practical applications of coalition formation. It still should be mentioned that Berghammer and Fronk in [6] develop a refinement of our programs that avoids the use of set inclusion and can be used for graphs consisting of about 100 vertices in general and even more in advantageous cases.

4.4 The Example Revisited

In the following, we want to demonstrate an application of the RelView-programs we have developed so far. As input we assume the dominance relation, the transitive reduction of which graphically is depicted in Fig. 1.

Following the general procedure of Section 4.1, in the first step we have to compute the initial SCCs using the RelView-program `InitSccs` of Section 4.2. The graph of Fig. 1 possesses exactly one initial SCC. Its RelView-represen-

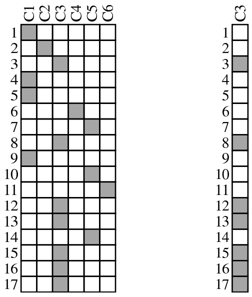

Fig. 2. SCCs and initial SCC of the former example

tation as Boolean vector is shown on the right-hand side of Fig. 2. To give an impression how a column-wise enumeration of sets of subsets looks in RelView, on the left-hand side of the figure we additionally show the six SCCs of the input as 17×6 matrix. In both cases labels are added to rows and columns by a specific feature of the tool for illustration purposes. In RelView a black square of a Boolean matrix means 'true' and a white square means 'false'. Hence, the SCCs of the input are $\{1, 4, 5, 9\}$, $\{2\}$, $\{3, 8, 12, 13, 15, 16, 17\}$, $\{6\}$, $\{7, 10, 14\}$, and $\{11\}$. The only initial SCC is $C_3 = \{3, 8, 12, 13, 15, 16, 17\}$.

Next, we perform Step a) of the general procedure to the initial SCC. C_3 contains the cycles $\{12,\ 16\}$, $\{3,\ 8\}$, $\{8,\ 12,\ 15\}$, $\{3,\ 8,\ 12,\ 15\}$, $\{8,\ 12,\ 16,\ 13\}$, $\{8,\ 12,\ 16,\ 15\}$ and $\{3,\ 8,\ 12,\ 16,\ 15\}$. By means of the RelView-program `MfvsVec` of Section 4.3 we obtain two minimum FVSs, viz. $\{8, 16\}$ and $\{8, 12\}$. The Boolean RelView-matrix of Fig. 3 column-wisely enumerates these sets.

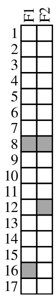

Fig. 3. Minimum FVSs of the initial SCC

Since Step a) of the general procedure of Section 4.1 demands to compute the minimum FVSs of the subgraph generated by the initial SCC C_3, strictly speaking we first get a relation of type $[C_3 \leftrightarrow \mathfrak{F}]$ as result, which means that the elements of $\mathfrak{F}$ are considered as subsets of C_3. The matrix of Fig. 3 is obtained from this result by multiplying it from the left with $inj(v)^{\mathsf{T}}$, where the vector $v : G^* \leftrightarrow \mathbf{1}$ describes the SCC C_3. Thus, the computed minimum FVSs become subsets of the set G^*.

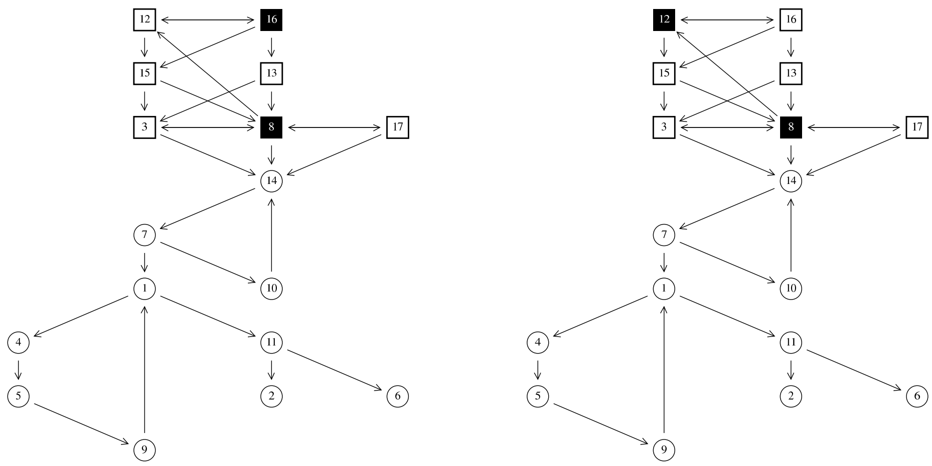

Fig. 4. The original graph marked with the minimum FVSs

That $\{8, 16\}$ and $\{8, 12\}$ are indeed the only minimum FVS hopefully becomes clear if we consider Fig. 4. It shows two copies of the input graph of Fig. 1. In both cases we have instructed RelView to draw the vertices of the initial SCC as squares and additionally to indicate a minimum FVS by the colour black. In the graph on the left-hand side we identify the minimum FVS $\{8, 16\}$ and in the other graph the minimum FVS $\{8, 12\}$. From Fig. 4 we also see that five arcs lead from outside into the FVS $\{8, 12\}$, but only four arcs lead from outside into $\{8, 16\}$. Hence, by Steps b) and c) of the general procedure we have to remove the vertices 8 and 12 from the graph, which leads to 16 and 17 as new sources, i.e., as governments that can be considered as rather stable. What government finally is chosen depends on specific circumstances. Here social choice rules or bargaining can help; we discuss this point in the next subsections.

4.5 Application of Social Choice Theory

According to the procedure described in Section 4.1, if the application of graph theory does not give a unique solution, we select the final government from among the 'graph-theoretical' results by applying social choice rules or bargaining theory. This subsection concerns an application of some well-known social choice rules, like *Plurality Rule* (*Most Votes Count*), *Majority Rule* (*Pairwise Comparison*), *Borda Rule*, and *Approval Voting*, to our choice problem. For an overview and comparison of social choice rules see, for instance, Brams and Fishburn [9], and de Swart et al. [15].

The input for an application of social choice theory consists of: (at least two) selected governments (from which we have to choose one), parties forming these governments, and preferences of the parties over the governments. Moreover, for each government each party either accepts (approves of) or does not accept (disapproves of) it. We consider four rules:

1. *Plurality Rule:* Under this rule only the first preference of a party is considered. A government g is collectively preferred to a government h if the number of parties that prefer g most is greater than the number of parties that prefer h most. The government chosen under the plurality rule is the government which is put first by most parties.
2. *Majority Rule:* This rule is based on the majority principle. A government g is collectively preferred to h if g defeats h, i.e., the number of parties that prefer g to h is greater than the number of parties that prefer h to g. If there is a government that defeats every other government in a pairwise comparison, this government is chosen, and it is called a Condorcet winner.
3. *Borda Rule:* Here weights are given to all the positions of the governments in the individual preferences. For n governments, every party gives n points to its most preferred government, $n-1$ points to its second preference, etc., and 1 point to its least preferred government. A decision is made based on the total score of every government in a given party profile.
4. *Approval Voting Rule:* Under Approval Voting (Brams and Fishburn [8]), each party divides the governments into two classes: the governments it approves of and the ones it disapproves of. Each time a government is approved of by a party is good for one point. The government chosen is the one that receives most points.

Let us apply these rules to our example. We have two governments chosen by the 'graph theoretical part' of the procedure described in Section 4.1: governments 16 and 17, and denote them by g_{16} (formed by parties A and C) and g_{17} (formed by parties A and B), respectively. Let $\succ_i$ denote the preference relation $R^{(i)}$ of party $i \in \{A, B, C\}$ over the set $\{g_{16}, g_{17}\}$. In our example, we have:

$$g_{17} \succ_A g_{16} \qquad g_{16} \succ_B g_{17} \qquad g_{16} \succ_C g_{17}$$

Moreover, all three parties accept both governments, except party C which does not approve of g_{17}. In the case of two alternatives, the Plurality Rule, the Majority Rule and the Borda Rule give the same result: government g_{16}. Moreover, g_{16} is approved of by all three parties in question, while g_{17} is approved of only by two parties.

4.6 Application of Bargaining Theory

We also like to mention another way for choosing one final government from among (at least two) governments selected by the 'graph-theoretical part' of our procedure. This alternative method is based on bargaining theory. In Rusinowska and de Swart [12], the authors define six bargaining games in which parties belonging to stable governments (it is assumed that there are at least two stable ones) bargain over the choice of one stable government. Subgame perfect equilibria of the games are investigated. Of course, the result of a bargaining game depends not only on the bargaining procedure, but also on the order in which parties bargain. In Rusinowska and de Swart [12], a procedure for choosing the order of parties for a given game is also proposed.

In this paper, we apply only some bargaining games analyzed in Rusinowska and de Swart [12], to show how such an application of bargaining to our choice problem may look like. Since we have only two governments and only three parties, the games are very simple. There are several common assumptions for our bargaining games. First of all, it is assumed that a party, when submitting an offer, may propose only one government. Moreover, the same offers are not repeated: a party cannot propose a government which has been already proposed before. Finally, it is assumed that choosing no government is the worst outcome for each party. Our bargaining games differ from each other with respect to the bargaining procedures and the bargaining costs. Here we consider the games in which a party prefers to form a government it likes most with a delay, rather than to form immediately (with no delay) a less preferred government.

Let us consider one of the bargaining games for which the parties' order chosen by a special procedure is (A, B, C). This is the order of parties according to the number of seats in Parliament. It is assumed in this game that a party, when submitting an offer, may propose only a government the party belongs to. The bargaining procedure for this game is the following. First, party A proposes either government g_{16} or government g_{17}. If g_{16} is proposed, then party C (which is involved in g_{16}) either accepts of rejects the proposal. Since there are no more parties 'responsible' for g_{16}, if party C accepts the offer, government g_{16} is chosen. Otherwise, no government is created, since party C is involved in no more governments. On the other hand, if A proposes g_{17}, it is party B which has to react. Similarly, the acceptance of this offer causes g_{17} to be formed, and the rejection leads to no government formed. There is only one subgame perfect equilibrium for this game and it leads to the choice of government g_{17}, the most preferred result of party A.

The other two games we like to mention are less profitable for the strongest party A. One of them gives more room for parties other than the strongest one. Let us assume that a party does not have to belong to the government it proposes, and all parties have to react to each offer. This means that if A submits an offer (g_{16} or g_{17}), both parties B and C must either accept or reject the offer. For some orders of the parties, this game has more than one subgame perfect equilibria, but they always lead to the creation of g_{16}, i.e., the government most preferred by parties B and C.

Finally, let us assume that only the strongest party, i.e., party A, may submit an offer, and the other party forming the proposed government has to react. The subgame perfect equilibrium of this game also results in the choice of government g_{16}.

5 Conclusions

The central concepts of the coalition formation model are the notion of (feasible) government and the notion of stable government. The latter is defined as a feasible government dominated by no feasible government. In the present paper, we aim to answer the question which government should be chosen if there is no stable government (that is, if the dominance graph has no source). The attractiveness and novelty of our approach consists in: 1. the clever combination of notions from partly different domains (relational algebra, graph theory, social choice theory and bargaining), and 2. the immediate and easy support by the computer system RelView for computing solutions and for visualizing the results. Given a dominance graph without a source, first we compute all initial strongly connected components. The governments of an initial strongly connected component can be seen as a cluster which is not dominated from outside. Next, for each initial strongly connected component, we compute the set of all minimum feedback vertex sets, where a minimum feedback vertex set is a minimal set of vertices which breaks all cycles. Next, we choose a specific minimum feedback vertex set according to the following rule. First, we choose the set(s) for which the number of ingoing arcs is maximal. Since an ingoing arc denotes that a government is dominated, such a choice means selecting governments dominated most frequently. Next, if there are at least two such sets, we choose one for which the number of outgoing arcs is minimal, meaning the choice of the governments which dominate other governments least frequently. Next, we break all cycles by removing the chosen set of governments. One may say that we remove governments which are least attractive for two reasons: because they are most frequently dominated and they dominate other governments least frequently. According to our procedure, if there is more than one initial strongly connected component, we select the final stable government (from the results of the procedure described above) by applying bargaining or some well-known social choice rules. Concerning the application of bargaining, we construct several bargaining games and choose the government which is a subgame perfect equilibrium result. Concerning the application of social choice theory, we apply the plurality rule, the majority rule, the Borda rule, or approval voting. Of course, some of these applications may also lead to a non-unique solution. In this case, we propose to combine several techniques and to apply a several-steps method consisting of, for instance, a social choice rule in the first step, and a bargaining game in the second step.

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