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HETEROSKEDASTICITY IN CROP YIELD MODELS

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Abstract

This study examines three alternative models of heteroskedasticity in crop yield models. Non-nested test results suggest that modeling the sources of heteroskedasticity is the preferred procedure. Including potential sources of heteroskedasticity as explanatory variables can remove the heteroskedasticity in wheat yields. The results also suggest that the GARCH specification is a promising model of heteroskedasticity when the sources cannot be identified. The time-trend variance model alone may misspecify the true variance structure.

HETEROSKEDASTICITY IN CROP YIELD MODELS

Understanding the behavior of crop yields becomes increasingly important for modeling production functions, forecasting price movements, and understanding farmers' response to government programs. Variability in crop yields is the principal source of instability in production levels (Hazell 1985), and most studies use the coefficient of variation around the trend to measure the variability in crop production (Hazell 1984; Weber and Sievers; Singh and Byerlee). The maintained assumption of using the coefficients of variation for analysis is that detrended yields are homoskedastic within the sample period.

A phenomenon usually confronted in statistical analysis of crop yields is heteroskedasticity, which seems to be characterized mainly by systematic changes in yield variation over time. Nevertheless, heteroskedasticity has received less attention and frequently has been handled inadequately in empirical analysis.

Hazell (1984) and, similarly, Singh and Byerlee recognized heteroskedasticity in detrended yields and partitioned the residuals into subsequent periods. However, the measured coefficient of variation could be misleading if heteroskedasticity occurs more frequently. Gallagher found that the detrended yields have an upward trend in variation and standardized the data with the predicted standard deviation from a regression against a time-trend to correct for heteroskedasticity[1]. This standardization procedure results in heteroskedasticity-adjusted estimates only if the true error structure is known. If the variation is cyclical or changing with some systematic patterns other than trend, this procedure would not correct fully for heteroskedasticity.

Variation in yields may have some patterns due to autocorrelated weather and/or gradual adaptation of new technology. If this is the case, the GARCH (Generalized Autoregressive Conditional Heteroskedastic) process developed by Bollerslev can well serve as an alternative to the time-trend variance model. The GARCH (p,q) process is equivalent to an ARMA process with m=max{p,q} and p in the squared disturbances (Bollerslev) and is useful to model systematic changes in yield variation. The GARCH model is analogous to Just and Pope's stochastic production function, which allows the relationships of inputs with risk to be independent of the relationships of inputs with production.

Conventional time-trend variance or the GARCH models may explain the variation in crop yields. However, heteroskedasticity may result from model misspecification, most likely due to omitted variables (Judge et al.). Offutt et al. found that variability of corn yield around a trend increases over time, but inclusion of weather variables is likely to remove the heteroskedasticity. Engle (p. 990) claimed:

> ... The existence of ARCH effect would be interpreted as evidence of misspecification. ... the ARCH may be a better approximation to reality than making standard assumptions about the disturbance, but trying to find the omitted variables or determine the nature of the structural change would be even better.

An appropriate way to model heteroskedastic yields is to incorporate possible sources of heteroskedasticity as a priori information. This approach should be preferred, as Engle suggested, to the models that allow heteroskedasticity and approximate the true error structure with time-varying variance models such as the time-trend variance or the GARCH models. If the analysis fails to identify those sources, then it becomes important to determine how the variance behaves over time to correctly standardize the data. If crop yields follow a GARCH process, ignoring it would bias estimated standard errors and test results (Diebold).

The objective of this study is to determine the appropriate model to correct for heteroskedasticity in the sample data. In this analysis, we conduct a non-nested test for three alternative models of correcting heteroskedasticity in crop yields: the time-trend variance model, the GARCH specification and an econometric model that explicitly includes the potential sources of heteroskedasticity.

The next section specifies the candidate model and test procedure. The estimated models and test results are reported in the following section. The final section summarizes this study and discusses the implications of findings.

Model specifications and test procedures

Let y_t be the sample yield and x_{1t} be a vector of explanatory variables for the mean process. The three alternative models to be tested are specified as follows. The time-trend variance model is

$$[1] \quad y_t = x_t b + \epsilon_t$$
$$\epsilon_t^2 = \alpha_0 + \alpha_1 T$$

where b and αs are parameters, ϵ_t is a disturbance term, and T is a trend variable.

On the other hand, the alternative GARCH process, which is used extensively for heteroskedastic time series data (Akgiray; Aradhyula and Holts; Yang), endogenizes the conditional variance. Following Bollerslev's proposition, we use the GARCH(1,1) specification as follows:

$$[2] \quad y_t = x_t b + \epsilon_t$$

$$h_t = \beta_0 + \beta_1 \epsilon^2_{t-1} + \beta_2 h_{t-1}$$

$$\epsilon_t \sim N(0, h_t),$$

where βs are parameters and ϵ_t follows conditionally a normal distribution with zero mean and time-varying variance, h_t, which is determined by past realization of second moments of disturbances and its own lag. The variance equation is equivalent to an ARMA specification in ϵ^2_t.

Finally, the econometric model is specified such that the disturbances ϵ_t in the above equations have a constant variance, i.e., $h_t = \sigma^2$, for all t. In this alternative model, x_t is a vector of explanatory variables, which cause heteroskedasticity in crop yields.

Potential sources that may cause heteroskedasticity are changes in technology, government programs, and climatological variables. The time-trend mean process, which is most commonly used in crop yield analysis, models the technology change with a trend, interpreting the coefficient as the productivity growth rate. However, in addition to yield increases, technology should change yield variability. For example, the objective of most crop breeding programs is to improve disease resistance and quality. The variability in yields should diminish as disease resistance improves. A spline function (Singh and Byerlee) capturing different rates of change in yield in different periods implies heteroskedasticity induced by technology.

Government programs have a significant impact on yields in a particular crop year (Houch and Gallagher). Marginal shifts in area planted (due mainly to government programs) are typically on the less productive land. Thus, variation in the area planted on marginal land should impact the variability in yields.

The variation in key climatological variables appears to be non-constant. For example, weather patterns in the 1980s differ significantly compared to previous decades, in which greater variability appears in both temperature and precipitation in key growing regions in the United States. Thus, changes in weather patterns induce apparent changes in variability in yield trends. Weather-related heteroskedasticity seems to be more profound with regional crop yield data than aggregated national data, which tend to average out regional weather effects (Offutt et al.). We chose two key climatological variables representing temperature and moisture.

Finally, we included a lagged dependent variable to remove autocorrelation which makes test inferences unreliable. This lagged variable also models persistence in average productivity, which may be due either to continuous cropping patterns or to fertilizer and moisture remaining from the previous years. This persistence would reduce yield variability.

The mean process for the econometric model is then specified as

[3] $y_t = b_0 + b_1T + b_2T^2 + b_3T^3 + b_4y_{t-1} + b_5A_t + b_6JT_t + b_7TM_t + e_t$

where A_t is the planted acres; JT_t, the mean of June's daily maximum temperature; and TM_t, the sum of growing season precipitation (May to July) and the recharged precipitation (October to April)[2]. A cubic function of trend is introduced to capture nonlinearity in growth rates.

The first step of the non-nested Wald test, which is valid for small samples, is to nest the alternative models in a more general specification. The general model is the same as Model [3] with a time-varying variance equation specified as

[4] $h_t = \gamma_0 + \gamma_1 e^2_{t-1} + \gamma_2 h_{t-1} + \gamma_3 T$

$e_t \sim N(0,h_t)$

where γs are parameters.

The hypothesis of the time-trend model is Ho:γ_1=0 and γ_2=0 while that of the GARCH model is Ho:γ_3=0. The hypothesis of the econometric model is Ho:γ_1=0, γ_2=0 and γ_3=0. As is always true of non-nested tests, the test may reject all candidate models or fail to reject any of them. Eight cases of test results are possible.

Heteroskedasticity in the data may not conform to systematic patterns which the candidate models represent. Thus, we apply the BDS test[3] (Brock, Dechert and Sheinkman) to the residuals (not standardized) of the general model. The BDS test detects both linear and nonlinear dependence, and the rejection of i.i.d. may be due to linear dependence, i.e., autocorrelation. Thus, the Ljung-Box test for serial correlation[4] is conducted to the residuals because our interest is heteroskedasticity.

The data considered in this study are average yields of durum and hard red spring (HRS) wheat produced in North Dakota during 1929 to 1988. We use these state-wide data to avoid the possibility that the sources of heteroskedasticity would be neutralized or masked in the nation-wide data. The data for yields and planted acres are obtained from various issues of North Dakota Agricultural Statistics, and data for the climatological variables are taken from the U.S. Department of Commerce, National Oceanic and Atmosphere Administration.

Time-trend variance vs. GARCH models

We first estimated a time-trending GARCH model to confirm heteroskedasticity in the data since the null hypothesis of the econometric model is no heteroskedasticity. The non-nested test between the time-trend variance and the GARCH models is also important since the GARCH specification for the heteroskedastic crop yields is a new effort in literature. The test is performed with the time-series (Bessler) and time-trend mean processes, which were widely used in past studies.

Time-series mean process

The mean process of each crop yield is identified as an ARMA(1,1) process through autocorrelation and partial autocorrelation functions (Box and Jenkins). We estimated the ARMA(1,1) process for the mean equation with the same variance equation as in Equation [4].

Table 1 shows the estimated results of the model for both crops. Both data are serially correlated. Estimated coefficients of the autoregressive and moving average terms are significant at the 5% level for each crop. The Ljung-Box test does not reject the null hypothesis of no serial correlation in the residuals for each model. The model removes autocorrelation in the sample data.

The results of non-nested tests differ for each crop. The test rejects the time-trend variance model in favor of the GARCH alternative in the case of durum, i.e., the GARCH term (h_{t-1}) is significant while the trend is not statistically different from zero at the conventional levels. On the other hand, the test rejects neither time-trend nor GARCH alternatives for HRS, i.e., all variables are significant.

Time-trend mean process

The sample data are autocorrelated as found in the time-series model, so we included a lagged dependent variable. To avoid heteroskedasticity due to misspecified functional form, trend is introduced into the model using a cubic function as in the general model [4].

The estimated results in Table 1 indicate significant changes in productivity due to time-dependent technical changes in each crop. However, the growth rate patterns differ. Yields for HRS increase linearly over time, while those for durum increase at first an increasing rate and, then, at a decreasing rate. Thus, the second- and third-order terms are not included in the final models for HRS.

The non-nested test results are consistent with those of time-series mean models. In case of durum, only the GARCH term is significant, rejecting the time-trend variance model in favor of the GARCH alternative, while neither model is rejected for HRS.

The results of these tests indicate that the conventional use of a time-trend in variance misspecifies the variance structure. The GARCH effects should be considered when modeling heteroskedastic crop yields. As indicated by the Ljung-Box test and Kolmogorov-Smirnov test of fit (D_{max}) for diagnostic check, the standardized residuals (e_t/h_t) satisfy the maintained assumption of i.i.d. normal. The estimated model is a valid specification for the sample data.

Non-nested test among the three alternatives

We have shown that the residuals in conventional time-series and time-trend mean processes are heteroskedastic. We now estimate the general model and conduct non-nested tests among the three alternative models. The trend variable in the variance equation for durum is dropped according to the previous findings since inclusion of an irrelevant variable reduces efficiency.

The estimated results are reported in Table 2 along with those from OLS estimation for comparison. In both cases, planted acres had a negative impact on yields as expected. These negative coefficients imply that these crops are produced with over-utilization of land[5]. Climatological variables are all significant in explaining yield movements for each crop. Moisture affects yields positively, while temperature affects yields negatively.

The non-nested tests support the econometric model against the other two time-varying variance models. Neither the ARCH nor GARCH term is significant at conventional levels for both crops. Also, the trend is not significant in the HRS model. The null hypothesis of no heteroskedasticity also is supported by the likelihood-ratio (LR) test (Ho: γ_1=0, γ_2=0 and γ_3=0) for the two crops.

To confirm the non-nested test results, we apply the BDS test for the residuals. The BDS test detects no dependence left for HRS, while the i.i.d. hypothesis is rejected for durum at an embedding dimension, m=5. However, the Ljung-Box test does not detect quadratic dependence in the durum residuals (e_t^2). This implies that the BDS rejection is due to nonlinear dependence at higher-than-second moments, e.g., heterokurtic. These test results indicate that the econometric model specified in this study corrects for heteroskedasticity found in the time-series and time-trend mean processes.

The standard errors of GARCH estimates are compared to those of OLS. Since the econometric specification removed autocorrelation and heteroskedasticity, OLS estimates would be more efficient. The results support this proposition for the durum model in which all standard errors from OLS are smaller than those from the GARCH model. However, results for HRS are mixed, i.e., some were smaller and some larger.

Summary and conclusions

Most studies on crop yields tend to ignore heteroskedasticity or to handle it improperly. Simple linear time-trend and time-series models usually encounter variances that change over time. The conventional way to correct for heteroskedasticity is to standardize the data with their predicted standard deviation. However, we suggest that including factors that cause the systematic changes in yield variation should be preferred to the model allowing heteroskedasticity and approximating the variance structure. The underlying hypothesis is that the detected heteroskedasticity results from model misspecification mostly are due to omitted variable explaining the systematic changes in yield variability. To confirm this proposition, we conducted non-nested tests among the three alternatives: the time-trend variance, the GARCH, and the econometric models, including potential sources of heteroskedasticity for durum and HRS wheat produced in North Dakota during 1929 to 1988.

The results indicate that the econometric model including the trend as a proxy for technology, planted acres and climatological variables removes heteroskedasticity found in time-trend and time-series mean models. Conventional use of the OLS estimation is acceptable for these sample data, which is convenient in empirical study. Among the variables included, the climatological variables seem to be most important in explaining heteroskedasticity. Models only included the climatological variables provided the same inferences in terms of heteroskedasticity, while it is not true for planted acres and trend (as shown in the third section)[6].

Efforts identifying the sources of heteroskedasticity may not necessarily be so desperate as many researchers seem to perceive. Climatological variables used in this study can be located easily, even if they may be unfamiliar to agricultural economists.

Another important implication of this study is that the GARCH specification is promising to model heteroskedastic yields. The GARCH effects were significant for the models that did not include those econometric variables. The conventional time-trend variance model alone cannot correctly approximate the systematic changes in the sample yield variation. If the sources cannot be identified, the GARCH specification or the GARCH with a trend in variance may be appropriate to correct for heteroskedasticity.

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Footnotes

1 Hereafter we call this model a time-trend variance model.

2 In sum, TM is the total precipitation during the year except August and September when precipitation almost evaporates.

3 Under the null hypothesis that a time series $\{x_t\}$ is i.i.d., the BDS statistic is $W_m(\epsilon,T)=T^{1/2}[C_m(\epsilon,T) - C_1(\epsilon,T)]/\sigma_m(\epsilon)$, where m is embedding dimensions, T the sample size, C_m the correlation integral evaluated at T and ϵ, a very small number, and σ_m the variance of the numerator. This statistic converges in distribution to a standard normal random variable under the null hypothesis. For more details, see Sheinkman and LeBaron.

4 Under the null hypothesis of no serial correlation, the Ljung-Box test statistic with sample size T is $Q(\tau)=T(T+2)\Sigma_\tau \rho(\tau)^2/(T-\tau)$, $\tau \leq K$, where $\rho(\tau)$ is the τ^{th} autocorrelation coefficient. When applied to residuals, Q has asymtotically a chi-square distribution with K-s degrees of freedom to adjust for the estimated parameters, and s is the number of lagged dependent variables.

5 The coefficient of planted acres is the first differentiation of yield with respect to the variable. Since yields are average products of land ($y=Q/A \equiv AP_L$), the negative coefficient implies that $(MP_L-AP_L)/A<0$ and $MP_L<AP_L$. The production exhibits decreasing returns to scale w.r.t. land.

6 The mean equations only with the planted acres still have the GARCH effects. We do not report these results for space limitation.

Table 1. Estimated results of time-series and time-trend models with the time-varying variance equations[a]

	Time-Series[b]		Time-Trend[b]	
	Durum	HRS	Durum	HRS
Mean				
Intercept	1.68	1.69	8.96*	3.88*
	(1.16)	(1.15)	(2.15)	(1.82)
AR(1)	0.92*	0.90*		
	(0.07)	(0.07)		
MA(1)	-0.26*	-0.39*		
	(0.11)	(0.17)		
y_{t-1}			0.37*	0.42*
			(0.17)	(0.13)
T			-0.64*	0.22*
			(0.24)	(0.07)
T^2			0.04*	-
			(0.01)	-
T^3			-0.0005*	-
			(0.0001)	-
Variance				
Intercept	-0.89	-0.97	13.01	-0.71
	(1.62)	(1.00)	(9.94)	(1.09)
e^2_{t-1}	-0.11	-0.13*	0.30	-0.11
	(0.15)	(0.05)	(0.17)	(0.07)
h_{t-1}	0.92*	0.99*	-0.73*	0.99*
	(0.16)	(0.10)	(0.27)	(0.11)
T	0.18	0.13*	0.53	0.10*
	(0.12)	(0.04)	(0.36)	(0.04)
Log-Likelihood	-173.60	-165.27	-168.91	-164.05
Ljung-Box test(Lag=10)				
e_t/h_t	5.17	7.29	14.65	13.22
$(e_t/h_t)^2$	3.63	7.15	6.10	13.17
Normality test				
Skewness	0.540*	0.331	-0.197	0.189
Kurtosis	0.463	0.168	-0.692	0.541
D_{max}	0.078	0.075	0.058	0.069

[a]Standard errors in parentheses.
[b]Statistically significant at a 5% level.

Table 2. Estimated results of the general model for the non-nested tests[a]

	Durum[c]		HRS[c]	
	GARCH	OLS	GARCH	OLS
Mean				
Intercept	33.05*	28.02*	25.31*	32.39*
	(15.38)	(10.39)	(4.73)	(9.13)
y_{t-1}	0.34*	0.40*	0.43*	0.29*
	(0.11)	(0.10)	(0.12)	(0.10)
T	-1.07*	-0.83*	0.13*	0.17*
	(0.44)	(0.34)	(0.05)	(0.06)
T^2	0.05*	0.04*	-	-
	(0.017)	(0.012)	-	-
T^3	-0.0005*	-0.0004*	-	-
	(0.0002)	(0.0001)	-	-
Acre	-1.76	-1.14	-0.93*	-1.12*
	(0.97)	(0.78)	(0.21)	(0.38)
JT	-0.37*	-0.37*	-0.30*	-0.38*
	(0.16)	(0.11)	(0.06)	(0.08)
TM	0.88*	0.95*	0.65*	0.73*
	(0.27)	(0.18)	(0.12)	(0.15)
Variance				
Intercept	12.91*	11.59*	3.52	7.98*
	(4.62)	(2.31)	(5.18)	(1.57)
e^2_{t-1}	0.09	-	0.44	-
	(0.08)	-	(0.28)	-
h_{t-1}	-0.28	-	-0.49	-
	(1.30)	-	(0.44)	-
T	-	-	0.23	-
	-	-	(0.14)	-
Log-Likelihood	-151.96	-153.55	-144.02	-145.06
LR test	3.18	-	2.08	-
Adjusted R-square	-	0.83	-	0.87
Ljung-Box test				
e_t	16.35	-	5.71	-
e^2_t	7.98	-	4.26	-
e_t/h_t	15.02	-	5.32	-
$(e_t/h_t)^2$	5.86	-	3.04	-
BDS test(ϵ=0.5σ)[b]				
m=2	0.11	-	0.10	-
m=3	-0.38	-	-0.78	-
m=4	-1.37	-	-1.02	-
m=5	-2.11*	-	-1.41	-
Normality test[b]				
Skewness	-0.113	-	-0.141	-
Kurtosis	1.109*	-	-0.418	-
D_{max}	0.088	-	0.063	-

[a]Standard errors in parentheses. Acres are divided by 1000 to avoid the scaling problem.
[b]Tests applied for the residuals.
[c]Significant at a 5% level.