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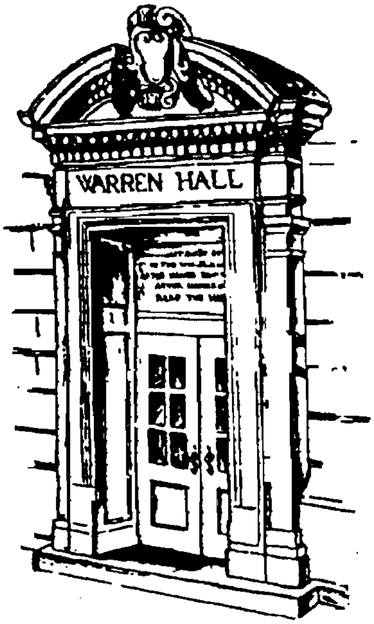

Staff Paper

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USING A JOINT-INPUT, MULTI-PRODUCT FORMULATION TO IMPROVE SPATIAL PRICE EQUILIBRIUM MODELS

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Phillip M. Bishop, James E. Pratt, and Andrew M. Novakovic*

ABSTRACT

Mathematical programming models, as typically formulated for international trade applications, may contain certain implied restrictions which lead to solutions which can be shown to be technically infeasible, or if feasible, then not actually an equilibrium. An alternative formulation is presented which allows joint-inputs and multi-products, with pure transshipment and product substitution forms of arbitrage.

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USING A JOINT-INPUT, MULTI-PRODUCT FORMULATION TO IMPROVE SPATIAL PRICE EQUILIBRIUM MODELS

INTRODUCTION

The potential impacts stemming from the liberalization of trade have received much attention in recent years. Indeed, the new General Agreement on Tariffs and Trade (GATT), and the North American Free Trade Agreement (NAFTA), have generated a plethora of studies seeking to explain the implications of these, and other, trade agreements. Mathematical programming models are frequently chosen over statistically oriented models as the most appropriate tool to perform these analyses. Such a preference is obviated by the limitations of historical data when analyzing structural change. However, the manner in which mathematical models are typically formulated may contain several serious shortcomings. For example, the spatial models typically used to analyze dairy trade can invariably be shown to produce results which are actually technically infeasible, or if feasible, then not an equilibrium. Statistical models, while never intended to derive optimal solutions, may also suffer problems with infeasibility because their typical construction also fails to require mass balancing constraints. This paper discusses these shortcomings and describes a formulation of the spatial trade model which addresses these concerns. While the model has general applications in a joint-input, multi-product setting, the focus here is on dairy trade.

ISSUES AND PROBLEMS

The spatial price equilibrium model, as pioneered by Samuelson (1952), and Takayama and Judge (1964, 1971), enjoys a lengthy history in the analysis of trade. Its underlying assumptions are well understood. However, when the basic model is extended to include joint inputs, multiple products, and policy instruments, a number of interrelated problems may arise.

1. Functional Representation:

The first issue concerns the way in which supply and demand functions are represented in the model. Frequently, reduced-form net trade functions are used to represent the regions in a spatial trade model. Often this approach is used simply because excess supply and demand functions are easier to estimate than domestic, or internal, supply and demand functions. However, the method by which the model determines net trade positions turns out to be particularly crucial. A spatial model, one capable of reproducing trade flows between each pair of traders, is necessary if discriminatory trade policies are to be analyzed (Anania and McCalla, 1991). Models based on an a priori definition of the sets of importing and exporting regions simply assume away the possibility of switching from one side of the market to the other. Arbitrage is also constrained and/or impossible when reduced-form expressions are used. We come back to this point later. The model presented below assumes internal supply and demand functions.

2. Vertical Separation of Markets:

The notion of an equilibrium across the vertical levels of a market is well-known (Gardner, 1975). However, many agricultural trade analyses seem to ignore the existence of intermediate market levels. In the case of dairy, this issue becomes critically important because it is simply not the case that farmers trade directly with final consumers, as is implicitly assumed when intermediate market levels are ignored. Theoretical representations of derived supply or demand are misleading given the complexities of a multi-product world, and all the more so when jointness in production is also considered. Thus, the model in this paper includes a processing, or an intermediate, market level.

The processing sector plays several important roles in dairy markets. It is responsible for balancing the supply of milk with the demand for milk products. It plays an allocative role, designating the components of milk to the final product mix on the basis of where the components are most highly valued. It is often responsible for the marketing and distribution services required by the industry. And finally, the processing sector is often the point in the marketing channel at which government policies and support programs are administered.

Fully accounting for all uses of milk, while at the same time reflecting the scope of possible patterns of component usage, enables the milk supply sector to be properly represented in a dairy trade model. This seems obvious given the need to appropriately balance milk supply with total product demand. Yet surprisingly, many models fail to provide a mass balancing linkage for components between the milk supply sector and product demands. Baker (1991), for example, proposes a spatial equilibrium model to analyze the impacts of U.S.-Canada dairy trade liberalization. While the presentation of the model includes an input-output transformation constraint (p. 65, equation 2(c)), it is switched off for the actual analysis (p. 81-82). Statistical models, by design, typically ignore the problem completely. A model which includes a processing sector is more able to adequately address such issues.

3. Component Based Measurement Units:

Milk consists of several components. The various products derived from milk utilize these components in different proportions. For instance, butter is highly fat intensive and contains few other milk solids. Nonfat Dry Milk (NDM), on the other hand, contains practically no fat but is comprised primarily of proteins and carbohydrates, principally lactose. Cheese contains both fat and protein but relatively small amounts of carbohydrates. Most dairy models include a farm milk supply sector and some representation of the markets for the products derived from milk. Hence it is necessary to express raw milk and the various demanded milk products in equivalent units. The problems associated with this seemingly simple task arise from the joint-input, multiple-output structure of the dairy sector. The composition of dairy products is simply too variable and complex for any particular milk equivalent coefficient to be everywhere appropriate. Fluid milk products, for example, are not even comprised of components in the same proportions as their raw milk input, and, for that matter, raw milk supplies show substantial variation, with respect to composition, across regions.

In the past, computational complexities often necessitated the use of homogeneous measurements in multi-product models, such as the use of milkfat-based milk equivalents in dairy models (eg. OECD, 1991). A milk equivalent unit of measurement is essentially a single component measurement. Although the use of milk equivalents can make the problem more tractable, it imposes unrealistic restrictions on the process of allocating milk to the various products produced from milk. A simple example illustrates how this is so.

Consider a model which has a milk supply sector and demand markets for four products; butter, NDM, cheese, and fluid milk. Suppose, for the purpose of simplicity, that the supply of raw milk is held constant, and that milk as well as all four products are expressed in milkfat equivalent units. Now, suppose the demand for NDM increases due to some exogenous factor. In order to satisfy the increased demand for NDM, and because the supply of raw milk cannot change, the model must reallocate milk away from one of the other three products. The fat equivalent of NDM is very small; thus the model would grossly understate the amount of milk actually reallocated. In this case, one would expect a small increase in the price of NDM, a small increase in the milk equivalent used in producing NDM, and offsetting reductions in the milk equivalents allocated to other products. In reality, an increase in NDM demand will increase the price of skim solids, decrease the value of milkfat, increase the milk allocated to NDM and butter production, increase the price of skim milk relative to whole milk, and decrease the amount of milk available to make cheese.

The reverse would be true, albeit to a lesser extent, if the increased demand were of butter. Furthermore, and more importantly, many dairy products are actually jointly produced. An increase in the production of butter actually results in the availability of more nonfat milk solids which are often used in the manufacture of NDM; an increase in the production of fluid milk products most often results in surplus cream. These complex interactions are difficult, if not impossible, to represent in an aggregated product and/or milk equivalent formulation. Hence, the model in this paper expresses milk and milk products with a multiple component unit of measure.

In addition to the allocative problem outlined above, the single component formulation also presents difficulties when models are required to assign values to milk and milk products, and to the components of milk. The value of milk components, whether implied or explicit, is a signaling device allowing producers to respond to changing consumer preferences. In most developed countries, per capita consumption of fat intensive dairy products, with the exception of cheese, has waned over the last two decades. Hence, the fat component has become less valuable relative to the nonfat components. Unwanted milkfat is more often than not made into butter and then exported at a price sufficiently low that its disposal is assured. Unless a multiple component formulation is used, then quite obviously it is not possible for a model to express diverging component valuations.

A model's ability to reasonably anticipate the direction of price changes in a joint-input, multi-product market is severely restricted when a single component structure is used. Such a structure presupposes that a price increase for one product necessarily implies a price increase (or at least not a price decrease) for all other products included in the model. Such limits on price responsiveness do not exist in the marketplace and the price paths for butter and NDM in the U.S. illustrate this.

Figure 1 shows indices for the prices of butter and NDM in the U.S. for the period 1970-92. These two prices have been highly positively correlated since as far back as the 1940's. However, in the late 80's they clearly diverged. Underlying this was a change in the relative component values. The price of butter declined during the 1980's implying a decrease in the value of milkfat. At the same time though, purchases of both butter and NDM, and cheese for that matter, under the dairy price support program were being used to maintain the support price for farm milk. If the sum of the values of all components is to be held constant to maintain the support price, and the value of one component declines, then the value of the others must increase. This is precisely what can be seen in Figure 1 as the price of NDM increases while the price of butter decreases. A model employing a single component based measurement is clearly unable to anticipate and/or replicate this type of price pattern because the value of the single component could not simultaneously increase and decrease. Furthermore, a joint-input model which fails to require mass balancing constraints, while allowing such a price pattern, will almost certainly lead to technically infeasible results. In other words, a market solution requires that the quantity of milk components available in the raw milk supply must be constrained to equal the quantity used in the manufacture of products.

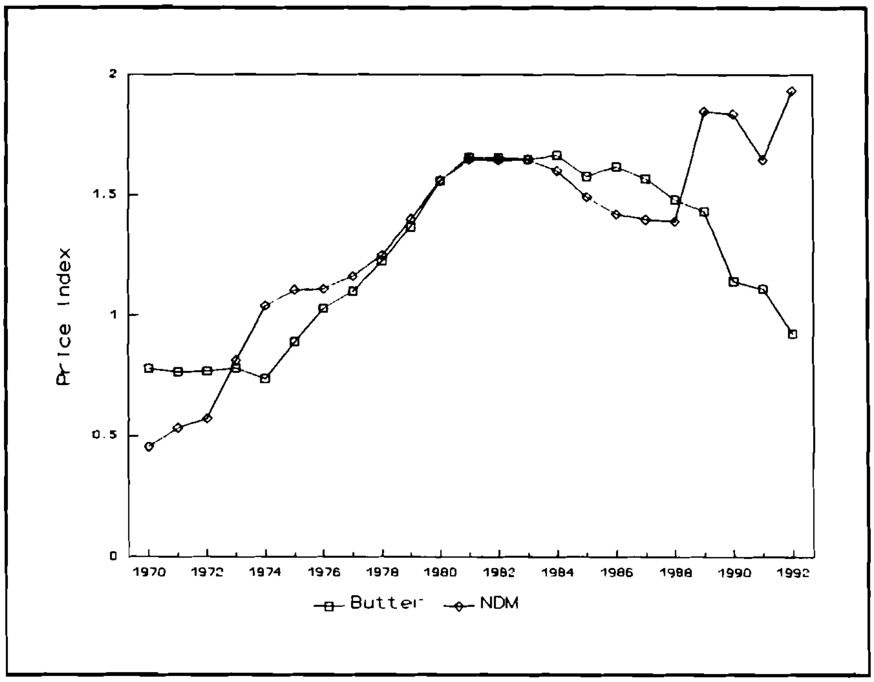

Figure 1: U.S. Butter and NDM Price Indices, 1970-92
(1970-80 average = 1)
Source: USDA, Dairy Market News

4. Product Aggregation:

The question of product specificity, or the level of aggregation, becomes one of significant importance in a dairy model. In general, three factors enter the choice of aggregation level; the need to accurately capture the policy issues being analyzed, the limits of data availability, and the need to constrain computational costs via the use of a model that readily admits manipulation.

From a practical standpoint, product aggregation leads to a number of modeling efficiencies, especially in a multi-sector model. The OECD's Ministerial Trade Mandate model, for example, represents the demand side of the dairy sector with just two products; fluid milk and manufacturing milk (OECD, 1991). Aggregation can alleviate the need to resolve the issues related to homogeneous units of measurement in multi-product models. The more it does so, however, the more it exacerbates problems caused by the fact that the demands for different products imply quite different demands for components and this can result in product prices moving in opposite directions. A high degree of aggregation would clearly mask some important empirical outcomes. The logical solution to the aggregation problem is to model the dairy sector as a multi-product sector although such a resolution may present further difficulties, for example, increased data requirements. But even when milk products are aggregated, their aggregate composition can vary markedly between countries and even between areas within a country. Aggregation of products can clearly eliminate some of the technical difficulties which arise when constructing a model, but not without implicitly forcing restrictions on the composition of the aggregated groupings. Such restrictions can lead to infeasible outcomes.

When policy instruments affect a particular product, it is prudent to separate that product in the model from other products not so affected. This applies to both domestic and trade policies. Policies targeted at one product usually have secondary effects on other products and/or on the supply of milk. This suggests a disaggregated product structure for a model intent on policy analysis. Even the fluid milk sector, a sector often excluded from dairy trade analyses, ought to be accounted for in a trade model because it is inextricably linked to manufactured product markets through milk component balancing. In developed countries, where per capita milkfat consumption is steadily declining, the fluid milk market usually supplies significant quantities of excess fat solids to the manufacturing milk sector. Disaggregation of products, to the extent allowed by the data, enables the response in and the relationship between each individual market to be better understood.

5. Policy Specificity:

The choices concerning the degree of policy specificity in a dairy trade model, like those to be made with respect to product aggregation, have important implications. A wide array of policy vehicles exist in dairy markets. Among them are price support programs, restrictions on imports such as quotas and tariffs, export subsidies, production subsidies, production quotas, deficiency payments, payment based quotas, levies and assessments, and classified pricing schemes. Frequently, for ease of modeling purposes, policies are aggregated into some kind of price wedge representation. Alternatively, models that derive solutions using net trade functions may attempt to capture policy effects in the elasticities of those functions or, in the case of price transmission equations, in the elasticities of such equations. The difficulty with these approaches is that the way in which individual policy instruments affect domestic and international markets is grossly simplified. Consequently, the information derived from such analyses is often not very specific.

Price support policies often target a specific product but are designed to have secondary, although no less important, effects on other products. The classified pricing and pooling, and the support price systems in the U.S. are cases in point. For example, changes in the administratively determined Class I (fluid) prices paid by processors may have substantial impacts on the more market oriented prices paid by the processors of manufactured products. Similarly, Class III products meeting certain specifications are purchased by the government at what amounts to a floor price. However, these government purchases are intended to provide a minimal, although less well-defined, price for non-Class III products. If eliciting the effects of specific policies is a research goal, then policies need to be explicitly modeled. A disaggregated product structure, inclusion of the processing sector, and the use of internal response functions all facilitate this task.

6. Arbitrage:

The explicit treatment of arbitraging behavior in spatial trade models is often necessary for the models to produce valid results (Anania and McCalla, 1991). Even though the potential for arbitrage arises with any trade agreement that establishes a preference for a subset of trading partners, this issue seems to be ignored in much of the trade liberalization literature. Mathematically, a non-consistent generalized transfer matrix is a necessary, but not sufficient, condition for arbitraging to occur. Anania and McCalla have already pointed-out how typical spatial models implicitly set restrictions on the potential for arbitrage. Inclusion of intermediate market levels in a trade model enables arbitrage to be dealt with relatively easily.

The typical design of a spatial trade model does not, and indeed can not, allow the direct linkage, through trade, of demand regions. Thus, it becomes impossible for region A to import from region B and then re-export, or transship, the product to region C. However, modeling the processing sector, in a dairy model for example, provides an additional market level which in turn makes such linkages mathematically feasible. Furthermore, inclusion of the processing sector and the multiple component representation used in the present model, together, allow for the possibility of reprocessing and consumption substituting forms of arbitrage. For example, country A may import NDM from one country and thereby facilitate exports of cheese to another. This can occur even with factor price equalization. This issue is particularly relevant when analyzing the impact of NAFTA on the U.S. dairy industry, given Mexico's historical reliance on dairy imports from Europe and New Zealand.

THE MODEL

In this section of the paper, a joint-input, multi-product spatial trade model is described. The model is formulated as a transshipment problem and is solved within a generalized complementarity programming framework yielding supply and demand prices and quantities, milk component values, and interregional trade flows. Conceptually, the model derives an equilibrium across spatially dispersed markets, for raw milk and the range of products produced from such milk, in which milk component quantities are balanced, and purchase and selling prices equate after taking account of generalized transfer costs. Operationally, a solution may be obtained using the method of variational inequalities (See Nagurney et al., 1993) or Rutherford's (1992) extension to the GAMS program. The model is completely described by the following set of equilibrium conditions. While Bishop et al. (1993) present the model as a traditional Takayama-Judge type of welfare maximization problem, the treatment of ad valorem tariffs in such a formulation is problematic (Nagurney et al., 1993).

Supply price of raw milk equals the market price of raw milk

$$p_i^s(s_i^*) = (>) \lambda_{1,i}^* \text{ if } s_i^* > (=) 0 \qquad (1)$$

Market price of a raw milk unit equals the sum of the value of its constituent components

$$\lambda_{1,i}^* = (>) \sum_{n=1}^{N} (\lambda_{2,i\ell n}^* * \psi_{i\ell n}) \text{ if } xs_{i\ell}^* > (=) 0 \qquad (2)$$

Market price of an available milk component equals the market price of a component used

$$\lambda^*_{2,i\ell n} = (>)\ \lambda^*_{3,i\ell' n} \text{ if } xc^*_{i\ell\ell' n} > (=)\ 0 \tag{3}$$

Sum of the values of the components used in a product plus the generalized transfer cost equals the market price of that product (in the case of product shipments)

$$[\sum_{n=1}^{N} (\lambda^*_{3,i\ell n} * \xi_{i\ell njk}) + (t_{ijk} + w_{ik} + \pi_{ijk} - \sigma_{ijk})](1 + \tau_{ijk}) = (>)\ \lambda^*_{4,jk} \text{ if } x^*_{ijk} > (=)\ 0 \tag{4}$$

Sum of the values of the components used in a product plus the generalized transfer cost equals the market price of that product (in the case of product transshipments)

$$[[\sum_{n=1}^{N} (\lambda^*_{3,i\ell n} * \xi_{i\ell njk}) + (t_{imk} + w_{ik} + \pi_{imk} - \sigma_{imk})](1 + \tau_{imk}) + (t_{mjk} + \pi_{mjk} - \sigma_{mjk})](1 + \tau_{mjk}) = (>)\ \lambda^*_{4,jk} \text{ if } xx^*_{imjk} > (=)\ 0 \tag{5}$$

Market price of a product equals the demand price of the product

$$\lambda^*_{4,jk} = (>)\ p^d_{jk}(y^*_{jk}) \text{ if } y^*_{jk} > (=)\ 0 \tag{6}$$

Quantity of raw milk supplied in each region equals the sum of all raw milk flows to plants in that region

$$s^*_i = \sum_{\ell=1}^{L} xs^*_{i\ell} \tag{7}$$

Quantity of components in raw milk flow to each plant equals the quantity of components available at that plant

$$\psi_{i\ell n} * xs^*_{i\ell} = \sum_{\ell'=1}^{L} xc^*_{i\ell\ell' n} \tag{8}$$

Quantity of components used in each plant equals the quantity of components in all products shipped and/or transshipped from that plant

$$\sum_{\ell=1}^{L} xc^{*}_{i\ell\ell'n} = \sum_{j=1}^{J}\sum_{k=1}^{K}(\xi_{i\ell njk} * x^{*}_{ijk}) + \sum_{m=1}^{M}\sum_{j=1}^{J}\sum_{k=1}^{K}(\xi_{i\ell njk} * xx^{*}_{imjk}) \tag{9}$$

The sum of product shipments and transshipments into a region equals the quantity demanded in that region

$$\sum_{i=1}^{I} x^{*}_{ijk} + \sum_{i=1}^{I}\sum_{m=1}^{M} xx^{*}_{imjk} = y^{*}_{jk} \tag{10}$$

where:
i,j,m denotes the milk supply, product demand, and transhipping regions respectively;
k denotes products;
n denotes the set of milk components;
ℓ denotes the milk processors (one for each product produced in each region i);
s_i denotes raw milk production;
$xs_{i\ell}$ denotes the flow of milk from farm to processors;
$xc_{i\ell\ell n}$ denotes the flow of components within and between processors;
x_{ijk} denotes trade flows;
xx_{imjk} denotes transhipment flows where $i \neq m \neq j$;
y_{jk} denotes the quantity of product demanded;
λ's denote shadow price variables. Precise definitions for individual λ's will become clear shortly;
t_{ijk} denotes the per unit transportation cost;
w_{ik} denotes per unit production cost;
π_{ijk} denotes the per unit import tariff;
σ_{ijk} denotes the per unit export subsidy;
τ_{ijk} denotes an ad valorem import tariff rate;
$\psi_{i\ell n}$ denotes the quantity of the n^{th} milk component in a unit of milk delivered to the ℓ^{th} processor in the i^{th} region;
$\xi_{i\ell njk}$ denotes the quantity of the n^{th} component in a unit of the k^{th} product demanded in the j^{th} region. Clearly, $\xi_{i\ell njk}$ is able to vary according to i, the region supplying the product;
p(.) denote price response functions.

Additional policies such as supply and import quotas, while not illustrated, are easily included. Stating the model in terms of the equilibrium conditions facilitates the discussion to follow. The shadow prices from the primal problem explicitly enter this formulation as variables, and may be considered "market" prices (Thore, 1991). It can be seen from equation (1) that if the i^{th} supply region produces milk, then the market price of raw milk, $\lambda^{*}_{1,i}$, must be equal to the supply price of milk, $p_i^s(s_i^*)$, in that region. If the supply price is greater than the market price, no milk will be produced and s_i^* would equal zero. Similarly, (6) indicates that if the j^{th} demand region receives product k, then its market price, $\lambda_{4,jk}^*$, must equal the demand price, $p_{jk}^d(y_{jk}^*)$. If the market price of the k^{th} product in region j is greater than the demand price, no product k will be received in region j.

Equation (2) requires that for all milk flows to all processors within a particular region, the market price of a unit of milk, $\lambda_{1,i}^*$, be equal to the sum of the market prices of its constituent components each multiplied by the total quantity of those components in that unit of milk, $\Sigma_{n=1}^N(\lambda_{2,i\ell n}^* * \psi_{i\ell n})$. Thus, the model will continue to allocate milk among that region's processors and move components between processors, until the market price of a unit of milk and the total value of that unit's components, at each processor, within each region, is equalized. If the total value of the components of a unit of milk, $\Sigma_{n=1}^N(\lambda_{2,i\ell n}^* * \psi_{i\ell n})$, does not attain the market price of milk for that region, then no milk is delivered to that processor and $xs_{i\ell}$ equals zero. Equation (4) requires that when $x_{ijk}^* > 0$, the sum of the market prices of the components used by the ℓ^{th} processor to produce a unit of the k^{th} product multiplied by the quantity of components in that product, $\Sigma_{n=1}^N(\lambda_{3,i\ell n}^* * \xi_{i\ell njk})$, plus the generalized transfer cost, be equal to the market price of that product in the j^{th} region, $\lambda_{4,jk}^*$. Equation (5) does the same thing for the transshipment flows.

Equation (3) indicates that if the n^{th} component is used by the ℓ^{th} processor, or that processor ships it to another processor, then the market price of that component in the milk-supply/product-processing market, $\lambda_{2,i\ell n}^*$, must be equal to the market price of that component in the product-processing/product-demand market, $\lambda_{3,i\ell' n}^*$. If the product-processing/product-demand market price is less than the milk-supply/product-processing market price, then none of that component will be allocated for use by that processor and $xc_{i\ell\ell' n}$ will be zero. Equations (7) through (10) are quantity flow constraints with (8) and (9) being the critical component balancing constraints.

The importance of the multi-component based formulation should now be apparent. Consider the market for NDM, say, in the j^{th} region and suppose that some exogenous factor causes a rightward shift of the inverse demand function, $p_{j,k=NDM}^d(y_{j,k=NDM}^*)$. By equation (6), ceteris paribus, the market price of NDM, $\lambda_{4,jk=NDM}^*$, must increase if $y_{j,k=NDM}^*$ is greater than zero. It's already been shown above that the sum of the value of all the components used in a unit of NDM demanded in region j, $\Sigma_{n=1}^N(\lambda_{3,i\ell n}^* * \xi_{i\ell njk})$, plus the generalized transfer cost, would also increase because it must equal $\lambda_{4,jk=NDM}^*$. Thus, when N, the number of elements in n, exceeds one, the direction of change for the n^{th} $\lambda_{3,i\ell n}^*$ for all processors shipping NDM to region j is indeterminate. We know only that the summation of the total value, $\Sigma_{n=1}^N(\lambda_{3,i\ell n}^* * \xi_{i\ell njk})$, must increase implying that individual components can change in value by different proportions and/or in opposite directions. However, if there is only one component, its market price, $\lambda_{3,i\ell n}^*$, is required to increase for all processors shipping NDM to the j^{th} region. But, this implies that following a price increase for NDM in region j, the prices for all other products will increase, or at least not decrease. Appreciating that N is indeed greater than one, it is quite clear that the result obtained for $N=1$ is unrealistic and would lead to inaccurate conclusions.

CONCLUDING REMARKS

A model has been presented which offers a number of advantages over the typical approaches to formulating spatial trade models. The major feature is the method by which a single input, consisting of more than one desired component, may be allocated to the various products which require those components. A simple example illustrated how this can lead to preferred results. This method has been used by the authors in a 12 region, 6 product model developed to analyze the recent GATT's impact on world dairy markets.

The inclusion of a processing sector enhances the realism of the model and provides the flexibility for further improvements. For instance, traded products which are actually intermediate goods could easily be further processed into final goods.

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