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# Taxes and quality: A market-level analysis<sup>†</sup>

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A conventional assumption of product homogeneity when the commodity of interest is actually heterogeneous will lead to errors in an analysis of the incidence of policies, such as taxes. In this article, an equilibrium displacement model is used to derive analytical solutions for price, quantity, and quality effects of ad valorem and per unit taxes. The results show how parameters determine the effects of tax policies on quality. The potential for tax-induced distortions in quality, and the distributive consequences of those distortions, are illustrated in a case study of the market for Australian wine.

## 1. Introduction

When conducting policy analyses, economists often use a model of an homogeneous good. However, commodities are increasingly heterogeneous and policy effects are likely to differ among various qualities of a particular commodity. An homogeneous-good model will fail to account for the different effects, and for policy-induced changes in the distribution of quality (or average quality). This article introduces a useful approach to modelling quality and applies it to formally link quality changes to tax policies, with a view to increasing our understanding of how policies influence quality at the market level.

Quality variation is incorporated in an equilibrium displacement model in which the commodity of interest is available in two qualities. This representation simplifies the nature of heterogeneity of most agricultural

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<sup>†</sup> The authors gratefully acknowledge data provided by Kym Anderson and Glyn Wittwer, and helpful comments provided by David Edgerton, Roley Piggott, Jim Vercammen, and participants at the Wine Economics Workshops in Adelaide and Napa, and the Microeconomic Theory Workshop at the University of Melbourne. Partial funding support for this work was provided by the Giannini Foundation of Agricultural Economics.

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commodities, but two qualities are sufficient to demonstrate how quality responds to various policies, while keeping the model as simple as possible. The model is used to show the theoretical price, quantity, and quality effects of ad valorem and per-unit tax policies in a closed economy. Analytical expressions for the errors from assuming product homogeneity are derived under the assumption that the high- and low-quality goods of interest comprise a weakly separable group. Solutions from this analytical model show which parameters play a key role in determining quality responses, and when these quality responses may be important components of policy outcomes.

The potential importance of quality responses to tax policies is illustrated with an application to Australian wine. When the goods and services tax (GST) was introduced in 2000, the existing ad valorem wholesale taxes on wine were to be replaced with a wholesale tax that, when combined with the 10 per cent GST, would be tax revenue neutral. Policy decisions included the size of the revenue-neutral tax, as well as how the tax should be specified – on a per unit or ad valorem basis. Based on intuition about individual behaviour, a per unit tax would be expected to increase the incentive to produce high-quality wine, relative to lower-quality cask wines. This paper derives theoretical results that show the conditions under which this argument is (and is not) valid at the market level, and presents results from empirical analysis in a stylised model of the Australian wine market. The numerical analysis demonstrates the potential empirical importance of tax-induced distortions in quality premiums, and the distributive consequences of those distortions. While the government has already made the decision to implement an ad valorem tax, the issue of distributional consequences of different types of wine taxes will surely arise again. Hence, it is useful to seek to understand the effects of the recent policy choice, with a view to providing information that may be helpful in future choices.

The contribution of this paper is more general. The fact that agricultural commodities are becoming more heterogeneous and output quality is becoming more important relative to output quantity means that we should re-think how we conduct policy analysis. Questions about how tax (or other) policies affect different qualities of a commodity differentially and about how various specifications of those policies may affect their price, quantity, quality, and welfare outcomes are certain to arise in Australia and elsewhere. This paper contributes to this more general set of questions by its development of an approach to modelling quality variation, by decomposing policy responses into scale and substitution effects about which we may have some intuition (e.g., their directions and relative sizes), and by identifying key parameters that are important determinants of quality responses to policy. Using this framework, we find that: (i) there are important differences in quality implications between ad valorem and per unit taxes, (ii) even

ad valorem taxes have quality implications once we allow for fairly general representations of consumer preferences for goods like wine, and (iii) in the case of Australian wine taxes, it is likely that the quality effects are large and the distributional implications are serious.

## 2. Previous models of quality responses to policies

Perhaps the best-known example of quality responses is the Alchian and Allen theorem, or why we 'ship out the good apples' (Borcherding and Silberberg 1978). The theorem postulates the effects of transportation costs on the relative consumption of high-quality and low-quality goods. The original example given by Alchian and Allen (1964) concerned 'good' and lower-priced 'bad' grapes, both grown in California. They noted that the cost of transporting grapes to, say, New York is the same for all shipments of grapes, regardless of their quality. From an individual consumer's perspective, prices are fixed so that the price of each quality of grapes increases by the amount of the transportation costs (per unit) for consumers in New York. Thus, good grapes become relatively cheaper for a consumer in New York, and hence, a New Yorker will consume a larger proportion of good grapes relative to a person in California who has identical preferences. While Alchian-Allen effects are usually discussed in the context of transportation costs, the hypothesised increase in the consumption share of high-quality goods could occur as a result of many other types of per-unit costs (as discussed by Umbeck 1980). The primary criteria for a per-unit cost to generate the Alchian-Allen result are that whatever gives rise to the cost does not change the good itself, and that it does not have any inherent economic value in and of itself – i.e., the cost acts just like a per-unit tax.

The reasoning behind the Alchian-Allen effect is that changes in relative prices drive changes in relative consumption. While intuitive, the effect is theoretically unambiguous only in a two-good world with no income effects (Gould and Segall 1969). Borcherding and Silberberg (1978) argued that while it is possible for the Alchian-Allen theorem to be negated with the introduction of a third good, unless the high- and low-quality products have very different consumption relationships with the third good, the standard Alchian-Allen result will hold. Notably, while most of the work in this area has focused on the effects on consumer choices, a similar analysis of a profit-maximising firm would generate the same prediction (an increase in average quality when a per unit cost is incurred). Each of these studies focused on the behaviour of an individual consumer (or producer) for whom prices are exogenous, such that the economic agent of interest absorbs the entire per unit cost. At the market level, however, such costs are shared by consumers and producers. While Alchian-Allen effects are often observed in

market-level behaviour (Bertonazzi *et al.* 1993), previous work has not shown the theoretical conditions under which such effects would be found at the market level.

Barzel (1976) used a different theoretical framework to analyse the problem at the market level in his alternative approach to taxation. The basis of his approach is that every commodity is more or less a bundle of characteristics, similar to Lancaster's (1966) 'new' approach to consumer theory. Barzel (1976) noted that an ad valorem tax applies to a commodity's entire value, so it taxes all of the commodity's characteristics. In contrast, if a per unit tax is imposed the tax statute will use a subset of characteristics to define the commodity, assuming that an exhaustive description is either impossible or very costly. As a result, the per unit tax actually taxes only those characteristics used to define the commodity. Barzel (1976) showed that a predictable outcome is that the quantity of the defining characteristics (specified in the tax statute) will decrease in response to a per unit tax, and the additional characteristics, which are not subject to the tax because they are not specified in the statute, will increase on a per unit basis – an increase in quality.

The work by Barzel (1976) provides valuable insight into quality responses at the market level. However, an explicit representation of product characteristics requires some specification of how the characteristics are combined to make units of the commodity. Work inspired by Barzel's (1976) alternative approach to taxation demonstrated the importance of this specification: changes in how characteristics were bundled could even reverse the quality effects that Barzel (1976) found.<sup>1</sup> Of particular importance is the degree of substitutability between quantity and quality, which is usually implicit in the particular functional form for the hedonic price function in such studies. Barzel's (1976) specification implied an elasticity of substitution between quantity and quality equal to one, whereas Kay and Keen (1987) implicitly specified no substitutability between quantity and quality, and found the converse effect.

Hedonic models have been used widely to represent and measure the price premiums paid for particular quality attributes of a range of goods, including wine.<sup>2</sup> These studies have given us an idea of the relative magnitudes of premiums for various quality attributes, and the role labelling or reputation plays in determining price. However, hedonic models have a number of limitations in relation to the analysis of the market-level price, quantity, and

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<sup>1</sup> See Bohanon and Van Cott (1984, 1991), Kaempfer and Brastow (1985), Kay and Keen (1987, 1991), and Cowen and Tabarrok (1995). A more detailed discussion of these papers and the related modelling issues may be found in James (2000).

<sup>2</sup> See Oczkowski (2001) for a comprehensive list of hedonic studies of wine prices, and a review of their main empirical findings.

quality impacts of policies. Hanemann (1982) pointed out that the need to specify a functional form for the hedonic price function was a drawback to using these models for applied demand analysis. In addition, the specification of the hedonic price function determines the findings regarding policy outcomes, which imposes a serious limitation on the usefulness of hedonic models in studies of the effects of policies. Further, incorporating policy instruments is difficult, as shown in Rosen (1974), where a number of simplifying assumptions had to be imposed to derive analytical results.

In the hedonic approach, quality of commodities changes continuously, with continuous changes in their characteristics. In this paper, we model discrete quality variation; different qualities are represented as distinct goods. It remains necessary to aggregate across qualities within the good being modelled, and to treat them as homogeneous. This approach is similar to the studies of the Alchian-Allen effect in that it treats different qualities of a commodity as distinct goods, and allows for conventional multi-market modelling approaches to be applied. This approach avoids the need to make assumptions about how characteristics are bundled into goods, and it has the further virtue that the specific assumptions that are imposed, such as the number of qualities included and the separability assumptions, are fairly transparent.

### **3. A multi-market approach to modelling quality**

The effects of taxes in a closed economy are modelled by specifying a multi-market equilibrium displacement model, as used, for example, by Buse (1958), Muth (1964), Perrin (1980), Alston (1986, 1991), Piggott (1992), and Alston *et al.* (1995). The commodity of interest is assumed to be available in two qualities, low and high, with some substitution between the two qualities, in both demand and supply. Changes in the distribution of consumption and production between low and high qualities are interpreted as changes in the average quality of the general commodity type. In this section we present the two-commodity model, which we use to derive theoretical results showing the effects of ad valorem and per unit taxes on average quality and quality premiums. In section 4 we present results from a numerical simulation using a three-commodity extension of the model (described in the appendix) applied to the Australian wine industry.

#### **3.1 Structure of the model and the general solution for two qualities**

Because the two qualities are related in consumption and production, the quantity demanded and supplied of each quality depends on its own price

and the price of the other quality. Other demand and supply shifters, such as income, demographic variables, and production technology, are treated as fixed in the analysis, and are therefore not included as arguments. The demand and supply relationships can be written in general form as:

$$C_L = C_L(P_L^D, P_H^D) \quad (1)$$

$$C_H = C_H(P_L^D, P_H^D) \quad (2)$$

$$Q_L = Q_L(P_L^S, P_H^S) \quad (3)$$

$$Q_H = Q_H(P_L^S, P_H^S), \quad (4)$$

where  $C$  denotes quantities consumed,  $Q$  denotes quantities produced, and  $P$  denotes prices. Subscripts  $L$  and  $H$  denote quantities and prices in the low- and high-quality markets, and superscripts  $D$  and  $S$  denote prices along the demand and supply curves, respectively. The market-clearing conditions are:

$$C_L = Q_L \quad (5)$$

$$C_H = Q_H \quad (6)$$

$$P_L^D = P_L^S(1 + t_L) \quad (7)$$

$$P_H^D = P_H^S(1 + t_H), \quad (8)$$

where  $t_L$  and  $t_H$  are proportional taxes in the low- and high-quality markets, and are initially equal to zero. Increasing either  $t_i$  term creates a wedge between the consumer price  $P_i^D$  and the producer price  $P_i^S$  in that market.

Totally differentiating equations (1) through (8) and transforming the results yields:

$$d \ln C_L = \eta_{LL} d \ln P_L^D + \eta_{LH} d \ln P_H^D \quad (9)$$

$$d \ln C_H = \eta_{HL} d \ln P_L^D + \eta_{HH} d \ln P_H^D \quad (10)$$

$$d \ln Q_L = \epsilon_{LL} d \ln P_L^S + \epsilon_{LH} d \ln P_H^S \quad (11)$$

$$d \ln Q_H = \epsilon_{HL} d \ln P_L^S + \epsilon_{HH} d \ln P_H^S \quad (12)$$

$$d \ln C_L = d \ln Q_L \quad (13)$$

$$d \ln C_H = d \ln Q_H \quad (14)$$

$$d \ln P_L^D = d \ln P_L^S + t_L \quad (15)$$

$$d \ln P_H^D = d \ln P_H^S + t_H, \quad (16)$$

where  $d \ln X \approx dX/X$  denotes a proportional change in the variable  $X$ . For instance,  $d \ln Q_L \approx dQ_L/Q_L$  is the proportional change in the quantity produced of the low-quality product. Coefficients on the  $d \ln P_i$  terms are elasticities:  $\eta_{ij}$  is the elasticity of demand for quality  $i$  with respect to the price of quality  $j$ , and  $\epsilon_{ij}$  is the elasticity of supply of quality  $i$  with respect to the

price of quality  $j$ .<sup>3</sup> Equations (9) through (16) implicitly define the eight endogenous variables (the proportional changes in quantities demanded and supplied and the proportional changes in consumer and producer prices in each of the two markets) as functions of the two exogenous tax rates,  $t_L$  and  $t_H$ .

Imposing the market-clearing conditions in equations (13) and (14), the proportional quantity changes may be represented by the  $d \ln Q_i$  terms alone, and the remaining six equations may be specified in matrix notation as:

$$\begin{bmatrix} 1 & 0 & -\eta_{LL} & -\eta_{LH} & 0 & 0 \\ 0 & 1 & -\eta_{HL} & -\eta_{HH} & 0 & 0 \\ 1 & 0 & 0 & 0 & -\epsilon_{LL} & -\epsilon_{LH} \\ 0 & 1 & 0 & 0 & -\epsilon_{HL} & -\epsilon_{HH} \\ 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} d \ln Q_L \\ d \ln Q_H \\ d \ln P_L^D \\ d \ln P_H^D \\ d \ln P_L^S \\ d \ln P_H^S \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ t_L \\ t_H \end{bmatrix}, \quad (17)$$

or  $\mathbf{A}\mathbf{y} = \mathbf{x}$ . Inverting the coefficient matrix,  $\mathbf{A}$ , and pre-multiplying both sides of equation (17) by the inverse,  $\mathbf{A}^{-1}$ , yields an explicit expression of the endogenous variables as functions of the elasticities and exogenous tax rates, i.e.,  $\mathbf{y} = \mathbf{A}^{-1}\mathbf{x}$ . The solution for the endogenous variables is:

$$\begin{bmatrix} d \ln Q_L \\ d \ln Q_H \\ d \ln P_L^D \\ d \ln P_H^D \\ d \ln P_L^S \\ d \ln P_H^S \end{bmatrix} = \frac{1}{D} \begin{bmatrix} \eta_{LL}(\epsilon_{LL}\epsilon_{HH} - \epsilon_{LH}\epsilon_{HL}) - \epsilon_{LL}(\eta_{LL}\eta_{HH} - \eta_{LH}\eta_{HL}) \\ \eta_{HL}(\epsilon_{LL}\epsilon_{HH} - \epsilon_{LH}\epsilon_{HL}) - \epsilon_{HL}(\eta_{LL}\eta_{HH} - \eta_{LH}\eta_{HL}) \\ \epsilon_{LL}(\epsilon_{HH} - \eta_{HH}) + \epsilon_{HL}(\eta_{LH} - \epsilon_{LH}) \\ \eta_{HL}\epsilon_{LL} - \eta_{LL}\epsilon_{HL} \\ \eta_{LL}(\epsilon_{HH} - \eta_{HH}) + \eta_{HL}(\eta_{LH} - \epsilon_{LH}) \\ \eta_{HL}\epsilon_{LL} - \eta_{LL}\epsilon_{HL} \end{bmatrix} t_L + \frac{1}{D} \begin{bmatrix} \eta_{LH}(\epsilon_{LL}\epsilon_{HH} - \epsilon_{LH}\epsilon_{HL}) - \epsilon_{LH}(\eta_{LL}\eta_{HH} - \eta_{LH}\eta_{HL}) \\ \eta_{HH}(\epsilon_{LL}\epsilon_{HH} - \epsilon_{LH}\epsilon_{HL}) - \epsilon_{HH}(\eta_{LL}\eta_{HH} - \eta_{LH}\eta_{HL}) \\ \eta_{LH}\epsilon_{HH} - \eta_{HH}\epsilon_{LH} \\ \epsilon_{HH}(\epsilon_{LL} - \eta_{LL}) + \epsilon_{LH}(\eta_{HL} - \epsilon_{HL}) \\ \eta_{LH}\epsilon_{HH} - \eta_{HH}\epsilon_{LH} \\ \eta_{HH}(\epsilon_{LL} - \eta_{LL}) + \eta_{LH}(\eta_{HL} - \epsilon_{HL}) \end{bmatrix} t_H \quad (18)$$

where:

$$D = (\epsilon_{LL} - \eta_{LL})(\epsilon_{HH} - \eta_{HH}) - (\eta_{LH} - \epsilon_{LH})(\eta_{HL} - \epsilon_{HL}).$$

<sup>3</sup> Equations (15) and (16) are derived by totally differentiating the market-clearing condition for the price of each quality and dividing each side of the expression by the consumer price, noting that  $P_i^D = P_i^S(1 + t_i)$ ,  $d(1 + t_i) = dt_i$ , and that  $dt_i = t_i$  (because the initial tax rate is zero).



The sign and size of the proportional change for each endogenous variable in equation (18) is determined by the signs, and in some cases the magnitudes, of the supply and demand elasticities, most of which cannot be determined in a strictly theoretical approach. In addition, it is not clear how to link these general results from this two-market specification of a differentiated good with those from a single-market representation of an homogeneous good. This link is made by assuming that the low- and high-quality goods comprise a weakly separable group, and that all other goods comprise another weakly separable group and may be aggregated into a composite commodity.

### 3.2 Elasticity decompositions under the assumption of weak separability

A group of goods is weakly separable if the marginal rates of substitution among commodities in that group are independent of the individual prices and quantities of goods not in the group. Imposing this assumption allows for the expression of the elasticities of demand and supply for low- and high-quality varieties as functions of fundamental demand and supply parameters. This approach is often associated with Armington (1969), because he used a special case of the weak separability assumption in his model of demand for goods distinguished by their country of origin. Muth (1966) provided an earlier justification for invoking the assumption, using a model of household production.

If two groups of goods are weakly separable and if the price indexes used for the two groups of commodities are invariant to income, then the consumer's budgeting process may be represented in two stages. In the first stage, total expenditure is allocated between the two groups, depending on the group price indexes. In the second stage, the expenditure for each group is allocated among the individual commodities in that group. Many of the results derived below are discussed in terms of the first- and second-stage effects of price changes, although the sufficient conditions for two-stage budgeting are stronger than those necessary for the elasticity decompositions used here (see Edgerton 1997 for derivations and discussion).<sup>4</sup>

Under the assumption that low- and high-quality varieties comprise a weakly separable group, the elasticities of demand for the individual

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<sup>4</sup> Carpentier and Guyomard (2001) note that these elasticity decompositions violate symmetry of the Slutsky substitution matrix, except in the special case of homothetic separability. While we expect Slutsky symmetry to hold at the level of the individual consumer or firm, it need not hold at the market level (nor should we expect it to do so, unless we adhere strictly to the representative consumer hypothesis).

commodities with respect to the individual prices can be expressed as:

$$\eta_{LL} = s_L \gamma_L \eta - s_H \sigma \quad (19)$$

$$\eta_{LH} = s_H (\gamma_L \eta + \sigma) \quad (20)$$

$$\eta_{HL} = s_L (\gamma_H \eta + \sigma) \quad (21)$$

$$\eta_{HH} = s_H \gamma_H \eta - s_L \sigma, \quad (22)$$

where  $s_i$  is the budget share of quality  $i$  (i.e.,  $s_i = (P_i Q_i)/PQ$ , where the absence of subscripts denotes aggregate price or quantity). First-stage effects are represented by  $\eta$ , the overall elasticity of demand, or the elasticity of demand for the aggregate quantity with respect to the aggregate price ( $\eta < 0$ ). There are two second-stage effects. The second-stage substitution effect is determined by  $\sigma$ , the elasticity of substitution between low- and high-quality commodities ( $\sigma > 0$ ). The second-stage expansion effects are determined by the  $\gamma_i$  terms, where  $\gamma_i$  is the elasticity of demand for quality  $i$  with respect to group expenditure ( $\gamma_i > 0$ ).

Elasticities of supply of the individual commodities with respect to individual prices can be expressed similarly, as:

$$\epsilon_{LL} = s_L \rho_L \epsilon - s_H \tau \quad (23)$$

$$\epsilon_{LH} = s_H (\rho_L \epsilon + \tau) \quad (24)$$

$$\epsilon_{HL} = s_L (\rho_H \epsilon + \tau) \quad (25)$$

$$\epsilon_{HH} = s_H \rho_H \epsilon - s_L \tau, \quad (26)$$

where  $\epsilon$  is the overall elasticity of supply with respect to the group price index, and represents the first-stage effect ( $\epsilon > 0$ ). The second-stage substitution effect is determined by  $\tau$ , the elasticity of transformation between low-quality and high-quality varieties in the production process ( $\tau < 0$ ), and  $\rho_i$  is an expansion elasticity, and determines the second-stage expansion effect ( $\rho_i > 0$ ).

One advantage of using these decompositions instead of the general elasticities is that the number of parameters is reduced, and all of the parameters are of known sign. The eight elasticities are replaced by seven underlying parameters:  $\eta, \epsilon, \sigma, \tau, \gamma_H, \rho_H$ , and  $s_H$ , noting that  $s_L + s_H = 1$ ,  $s_L \gamma_L + s_H \gamma_H = 1$ , and  $s_L \rho_L + s_H \rho_H = 1$ . Another important advantage is that the elasticity decompositions nest two special cases. The first is the case of homothetic separability, used in Armington (1969) trade models.<sup>5</sup> In addition to the assumptions imposed by weak separability, homothetic separability restricts the elasticities of demand with respect to changes in group

<sup>5</sup> These models have been used extensively in models of trade in agricultural commodities. An early and notable example is the analysis of the international wheat market by Grennes *et al.* (1978). Also, see Johnson (1971), Alston (1986), Alston *et al.* (1990), MacLaren (1990), Davis and Kruse (1993), and Sumner *et al.* (1994) for discussions of the Armington model, specific studies, and related econometric issues.

expenditure, and the expansion elasticities of supply, to be equal to one for both qualities (i.e.,  $\gamma_L = \gamma_H = \rho_L = \rho_H = 1$ ) such that the quantities consumed of the different qualities change by the same proportion, unless their relative prices change; similarly for quantities produced. These additional restrictions eliminate the second-stage expansion effects from the elasticity terms.

The second special case is that of product homogeneity. If, in addition to eliminating the second-stage expansion effects as in the case of homothetic separability, the second-stage substitution effects are also eliminated from the elasticity terms, then only the first-stage effects remain. These first-stage effects represent changes in aggregate prices and quantities that would be predicted from a model of an homogeneous good, without regard for how the composition of the aggregate might change. Estimated policy outcomes under the assumption of product homogeneity are thus found by evaluating the more general price and quantity effects under the assumptions that the expenditure and expansion elasticities of the two qualities are equal to one (as in the Armington case), and that there is no substitution between qualities (i.e.,  $\sigma = \tau = 0$ ). This special case will be used as a basis for determining the errors in the estimated policy effects caused by ignoring quality responses to those policies.

### 3.3 Price and quantity effects of ad valorem and per unit taxes

After substituting the expressions in equations (19) through (26) into equation (18), the solution for changes in prices and quantities in response to the introduction of taxes can be written as:

$$\begin{aligned}
 \begin{bmatrix} d\ln Q_L \\ d\ln Q_H \\ d\ln P_L^D \\ d\ln P_H^D \\ d\ln P_L^S \\ d\ln P_H^S \end{bmatrix} &= \underbrace{\begin{bmatrix} \eta\epsilon \\ \eta\epsilon \\ \epsilon \\ \epsilon \\ \eta \\ \eta \end{bmatrix} \frac{(s_L t_L + s_H t_H)}{\epsilon - \eta}}_{\text{first-stage effects}} + \underbrace{\begin{bmatrix} s_H \sigma \tau \\ -s_L \sigma \tau \\ -s_H \tau \\ s_L \tau \\ -s_H \sigma \\ s_L \sigma \end{bmatrix} \frac{(t_L - t_H)}{\sigma - \tau}}_{\text{second-stage substitution effects}} \\
 &+ \underbrace{\begin{bmatrix} s_H [\sigma(\rho_L - \rho_H) + \tau(\gamma_H - \gamma_L)] \\ -s_L [\sigma(\rho_L - \rho_H) + \tau(\gamma_H - \gamma_L)] \\ -\frac{s_H}{s_L}(\gamma_H - \rho_H) \\ (\gamma_H - \rho_H) \\ -\frac{s_H}{s_L}(\gamma_H - \rho_H) \\ (\gamma_H - \rho_H) \end{bmatrix} \frac{\eta\epsilon(s_L t_L + s_H t_H)}{(\epsilon - \eta)(\sigma - \tau)}}_{\text{second-stage expansion effects}} \quad (27)
 \end{aligned}$$

The first element of the solution, the first-stage effects, shows the price and quantity effects that would be predicted from a single-market model of an homogeneous good, in which the tax rate is a value-share weighted sum of the individual tax rates. The second element of the solution represents the substitution effects from the second stage of the budgeting process, in which expenditure is allocated between the different qualities of the good. The first two terms combined comprise the solutions under the Armington assumption of homothetic separability. Finally, the third element in the solution represents terms that adjust the Armington solutions for the different expansion effects in the low- and high-quality markets. The solution in equation (27) holds for both ad valorem and per-unit tax policies, but the specific results vary because the different types of taxes have different implications for the tax-rate parameters,  $t_L$  and  $t_H$ . The signs of the first- and second-stage effects (when known) for the ad valorem and per-unit tax policies are summarised in table 1.

Consider first a uniform ad valorem tax policy, where a 100*t* per cent tax is imposed on both qualities. In this case,  $t_L = t_H = t$ , which means the second-stage substitution effects vanish, and the second-stage expansion effects alone represent the error from assuming product homogeneity. When the expansion elasticities of the two qualities are equal, the second-stage expansion effects also vanish, and there is no error from using a model of an homogeneous good to estimate the effects of an ad valorem tax. However, when the two qualities comprise a weakly separable but not a homothetically separable group, the second-stage expansion effects adjust the predicted effects from the single-market model. The signs of the second-stage expansion effects depend on the relative sizes of the expansion elasticities, which are not known for this general case. However, for typical goods, such as wine, it seems likely that the expenditure elasticity of demand

**Table 1** Directions of effects on endogenous variables from ad valorem and per-unit taxes

Variable	First-stage effects (both policies)	Second-stage substitution effects		Second-stage expansion effects (both policies)	
		Ad valorem tax	Per-unit tax	$\gamma_H > \rho_H$	$\gamma_H < \rho_H$
$Q_L$	—	0	—	?	?
$Q_H$	—	0	+	?	?
$P_L^D$	+	0	+	+	—
$P_H^D$	+	0	—	—	+
$P_L^S$	—	0	—	+	—
$P_H^S$	—	0	+	—	+

Note: “?” indicates that the direction of the effect is unknown.

for the high-quality variety will be larger than that of the low-quality variety, so that  $\gamma_H > 1$  and  $\gamma_L < 1$  (a higher income elasticity of demand for higher quality), while the relationship on the supply side seems likely to be the converse,  $\rho_H < 1$  and  $\rho_L > 1$  (it is relatively difficult to expand production of higher quality). These conditions imply that  $\gamma_H > \rho_H$ , which implies that the price effects will be more pronounced in the market where the expansion effect is smaller: the expansion effects accentuate the consumer price increase for the low-quality good and the producer price decrease for the high-quality good.

Next, consider the effects of a uniform tax of  $T$  per unit. The initial quality-specific prices are used to convert the tax to proportional terms, and the two tax rates are specified as  $t_L = T/P_L$  and  $t_H = T/P_H$ , where  $P_L$  and  $P_H$  are the initial prices, and  $P_i^D = P_i^S$  at the initial equilibrium, so the superscripts may be dropped. The algebra for the effects of this tax policy can be condensed somewhat by defining  $P$  as the average unit value of the total quantity at the initial equilibrium, i.e.,  $P = (P_L Q_L + P_H Q_H)/(Q_L + Q_H)$ , which implies that  $s_L t_L + s_H t_H = T/P$  and  $t_L - t_H = (P_H - P_L)T/P_L P_H$ . Further, prices of individual qualities relative to the average unit value are defined as  $\tilde{P}_i = P_i/P$ . Using these definitions, the effects of the per-unit tax policy can be written as:

$$\begin{aligned}
 \begin{bmatrix} d \ln Q_L \\ d \ln Q_H \\ d \ln P_L^D \\ d \ln P_H^D \\ d \ln P_L^S \\ d \ln P_H^S \end{bmatrix} &= \underbrace{\begin{bmatrix} \eta \epsilon \\ \eta \epsilon \\ \epsilon \\ \epsilon \\ \eta \\ \eta \end{bmatrix} \frac{1}{(\epsilon - \eta)} \frac{T}{P}}_{\text{first-stage effects}} + \underbrace{\begin{bmatrix} s_H \sigma \tau \\ -s_L \sigma \tau \\ -s_H \tau \\ s_L \tau \\ -s_H \sigma \\ s_L \sigma \end{bmatrix} \frac{1}{(\sigma - \tau)} \frac{(\tilde{P}_H - \tilde{P}_L) T}{\tilde{P}_L \tilde{P}_H P}}_{\text{second-stage substitution effects}} \\
 &+ \underbrace{\begin{bmatrix} s_H [\sigma(\rho_L - \rho_H) + \tau(\gamma_H - \gamma_L)] \\ -s_L [\sigma(\rho_L - \rho_H) + \tau(\gamma_H - \gamma_L)] \\ -\frac{s_H}{s_L} (\gamma_H - \rho_H) \\ (\gamma_H - \rho_H) \\ -\frac{s_H}{s_L} (\gamma_H - \rho_H) \\ (\gamma_H - \rho_H) \end{bmatrix} \frac{\eta \epsilon}{(\epsilon - \eta)(\sigma - \tau)} \frac{T}{P}}_{\text{second-stage expansion effects}} \quad (28)
 \end{aligned}$$

The first-stage effects in equation (28) are equivalent to the changes in prices and quantities that would be estimated using a single-market model for a per-unit tax of  $T$  and an initial price of  $P$ . In contrast to the ad valorem

tax, the second-stage substitution effects are no longer equal to zero, because the proportional tax rates differ. By definition, the price of the high-quality good is larger than that of the low-quality good, and all of the other elements in the second set of terms are of known sign. As shown in the third column of table 1, the second-stage substitution effects mean that the quantity reduction in the low-quality market is greater and the quantity reduction in the high-quality market is smaller than those predicted in a single-market model of a per unit tax applied to an homogeneous good. Finally, the terms representing the second-stage expansion effect take the same signs as they did for the ad valorem tax, for the same relative parameter values.

### 3.4 Quality effects of ad valorem and per unit tax policies

The use of a single-market model to represent an aggregate of various qualities implicitly assumes that the policy impacts will be the same for all qualities, so that the average quality of the aggregate is constant. Defining average quality as  $Q_H/Q_L$ , the proportional change in average quality resulting from the tax policy is measured as the difference between the proportional quantity changes,  $d \ln Q_H - d \ln Q_L$ .<sup>6</sup> Similarly, consumer and producer price premiums for high quality can be expressed as the ratio of the price of the high-quality product to the price of the low-quality product (i.e.,  $P_H^D/P_L^D$  and  $P_H^S/P_L^S$ , respectively). Thus, the proportional changes in the price premiums will equal the difference between the proportional price changes for the individual goods,  $d \ln P_H^i - d \ln P_L^i$ , for  $i = D, S$ . The degree to which these measures differ from zero is a further indication of the errors caused by assuming product homogeneity.

The proportional changes in average quality and the price premiums for either tax policy are given by:

$$d \ln \left( \frac{Q_H}{Q_L} \right) = -\frac{\sigma\tau}{\sigma - \tau} (t_L - t_H) - \frac{\eta\epsilon[\sigma(\rho_L - \rho_H) + \tau(\gamma_H - \gamma_L)]}{(\epsilon - \eta)(\sigma - \tau)} (s_L t_L + s_H t_H) \quad (29)$$

$$d \ln \left( \frac{P_H^D}{P_L^D} \right) = \frac{\tau}{\sigma - \tau} (t_L - t_H) + \frac{\eta\epsilon(\gamma_H - \rho_H)}{s_L(\epsilon - \eta)(\sigma - \tau)} (s_L t_L + s_H t_H) \quad (30)$$

$$d \ln \left( \frac{P_H^S}{P_L^S} \right) = \frac{\sigma}{\sigma - \tau} (t_L - t_H) + \frac{\eta\epsilon(\gamma_H - \rho_H)}{s_L(\epsilon - \eta)(\sigma - \tau)} (s_L t_L + s_H t_H). \quad (31)$$

<sup>6</sup> Average quality could be defined in a number of ways. The definition used here is particularly convenient.

The directions and magnitudes of each of these changes hinge on the second-stage effects in equation (27). The first terms in equations (29) through (31) measure the differences between the substitution effects in the low- and high-quality markets, and the second terms measure the differences between the expansion effects in the two markets.

For a uniform ad valorem tax, there are no substitution effects in either market, so the first terms in equations (29) through (31) are eliminated. When the Armington assumptions are appropriate, the second terms equal zero as well; average quality and both the consumer and the producer quality premiums remain constant, as would be implicitly predicted from a single-market model of an homogeneous product. However, these changes will not equal zero under the assumption of (nonhomothetic) weak separability, even for the case of an uniform ad valorem tax. This is a somewhat unexpected result: even an ad valorem tax can distort relative prices and the incentives to produce and consume quality when the more general (and more realistic) supply and demand relationships are incorporated in the analysis. These quality effects are summarised in the upper panel of table 2.

For the case of a weakly separable group of goods, when  $\gamma_H > \rho_H$  (where the expansion effects for the higher-quality good on the demand side exceed those on the supply side), the quality premiums for consumers and producers both decrease as a result of the tax policy (column three of table 2). This would create an incentive for consumers to increase the quality of their consumption, and for producers to decrease the quality of their production. The direction of the effect on average quality, which must be the same for both consumers and producers, is indeterminant. It depends on the relative expansion effects between the low- and high-quality commodities for

**Table 2** Quality effects of ad valorem and per-unit taxes

Quality variable	Homogeneous product	Homothetic separability	Weak separability	
			$\gamma_H > \rho_H$	$\gamma_H < \rho_H$
<i>Ad valorem tax</i>				
$Q_H/Q_L$	0	0	?	?
$P_H^D/P_L^D$	0	0	—	+
$P_H^S/P_L^S$	0	0	—	+
<i>Per-unit tax</i>				
$Q_H/Q_L$	0	+	?	?
$P_H^D/P_L^D$	0	—	—	?
$P_H^S/P_L^S$	0	+	?	++

Note: “++” (“—”) indicates a larger increase (decrease) relative to “+” (“—”), “?” indicates that the direction of the effect is unknown.

consumers and for producers, and on the relative differences in the expansion effects between consumers and producers for each quality.

For a per-unit tax, the quality effects are:

$$d\ln\left(\frac{Q_H}{Q_L}\right) = -\frac{\sigma\tau}{\sigma-\tau}\frac{(\tilde{P}_H-\tilde{P}_L)T}{\tilde{P}_L\tilde{P}_H P} - \frac{\eta\epsilon[\sigma(\rho_L-\rho_H)+\tau(\gamma_H-\gamma_L)]}{(\epsilon-\eta)(\sigma-\tau)}\frac{T}{P} \quad (32)$$

$$d\ln\left(\frac{P_H^D}{P_L^D}\right) = \frac{\tau}{\sigma-\tau}\frac{(\tilde{P}_H-\tilde{P}_L)T}{\tilde{P}_L\tilde{P}_H P} + \frac{\eta\epsilon(\gamma_H-\rho_H)}{s_L(\epsilon-\eta)(\sigma-\tau)}\frac{T}{P} \quad (33)$$

$$d\ln\left(\frac{P_H^S}{P_L^S}\right) = \frac{\sigma}{\sigma-\tau}\frac{(\tilde{P}_H-\tilde{P}_L)T}{\tilde{P}_L\tilde{P}_H P} + \frac{\eta\epsilon(\gamma_H-\rho_H)}{s_L(\epsilon-\eta)(\sigma-\tau)}\frac{T}{P}. \quad (34)$$

The directions of these quality effects are shown in the lower panel of table 2. Unlike the ad valorem tax, under the assumption of homothetic separability, a per-unit tax has unambiguous quality effects unless both  $\sigma$  and  $\tau$  are equal to zero. As long as there are some substitution possibilities in both consumption and production, and  $\sigma$  and  $\tau$  take on their normal signs (i.e.,  $\sigma > 0$ ,  $\tau < 0$ ), the proportional quantity reduction in the high-quality market will be smaller than that in the low-quality market, and average quality will increase as a result of the tax. The consumer's quality premium decreases, and the producer's quality premium increases. These effects are intuitive. If average quality increases as a result of the tax, consumers require an incentive to consume higher quality: a lower quality premium. Similarly, producers require an incentive to produce higher quality: a higher quality premium.

The results for the case of homothetic separability offer proof of the Alchian-Allen theorem at the market level. However, this 'proof' must be qualified, as it relies on the assumption of homothetic separability; results are ambiguous in a more general setting. As noted by Gould and Segall (1969) and Borchering and Silberberg (1978) for the individual consumer problem, the Alchian-Allen effect is unambiguous only in a two-good world with no income effects. Similarly, at the market level, the assumption of homotheticity in effect restricts the roles that income and third goods can play, as they enter the conditional demand functions through the expenditure term. The quality effects in the more-general setting, when the assumption of homotheticity is relaxed, are shown in the third column of table 2 for the case where  $\gamma_H > \rho_H$ , and in the fourth column for the less-likely case where  $\gamma_H < \rho_H$ . Under the more-general assumption of (nonhomothetic) weak separability, only one of the three quality effects can be determined unambiguously.

#### 4. Taxes on Australian wine

Prior to June 2000, Australian wine was subject to a 41 per cent wholesale tax. With the introduction of the 10 per cent GST, the previous tax policy



was also reformed such that the combined set of changes would be tax-revenue neutral. There was some debate about whether the new wholesale tax should be specified on a per-unit or ad valorem basis (Wittwer and Anderson 1998 and 2002; Berger and Anderson 1999). In discussions of the relative merits of the different tax policies, the effects on producers and consumers of different qualities of wine were often mentioned. Alchian-Allen type arguments were made, suggesting that costs from a per-unit tax would be relatively higher for producers and consumers of low-quality wine, and thus would favour high-quality producers (Berger and Anderson 1999). The analysis presented here serves to evaluate whether such statements, based on intuition about individual consumer and producer optimisation, are true at the market level, and to quantify the extent to which price, quantity, and welfare impacts differ among the markets for lower-quality (cask) and premium wines.

In our model of the market for Australian wine, wine is aggregated into three groups: cask, premium white, and premium red.<sup>7</sup> This representation aggregates wine that is certainly heterogeneous into each of the three composites, but this aggregation is justified on at least three grounds. First, it is more accurate than treating all wine as a single aggregate. Second, it allows us to check our intuition about how the responses to taxes may differ among a few classes of wine, whereas including more detail would tend to make the results difficult to decipher. Third, it is a level of disaggregation for which price and quantity data are available.

The particular representation of the wine market is stylized, and it abstracts from a number of details that might be important if the analysis were to be used for policy prescription rather than illustration. The model does not distinguish between wholesale and retail markets for wine, which would be important for purposes of prescribing policy (i.e., finding the revenue-neutral pre- and post-GST tax rates, as done by Wittwer and Anderson 1998 and 2002) because pre-2000 taxes were all imposed at the wholesale level, while the new GST is a retail tax. The wholesale-retail distinction could be incorporated by specifying a mark-up pricing rule, but that would be an unhelpful complication for the present purpose. A further simplification in the model is that it does not explicitly incorporate international trade. Export demand is included implicitly by specifying the total demand for Australian wine in each quality category. A more realistic model might have differential tax rates applied to exports versus domestic consumption. This would mean that a tax policy change might give rise to

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<sup>7</sup> How to disaggregate Australian wine is arbitrary, given the absence of any empirical evidence for particular separability assumptions. Ultimately, the decision is governed by the availability of data. We have used the same categories as Wittwer and Anderson (2001), from whom we obtained the data.

differences in the mixes of quality produced, consumed, and traded. While foreign wine imported into Australia is omitted, imported wine accounts for a small share of wine consumed domestically, and any errors caused by omitting wine imports are expected to be small.

The general structure of the three-quality model is similar to that of the two-quality model described in section 3.1, and the full details are provided in the appendix. Demand and supply of each quality depend on all of the quality-specific prices. Taxes are represented as differences between consumer and producer prices, and the proportional changes in the quality-specific prices and quantities are expressed as functions of the exogenous, quality-specific tax rates. As for the two-quality case, an assumption of weak separability is imposed to simplify the results. In particular, cask and premium wine are assumed to comprise a weakly separable group, and then red and white premium wines are assumed to be weakly separable from cask wine. This separability structure adds an additional stage to the budgeting process, in which the expenditure on production and consumption of premium wine is allocated between red and white. In this additional stage, total response to price changes includes both substitution and expansion effects, much like the second-stage effects described earlier. The model is structured such that the two-quality model is a special case in which there are no substitution possibilities between red and white premium wines, and their expansion effects are equal. The extension of the model from two to three qualities illustrates how this type of approach to modelling quality can be applied to cases with more than three qualities, or with different separability structures.

#### **4.1 Price, quantity, and quality effects of alternative wine taxes**

The model of the three qualities of wine described earlier (and in the appendix) is specified using price and quantity data for 1999, shown in table 3. While several studies of the demand for wine in different countries have been conducted (see Larivière *et al.* 2000 for a review), very few have focused on demand for Australian wine. Abdalla and Duffus (1988) estimated the demand for cask and premium wines, and found own-price elasticities of  $-1.50$  and  $-0.02$ , respectively (information obtained from Shepherd, O'Donnell and Abdalla 1999). Clements and Johnson (1983) estimated the aggregate demand for wine, and found an own-price elasticity of  $-0.43$ . Wittwer and Anderson (2002) note, however, that the parameter estimates of these studies may no longer apply, because of the subsequent expansion of the industry, particularly in the production and consumption of higher-quality wines. Estimates of supply elasticities are even more rare in the literature.

**Table 3** Prices and quantities of Australian wine, 1999

Type of wine	Retail price	Quantity <sup>a</sup>	Value share
	<i>A\$ per litre</i>	<i>millions of litres</i>	<i>proportion</i>
Cask	3.71	270.6	0.22
Premium <sup>b</sup>	12.28	294.3	0.78
White	11.46	148.2	0.37 (0.47 of premium)
Red	13.11	146.1	0.41 (0.53 of premium)

Source: Wittwer and Anderson (2001).

Notes: <sup>a</sup> Quantities refer to quantities of Australian wine consumed in 1999, and include domestic and foreign consumption.

<sup>b</sup> Price for the premium aggregate is defined as the quantity-share weighted sum of the prices for premium white and red wines. Quantity for the premium aggregate is defined as the simple sum of the quantities of premium white and red.

The approach taken here is a common one in studies of commodity markets and policies. A model is specified and parameterised based on consumption, production, and prices in a particular year (or a representative year), combined with a set of elasticities. In most cases, few if any of the elasticities are estimated directly within a policy study, and usually it is not possible (or sensible) simply to take elasticities from the literature. Instead, relevant elasticities are ‘guestimated’ using a combination of results in the literature, economic theory, and intuition. The problem of limited availability of specific elasticity estimates for parameterising a policy model becomes more serious as we move in the direction of using less aggregative models. That this is so can be seen in the studies by Wittwer and Anderson (1998, 2001 and 2002) that model the Australian wine market.<sup>8</sup> While these studies use the same three categories of wine, our model is structured differently such that the specific elasticities refer to different concepts (e.g., Hicksian versus Marshallian demand elasticities, domestic versus total demand, and possibly different lengths of run); hence, the elasticities are not directly comparable. Even though they are not directly comparable without significant effort, we believe that the set of elasticities used here is broadly consistent with that used by Wittwer and Anderson (1998, 2001 and 2002), and neither set is clearly better or worse than the other.<sup>9</sup>

Some economists appear to believe that econometrically estimated elasticities are intrinsically more accurate and otherwise superior to ‘calibrated’ or ‘guestimated’ elasticities of the sort typically used in applied policy

<sup>8</sup> See also Zhao, Anderson and Wittwer (2002).

<sup>9</sup> Wittwer and Anderson (1998, 2001 and 2002) also had to use a combination of economic theory and their own intuition to derive the elasticities they used.

analysis, but econometric estimates have their own set of deficiencies – such as implausible magnitudes, wrong signs, and inconsistencies with theory. A virtue of the introspective approach to estimating elasticities is that at least these drawbacks can be avoided. In his discussion of ways in which *noise* affects markets and colors our investigations of them, Fischer Black (1986) said ‘Sometimes I wonder if we can draw any conclusions at all from the results of regression studies ... [The] slopes of demand and supply curves are so hard to estimate that they are essentially unobservable. Introspection seems as good a method as any in trying to estimate them’ (pp. 535–536). Even if we wished to estimate the supply and demand elasticities econometrically, sufficient data are simply not available for the quality categories of interest. We have to rely on a few estimates from the literature and introspection.

While there is little empirical evidence to support any particular elasticity values, we do have some intuition about the relative values of certain parameters; and once some parameter values are specified, others are determined by theoretical restrictions. The values used for the underlying supply and demand parameters are shown in table 4, and the own- and cross-price elasticities of demand and supply implied by the underlying parameter values and the value shares are shown in table 5. Demand is assumed to be more elastic than supply, in terms of the overall elasticities. The remaining parameter values were chosen to ensure that the different qualities of wine were substitutes in demand and in supply, while imposing the adding-up conditions on the expansion elasticities. The elasticity of substitution in demand between cask and premium wines and that between red and white premium wines were both set equal to 3. On the supply side, cask and

**Table 4** Parameter values used for model of Australian wine taxes

Effect represented by parameter	Demand parameters		Supply parameters	
	Symbol	Value	Symbol	Value
Stage 1 effects				
Overall elasticity	$\eta$	-1.5	$\epsilon$	1.0
Stage 2 effects				
Substitution effect	$\sigma$	3.0	$\tau$	-2.0
Scale effect – cask	$\gamma_L$	0.3	$\rho_L$	1.7
Scale effect – premium	$\gamma_H$	1.2	$\rho_H$	0.8
Stage 3 effects				
Substitution effect	$\sigma_{WR}$	3.0	$\tau_{WR}$	-1.5
Scale effect – white premium	$\gamma_W$	0.8	$\rho_W$	1.0
Scale effect – red premium	$\gamma_R$	1.2	$\rho_R$	1.0

**Table 5** Demand and supply elasticities used in the model of Australian wine

	Elasticity with respect to the price of:		
	Cask	Premium white	Premium red
Elasticity of demand for:			
Cask	-2.44	0.95	1.07
Premium white	0.20	-2.34	0.74
Premium red	0.31	0.25	-2.72
Elasticity of supply of:			
Cask	1.94	-0.10	-0.12
Premium white	-0.26	1.29	-0.23
Premium red	-0.26	-0.21	1.27

premium wines were assumed to be more easily transformed from one to the other (by altering production practices) than are red and white premium wines (because such a transformation would require grafting or replanting a vineyard).

Elasticities of demand with respect to expenditure are assumed to be larger for higher-priced categories of wine. The expenditure elasticity of premium wine was set equal to 1.2, and the cask-wine elasticity was recovered using the adding-up condition:  $s_L\gamma_L + s_H\gamma_H = 1$ . Similarly, the elasticity of demand for premium red wine with respect to expenditure on premium wine was set equal to 1.2, and the expenditure elasticity for premium white wine was recovered using a similarly defined adding-up condition. On the supply side, expansion of high-quality wine was assumed to be less elastic (owing to more limiting specialised factors, such as land and management), and so the expansion elasticity for cask wine is larger than that of premium wine. The expansion elasticity of premium wine was set equal to 0.8, and the corresponding elasticity for cask wines was recovered using the adding-up condition. The expansion elasticities of supply are assumed to be equal to one for both red and white premium wines.

The own- and cross-price elasticities derived from these values, shown in table 5, have signs (and magnitudes) that are consistent with our intuition. Own-price elasticities of demand are all negative, and are larger for the premium wines than for cask wines. Cross-price elasticities are all positive, indicating that the wines are gross substitutes, and demand for cask wine is more responsive to changes in the prices of premium wines than the converse. Own-price elasticities of supply are all positive, and larger for cask wine than for premium wines. Cross-price elasticities of supply are all negative, indicating that the wines are substitutes. The analytical solution in the appendix can be used to see how the results might differ for a different set of parameter values.

**Table 6** Price and quantity effects of ad valorem and per litre wine taxes, decomposed into first-, second-, and third-stage effects

Variable	First-stage effect	Second-stage effects		Third-stage effects		Total effect
		Subst.	Expan.	Subst.	Expan.	
<i>percentage change</i>						
<i>10 per cent Ad Valorem Tax</i>						
$Q_L$	-6.00	0.00	-0.86	0.00	0.00	-6.86
$Q_W$	-6.00	0.00	0.24	0.00	0.43	-5.33
$Q_R$	-6.00	0.00	0.24	0.00	-0.38	-6.14
$P_L^D$	4.00	0.00	1.73	0.00	0.00	5.73
$P_W^D$	4.00	0.00	-0.48	0.00	0.29	3.81
$P_R^D$	4.00	0.00	-0.48	0.00	-0.26	3.26
$P_L^S$	-6.00	0.00	1.73	0.00	0.00	-4.27
$P_W^S$	-6.00	0.00	-0.48	0.00	0.29	-6.19
$P_R^S$	-6.00	0.00	-0.48	0.00	-0.26	-6.74
<i>percentage change</i>						
<i>\$0.91 Per Litre Tax</i>						
$Q_L$	-6.70	-16.12	-0.96	0.00	0.00	-23.78
$Q_W$	-6.70	4.48	0.27	-0.53	0.15	-2.34
$Q_R$	-6.70	4.48	0.27	0.47	-0.13	-1.61
$P_L^D$	4.46	5.37	1.93	0.00	0.00	11.77
$P_W^D$	4.46	-1.49	-0.54	0.18	0.10	2.71
$P_R^D$	4.46	-1.49	-0.54	-0.16	-0.09	2.19
$P_L^S$	-6.70	-8.06	1.93	0.00	0.00	-12.83
$P_W^S$	-6.70	2.24	-0.54	-0.35	0.10	-5.25
$P_R^S$	-6.70	2.24	-0.54	0.31	-0.09	-4.77

The upper panel of table 6 shows the percentage change in the quantity, consumer price, and producer price of each quality of wine for a 10 per cent ad valorem tax. The values of the first-, second-, and third-stage effects are shown, as well as the net effects (in the last column). It is clear that, even when there are no substitution effects, the expansion effects cause the changes in price and quantity to differ among the various qualities. Using a model of an homogeneous good as an approximation, or relying on intuition from individual consumer or producer problems, one would expect each of the price and quantity effects to be the same for all three qualities, which is clearly not the case.

Effects of a per-litre tax that would generate the same tax revenue as the 10 per cent ad valorem tax are shown in the lower panel of table 6. The tax of \$0.91 per litre was found by equating tax revenue from the 10 per cent ad valorem tax to that of the per-litre tax, using the model of three qualities of wine. The differences among the effects in the three markets are revealed in the last column, and are much more pronounced than they were for the

**Table 7** Quality effects of ad valorem and per litre wine taxes

Quality measure <sup>a</sup>	10 per cent ad valorem	\$0.91 per litre
	<i>percentage change</i>	
Average quality		
White	1.54	21.44
Red	0.72	22.17
Consumer quality premium		
White	-1.92	-9.05
Red	-2.46	-9.57
Producer quality premium		
White	-1.92	7.58
Red	-2.46	8.06

<sup>a</sup> Each quality effect is calculated assuming that proportional changes in quantities and prices are equal for white and red cask wines.

ad valorem tax. The proportional decrease in the quantity of cask wine is roughly ten times those of the premium wines. Differences in the proportional changes in consumer and producer prices are also large, with the consumer price of cask wine increasing by about 12 per cent, and the consumer prices of white and red premium wines increasing by approximately 2 and 3 per cent, respectively. The differences in the price and quantity outcomes among the three markets are driven by the second- and third-stage effects, particularly the second-stage substitution effects.

The quality effects of the two taxes are summarised in table 7, which shows the percentage change in average quality and the consumer and producer quality premiums, for white and red wines. These quality effects are calculated assuming that proportional effects on white and red cask wines are equal, and using expressions similar to equations (29) through (34). For the ad valorem tax, average qualities of white and red wines increase somewhat, but the increases in average quality are much more pronounced for the revenue-neutral per-unit tax (both increasing by over 20 per cent). For both tax policies and both red and white wines, consumer quality premiums decrease, although these decreases are larger for the per-unit tax policy. Finally, producer quality premiums for both red and white wines decrease for the ad valorem tax, but increase for the per unit tax.

Table 8 shows the tax revenue collected in each market from each tax policy, in millions of dollars and as a percentage of the total tax revenue collected. For the ad valorem tax, the tax burden is relatively higher in the premium wine markets than in the cask-wine market, with only 22 per cent of the tax revenue collected on cask wine sales. In contrast, for the per-litre tax, the tax burden is relatively higher in the cask-wine market, with

**Table 8** Tax revenue collected from ad valorem and per-litre wine taxes

Market	10 per cent ad valorem tax		\$0.91 per-litre tax	
	<i>\$ million</i>	<i>share of tax revenue (per cent)</i>	<i>\$ million</i>	<i>share of tax revenue (per cent)</i>
Cask	98.86	21.90	188.19	41.69
White	166.91	36.98	132.06	29.26
Red	185.64	41.12	131.16	29.06
Total	451.41	100.00	451.41	100.00

42 per cent of the tax revenue collected. Thus, although the two tax policies generate the same amount of tax revenue, the incidence of the costs differs substantially between the two policies.

### 5. Concluding remarks

While the assumption of product homogeneity is convenient, it is important to recognise that it may not always be appropriate. The analytical results presented here indicate that tax policies may induce distortions in the quality mix of units sold and in quality premiums. Under the assumption of homothetic separability, effects of taxes are very similar to the Alchian-Allen effects discussed in the literature in the context of individual consumer behaviour: an ad valorem tax leaves average quality and quality premiums unchanged, while a per unit tax increases average quality, decreases the consumer quality premium, and increases the producer quality premium.

In contrast, when we allow for more general demand and supply conditions, market-level effects of taxes are not entirely consistent with our expectations, based on intuition about individual behaviour from Alchian and Allen (1964). When the qualities of interest comprise a (nonhomothetic) weakly separable group, even an ad valorem tax can distort quality. Because the quality effects are second-order effects, they will not always be important. When the quality effects are small, a single-market model for an aggregate good may reasonably approximate the actual policy effects in the markets for heterogeneous products. However, the larger are those quality effects, the less accurate will be the results from a model of an homogeneous good. Relatively large quality effects mean that the errors from using an model of an homogeneous good to estimate policy impacts would be large as well.

Results from the analytical model demonstrate which parameters are important in determining the direction and magnitude of the errors caused by



ignoring quality variation. In general, the errors from assuming product homogeneity increase as the degree of substitutability between qualities (in consumption or production) increases, as the difference in prices of high and low qualities increases, and as the size of the tax increases. Differences in the expansion effects between qualities also influence the errors from incorrectly assuming product homogeneity. An empirical analysis using price and quantity data and reasonable parameter values to represent the Australian wine market indicated that per unit taxes would have substantial effects on average quality and quality premiums, which differed significantly from those of ad valorem taxes. Hence, ad valorem and per unit tax policies that are equivalent in terms of tax revenue collected have very different implications for how the costs of the policies are distributed among producers and consumers of different qualities.

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## Appendix

### A three-quality model of the Australian wine market

The three-quality model is structured as above for two qualities, where demand for and supply of each quality of wine are functions of the prices of all three qualities, or:

$$C_i = C_i(P_L^D, P_W^D, P_R^D) \quad (35)$$

$$Q_i = Q_i(P_L^S, P_W^S, P_R^S), \quad (36)$$

for  $i = L, W$ , and  $R$ , where subscript  $L$  refers to cask wine (including both white and red), and subscripts  $W$  and  $R$  denote premium white and premium red wines, respectively. Equilibrium conditions are specified for each of the three markets, as:

$$C_i = Q_i \quad (37)$$

$$P_i^D = P_i^S(1 + t_i). \quad (38)$$

These supply and demand functions and equilibrium conditions can be totally differentiated and transformed to logarithmic differential form, so that the nine endogenous variables (quantity, consumer price, and producer

price for each of the three qualities) are implicitly defined as functions of parameters and the three tax rates. In matrix notation:

$$\begin{bmatrix} 1 & 0 & 0 & -\eta_{LL} & -\eta_{LW} & -\eta_{LR} & 0 & 0 & 0 \\ 0 & 1 & 0 & -\eta_{WL} & -\eta_{WW} & -\eta_{WR} & 0 & 0 & 0 \\ 0 & 0 & 1 & -\eta_{RL} & -\eta_{RW} & -\eta_{RR} & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & -\epsilon_{LL} & -\epsilon_{LW} & -\epsilon_{LR} \\ 0 & 1 & 0 & 0 & 0 & 0 & -\epsilon_{WL} & -\epsilon_{WW} & -\epsilon_{WR} \\ 0 & 0 & 1 & 0 & 0 & 0 & -\epsilon_{RL} & -\epsilon_{RW} & -\epsilon_{RR} \\ 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} d\ln Q_L \\ d\ln Q_W \\ d\ln Q_R \\ d\ln P_L^D \\ d\ln P_W^D \\ d\ln P_R^D \\ d\ln P_L^S \\ d\ln P_W^S \\ d\ln P_R^S \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ t_L \\ t_W \\ t_R \end{bmatrix}. \quad (39)$$

The general solution for the endogenous variables can be found by pre-multiplying each side of equation (39) by the inverse of the left-hand side matrix.

Cask and premium wines are assumed to comprise a weakly separable group and, within that group, red and white premium wines are assumed to comprise another weakly separable group (i.e., the marginal rate of substitution between red and white premium wines is independent of the consumption of cask wine). This additional separability assumption adds a third stage to the budgeting process, in which expenditure on premium wine is allocated between red and white wines. Accordingly, the elasticities of demand and supply for each of the premium wines include third-stage substitution and expansion effects in addition to the first- and second-stage effects.

The elasticities of demand for the different qualities of wine with respect to individual price changes are:

$$\eta_{LL} = s_L \gamma_L \eta - s_H \sigma \quad (40)$$

$$\eta_{Lj} = s_j (\gamma_L \eta + \sigma) \quad (41)$$

$$\eta_{jL} = s_L \gamma_j (\gamma_H \eta + \sigma) \quad (42)$$

$$\eta_{ij} = s_{(H)j} [\gamma_i (s_H \gamma_H \eta - s_L \sigma) + \sigma_{WR}] \quad \text{for } i \neq j \quad (43)$$

$$\eta_{ii} = s_{(H)i} \gamma_i (s_H \gamma_H \eta - s_L \sigma) - (1 - s_{(H)i}) \sigma_{WR} \quad (44)$$

for  $i, j = W, R$ , where  $s_{(H)W}$  is the expenditure on white premium wine as a share of total expenditure on premium wine (i.e.,  $s_{(H)W} = (P_W Q_W) / (P_H Q_H)$ ), and  $s_{(H)R}$  is defined similarly. In addition,  $\gamma_i$  for  $i = W, R$  is the elasticity of demand for the  $i$ th wine with respect to expenditure on premium wine, and  $\sigma_{WR} > 0$  is the elasticity of substitution between white and red premium wines. All other parameters are defined as in the two-quality case. The

elasticities of supply for the individual qualities with respect to changes in price are:

$$\epsilon_{LL} = s_L \rho_L \epsilon - s_H \tau \quad (45)$$

$$\epsilon_{Lj} = s_j (\rho_L \epsilon + \tau) \quad (46)$$

$$\epsilon_{jL} = s_L \rho_j (\rho_H \epsilon + \tau) \quad (47)$$

$$\epsilon_{ij} = s_{(H)i} [\rho_i (s_H \rho_H \epsilon - s_L \tau) + \tau_{WR}] \quad \text{for } i \neq j \quad (48)$$

$$\epsilon_{ii} = s_{(H)i} \rho_i (s_H \rho_H \epsilon - s_L \tau) - (1 - s_{(H)i}) \tau_{WR} \quad (49)$$

for  $i, j = W, R$ , where  $\rho_i$  represents the third-stage expansion effect in either the premium white or premium red market, and  $\tau_{WR} < 0$  is the elasticity of transformation between white and red premium wines.

The analytical solution to the system of equations (39) obtained using these elasticity decompositions is given in equation (50). Comparing the two- and three-quality solutions (i.e., equations (27) and (50)), the nested nature of the results is revealed. The first- and second-stage effects from the two-good solution are modified slightly for the three-good case, and third-stage substitution and expansion effects are added. Notably, there are no third-stage effects for the quantity and producer and consumer prices of the low-quality good, as the third-stage effects allocate expenditure between white and red premium wines, just as the second-stage effects allocate expenditure between cask and premium wines.

$$\begin{aligned} \begin{bmatrix} d \ln Q_L \\ d \ln Q_W \\ d \ln Q_R \\ d \ln P_L^D \\ d \ln P_W^D \\ d \ln P_R^D \\ d \ln P_L^S \\ d \ln P_W^S \\ d \ln P_R^S \end{bmatrix} &= \underbrace{\begin{bmatrix} \eta \epsilon \\ \eta \epsilon \\ \eta \epsilon \\ \epsilon \\ \epsilon \\ \epsilon \\ \eta \\ \eta \\ \eta \end{bmatrix} \frac{(s_L t_L + s_W t_W + s_R t_R)}{(\epsilon - \eta)}}_{\text{first-stage effects}} \\ &+ \underbrace{\begin{bmatrix} s_H \sigma \tau \\ -s_L \sigma \tau \\ -s_L \sigma \tau \\ -s_H \tau \\ s_L \tau \\ s_L \tau \\ -s_H \sigma \\ s_L \sigma \\ s_L \sigma \end{bmatrix} \frac{(t_L - (s_{(H)W} t_W + s_{(H)R} t_R))}{(\sigma - \tau)}}_{\text{second-stage substitution effects}} \end{aligned}$$

$$\begin{aligned}
& + \underbrace{\begin{bmatrix} s_H[\sigma(\rho_L - \rho_H) + \tau(\gamma_H - \gamma_L)] \\ -s_L[\sigma(\rho_L - \rho_H) + \tau(\gamma_H - \gamma_L)] \\ -s_L[\sigma(\rho_L - \rho_H) + \tau(\gamma_H - \gamma_L)] \\ -\frac{s_H}{s_L}(\gamma_H - \rho_H) \\ (\gamma_H - \rho_H) \\ (\gamma_H - \rho_H) \\ -\frac{s_H}{s_L}(\gamma_H - \rho_H) \\ (\gamma_H - \rho_H) \\ (\gamma_H - \rho_H) \end{bmatrix}}_{\text{second-stage expansion effects}} \frac{\eta\epsilon(s_L t_L + s_W t_W + s_R t_R)}{(\epsilon - \eta)(\sigma - \tau)} \\
& + \underbrace{\begin{bmatrix} 0 \\ s_{(H)R}\sigma_{WR}\tau_{WR} \\ -s_{(H)W}\sigma_{WR}\tau_{WR} \\ 0 \\ -s_{(H)R}\tau_{WR} \\ s_{(H)W}\tau_{WR} \\ 0 \\ -s_{(H)R}\sigma_{WR} \\ s_{(H)W}\sigma_{WR} \end{bmatrix}}_{\text{third-stage substitution effects}} \frac{(t_W - t_R)}{(\sigma_{WR} - \tau_{WR})} \\
& + \underbrace{\begin{bmatrix} 0 \\ \{s_{(H)R}[\sigma_{WR}(\rho_W - \rho_R) \\ -\tau_{WR}(\gamma_W - \gamma_R)]\} \\ \{-s_{(H)W}[\sigma_{WR}(\rho_W - \rho_R) \\ -\tau_{WR}(\gamma_W - \gamma_R)]\} \\ 0 \\ \frac{s_R}{s_W}(\rho_R - \gamma_R) \\ -(\rho_R - \gamma_R) \\ 0 \\ \frac{s_R}{s_W}(\rho_R - \gamma_R) \\ -(\rho_R - \gamma_R) \end{bmatrix}}_{\text{third-stage expansion effects}} \frac{\left[ \begin{array}{l} \eta\epsilon(\sigma\rho_H - \tau\gamma_H)(s_L t_L + s_W t_W + s_R t_R) \\ -s_L\sigma\tau(\epsilon - \eta)\{t_L - (s_{(H)W}t_W + s_{(H)R}t_R)\} \end{array} \right]}{(\epsilon - \eta)(\sigma - \tau)(\sigma_{WR} - \tau_{WR})}
\end{aligned}
\tag{50}$$