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Optimal management of annual ryegrass (*Lolium rigidum* Gaud.) in phase rotations in the Western Australian Wheatbelt*

Graeme J. Doole†

Lucerne (*Medicago sativa* L.) helps to prevent soil salinisation in the Western Australian Wheatbelt by reducing recharge to saline water tables. There is broad consensus, though, that it is not sufficiently profitable to motivate producers to plant it at the intensity at which considerable off-site benefits would be conferred. This paper employs a multiple-phase optimal control model to explore the value of this perennial pasture for the management of herbicide-resistant annual ryegrass (*Lolium rigidum* Gaud.) in a crop–pasture rotation, given the difficulty of observing this value in practice. The availability of selective herbicides for efficient weed control is found to determine whether or not it is profitable to adopt lucerne pasture under optimal management. Herbicide resistance requires producers to employ costly, non-selective treatments for in-crop weed control. Thus, it motivates the adoption of perennial pasture in which cost-effective forms of control can be implemented. Moreover, this result is robust to feasible changes in the current economic environment.

Key words: annual ryegrass, herbicide resistance, multiple-phase optimal control.

1. Introduction

Low livestock profitability, wide-scale adoption of reduced cultivation and the introduction of selective herbicides have encouraged large-scale planting of grain crops across the Wheatbelt of Western Australia over the last 30 years. However, frequent application of selective herbicides in prolonged crop sequences has promoted herbicide resistance among a number of important crop weeds in this region (Monjardino *et al.* 2004; Owen *et al.* 2007). Herbicide resistance motivates the use of more expensive in-crop forms of weed control by directly reducing the efficacy of efficient selective herbicides. In addition, crop yield declines if insufficient weed control is attained. Continued reliance on shallow-rooted annual crops and pastures in rotations has also encouraged the onset of dryland soil salinisation by allowing a higher proportion of

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† Dr Graeme Doole (email: gdoole@cyllene.uwa.edu.au), Research Fellow, Centre for Environmental Economics and Policy, School of Agricultural and Resource Economics, Faculty of Natural and Agricultural Sciences, University of Western Australia, Crawley, Western Australia, Australia.

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rainfall to recharge saline water tables, relative to that occurring under native vegetation (Pannell and Ewing 2006; Ward 2006).

Inclusion of a pasture phase in a rotation may aid the management of herbicide resistance by killing resistant weeds or by delaying its onset through the diversification of weed treatments in an integrated weed management (IWM) strategy (Powles *et al.* 1997). Weed control methods available during a pasture phase are as follows: grazing, hay and silage production, killing of an actively growing sward using non-selective herbicides (brown-manuring) or cultivation (green-manuring), application of a non-selective herbicide to flowering weeds to sterilise seed (pasture-topping), and the use of a non-selective herbicide to kill weeds before seed-set (winter-cleaning) (Doole and Pannell 2008).

In addition, incorporating phases of perennial pasture species, such as lucerne (*Medicago sativa* L.), between long cropping sequences in 'phase rotations' has been identified as the 'most promising system' (Cocks 2001, p. 137) for reducing groundwater recharge on broadacre farms in the Western Australian Wheatbelt. Lucerne is intolerant of the waterlogging that typically accompanies salinisation, and its roots are more sensitive to soil salts than barley, the most salt-tolerant conventional crop in Western Australia (Cocks 2001). The value of lucerne for salinity mitigation in a phase sequence therefore lies in its capacity to grow in response to summer rainfall and to create a buffer of dry soil that may intercept recharge occurring underneath annual crops that follow in the rotation. There is considerable evidence that, in most areas of the Western Australian Wheatbelt, phase farming is not profitable enough to prompt sufficient adoption to yield considerable off-site benefits through recharge reduction (Pannell and Ewing 2006). However, the profitability of lucerne–cereal rotations has not been assessed in light of the value of perennial pasture for the management of herbicide-resistant weeds. This value may be considerable, but it is difficult to gauge without bioeconomic analysis.

Previous studies conducted in relation to weed management in Wheatbelt cropping systems have focused solely on the crop phase (e.g. Gorddard *et al.* 1995) or have been limited to the study of annual pastures (e.g. Monjardino *et al.* 2004; Pannell *et al.* 2004). The objective of this paper is consequently to identify the value of lucerne for weed management in phase rotations in the Wheatbelt of Western Australia. The dearth of previous analysis motivates a conceptual approach to modelling, whereby general principles are drawn from a concise framework. This investigation is the first application of the regime-programming algorithm of Doole (2007) and aids the interpretation of a more detailed model in related work (Doole and Pannell 2008).

Section 2 introduces a multiple-phase control system. Section 3 describes a two-stage control model of a stylised crop-rotation problem. It also outlines parameter estimation and presents the different scenarios investigated in the study. Section 4 reports on and discusses the results of the analysis. Key findings are summarised in Section 5. The regime-programming method used to solve the multiple-phase control problem defined in the paper is described in an Appendix.

2. Regime programming

The analysis employs multiple-phase optimal control. This concerns the optimisation of a system incorporating a number of alternate regimes of which only one may be active at each point in time. Each of these distinct stages may possess a different objective functional and/or set of state equations to the other regimes. A multiple-phase optimal control solution algorithm determines the optimal way to control each individual regime and selects which phase is active at each point in time. The duration of each stage is controlled through a switching time that activates the next stage in a specified sequence. A fixed sequence is adopted due to the geometric increase in system size as the number of potential phases is enlarged in a free-sequencing problem. The inclusion of transition costs (those that occur at the point of switching) also complicates gradient formulation if the regime sequence is endogenously determined.

This study employs the regime-programming algorithm of Doole (2007) given the insufficiency of alternative solution methods. Gradient-based methods are difficult to apply as the state and adjoint equations in a multiplephase control problem are piecewise defined and the objective functional has discontinuous derivatives with respect to the control variables in each stage. The method of Mueller *et al.* (1999) also requires state and adjoint equations that have explicit solutions, thereby limiting its application to more abstract problems than that considered here.

A general multiple-phase system is assumed to incorporate an m-dimensional state vector $x(t) = \{x^1(t), x^2(t), \ldots, x^m(t)\}$ of continuous functions. The state variables are assumed fixed at the initial time and are denoted x_0. The state variables free at the terminal time are denoted x_n^i for $i = [1, 2, \ldots, d]$. Terminal state variables x_n^i, for $i = [d+1, \ldots, m]$, are fixed.

A multiple-phase system is defined as $\Xi = \{T, K, \Omega\}$, where T is a set of discrete controls known as switching times that dictate the termination of one phase and the start of the next and $K = \{k_1, k_2, \ldots, k_n\}$ is a finite, fixed and exogenously determined sequence of discrete (integer) states over the closed interval $j = [1, 2, \ldots, n]$ that indexes individual continuous dynamical systems $\Omega = \{\Omega_k\}_{k \in K}$. Here, $\Omega_k = \{X, f_k, U_j\}$, where X is a continuous state space with $X \in R^m$, f_k is the vector of state equations for each stage k and U_j is a set of admissible controls for each j in $j = [1, 2, \ldots, n]$. Each set U_j lies in R^{v_j}, where v_j is the dimensionality of the control vector for phase j.

A control input for a multiple-phase switching system Ξ consists of a set of vectors $\chi_\Xi = \{u, t\}$, where $u = \{u_1, u_2, \ldots, u_n\}$ is a collection of control functions defined for each stage in sequence K and $t = \{t_1, t_2, \ldots, t_{n-1}, t_n\}$ is a sequence of real numbers denoting the terminal time t_n and the switching times $t_1, t_2, \ldots, t_{n-1}$. Switching time t_j denotes the time at which stage k_j is terminated and the stage k_{j+1} becomes active. It follows that regime k_j is active over the interval $[t_{j-1^+}, t_{j^-}]$, where t_{j-1^+} is the moment just after t_{j-1} and t_{j^-} is the moment just before t_j. It may be optimal for two consecutive switching times, such as t_j and t_{j+1}, to coalesce (that is, $t_j = t_{j+1}$), in which case, movement from k_j to k_{j+2} will occur without the activation of k_{j+1}.

A trajectory (Γ) for a multiple-phase switching system Ξ and control sequence χ_Ξ is admissible over the interval $t = [t_0, t_1, \ldots, t_{n-1}, t_n]$ if it satisfies the continuous dynamics $\dot{x} = f_j(x(t), u_j(t), t)$, for $[t_{j-1^+}, t_{j^-}]$ and $j \in J$, for a predefined switching sequence $K = \{k_1, k_2, \ldots, k_n\}$.

A multiple-phase optimal control problem may subsequently be defined as shown below.

Problem 1. For a multiple-phase system Ξ identify an admissible trajectory that maximises the objective function:

$$J = e^{-rt_n}G(x(t_n), t_n) - \sum_{j=1}^{n-1} e^{-rt_j}C_j(x(t_j)) + \sum_{j=1}^{n}\left[\int_{t_{j-1^+}}^{t_{j^-}} [e^{-rt}F_j(x(t), u_j(t))]dt\right], \quad (1)$$

subject to,

$$\dot{x} = f_j(x(t), u_j(t), t) \text{ for } [t_{j-1^+}, t_{j^-}] \quad \text{and} \\ j = [1, 2, \ldots, n] \text{ given } K = \{k_1, k_2, \ldots, k_n\}, \quad (2)$$

$$t_j \text{ free for } j = [1, 2, \ldots, n], \quad (3)$$

$$x(t_j) \text{ free for } j = [1, 2, \ldots, n-1], \quad (4)$$

$$x_0 \text{ fixed}, \quad (5)$$

$$x_n^i(t_n) \text{ free for } i = [1, \ldots, d], \quad \text{and} \quad (6)$$

$$x_n^i(t_n) \text{ fixed for } i = [d+1, \ldots, m]. \quad (7)$$

Here r is a discount rate, $G(x(t_n), t_n)$ is a terminal reward function, $C_j(x(t_j))$ is a switching cost function for the jth phase and $F_j(x(t), u_j(t))$ is a single-valued reward function on $X^m \times U^v$ for the jth phase. Functions $G(\cdot)$, $C(\cdot)$ and $F(\cdot)$ are all real-valued functions that are twice continuously differentiable in the relevant arguments. The terminal value function G is defined for $x_n^i(t_n)$, where $i = [1, \ldots, d]$.

The necessary conditions required for the optimisation of Problem 1 and the regime-programming algorithm of Doole (2007) are presented in the Appendix.

3. Model

3.1 Model description

It is assumed that a producer wishes to determine the optimal management of a single field on a good sandplain soil in the Central Wheatbelt of Western

Australia. The goal of the producer is to determine the optimal management of two phases in a steady-state field rotation. The initial phase involves lucerne pasture, and the second phase involves wheat (*Triticum aestivum* L.) cropping. The model explicitly studies the management of weed control inputs and phase length across the steady-state cycle. Time notation is omitted where not required in the following discussion for notational simplicity.

It is assumed that:

1. Wheat only competes with a single weed (annual ryegrass).
2. This weed may develop resistance to a single Group A selective herbicide (diclofop methyl) (Owen *et al.* 2007).

Assumption 1 is appropriate when given the large influence that this weed has on crops in the study region and its recognition as the 'world's most severe example of herbicide resistance' (Pannell *et al.* 2004, p. 306). Assumption 2 helps to sharpen the focus on the optimal management of herbicide use.

There is one switching time (t_1) and the terminal time (t_2) is free. The free terminal time determines the length of the second phase in the rotation.

Two state variables represent the annual ryegrass seed population (Gorddard *et al.* 1995). The seed population per square metre susceptible to the selective herbicide following germination is denoted by $x^s(t)$. The seed population per square metre resistant to the Group A herbicide following germination is denoted by $x^h(t)$.

Four standard control variables representing weed management are represented, two in each regime. An additional control variable that determines phase length in the terminal regime is described in Section 3.3. Subscripts denote whether a control variable is used in the lucerne pasture (α) or crop (β) phase.

The control variables that influence the weed populations in the lucerne phase are:

1. the sheep stocking rate (u_α^1); and
2. non-selective herbicide (glyphosate) application in a standard winter-cleaning operation, measured in kilograms of active ingredient per hectare (u_α^2).

The non-selective herbicide application is representative of the effective non-selective strategies available during a pasture phase that do not contribute to herbicide resistance. Other methods (e.g. hay and silage production) are too costly for regular implementation. Resistance to non-selective herbicides (e.g. glyphosate) is not represented in the model because it is rare in Western Australia and may be prevented through appropriate management (Weersink *et al.* 2005).

In comparison, the control variables that influence the weed populations in the crop phase are (Gorddard *et al.* 1995):

1. selective herbicide (diclofop methyl) application, measured in kilograms of active ingredient per hectare (u_β^1); and
2. the percentage of the total annual ryegrass population killed by non-selective weed treatments used during a crop phase (u_β^2).

The last control variable (u_β^2) is a composite variable representing weed mortality occurring through non-selective herbicide application and the use of cultural treatments.

3.1.1 State transition equations

The general growth equation for each seed population in the absence of weed control is:

$$\dot{x} = x(-g - (1-g)M_{\text{seed}} + g(1 - M_{\text{plant}})R), \tag{8}$$

where g is the germination rate, M_{seed} is the natural mortality rate of ungerminated seeds, M_{plant} is the natural mortality rate of germinated seeds (i.e. plants) and R is the mean number of seeds produced by an individual plant. This equation is manipulated for each phase to reflect differences in control mechanisms.

The state-transition equation for each seed population in the lucerne phase is:

$$\dot{x}_\alpha^\varphi = x_\alpha^\varphi\left(v_1 + v_2\left(1 - \frac{u_\alpha^1}{u_\alpha^1 d + l}\right)(1 - e^{-\vartheta u_\alpha^2})R\right), \tag{9}$$

where $\varphi = \{s, h\}$, $v_1 = -g - (1-g)M_{\text{seed}}$, $v_2 = g(1 - M_{\text{plant}})$, d and l are parameters describing a relationship between grazing rate and weed control and ϑ is a parameter designating the strength of the relationship between ryegrass mortality and non-selective herbicide application. The function described by d and l is concave and increasing, asymptotically approaching a maximum level of weed control. Weed invasion occurs at low grazing rates due to selective grazing by sheep. Increasing the stocking rate helps to overcome selective grazing, but this marginal benefit declines as the maximum level of weed control is approached (Pratley and Godyn 1991).

In comparison, the state-transition equation for the susceptible seed population in the crop phase is $\dot{x}_\beta^s = x_\beta^s(v_1 + v_2 e^{-qu_\beta^1}(1 - u_\beta^2)R)$, where q is a parameter describing selective herbicide efficacy. The motion equation for the resistant seed population in this regime is $\dot{x}_\beta^h = x_\beta^h(v_1 + v_2(1 - u_\beta^2)R)$. The only difference between these equations is that the resistant seed population is unaffected by selective herbicide dose.

3.1.2 Objective functionals

Light grazing is required in the initial year of a lucerne phase because of its slow establishment. Full production is generally achieved in the third year, but the stand will seldom persist past four or five years of age in the study region because of plant disease and low rainfall. A logistic function (Φ_α) is consequently used to represent the productivity of the lucerne pasture over time. This defines production as a proportion of its potential (i.e. $\Phi_\alpha = [0, 1]$). This function is:

$$\Phi_\alpha = \int_{t_0}^{t_{1^-}} \zeta t\left(1 - \frac{t}{\tau}\right)dt. \tag{10}$$

Here ς is a parameter and τ is the maximum productive length of a lucerne phase.

A concave function is generally used to describe the relationship between grazing density and animal production on a given area of pasture in the absence of supplementary feed (Mott 1960). Feed quality declines at low stocking rates due to weed invasion resulting from selective grazing (Pratley and Godyn 1991). Also, production decreases beyond an optimal stocking rate because of overgrazing, which promotes weed infestation and the compaction and/or erosion of soil. These factors motivate the use of a logistic function to represent grazing profit (π_α) as a function of the stocking rate (u_α^1). This function is:

$$\pi_\alpha = \int_{t_0}^{t_{1^-}} e^{-rt} a u_\alpha^1\left(1 - \frac{u_\alpha^1}{b}\right)dt. \tag{11}$$

Here a and b are shape parameters.

Crop yield (y_β) (measured in tonnes) is multiplied by a constant price (p) (defined per tonne) to obtain total revenue for the crop phase. Crop yield is defined as (Pannell *et al.* 2004):

$$y_\beta = \int_{t_{1^+}}^{t_2} e^{-rt}\left(y_0(1 - \eta u_\beta^1)\left((1 - z) + z\left(\frac{m}{m + kw(t)}\right)\right)\right)dt. \tag{12}$$

Here y_0 is weed-free yield, η is the proportion of yield lost to phytotoxic damage for a given dose of selective herbicide, z is the maximum proportion of grain yield lost at high weed density, m is a crop-density parameter, k is a constant representing the degree of competition between the weed population and the wheat crop and $w(t)$ is the total weed population at time t. The total weed population is defined as $w = w^s + w^h$, where w^s is the susceptible weed population and w^h is the herbicide-resistant weed population. These are related to the susceptible and resistant seed populations through the relationships $w_\beta^s = x^s g(1 - M_{\text{plant}})e^{-qu_\beta^1}(1 - u_\beta^2)$ and $w_\beta^h = x^h g(1 - M_{\text{plant}})(1 - u_\beta^2)$.

The cost of active ingredient for the non-selective herbicide is denoted $c_{\alpha,\text{dose}}^2$. For the selective herbicide it is denoted by $c_{\beta,\text{dose}}^1$. The cost of herbicide application ($c_{\beta,\text{appl}}^1$ and $c_{\alpha,\text{appl}}^2$) is incurred whenever herbicide is applied.

A moderate level of weed control may be achieved in the cereal phase through shallow cultivation and the application of non-selective herbicides prior to seeding. However, attaining extremely high rates of control (around 98 per cent) requires cultural methods, such as the green-manuring of crops (Pannell *et al.* 2004). These are extremely costly since crop yield is sacrificed in that year. These factors motivate the definition of an increasing marginal control cost (c_β^2) for non-selective strategies that increases sharply at high levels of control (Gorddard *et al.* 1995):

$$c_\beta^2 = \frac{c_{\beta,\text{dest}}^2 u_\beta^2}{(1 - u_\beta^2)}. \tag{13}$$

Here $c_{\beta,\text{dest}}^2$ is the cost of killing 50 per cent of weeds using non-selective methods in the crop phase.

A fixed establishment cost (c_{lest}) is incurred at the inception of the lucerne phase. In contrast, the establishment cost for wheat (c_{cest}) is incurred each year during the cereal phase. Removing lucerne is difficult and requires the heavy application of non-selective herbicides (Devenish 2001). A switching-cost function for t_1 is therefore defined as $e^{-rt_1}c_{\text{lrem}}$, where c_{lrem} is the fixed cost of lucerne removal.

3.1.3 Multiple-phase control problem

The stationarity of the rotation is imposed through enforcing equality between the initial (x_α^0) and terminal (x_β^2) state vectors. Numerical experiments with the standard model identify that the incorporation of multiple rotations between lucerne and wheat in a single problem leads to cycling (repeated expressions of the same optimal solution for each phase, regardless of its order in the phase sequence) but with optimal management in the initial and terminal phases heavily biased by the initial and terminal conditions (data not shown). Defining a stationary state removes this bias and is more computationally efficient. It also removes the need to identify an appropriate terminal value function, for which there is little available information.

The free terminal time is incorporated through the definition of an additional control variable u_β^3. This variable may take any value in the set $U_\beta^3 = [0, \ldots, t_{\max}]$, where $t_{\max}$ is the maximum length of the cereal phase.

The multiple-phase control problem may be defined, in accordance with Problem 1, as:

$$\max_{u_\alpha^1, u_\alpha^2, u_\beta^1, u_\beta^2, u_\beta^3, t_1} J = J_\alpha + J_\beta - e^{-rt_1}c_{\text{lrem}}, \tag{14}$$

subject to,

$$\dot{x}_\alpha^s = x_\alpha^s\left(v_1 + v_2\left(1 - \frac{u_\alpha^1}{u_\alpha^1 d + l}\right)e^{-\vartheta u_\alpha^2}R\right), \tag{15}$$

$$\dot{x}_\alpha^h = x_\alpha^h\left(v_1 + v_2\left(1 - \frac{u_\alpha^1}{u_\alpha^1 d + l}\right)e^{-\vartheta u_\alpha^2}R\right), \tag{16}$$

$$\dot{x}^s_\beta = u^3_\beta x^s_\beta (v_1 + v_2 e^{-qu^1_\beta}(1 - u^2_\beta)R), \tag{17}$$

$$\dot{x}^h_\beta = u^3_\beta x^h_\beta (v_1 + v_2(1 - u^2_\beta)R), \tag{18}$$

$$x^0_\alpha = \{x^s(t_0), x^h(t_0)\}, \tag{19}$$

$$x^2_\beta = \{x^s(t_0), x^h(t_0)\}, \tag{20}$$

where,

$$J_\alpha = \int_{t_0}^{t_{1^-}} e^{-rt}\left(\varsigma t\left(1 - \frac{t}{\tau}\right)au^1_\alpha\left(1 - \frac{u^1_\alpha}{b}\right) - c^2_{\alpha,\text{dose}}u^2_\alpha - c^2_{\alpha,\text{appl}}\right)dt - c_{\text{lest}}, \text{ and} \tag{21}$$

$$J_\beta = \int_0^1 u^3_\beta e^{-r(t_1+u^3_\beta t)}\left(\begin{array}{l} py_0(1-\eta u^1_\beta)\left((1-z) + z\left(\dfrac{m}{m+kw}\right)\right) \\ - c^1_{\beta,\text{dose}}u^1_\beta(t) - c^1_{\beta,\text{appl}} - c^2_{\beta,\text{dest}}\left(\dfrac{u^2_\beta}{(1-u^2_\beta)}\right) - c_{\text{cest}} \end{array}\right)dt. \tag{22}$$

Equations (15) and (16) are the state equations for the lucerne phase. Equations (17) and (18) are the state equations for the wheat stage. The boundary conditions for the problem are defined in Equations (19) and (20). The objective functionals for each phase are stated in Equations (21) and (22).

3.2 Parameter estimation

The standard parameter values used in the analysis are listed in Table 1. Many of the parameters are taken from the resistance and integrated management (RIM) model (Pannell *et al.* 2004), a framework constructed to compare the profitability of different IWM strategies for annual ryegrass control in the study area. The following discussion focuses on the estimation procedures for those parameters that are not taken directly from other sources.

The average rate of seed production per plant ($R = 100$) is well below maximum seed production, which is around 1350 seeds plant^{-1} according to the seed production model reported in Pannell *et al.* (2004). However, it is appropriate as competition with wheat adversely affects ryegrass seed production and wheat density is assumed to be 100 plants m^{-2}.

The maximum level of annual ryegrass control achieved by sheep grazing lucerne is assumed to be 90 per cent. This estimate is drawn from data for French serradella (*Ornithopus sativus* Brot.) in Pannell *et al.* (2004). It is conservative as lucerne is generally more competitive with weeds than annual pastures, such as serradella (Roy Latta, pers comm., 2005). This estimated level of the maximum rate of weed control obtained by grazing yields

Table 1 Parameter values for the two-phase model

Parameter	Description	Value	Source
d, l	Parameters describing weed control by grazing	$d = 1.11$, $l = 0.5$	See text for details
g	Germination rate	$g = 0.8$	RIM model
M_{seed}	Rate of seed mortality	$M_{\text{seed}} = 0.05$	RIM model
M_{plant}	Rate of plant mortality	$M_{\text{plant}} = 0.05$	RIM model
R	Seed production per plant	$R = 100$ seeds plant^{-1}	See text for details
q	Efficacy of selective herbicide	$q = 7.45$	Gorddard *et al.* (1995)
ϑ	Efficacy of non-selective herbicide	$\vartheta = 7.87$ (s.e. = 0.58)*,†	See text for details
r	Discount rate	$r = 0.05$	Doole (2007)
ς	Parameter describing lucerne productivity	$\varsigma = 0.78$ (s.e. = 0.04)	See text for details
τ	Maximum productive length of a lucerne phase	$\tau = 4.96$ (s.e. = 0.18)	See text for details
a, b	Profit function parameters for pasture phase	$a = 25.32$ (s.e. = 1.27), $b = 14.88$ (s.e. = 0.54)	See text for details
$c^2_{\alpha,\text{dose}}$	Cost of chemical for non-selective herbicide	$c^2_{\alpha,\text{dose}} = \$12.50$	Agriculture Western Australia (2004)
c_{lest}	Cost of lucerne establishment	$c_{\text{lest}} = \$88.17$	See text for details
c_{lrem}	Cost of lucerne removal	$c_{\text{lrem}} = \$18.90$	See text for details
p	Price per tonne of wheat	$p = \$185$ t^{-1}	RIM model
y_0	Weed-free yield	$y_0 = 1.82$ t ha^{-1}	Doole (2007)
η	Parameter describing rate of phytotoxic damage	$\eta = 0.14$	Gorddard *et al.* (1995)
z, m, k	Wheat yield function parameters	$z = 0.6$, $m = 105$, $k = 0.33$	Pannell *et al.* (2004)
$c^2_{\alpha,\text{appl}}$, $c^1_{\beta,\text{appl}}$	Cost of herbicide application	$c^2_{\alpha,\text{appl}} = c^1_{\beta,\text{appl}} = \$2.50$	RIM model
$c^1_{\beta,\text{dose}}$	Cost of chemical for selective herbicide	$c^1_{\beta,\text{dose}} = \$40$	Agriculture Western Australia (2004)
$c^2_{\beta,\text{dest}}$	Cost of non-selective control methods in crop	$c^2_{\beta,\text{dest}} = \$1.09$ (s.e. = 0.84)	See text for details
c_{cest}	Cost of cereal establishment	$c_{\text{cest}} = \$82$	Doole (2007)

* The term s.e. denotes standard error.
† All reported results from non-linear regression procedures are rounded to two decimal places.

$d = 1/0.9 = 1.11$. Data on the efficacy of grazing for ryegrass control is taken from Pearce and Holmes (1976) and the RIM model. The remaining parameter (l) in this function is then varied, with d held fixed, until the relationship adequately fits this data. This is achieved at $l = 0.5$. The resulting function is shown in Figure 1a. This method of estimation is heuristic, but it is necessary given the lack of appropriate data.

All non-linear regression results reported below are estimated using the LSQCURVEFIT function in the Optimisation toolbox of MATLAB version 7.1 (Miranda and Fackler 2002). The residual sum of squares is minimised over multiple initial guesses for each parameter value to reduce the probability that the estimation algorithm converges to local minima.

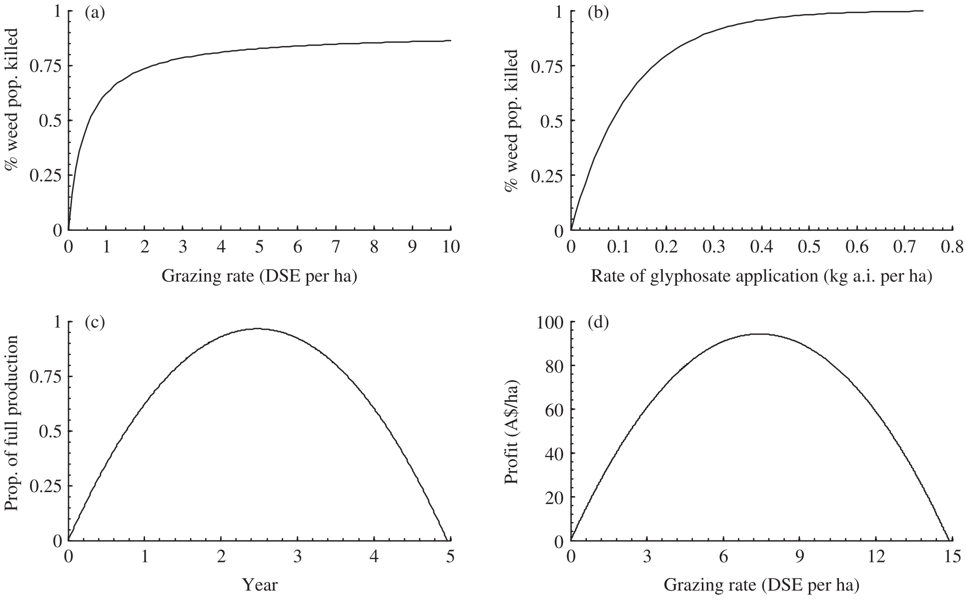

Figure 1 Functions in the standard model representing the relationships between (a) the stocking rate and the proportion of the weed population killed by grazing, (b) the rate of non-selective herbicide application in the lucerne phase and the proportion of the weed population killed by this chemical, (c) the length of the lucerne phase and its production represented as a proportion of its potential, and (d) the stocking rate in the lucerne phase and profit.

The parameter (ϑ) describing the efficacy of glyphosate applied during the lucerne phase is identified from non-linear regression of data ($n = 40$) from Wakelin *et al.* (2004). The sum of squares is 0.4, indicating an excellent fit. The estimated function is shown in Figure 1b.

The parameters (ζ, τ) describing the productivity of lucerne as a function of time are estimated through non-linear regression of production estimates outlined in Department of Agriculture and Food Western Australia (DAFWA) (Keith Devenish and Roy Latta unpublished data). The sum of squares is 3307 for $n = 10$. The resulting function is shown in Figure 1c.

An estimate of the gross margin received for livestock production in the study region is taken from the RIM model. This estimate is $15 per dry sheep equivalent per hectare (DSE ha^{-1}). DSE is a measure of the stocking rate and represents an average-sized, non-lactating sheep (Doole and Pannell 2008). A constant stocking rate ($sr = 7.7$ DSE ha^{-1}) corresponding to that used to rotationally graze an established lucerne stand is estimated from data in Devenish (2001). Mott (1960) presented a general relationship between stocking rate and animal production, both as a proportion of a standard profitable level. The levels listed above and this curve are used to identify 35 data points that are regressed with non-linear least squares to identify a and b. The sum of squares is 4820 for $n = 35$. The estimated function is depicted in Figure 1d.

The cost of a kilogram of active ingredient (kg a.i.) of the non-selective herbicide ($c^2_{\alpha,\text{dose}}$) is \$12.50 (Agriculture Western Australia 2004). The cost of a kilogram of active ingredient of the selective herbicide ($c^1_{\beta,\text{dose}}$) is \$40 (Agriculture Western Australia 2004).

The cost of non-selective control methods available during a cropping phase and their efficacy are obtained from the RIM model. Equation (13) is fitted to this data using non-linear least squares. Only the cheapest destructive technique available during the cropping phase (green-manuring) is incorporated given the relatively high cost of hay and silage production. The value of weed-free yield is included as an opportunity cost given that this method of control sacrifices crop yield. The sum of squares for this non-linear regression is 5862 for $n = 5$. Alternative functional forms did not improve the fit of this equation.

The total cost of lucerne establishment ($c_{\text{lest.}}$) is \$88.17 ha^{-1} (Doole 2007). The total cost to remove lucerne (c_{lrem}) is \$18.90 ha^{-1} (Doole 2007). Lucerne is removed in spring with a mixture of 1 litre of Glyphosate CT® (0.4 kg a.i. ha^{-1}) and 1.5 L of 2,4-D Amine 625® (0.9375 kg a.i. ha^{-1}) per hectare (Devenish 2001). The glyphosate component of this mixture is incorporated for its high efficacy against weeds given lucerne's relative tolerance of this chemical.

3.3 Model scenarios

The model is used to investigate a range of different scenarios (Table 2). The standard model incorporates initial conditions of 50 s. s. m^{-2} (susceptible seeds per square metre) and 25 r. s. m^{-2} (herbicide-resistant seeds per square metre). This population may be adequately controlled under standard management. It may become troublesome, though, since the proportion of the population consisting of resistant seeds is higher than that level inferred by genetic probability—around 1 : 1000 000 (Gressel and Segel 1990). Such a high proportion occurs through the interbreeding of herbicide-resistant weeds. This provides insight into the optimal management of herbicide-resistant weeds and is necessary because an equilibrium framework is not suited to representing the development of herbicide resistance.

The highest initial seed densities considered in this model are 200 s. s. m^{-2} and 100 r. s. m^{-2}. It is difficult to undertake a meaningful analysis of higher initial seed burdens because the model does not incorporate control variables, such as burning, that directly affect the seed bank.

The effect of improving the efficacy of weed control in the lucerne phase is investigated given its importance to results reported in Sections 4.1 and 4.2 and to highlight the relative benefit of conducting research in this area. An additional term ($1 - \bar{U}$) is incorporated into equations 15 and 16 to represent this scenario. Here $\bar{U}$ is a parameter representing the hypothetical proportional increase in weed control that is attained in a lucerne phase. For example, $\bar{U} = 0.05$ represents a 5 per cent increase in weed control efficacy in the

Table 2 Scenarios evaluated in the model

Description	Parameter Value
Section 4.1: Optimal weed control with herbicide resistance	
Low initial weed density*	26 s. s. m^{-2} and 13 r. s. m^{-2}
Standard initial weed density	50 s. s. m^{-2} and 25 r. s. m^{-2}
High initial weed density	200 s. s. m^{-2} and 100 r. s. m^{-2}
Section 4.2: Optimal weed control without herbicide resistance	
Low initial weed density	26 s. s. m^{-2}
Standard initial weed density	50 s. s. m^{-2}
High initial weed density	200 s. s. m^{-2}
Section 4.3: Higher efficacy of weed control in lucerne phase	
With herbicide resistance	$\bar{U} = \{0.05, 0.1, 0.15, 0.2\}$
Without herbicide resistance	$\bar{U} = \{0.05, 0.1, 0.15, 0.2\}$
Section 4.4: Sensitivity analysis	
Low wheat price	p_{low} = \$148 t^{-1}
High wheat price	p_{high} = \$222 t^{-1}
Low stocking rate	sr = 5.7 DSE ha^{-1}
Profit function parameters	a = 20.07 (s.e. = 0.60)†,‡
	b = 15.66 (s.e. = 0.51)
	n = 35
	SS§ = 1097
High stocking rate	sr = 9.7 DSE ha^{-1}
Profit function parameters	a = 37.72 (s.e. = 1.85)
	b = 13.22 (s.e. = 0.5)
	n = 35
	SS = 10 237
Low cost of destructive treatments in crop phase	$c^2_{\beta,\text{dest}}$ = \$0.82
High cost of destructive treatments in crop phase	$c^2_{\beta,\text{dest}}$ = \$1.36

* The initial seed densities are labelled {low, standard, high} according to their magnitude relative to one another.
† The term s.e. denotes standard error.
‡ All reported results from non-linear regression procedures are rounded to two decimal places.
§ SS denotes the sum of squares accruing to each non-linear regression result.

pasture phase. The introduction of this parameter is equivalent to improving the competitiveness of the perennial pasture without impacting production or weed management costs.

The implications of large changes occurring in the profitability of livestock husbandry are analysed through the estimation of new profit functions for the pasture phase. The stocking rate sr used to formulate the profit function for the lucerne phase (see Section 3.2) is decreased or increased by 2 DSE ha^{-1} to formulate a low- and high-profitability scenario, respectively. These manipulations are equivalent to decreasing or increasing livestock profitability by a quarter. The updated values for sr and the proportions of Mott (1960) are used to re-estimate the parameters for π_α through non-linear regression.

The cost of achieving 50 per cent weed control using non-selective treatments in the crop phase ($c^2_{\beta,\text{dest}}$) is varied by 25 per cent from its standard value. This is done because this form of control is used intensively when herbicide resistance constrains selective herbicide efficacy.

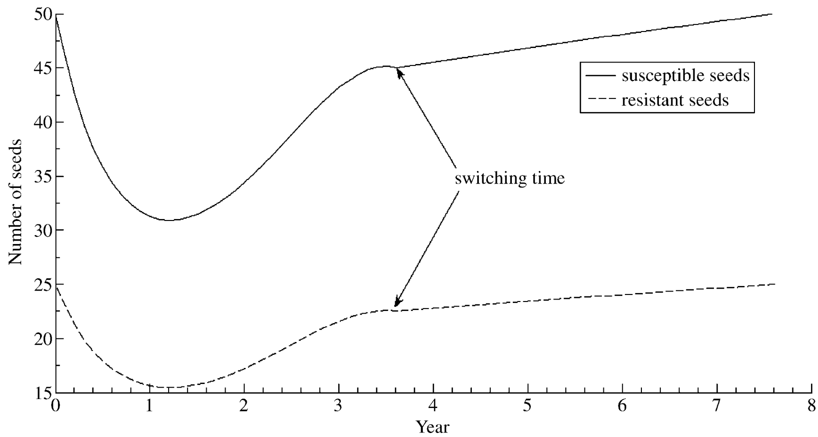

Figure 2 Optimal seed trajectories for a lucerne–wheat rotation with initial seed populations of 50 s. s. m^{-2} and 25 r. s. m^{-2}.

4. Results and discussion

4.1 Optimal weed control with herbicide resistance

This scenario is referred to throughout the discussion as the standard solution. The optimal switching time is 3.6 years, and the optimal transition states (i.e. the level of the state variables at the optimal switching time) are 45 s. s. m^{-2} and 23 r. s. m^{-2}. Both seed populations are lower at the switching time than at the start of the horizon (Figure 2). This reflects the combined efficacy of grazing and non-selective herbicide application for weed control in the pasture phase. Grazing is maintained at an optimal rate of 7.46 DSE ha^{-1} over the most productive years of the lucerne stand. It is necessary to constrain the weed population over the lucerne phase; however, maintaining sustained, intensive control over this period would greatly increase weed-management costs. Accordingly, glyphosate application decreases from a rate of 0.67 kg a.i. ha^{-1} to 0.29 kg a.i. ha^{-1} over the duration of this stage, giving rise to the convexity of the state trajectories observable in Figure 2.

Only one selective herbicide is represented in this model, so only non-selective methods are effective against the total weed population if any resistant weeds are present. Accordingly, the selective herbicide is never applied if a positive population of resistant seeds is present at the beginning of the period; instead, intensive non-selective control is maintained across the cereal phase. These treatments are used to kill, on average, 98 per cent of ryegrass plants over the duration of this regime. The intensive use of non-selective treatments is motivated by the strong economic incentive to maintain a low number of seeding plants given the competitiveness and large seed production of annual ryegrass.

The lucerne pasture and cropping phase are also adopted at initial conditions of (1) 26 s. s. m^{-2} and 13 r. s. m^{-2}, and (2) 200 s. s. m^{-2} and 100 r. s. m^{-2}.

The optimal switching time for scenario 1 is 3.4 years, and the optimal transition states are 23 s. s. m^{-2} and 11 r. s. m^{-2}. In comparison, the optimal switching time for scenario 2 is 3.9 years, and it is optimal to switch at 171 s. s. m^{-2} and 85 r. s. m^{-2}. This demonstrates the value of the IWM strategy adopted during the pasture phase. The selective herbicide is never applied at any of these different initial conditions.

4.2 Optimal weed control without herbicide resistance

It is optimal to bypass the pasture phase and begin cropping immediately if there are no herbicide-resistant weeds at the initial time t_0. The cereal phase is maintained for four years in this scenario, and the weed population is controlled with a combination of selective herbicide (applied at a mean dose rate of 0.503 kg a.i. ha^{-1}) and non-selective control (used to kill an average of 49 per cent of ryegrass plants at each point in time). This solution is identified through the identification of $H_{\text{lucerne}}(t_{1^-}) + re^{-rt}c_{\text{lrem}} \leq H_{\text{crop}}(t_{1^+})$ for all t_j during the initialisation stage of the regime-programming algorithm. This corresponds to $t_0 = t_1$ according to necessary condition A.9 in Theorem 1 in the Appendix. This result is observed for all initial ryegrass populations.

The cereal crop is always more profitable than the pasture regime in the absence of herbicide resistance as the selective herbicide permits efficient control of the weed population. Consequently, though grazing income is important in a lucerne phase, at the parameter values used in this study, the susceptibility of annual ryegrass to the selective herbicide directly determines whether or not perennial pasture phases should be adopted in the optimal rotation. There is only a single selective herbicide represented here, so this result is specific to where no selective herbicide options are available to the producer. This extends the findings of Monjardino *et al.* (2004), who identified that annual pasture was only sufficiently profitable to adopt in crop rotations once ryegrass had developed resistance to both Group A and B herbicides.

This relationship between the presence of herbicide resistance and the optimal adoption of perennial pasture is robust to large changes in the important economic parameters used in the model. For example, it is only profitable to adopt lucerne pasture at a wheat price below \$101 per tonne, a decrease of over 45 per cent relative to the parameter used in the standard model. This decrease is too large to foresee its occurrence in the near-term, given forecasts of increased demand for Australian grain in developing countries (Food and Agriculture Organisation 2002). Removing the establishment cost for lucerne ($c_{\text{lest}} = \$88.17$) is also insufficient to warrant use of perennial pasture if no herbicide resistance is present.

However, lucerne is adopted if the stocking rate is increased to 17 DSE ha^{-1}, which is 9.54 DSE ha^{-1} higher than the optimal stocking rate reported in the standard set of results (7.46 DSE ha^{-1}), or following an \$18.11 increase in the current gross margin received for sheep production (\$15 DSE^{-1}). Using a variant of the RIM model incorporating both wild radish and annual ryegrass,

Table 3 Optimal output as the efficacy of weed control in the lucerne phase ($\bar{U}$) is improved in the presence of herbicide resistance

$\bar{U}$	t_1	$x^s(t_1)$	$x^h(t_1)$
0 (standard value)	3.6	45	23
0.05	3.4	40	20
0.1	3.1	38	19
0.15	2.8	36	18
0.2	2.6	32	16

Monjardino *et al.* (2004) identified that the most-valuable rotation incorporating annual pasture required an increase in the carrying capacity of pasture of 7 DSE ha^{-1}, or a \$15.50 DSE^{-1} increase in the sheep gross margin, to be as profitable as a continuous-cropping sequence in the absence of herbicide resistance. These increases are 33 and 17 per cent lower, respectively, than those identified in the current study. This result is logical since lucerne requires a greater level of profitability to offset its larger establishment cost relative to the annual pasture species studied by Monjardino *et al.* (2004). However, either set of improvements appear unlikely to occur in the near future as though sheepmeat prices are forecast to increase by around a third by 2020 (Kingwell and Pannell 2005), wool prices are expected to fall by up to 50 per cent by 2030 (Sackett 2004).

4.3 Higher efficacy of weed control in the lucerne phase

Improving the efficacy of weed control in the lucerne phase in the presence of herbicide resistance decreases the optimal length of the pasture phase and lowers the weed population at the switching time (Table 3). This reflects an improved capacity to reduce the weed population prior to grain production. Profit increases in the cereal phase as control efficacy in the pasture phase is improved. For example, there is a 12 per cent increase in profit in the wheat stage when $\bar{U} = 0.2$ as there is a lower weed burden and thus a reduced need to employ costly in-crop, non-selective treatments. In contrast, an increase in control efficacy is insufficient to warrant the adoption of lucerne pasture in the absence of herbicide resistance. This reflects the efficient in-crop weed control offered by selective herbicides, relative to the expense of establishing a pasture and implementing an IWM strategy therein.

4.4 Sensitivity analysis

4.4.1 Wheat price

The low wheat price ($p_{low} = \$148t^{-1}$) motivates an increase in the optimal switching time from 3.4 years to 4.8 years (Figure 3). The optimal transition states also increase from 45 s. s. m^{-2} and 23 r. s. m^{-2} to 49 s. s. m^{-2} and 24 r. s. m^{-2},

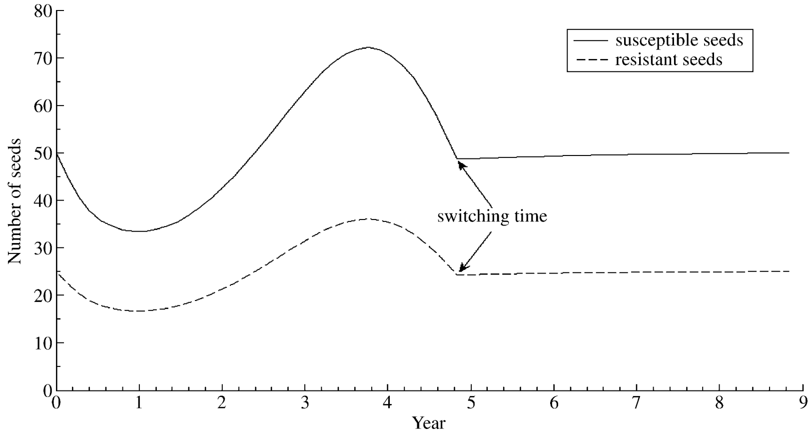

Figure 3 The optimal seed trajectories across a lucerne–wheat rotation for a low wheat price (p_{low} = \$148 t^{-1}).

respectively. This reflects a reduction in the marginal value of weed control conducted during the pasture phase. The curvature of the state trajectories in Figure 3 reflects the decreased value of weed control in the pasture phase and the cost of maintaining intensive control over the duration of this extended regime.

In contrast, the optimal switching time and transition states are lower at the high wheat price. The optimal switching time is reduced to three years, and the optimal transition states decrease from 45 s. s. m^{-2} and 23 r. s. m^{-2} to 39 s. s. m^{-2} and 19 r. s. m^{-2}, respectively. These results follow logically from the higher value of the cereal crop, which increases the marginal value of the IWM strategy implemented during the pasture stage.

The optimal stocking rate (u_α^1) remains unchanged following both price changes. The marginal contribution of an additional grazing unit to weed control is rapidly diminishing at higher stocking intensities (Figure 1a). Moreover, increasing grazing intensity is not profitable due to a decline in the productivity of livestock at high stocking rates because of overgrazing (Figure 1d).

In contrast, a change in cereal price causes a marked change in optimal glyphosate application (u_α^2). The reductions in the weed populations at the beginning and the end of the pasture phase at p_{low} (Figure 3) are achieved through high rates of glyphosate application (Figure 4). In comparison, the shorter pasture phase adopted at p_{high} encourages the producer to maintain low seed populations throughout its duration using a primarily declining dose rate (Figure 4).

4.4.2 Livestock profitability

The state trajectories are very similar to those of the standard model at the low level of livestock profitability. In fact, the resistant seed populations are

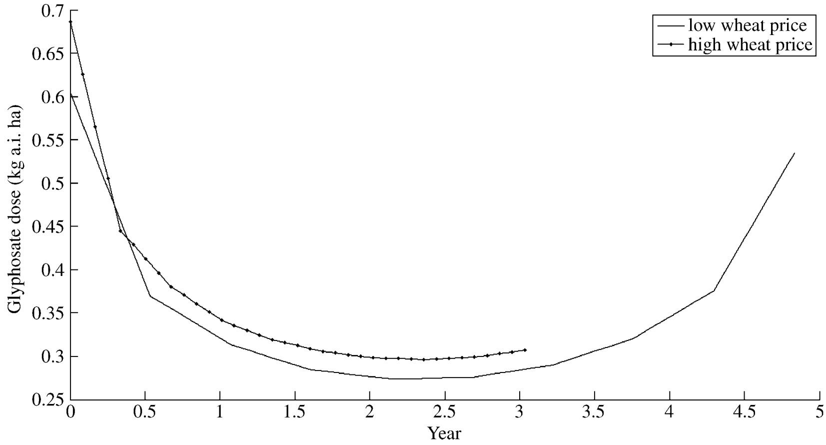

Figure 4 Kilograms of glyphosate applied during the pasture phase for a low wheat price (p_{low} = \$148 t^{-1}) and a high wheat price (p_{high} = \$222 t^{-1}).

equivalent at the switching times, while the susceptible seed population at the switching time is higher by two seeds following the price change. However, the optimal length of the pasture phase decreases from 3.6 to 2.8 years. Although shorter than before, the pasture phase is still of considerable length, highlighting the importance of animal production to farm profit in the presence of severe herbicide resistance.

In contrast, the higher level of livestock profitability motivates the adoption of continuous pasture in phases of a mean duration of 3.17 years, with an optimal mean grazing rate of 6.58 DSE ha^{-1} $year^{-1}$. This demonstrates that resistance to selective herbicides may motivate a complete movement away from crop production to livestock husbandry if the latter is sufficiently profitable.

4.4.3 Cost of non-selective control

The high intensity of non-selective control applied under optimal management remains unchanged at $c^2_{\beta,\text{dest}}$ = \$0.82 and $c^2_{\beta,\text{dest}}$ = \$1.36. This is driven by strong economic incentives to minimise in-crop competition, particularly given the high seed production of individual plants. Accordingly, the optimal transition states experience little change, with a maximum adjustment of around 4 per cent. However, a decrease (cf. increase) in the cost of non-selective control motivates the adoption of a shorter (cf. longer) pasture phase. For example, the optimal switching time decreases from 3.6 to 2.55 years with $c^2_{\beta,\text{dest}}$ = \$0.82 and increases from 3.6 to 4.15 years with a cost of $c^2_{\beta,\text{dest}}$ = \$1.36. These changes reflect a direct relationship between the cost of non-selective control and the profitability of cropping, relative to livestock husbandry, in the presence of herbicide resistance.

4.5 Limitations

The model represents two alternative land uses on a single field. This approach disregards other land uses (such as annual pastures), different soil types and the implications of the management of this field for whole-farm profit and organisation. This allows a greater focus on optimal weed management since other studies (e.g. Bathgate and Pannell 2002) have considered the whole-farm implications of perennial pasture adoption.

The value of lucerne for the prevention of dryland salinisation is also not dealt with. The length of the lucerne phases in the rotations containing perennial pasture in Sections 4.1–4.4 are sufficient to prevent the development of dryland salinisation on the field of interest. This is identified through the calculation of mean recharge over the last century for each rotation using the Leakage-Buffer model (Ward 2006) (data not shown). Furthermore, Doole (2007) identified that, if the value of lucerne phases for weed management is not considered and the only effect of salinisation is the loss of agricultural production, perennial pasture is only profitable to adopt for salinity prevention if a saline water table is less than 3.5 m from the soil surface. Hence, at current commodity prices, the adoption of lucerne for salinity prevention is likely to be profitable only in extreme circumstances, particularly given the limited spatial impact of perennials in the Western Australian Wheatbelt (Pannell and Ewing 2006).

It is never profitable to implement large reductions in the weed seed banks in model output. Use of an equilibrium framework may conceptually dampen the marginal benefit accruing to any weed control. However, a steady-state approach is retained because of its greater computational efficiency and the lack of an appropriate terminal value function (see Section 3.1.3). Moreover, the lack of any such large reductions arises primarily from the definition of this model in continuous time because the impact of weed treatments and biological processes on the seed populations are not temporally distinct (see, for example, Equation (9)). This permits the weed population to continually respond to intensive control through seed production, and thus promotes the intensity at which weed treatments are applied under optimal management. Control variables that directly influence the seedbank (e.g. burning) are also not included as they are not regularly used in the study region. Another impact of defining the model in continuous time is that optimal phase duration is calculated with a precision that is not entirely practicable; for example, the optimal length of the lucerne phase is 3.6 years in the standard solution.

Despite these limitations, a continuous-time framework is retained as:

1. This model still provides important and intuitive conceptual insight. For example, a perennial pasture phase of 3.6 years duration under optimal management in the presence of established herbicide resistance suggests that incorporating a lucerne phase of standard length (three to four years) in a rotation is profitable, relative to continuous cropping, in these circumstances.

2. It aids interpretation of output from a more detailed model in Doole and Pannell (2008). For example, the central relationship between lucerne's value to phase rotations and the severity of herbicide resistance highlighted here is also observed in Doole and Pannell (2008).
3. Other studies of IWM in Western Australian cropping systems (e.g. Gorddard *et al*. 1995; Monjardino *et al*. 2004) have not considered optimal rotation length. This motivates the definition of this problem as a multiple-phase control system, of which it is a relevant and interesting example.
4. No suitable discrete-time algorithms for multiple-phase control have been developed.

5. Summary and conclusions

This study employs the regime-programming algorithm of Doole (2007) to analyse the optimal management of herbicide-resistant annual ryegrass in lucerne–wheat rotations in the Central Wheatbelt of Western Australia. This is important as the value of perennial pastures for weed control in this region is unknown. This could promote the adoption of lucerne over greater areas of the Wheatbelt with subsequent benefits for the prevention of dryland salinisation. Bioeconomic analysis is employed as the decision facing producers is complex, incorporating multiple land uses, a temporal dimension, numerous non-linearities and many alternative forms of weed control.

In results from the standard model, the presence of severe herbicide resistance motivates the intensive use of in-crop, non-selective treatments to reduce weed competition during the cereal phase. The low efficiency of these treatments, relative to selective herbicides, promotes the use of an IWM strategy, consisting of grazing and a winter-cleaning application, in a perennial pasture in the optimal rotation. However, with the availability of selective herbicides for efficient weed control, lucerne pasture is only profitable to include in rotations at very high livestock prices.

The low value of lucerne pasture phases for IWM in the absence of severe herbicide resistance is unlikely to motivate wide-scale planting of this perennial in the near future. This has direct implications for the prevention of dryland salinisation in the study region. However, recent research highlights the potential efficacy of alternative perennial legumes, such as hairy canary clover (*Dorycnium hirsutum* L.) (Bell *et al*. 2007), that may be suitable for recharge reduction in the Central Wheatbelt. This study suggests that the benefit of these pastures for IWM could be an important driver for their adoption. It therefore requires consideration in any economic evaluations conducted for these species.

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Appendix

Theorem 1 presents the set of necessary conditions required for the solution of the multiple-phase control problem described in Problem 1.

Theorem 1. For $j = [1, 2, \ldots, n]$ and switching sequence $K = \{k_1, k_2, \ldots, k_n\}$, let $(x^*(t), u_j^*(t), t_j^*)$ denote the admissible trajectory that maximises the value of J in Problem 1. This is the optimal trajectory Γ^*.

Define a Hamiltonian function for each regime k_j as:

$$H_j(x(t), u_j(t), \lambda_j(t), t) = e^{-rt} F_j(x(t), u_j(t)) + \lambda_j(t) f_j(x(t), u_j(t), t), \qquad \text{(A.1)}$$

across the interval $[t_{j-1^+}, t_{j^-}]$.

An optimal trajectory Γ^* requires:

1. initial condition $x_0 = x(t_0)$ for fixed initial state variable(s) x_0, (A.2)
2. n m-dimensional vectors of real-valued, piecewise-continuous adjoint functions $\lambda_j(t) = \{\lambda_j^1(t), \lambda_j^2(t), \ldots, \lambda_j^m(t)\}$, defined across $j = [1, 2, \ldots, n]$ and piecewise continuously differentiable over the interval $[t_{j-1^+}, t_{j^-}]$, that satisfy,

$$\dot{\lambda}_j^T(t) = -\frac{\partial H_j(x(t), u_j(t), \lambda_j(t), t)}{\partial x(t)}, \qquad \text{(A.3)}$$

   where $\lambda_j^T(t)$ denotes the transpose of the n adjoint vectors,
3. optimal control function(s) that satisfy,

$$\max_{u_j(t)} H_j(x(t), u_j(t), \lambda_j(t), t) \text{ for all } t \in [t_{j-1^+}, t_{j^-}], \qquad \text{(A.4)}$$

4. a terminal adjoint vector $\lambda_n(t_n)$ that satisfies,

$$\lambda_n^T(t_n) = \frac{\partial e^{-rt_n} G(x(t_n), t_n)}{\partial x(t_n)}, \qquad \text{(A.5a)}$$

   for state variables $x_n^i(t_n)$, where $i = [1, \ldots, d]$, free at the terminal time and defined in G,
   NOTE: $\lambda_n^T(t_n) = 0$ replaces Equation (A.5a) for those state variables $x_n^i(t_n)$, where $i = [1, \ldots, d]$, that are not defined in G, (A.5b)
   NOTE: $x_n^i(t_n) = x(t_n)$ replaces Equations (A.5a) and (A.5b) for fixed terminal state variables $x_n^i(t_n)$, where $i = [d + 1, \ldots, m]$, (A.5c)

5. a terminal time that satisfies,

$$H_n(x(t), u_n(t), \lambda_n(t), t)|_{t_n} + \frac{\partial e^{-rt_n} G(x(t_n), t_n)}{\partial t_n} = 0, \tag{A.6a}$$

if no terminal value function is defined, then the equivalent of Equation (A.6a) is,

$$H_n(x(t), u_n(t), \lambda_n(t), t)|_{t_n} = 0, \tag{A.6b}$$

if, instead, the terminal time is fixed, then $t = t_n$, (A.6c)

6. adjoint vectors that satisfy the boundary conditions,

$$\lambda_j^T(t_{j^-}) + \frac{\partial e^{-rt_j} C_j(x(t_j))}{\partial x(t_j)} = \lambda_{j+1}^T(t_{j^+}), \tag{A.7}$$

at switching times $t = \{t_1, t_2, \ldots, t_{n-1}\}$ and $j = [1, 2, \ldots, n-1]$,

7. $$H_j(x(t), u_j(t), \lambda_j(t), t)|_{t_{j^-}} - \frac{\partial e^{-rt_j} C_j(x(t_j))}{\partial t_j} = H_{j+1}(x(t), u_{j+1}(t), \lambda_{j+1}(t), t)|_{t_{j^+}}, \tag{A.8}$$

for those switching times in $t = \{t_1, t_2, \ldots, t_{n-1}\}$ for which $t_{j-1} < t_j < t_{j+1}$ holds,

8. $$H_j(x(t), u_j(t), \lambda_j(t), t)|_{t_{j^-}} - \frac{\partial e^{-rt_j} C_j(x(t_j))}{\partial t_j} \le H_{j+1}(x(t), u_{j+1}(t), \lambda_{j+1}(t), t)|_{t_{j^+}}, \tag{A.9}$$

for those switching times in $t = \{t_1, t_2, \ldots, t_{n-1}\}$ for which $t_{j-1} = t_j < t_{j+1}$ holds, and

9. $$H_j(x(t), u_j(t), \lambda_j(t), t)|_{t_{j^-}} - \frac{\partial e^{-rt_j} C_j(x(t_j))}{\partial t_j} \ge H_{j+1}(x(t), u_{j+1}(t), \lambda_{j+1}(t), t)|_{t_{j^+}}, \tag{A.10}$$

for those switching times in $t = \{t_1, t_2, \ldots, t_{n-1}\}$ for which $t_{j-1} < t_j = t_{j+1}$ holds.

Proof. See Doole (2007).

The structure of Theorem 1 motivates the decomposition of the regime-programming algorithm into two parts: (i) solution of each phase as an independent control problem at each iteration, and (ii) updating of the estimates of the optimal switching times and transition states using Conditions (A.7) and (A.8) and a bisection technique (Miranda and Fackler 2002). The presence of no optimal switching times, as described by Equations (A.9) and (A.10), appears in the initialisation stage when Hamiltonian values that alternate in sign cannot be identified. This application is programmed in MATLAB version 7.1 (Miranda and Fackler 2002). Each individual phase is solved using a variant of the MISER control parameterisation algorithm of Teo *et al*. (1991). The code for this procedure is available from the authors on request.

Regime-programming algorithm

Initialisation:

a. Let numeric superscripts denote the iteration number for ease of reference. Let $(\cdot)_x$ denote the derivative of the term enclosed in brackets with respect to the subscripted variable (x in this example). Determine a fixed stage sequence K. Define the maximum number of permissible iterations ($\hat{\imath}$). Define the stopping tolerance ε. Define a set of initial conditions $\Lambda = \{t_0, x_0\}$. Provide estimates of the optimal switching times (t_j^i for $j = [1, 2, \ldots, n-1]$) and the transition states ($x(t_j^i)$ for $j = [1, 2, \ldots, n-1]$) for $i = \{1, 2\}$. Ensure $t_j^1 < t_j^2$ and $x(t_j^1) < x(t_j^2)$.

b. Optimise each phase k_j, for $j = [1, 2, \ldots, n-1]$, as a fixed-point control problem using Conditions (A.1)–(A.4) and (A.5c) and (A.6c). Conditions (A.5c) and (A.6c) are determined by the estimates of t_j^i and $x(t_j^i)$. Optimise the terminal stage using Conditions (A.1)–(A.4) and the relevant terminal conditions from (A.5)–(A.6). Obtain $\lambda_j^T(t_j)$ and compute $H_j(t_j)$ for all j. Do for $i = \{1, 2\}$.

c. Ensure that $(H_{t_j}^1(t_j) - (e^{-rt_j}C_j(\cdot))_{t_j}^1 - H_{j+1}^1(t_j))(H_{t_j}^2(t_j) - (e^{-rt_j}C_j(\cdot))_{t_j}^2 - H_{j+1}^2(t_j))$ < 0 and $(\lambda_j^1(t_j) + (e^{-rt_j}C_j(\cdot))_{x(t_j)}^1 - \lambda_{j+1}^1(t_j))(\lambda_j^2(t_j) + (e^{-rt_j}C_j(\cdot))_{x(t_j)}^2 - \lambda_{j+1}^2(t_j))$ < 0 before starting the main computation.

Main computation:

For $i = 3$ to $\hat{\imath}$:

1. Form switch points using the midpoint formulas $t_j^i = t_j^{i-2} + (t_j^{i-1} - t_j^{i-2})/2$ and $x(t_j^i) = x(t_j^{i-2}) + (x(t_j^{i-1}) - x(t_j^{i-2}))/2$.

2. Optimise each phase k_j for $j = [1, 2, \ldots, n-1]$ as a fixed-point control problem using Conditions (A.1)–(A.4) and (A.5c) and (A.6c). Optimise the terminal stage using Conditions (A.1)–(A.4) and the relevant terminal conditions in (A.5)–(A.6). Obtain $\lambda_j^T(t_j)$ and compute $H_j(t_j)$ for all j.

3. If $(\lambda_j^i(t_j) + (e^{-rt_j}C_j(\cdot))_{x(t_j)}^i - \lambda_{j+1}^i(t_j))(\lambda_j^{i-2}(t_j) + (e^{-rt_j}C_j(\cdot))_{x(t_j)}^{i-2} - \lambda_{j+1}^{i-2}(t_j)) > 0$, then $x(t_j^i) = x(t_j^{i-2})$ and $x(t_j^{i-1}) = x(t_j^{i-1})$. Else, $x(t_j^i) = x(t_j^{i-1})$ and $x(t_j^{i-2}) = x(t_j^{i-2})$.

4. If $(H_{t_j}^i(t_j) - (e^{-rt_j}C_j(\cdot))_{t_j}^i - H_{j+1}^i(t_j))(H_{t_j}^{i-2}(t_j) - (e^{-rt_j}C_j(\cdot))_{t_j}^{i-2} - H_{j+1}^{i-2}(t_j)) > 0$, then $t_j^i = t_j^{i-2}$ and $t_j^{i-1} = t_j^{i-1}$. Else, $t_j^i = t_j^{i-1}$ and $t_j^{i-2} = t_j^{i-2}$.

5. Stop and print output if $t_j^i - t_j^{i-1} < \varepsilon$ and $x(t_j^i) - x(t_j^{i-1}) < \varepsilon$ for all j, or $(\lambda_j^i(t_j) + (e^{-rt_j}C_j(\cdot))_{x(t_j)}^i - \lambda_{j+1}^i(t_j)) < \varepsilon$ and $(H_j^i(t_j) - (e^{-rt_j}C_j(\cdot))_{t_j}^i - H_{j+1}^i(t_j)) < \varepsilon$.

6. If $i = \hat{\imath}$, then stop and report progress; else, go to Step 1.