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Measuring producer welfare under output price uncertainty and risk non-neutrality

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Procedures to measure the producer welfare effects of changes in an output price distribution under uncertainty are reviewed. Theory and numerical integration methods are combined to show how for any form of Marshallian risk-responsive supply, compensating variation of a change in higher moments of an output price distribution can be derived numerically. The numerical procedure enables measurement of producer welfare effects in the many circumstances in which risk and uncertainty are important elements. The practical ease and potential usefulness of the procedure is illustrated by measuring the producer welfare effects of USA rice policy.

Key words: price uncertainty, risk non-neutrality, welfare economics.

1. Introduction

Extensive published literature exists on evaluating producer welfare consequences of changes in agricultural policies, prices and technology. Typically, the classical producer surplus measure, the geometric area behind an ordinary supply curve generated assuming non-random prices, is used to identify the welfare consequences of such changes (e.g., Cramer *et al.* 1990). However, reality is often more complicated than this simple procedure can address. Producers face price uncertainty, and policy changes can alter the entire price distribution. The question is how to measure the *ex ante* producer welfare effects of such a change in the price distribution when the producer is not risk-neutral. Unfortunately, in this case, the classical producer surplus measure does not provide a meaningful estimate of the welfare impact of the policy or price change.[1]

One objective of the present paper is to review the methods that measure welfare changes when a change in policy leads to a change in the price distribution. This review allows us to discuss why little empirical research using these procedures exists. We then combine theory with numerical integration methods to show how, for any form of Marshallian risk-responsive supply, compensating variation of a change in higher moments of an output price distribution can be derived. Our procedure can help policy analysts measure producer welfare effects in circumstances in which risk is

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[1] We distinguish between cases of price uncertainty and price variability, and focus on price uncertainty. When prices vary but are known, producers' welfare impacts may be measured by the classical 'producer surplus' area behind ordinary supply curves.

an important element. To illustrate the practical ease and usefulness of our procedure we apply it to measure the producer welfare effects of USA rice policy.

2. Model, notation and definitions

It is convenient to establish notation and definitions for the commonly used expected utility model developed by Baron (1970), Sandmo (1971), Batra and Ullah (1974), and others.[2] The producer's objective is to solve the maximisation problem:

$$\max_{\mathbf{x}} E\{U(W + \mathbf{pf}(\mathbf{x}) - \mathbf{rx}) \mid \gamma\}, \tag{1}$$

where E is an expectations operator, U is a utility function, $\mathbf{p}$ is an m-dimensional vector of output prices, $\mathbf{f}(\mathbf{x})$ is a corresponding m-dimensional vector of (non-joint) production functions, $\mathbf{r}$ is an l-dimensional vector of input prices, $\mathbf{x}$ is a corresponding l-dimensional vector of input quantities, W is non-random and exogenously determined wealth, and the n pertinent parameters are contained in a vector $\boldsymbol{\gamma} \in \mathrm{R}^{\mathrm{n}}$. The price vectors $\mathbf{p}$ and $\mathbf{r}$, and the vector of production functions $\mathbf{f}(\mathbf{x})$ may be considered random or non-random variables, depending on the case at hand. The moments of the distribution(s) of the random variables $\mathbf{p}$, $\mathbf{r}$, and $\mathbf{f}(\mathbf{x})$ in (1) are included as elements of $\boldsymbol{\gamma}$, and we explicitly denote that expectations depend on $\boldsymbol{\gamma}$. Initial wealth W is also an element of $\boldsymbol{\gamma}$.

In Appendix 1 (Bullock *et al.* 2005), we derive and define concepts common in the literature: the indirect certainty equivalent function $L^*(\boldsymbol{\gamma})$, the vector of (Marshallian) risk-responsive input demand functions $\mathbf{x}^*(\boldsymbol{\gamma})$, and the (Marshallian) profit function $\pi^*(\boldsymbol{\gamma})$. Let $\boldsymbol{\gamma}_1$ denote a vector of parameters in an initial situation, and $\boldsymbol{\gamma}_2$ denote a vector of model parameters in a subsequent situation. An intuitively appealing welfare measure of the effect on the firm of a change in parameters from $\boldsymbol{\gamma}_1$ to $\boldsymbol{\gamma}_2$ is compensation-dependent *ex ante* compensating variation, implicitly defined by identity (2):[3]

$$\underbrace{E\{U(W_1 + \mathbf{pf}(\mathbf{x}^*(\boldsymbol{\gamma}_1)) - \mathbf{rx}^*(\boldsymbol{\gamma}_1)) \mid \boldsymbol{\gamma}_1\}}_{EU^*(\boldsymbol{\gamma}_1)}$$

$$\equiv E\Bigg\{ U\Bigg[W_2 - c(\boldsymbol{\gamma}_1, \boldsymbol{\gamma}_2, EU^*(\boldsymbol{\gamma}_1))$$

$$+ \underbrace{\overbrace{\mathbf{pf}(\mathbf{x}^*(\boldsymbol{\gamma}_{-W_2}, W_2 - c(\boldsymbol{\gamma}_1, \boldsymbol{\gamma}_2, EU^*(\boldsymbol{\gamma}_1))))}^{\mathbf{y}_c(\boldsymbol{\gamma}_1, \boldsymbol{\gamma}_2, EU^*(\boldsymbol{\gamma}_1))} - \overbrace{\mathbf{rx}^*(\boldsymbol{\gamma}_{-W_2}, W_2 - c(\boldsymbol{\gamma}_1, \boldsymbol{\gamma}_2, EU^*(\boldsymbol{\gamma}_1)))}^{\mathbf{x}_c(\boldsymbol{\gamma}_1, \boldsymbol{\gamma}_2, EU^*(\boldsymbol{\gamma}_1))}}_{\pi_c(\boldsymbol{\gamma}_1, \boldsymbol{\gamma}_2, EU^*(\boldsymbol{\gamma}_1))} \Bigg] \Bigg| \boldsymbol{\gamma}_2 \Bigg\}. \tag{2}$$

[2] Initially our model, notation, and definitions follow Pope *et al.* (1983), though our notation is slightly less sparse to aid in explanation.

[3] Equivalent variation has equally intuitive appeal, but we omit further discussion of equivalent variation to save space.

The compensation-dependent compensating variation function $c(\boldsymbol{\gamma}_1, \boldsymbol{\gamma}_2, EU^*(\boldsymbol{\gamma}_1))$ shows the maximum amount of money the firm would pay,[4] (before making any input/output decisions) to face parameter vector $\boldsymbol{\gamma}_2$ instead of $\boldsymbol{\gamma}_1$. Also defined in (2) are the Hicksian profit function $\pi_c(\boldsymbol{\gamma}_1, \boldsymbol{\gamma}_2, EU^*(\boldsymbol{\gamma}_1))$, the vector of Hicksian risk-responsive supply functions $\mathbf{y}_c(\boldsymbol{\gamma}_1, \boldsymbol{\gamma}_2, EU^*(\boldsymbol{\gamma}_1))$, and the vector of Hicksian risk-responsive input demand functions, $\mathbf{x}_c(\boldsymbol{\gamma}_1, \boldsymbol{\gamma}_2, EU^*(\boldsymbol{\gamma}_1))$. These functions show the profits, supplies, and input demands of a firm facing parameter vector $\boldsymbol{\gamma}_2$ and receiving/paying compensation $c(\boldsymbol{\gamma}_1, \boldsymbol{\gamma}_2, EU^*(\boldsymbol{\gamma}_1))$ before taking any action.

Conceptually, the compensating variation function $c(\boldsymbol{\gamma}_1, \boldsymbol{\gamma}_2, EU^*(\boldsymbol{\gamma}_1))$ defined in (2) can be applied in numerous situations. A change from $\boldsymbol{\gamma}_1$ to $\boldsymbol{\gamma}_2$ can represent changes in the distribution(s) of output and input prices $(\mathbf{p}, \mathbf{r})$, and/or in the parameters of production functions $\mathbf{f}(\mathbf{x})$. In the present paper, we follow the main thrust of the literature by focusing on situations in which the distribution of a single output price changes. (The framework presented can be extended to more general changes in the joint distribution of all prices and production functions, but only at the expense of notational complication.)

3. Literature

The producer welfare measure defined in (2), the *ex ante* compensating variation function, is based on the risk preferences of the firm as revealed by the utility function U. Because risk preferences are not observed directly from market data, empirical application of this stochastic welfare measure has been restricted. Most papers have focused on imposing restrictive assumptions on changes in the price distribution and on producers' risk preferences to derive procedures that relate data to theoretical measures of changes in producer welfare under uncertainty and risk aversion. Little attempt has been made to apply this theory to real-world data.

3.1 Chavas and Pope (1981)

Chavas and Pope (1981) examined the use of risk-responsive supply functions to measure the welfare effects of a change in the mean of the output price distribution. Their measure of compensating variation differs slightly from that defined implicitly in (2) in that they assume that the producer's decisions do not depend on the compensation itself. We call this compensation-independent compensating variation, and denote it c^{ci}. This measure is defined implicitly by Equation (3):

$$\underbrace{E\{U(W_1 + \mathbf{pf}(\mathbf{x}^*(\boldsymbol{\gamma}_1)) - \mathbf{rx}^*(\boldsymbol{\gamma}_1))|\boldsymbol{\gamma}_1\}}_{EU^*(\boldsymbol{\gamma}_1)}$$

$$\equiv E\{U[W_2 + \mathbf{pf}(\mathbf{x}^*(\boldsymbol{\gamma}_{-W_2}, W_2)) - \mathbf{rx}^*(\boldsymbol{\gamma}_{-W_2}, W_2) - c^{ci}(\boldsymbol{\gamma}_1, \boldsymbol{\gamma}_2, EU^*(\boldsymbol{\gamma}_1))] \,|\, \boldsymbol{\gamma}_2\}. \quad (3)$$

[4] Or, if negative, its absolute value is the minimum amount the firm would be willing to receive.

Chavas and Pope (1981) showed that Equation (3) implies that compensation-independent compensating variation is identical to the change in the indirect certainty equivalent:

$$c^{ci}\left(\boldsymbol{\gamma}_1, \boldsymbol{\gamma}_2, EU^*\left(\boldsymbol{\gamma}_1\right)\right) \equiv \underbrace{L^*\left(\boldsymbol{\gamma}_2\right) - L^*\left(\boldsymbol{\gamma}_1\right)}_{\Delta L^*(\boldsymbol{\gamma}_1, \boldsymbol{\gamma}_2)}. \tag{4}$$

Chavas and Pope (1981) addressed the situation in which only one element of $\boldsymbol{\gamma}$, the mean of the output price distribution, changes. Write $\boldsymbol{\gamma}_1 = (\mu_1, \boldsymbol{\gamma}_{-\mu})$ and $\boldsymbol{\gamma}_2 = (\mu_2, \boldsymbol{\gamma}_{-\mu})$ to reflect the change in the parameter vector between situations 1 and 2. Let $y^*(\mu, \boldsymbol{\gamma}_{-\mu})$ represent the 'risk-responsive supply function', which shows how producers respond to changes in the parameters in question. Chavas and Pope's (1981) analysis focused on the welfare implications of the geometric area behind the 'risk-responsive supply curve' between the means of two price distributions (Figure 1), and is expressed as:

$$\int_{\mu_1}^{\mu_2} y^*\left(\mu, \boldsymbol{\gamma}_{-\mu}^0\right) d\mu. \tag{5}$$

This geometric area seems familiar, because it resembles the classical change in 'producer surplus.' However, there are two differences. In Figure 1 the mean of the price distribution, and not the price itself, is the variable on the vertical axis, and the integral is taken behind a 'risk-responsive' supply curve, not an ordinary supply curve.

Chavas and Pope (1981) showed that when a firm is risk-averse, the geometric area represented by the integral in (5) equals compensation-independent compensating variation in (3) and (4), under the assumptions of (a) constant absolute risk aversion

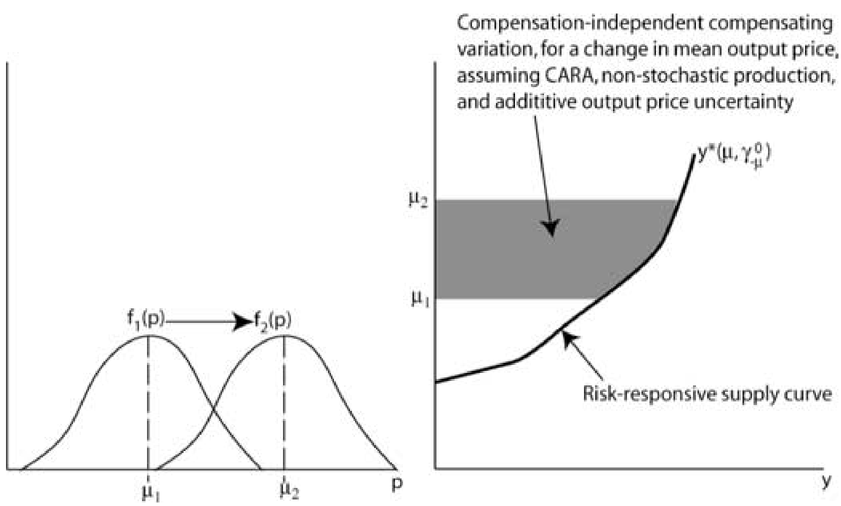

Figure 1 Chavas and Pope's (1981) measure of the change in producer welfare when the mean of the output price distribution changes and all other moments remain unchanged.

(CARA), (b) non-stochastic production, and (c) additive output price uncertainty. Under these assumptions,

$$c^{ci}\left(\left(\mu_1, \boldsymbol{\gamma}^0_{-\mu}\right), \left(\mu_2, \boldsymbol{\gamma}^0_{-\mu}\right), EU^*\left(\mu_1, \boldsymbol{\gamma}^0_{-\mu}\right)\right) \equiv \int_{\mu_1}^{\mu_2} y^*\left(\mu, \boldsymbol{\gamma}^0_{-\mu}\right) d\mu. \tag{6}$$

The welfare measure shown in (6) is restrictive in three ways. First, it only allows one parameter in $\boldsymbol{\gamma}$ to change. Second, the assumptions of constant absolute risk aversion, non-stochastic production, and additive output price uncertainty are restrictive, and even unreasonable in many applications. Third, using compensation-independent compensating variation as the welfare measure may be limiting as it assumes that the producer is not fully aware, before making production decisions, of potential compensation.

3.2 Pope *et al.* (1983)

Pope *et al.* (1983) derived measures of producer welfare change under more general changes in the parameter vector $\boldsymbol{\gamma}$. Using line integral theory (cf. Kaplan 1984) and taking partial derivatives of Equation (A.1.5) in Appendix 1 of Bullock *et al.* (2005), Pope *et al.* (1983) derived Equation (7) below, which shows the change in the indirect certainty equivalent implied by a change in the parameters from $\boldsymbol{\gamma}_1$ to $\boldsymbol{\gamma}_2$.

$$\begin{aligned}\Delta L^*(\boldsymbol{\gamma}_1, \boldsymbol{\gamma}_2) &\equiv L^*(\boldsymbol{\gamma}_2) - L^*(\boldsymbol{\gamma}_1)\\ &\equiv \int_{\boldsymbol{\gamma}_1}^{\boldsymbol{\gamma}_2} \frac{\partial L^*(\mathbf{z})}{\partial \mathbf{z}} \mathbf{dz}\\ &\equiv \int_{\boldsymbol{\gamma}_1}^{\boldsymbol{\gamma}_2} [U^{-1'}()] \left[E\left\{ U'(), \frac{\partial \pi^*(\mathbf{z})}{\partial \mathbf{z}} \,\middle|\, \mathbf{z} \right\}\right] \mathbf{dz}\\ &\equiv \int_{\boldsymbol{\gamma}_1}^{\boldsymbol{\gamma}_2} [U^{-1'}()] \left[E\{U'(), \mid \mathbf{z}\} E\left\{ \frac{\partial \pi^*(\mathbf{z})}{\partial \mathbf{z}} \,\middle|\, \mathbf{z} \right\} + \mathrm{cov}\left(U'(), \frac{\partial \pi^*(\mathbf{z})}{\partial \mathbf{z}} \right)\right] \mathbf{dz}.\end{aligned} \tag{7}$$

Pope *et al.*'s (1983) measure is quite general, placing no restrictions on risk preferences, the form of the distribution of output prices, or the type of output price uncertainty (e.g., additive, multiplicative);[5] it also allows output to be stochastic. In (7), $\mathbf{z}$ is a vector of dummy variables of integration holding the place of the vector of arguments $\boldsymbol{\gamma}$ of the function $L^*(\boldsymbol{\gamma})$. Also, $U^{-1}(\cdot) \equiv U^{-1}(E\{W + \pi^*(\mathbf{z}) \mid \mathbf{z}\})$, and $U(\cdot) \equiv U(W + \pi^*(\mathbf{z}))$. Because the line integral in (7) is path independent (Kaplan 1984, pp. 291–298), the path of integration is an arbitrary one between endpoints $\boldsymbol{\gamma}_1$ and $\boldsymbol{\gamma}_2$. However, (7) is difficult to apply to real-world research, because estimation of the covariance term $\mathrm{cov}(U'(), \frac{\partial \pi^*(\mathbf{z})}{\partial z})$ is problematic.

[5] Under additive uncertainty output price typically takes the form $p = f(\cdot) + \varepsilon$; under multiplicative uncertainty, output price typically takes the form $p = f(\cdot)\varepsilon$, where in either case $f(\cdot)$ is a function of various parameters, and ε is a random variable.

Pope *et al.* (1983) also derived an expression for compensation-dependent compensating variation (Kaplan 1984).[6]

$$
\begin{aligned}
&c(\boldsymbol{\gamma}_1, \boldsymbol{\gamma}_2, EU^*(\boldsymbol{\gamma}_1))\\
&\equiv c(\boldsymbol{\gamma}_1, \boldsymbol{\gamma}_2, EU^*(\boldsymbol{\gamma}_1)) - \underbrace{c(\boldsymbol{\gamma}_1, \boldsymbol{\gamma}_1, EU^*(\boldsymbol{\gamma}_1))}_{0}\\
&\equiv \int_{\boldsymbol{\gamma}_1}^{\boldsymbol{\gamma}_2} \left[\sum_{j=1}^{n} \frac{\partial c(\boldsymbol{\gamma}_1, \mathbf{z}, EU^*(\boldsymbol{\gamma}_1))}{\partial z_j} dz_j \right]\\
&\equiv \int_{\boldsymbol{\gamma}_1}^{\boldsymbol{\gamma}_2} \left[\sum_{j=1}^{n} \left(\frac{\partial W}{\partial z_j} + E\left\{ \frac{\partial \pi_c(\boldsymbol{\gamma}_1, \mathbf{z}, EU^*(\boldsymbol{\gamma}_1))}{\partial z_j} \middle| \mathbf{z} \right\} \right.\right.\\
&\left.\left. + \frac{\operatorname{cov}\left((U'(W - c(\boldsymbol{\gamma}_1, \mathbf{z}, EU^*(\boldsymbol{\gamma}_1)) + \pi_c(\boldsymbol{\gamma}_1, \mathbf{z}, EU^*(\boldsymbol{\gamma}_1))), \frac{\partial \pi_c(\boldsymbol{\gamma}_1, \mathbf{z}, EU^*(\boldsymbol{\gamma}_1))}{\partial z_j} \right)}{E\{U'(W - c(\boldsymbol{\gamma}_1, \mathbf{z}, EU^*(\boldsymbol{\gamma}_1)) + \pi_c(\boldsymbol{\gamma}_1, \mathbf{z}, EU^*(\boldsymbol{\gamma}_1))) \mid \mathbf{z}\}} \right) dz_j \right].
\end{aligned}
\tag{8}
$$

As with (7), Equation (8) holds for the general case identified immediately above, but contains a complicated covariance term that is difficult to estimate in applied research.

Pope *et al.* (1983) pointed out that if we assume additive output price uncertainty and non-stochastic production, an implication of (8) and the envelope theorem is that compensation-dependent compensating variation for the change from $\boldsymbol{\gamma}_1 = (\mu_1, \boldsymbol{\gamma}^0_{-\mu})$ to $\boldsymbol{\gamma}_2 = (\mu_2, \boldsymbol{\gamma}^0_{-\mu})$ may be found by taking a definite integral behind the Hicksian risk-responsive supply curve:

$$
c\left(\overbrace{(\mu_1, \boldsymbol{\gamma}^0_{-\mu})}^{\gamma_1} \overbrace{(\mu_2, \boldsymbol{\gamma}^0_{-\mu})}^{\gamma_2}, EU^*(\boldsymbol{\gamma}_1) \right) \equiv \int_{\mu_1}^{\mu_2} \mathbf{y}_c \left(\overbrace{(\mu_1, \boldsymbol{\gamma}^0_{-\mu})}^{\gamma_1} (\mu, \boldsymbol{\gamma}^0_{-\mu}), EU^*(\boldsymbol{\gamma}_1) \right) d\mu. \tag{9}
$$

[6] To explain the derivation in our notation, first differentiate compensation-*dependent* compensating variation in (2) to obtain a term for $\partial c(\boldsymbol{\gamma}_1, \boldsymbol{\gamma}_2, EU^*(\boldsymbol{\gamma}_1))/\partial \boldsymbol{\gamma}_2 = \partial c(\boldsymbol{\gamma}_1, \boldsymbol{\gamma}_2, EU^*(\boldsymbol{\gamma}_1))/\partial \gamma_{2_1}, \ldots, (\partial c(\boldsymbol{\gamma}_1, \boldsymbol{\gamma}_2, EU^*(\boldsymbol{\gamma}_1))/\partial \gamma_{2_n})$. Then, let a vector of dummy variables $\mathbf{z}$ stand in place of $\boldsymbol{\gamma}_2$, which is the second vector of arguments of function $c(\boldsymbol{\gamma}_1, \boldsymbol{\gamma}_2, EU^*(\boldsymbol{\gamma}_1))$. Keeping the first vector of arguments $\boldsymbol{\gamma}_1$ and the last argument $EU^*(\boldsymbol{\gamma}_1)$ constant, allow $\mathbf{z}$ to move over a path of integration that has endpoints $\boldsymbol{\gamma}_1$ and $\boldsymbol{\gamma}_2$, and take a line integral of $\partial c(\boldsymbol{\gamma}_1, \mathbf{z}, EU^*(\boldsymbol{\gamma}_1))/\partial \mathbf{z} = (\partial c(\boldsymbol{\gamma}_1, \mathbf{z}, EU^*(\boldsymbol{\gamma}_1))/\partial z_1, \ldots, \partial c(\boldsymbol{\gamma}_1, \mathbf{z}, EU^*(\boldsymbol{\gamma}_1))/\partial z_n)$ to obtain a term for compensation-dependent compensating variation. To ease the notational burden we assume W to be an element of vector $\boldsymbol{\gamma}$. In Pope *et al.* (1983), $\boldsymbol{\gamma}$ represents all parameters except W. Our formula in (8) has the term $\partial W/\partial z_j$, and Pope *et al.*'s (1983) Equation 9 lacks this term only because of the notational difference.

Note that (9) requires no restrictions on risk preferences. Therefore, in addition to providing a measure of the theoretically preferable compensation-dependent compensating variation (9) is slightly more general than Chavas and Pope's (1981) measure in (6), which was derived by assuming additive output price uncertainty, non-stochastic production, and CARA. Also, because only parameter μ is changed in (9), the covariance terms in (7) and (8) disappear. However, identity (9) has two limitations. First, to apply (9) the functional form of the Hicksian supply curves must be known, and estimation of a Hicksian supply curve when utility is unobservable presents a challenge. Second, (9) only considers a change in the mean output price parameter.

Pope *et al.* (1983) partially addressed the first limitation by showing that when a CARA firm faces a change in one or more of the means of the output price distributions (these means are the elements of a vector $\boldsymbol{\mu}$), the Hicksian and Marshallian risk-responsive supply curves are the same. The implication is that with CARA utility and non-stochastic production, the compensation-dependent compensating variation in (9), the difference in the certainty equivalent and compensation-independent compensating variation in (6) are equal,[7] and can be represented by an integral similar to the classic producer surplus integral, except that the means of the price distributions, and not actual prices, are used:[8]

$$\begin{aligned}
\Delta L^*\left(\overbrace{(\mu_1,\boldsymbol{\gamma}^0_{-\mu})}^{\gamma_1},\overbrace{(\mu_2,\boldsymbol{\gamma}^0_{-\mu})}^{\gamma_2}\right) &\equiv \int_{\mu_1}^{\mu_2}\mathbf{y}^*(\mu,\boldsymbol{\gamma}_{-\mu_1})\,d\mu \\
&\equiv \int_{\mu_1}^{\mu_2} y_c\left(\overbrace{(\mu_1,\boldsymbol{\gamma}^0_{-\mu})}^{\gamma_1},(\mu,\boldsymbol{\gamma}^0_{-\mu}),EU^*(\mu_1,\boldsymbol{\gamma}_1)\right)d\mu \\
&\equiv c\left(\overbrace{(\mu_1,\boldsymbol{\gamma}^0_{-\mu})}^{\gamma_1},\overbrace{(\mu_2,\boldsymbol{\gamma}^0_{-\mu})}^{\gamma_2},EU^*(\mu_1,\boldsymbol{\gamma}_1)\right) \\
&\equiv c^{ci}\left(\overbrace{(\mu_1,\boldsymbol{\gamma}^0_{-\mu})}^{\gamma_1},\overbrace{(\mu_2,\boldsymbol{\gamma}^0_{-\mu})}^{\gamma_2},EU^*(\mu_1,\boldsymbol{\gamma}_1)\right).
\end{aligned} \tag{10}$$

In (10), $\boldsymbol{\gamma}_1 = (\mu_1,\boldsymbol{\gamma}_{-\mu_1})$ and $\boldsymbol{\gamma}_2 = (\mu_2,\boldsymbol{\gamma}_{-\mu_1})$ as only the means of the output price distributions change. In (10), Pope *et al.* (1983) derived what in many cases seems like an empirically practical measure of producer welfare change under uncertainty: the measure employs Marshallian instead of Hicksian supply curves, and there is no

[7] When vector μ in has more than one element, we must reinterpret (10) as follows. The single supply function $y^*()$ should be changed to a vector of supply functions $y^*()$, and $\mathbf{d}\boldsymbol{\mu}$ is a vector with the same dimensions. The definite integral is a path-independent line integral, which can be converted to a sum of definite integrals by letting the variables in μ change sequentially between endpoints μ_1 and μ_2. See Pope *et al.* (1983, footnote 2) and Just *et al.* (1982, pp. 338–341).

[8] Pope *et al.* (1983) showed that under CARA and-non-stochastic production, this integral also represents equivalent variation.

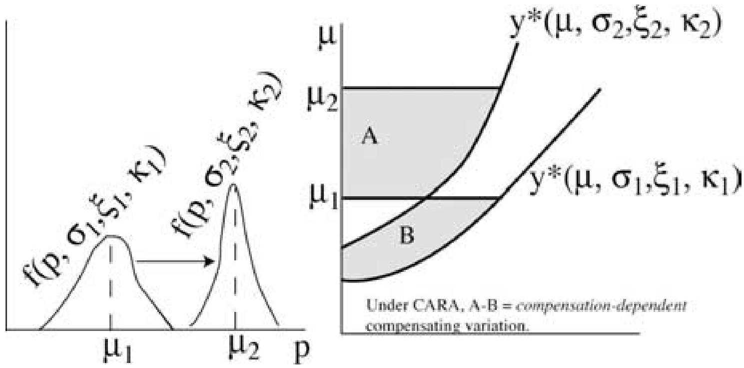

Figure 2 Pope and Chavas (1983) use the "shutdown price" and CARA assumptions to derive a measure of the change in welfare due to a change in more than one moment of the output price distribution.

complicated covariance term to estimate such as those appearing in (7) and (8). But the practical applicability of (10) was achieved through the restrictive assumptions of CARA and that only $\boldsymbol{\mu}$ changes.[9]

To address the limitation that (9) and (10) are not applicable to changes in higher moments of the output price distribution, Pope *et al.* (1983) suggested using the concept of shutdown prices to measure producer welfare changes associated with general changes in the moments of the output price distribution. They argue (Figure 2) that the difference in geometric areas behind two two-dimensional Marshallian risk-responsive supply curves, where each curve is drawn as dependent on the mean output price, may be used to measure the change in welfare.[10]

Pope *et al.* (1983) identified two unanswered research questions (Figure 3). The first question was whether for practical purposes the Marshallian risk-responsive supply curve closely approximates the Hicksian risk-responsive supply curve. Citing arguments consistent with Willig (1976), they suggested that even if the CARA assumption does not hold, only a small error is obtained from integrating behind (observable) Marshallian supply curves, instead of integrating behind (unobservable) Hicksian supply curves to obtain compensating variation.

Pope *et al.*'s (1983) second unanswered research question was whether it was possible, following Hausman (1981), to analytically solve ordinary differential equations to measure the geometric areas behind (unobserved) Hicksian curves using just the information revealed by (observed) Marshallian curves. For the consumer case and only one price change, Hausman's (1981) method derives an 'exact' measurement of welfare change. This method may be useful in the measurement of dead-weight losses, which may not be well approximated even when Willig's (1976) bounds imply that the total change in consumer welfare is closely approximated (Hausman 1981).

[9] Chavas and Holt (1990, p. 536) report empirical findings that cast doubt on the use of CARA utility functions.

[10] In Appendix 2 (Bullock *et al.* 2005), we offer our own detailed explanation of the shutdown price method.

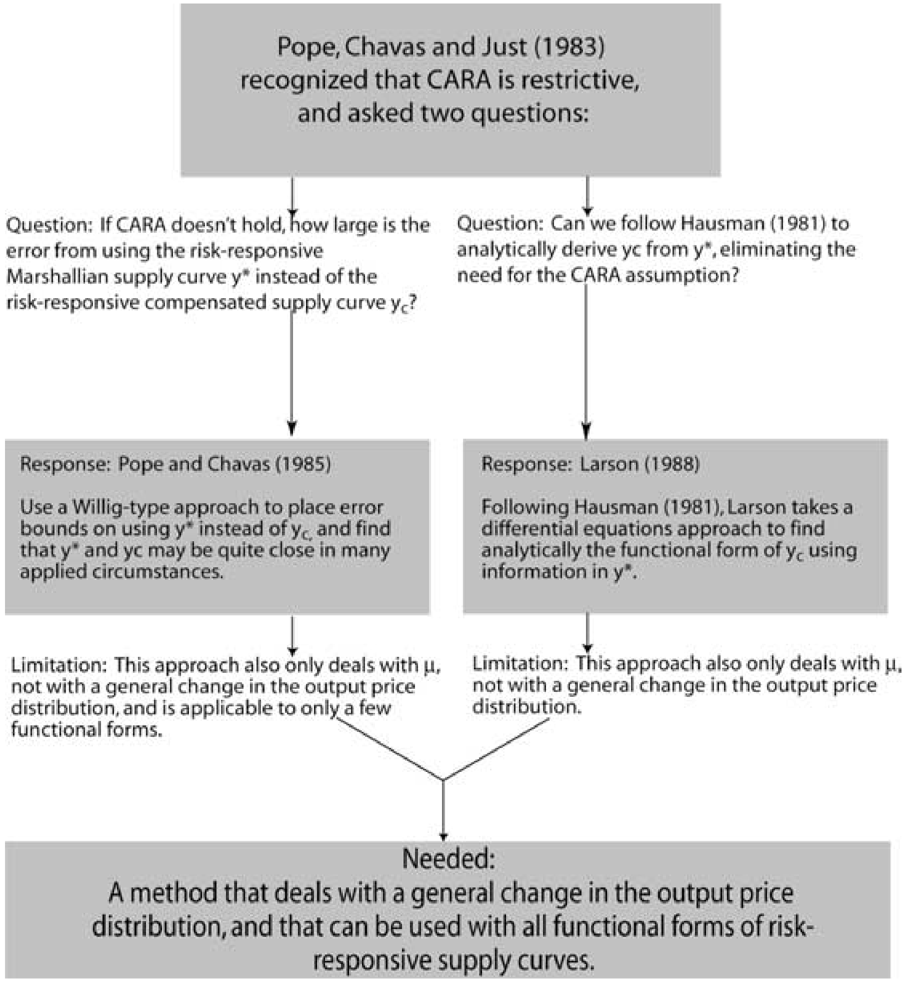

Figure 3 Progress in measuring welfare changes under output price uncertainty.

3.3 Pope and Chavas (1985)

Pope and Chavas (1985) investigated Pope *et al.*'s (1983) first unanswered question (Figure 3), and proposed an approximation to compensating variation for a change in the mean output price distribution, assuming decreasing absolute risk aversion (DARA). The authors placed Willig-type error bounds on compensating variation using the change in classical producer surplus. They derived bounds for the 'percentage error' of using the integral behind the Marshallian risk-responsive supply curve. They asserted that for most reasonable policy settings the bounds are narrow, providing a priori evidence that the integral behind the Marshallian risk-responsive supply curve may be a good approximation to compensating variation (less than 5% error).

There are limitations to this Willig-type approach. First, it is not applicable to welfare measurement associated with changes in price risk, that is, with higher moments of the price distribution. Second, for many firms a change in the output price distribution might have considerable wealth effects, causing the approximation error to be large, discouraging the use of a Willig-type approach. Third, even when the Willig-type approach provides good approximations for changes in one group's welfare, it may provide a poor approximation of changes in dead-weight costs (Hausman 1981).

3.4 Larson (1988)

To address Pope *et al.*'s (1983) second unanswered research question (Figure 3), and to address the limitations of the Willig-type approach, Larson (1988) followed Hausman (1981) to show how ordinary differential equations can be solved to analytically derive closed-form expressions of compensating variation associated with a change in the mean of an output price distribution.

Larson's (1988) procedure is more general than that of Pope *et al.* (1983) in that it relaxes their assumptions of CARA or DARA. However, Larson's (1988) method can be used only with Marshallian risk-responsive supply functions from which the functional form of the Hicksian risk-responsive supply function can be analytically derived. Also, Larson (1988) showed how his method could be used to find compensation-dependent compensating variation from a change in only the mean of a single output price's distribution. He did not consider the welfare effects of changes in other parameters.

3.5 Tsur (1993)

Tsur (1993) argued that the procedures reviewed above were difficult to implement and rarely used. He asserted that the major difficulty was estimation of the risk-responsive Marshallian supply function $y^*(\boldsymbol{\gamma})$, or the risk-responsive Hicksian supply function $y_c(\boldsymbol{\gamma}_1, \boldsymbol{\gamma}_2, EU^*(\boldsymbol{\gamma}_1))$ used in Equations (6), (9), and (10). In response, he developed 'a simple and practical technique for evaluating producer welfare under price uncertainty' (p. 44). Tsur's (1993) model has the producer solving an expected utility maximisation problem, similar to (1) except the producer chooses output y directly, and not indirectly through input choice x:

$$\underset{y}{\text{Max}} \left\{ \int_0^\infty U\left(py - C\left(y, \mathbf{c}\right);\ \mathbf{b}\right) g\left(p; \mathbf{a}\right)\, dp \right\}. \tag{1$'$}$$

The problem's parameters may be placed into three categories: $\boldsymbol{\gamma} = (\mathbf{a}, \mathbf{b}, \mathbf{c})$, where $\mathbf{a}$ is a vector of parameters in the probability density function $g(\,)$ of random output price p; $\mathbf{b}$ is a vector of parameters in the utility function $U(\,)$ and may include exogenous wealth; and $\mathbf{c}$ is a vector of parameters of the cost function $C(\cdot)$. Tsur (1993) developed analytical solutions for $y^*(\mathbf{a}, \mathbf{b}, \mathbf{c})$, and for the indirect certainty equivalent because of a change in the output price distribution assuming: (i) CARA preferences; (ii) a constant-returns-to-scale cost function; and (iii) the output price takes on either a gamma distribution or a truncated normal distribution. Implementation of Tsur's (1993) method requires knowledge of the marginal cost function, the output price distribution, and the absolute risk aversion coefficient.

The usefulness of Tsur's (1993) procedure relative to estimating a risk-responsive supply function can be questioned. Cost functions require data on input prices, output levels, and some measure of the level of technology. Estimation of cost functions with related input demand functions is facilitated by theoretical restrictions on the parameters, and by our profession's experience with various flexible forms. However, problems still exist in defining the specification and aggregation of input prices and quantities, and the specification of technical change over time. Also, cost functions are commonly estimated and reflect behaviour at different levels of aggregation (e.g., producers, geographical regions). Tsur's (1993) procedure appears to call for producer-level data, which requires a 'representative' sample, and similar accounting techniques to ensure meaningful parameters. In contrast, risk-responsive supply functions are estimated at market level, and require specification of price expectations and risk measures, as well as the functional form, and the nature of technical change.

In many cases, estimating risk-responsive supply functions is more practicable than Tsur (1993) asserted. A large number of risk-responsive supply functions have been estimated under various price expectations mechanisms, risk measures, and for a variety of crop and livestock commodities (Just 1974, 1975; Traill 1978; Hurt and Garcia 1982; Antonovitz and Green 1986, 1990; Brorsen *et al.* 1987; Seale and Shonkwiler 1987; Aradhyula and Holt 1989; Holt 1989, 1994; Chavas and Holt 1990, 1996; Holt and Aradhyula 1990, 1998; Holt and Moschini 1992; Park and Garcia 1994; Krause and Koo 1996; Rambaldi and Simmons 2000).

Another aspect of Tsur's (1993) method that may be difficult to develop empirically is a measure of the absolute risk aversion coefficient.[11] He offered a procedure to develop the risk coefficient by estimating production as a function of wealth, socioeconomic characteristics, and the parameters of the price distribution. However, Tsur's (1993) approximation offers little empirical guidance regarding the variables, their expected relationships, and ease of estimation. Further, determining the reasonableness of the risk coefficient is not trivial as comparisons to coefficients estimated using elicitation procedures may be suspect (Robison 1982; Pennings and Garcia 2001).

Finally, Tsur's (1993) procedure requires restrictive assumptions about the output price distribution, preferences, and the producer's cost function. Tsur (1993) showed how to implement his procedure if the price distribution shifts from one gamma distribution to another, or from one truncated normal distribution to another, but not for a more general shift from one type of distribution to another. Tsur's (1993) procedure also relies on CARA preferences and constant returns to scale technology, which may be untenable for many producers and industries.

3.6 Dual approach: using indirect expected utility

The dual method offers an alternative approach to welfare measurement under uncertainty. Similar to the dual approach under certainty, a form of the indirect expected utility function is assumed. Applying a version of Roy's identity provides output supply

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[11] Tsur (1993) provides a comparative analysis of the relative attractiveness of alternative programs that change the underlying price distributions under constant returns to scale production. In this case, the magnitude of the absolute risk coefficient is of concern only if the absolute welfare gains and the supply response to changes in uncertainty are of interest. For non-constant returns to scale technology or when the actual welfare or supply response measures are needed, the absolute risk aversion coefficient must be estimated or parameterised.

and input demand equations (Holt and Chavas 2002, p. 231). To date, few empirical applications exist (Appelbaum and Kohli 1997; Coyle 1999). The dual approach has advantages and disadvantages compared to other approaches. An advantage is that it uses symmetry from theory to reduce the number of estimated parameters in an applied model. A limitation is that it specifies the functional forms of the output supply and input demand equations 'rather than allowing the functional form to be determined by the observed data' (Slesnick 1998, p. 2114; Sheffrin and Turner 2001, p. 629). Pope and Saha (2002) write, '... duality is not at issue conceptually so much as the essence of the tradeoffs between method used, light shed on the problem, and the cost of using each approach.' (pp. 139–140).

In an applied context, Holt and Chavas (2002, p. 237) identify that while dual models under uncertainty exist, no serious attempt has been made 'to estimate a consistent set of output supply and input demand equations that incorporate reciprocity conditions.' Consequently, it is difficult to determine if these procedures offer further insights or explanatory power for the relationship between risk and producer response. Similarly, in light of limited information, our ability to use these procedures to effectively measure producer welfare in the presence of government programs that might alter higher moments of the price distribution is uncertain and requires further research.

4. Numerical procedure to measure producer welfare changes under price uncertainty

There is a gap in the published literature reviewed. There is no way to measure the change in producer welfare under general changes in the output price distribution for general functional forms of supply and non-restricted producer risk preferences. We narrow this gap by showing how numerical methods can be employed to measure welfare changes under price uncertainty. Our method offers several advantages over existing methods of estimating producer welfare changes under price uncertainty. Unlike Pope *et al.* (1983) and Tsur (1993), our method does not impose any restriction on risk preferences. Unlike Larson (1988), our procedures permit complete flexibility in specification of the functional form of the risk-responsive Marshallian supply function, and are applicable under general changes in the output price distribution – not just changes in the mean. Unlike Tsur (1993), our method is not limited to analysing changes from a distribution in one family of distributions to another distribution in that same family (such as from one gamma distribution to another). Unlike the dual approaches, 'because a closed form is not necessary' (Slesnick 1998, p. 2115), our method allows the functional form of the risk-responsive supply function to be determined by the data.

Numerical methods for applied welfare economics were introduced by Vartia (1983), who presented a method of deriving Hicksian from Marshallian curves. Several applications of Vartia's (1983) method to the measurement of consumer welfare changes under price certainty have appeared (Hayes and Porter-Hudak 1987; Fan *et al.* 1998; Minot 1998; Lavergne *et al.* 2001). Wright and Williams (1988) also have used numerical procedures in an indirect utility function framework to investigate the impact of price changes on consumer welfare under uncertainty. It is a natural step to extend

Vartia-type methods used to study changes in consumer welfare to measure the impact of price changes on producers. We take this step, focusing on the impact of price changes on producer welfare under uncertainty.[12] Our method provides a specific response to Sheffrin and Turner's (2001) critique that 'the numerical approximation methods of Vartia (1983) [do not] ... readily adapt to capture changes in the moments of the price distribution' (p. 629). Indeed we do adapt Vartia's (1983) method to approximate the compensation-dependent compensating variation for changes in the first and second moments of the output price distribution from information in a risk-responsive Marshallian supply function.[13] Though our procedure can be generalised, for purposes of illustration we confine our analysis to changes in three or fewer parameters.

4.1 Change in the parameter vector from (W_1, μ_1, σ_1) to (W_1, μ_2, σ_1)

First, we develop a procedure to find compensating variation for a change in parameter vector from (W_1, μ_1, σ_1) to (W_1, μ_2, σ_1). This is the same parameter change analysed by Chavas and Pope (1981), Pope *et al.* (1983), and Larson (1988). However, our numerical approach provides a number of advantages. Like Larson's (1988) method, our method does not restrict preferences to be CARA. Our method allows for use of any functional form of Marshallian risk-responsive supply, whereas Larson's (1988) differential equations approach can only be used with a few assumed functional forms. An advantage of our method over Chavas and Pope's (1981) is that we find compensation-dependent compensating variation, not compensation-independent. An advantage of our method over Chavas and Pope's (1985) and Tsur's (1993) is that we do not require the CARA assumption. Finally, our method has an advantage over Pope *et al.* (1983) in that we require only knowledge of the risk-responsive Marshallian supply function, whereas Pope *et al.* (1983) require knowledge of the risk-responsive Hicksian supply function, which is more difficult to estimate as utility is not observable.

Let t denote an auxiliary variable such that $0 = t = 1$, and let $\mu(t)$ be a differentiable function connecting $\mu_1 = \mu(0)$ to $\mu_2 = \mu(1)$. Let $h(t) \equiv c(\boldsymbol{\gamma}_1; W_1, \mu(t), \sigma_1; EU^*(\boldsymbol{\gamma}_1))$. That is, $h(t)$ is compensating variation for a change in the parameter vector from (W_1, μ_1, σ_1) to $(W_1, \mu(t), \sigma_1)$. Differentiating this identity with respect to t, using (8), the definition of Hicksian supply in (2), and the Fundamental Theorem of Calculus, we

[12] Earlier work used Vartia-type procedures to measure the welfare change from changes in certain prices. Because errors in measurement of welfare changes can result from econometric estimation of demand or supply curves, it is therefore desirable to estimate confidence intervals for estimates of welfare change. Hausman (1981, p. 669) discussed how analytical methods that solve differential equations could be used to construct confidence intervals for an estimate of compensating variation from a change in price faced by consumers with certainty. This work was gradually extended to the measurement of the mean and variance of the distribution of compensation variation for changes in n (certain) prices in an n-equation demand system (Porter-Hudak and Hayes 1986, 1991; Breslaw and Smith 1995). In a more related context, Wright and Williams (1988) use Vartia's numerical algorithm and an indirect utility function approach to derive an exact measure, the expected (*ex ante*) equivalent variation, of consumer gains from market stabilisation under price uncertainty.

[13] Similar numerical algorithms, such as those appearing in Breslaw and Smith (1995) or Porter-Hudak and Hayes (1986, 1991), can be adapted.

have a first-order differential equation in $h(t)$:

$$\begin{aligned}\frac{dh(t)}{dt} &\equiv \frac{\partial c(\boldsymbol{\gamma}_1; W_1, \mu(t), \sigma_1; EU^*(\boldsymbol{\gamma}_1))}{\partial \mu}\frac{d\mu(t)}{dt} \\ &\equiv y_c(\boldsymbol{\gamma}_1; W_1, \mu(t), \sigma_1; EU^*(\boldsymbol{\gamma}_1))\frac{d\mu(t)}{dt} \\ &\equiv y^*(W_1 - h(t), \mu(t), \sigma_1)\frac{d\mu(t)}{dt}. \end{aligned} \tag{11}$$

In (11), $\mu(t)$ and $d\mu(t)/dt$ are known functions, and $h(t)$ is an unknown function to be found. By noting that $h(0) = 0$ and integrating (11) with respect to t, we have:

$$\begin{aligned} h(t) &= h(t) - h(0) \\ &= \int_0^t \frac{dh(t)}{dt}dt = \int_0^t y^*(W_1 - h(t), \mu(t), \sigma_1)\frac{d\mu(t)}{dt}\, dt. \end{aligned} \tag{12}$$

Compensating variation for the change from μ_1 to μ_2, $h(1)$, is the solution to (12) if $t = 1$.

Next, we develop a practical method for calculating or approximating $h(1)$. For any functional form of supply, Equation (12) can be solved numerically. Following Vartia (1983), an algorithm that provides a numerical solution to (12) can be described. By choosing numbers $t_1, t_2, \ldots, t_{N+1}$ such that $0 = t_1 < t_2 < \cdots < t_{N+1} = 1$, we derive from (12) the following:

$$\begin{aligned} h(1) = h(t_{N+1}) &= \sum_{i=2}^{N+1} [h(t_k) - h(t_{k-1})] \\ &= \sum_{i=2}^{N+1} \int_{t_{k-1}}^{t_k} y^*(W_1 - h(t), \mu(t), \sigma_1)\frac{d\mu(t)}{dt}\, dt. \end{aligned} \tag{13}$$

Examining the sum of integrals in (13), when $t_k - t_{k-1}$ is small (i.e., when N is large) we approximate $y^*(W_1 - h(t), \mu(t), \sigma_1)$ by the mean of its values at the limits of the integral: $y^*(W_1 - h(t), \mu(t), \sigma_1) \approx 0.5[y^*(W_1 - h(t_{k-1}), \mu(t_{k-1}), \sigma_1) + y^*(W_1 - h(t_k), \mu(t_k), \sigma_1)]$, resulting in,

$$\begin{aligned} &\int_{t_{k-1}}^{t_k} y^*(W_1 - h(t), \mu(t), \sigma_1))\frac{d\mu(t)}{dt}dt \\ &\approx 0.5[y^*(W_1 - h(t_{k-1}), \mu(t_{k-1}), \sigma_1) + y^*(W_1 - h(t_k), \mu(t_k), \sigma_1)]\int_{t_{k-1}}^{t_k} \frac{d\mu(t)}{dt}\, dt \\ &= 0.5[y^*(W_1 - h(t_{k-1}), \mu(t_{k-1}), \sigma_1) + y^*(W_1 - h(t_k), \mu(t_k), \sigma_1)] \cdot [\mu(t_k) - \mu(t_{k-1})]. \end{aligned} \tag{14}$$

Next, assume that $\mu(t)$ is linear: $\mu(t) = \mu_1 + t[\mu_2 - \mu_1]$, $0 \le t \le 1$. For a given integer N, and for every $k = 1, \ldots, N + 1$, let $t_k = (k-1)/N$. To shorten the notation, let $h_k = h(t_k)$ and let $y_k^* = y^*(W_1 - h_k, \mu(t_k), \sigma_1)$, $k = 1, \ldots, N$. First begin with starting values of $\mu(t_1) = \mu_1$ and $h_1 = 0$. Then generate a sequence $h_2, \ldots, h_{N+1}$ such that

$$h_k = h_{k-1} + \frac{1}{2}\left[\underbrace{y^*(W_1 - h_{k-1}, \mu(t_{k-1}), \sigma_1)}_{y_{k-1}^*} + \underbrace{y^*(W_1 - h_k, \mu(t_k), \sigma_1)}_{y_k^*}\right] \times [\mu(t_k) - \mu(t_{k-1})]. \tag{15}$$

Because the term h_k appears on both sides of (15), it must be determined iteratively. Define $h_k^{(1)} = h_{k-1}$, and for $k = 2, \ldots, N+1$, and $m = 2, 3, \ldots,$ let

$$h_k^{(m)} = h_{k-1} + \frac{1}{2}\Big[y^*(W_1 - h_{k-1}, \mu(t_{k-1}), \sigma_1) + y^*\Big(W_1 - h_k^{(m-1)}, \mu(t_k), \sigma_1\Big)\Big] \times [\mu(t_k) - \mu(t_{k-1})]. \tag{16}$$

As the number m increases, $|h_k{}^{(m)} - h_k^{(m-1)}|$ will become negligibly small. When at some number M_k, $|h_k^{(Mk)} - h_k^{(Mk-1)}|$ is deemed sufficiently small, we can use (15) to write

$$h_k \approx \frac{1}{2}\Big[y^*(W_1 - h_{k-1}, \mu(t_{k-1}), \sigma_1) + y^*\Big(W_1 - h_k^{(M_k)}, \mu(t_k), \sigma_1\Big)\Big] \times [\mu(t_k) - \mu(t_{k-1})], \tag{17}$$

and start the calculation for $k + 1$.

Once the value of h_k is approximated for $k = 1, \ldots, N+1$, the compensation-*dependent* compensating variation for the change from μ_1 to μ_2 can be approximated following (13) as,

$$c(W_1, \mu_1, \sigma_1; W_1, \mu_2, \sigma_1; EU^*(\boldsymbol{\gamma}_1)) \approx \sum_{k=2}^{N+1} 0.5\Big[y^*\Big(W_1 - h_{k-1}^{(M_{k-1})}, \mu(t_{k-1}), \sigma_1\Big) + y^*\Big(W_1 - h_k^{(M_k)}, \mu(t_k), \sigma_1\Big)\Big] \times [\mu(t_k) - \mu(t_{k-1})]. \tag{18}$$

Letting N grow arbitrarily large allows (18) to provide an arbitrarily close approximation of compensating variation.

4.2 Change in the parameter vector from (W_1, μ_1, σ_1) to (W_2, μ_2, σ_2)

Now we consider the more general case in which the whole parameter vector changes from (W_1, μ_1, σ_1) to (W_2, μ_2, σ_2). This is the case analysed by Pope *et al.* (1983). The advantage of our numerical procedure is that it does not require knowledge of the risk-responsive Hicksian supply function, but instead employs the risk-responsive Marshallian supply function, which in principle is easier to estimate empirically.

Furthermore, our procedure can be easily generalised to analyse the producer welfare effect of changes in third and higher moments of the output price distribution.

By assuming the existence of appropriate shutdown mean prices, we can derive a formula for compensating variation for the general parameter change (see Equation (A.2.6) in Appendix 1, Bullock *et al.* 2005):

$$c\left(\overbrace{\mu_1, \sigma_1, W_1}^{\boldsymbol{\gamma}_1}; \overbrace{\mu_2, \sigma_2, W_2}^{\boldsymbol{\gamma}_2}; EU^*(\boldsymbol{\gamma}_1)\right)$$

$$\equiv \underbrace{\int_{W_1}^{W_2} 1 \cdot dW}_{W_2 - W_1} + \underbrace{\int_{\mu_1}^{\mu^*(W_2, \sigma_1, EU^*(\boldsymbol{\gamma}_1))} y_c(\boldsymbol{\gamma}_1; W_2, \mu, \sigma_1; EU^*(\boldsymbol{\gamma}_1))\, d\mu}_{c(W_2, \mu_1, \sigma_1; W_2, \mu^*(W_2, \sigma_1, EU^*(\boldsymbol{\gamma}_1)), \sigma_1; EU^*(\boldsymbol{\gamma}_1))}$$

$$+ \underbrace{\int_{\mu^*(W_2, \sigma_2, EU^*(\boldsymbol{\gamma}_1)))}^{\mu_2} y_c(\boldsymbol{\gamma}_1; W_2, \mu, \sigma_2; EU^*(\boldsymbol{\gamma}_1))\, d\mu}_{c(W_2, \mu^*(W_2, \sigma_2, EU^*(\boldsymbol{\gamma}_1)), \sigma_2; \mu_2, \sigma_2, W_2; EU^*(\boldsymbol{\gamma}_1))} . \quad (19)$$

We can approximate the second integral on the right-hand side of (19) in a manner similar to (18), except that instead of finding compensating variation for a parameter change from (W_1, μ_1, σ_1) to (W_1, μ_2, σ_1), we must find compensating variation for a parameter change from (W_2, μ_1, σ_1) to $(W_2, \mu^*(W_2, \sigma_1, EU^*(\boldsymbol{\gamma}_1)), \sigma_1)$, where $\mu^*(W_2, \sigma_1, EU^*(\boldsymbol{\gamma}_1))$ is the shutdown mean price, that is, the price μ at which $y_c(\boldsymbol{\gamma}_1; W_2, \mu, \sigma_1; EU^*(\boldsymbol{\gamma}_1)) = y^*(W_2 - c(\boldsymbol{\gamma}_1; W_2, \mu, \sigma_1; EU^*(\boldsymbol{\gamma}_1)), \mu, \sigma_1) = 0$. Similarly, the third integral on the right-hand side of (19) can be found numerically.

4.3 Example

To provide an example of how our numerical methods may be applied to welfare analysis, we use Shin's (1999) estimation of the USA risk-responsive rice supply curve. Shin (1999) used a rational-expectations approach and data from 1976 to 1995 to estimate the supply function as $y^*(W, \mu, \sigma) = -23\ 399 + 17.196\mu - 17.899\sigma^2 + 0.0046161\,W + 106.09ALR + 11.711\,YEAR$.

The variable ALR represents a government acreage reduction policy, and $YEAR$ is used to proxy technological change. The quantity variable is stated in millions of cwt, μ is in 1982–1984 US dollars/cwt, and σ^2 is in 1982–1984 US dollars/cwt squared. Shin (1999) uses the 1991–1995 means of the exogenous variables above to obtain a baseline risk-responsive supply function. Schnepf and Just (1995) report that the most efficient USA rice producers have around \$3.00/cwt of variable cash expenses (in 1992 dollars). When deflated to 1982–1984 dollars using the CPI, this shutdown price is roughly \$2.00/cwt. The 1991–1995 mean of the ALR and $YEAR$ variables was 0.80. Substituting these values and the \$2.00 shutdown price into the equation above, we obtain a baseline Marshallian risk-responsive supply function of

$$y^*_{\text{base}}(W, \mu, \sigma) = \begin{cases} 25.895 + 17.196\mu - 17.899\sigma^2 + 0.0046161\,W & \text{if } \mu \geq 2.00 \\ 0 & \text{if } \mu < 2.00. \end{cases} \quad (20)$$

Shin (1999) also estimated rice demand (not reported here), and assumed rational expectations to conduct numerous policy simulations. By way of example, we will apply our numerical welfare measurement method to obtain the producer welfare effects for one of Shin's (1999) policy simulations, in which he examined the effect of raising the target price policy variable from \$10.00/cwt to \$11.00/cwt (in nominal dollars) while maintaining the acreage control variable at 0.80. Raising the target price variable truncates the producer price distribution, both raising its mean and lowering its variance. In his simulated equilibrium (Shin 1999, Table 6.1.3, p. 136), the policy of a \$10.00/cwt target price and a 0.80 acreage control results in a producer price distribution with mean $\mu_1 = \$7.888$/cwt (in 1982–1984 dollars) and a variance of $\sigma_1^2 = 0.57903$. The \$11.00/cwt target price policy results in a producer price distribution with mean $\mu_2 = \$8.207$/cwt (in 1982–1984 dollars) and a variance of $\sigma_2^2 = 0.28442$.

Our task is to use our knowledge about Marshallian risk-responsive supply to calculate compensating variation for this policy change that raises the target price from \$10.00/cwt to \$11.00/cwt. Using an SAS program (see Appendix 3, Bullock *et al.* 2005), we numerically calculated USA rice producer compensating variation because of this policy change as \$89.107 million (in 1982–1984 dollars). The total change in μ, the mean of the output price distribution, from \$7.888/cwt to \$8.207/cwt is divided into 1200 increments. The change in the target price is assumed not to change wealth, and so the wealth variable remains constant at $W_1 = W_2 = 3574.843$ everywhere in the program. But the program is written such that if the wealth were to change to a new amount W_2, this change is easily calculated by placing a new value for wealth 'wtwo' in lines 15 and 100. Risk-responsive Hicksian supply curves $y_c(W_2, \mu, \sigma_1, EU^*(\boldsymbol{\gamma}_1))$ and $y_c(W_2, \mu, \sigma_2, EU^*(\boldsymbol{\gamma}_1))$ are traced all the way down to assumed shutdown price of \$2.00. Note that while Shin (1999) assumed a linear functional form, our computer program easily could be applied to any functional form of Marshallian risk-responsive supply by simply substituting the desired functional form for the linear form in lines 30, 55, 60, and 63 of the program. This provides a distinct advantage over previous analytical methods, which are limited to a few functional forms of supply.

The model analysed above provides an excellent example of the potential importance of accounting for changes in risk in policy analysis. If we ignore the change in risk, and only consider the change in the mean of the output price distribution from $\mu_1 = \$7.888$/cwt to $\mu_2 = \$8.207$/cwt (in 1982–1984 dollars), then Larson's (1988, p. 600) analytical method can be employed to calculate that the compensating variation for this change in mean output price to be \$53.809 million (in 1982–1984 dollars) from the following formula:

$$c = \frac{\beta}{\delta}[\mu_1 - \mu_0] - \frac{1}{\delta}\left[q_0 - \frac{\beta}{\delta}\right]\left[e^{-\delta[\mu_1-\mu_0]} - 1\right], \tag{21}$$

where $\beta = 17.196$, $\delta = 0.00416161$, $\mu_0 = 7.888$, $\mu_1 = 8.207$, and $q_0 = y^*_{\text{base}}$ ($W = 3574.843$, $\mu = 7.888$, $\sigma = \sqrt{0.57903}$) $= 166.05$. Hence, accounting for the reduction in risk that is brought about by the rise in the target price raises the estimated gain in producer welfare from \$53.809 million to \$89.107 million, which is almost 66 per cent.

While our analysis is only illustrative, it does confirm the importance of considering the change in risk in applied analysis.

5. Conclusions and limitations

Our review traced the development of producer welfare measurement, focusing on the theoretical restrictions and empirical procedures needed to develop appropriate measures. We have presented a new procedure that combines theory with numerical integration methods to measure compensating variation of a change in endowed wealth and in the first two moments of an output price distribution for any form of Marshallian risk-responsive supply. Our procedure can be adapted to evaluate producer welfare effects from changes in third and higher moments of the output price distribution.

When effects of second and higher moments are analysed, our approach is limited as it requires the existence of shutdown prices. This poses no problem when a firm produces only one product, for an output price of zero will always force a shutdown. However, as discussed in Pope *et al.* (1983), if a firm produces more than one good, even a zero price of one good may not cause the firm to stop producing all goods and so shut down. Additionally, it is often the case that there are no price-quantity observations in the data in the neighbourhood of the shutdown price, which causes standard deviations of estimates of the vertical intercepts of supply curves to be large. Thus, in a statistical sense, the confidence we can place in welfare measures that rely on accurate estimation of the entire supply curve may be limited (Just *et al.* 1982, pp. 165–175). Hence, the statistical accuracy of welfare measures using shutdown prices should be monitored using bootstrapping techniques to draw statistical inferences about welfare change measurements.

While our method offers improvements over earlier methods, further theoretical and empirical efforts are needed to stimulate a more common usage in applied work of welfare measures in the presence of risk. In the context of price risk, our procedure only requires the development of risk-responsive Marshallian supply functions, but in the presence of multiple products and changes in higher moments of the subjective probability distribution is limited by the shutdown price problem. Tsur's (1993) procedure does not suffer from the shutdown price problem, but is limited by its CARA assumption, the difficulties in developing reliable measures of the absolute risk coefficient, and the absence of an empirical template that would assist researchers in its implementation. More generally, both procedures are limited as they do not explicitly consider the presence of stochastic production, nor do they consider the measurement of producer welfare in the presence of dynamic output and input adjustments.

Hence, why these procedures have not been applied to measure changes in producer welfare in the real world is evident, but we summarise. The complex theory and empirical procedures needed to appropriately measure welfare changes in the presence of certain types of risk are still in development, and often the techniques are only appropriate in specific situations. In addition, usage is limited by the absence of comparative empirical evidence that would provide more definitive conclusions regarding the relative attractiveness of these procedures in applied situations. Further, the measurement of welfare under risk requires sophisticated and in-depth knowledge of procedures that may not overlap. That is, a disconnection may exist between policy analysts and the individuals who model risk in markets – or at least between the skills necessary to implement risk measurement in policy analysis. It may well be that the procedures reviewed here have been only rarely applied because policy researchers have not understood them, or do not have the technical skills to implement them. We hope that our study strengthens the needed connection.

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